

1. INTRODUCTION

Consider a basketball player coming the court. They arrive at the three point line and have a decision to make: should they cut to the basket or run to the left corner. Depending on the positions of other players, the state in the game, or whether they are playing at home or away, the player might make a different decision. Scenarios like these produce data generated by complex, nonlinear dynamical systems. Instead of trying to produce the exact equations that lead to dynamics like those described above, we can instead understand these systems by decomposing them into multiple, simpler dynamical systems.

1.1. Switching Linear Dynamical Systems. A switching linear dynamical system (SLDS) has the following components for each time step $t = 1, 2, \dots, T$: a discrete latent state z_t , a continuous latent state x_t , and an observation y_t . The underlying state z_t is in the set $\{1, 2, \dots, K\}$ and is driven by a Markov process:

$$\mathbb{P}(z_{t+1} = j \mid z_t = i) = \pi_{ij},$$

where $[\pi_{ij}]_{K \times K}$ is the transition matrix of the process. The underlying continuous state x_t is in \mathbb{R}^M and is driven by conditionally linear dynamics, dependent on the discrete latent state z_{t+1} :

$$x_{t+1} = A_{z_{t+1}}x_t + b_{z_{t+1}} + v_t,$$

where $A_k \in \mathbb{R}^{M \times M}$, $b_k \in \mathbb{R}^M$, and $v_t \stackrel{\text{iid}}{\sim} \mathcal{N}(0, Q_{z_{t+1}})$, where $Q_k \in \mathbb{R}^{M \times M}$ is the covariance matrix of the normal distribution. Finally, the observation y_t is generated from the corresponding discrete continuous and discrete latent states:

$$y_t = C_{z_t}x_t + d_{z_t} + w_t,$$

where $C_k \in \mathbb{R}^{N \times M}$, $d_k \in \mathbb{R}^N$, and $w_t \stackrel{\text{iid}}{\sim} \mathcal{N}(0, S_{z_t})$, where $S_k \in \mathbb{R}^{N \times N}$ is the covariance matrix of the normal distribution.

1.2. Fitting SLDSs. Talk about stick-breaking ... and Polya-gamma ...

1.3. Recurrent Switching Linear Dynamical Systems (rSLDSs). If a switch in the discrete state of a system should occur when the trajectory enters (or leaves) a particular region, the model will be unable to capture this behavior since the discrete state is a function only of the previous discrete state. To rectify this, we instead consider a recurrent switching linear dynamical system (rSLDS).

1.4. Conclusions from the Paper. Discuss the mathematical or scientific question that is addressed in the paper. Cite at least two other articles