

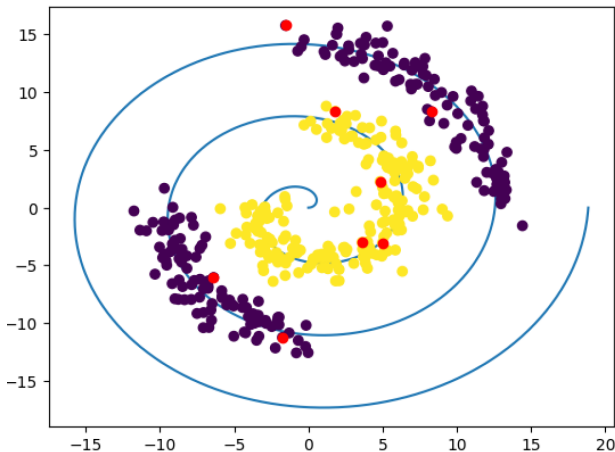
Graphical Semisupervised Learning

We should probably write something here

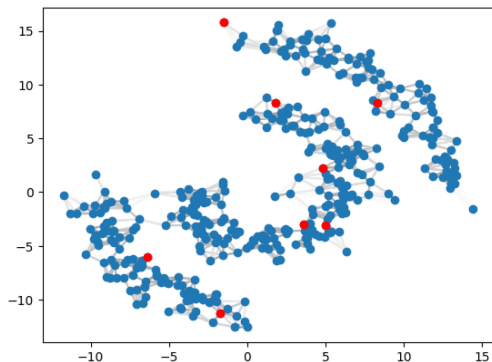
Rohin Gilman Alex Johnson Kaitlynn Lilly

May 22, 2023

- Goal in SSL is, given $\{x_i, y_i\}_{i=1}^M$ and $\{x_i\}_{i=M+1}^N$, to predict $y_i(x_i)$ for $i = M + 1, \dots, N$ where $M \ll N$.



- We leverage spectral geometric properties of the graph Laplacian matrix L .
- First, we construct our graphs by KNN or Proximity graph construction.



- General optimization

$$\vec{f}^* = \arg \min_{\vec{f} \in \mathbb{R}^m} \mathcal{L}(\vec{f}; y) + \lambda \vec{f}^T C^{-1} \vec{f}$$

where \mathcal{L} is a loss function, either Probit or Regression loss in our case.

- Probit loss

$$\mathcal{L}(\vec{f}, y) = - \sum_{j=1}^M \log \Psi(\vec{f}_j y_j)$$

- Regression loss

$$\mathcal{L}(\vec{f}, y) = \sum_{j=1}^M \left(\vec{f}_j - y_j \right)^2$$

- General optimization

$$\vec{f}^* = \arg \min_{\vec{f} \in \mathbb{R}^m} \mathcal{L}(\vec{f}; y) + \lambda \vec{f}^T C^{-1} \vec{f}$$

where $C = (L + \tau^2 I)^{-\alpha}$, where L is the graph Laplacian.

- In class, we showed C is a kernel matrix. In fact, C belongs to a class of kernels called the Matérn family that have the form

$$K(x, y) = \kappa(\|x - y\|), \kappa(t) = \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\sqrt{2\nu} \frac{t}{\gamma} \right) K_\nu \left(\sqrt{2\nu} \frac{t}{\gamma} \right)$$

where Γ is the Gamma function and K_ν is the modified Bessel function of the second kind.



Results

Three Cluster Case

This is one of Rohin's Slides

This is one of Rohin's Slides



Conclusions and Further Questions

This is one of Rohin's Slides



Viacheslav Borovitskiy, Iskander Azangulov, Alexander Terenin, Peter Mostowsky, Marc Peter Deisenroth, and Nicolas Durrande.

Matérn Gaussian processes on graphs, April 2021.



Franca Hoffmann, Bamdad Hosseini, Zhi Ren, and Andrew M. Stuart.

Consistency of semi-supervised learning algorithms on graphs: Probit and one-hot methods, March 2020.



Daniel Sanz-Alonso and Ruiyi Yang.

The SPDE approach to Matérn fields: Graph representations, April 2021.