Graphical Semisupervised Learning

We should probably write something here

Rohin Gilman Alex Johnson Kaitlynn Lilly

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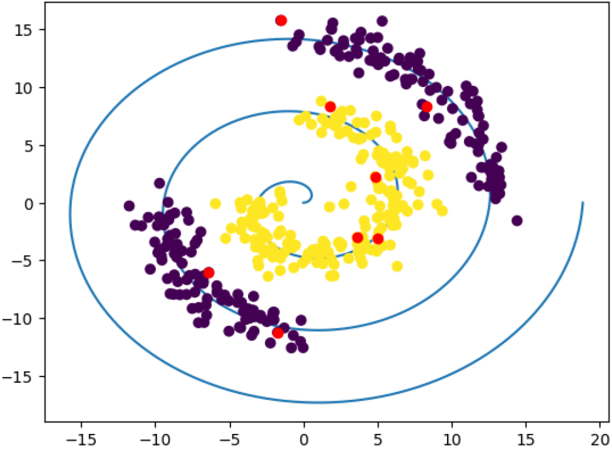
UNIVERSITY of WASHINGTON | Applied Mathematics

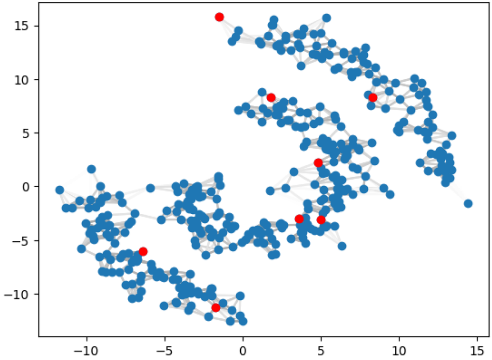
Introduction to Problem

- Goal in SSL is, given $\{x_i, y_i\}_{i=1}^{M}$ and $\{x_i\}_{i=M+1}^{N}$, to predict $y_i(x_i)$ for $i = M+1, \ldots, N$ where $M \ll N$.



GSSL

- We leverage spectral geometric properties of the graph Laplacian matrix L.
- First, we construct our graphs by KNN or Proximity graph construction.



Optimization Problem
Loss functions

- General optimization

$$\vec{f}^* = \underset{\vec{f}\in\mathbb{R}^m}{\arg\min}\, \mathcal{L}(\vec{f}; y) + \lambda \vec{f}^T C^{-1} \vec{f}$$

  where $\mathcal{L}$ is a loss function, either Probit or Regression loss in our case.

- Probit loss

$$\mathcal{L}(\vec{f}, y) = -\sum_{j=1}^{M} \log \Psi(\vec{f_j} y_j)$$

- Regression loss

$$\mathcal{L}(\vec{f}, y) = \sum_{j=1}^{M} \left(\vec{f_j} - y_j\right)^2$$

Optimization Problem
Matérn kernel-regularization

- General optimization

$$\vec{f}^* = \arg\min_{\vec{f}\in\mathbb{R}^m} \mathcal{L}(\vec{f}; y) + \lambda \vec{f}^T C^{-1} \vec{f}$$

where $C = (L + \tau^2 I)^{-\alpha}$, where L is the graph Laplacian.

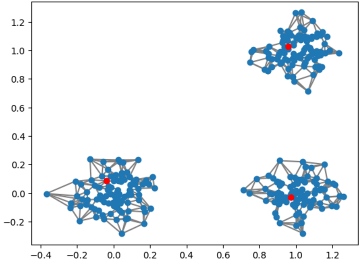
- In class, we showed C is a kernel matrix. In fact, C belongs to a class of kernels called the Matérn family that have the form

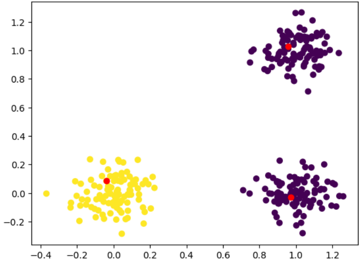
$$K(x, y) = \kappa(\|x - y\|), \kappa(t) = \frac{2^{1-\nu}}{\Gamma(\nu)}\left(\sqrt{2\nu}\frac{t}{\gamma}\right) K_\nu\left(\sqrt{2\nu}\frac{t}{\gamma}\right)$$

where Γ is the Gamma function and K_ν is the modified Bessel function of the second kind.

Results
Three Cluster Case

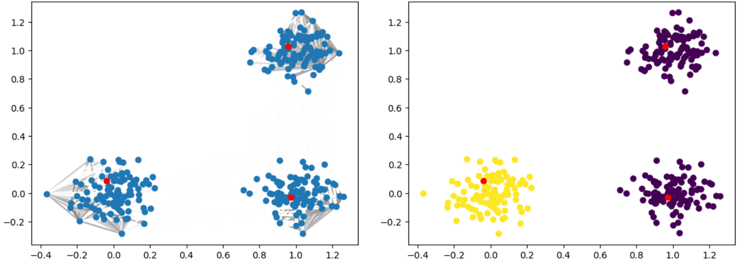
Using the K-Nearest Neighbors Graph with Uniform Weights





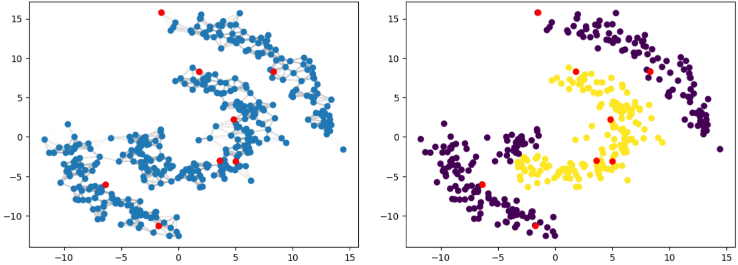
Loss Function	Classification Accuracy (over 50 trials)
Probit	100%
Regression	100%

Using the Proximity Graph with RBF Weights



Loss Function	Classification Accuracy (over 50 trials)
Probit	100.0%
Regression	100.0%

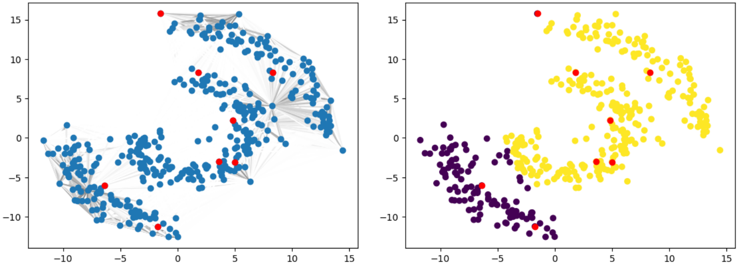
Using the K-Nearest Neighbors Graph with RBF Weights



Loss Function	Classification Accuracy (over 50 trials)
Probit	96.3%
Regression	95.8%

Results
Spiral Case

Using the Proximity Graph with RBF Weights



Loss Function	Classification Accuracy (on 1 trial)
Probit	73.0%
Regression	63.0%

Conclusions and Further Questions

- Conclusions
  - For well separated clusters, the method is effective, even without careful tuning
  - Not the case for more difficult scenarios
- Challenges Encountered
  - Tuning of paramters - especially for real data
  - Majority labelling
- Next Steps
  - Plan to apply this approach to MNIST to test on real data
  - Anticipate tuning challenges for more difficult pairs of numbers

Bibliography

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