AI 1103 - Assignment 8

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Download all python codes from

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https://github.com/rohanthota/Assignment_8/codes
/Assignment_8.py
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and latex codes from

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https://github.com/rohanthota/Assignment_8/
Assignment 8.tex
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Question

Suppose n units are drawn from a population of N units sequentially as follows. A random sample

$$U_1, U_2, ...U_N \text{ of size N, drawn from } U(0,1) \tag{0.0.1}$$

The k-th population unit is selected if

$$U_k < \frac{n - n_k}{N - k + 1}, k = 1, 2, ..N.\text{where, } n_1 = 0, n_k = \tag{0.0.2}$$

number of units selected out of first k-1 units for each k = 2, 3, ..N. Then,

1) The probability of inclusion of the second unit in the sample
$$\text{is } \frac{n}{N} \tag{0.0.3}$$
2) The probability of inclusion of the first and the second unit in the sample
$$\text{is } \frac{n(n-1)}{N(N-1)} \tag{0.0.4}$$
3) The probability of not including the first and including the second unit in the sample
$$\text{is } \frac{n(N-n)}{N(N-1)} \tag{0.0.5}$$
4) The probability of including the first and not including the second unit in the sample
$$\text{is } \frac{n(n-1)}{N(N-1)} \tag{0.0.6}$$

Solution

$$\text{Defining random variable } X \in \{0, 1, 2, ..N\} \tag{0.0.7}$$

$$\text{Where, } X = i \text{ refers the ith unit being included.} \tag{0.0.8}$$

The first unit in the sample is included if

$$U_1 < \frac{n - n_1}{N - 1 + 1} \left(= \frac{n}{N}\right) \tag{0.0.9}$$

$$\therefore \Pr(X = 1) = \frac{n}{N} \tag{0.0.10}$$

For k=2,

$$n_2 = 1 \text{ when, first unit is included.} \tag{0.0.11}$$

$$U_2 < \frac{n - n_2}{N - 2 + 1} \left(= \frac{n-1}{N-1}\right) \tag{0.0.12}$$

$$\therefore \Pr(X = 2 \mid X = 1) = \frac{n-1}{N-1} \tag{0.0.13}$$

$$\therefore \Pr(X = 1, X = 2) = \frac{n(n-1)}{N(N-1)} \tag{0.0.14}$$

Hence, option 2 is correct.

$$n_2 = 0 \text{ when, first unit is not included.} \tag{0.0.15}$$

$$U_2 < \frac{n - n_2}{N - 2 + 1} \left(= \frac{n}{N-1}\right) \tag{0.0.16}$$

$$\therefore \Pr(X = 2 \mid X \neq 1) = \frac{n}{N-1} \tag{0.0.17}$$

$$\therefore \Pr(X \neq 1, X = 2) = \left(1 - \frac{n}{N}\right) \times \frac{n}{N-1} = \frac{n(N-n)}{N(N-1)} \tag{0.0.18}$$

Hence, option 3 is correct.

From (0.0.14) and (0.0.18)

$$\Pr(X=2) = \frac{n(n-1)}{N(N-1)} + \frac{n(N-n)}{N(N-1)} = \frac{n}{N} \quad (0.0.19)$$

Hence, option 1 is correct.

$$\Pr(X=1, X \neq 2) = \frac{n}{N} \times \left(1 - \frac{n}{N}\right) = \frac{n(N-n)}{N^2} \quad (0.0.20)$$

Hence, option 4 is incorrect.
Therefore, Options 1, 2, 3 are correct