Design and Analysis of Algorithms I

Data Structures

Hash Tables: Some Implementation Details

Hash Table: Supported Operations

Purpose : maintain a (possibly evolving) set of stuff.
(transactions, people + associated data, IP addresses, etc.)

Insert : add new record

Using a "key"

Delete : delete existing record

AMAZING GUARANTEE

Lookup : check for a particular record
(a "dictionary")

All operations in O(1) time ! *

* 1. properly implemented 2. non-pathological data

High-Level Idea

Setup : universe U [e.g., all IP addresses, all names, all chessboard configurations, etc.]
[generally, REALLY BIG]

Goal : want to maintain evolving set $S \subseteq U$
[generally, of reasonable size]

Solution : 1.) pick n = # of "buckets" with

n ~ size of S.
Assume n=2*S

(for simplicity assume |S| doesn't vary much)

2.) choose a hash function $h : U \rightarrow \{0, 1, 2, ..., n-1\}$

3.) use array A of length n, store x in A[h(x)]

Hash function tells
us where to store the given key
x belonging to universe U

can resize size of hash table if ISI is dynamic in pratice

Naïve Solutions

1. Array-based solution
   [indexed by u]
   - O(1) operations but $\theta(|U|)$ space
2. List –based solution

   something like linked list

   - $\theta(|S|)$ space but $\theta(|S|)$ Lookup

Tim Roughgarden

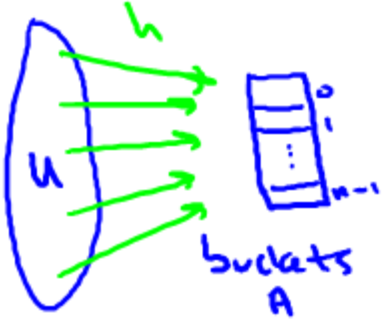
Consider n people with random birthdays (i.e., with each day of the year equally likely). How large does n need to be before there is at least a 50% chance that two people have the same birthday?

- 23 — 50 %
- 57 — 99 %
- 184 — 99.99....%
- 367 — 100%

BIRTHDAY "PARADOX"

Implication:
If you had 10000 indexes in your array,
just 100 keys are enough to have probability of collision=50%

Resolving Collisions



Collision: distinct $x, y \in U$ such that $h(x) = h(y)$

Solution # 1 : (separate) chaining

-keep linked list in each bucket

- given a key/object x, perform Insert/Delete/Lookup in the list in A[h(x)]

Linked list for x

Bucket for x

Solution #2 : open addressing. (only one object per bucket)

-Hash function now specifies probe sequence $h_1(x), h_2(x), ..$

(keep trying till find open slot)

Use 2 hash functions

- Examples : linear probing (look consecutively), double hashing

Tim Roughgarden

What Makes a Good Hash Function?

Note : in hash table with chaining, Insert is $\theta(1)$

$\theta(list\ length)$ for Insert/Delete.

Insert new object x at front of list in A[h(x)]

Equal-length lists

could be anywhere from $\boxed{m/n}$ to m for $\boxed{m}$ objects

All objects in same bucket

Point : performance depends on the choice of hash function!

(analogous situation with open addressing)

Properties of a "Good" Hash function

1. Should lead to good performance => i.e., should "spread data out" (gold standard – completely random hashing)
2. Should be easy to store/ very fast to evaluate.

Completely random hashing is as bad as a vanilla linked list to store/evaluate.
We would have to remember entire hash map, and loop up/evaluating hash function would take O(n) time

Bad Hash Functions

Example : keys = phone numbers (10-digits). $|u| = 10^{10}$

-Terrible hash function : $h(x) = 1^{st}$ 3 digits of x choose $n = 10^3$

(i.e., area code)

- mediocre hash function : $h(x) =$ last 3 digits of x

[still vulnerable to patterns in last 3 digits]

Example : keys = memory locations. (will be multiples of a power of 2)

-Bad hash function : $h(x) = x \bmod 1000$ (again $n = 10^3$)

=> All odd buckets guaranteed to be empty.

Quick-and-Dirty Hash Functions

"hash code"

e.g., subroutine to convert strings to integers

"comparison function "

like the mod n function

How to choose n = # of buckets

1. Choose n to be a prime (within constant factor of # of objects in table)
2. Not too close to a power of 2
3. Not too close to a power of 10