$$\text{Q2.} \quad [\dot{B}] = \frac{\beta [B][A]}{N} - \gamma [B] \quad \text{—— (i)}$$

$$[\dot{A}] = -\frac{\beta [B][A]}{N} + \gamma [B] \quad \text{—— (ii)}$$

$$[A] + [B] = N. \quad \text{—— (iii)}$$

Using (i) & (iii)

$$[\dot{B}] = \frac{\beta [B]}{N}\left[N - [B]\right] - \gamma [B]$$

To find fixed points, let $[\dot{B}] = 0$.

$$0 = -\frac{\beta [B]^2}{N} + \beta [B] - \gamma [B]$$

$$0 = -\frac{\beta [B]^2}{N} + [B](\beta - \gamma)$$

$$\frac{\beta [\cancel{B}]^{\cancel{2}}}{N} = [\cancel{B}][\beta - \gamma] \qquad \left[\text{When } [B] \,!= 0\right]$$

$$[B] = N\left(1 - \frac{\gamma}{\beta}\right)$$

$$[B] = N\left(1 - \frac{1}{R_0}\right) \qquad \left[R_0 = \frac{\beta}{\gamma}\right]$$

$\therefore$ fixed points for $[B] = 0, \; N\left(1 - \frac{1}{R_0}\right)$

Taking eq^n (ii) & (iii)

$$[\dot{A}] = -\beta \frac{(N - [A])[A]}{N} + \gamma [N - [B]]$$

To find fixed points, let $[\dot{A}] = 0$

$$\beta \frac{[N - [A]]\,[A]}{N} = \gamma \,[N - [B]]$$

When $[A] \,!= N$

$$\beta \frac{[A]}{N} = \gamma$$

$$[A] = N \frac{\gamma}{\beta}$$

$$[A] = \frac{N}{R_0}.$$

$\therefore$ The overall fixed points of the system are.

$$\boxed{(N, 0) \quad \text{and} \quad \frac{N}{R_0}, \; N\left(1 - \frac{1}{R_0}\right)}$$

According to the fixed points,

Non-zero fixed point $\boxed{B^* = N - \frac{N}{K_0}}$

Stability of the fixed points.

- Finding the Jacobian of the equations

$$J = \begin{bmatrix} f_{[A]}([A],[B]) & f_{[B]}([A],[B]) \\ g_{[A]}([A],[B]) & g_{[B]}([A],[B]) \end{bmatrix}$$

where $f_{[A]}([A],[B]) = \frac{\partial([\dot{A}])}{\partial [A]}$ $f_{[B]}([A],[B]) = \frac{\partial([\dot{A}])}{\partial [B]}$

$g_{[A]}([A],[B]) = \frac{\partial([\dot{B}])}{\partial [A]}$ $g_{[B]}([A],[B]) = \frac{\partial([\dot{B}])}{\partial [B]}$

$\therefore$

$$J = \begin{bmatrix} -\frac{\beta[B]}{N} & -\frac{\beta[A]}{N} + \gamma \\ \frac{\beta[B]}{N} & \frac{\beta[A]}{N} - \gamma \end{bmatrix}$$

at $(N, 0)$

$$J(N,0) = \begin{bmatrix} 0 & -\beta+\gamma \\ 0 & \beta-\gamma \end{bmatrix}$$

at $N/R_0,\ N(1-1/R_0)$

$$J(A^*, B^*) = \begin{bmatrix} -\frac{\beta \times N}{N}\left(1-\frac{\gamma}{\beta}\right) & -\frac{\beta}{N}\frac{N\gamma}{\beta}+\gamma \\ \frac{\beta N}{N}\left(1-\frac{\gamma}{\beta}\right) & \frac{\beta}{N}\frac{N\gamma}{\beta}-\gamma \end{bmatrix}$$

$$J(A^*, B^*) = \begin{bmatrix} -\beta\left(1-\frac{\gamma}{\beta}\right) & 0 \\ \beta\left(1-\frac{\gamma}{\beta}\right) & 0 \end{bmatrix}$$

Finding the Eigen values.

$$|A-\lambda I| = 0.$$

For $J(N,0)$

$$\begin{vmatrix} -\lambda & -\beta+\gamma \\ 0 & \beta-\gamma-\lambda \end{vmatrix} = 0.$$

$$(-\lambda)(\beta-\gamma-\lambda) = 0$$

when $\lambda \,!= 0$

$$\beta-\gamma-\lambda = 0$$

$$\lambda = \beta-\gamma$$

classmate

Date

Page

$\therefore \quad \lambda_1 = 0$

$\lambda_2 = \beta - \gamma$

only if $\beta < \gamma$, the system is stable!

i.e. $R_0 = \frac{\beta}{\gamma} < 1$.

For $J(A^*, B^*)$

$|A - \lambda I| = 0$

$$\begin{vmatrix} -\beta\left(1-\frac{\gamma}{\beta}\right) - \lambda & 0 \\ \beta\left(1-\frac{\gamma}{\beta}\right) & -\lambda \end{vmatrix} = 0$$

$$\left(-\beta\left(1-\frac{\gamma}{\beta}\right) - \lambda\right)(-\lambda) = 0$$

when $\lambda \, != 0$

$$-\beta\left(1-\frac{\gamma}{\beta}\right) = \lambda$$

$\lambda = -\beta + \gamma$

$\lambda = \gamma - \beta$

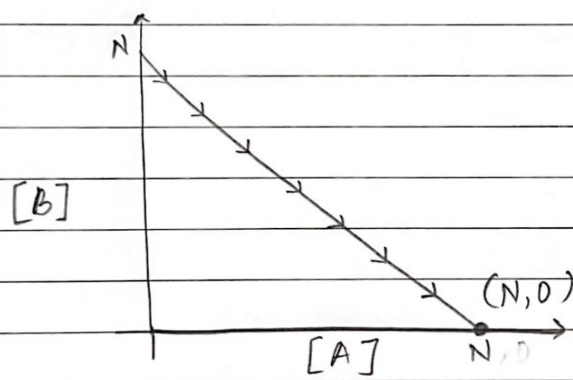
$\therefore \lambda = 0, \gamma - \beta$

$\therefore$ only if $\gamma < \beta$, the system is stable

i.e. $R_0 = \frac{\beta}{\gamma} > 1$

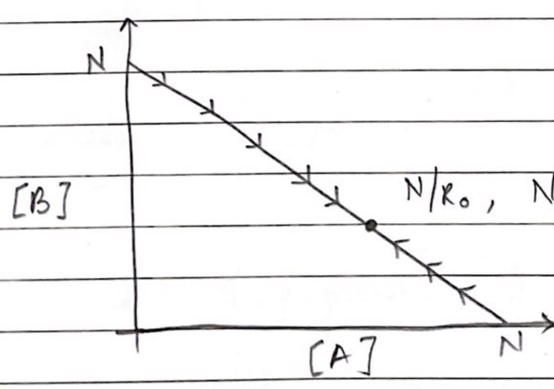
PHASE PORTRAIT.

1)



When $\gamma > \beta$ i.e. $R_0 \leq 1$

2)



When $\gamma < \beta$ i.e. $R_0 > 1$