AI 1103 - Assignment 4

T. Rohan
CS20BTECH11064

Download all python codes from

https://github.com/rohanthota/Assignment_4/codes/Assignment_4.py

and latex codes from

https://github.com/rohanthota/Assignment_4/Assignment 4.tex

Question

Let Z be the vertical coordinate, between -1 and 1, of a point chosen uniformly at random on the surface of a unit sphere in R^3. Then, $\Pr\left(\frac{-1}{2} \le Z \le \frac{1}{2}\right)$ is

Solution

The equation of the sphere can be written as : $x^2 + y^2 + z^2 = 1$. Now,

$$\Pr\left(\frac{-1}{2} \le z \le 0\right) = \Pr\left(0 \le z^2 \le \frac{1}{4}\right) \tag{0.0.1}$$

$$\Pr\left(0 \le z \le \frac{1}{2}\right) = \Pr\left(0 \le z^2 \le \frac{1}{4}\right) \tag{0.0.2}$$

$$\therefore \Pr\left(\frac{-1}{2} \le z \le \frac{1}{2}\right) = 2 \times \Pr\left(0 \le z^2 \le \frac{1}{4}\right) \tag{0.0.3}$$

$$\Pr\left(0 \le z^2 \le \frac{1}{4}\right) = \Pr\left(\frac{3}{4} \le x^2 + y^2 \le 1\right) \tag{0.0.4}$$

Taking, $x^2 + y^2 = r^2$. (0.0.5)

$$\Pr\left(\frac{3}{4} \le r^2 \le 1\right) = \frac{1}{4} \tag{0.0.6}$$

$\left(\text{Since, } r^2 \text{ is uniform between 0 and 1}\right)$

$$\therefore \Pr\left(\frac{-1}{2} \le Z \le \frac{1}{2}\right) = 2 \times \frac{1}{4} = \frac{1}{2} \tag{0.0.7}$$