Abstract

The objective of the report is to analyze five aircraft performance aspects of the Cessna 172R Skyhawk aircraft and comment on the aircrafts design. These aspects are to be analyzed using various approximation methods learned in the Aircraft Performance course offered at Ryerson University. The five aircraft performance aspects are steady level flight envelope, takeoff, landing rate of climb and range-payload diagram. For the analysis of the steady level flight envelope a simple envelope was created using information found through multiple sources and the methods learned in class. The envelope showed no concern for design limitations but only improvements for future designs. Consequently, takeoff and landing performance were judged on the bias of takeoff and landing distance respectively. This parameter was analyzed under different altitude and wind conditions. The takeoff analysis numbers were close to the actual takeoff data found in other sources. However, landing analysis did not produce the similar behavior even though it was modeled for the stall-on and flown-on landing approaches. Nonetheless, the analysis proved that these landing distances are reasonable for the size of the aircraft. The next aspect analyzed was the rate of climb of the aircraft and two different methods were used. The results found were higher than the actual but still followed the demanded trend required for design analysis. The design of the aircraft again was once proved as mediocre. Last of all was the range payload diagram. This analysis was effective as well and lead to the conclusion that the aircraft had excellent cruise range with varying payload at an altitude of $8000\ ft$. It was concluded that the aircraft overall was mediocre but as a personal use aircraft it was far better than the norm. Nevertheless, it could be improved with the use of slats, composite material, and a more powerful engine.

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Nomenclature

Table 1 - Nomenclature

Parameter Symbol	Parameter Name
b	Wing Span
S	Wing Area
h	Mean Height of Wing
e	Oswald Efficiency Factor
$MTOW$	Maximum Takeoff Weight
W_E	Empty Weight
P_S	Shaft Power (Brake Horsepower)
d_p	Diameter of Propeller
n	Engine Speed
$Service\ Ceiling$	Maximum Aircraft Altitude
$v_{max}\ at\ sea\ level$	Max Velocity at Sea Level
W	Weight
ρ_{SL}	Density at Sea Level
ρ	Density
$C_{L,max}$	Max Lift Coefficient
$C_{L,ground\ roll}$	Ground Roll Lift Coefficient
C_{Do}	Zero-Lift Drag Coefficient
μ	Coefficient of Friction
T_o	Static Thrust
θ_R	Runway Slope
h_{sc}	Height of Obstacle
g	Earth's Gravitational Constant
$C_{L,max\ landing}$	Max Lift Coefficient of Landing
μ_{decel}	Coefficient of Friction in Deceleration
$\gamma_{D\ deg}$	Landing Decent Angle
$\frac{C_{L,flare}}{C_{L,approach}}$	Lift Coefficient Ratio
Δt_{delay}	Time Delay
v_{min}	Minimum Velocity
ρ_{fuel}	Density of Fuel
v_{cruise}	Cruise Velocity
η_{pr}	Propeller Efficiency
J	Advanced Ratio
AR	Aspect Ratio
K	Lift-Induced Drag Factor
θ	Temperature Ratio
σ	Density Ratio
T	Temperature/Thrust
T_{SL}	Temperature at Sea Level
v_{gLOF}	Ground Liftoff Speed

$C_{D_{GE}}$	Ground Effect Drag Coefficient
v_{STO}	Takeoff Stall Speed
v_{LOF}	Liftoff Speed
ϕ	Ground Effect Factor
v_2	Climb/Decent Speed
v_w	Wind Speed
v_{SLD}	Landing Stall Speed
γ	Climb Angle
R/C	Rate of Climb
a	Parabolic Thrust Coefficient
v_∞	Airspeed
$BSFC$	Brake Specific Fuel Consumption
R	Range

1.0 Introduction

Aircraft performance is important to acknowledge and understand. Just like cars one must look at performance specifications of an aircraft to understand them before purchasing them. Likewise, in design it is important to set out the desired aircraft performances because these dictate a major role in how the aircraft will turn out. It is also important for pilots because the performance stats tell them if in a case of emergency what they can do and what should they not do. Therefore, the following report's general topic is about aircraft performance.

In particular this report is about analyzing the aircraft performance of the Cessna 172R Skyhawk. This is a lightweight small aircraft with a 160hp (at 2700 rpm) Textron Lycoming IO-360-L2A four cylinder engine. Which powers a two blade fixed propeller. In general this plane is used by people for personal use or at flight schools. Overall, this plane is best described as an awesome all rounder. However, to better understand why this statement is true five performance aspects were analyzed. Starting with the steady level flight envelope and then followed by takeoff and landing performances. After these three aspects are the climb performance and the range-payload diagram in that respective order. Each aspect will be broken down into three sections. The first section is introduction which will cover theory, how the performance aspect were analyzed and what information was used to perform the analysis. This will be followed by a section summarizing all the results. Last of all is a small discussion on how the analysis compared with other analyses found, how good is the aircraft and what can be done to make it better if possible. It is important to note that each aspect is analyzed using equations, tables and graphs. These analysis at most are approximations for two reasons. The first is that the equations used to perform the analysis are simplified so they can be performed with more ease. Also, the information needed at times to perform these analysis had to be approximated using information from a different model of the Cessna 172 or reasonably fabricated. Also, all analysis was done using MATLAB and the codes for each analysis can be found in the appropriate Appendix section. Note that imperial units were used due to the fact that most information on the aircraft is already in imperial units.

2.0 Theory

As stated above the Cessna 172R is a light weight small aircraft due its max takeoff weight of $2450\ lbs$ and its relatively small wingspan of about $36\ ft$. Table 2 provides more physical characteristics of the aircraft followed by Figure 1 which, provides an image of the Cessna 172R. Please refer to the Nomenclature section (Page V) to further better understand the meaning of each variable. Also, for more theory on a certain topic relating to a variable please seek other sources as it is not covered in this report.

Table 2 - General Specifications of Cessna 172R Skyhawk [1] [2][3][4]

Parameter Symbol	Value
$\boldsymbol{b}$	$36\ ft$
$\boldsymbol{S}$	$175.5\ ft^2$
$\boldsymbol{h}$	$8\ ft$
$\boldsymbol{e}$	0.77
$\boldsymbol{MTOW}$	$2450\ lbs$
$\boldsymbol{W_E}$	$1679\ lbs$
$\boldsymbol{P_S}$	$160\ hp$
$\boldsymbol{d_p}$	$6.25\ ft$
$\boldsymbol{n}$	$2700\ rpm$
$\boldsymbol{Service\ Ceiling}$	$13500\ ft$
$\boldsymbol{v_{max}\ at\ sea\ level}$	$212.66\frac{ft}{s}$

Figure 1 - Cessna 172R Skyhawk Takeoff [5]

It is a given fact that performance of an aircraft vary at different altitudes and the Cessna 172R is no different. Changes in altitude cause the air properties to change and the two main properties are density and temperature. Therefore, these analysis require the calculation of density and temperature at different altitudes. Throughout, the analysis these values were calculated using Equation 1.1 to Equation 1.4. A MATLAB code was written for these calculations and can be found in Appendix A1: Density Function (page 27).

$$\theta = 1 - 2.26 \times 10^{-5}(h) \tag{1.1}$$

$$\theta = \frac{T}{T_{SL}} \tag{1.2}$$

$$\sigma = \theta^{4.256} \tag{1.3}$$

$$\sigma = \frac{\rho}{\rho_{SL}} \tag{1.4}$$

Above Equations Taken From[6] *Unless Stated.*

It is to be noted that Equation 1.1 is used to calculate the temperature ratio (θ) using altitude (h) in meters which, is then used to calculate the density ratio (σ). Equation 1.1 can only be used from $0\ km$ to $11\ km$ and this range is acceptable because the service ceiling of the 172R is well below that. With the use of these equations the basis of the analysis has been set.

3.0 Analysis

3.1 Steady Level Flight Envelope

3.1.1 Theory

A steady level flight envelope represents an area of safe control in terms of the aircraft speed. It is represented in a velocity verses altitude graph which is enclosed hence the envelope name. Any speeds outside of the envelope result in the aircraft not being in steady state or also known as not in ideal control. Generally, a larger envelope is better as it displays that the aircraft is easy to control because it can reach steady level flight easily and stay there. On the other hand, the steady level flight envelope is highly useful in judging the design capabilities.

In general these graphs are broken down into three types of lines which are stall, max altitude and top speed. The stall line represents the stall velocity from sea level all the way up to the service ceiling of the aircraft. The max altitude line represents the service ceiling of the aircraft and last of all top speed represents the conditions that allow the aircraft to go the fastest. Stall and top speed line were constructed using Equation 3.1. The only difference is the different lift coefficients which are displayed in Table 3 along with values of the other variables. The lift coefficient ($C_{L,min}$) could not be found therefore it was calculated using Equation 3.2 where (v_{max}) was known at sea level. Likewise the lift coefficient ($C_{L,max}$) for stall could not be found so it was calculated using maximum value with no flap [3]. The max altitude line was difficult to calculate because of other factors such as engine performance relative to speed. Therefore, it was approximated as a constant line at the service ceiling height [7]. All the calculations were done using MATLAB code which can be found in Appendix A2: Steady Level Flight Envelope MATLAB Code (Page 27).

$$v = \sqrt{\frac{2W}{\rho S C_L}} \tag{3.1}$$

$$C_{L,min} = \frac{2W}{\rho_{SL} S {v_{max}}^2} \tag{3.2}$$

Above Equations Taken From [6] *Unless Stated.*

Table 3 - Initial Parameters Used to Calculate Steady Level Flight Envelope [1] [2][3][4]

Parameter Symbol	Values
$\boldsymbol{W = MTOW}$	$2450\ lbs$
$\boldsymbol{\rho_{SL}}$	$0.002378 \frac{slug}{ft^3}$
$\boldsymbol{\rho}$	$Calculated\ Using\ Equation\ 1.1 - 1.4$
$\boldsymbol{v_{max}}$	$212.66 \frac{ft}{s}$
$\boldsymbol{S}$	$175.5\ ft^2$
$\boldsymbol{C_{L,max}}$	2.14
$\boldsymbol{Service\ Ceiling}$	$13500\ ft$

3.1.2 Results

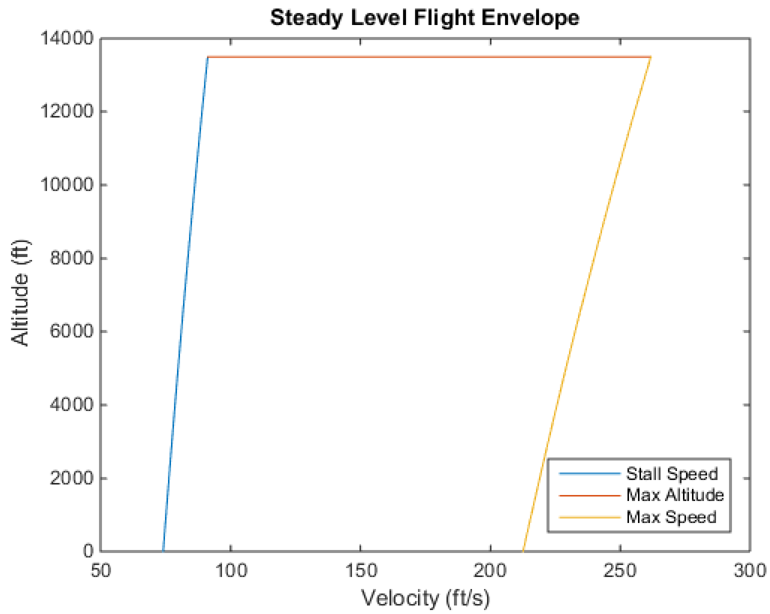

Figure 2 - Cessna 172R Skyhawk Steady Level Flight Envelope (Calculated)

Table 4 - Lift Coefficient Values of Each Line

Line	Lift Coefficient Value
Stall Speed	2.14
Max Altitude	N/A
Max Speed	0.2596

3.1.3 Discussion

The steady level flight envelope shown in Figure 2 is a close approximation of the actual steady level flight envelope of the 172R. Evidently, the envelope is not large because of the low service ceiling and the speed capabilities of this lightweight aircraft. However, this is good enough for a small aircraft intended for personal use. Similarly, it is to be noted that the top of the graph has a slightly larger range of acceptable velocity and this is due to the air becoming thinner and decreasing at non-linear rate. Thus according to Equation 3.1 the velocity increases. Not too much stock should be taken from this point because in reality as the density decreases the performance of the engine decreases. Although for the Cessna 172R it is expected to not have that much of an engine efficiency drop because it is not turbo or super charged. Likewise, an increase in velocity also leads to a drop in engine efficiency. As well, Figure 2 has another interesting point. Although both the stall speed and max speed were constructed using the same equation the max speed is steeper and shows sign of curving. This is probably due to the larger lift coefficient (C_L). Table 4 summarizes the two values. Nonetheless, as stated before the steady level flight envelope is satisfactory but could be better. It could be improved by changing the lift coefficient which can be accomplished with different wings that produce better lift. Another way to improve the envelope is through a morphing wing which is in the research phase. This unique wing produces more stability depending on the flight condition. Unfortunately this wing is years into the future.

3.2 Takeoff Performance

3.2.1 Introduction

Unlike steady level flight envelope, takeoff performance is very easy to analyze. The simplest way to judge takeoff performance is by analyzing the amount of distance it takes for an aircraft to takeoff. Obviously, the shorter a takeoff distance the better. However, takeoff distances are effected by multiple factors such as wind and altitude. This report will only focus on headwind, tailwind and altitude changes. Since the Cessna 172R is a single engine aircraft, a result in engine failure would mean that the aircraft cannot continue takeoff procedure. Therefore, no engine out (balanced field length) cases were calculated for.

Analyzing takeoff performance is easy but calculating takeoff distance is not. The takeoff of an aircraft can be broken down into three segments and requires the information summarized in Table 5. The first segment is the ground roll portion and it can be calculated using two methods which are method A and B but in this report method B was used. Method B uses Equation 3.3 to calculate the distance of the ground roll segment. This equation requires the knowledge of the drag, lift, weight and thrust force of the aircraft at this moment. Equation 3.3 is an approximation therefore it can use the mean force values which can be calculated using Equation 3.4 to Equation 3.6. However to calculate these forces Equations 3.7 to Equation 3.16 are required first. Equations 3.3 to Equation 3.16 only gives the ground roll segment which is a majority of the takeoff distance.

Table 5 - Initial Parameters Used to Calculate Takeoff Performance [1] [2][3][4]

Parameter Symbol	Values
$W = MTOW$	$2450\ lbs$
ρ_{SL}	$0.002378 \frac{slug}{ft^3}$
ρ	$Calculated\ Using\ Equation\ 1.1 - 1.4$
e	0.77
S	$175.5\ ft^2$
h	$8\ ft$
b	$36\ ft$
$C_{L,max}$	2.1
n	$2700\ rpm$
$C_{L,ground\ roll}$	0.9^*
C_{Do}	0.032
μ	0.02^*
T_o	$531.911\ lbf^{**}$
d_p	$6.25\ ft$
θ_R	$0°$
h_{sc}	$50\ ft$
g	$32.2 \frac{ft^2}{s}$

**Taken from Problem Set Question 1 & 2* [8]

***Calculated Value* [9]

$$S_{to_G} = \frac{{v_{gLOF}}^2}{\frac{g}{W}[\bar{T} - \bar{D} - \mu(W - \bar{L}) - W \sin\theta_R]} \tag{3.3}$$

$$\bar{D} = \frac{1}{2}\rho\bar{v}^2 S C_{D_{GE}} \tag{3.4}$$

$$\bar{L} = \frac{1}{2}\rho\bar{v}^2 S C_{L,ground\ roll} \tag{3.5}$$

$$\bar{T} = T_o(1 - 0.3\bar{J}) \tag{3.6*}$$

$$v_{STO} = \sqrt{\frac{2W}{\rho S C_{L,max}}} \tag{3.7}$$

$$v_{LOF} = 1.15 v_{STO} \tag{3.8*}$$

$$v_{gLOF} = v_{LOF} - v_w \tag{3.9}$$

$$\bar{v} = \sqrt{\frac{{v_{LOF}}^2 - v_w|v_w|}{2}} \tag{3.10}$$

$$\bar{J} = \frac{\bar{v}}{n d_p} \tag{3.11}$$

$$n = \frac{n\ (rpm)}{60} \tag{3.12}$$

$$C_{D_{GE}} = C_{Do} + \phi K {C_{L,ground\ roll}}^2 \tag{3.13}$$

$$K = \frac{1}{\pi e AR} \tag{3.14}$$

$$AR = \frac{b^2}{S} \tag{3.15}$$

$$\phi = 1 - \frac{\left(1 - 1.32\left(\frac{h}{b}\right)\right)}{\left(1.05 + 7.4\left(\frac{h}{b}\right)\right)} \tag{3.16}$$

Above Equations Taken From [6] *Unless Stated.*

**Taken from Problem Set Question 1* [8].

The next segment is the first airborne transition. In this segment the aircraft has just lifted off the ground. It can be calculated using Equations 3.16 to Equations 3.24. This segment is very similar to method B of the ground roll segment. The only difference is in Equation 3.21 where the ground effect factor (ϕ) is equal to one because the aircraft is off the ground.

$$S_{to_{AI}} = \frac{\frac{{v_2}^2 - {v_{LOF}}^2}{2}}{\frac{g}{W}[\bar{T} - \bar{D}]} \tag{3.17}$$

$$v_2 = 1.3 v_{STO} \tag{3.18*}$$

$$\bar{v} = \sqrt{\frac{{v_{LOF}}^2 - {v_2}^2}{2}} \tag{3.19}$$

$$\bar{J} = \frac{\bar{v}}{n d_p} \tag{3.20}$$

$$\bar{T} = T_o(1 - 0.3\bar{J}) \tag{3.21}$$

$$\overline{C_D} = C_{Do} + \phi K \overline{C_L}^2 \tag{3.22}$$

$$\overline{C_L} = \frac{2W}{\rho \bar{v}^2 S} \tag{3.23}$$

$$\bar{D} = \frac{1}{2}\rho \bar{v}^2 S \overline{C_D} \tag{3.24}$$

Above Equations Taken From [6] *Unless Stated.*
**Taken from Problem Set Question 1* [8].

Last of all, is the second airborne transition in which an obstacle has to be cleared. The obstacle height to be cleared was set to $50\ ft$. This segment can be calculated using Equation 3.25 to Equation 3.30.

$$S_{to_{AII}} = \frac{h_{sc}}{\tan \gamma_{deg}} \tag{3.25}$$

$$\gamma_{rad} = \frac{T}{W} - \frac{C_D}{C_L} \tag{3.26}$$

$$T = T_o(1 - 0.3J) \tag{3.27}$$

$$J = \frac{v_2}{n d_p} \tag{3.28}$$

$$C_L = \frac{2W}{\rho {v_2}^2 S} \tag{3.29}$$

$$C_D = C_{Do} + \phi K {C_L}^2 \quad (3.30)$$

Above Equations Taken From[6] *Unless Stated.*

After, all the segments are calculated they can be added together to get the final takeoff distance. This was the procedure used to calculate the takeoff distances for the analysis of the Cessna 172R. However, this is the procedure for a no wind case. Equations 3.31 to Equations 3.34 can be used to calculate the takeoff distance with wind. Note that only the airborne segments need to be altered because ground roll equations already accounted for wind. For the analysis of the Cessna the calculations were done using MATLAB code which can be found in Appendix A:3 Takeoff Performance MATLAB Code (Page 28).

$$S_{to_{AI}} = S_{to_{AI}}(0) - v_w \Delta t_{to_{AI}} \quad (3.31)$$

$$\Delta t_{to_{AI}} = \frac{v_2 - v_{LOF}}{\frac{g}{W}[\bar{T} - \bar{D}]} \quad (3.32)$$

$$S_{to_{AII}} = S_{to_{AII}}(0) - v_w \Delta t_{to_{AII}} \quad (3.33)$$

$$\Delta t_{to_{AII}} = \frac{S_{to_{AII}}(0)}{v_2 \tan \gamma_{deg}} \quad (3.34)$$

3.2.2 Results

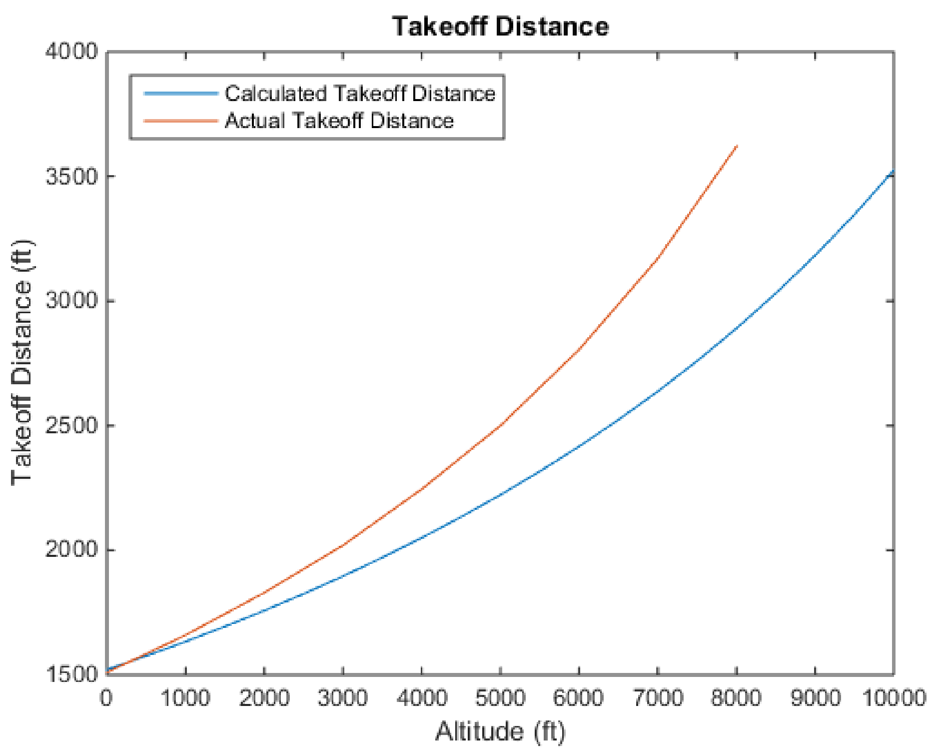

Figure 3 - Takeoff Distance Vs Altitude of Cessna 172R Skyhawk [1]

Table 6 - Actual Takeoff Distances of Cessna 172R Skyhawk [1]

Altitude (ft)	Takeoff Distance at $0℃$ (ft)	Takeoff Distance at $10℃$ (ft)
Sea Level	1510	1625
1000	1660	1790
2000	1830	1970
3000	2020	2185
4000	2245	2430
5000	2500	2715
6000	2805	3060
7000	3170	3470
8000	3620	3975

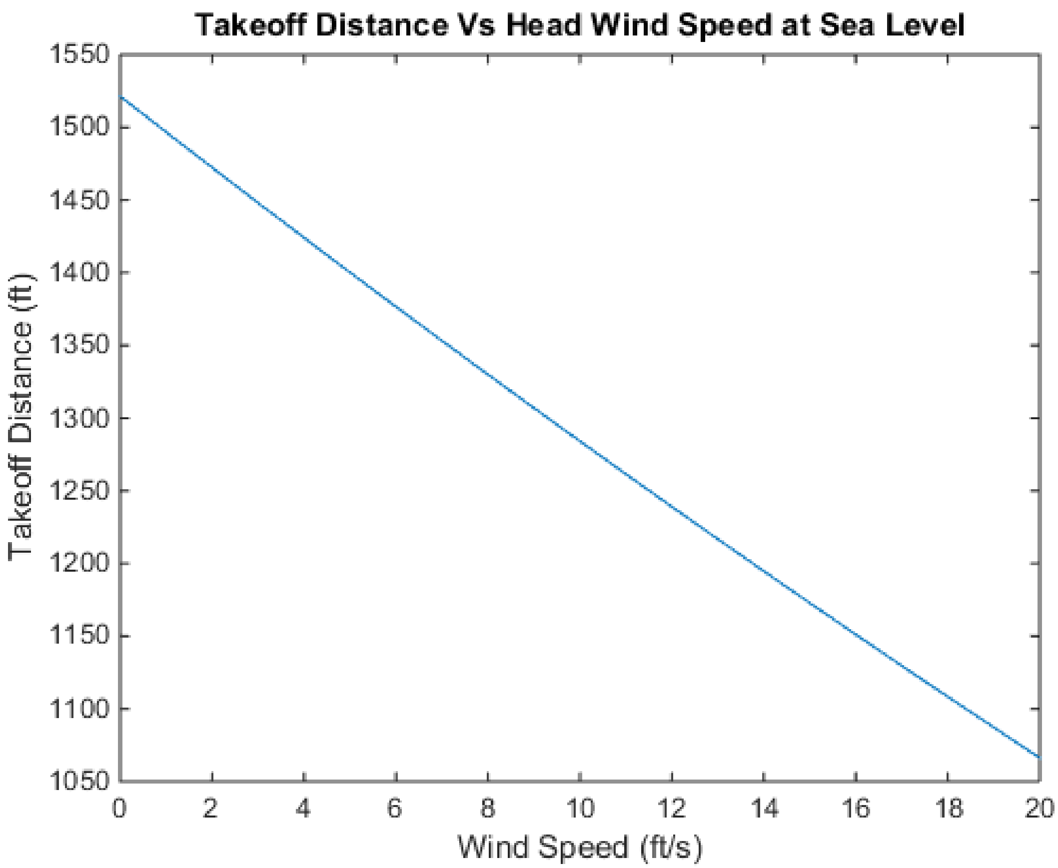

Figure 4 - Takeoff Distance Vs Head Wind Speed of Cessna 172R Skyhawk (Sea Level)

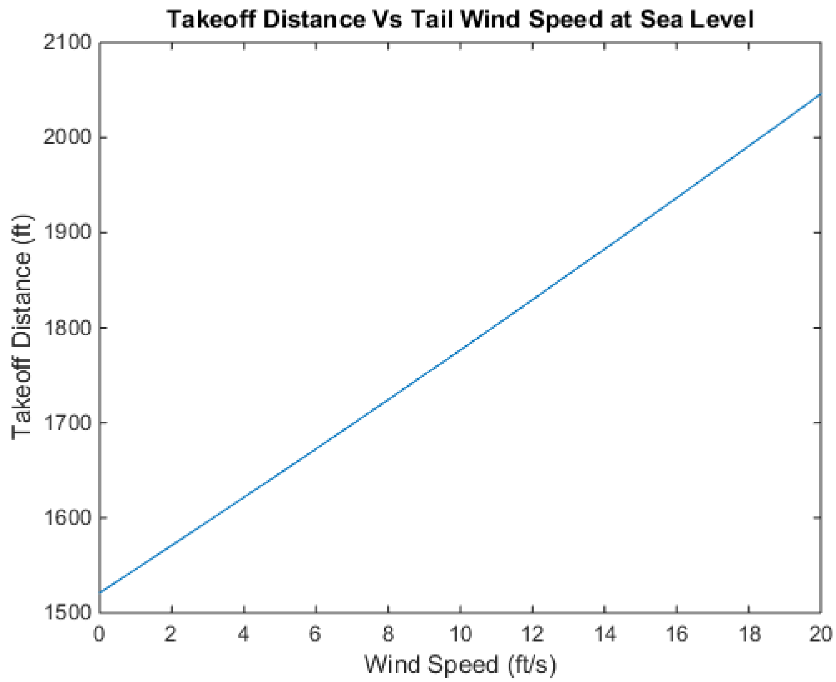

Figure 5 - Takeoff Distance Vs Tail Wind Speed of Cessna 172R Skyhawk (Sea Level)

3.2.3 Discussion

The takeoff performance of the Cessna 172R can be best judged by Figure 3 which shows the takeoff distances at different altitudes. It is clear at lower altitudes the approximated takeoff distance calculations are a lot more accurate. This is primarily due to the assumptions made for thrust and velocity as they change at higher altitudes. Therefore, the equations of those could be adjusted to produce a better approximation model. Also, temperature was not accounted for in the calculation process. Obviously excluding the density calculations. Nevertheless temperature can play a larger role at higher altitudes as displayed in Table 6 which, shows that the takeoff distance difference is far greater at higher altitudes. Note that Table 6 only displays a change in temperature of $10°\text{C}$. However, the approximated values do follow the same pattern and give reasonable values which can be used to analyze the effects of different design parameters.

Generally, an aircraft will experience some sort of wind during takeoff. Depending on the direction of the wind this can be beneficial or not. Figure 4 and Figure 5 show how two types of wind can affect the Cessna 172R takeoff distances. It is important to note that head wind is considered as wind contacting the aircraft straight on in the opposite direction of travel. These winds are usually unwanted in different forms of transportation as they slow the object in motion down. However in a takeoff situation these winds are beneficial as demonstrated in Figure 4 cause they decrease the overall take off distance. This occurs due to the head wind velocity generating more lift. It does this because the head wind hits the wings of the aircraft straight on thus creating lift force. The opposite occurs with tail wind cause the shape of the airfoil does not allow it to exploit this wind in the same manner. In fact, tail winds cause a down force on the wings thus increasing the weight of the aircraft. Thus ,resulting in a larger takeoff distance as shown in Figure 5.

The takeoff performance of the Cessna 172R is another design performance that is satisfactory. Besides changing the wing design to generate more lift there are two other options available. The simplest solution is to use more composite material throughout the aircraft to reduce the overall weight thus allowing it to takeoff sooner. The other is to use more powerful engines or turbochargers which will provide more thrust and therefore shorten the takeoff distances as well.

3.3 Landing Performance

3.3.1 Introduction

Landing performance analysis is very similar to takeoff performance except the whole process is reversed and some minor changes. During landing there are different drag and lift coefficients ($\boldsymbol{C_D}$&$\boldsymbol{C_L}$). Also, in landing forward thrust is not wanted as this would increase the landing distance. Reverse thrust is ideal however for the Cessna 172R there is no way to produce reverse thrust therefore there is no thrust force at all. Similar to the takeoff performance the only factors that will effect landing distance that were analyzed were head wind, tail wind and altitude changes. Conversely, unlike takeoff performance there are two types of landing analysis. The first type is called stalled-on landing approach in which the aircraft stays airborne even after the flare portion of the landing procedure until it reaches aerodynamic stall. Table 7 displays the required parameters to solve a stalled on landing analysis and Equations 3.35 to equation 3.51 are used to reach the conclusion of the analysis. These equations are for a no wind scenario but with the addition of Equation 3.52 to Equation 3.54 the landing distance with initial wind condition could be solved. The other landing approach is the flown on case in which the aircraft touches down right after the flare portion of the landing procedure. Both types of landing procedure were analyzed but only the equations for stalled on are described as the process is very similar. The calculations were done using MATLAB code which can be found in Appendix A6: Landing Performance MATLAB Code (Page 32).

Table 7 - Initial Parameters Used to Calculate Landing Performance [1] [2][3][4]

Parameter Symbol	Values
$W = MLW = MTOW$	$2450\ lbs$
ρ_{SL}	$0.002378 \frac{slug}{ft^3}$
ρ	$Calculated\ Using\ Equation\ 1.1 - 1.4$
e	0.77
S	$175.5\ ft^2$
h	$8\ ft$
b	$36\ ft$
$C_{L,max\ landing}$	2.1
n	$2700\ rpm$
$C_{L,ground\ roll}$	0.3*
C_{Do}	0.05*
μ_{decel}	0.25*
T_o	0
d_p	$6.25\ ft$
θ_R	$0°$
h_{sc}	$50\ ft$
g	$32.2 \frac{ft^2}{s}$
$\gamma_{D\ deg}$	$5°$*
$\frac{C_{L,flare}}{C_{L,approach}}$	1.15*
Δt_{delay}	$2\ s$

**Taken from Problem Set Question 3* [8].

$$v_{SLD} = \sqrt{\frac{2W}{\rho S C_{L,\max\ landing}}} \tag{3.35}$$

$$S_{L_{AI}} = \frac{h_{sc}}{\tan \gamma_{D\ deg}} \tag{3.36}$$

$$S_{L_{AII}} = R \sin\left(\frac{\gamma_{D\ deg}}{2}\right) + \frac{W}{2g} \frac{({v_{SLD}}^2 - {v_A}^2)}{(\bar{T} - \bar{D})} \tag{3.37}$$

$$v_A = 1.25 v_{SLD} \tag{3.38*}$$

$$R = \frac{{v_A}^2}{g\left(\frac{C_{L,flare}}{C_{L,approach}} - 1\right)} \tag{3.39}$$

$$\bar{D} = \frac{1}{2}\rho \bar{v}^2 S\overline{C_D} \tag{3.40}$$

$$\overline{C_D} = C_{Do} + \phi K \overline{C_L}^2 \tag{3.41}$$

$$\overline{C_L} = \frac{2W}{\rho \bar{v}^2 S} \tag{3.42}$$

$$\bar{v} = \sqrt{\frac{{v_{SLD}}^2 + {v_A}^2}{2}} \tag{3.43}$$

$$\phi = 1 - \frac{\left(1 - 1.32\left(\frac{h}{b}\right)\right)}{\left(1.05 + 7.4\left(\frac{h}{b}\right)\right)} \tag{3.44}$$

$$S_{L_{GI}} = v_{GS}\Delta \boldsymbol{t_{delay}} \tag{3.45}$$

$$v_{GS} = v_{SLD} - v_w \tag{3.46}$$

$$S_{L_{GII}} = \frac{1}{2B}\ln\left(\frac{A - B{v_{GS}}^2}{A}\right) \tag{3.47}$$

$$A = g\left(\frac{NT_o}{W} - \mu_{decel} - \sin\theta_R\right) \tag{3.48}$$

$$B = \frac{g}{W}\left[\frac{1}{2}\rho S\left(C_{D_{GE}} - \mu_{decel} C_{L,ground\ roll}\right) + Na\right] \tag{3.49}$$

$$C_{D_{GE}} = C_{Do} + \phi K {C_{L,ground\ roll}}^2 \tag{3.50}$$

$$a = \frac{T_o - \bar{T}}{\bar{v}} \tag{3.51}$$

Above Equations Taken From [6] *Unless Stated.*
**Taken from Problem Set Question 3* [8].

$$S_{L_A} = S_{L_{AI}} + S_{L_{AII}} \tag{3.52}$$

$$S_{L_A} = S_{L_A}(o) - v_w \Delta t_{L_A} \tag{3.53}$$

$$\Delta t_{L_A} = \frac{h_{sc}}{v_a \tan\gamma_{D\ deg}} + \frac{R}{v_a}\sin\left(\frac{\gamma_{D\ deg}}{2}\right) + \frac{W}{g}\frac{(v_{SLD} - v_A)}{(\bar{T} - \bar{D})} \tag{3.54}$$

3.3.2 Results

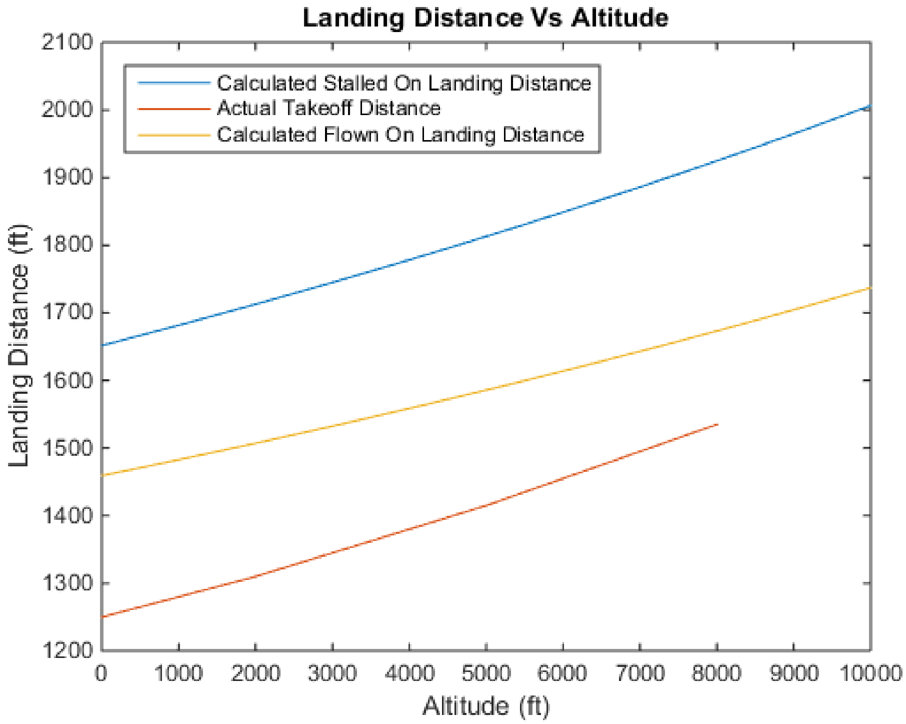

Figure 6 - Landing Distance Vs Altitude of Cessna 172R Skyhawk [1]

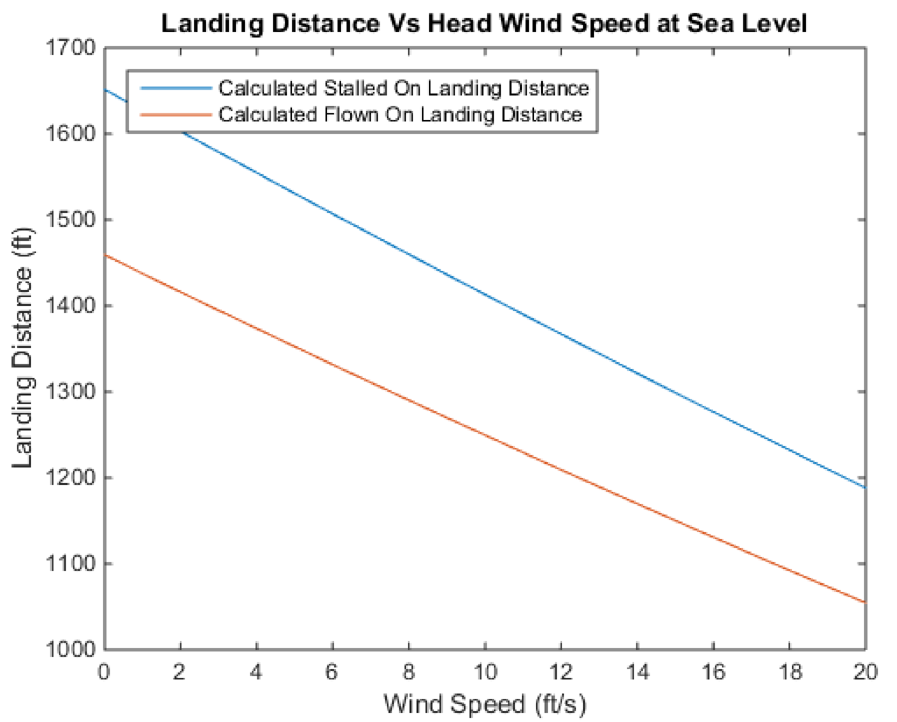

Figure 7 - Landing Distance Vs Head Wind Speed of Cessna 172R Skyhawk (Sea Level)

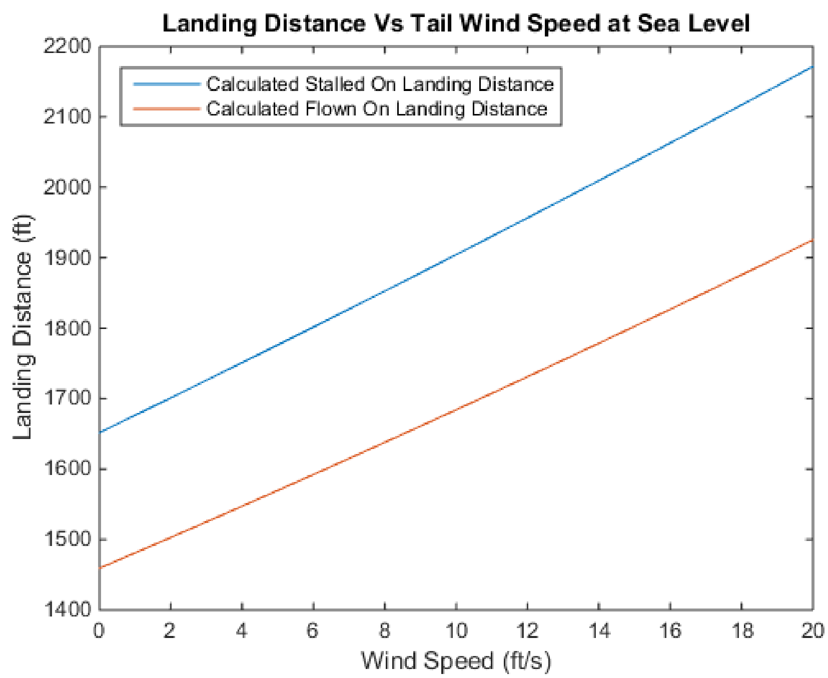

Figure 8 - Landing Distance Vs Tail Wind Speed of Cessna 172R Skyhawk (Sea Level)

3.3.3 Discussion

Primarily Figure 6 provides insight into the analysis of the landing performance. First of all, both approximations are way off and give a far greater landing distance. The flown on method is closer due to the fact that the airplane initiates the ground roll segment sooner. Which, for the 172R is more beneficial due to the fact that it has no reverse thrusters and relays more on the surface material to stop. One of the main reasons for the discrepancy of these calculations was due to more approximation assumptions made such as the lift and drag coefficients. Also, one big assumption factor is the time delay for a pilot to react once the airplane touches the ground which, was assumed to be two seconds. This may be shorter for more experienced pilots. Again temperature can play a role. On the other hand, the stalled on landing models the actual landing distances better than flown as shown in Figure 6. Therefore, neither one landing model seems to be a clear fit but they still provide results which would ensure safe landing in an actual situation.

Similarly to takeoff, Figure 7 and Figure 8 provide evidence that head winds results in shorter landing distance and tail wind provide longer landing distances. Again, the reason for this has to do on how the wind flows over the airfoil. The major difference from the takeoff situation are the forces the winds create. In the landing situation the flap configuration is down on an airfoil to increase the drag force thus allowing it to stop sooner. Thus, with head wind this force is increased cause the winds are 'pushing' the plane back. It is similar to two people running into each other but in this case the aircraft is the larger person and simply keeps moving forward. However, the aircraft took a hit in speed. Consequently, a tail wind is similar to a smaller person pushing a larger person forward. They would likely speed up. The same concept occurs with the tail wind on the flap and wing of the airfoil thus increasing the landing distance.

Evidently, there is nothing drastically wrong with the design parameters of the Cessna 172R related to landing. Nonetheless, improvements can still be made to the aircraft. The aircraft would have shorter landing distances if there was a way to provide reverse thrust and/or if it had slats. Reverse thrusters are unlikely on a propeller aircraft powered by a piston engine but slats can be possible for future models.

3.4 Climb Performance

3.4.1 Introduction

The best way to analyze the climb performance of an aircraft is through its rate of climb numbers. Generally, for a small personal use light weight aircraft the rate of climb are expected to be nothing impressive. Rate of climb in imperial units is measure in feet per minute and can be calculated using Equation 3.55 or Equation 3.61. For Equation 3.55 the max rate of climb can be found using Equation 3.56 which calculates the max rate of climb lift coefficient ($C_{L,max\ R/C}$). Equation 3.55 to Equation 3.60 were used to calculate individual values of max rate of climb whereas Equation 3.61 was used to make Figure 9. In the figure the flight path velocity is the x-axis and the graph starts at lowest steady level speed for sea level and goes to max steady level speed at sea level. Table 8 shows all the required parameters to use Equation 3.55 to Equation 3.62 and it is to be noted that all the calculations were conducted using MATLAB code found in Appendix A9: Climb Performance MATLAB Code (Page 37).

Table 8 - Initial Parameters Used to Calculate Climb Performance [1] [2][3][4]

Parameter Symbol	Values
$\boldsymbol{W = MLW = MTOW}$	$2450\ lbs$
$\boldsymbol{\rho_{SL}}$	$0.002378\frac{slug}{ft^3}$
$\boldsymbol{\rho}$	$Calculated\ Using\ Equation\ 1.1 - 1.4$
$\boldsymbol{e}$	0.77
$\boldsymbol{S}$	$175.5\ ft^2$
$\boldsymbol{h}$	$8\ ft$
$\boldsymbol{b}$	$36\ ft$
$\boldsymbol{v_{max}}$	$212.66\frac{ft}{s}$
$\boldsymbol{n}$	$2700\ rpm$
$\boldsymbol{v_{min}}$	$74.07\frac{ft}{s}$
$\boldsymbol{C_{Do}}$	0.032
$\boldsymbol{T_o}$	$531.911\ lbf^{**}$
$\boldsymbol{d_p}$	$6.25\ ft$
$\boldsymbol{g}$	$32.2\frac{ft^2}{s}$

***Calculated Value* [9]

$$R/C = v\sin\gamma = \sqrt{\frac{2W}{\rho S}}\left[\frac{T}{W}\frac{1}{\sqrt{C_L}} - \frac{C_{Do}}{C_L^{\frac{3}{2}}} - K\sqrt{C_L}\right] \tag{3.55}$$

$$C_{L,max\ R/C} = \sqrt{\frac{3C_{Do}}{K}} \tag{3.56}$$

$$T = T_o(1 - 0.3J) \qquad (3.57)^*$$

$$J = \frac{v_{max}}{nd_p} \tag{3.58}$$

$$K = \frac{1}{\pi e AR} \tag{3.59}$$

$$n = \frac{n\ (rpm)}{60} \tag{3.60}$$

Above Equations Taken From [6] *Unless Stated.*

**Taken from Problem Set Question 1* [8].

$$R/C = -\frac{2KW}{\rho S}\frac{1}{v_\infty} + \frac{\sigma T_o}{W}v_\infty - \frac{\sigma a}{W}{v_\infty}^2 - \frac{C_{Do}\rho S}{2W}{v_\infty}^3 \qquad (3.61)^{***}$$

$$a = \frac{T_o - T}{v_\infty} \tag{3.62}$$

Above Equations Taken From [6] *Unless Stated.*

*** *Taken from* [10].

3.4.2 Results

Table 9 - Max Rate of Climb at Sea Level for Cessna 172R Skyhawk [1]

Max Rate of Climb (Sea Level)	Value (fpm)
Calculated	398
Actual	720

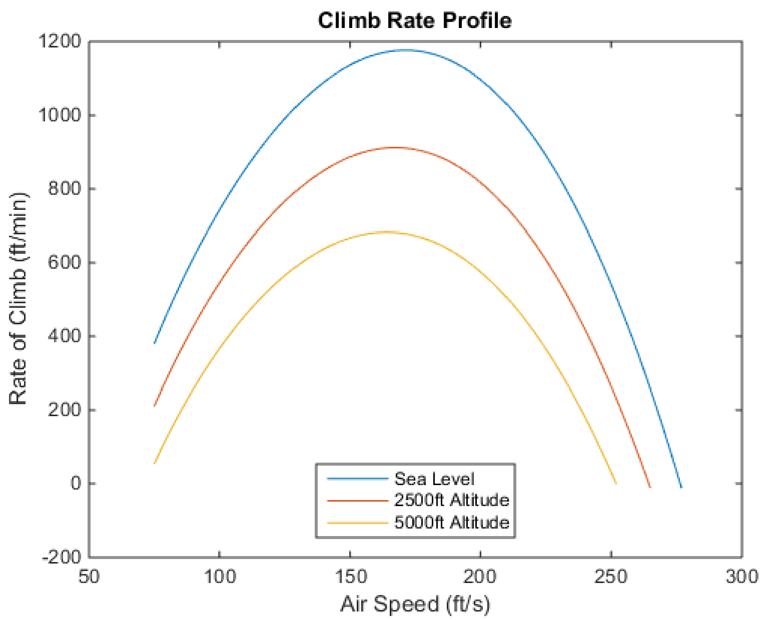

Figure 9 - Climb Rate Profile at Different Altitudes of Cessna 172R Skyhawk

3.4.3 Discussion

The max rate of climb was calculated to obtain the basis to perform a cruise performance analysis. The results are summarized in Table 9 and were calculated using Equation 3.55. The calculated value is drastically different from the actual max rate of climb at sea level. This is due to the fact that a lot of assumptions were made. Starting with the thrust value which was calculated. The next assumption was that the lift coefficient value for max rate of climb was also calculated using an approximation method. The last reason for the large difference could be due to the equation being an approximation. Nevertheless, this equation did not provide valid information for analysis of the aircraft.

Consequently, Figure 9 was calculated using a different equation as stated above and therefore provided a better solution. It is still a bit odd compared to other climb rate graphs of other aircrafts. However, it is still accurate and provides reasonable results even though the numbers are larger than actual. Equation 3.61 is also an approximated equation which uses airspeed. However, this data is still hard to compare to the actual test results of the aircraft as no reliable source was found. Consequently, the graph still shows the correct trend of greater altitude decreasing the rate of climb. This mainly has to do with a thinner density affecting the static thrust power of the aircraft. The other trend it shows correctly is the shape of the graph which like other similar graphs from other graphs represent a parabolic shape with the max rate of climb occurring at the peak of the parabolic. In conclusion, this graph did help determine the climb performance.

The rate of climb could be a lot better for the Cessna 172R. This could be accomplished with a more powerful engine or even an alternative engine which does not suffer a loss of static power as the density thins out. Another way to increase the rate of climb of the aircraft would be to decrease the weight of the aircraft and this could be accomplished with the use of composites. On the other hand, these changes would increase the price of the 172R which is a key marketing feature of the aircraft.

3.5 Range-Payload Diagram

3.5.1 Introduction

A Range-Payload diagram is very self-explanatory. This diagram displays how far the Cessna 172R Skyhawk can cruise for at a certain altitude depending on the amount of payload weight it is carrying. There are four main points on a diagram like this. The first is the max payload - zero range point which leads up to the next point which is the max range with max payload situation. This then drops down in a linear matter to the third point. The third point represents the situation where there is max fuel with a smaller payload. Last of all, is the max fuel with no payload situation. This fourth point represent the maximum range of the aircraft. Note, that this diagram is measured in nautical miles which is equivalent to about $1.15\,miles$ [11].The diagram was constructed using the initial parameters listed in Table 10 and calculated using the MATLAB code in Appendix A10: Range Payload Diagram MATLAB Code (Page 39) with the use of Equation 3.63 to Equation 3.76.

Table 10 - Initial Parameters Used to Range-Payload Diagram [1] [2][3][4] [12]

Parameter Symbol	Values
$\boldsymbol{n}$	$2700\ rpm$
$\boldsymbol{d_p}$	$6.25\ ft$
$\boldsymbol{C_{Do}}$	0.032
$\boldsymbol{\rho}$	$Calculated\ Using\ Equation\ 1.1 - 1.4$
$\boldsymbol{\rho_{fuel}}$	$6.209\frac{lbs}{gal}$
$\boldsymbol{e}$	0.77
$\boldsymbol{S}$	$175.5\ ft^2$
$\boldsymbol{b}$	$36\ ft$
$\boldsymbol{v_{cruise}}$	$194.098\ \frac{ft}{s}$
$\boldsymbol{Fuel\ Consumption}$	$57.1228\frac{lbs}{hr}$
$\boldsymbol{W_{max}}$	$2450\ lbs$
$\boldsymbol{W_{payload}}$	$607\ lbs$
$\boldsymbol{W_{empty}}$	$1679\ lbs$
$\boldsymbol{W_{total\ fuel}}$	$347.704\ lbs$
$\boldsymbol{W_{reserve\ fuel}}$	$\frac{Fuel\ Consumption}{2}$
$\boldsymbol{W_{TO\ fuel}}$	$\frac{Fuel\ Consumption}{3}$
$\boldsymbol{W_{LD\ fuel}}$	$(0.5)W_{TO\ fuel}$
$\boldsymbol{P_S}$	$180\ hp$
$\boldsymbol{\eta_{pr}}$	0.83

$$W_{fuel} = W_{total\ fuel} - W_{reserve\ fuel} - W_{TO\ fuel} - W_{LD\ fuel} \tag{3.63}$$

$$R = \frac{326\eta_{pr}\left(\frac{C_{L\,avg}}{C_{D\,avg}}\right)}{BSFC}\ln\left(\frac{W_{ini}}{W_{fin}}\right) \tag{3.64}$$

$$BSFC = \frac{Fuel\ Consumption}{0.8\eta_{pr}} \tag{3.65}$$

$$W_{avg} = \sqrt{(W_{ini}W_{fin})} \tag{3.66}$$

$$C_{L\,avg} = \frac{2W_{avg}}{\rho S v_{cruise}{}^2} \tag{3.67}$$

$$C_{D\,avg} = C_{Do} + K C_{L\,avg}{}^2 \tag{3.68}$$

Max Payload:

$$W_{ini} = W_{max} - W_{TO\,fuel} \tag{3.69}$$

$$W_{fo} = W_{max} - W_{empty} - W_{payload} \tag{3.70}$$

$$W_{fin} = W_{ini} - (W_{fo} - W_{LD\,fuel} - W_{reserve\,fuel}) \tag{3.71}$$

Max Fuel:

$$W_{ini} = W_{max} - W_{TO\,fuel} \tag{3.72}$$

$$W_{load} = W_{max} - W_{empty} - W_{fuel} \tag{3.73}$$

$$W_{fin} = W_{empty} + W_{load} + W_{LD\,fuel} + W_{reserve\,fuel} \tag{3.74}$$

Zero Payload:

$$W_{ini} = W_{empty} + W_{total\,fuel} - W_{TO\,fuel} \tag{3.75}$$

$$W_{fin} = W_{empty} + W_{LD\,fuel} + W_{reserve\,fuel} \tag{3.76}$$

Above Equations Taken From[6] *Unless Stated.*
Equation 3.69 to Equation 3.76 Were Created

To perform the calculations some parameters had to be estimated or found. For starters, the lift and drag coefficient were calculated using average weight. Also, the fuel consumption was taken on average basis which was then used in Equation 3.65. The weight of the fuel was found by converting the gallons into pounds. This was done by using the density of a recommended fuel which was Avgas 100LL (blue)[1]. This allowed the calculation of the weight of the reserve fuel. The amount of reserve fuel allocated within the calculations was with the FAR regulations [13]. As well, takeoff fuel usage was estimated using numbers from similar aircrafts and landing usage was estimated as half of the takeoff fuel because landing requires less fuel. Likewise, propeller efficiency was taken from Figure 10 using the advanced ratio calculated. Once, these parameters were set out it was possible to create the range-payload diagram.

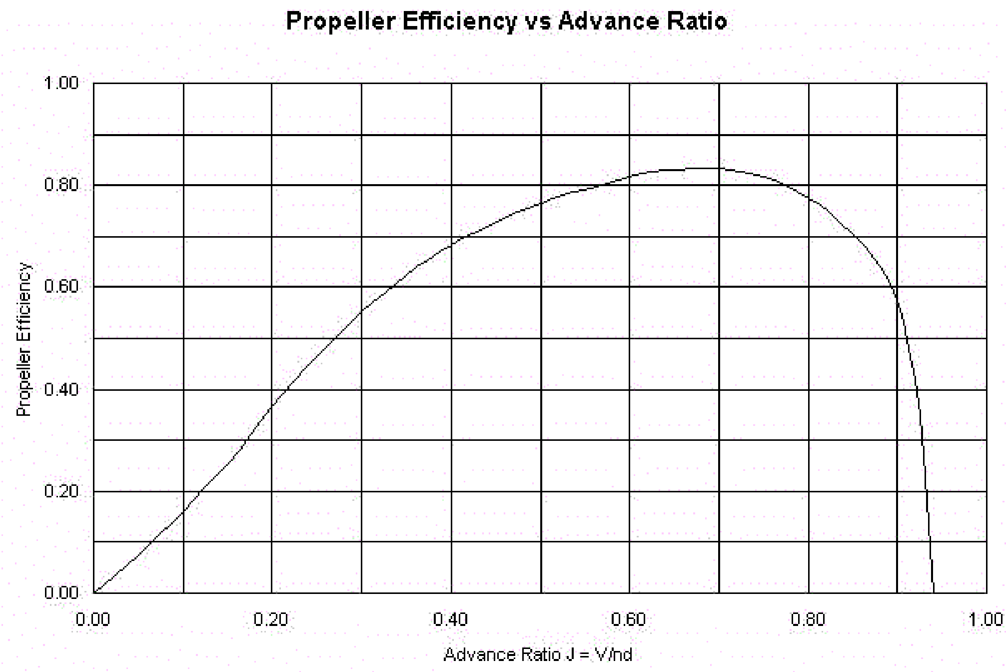

Figure 10 - Propeller Efficiency Vs Advance Ratio of Cessna 172R Skyhawk

3.5.2 Results

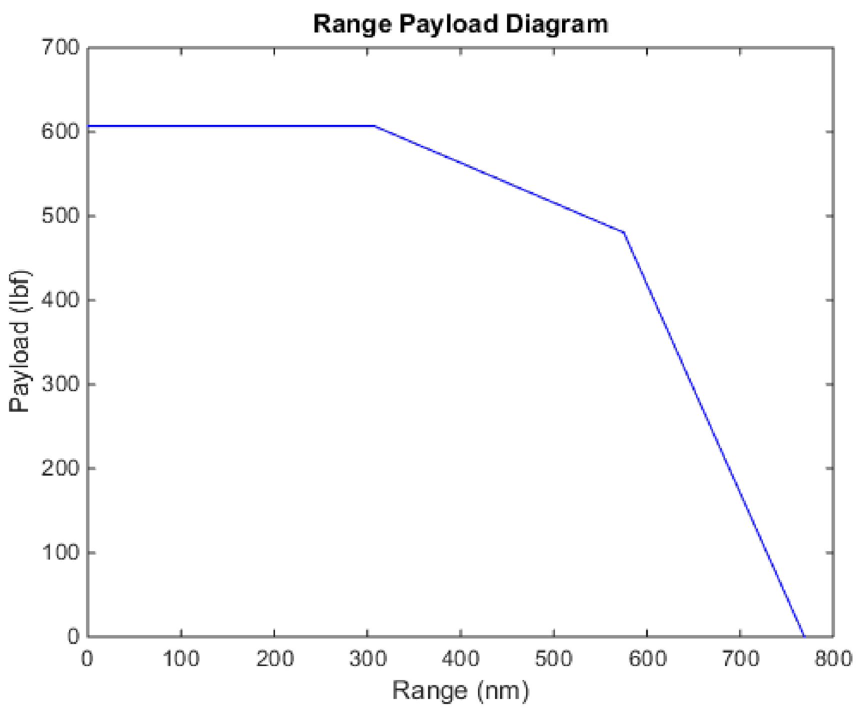

Figure 11 - Range - Payload Diagram at Sea Level of Cessna 172R Skyhawk

Table 11 - Range-Payload Diagram Construction Points

Points	Payload (lbf)	Range (nmi)
Max Payload - Zero Range	607	0
Max Payload	607	307.28
Max Fuel	480.42	575.02
Max Fuel - Max Range	0	768.95

3.5.3 Discussion

Through the use of some assumptions and estimations it was possible to create the Range-Payload Diagram of the Cessna 172R Skyhawk (Figure 11). The diagram is an accurate representation of the actual results of the 172R. For example, the maximum range of the aircraft is about $580\,nmi$ and Figure 11 shows a maximum range of about $768.95\,nmi$ [1]. Note that this is the range of the aircraft once it has reached an altitude of about $8000\,ft$ and the four key points used to construct the diagram are in Table 11. It is to be noted that the actual range had different initial conditions. These initial conditions included more fuel reserve and more accurate parameters. Interestingly, the calculated range is slightly close to the actual range considering all the assumptions stated. The range of the aircraft could be stated as adequate for a personal use aircraft especially, considering most of these planes are used for pilot training. Therefore, the 172R does not have to be improved for range. If it was improved then it would require a larger engine so other performances do not suffer from the added weight of the fuel for example. Another solution is to use composite material for the fuselage to decrease the overall weight of the aircraft. This is a drastic solution and mentioned for other performance improvements as well. Mainly due to its recent usage in modern aircrafts.

4.0 Conclusion

As a aircraft put to the standards of the modern era the Cessna 172R is mediocre. However, for an aircraft with a reasonable price and intended for personal use this aircraft is the only choice. The aircraft welcomes speed that are excellent for cruising or learning with as evident through the steady level flight envelope (Figure 2). As well, its takeoff and landing distance are good enough for most smaller sized airports. On the other hand, the climb rate could be improved as it varies too greatly at different altitudes and the fact that the actual rate of climb is a lot lower than calculated. Consequently, the range of these aircrafts for the given situation are adequate as demonstrated by the range-payload diagram (Figure 11). Nonetheless, the aircraft can be improved. It can be improved for stability and shorter landing distances with the use of slats. The takeoff distances and rate of climb could be improved with the use a more powerful engine. This could also be accomplished with the use of a turbocharged version of the Textron Lycoming. Last of all, most of the aircraft performances could be improved with the use of composite material for the aircraft structure thus making it lighter. In conclusion, the Cessna 172R is a solid small lightweight aircraft for its intended purpose but with some design changes it could be even better.

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Appendix

A1: Density Function

By: Rahul Rohatgi

Student Number: 500498833

Created Function

```
function[density]=density(alt)
format shorte
alt=alt*0.3048 %Feet to Meter conversion
density_slvl = 0.002378; %Density of Air at Sea Level In Imperial Units
temp_ratio = 1-2.26e-5*alt; %Calculating Temperature Ratio
density_ratio = temp_ratio^4.256; %Calculating Density Ratio
density = density_ratio*density_slvl; %Calculating New Density
end
```

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A2: Steady Level Flight Envelope MATLAB Code

By: Rahul Rohatgi

Student Number: 500498833

Initial Parameters

```
W = 2450; % Maximum Takeoff Weight
S = 175.5; % Wing Area
v_max = 212.664; %Max Velocity at Sea Level
Cl_max = 2.14; %Max Lift Coefficient
density_sl = 0.002378; % Density at Sea Level
Cl_min = (2*W)/(density_sl*v_max^2*S); % Calculation for Min Lift Coefficient
alt = 0; % Altitude
ceiling = 13500; % Service Ceiling
```

Stall Speed Curve Calculation

```
i=1;
while alt <= ceiling
    density(alt); %Function that Calculates Density According to Altitude
    v_stall=sqrt(W/(0.5*ans*S*Cl_max)); %Calculating Speed
    stall_line(1,i)= v_stall;
    stall_line(2,i) = alt;
    alt=alt+500;
    i=i+1;
end
```

Max Speed Curve Calculation

```
i=1;
alt=0;
while alt <= ceiling
    density(alt); %Function that Calculates Density According to Altitude
    v_max=sqrt(W/(0.5*ans*S*Cl_min)); %Calculating Speed
    max_line(1,i)= v_max;
    max_line(2,i) = alt;
    alt=alt+500;
    i=i+1;
end
```

Ceiling Calculation

```
ceiling_line = [v_stall v_max;ceiling ceiling];
```

Envelope

```
figure(1)
plot(stall_line(1,:),stall_line(2,:),ceiling_line(1,:),ceiling_line(2,:),max_line(1,:),max_line(2
,:))
title('Steady Level Flight Envelope')
ylabel('Altitude (ft)')
xlabel('Velocity (ft/s)')
legend('Stall Speed','Max Altitude','Max Speed','Location','southeast')
```

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A:3 Takeoff Performance MATLAB Code

By: Rahul Rohatgi

Student Number: 500498833

Initial Parameters

```
alt = 0; % Altitude
```

Calculated Takeoff Distance

```
i=1;
while alt <= 10000
    [x,y]=density_Thrust(alt); %Function that Calculates Density and Static Thrust According to
Altitude
    takeoff(1,i)= alt;
    takeoff(2,i) = Takeoff(x,0,y);
    alt=alt+500;
    i=i+1;
end
```

Actual Takeoff Distance

```
actual_takeoff=[1510 1660 1830 2020 2245 2500 2805 3170 3620;0 1000 2000 3000 4000 5000 6000 7000
8000];
```

Calculated Vs Actual Takeoff Distance Graph

```
figure(1)
plot(takeoff(1,:),takeoff(2,:),actual_takeoff(2,:),actual_takeoff(1,:))
title('Takeoff Distance')
ylabel('Takeoff Distance (ft)')
xlabel('Altitude (ft)')
legend('Calculated Takeoff Distance','Actual Takeoff Distance','Location','northwest')
```

Calculated Takeoff Distance with Head Wind

```
i=1;
alt=0;
vw=0;
while alt <= 10000
    [x,y]=density_Thrust(0); %Function that Calculates Density and Static Thrust According to
Altitude
    takeoff(1,i)= vw;
    takeoff(2,i) = Takeoff(x,vw,y);
    alt=alt+500;
    vw=vw+1; %Changes Wind Speed for Each Iteration
    i=i+1;
end
```

Takeoff Distance Vs Head Wind Graph

```
figure (2)
plot(takeoff(1,:),takeoff(2,:))
title('Takeoff Distance Vs Head Wind Speed at Sea Level')
ylabel('Takeoff Distance (ft)')
xlabel('Wind Speed (ft/s)')
```

Calculated Takeoff Distance with Tail Wind

```
i=1;
alt=0;
vw=0;
while alt <= 10000
    [x,y]=density_Thrust(0); %Function that Calculates Density and Static Thrust According to
Altitude
    takeoff(1,i)= -vw;
    takeoff(2,i) = Takeoff(x,vw,y);
    alt=alt+500;
    vw=vw-1; %Changes Wind Speed for Each Iteration
```

```
    i=i+1;
end
```

Takeoff Distance Vs Tail Wind Graph

```
figure (3)
plot(takeoff(1,:),takeoff(2,:))
title('Takeoff Distance Vs Tail Wind Speed at Sea Level')
ylabel('Takeoff Distance (ft)')
xlabel('Wind Speed (ft/s)')
```

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A4: Density & Thrust Function

By: Rahul Rohatgi

Student Number: 500498833

Created Function

```
function[density,thrust]=density_Thrust(alt)
format shorte
alt=alt*0.3048; %Feet to Meter conversion
density_slvl = 0.002378; %Density of Air at Sea Level In Imperial Units
To_slvl=531.911; %Static Thrust at Sea Level
temp_ratio = 1-2.26e-5*alt; %Calculating Temperature Ratio
density_ratio = temp_ratio^4.256; %Calculating Density Ratio
density = density_ratio*density_slvl; %Calculating New Density
thrust=density_ratio*To_slvl; %Calculating New Static Thrust
end
```

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A5: Takeoff Function

By: Rahul Rohatgi

Student Number: 500498833

Created Function

```
function[Sto]=Takeoff(rho,V_w,To)
%Input Parameters
%rho = Density
%V_w = Wind (headwind = postitive)
%To = Static Thrust at given density
% Initial Fixed Parameters
e = 0.77;
h = 8;
b = 36;
```

```
S = 175.5;
W = 2450;
Cl_max = 2.1;
rpm = 2700;
Cl_gr = 0.9;
CD_o = 0.032;
mu = 0.02;
theta_R = 0;
dprop = 6.25;
g = 32.2;
hsc = 50;
AR = b^2/S;
K = 1/(pi*AR*e);
n = rpm/60;
phi = 1-(1-1.32*(h/b))/(1.05+7.4*(h/b));
Vs_to = sqrt((2*W)/(rho*S*Cl_max));
V_Lof = 1.15*Vs_to;
V2 = 1.3*Vs_to;

if V_w == 0

    % Ground Roll (Method B)
    V_bar = sqrt(((V_Lof^2)+(V_w*(abs(V_w))))/2);
    CD_ge = CD_o+phi*K*Cl_gr^2;
    D_bar = 0.5*rho*(V_bar^2)*S*CD_ge;
    L_bar = 0.5*rho*(V_bar^2)*S*Cl_gr;
    J_bar = V_bar/(n*dprop);
    T_bar = (1-0.3*J_bar)*To;
    stog = (V_Lof^2/2)/((g/W)*(T_bar-D_bar-mu*(W-L_bar)-W*sin(theta_R)));

    % Airbone Segment I
    V_bar = sqrt(((V_Lof^2)+(V2^2))/2);
    J_bar = V_bar/(n*dprop);
    T_bar = (1-0.3*J_bar)*To;
    Cl_bar = W/(0.5*rho*V_bar^2*S);
    Cd_bar = CD_o+K*(Cl_bar)^2;
    D_bar = 0.5*rho*(V_bar^2)*S*Cd_bar;
    stoaI = ((V2^2-V_Lof^2)/2)/((g/W)*(T_bar-D_bar));

    % Airbone Segment II
    J = V2/(n*dprop);
    T = (1-0.3*J)*To;
    Cl = W/(0.5*rho*V2^2*S);
    Cd = CD_o+K*(Cl)^2;
    gamma = (T/W)-(Cd/Cl);
    stoaII = hsc/tan(gamma);

else

    % Ground Roll (Method B)
    V_bar = sqrt(((V_Lof^2)+(V_w*(abs(V_w))))/2);
    CD_ge = CD_o+phi*K*Cl_gr^2;
    D_bar = 0.5*rho*(V_bar^2)*S*CD_ge;
```

```
    L_bar = 0.5*rho*(V_bar^2)*S*Cl_gr;
    J_bar = V_bar/(n*dprop);
    T_bar = (1-0.3*J_bar)*To;
    Vg_Lof = V_Lof-V_w;
    stog = (Vg_Lof^2/2)/((g/W)*(T_bar-D_bar-mu*(W-L_bar)-W*sin(theta_R)));

    % Airbone Segment I
    V_bar = sqrt(((V_Lof^2)+(V2^2))/2);
    J_bar = V_bar/(n*dprop);
    T_bar = (1-0.3*J_bar)*To;
    Cl_bar = W/(0.5*rho*V_bar^2*S);
    Cd_bar = CD_o+K*(Cl_bar)^2;
    D_bar = 0.5*rho*(V_bar^2)*S*Cd_bar;
    stoaI_0 = ((V2^2-V_Lof^2)/2)/((g/W)*(T_bar-D_bar));
    t_toaI = (V2-V_Lof)/((g/W)*(T_bar-D_bar));
    stoaI = stoaI_0 - V_w*t_toaI;

    % Airbone Segment II
    J = V2/(n*dprop);
    T = (1-0.3*J)*To;
    Cl = W/(0.5*rho*V2^2*S);
    Cd = CD_o+K*(Cl)^2;
    gamma = (T/W)-(Cd/Cl);
    stoaII_0 = hsc/tan(gamma);
    t_toaII = stoaII_0/(V2*cos(gamma));
    stoaII = stoaII_0 - V_w*t_toaII;
end

% Total Takeoff Distance
Sto=stog+stoaI+stoaII;
```

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A6: Landing Performance MATLAB Code

By: Rahul Rohatgi

Student Number: 500498833

Initial Parameters

```
alt = 0; % Altitude
```

Calculated Landing Distance

```
i = 1;
while alt <= 10000
    [x] = density(alt); %Function that Calculates Density According to Altitude
    landing(1,i) = alt;
    landing(2,i) = stalled_on_landing(x,0);
    landing_flown(2,i) = flown_on_landing(x,0);
    alt = alt+500;
```

```
    i = i+1;
end
```

Actual Landing Distance

```
actual_landing = [1250 1280 1310 1345 1380 1415 1455 1495 1535;0 1000 2000 3000 4000 5000 6000
7000 8000];
```

Calculated Vs Actual Landing Distance Graph

```
figure(1)
plot(landing(1,:),landing(2,:),actual_landing(2,:),actual_landing(1,:),landing(1,:),landing_flown
(2,:))
title('Landing Distance Vs Altitude')
ylabel('Landing Distance (ft)')
xlabel('Altitude (ft)')
legend('Calculated Stalled On Landing Distance','Actual Takeoff Distance','Calculated Flown On
Landing Distance','Location','northwest')
```

Calculated landing Distance with Head Wind

```
i = 1;
alt = 0;
vw = 0;
while alt <= 10000
    [x] = density(0); %Function that Calculates Density According to Altitude
    landing(1,i) = vw;
    landing(2,i) = stalled_on_landing(x,vw);
    landing_flown(2,i) = flown_on_landing(x,vw);
    alt = alt+500;
    vw = vw+1; %Changes Wind Speed for Each Iteration
    i = i+1;
end
```

Landing Distance Vs Head Wind Graph

```
figure (2)
plot(landing(1,:),landing(2,:),landing(1,:),landing_flown(2,:))
title('Landing Distance Vs Head Wind Speed at Sea Level')
ylabel('Landing Distance (ft)')
xlabel('Wind Speed (ft/s)')
legend('Calculated Stalled On Landing Distance','Calculated Flown On Landing
Distance','Location','northwest')
```

Calculated Landing Distance with Tail Wind

```
i = 1;
alt = 0;
vw = 0;
```

```
while alt <= 10000
    [x,y] = density_Thrust(0); %Function that Calculates Density and Static Thrust According to
Altitude
    landing(1,i) = -vw;
    landing(2,i) = stalled_on_landing(x,vw);
    landing_flown(2,i) = flown_on_landing(x,vw);
    alt = alt+500;
    vw = vw-1; %Changes Wind Speed for Each Iteration
    i = i+1;
end
```

Landing Distance Vs Tail Wind Graph

```
figure (3)
plot(landing(1,:),landing(2,:),landing(1,:),landing_flown(2,:))
title('Landing Distance Vs Tail Wind Speed at Sea Level')
ylabel('Landing Distance (ft)')
xlabel('Wind Speed (ft/s)')
legend('Calculated Stalled On Landing Distance','Calculated Flown On Landing
Distance','Location','northwest')
```

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A7: Stalled On Landing Function

By: Rahul Rohatgi

Student Number: 500498833

Created Function

```
function[Sld]=stalled_on_landing(rho,V_w)
%Input Parameters
%rho = Density
%V_w = Wind (headwind = postitive)
% Initial Fixed Parameters
e = 0.77;
h = 8;
b = 36;
S = 175.5;
W = 2450;
Cl_max_landing = 2.1;
Cl_grl = 0.3;
CDo_l = 0.05;
mu_l = 0.25;
theta_R = 0;
g = 32.2;
hsc = 50;
Cl_ratio = 1.15;
a_angle = 5;
AR = b^2/S;
K = 1/(pi*AR*e);
```

```
Vs_ld = sqrt((2*W)/(rho*S*Cl_max_landing));
VA = 1.25*Vs_ld;

% Airbone Segment I
SlaI = hsc/(tand(a_angle));

% Airbone Segment II
phi = 1-(1-1.32*(h/b))/(1.05+7.4*(h/b));
R = VA^2/(g*(Cl_ratio-1));
V_bar = sqrt((((Vs_ld^2)+(VA^2))/2));
Cl_bar = W/(0.5*rho*V_bar^2*S);
Cd_bar = CDo_l+phi*K*(Cl_bar)^2;
D_bar = 0.5*rho*(V_bar^2)*S*Cd_bar;
T_bar = 0; % No Reverse or Forward Thrust
SlaII = R*sind(a_angle/2)+(W/(2*g))*((Vs_ld^2-VA^2)/(T_bar-D_bar));

if V_w == 0
    Sla = SlaI + SlaII;
else
    Sla = SlaI + SlaII;
    delta_t = hsc/(VA*tand(a_angle))+(R/VA)*sind(a_angle/2)+(W/g)*((Vs_ld-VA)/(T_bar-D_bar));
    Sla = Sla - V_w*delta_t;
end

% Ground Roll Segment I
t_delay = 2; % Initial Parameter
if V_w == 0
    SlgI = Vs_ld*t_delay;
else
    VGS = Vs_ld - V_w;
    SlgI = VGS*t_delay;
end

% Ground Roll Segement II
Cd_ge = CDo_l+phi*K*(Cl_grl)^2;
To = 0; % Due to No Thrust
N = 0; % No Engine Is Used
A = g*((N*To)/W-mu_l-sin(theta_R));
a = 0; % Due to No Thrust
B = (g/W)*(0.5*rho*S*(Cd_ge-mu_l*Cl_grl)+N*a);
if V_w == 0
    SlgII = (1/(2*B))*log((A-B*Vs_ld^2)/A);
else
    SlgII = (1/(2*B))*log((A-B*VGS^2)/A);
end
Slg = SlgI + SlgII;

%Total Landing Distance
Sld = Sla + Slg;
end
```

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A8: Flown On Landing Function

By: Rahul Rohatgi

Student Number: 500498833

Created Function

```
function[Sld]=flown_on_landing(rho,V_w)
%Input Parameters
%rho = Density
%V_w = Wind (headwind = postitive)
% Initial Fixed Parameters
e = 0.77;
h = 8;
b = 36;
S = 175.5;
W = 2450;
Cl_max_landing = 2.1;
Cl_grl = 0.3;
CDo_l = 0.05;
mu_l = 0.25;
theta_R = 0;
g = 32.2;
hsc = 50;
Cl_ratio = 1.15;
a_angle = 5;
AR = b^2/S;
K = 1/(pi*AR*e);
Vs_ld = sqrt((2*W)/(rho*S*Cl_max_landing));
VA = 1.3*Vs_ld;
t_delay = 2;

%Approach and Flare
R = VA^2/(g*(Cl_ratio-1));
if V_w == 0
    Sla = (hsc/tand(a_angle))+ R*sind(a_angle/2);
else
    Sla = (hsc/tand(a_angle))+ R*sind(a_angle/2);
    delta_t = (hsc/(VA*tand(a_angle)))+ (R/VA)*sind(a_angle/2);
    Sla=Sla-V_w*delta_t;
end

%Ground Roll Segement 1
if V_w == 0
    SlgI = VA*t_delay;
else
    VGA = VA - V_w;
    SlgI = VGA*t_delay;
end

%Ground Roll Segement 2
phi = 1-(1-1.32*(h/b))/(1.05+7.4*(h/b));
```

```
Cd_ge = CDo_l+phi*K*(Cl_grl)^2;
To = 0; % Due to No Thrust
N = 0; % No Engine Is Used
A = g*((N*To)/W-mu_l-sin(theta_R));
a = 0; % Due to No Thrust
B = (g/W)*(0.5*rho*S*(Cd_ge-mu_l*Cl_grl)+N*a);
if V_w == 0
    SlgII = (1/(2*B))*log((A-B*VA^2)/A);
else
    SlgII = (1/(2*B))*log((A-B*VGA^2)/A);
end

%Total Ground Roll
Slg = SlgI + SlgII;

%Total
Sld = Sla + Slg;
```

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A9: Climb Performance MATLAB Code

By: Rahul Rohatgi

Student Number: 500498833

Initial Parameters

```
e = 0.77;
h = 8;
b = 36;
S = 175.5;
W = 2450;
rpm = 2700;
CD_o = 0.032;
dprop = 6.25;
v_max = 212.66;
v_min = 74.07;
g = 32.2;
AR = b^2/S;
K = 1/(pi*AR*e);
n = rpm/60;
```

Max Rate of Climb Calculations

```
Cl_mrc = sqrt(((3*CD_o)/K)); %Calculating Max Lift Coefficient for RC
J = v_max/(n*dprop);
[rho,To] = density_Thrust(0);
T = To*(1-0.3*J);
RCmax = sqrt((2*W)/(rho*S))*((T/W)*(1/sqrt(Cl_mrc))-(CD_o/(Cl_mrc)^1.5)-K*sqrt(Cl_mrc));
```

Climb Performance Graph

```
% Climb Rate @Sea Level
i = 1;
v = ceil(v_min);
RC = 100;
while RC >= 0
    [rho,To,sigma] = density_Thrust2(0);
    T = To*(1-0.3*J);
    a = (To-T)/v^2;
    RC = -(((2*K*W)/(rho*S))*(1/v))+(((sigma*To)/W)*v)-(((sigma*a)/W)*v^2)-
(((CD_o*rho*S)/(2*W))*v^3);
    climb(1,i) = v;
    climb(2,i) = RC*60;
    v = v+1;
    i = i+1;
end

% Climb Rate @2500ft
i = 1;
v = ceil(v_min);
RC = 100;
while RC >= 0
    [rho,To,sigma] = density_Thrust2(2500);
    T = To*(1-0.3*J);
    a = (To-T)/v^2;
    RC = -(((2*K*W)/(rho*S))*(1/v))+(((sigma*To)/W)*v)-(((sigma*a)/W)*v^2)-
(((CD_o*rho*S)/(2*W))*v^3);
    climb2500(1,i) = v;
    climb2500(2,i) = RC*60;
    v = v+1;
    i = i+1;
end

% Climb Rate @500ft
i = 1;
v = ceil(v_min);
RC = 100;
while RC >= 0
    [rho,To,sigma] = density_Thrust2(5000);
    T = To*(1-0.3*J);
    a = (To-T)/v^2;
    RC = -(((2*K*W)/(rho*S))*(1/v))+(((sigma*To)/W)*v)-(((sigma*a)/W)*v^2)-
(((CD_o*rho*S)/(2*W))*v^3);
    climb5000(1,i) = v;
    climb5000(2,i) = RC*60;
    v = v+1;
    i = i+1;
end
```

Plotting Climb Rate Profile

```
figure(1)
plot(climb(1,:),climb(2,:),climb2500(1,:),climb2500(2,:),climb5000(1,:),climb5000(2,:))
xlabel('Air Speed (ft/s)')
ylabel('Rate of Climb (ft/min)')
title('Climb Rate Profile')
legend('Sea Level','2500ft Altitude','5000ft Altitude','Location','south')
```

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A10: Range Payload Diagram MATLAB Code

By: Rahul Rohatgi

Student Number: 500498833

Initial Parameters

```
v_cruise = 194.098;
rpm = 2700;
n = rpm/60;
dprop = 6.25;
J = v_cruise/(n*dprop);
n_pr = 0.83; %Propeller Efficiency
fuel_con = 57.1228;
bhp = 160;
CD_o = 0.032;
b = 36;
S = 175.5;
e = 0.77;
AR = b^2/S;
K = 1/(pi*AR*e);
[rho] = density(8000);
W_max = 2450;
W_payload = 607;
W_empty = 1679;
W_totalfuel = 347.704;
W_reservefuel = fuel_con/2;
W_tofuel = fuel_con/3;
W_landingfuel = 0.5*W_tofuel;
W_fuel = W_totalfuel-W_reservefuel-W_tofuel-W_landingfuel;
```

Calculating Range Points

```
BSFC = fuel_con/(0.8*bhp);

%Max Payload
W_ini = W_max-W_tofuel;
W_fo = W_max-W_empty-W_payload;
W_fin = W_ini-(W_fo-W_landingfuel-W_reservefuel);
W_avg = sqrt((W_ini*W_fin));
```

```
CL_avg = (2*W_avg)/(rho*(v_cruise^2)*S);
CD_avg = CD_o+K*(CL_avg)^2;
R1 = ((326*n_pr*(CL_avg/CD_avg))/(BSFC))*log((W_ini/W_fin));

%Max Fuel
W_ini = W_max-W_tofuel;
W_pload = W_max-W_empty-W_fuel;
W_fin = W_empty+W_pload+W_landingfuel+W_reservefuel;
W_avg = sqrt((W_ini*W_fin));
CL_avg = (2*W_avg)/(rho*(v_cruise^2)*S);
CD_avg = CD_o+K*(CL_avg)^2;
R2 = ((326*n_pr*(CL_avg/CD_avg))/(BSFC))*log((W_ini/W_fin));

%Max Fuel & Zero Payload
W_ini = W_empty+W_totalfuel-W_tofuel;
W_fin = W_empty+W_landingfuel+W_reservefuel;
W_avg = sqrt((W_ini*W_fin));
CL_avg = (2*W_avg)/(rho*(v_cruise^2)*S);
CD_avg = CD_o+K*(CL_avg)^2;
R3 = ((326*n_pr*(CL_avg/CD_avg))/(BSFC))*log((W_ini/W_fin));
```

Range - Payload Diagram

```
figure (1)
%Line 1
x1 = [0,R1];
y1 = [W_payload,W_payload];
%Line 2
x2 = [R1,R2];
y2 = [W_payload,W_pload];
%Line 3
x3 = [R2,R3];
y3 = [W_pload,0];
plot(x1,y1,'b',x2,y2,'b',x3,y3,'b')
xlabel('Range (nm)')
ylabel('Payload (lbf)')
title('Range Payload Diagram')
```

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