AI 1103 - Assignment 4

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Download all python codes from

https://github.com/rohanthota/Assignment_4/codes/Assignment_4.py

and latex codes from

https://github.com/rohanthota/Assignment_4/Assignment 4.tex

Question

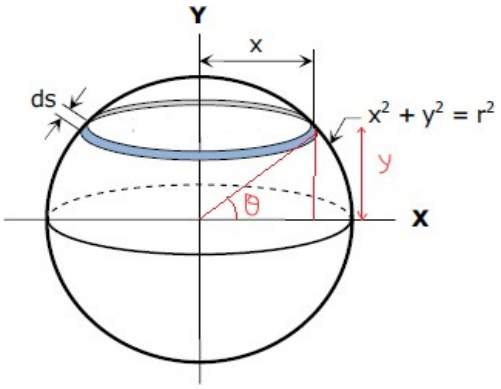
Let Z be the vertical coordinate, between -1 and 1, of a point chosen uniformly at random on the surface of a unit sphere in R^3. Then, $\Pr\left(\frac{-1}{2} \le Z \le \frac{1}{2}\right)$ is

Solution

The probabilities of various conditions, directly depend on the surface areas'.
Total surface area $A = 4\pi r^2 = 4\pi\left(1^2\right) = 4\pi$
Here, we define random variable $X \in \{0, 1\}$
Where,
$X = 0$ when a point with $\frac{-1}{2} \le Z \le \frac{1}{2}$ is picked.
$X = 1$ for all the other cases.

$$\Pr(X = 0) = \frac{\text{Area with } \frac{-1}{2} \le Z \le \frac{1}{2}}{\text{Total surface area}} \tag{0.0.1}$$

$$\text{Considering area } A' \text{ with } \frac{-1}{2} \le Z \le \frac{1}{2} \tag{0.0.2}$$



$$\text{Here } x = r\cos\theta \tag{0.0.3}$$

$$y = r\sin\theta \tag{0.0.4}$$

$$r^2 = x^2 + y^2 \tag{0.0.5}$$

$$ds = rd\theta. \tag{0.0.6}$$

$$\text{Area of strip } dA = 2\pi x \times ds = 2\pi r\cos\theta \times rd\theta. \tag{0.0.7}$$

$$\text{For } y = \frac{-1}{2},\ \theta = \frac{-\pi}{6}. \tag{0.0.8}$$

$$\text{For } y = \frac{1}{2},\ \theta = \frac{\pi}{6} \tag{0.0.9}$$

$$A' = \int_{y=\frac{-1}{2}}^{y=\frac{1}{2}} dA = \int_{\frac{-\pi}{6}}^{\frac{\pi}{6}} 2\pi r^2\cos\theta d\theta \tag{0.0.10}$$

$$A' = 2\pi\left(1\right)^2 \int_{\frac{-\pi}{6}}^{\frac{\pi}{6}} \cos\theta d\theta \tag{0.0.11}$$

$$A' = 2\pi\left[\sin\theta\right]_{\frac{-\pi}{6}}^{\frac{\pi}{6}} = 2\pi\left[\frac{1}{2} - \frac{-1}{2}\right] \tag{0.0.12}$$

$$\therefore A' = 2\pi \tag{0.0.13}$$

$$\therefore \Pr(X = 0) = \frac{A'}{A} = \frac{2\pi}{4\pi} = \frac{1}{2} \tag{0.0.14}$$