Fourier Neural Operator Network for the Signal Generation and Prediction

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***Abstract*—Various types of deep neural networks, like convolutional neural networks and recurrent neural networks, have been widely used for signal analysis, signal generation, and sequence signal prediction. However, traditional neural networks may face challenges in predicting unknown signals due to limited extrapolation capabilities. In this study, we employ a neural operator directly represented by an integral kernel in Fourier space to simulate sine wave signals with unknown amplitude, frequency and phase, and naming it SFNO. Experimental results show the SFNO method is effective to generate signals from the parameter space to the signal space, and its performance is better than that of the general DNN.**

***Index Terms*—Signal prediction; Deep Neural network; Fourier Neural Operator; Activation function; Parameters**

I. Introduction

Signal processing involves analyzing, manipulating, and transforming signals, commonly including extracting, enhancing, compressing, identifying, or reconstructing signal information. In the areas of communication, electronic devices, and signal processing, it is vital to generate a signal to transmitted information or test equipment.

Traditionally, we usually generate a signal through the specific mathematical expression for given parameters of signal[1]. This mathematical expression can vary depending on the type of signal being generated, such as a sine wave, square wave, or sawtooth wave. By manipulating the parameters of the expression, such as the amplitude, frequency, and phase, we can create a wide variety of signals with different characteristics. In addition to mathematical expressions, signals can also be generated using signal processing techniques, such as filtering, modulation, and sampling. These techniques can be used to manipulate existing signals or create new signals from scratch.

With the rapid development of deep learning, DNN-based method for generating various signal have draw more and more attention of many researchers[2]. Deep neural networks (DNNs) have shown promise in generating signals with complex patterns and characteristics, often surpassing traditional mathematical methods in certain applications.

One popular approach is using generative adversarial networks (GANs) for signal generation [3]. GANs consist of two neural networks, a generator and a discriminator, which are trained in a competitive manner. The generator learns to generate realistic signals, while the discriminator learns to distinguish between real and generated signals. Through this adversarial training process, GANs can produce high-quality signals that closely resemble real data. Another approach is using recurrent neural networks (RNNs) or convolutional neural networks (CNNs) for signal generation. These networks can learn the temporal or spatial dependencies in the data and generate signals with complex patterns and structures. Overall, deep learning methods offer a powerful and flexible framework for signal generation, allowing researchers to generate signals with a wide range of characteristics and applications. As the field of deep learning continues to advance, we can expect even more sophisticated and efficient methods for signal generation to emerge.

However, the classical and DNN-based methods will encounter some challenges for the signal generation with time-varying parameters. In this paper, we attempt solve this problem by means of Fourier neural operator network used to obtain the solution of partial differential equations. By incorporating Fourier transforms into neural networks, these networks can effectively capture the frequency domain information of signals, allowing for more robust and flexible signal generation. Fourier neural operator (FNO) networks combine the strengths of traditional signal processing techniques, such as Fourier transforms, with the power of deep learning methods[4]. This hybrid approach enables the network to learn complex relationships between signal parameters and their corresponding Fourier representations, leading to improved signal generation performance.

II. Preliminaries

This section provide the detailed description of the relevant problem and the formulae of FNO.

A. Problem setup

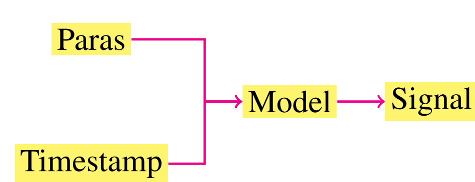


Fig. 1. A schematic diagram for signal generation

Fig. 1 deptics the process of generating signal for given parameters and timestamp. In this structure, the model stands

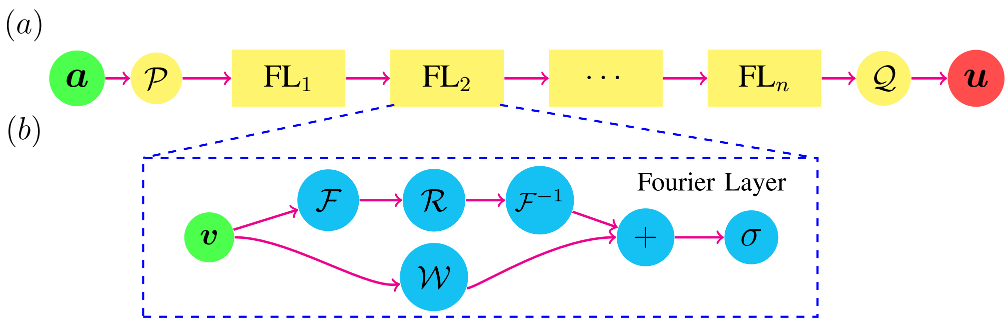


Fig. 2. The framework of Fourier Neural Operator. In the top branch, $\boldsymbol{a}$ and $\boldsymbol{u}$ are input and output, respectively. $\mathcal{P}$ and $\mathcal{Q}$ are two simply neural network used to lift and reduce feature dimension, respectively. FL stands for Fourier layer including Fourier convolution pipeline and linear pipeline(see branch (b)).

for the classical mathematical formulation that generating sine wave, square wave and sawtooth wave, etc.. On the other hand, it may be the DNN, CNN, RNN or GANs model in terms of deep learning. By providing prescribed parameters like amplitude, frequency, phase, the model will produce a sequential signal based on the timestamp.

B. Fourier Neural Operator

Generally, we need to model a nonlinear mapping $\mathcal{G} : \mathcal{H}_1 \to \mathcal{H}_2$ with an underlying bounded spacial domain. The neural operator, proposed by Li et al. in [4], parameterized the nonlinear mapping $\mathcal{G}$ into $\mathcal{G}_\theta : \mathcal{H}_1 \times \theta \to \mathcal{H}_2$, where θ represents the parameter set trained in the neural network.

Mathematically, the neural network operator with iterative layer architecture can be formulated as follows

$$\begin{cases} \boldsymbol{z}^0 = \mathcal{P}\boldsymbol{x} \\ \boldsymbol{z}^{\ell+1} = \mathcal{L}^\ell(\boldsymbol{z}^\ell) = \sigma\left(W^\ell \boldsymbol{z}^\ell + b^\ell + \mathcal{K}_\theta^\ell(\boldsymbol{z}^\ell)\right) \\ \qquad \text{for} \quad \ell = 1, 2, 3, \cdots, L-1 \\ \boldsymbol{s} = \mathcal{Q}\boldsymbol{z}^L \end{cases} \tag{1}$$

where the lifting and projection operators $\mathcal{P}$ and $\mathcal{Q}$ are two shallow neural network that encode the lower dimension function into higher dimensional space and vice versa. $\sigma : \mathbb{R} \to \mathbb{R}$ is a point-wise non-linear activation function, and $\mathcal{K}_\theta^\ell$ is a kernel integral operator. In form, FNO method utilize Fourier convolution operator to model $\mathcal{K}_\theta^\ell$, it is

$$\mathcal{K}_\theta(\boldsymbol{z}^\ell) = \mathcal{F}^{-1}\left(R^\ell \cdot \mathcal{F}(\boldsymbol{z}^\ell)\right)$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the Fourier transform and its inverse, respectively, which are computed using the FFT algorithm to each component of h separately. The Fourier-domain weight matrices $\{R^\ell | \ell = 1, 2, 3, \cdots, L-1\}$ take up most of the model size, requiring $O(LH^2M^D)$ parameters, where H is the hidden size, M is the number of top Fourier modes being kept, and D is the problem dimension. The FNO model is shown in Fig. 2.

III. FNO for Signal Generation and Prediction

In this section, we provide the implementation of signal generation and prediction by means of the above-mentioned FNO model.

A. Data Preparation

We generate some single-tone signals as our training set and test set by the following sine expression

$$s = A \cdot \sin(\omega t + \phi) \tag{2}$$

in which, the A, ω and ϕ represent the amplitude, frequency and phase of signal, respectively. N different amplitude $\{A_i\}_{i=1}^N$, frequencies $\{\omega_i\}_{i=1}^N$ and phases $\{\phi_i\}_{i=1}^N$ are sampled from $(0, A_s]$, $(0, \omega_s]$ and $(0, \phi_s]$, respectively. Then, N different signals can be obtain via equation (2) for the prescribe parameters A_i, ω_i and ϕ_i.

B. SFNO model and its optimization

In our propose model, the input parameters and timestamp of signal will be embedded into a high dimensional space to enrich the feature of input data. Generally, these parameters are time-varying and multi-scale for different signals, we then use an encoder represented by Multi Fourier Feature Network (MFF) to encode input data[5]. The function of output model is inverse, we also take a MFF to represent it. Moreover, the input data and label data need to be normalized for FNO model.

Algorithm 1 SFNO algorithm for signal processing.

1) Normalizing the input data $\mathcal{S}$ and label data $\mathcal{L}$;
2) Selecting two shallow MFF networks to model $\mathcal{P}$ and $\mathcal{Q}$, and determining the hyper-parameters of FNO;
3) Obtaining the output of FNO and computing the square l^2 loss function
$$Loss = \frac{\|\boldsymbol{s} - \boldsymbol{s}^*\|^2}{\|\boldsymbol{s}^*\|^2}$$
where $\boldsymbol{s}$ and $\boldsymbol{s}^*$ are the predicted signal and true signal, respectively. $\|\cdot\|$ stands for the 2-norm for given data.
4) Taking a suitable optimization method to update the internal parameters of DNN at the random point of $\tilde{\boldsymbol{x}}^k$, such as SGD method expressed as follows:
$$\boldsymbol{\theta}^{k+1} = \boldsymbol{\theta}^k - \alpha^k \nabla_{\boldsymbol{\theta}^k} Loss(\boldsymbol{x}^k; \boldsymbol{\theta}^k),$$
where the "learning rate" α^k decreases with k increasing.
5) Repeating steps 1-4 until the convergence criterion is satisfied or the objective function tends to be stable.

The goal is to find a set of parameters $\boldsymbol{\theta}$ such that the predicted signal $\boldsymbol{s}$ close the true signal. To obtain the $\boldsymbol{\theta}^*$, one can update the parameters $\boldsymbol{\theta}$ using stochastic gradient descent method for a few training samples at each iteration.

IV. Experiments

The goal of our experiments is to test the performance of our SFNO for generating and predicting signal. In addition, a DNN model is introduced to as the baseline.

A. Model setups

SFNO consists of 4 Fourier layer with convolution dimension 16 and width 64, the activation function is set as $gelu$. Its encoder and decoder are set as MFF network with three sub-pipelines, their network size is $(40, 30, 10)$ and their scale factor of pipelines are $1, 5$ and 10. Their activation functions for hidden layers are $sine$ and the output is linear. For DNN, its network size is $(400, 300, 200, 200, 100)$ and its activation function of hidden layers are $sine$, the output of DNN is linear.

B. Training setups

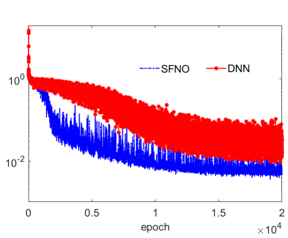
We train all neural networks by an Adam optimizer with an initial learning rate of 0.01, and the learning rate will be decayed by 3% for every 100 training epochs [6]. The batch size in training cycle for two models is 16 and the total epochs is 20000. A criteria is provided to evaluate model.

$$REL = \frac{\|\boldsymbol{s} - \boldsymbol{s}^*\|^2}{\|\boldsymbol{s}^*\|^2}$$

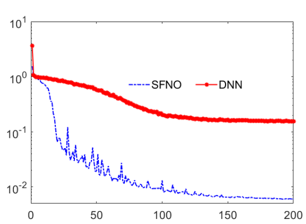
where $\boldsymbol{s}$ and $\boldsymbol{s}^*$ are the predicted signal and exact signal, respectively.

C. Experimental results

Firstly, we generate 5000 signals with different amplitudes, frequencies and phases in time interval $(0, 10s]$, these parameters are equidistantly generated from $(0, 50]$, $(0, 30]$ and $(0, 2\pi]$, respectively. Secondly, the former 4000 signals are used as training dataset and the remainder 1000 signals are used for testing model.



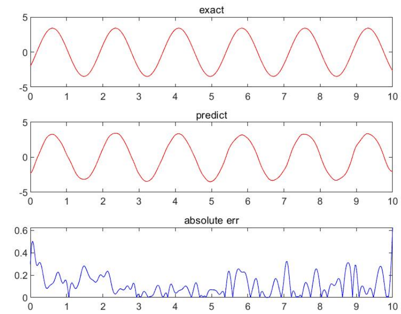
(a) losses



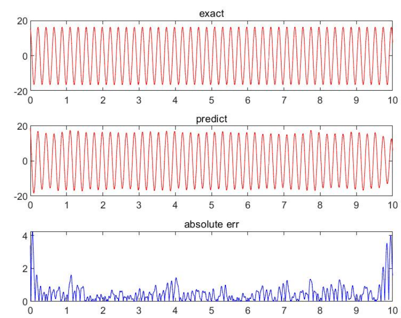
(b) REL for testing

Fig. 3. Losses VS training epoch and RELs VS testing epoch.

Based on the results, our SFNO model can well capture the feature of varying signal for different unknown signal parameters, and its performance is clearly superior to that of DNN. Additionally, our SFNO have favourable ability for generating multi signals with obvious different parameters.



(a) low frequency and small amplitude



(b) high frequency and big amplitude

Fig. 4. The true signals, predicted signal by SFNO and the absolute error of signals.

V. Conclusion

This paper develop a framework based on Fourier Neural Operator for signal generation and prediction. Computational results showed that the proposed method is feasible and efficient to predict signal for varying signal parameters.

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