2. Since $[\dot{B}]$ (mean field equation) does not have $[A]$ involved in it (i.e. it is only dependent on $[B]$) we can decouple the equation.

Considering on $[\dot{B}]$ to be representing the overall system,

$$[\dot{B}] = [B](\beta - \gamma) - \beta \frac{[B]^2}{N}$$

For fixed points $[\dot{B}] = 0$, thus,

$$0 = [B](\beta - \gamma) - \frac{\beta [B]^2}{N}$$

$$\frac{\beta [B]^{\cancel{2}}}{N} = \cancel{[B]}(\beta - \gamma) \quad [\text{when } [B] != 0]$$

$$\beta [B] = N(\beta - \gamma)$$

$$[B] = N\left(1 - \frac{\gamma}{\beta}\right)$$

$$[B] = N - \frac{N}{R_0} \quad \left[R_0 = \frac{\beta}{\gamma}\right]$$

$\therefore$ The fixed points of the system with respect to

$$\boxed{[B] = 0, \; N - \frac{N}{R_0}}$$

~~Since~~ $\boxed{B^* = N - \frac{N}{R_0}}$

Stability of the fixed points.

- To find the stability of the fixed points, taking the Jacobian of the ODE at the fixed points

$$\frac{d(f([B])}{d([B])}$$

$$J = (\beta - \gamma) - 2\frac{\beta [B]}{N}$$

(i) When $[B] = 0$

$$J(0) = \beta - \gamma$$

When $\gamma > \beta$, ~~J(0)~~ is stable
else ~~J(0)~~ is unstable

(ii) When $[B] = N - \frac{N}{R_0}$

$$J(B^*) = (\beta - \gamma) - \frac{2 \times \beta \times \cancel{N}}{\cancel{N}}\left(1 - \frac{1}{R_0}\right)$$

$$= \beta - \gamma - 2 \times \cancel{\beta}\frac{(\beta - \gamma)}{\cancel{\beta}}$$

$= \beta - \gamma - 2\beta + 2\gamma$

$= \gamma - \beta$

$\therefore$ if $\gamma > \beta$ $J(B^*)$ is unstable
else stable

Fixed points for $[B] = 0, \; N - \dfrac{N}{R_0}$.

Since $[A] + [B] = N$,
the overall fixed points in terms of $[A]$ & $[B]$ can be found.

$[A] = N - [B]$ [fixed points of $[A]$]

(i) When $[B] = 0$

$[A] = N$

(ii) When $[B] = N - \dfrac{N}{R_0}$

$$[A] = \cancel{N} - \cancel{N} + \frac{N}{R_0}$$

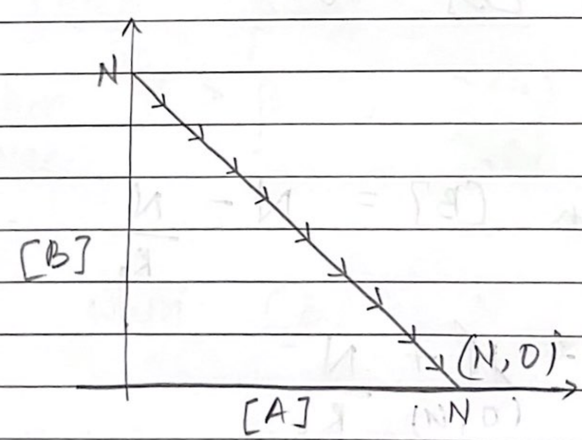
$$= \frac{N}{R_0}$$

∴ The fixed points of the overall system is

$(N, 0)$ & $\left(\frac{N}{R_0}, N - \frac{N}{R_0}\right)$

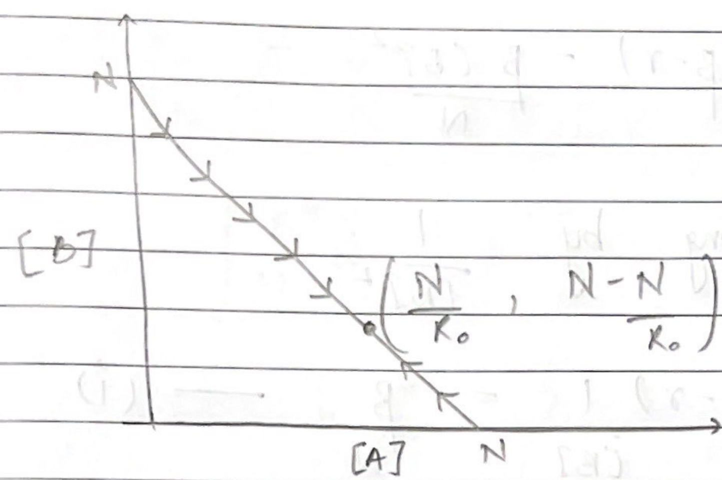
- There exists another fixed point which is $(0,0)$ but it is impossible to reach it unless we start at that point.
- $(0,0)$ is a trivial fixed point which cannot be reached.

PHASE PORTRAIT.

1) 


When $r > \beta$ i.e. $R_0 \leq 1$

2)




When $\gamma < \beta$ i.e. $R_0 > 1$