Cardiac Output (Liter/min) = Stroke Volume (Liter/beat) * Heart Rate (beats/min)

Typically 4-8 L/min

$$M = \int Q(t)C(t)dt$$

M = Amount of indicator (moles)
Q(t) = blood flow at time t (liter/min)
C(t) = measured concentration of indicator at time t (Mol/liter)

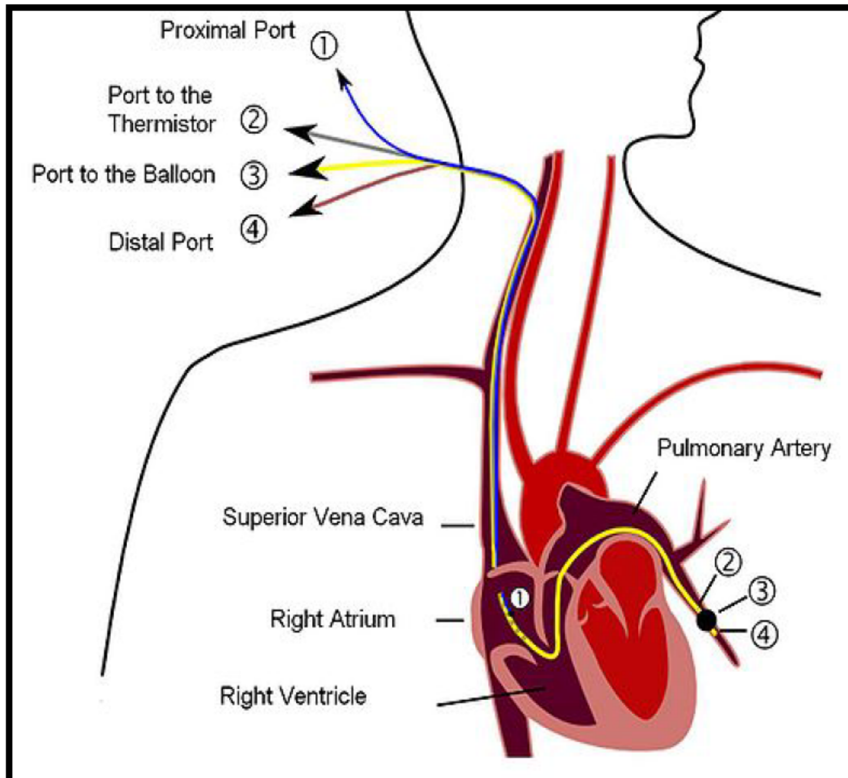
Assume blood flow Q constant. Then

$$Q = \frac{M}{\int C(t)dt}$$

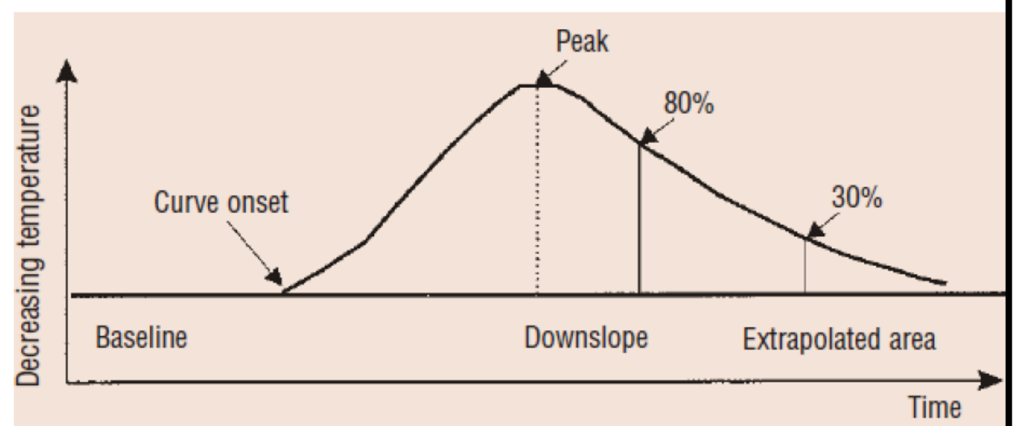
Heat is the indicator and is “removed” by bolus injection of cold fluid

This is the Stewart-Hamilton equation

Assume heat is the indicator and is added by bolus injection of cold fluid via a PAC



Pulmonary thermodilution (P-TD) with a PAC



The introduction of the cold injectate causes a rapid upslope to a peak, a gradual downslope, and an exponential decay of the thermal signal. The CO computer begins integration of the area under the TD curve until the exponential decay reaches a value of about 30%, and extrapolates the exponential decay to baseline in order to minimize artifacts due to recirculation of the indicator.

$$Q \propto \frac{T_B - T_I}{\int \Delta T_B(t)\,dt}$$

T_B is blood temp
T_I is bolus temp
ΔT_B is measured temp change

Thermodilution via PAC is considered “gold standard”

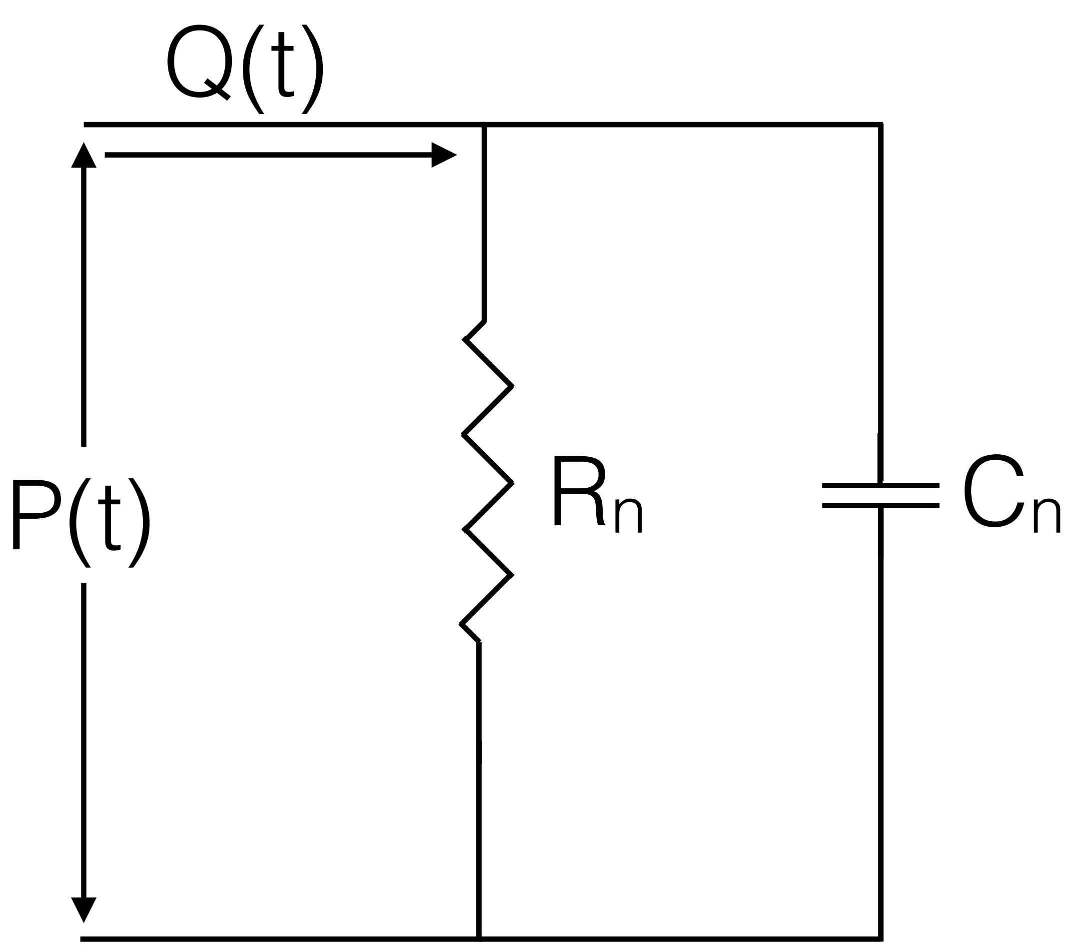
Disadvantages:

- Discrete measurements
- Invasive

How can we estimate CO non-invasively and continuously?

Parlikar et al (2007) Model-based estimation of cardiac output and total peripheral resistance. *Computers in Cardiol*. 34: 379

The world's simplest cardiovascular model



P(t) = Arterial blood pressure at aortic root

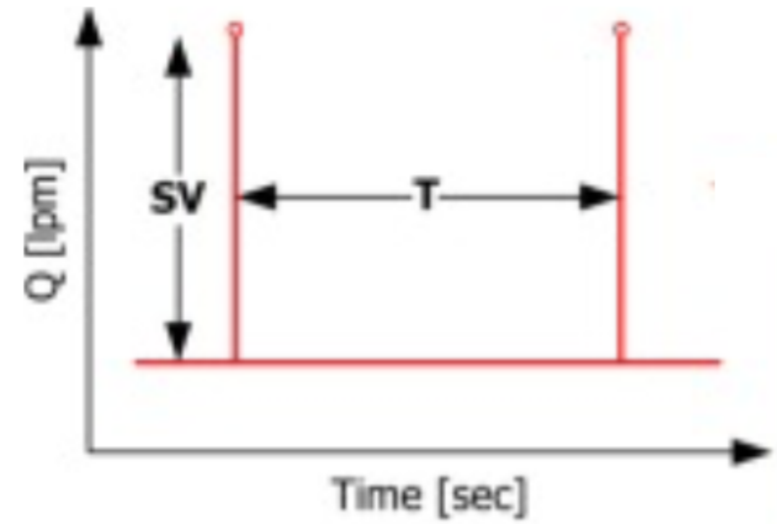
Q(t) = dirac-delta current pulse that deposits some volume SV_n of blood instantaneously on the capacitor C at n^{th} heart beat

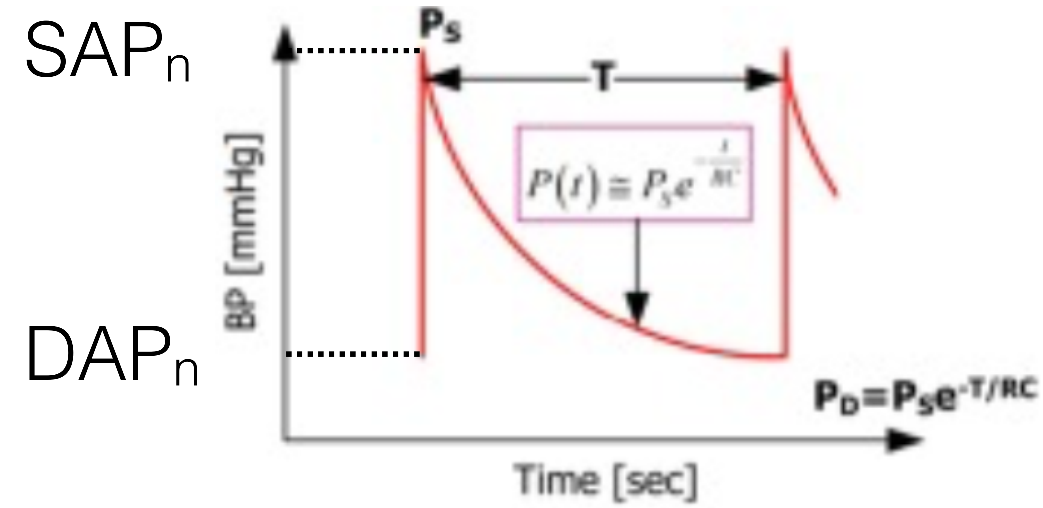
SV_n = Stroke volume on n^{th} beat

R_n = Total peripheral resistance n^{th} beat

C_n = Total elastic compliance of all blood vessels n^{th} beat

How can we estimate CO non-invasively and continuously?





Q=C*V (charge on C = C * voltage)

$$SV_n = C\left[SAP_n - DAP_n\right] = C_n PP_n$$

$$CO_n = \frac{SV_n}{T_n} \Rightarrow PP_n = \frac{CO_n T_n}{C}$$

T_n = n^{th} beat interval

$$CO_n = C\frac{PP_n}{T_n}$$

This is an estimate of CO on beat n
Doesn’t use any interbeat information

How can we estimate CO non-invasively and continuously?

Applying Kirchoff's Current Law to the ckt yields:

$$C_n \frac{dP(t)}{dt} + \frac{P(t)}{R_n} = \sum_n SV_n \delta\left(t - t_n\right)$$

Integrate over the n^{th} beat from $[t_n, t_{n+1})$

$$\frac{C}{T_n}\int_{t_n}^{t_{n+1}^-} \frac{dP(t)}{dt}dt + \frac{1}{T_n}\int_{t_n}^{t_{n+1}^-} \frac{P(t)}{R_n}dt = \frac{1}{T_n}\int_{t_n}^{t_{n+1}^-} \sum_n SV_n \delta\left(t - t_n\right) = \frac{SV_n}{T_n} = CO_n$$

$$\frac{C}{T_n}\left[P(t_{n+1}^-) - P(t_n)\right] + \frac{1}{R_n}\left(\frac{1}{T_n}\int_{t_n}^{t_{n+1}^-} P(t)dt\right) = CO_n$$

$$\frac{C}{T_n}\Delta P_n + \frac{1}{R_n}\bar{P}_n = CO_n \Rightarrow CO_n = C\left(\frac{\Delta P}{T_n}_n + \frac{1}{R_n C}\bar{P}_n\right)$$

Information about CO in time constant of decay "interbeat information"

Combining the above equation with our prior result $CO_n = C\frac{PP_n}{T_n}$ gives

$$\frac{\Delta P_n}{T_n} + \frac{\bar{P}_n}{R_n C_n} = \frac{PP_n}{T_n}$$

How can we estimate CO non-invasively and continuously?

Parlikar say they use the following approximation for PP_n:

$$PP_n = 2\left(\bar{P}_n - DAP_n\right)$$

Therefore

$$\frac{\Delta P_n}{T_n} + \frac{\bar{P}_n}{\tau_n} = \frac{2\left(\bar{P}_n - DAP_n\right)}{T_n}$$

τ_n is the unknown.

$$\tau_n = \frac{\bar{P}_n T_n}{2\left(\bar{P}_n - DAP_n\right) - \Delta P_n}$$

Would be exact if our measurements were exact, but they are not

How can we estimate CO non-invasively and continuously?

"Estimate τ_n by selecting an odd number of beats centered about n, assume τ_n is constant (τ) over this interval . Compute the least-squares solution for $\beta=1/\tau$"

They don't give details. Define:

$$\beta = \frac{1}{\tau}$$

$$x_i = \bar{P}_i$$

$$y_i = \frac{2\left(\bar{P}_i - DAP_i\right) - \Delta P_i}{T_i}$$

$$y_i = \beta x_i$$

Our estimate β won't satisfy any of the above linear relations exactly so the summed squared error is

$$E_n = \sum_{i=n-a}^{i=n+a} \left(y_i - \beta x_i\right)^2$$

How can we estimate CO non-invasively and continuously?

Differentiate E_n wrt β_n and set to zero:

$$\frac{dE_n}{d\beta} = \sum_{i=n-a}^{i=n+a} -2x_i\left(y_i - \beta x_i\right) = 0$$

$$-2\sum_{i=n-a}^{i=n+a} x_i y_i + 2\beta \sum_{i=n-a}^{i=n+a} x_i^2 = 0$$

$$\beta = \frac{\sum_{i=n-a}^{i=n+a} x_i y_i}{\sum_{i=n-a}^{i=n+a} x_i^2}$$

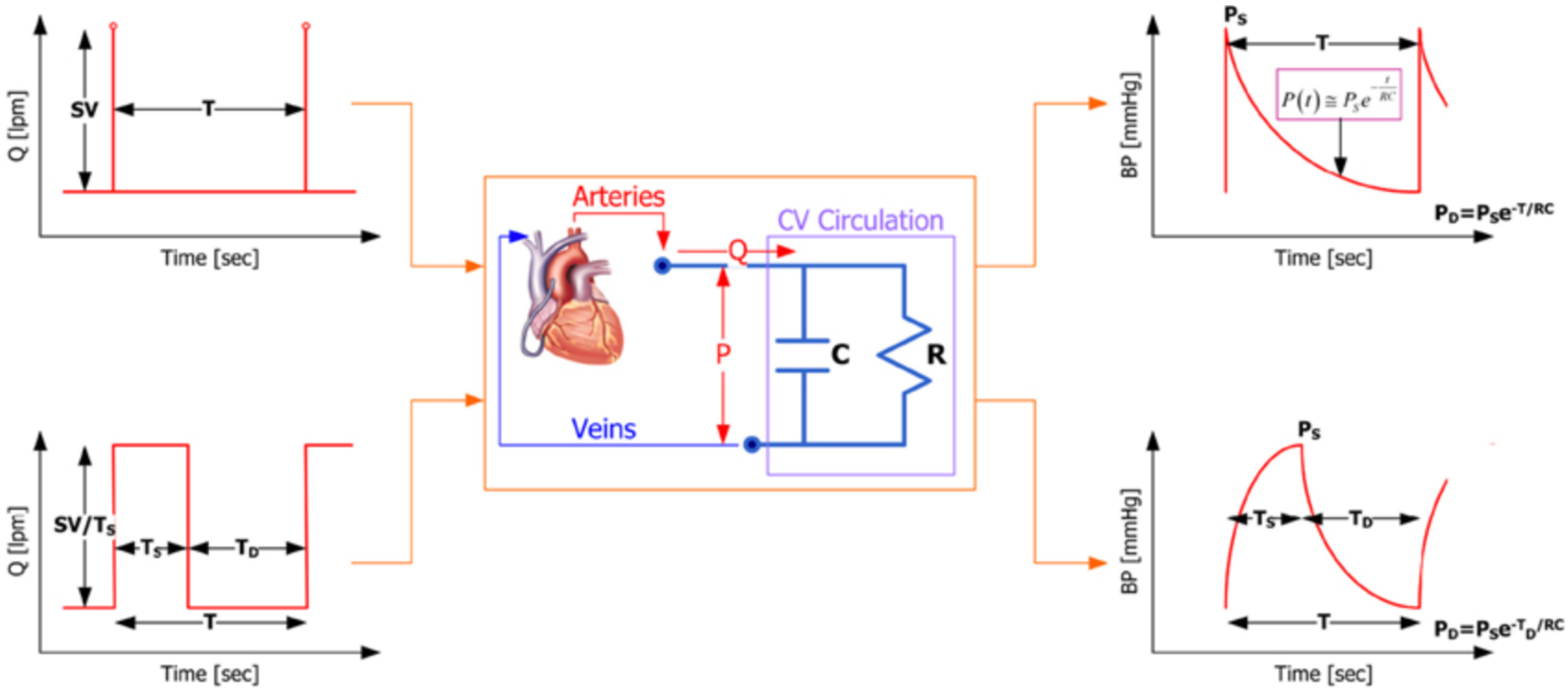
How can we estimate CO non-invasively and continuously?

Once time constant is estimated, we have

$$CO_n = C\left(\frac{\Delta P_n}{T_n} + \frac{\overline{P}_n}{\tau_n}\right)$$

Note we need the capacitance (compliance) Cn
Constant of proportionality
Need one or more measured values of CO to calibrate.
Choose to minimize error in measured vs estimated values

Another Approach - Use Square Waves not Impulses



Need to estimate SV, T_S, T_D

Fazeli and Hahn (2012) Estimation of cardiac output and peripheral resistance using square-wave approximated aortic flow signal. *Front. Comp. Physiol. Med.* http://dx.doi.org/10.3389/fphys.2012.00298

Another Approach - Use Square Waves not Impulses

Assume $\bar{P} = CO * R$

Note $\frac{SV}{T_S} = \frac{SV}{T}\frac{T}{T_S} = CO\frac{T}{T_S}$

$$P(t) = P_D e^{-\frac{t}{\tau}} + \frac{CO}{C}\frac{T}{T_S}\int_o^t e^{-\frac{t-s}{\tau}} ds = P_D e^{-\frac{t}{\tau}} + \bar{P}\frac{T}{T_S}\left[1 - e^{-\frac{t}{\tau}}\right]$$ on $0 \le t \le T_S$

$$P(T_S) = P_S = P_D e^{-\frac{T_S}{\tau}} + \bar{P}\frac{T}{T_S}\left[1 - e^{-\frac{T_S}{\tau}}\right]$$

$$P(t) = P_S e^{-\frac{t-T_S}{\tau}}$$ on $T_S < t \le T$

$$= \left(P_D e^{-\frac{T_S}{\tau}} + \bar{P}\frac{T}{T_S}\left[1 - e^{-\frac{T_S}{\tau}}\right]\right) e^{-\frac{t-T_S}{\tau}}$$

$$P(T_S + T_D) = P_D = \left(P_D e^{-\frac{T_S}{\tau}} + \bar{P}\frac{T}{T_S}\left[1 - e^{-\frac{T_S}{\tau}}\right]\right) e^{-\frac{T_D}{\tau}}$$

$$P_D = \bar{P}\frac{T}{T_S}\frac{e^{-\frac{T_D}{\tau}} - e^{-\frac{T}{\tau}}}{1 - e^{-\frac{T}{\tau}}}$$

Another Approach - Use Square Waves not Impulses

$$P_D = \bar{P}\frac{T}{T_S}\frac{e^{-\frac{T_D}{\tau}} - e^{-\frac{T}{\tau}}}{1 - e^{-\frac{T}{\tau}}}$$

Linear function of $\bar{P}$

$$P_S = P_D e^{-\frac{T_S}{\tau}} + \bar{P}\frac{T}{T_S}\left[1 - e^{-\frac{T_S}{\tau}}\right]$$

Linear function of $\bar{P}$

Mean arterial pressure $\bar{P}$ satisfies the following equation
Note P_D and P_S can be written in terms of $\bar{P}$

$$\bar{P} = \frac{1}{T}\left[\int_0^{T_S}\left(P_D e^{-\frac{t}{\tau}} + \bar{P}\frac{T}{T_S}\left[1 - e^{-\frac{t}{\tau}}\right]\right)dt + \int_{T_S}^{T_S+T_D}\left(P_S e^{-\frac{t-T_S}{\tau}}\right)dt\right]$$

Given C, τ, T_D, T_S:

- compute $\bar{P}$, P_D and P_S
- Compute error between estimates and data
- adjust parameters to minimize error