Design and Analysis of Algorithms I

Graph Primitives

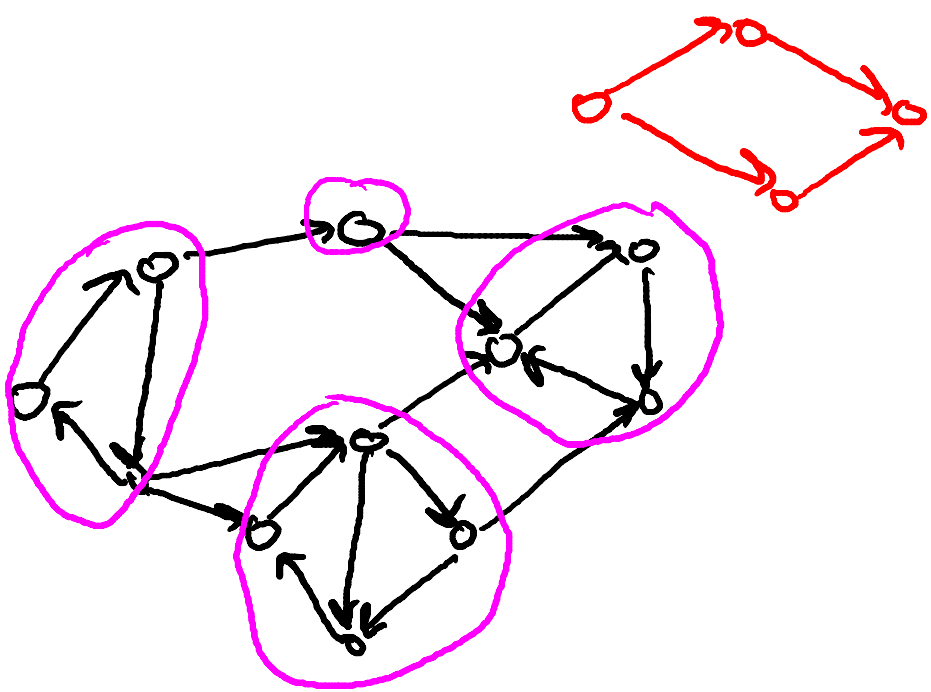
An O(m+n) Algorithm for Computing Strong Components

Strongly Connected Components

Formal Definition : the strongly connected components (SCCs) of a directed graph G are the equivalence classes of the relation

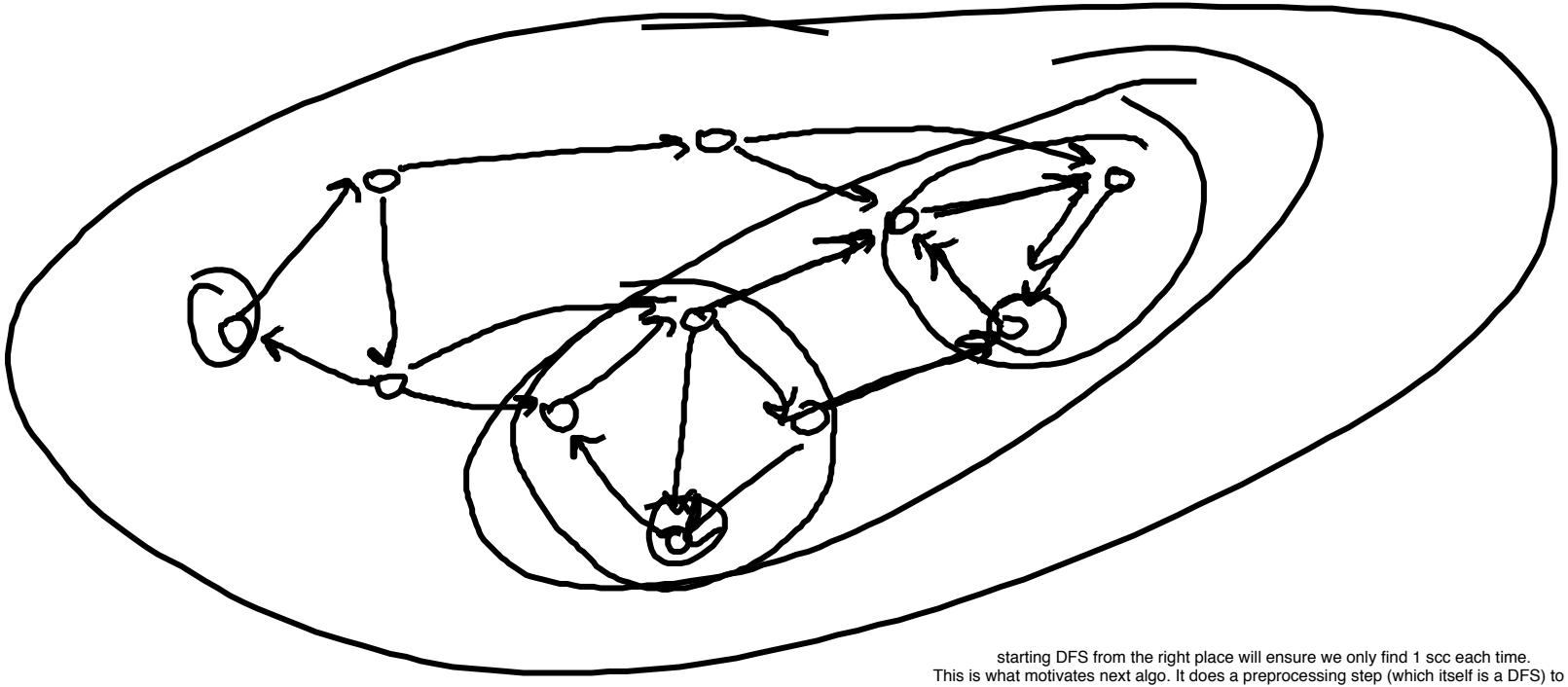
u<->v <==> there exists a path u->v and a path v->u in G

You check : <-> is an equivalence relation



calling DFS/BFS on any of the pink circles will find 1 or more scc circles,
depending on the nodes and kind of graph. It will find a union of 1 or more scc. In worst case it will explore the entire graph.
So single pass of DFS may not may not work

Why Depth-First Search?



starting DFS from the right place will ensure we only find 1 scc each time.
This is what motivates next algo. It does a preprocessing step (which itself is a DFS) to find the places from where we should start DFS.

Kosaraju's Two-Pass Algorithm

Theorem : can compute SCCs in O(m+n) time.

Algorithm : (given directed graph G)

1. Let Grev = G with all arcs reversed
2. Run DFS-Loop on Grev

   Let f(v) = "finishing time" of each v in V

1. Run DFS-Loop on G

   processing nodes in decreasing order of finishing times

[SCCs = nodes with the same "leader"]

Goal : compute "magical ordering" of nodes

Goal : discover the SCCs one-by-one

DFS-Loop

```
DFS-Loop (graph G)
Global variable t = 0
[# of nodes processed so far]
Global variable s = NULL
[current source vertex]
Assume nodes labeled 1 to n
For i = n down to 1
      if i not yet explored
          s := i
          DFS(G,i)
```

For finishing times in 1st pass on the reverse G

For leaders in 2nd pass on the forward G

```
DFS (graph G, node i)
-- mark i as explored
-- set leader(i) := node s
-- for each arc (i,j) in G :
          -- if j not yet explored
                -- DFS(G,j)
-- t++
-- set f(i) := t
```

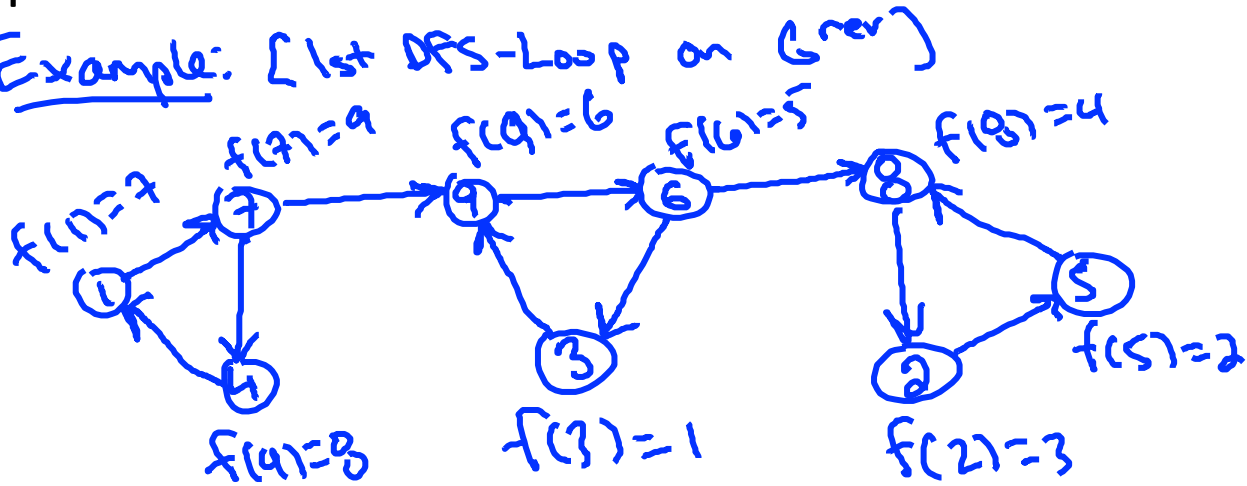
For rest of DFS-Loop

i's finishing time

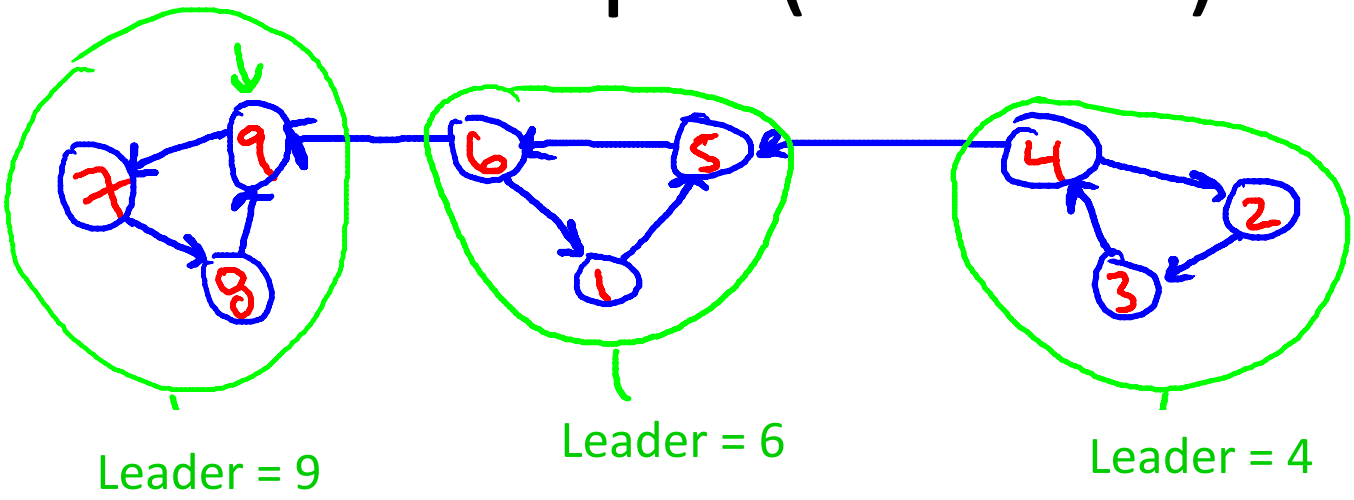
Tim Roughgarden

Only one of the following is a possible set of finishing times for the nodes 1,2,3,…,9, respectively, when the DFS-Loop subroutine is executed on the graph below. Which is it?

- 9,8,7,6,5,4,3,2,1
- 1,7,4,9,6,3,8,2,5
- 1,7,9,6,8,2,5,3,4
- 7,3,1,8,2,5,9,4,6



Example (2nd Pass)



Running Time : 2*DFS = O(m+n)