

**Bayesian Learning and Inference
in Recurrent Switching Linear Dynamical Systems**

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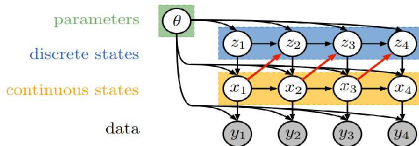
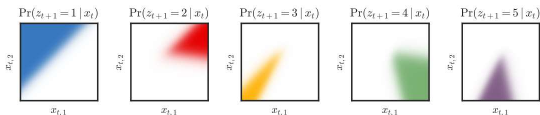
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SLDS and rSLDS

- Observation $y_t = Cx_t + d + w_t$, $w_t \stackrel{\text{iid}}{\sim} \mathcal{N}(0, S)$
- Continuous latent state $x_{t+1} = A_{z_{t+1}}x_t + b_{z_{t+1}} + \nu_t$, $\nu_t \stackrel{\text{iid}}{\sim} \mathcal{N}(0, Q_{z_{t+1}})$
- Discrete latent state $z_t \in \{1, \dots, K\}$
 - SLDS $z_{t+1}|z_t \sim \pi_{z_t}$
 - rSLDS $z_{t+1}|z_t, x_t \sim \pi_{SB}(\nu_{t+1})$, $\nu_{t+1} = R_{z_t}x_t + r_{z_t}$



Stick Breaking Logitstic Regression

- $p(z|x) \sim \pi_{SB}(\nu)$, $\nu = Rx + r$
- Link function: $\pi_{SB}(\nu) = \left(\pi_{SB}^{(1)}(\nu), \dots, \pi_{SB}^{(K)}(\nu) \right)$ (with $\sigma(x) = \frac{e^x}{1+e^x}$)

$$\pi_{SB}^{(k)}(\nu) = \begin{cases} \sigma(\nu_k) \prod_{j < k} \sigma(-\nu_j), & \text{if } k = 1, \dots, K-1 \\ \prod_{j=1}^K \sigma(-\nu_j), & \text{if } k = K \end{cases}$$
- $p(z|x) \sim \prod_{k=1}^K \sigma(\nu_k)^{\mathbb{1}[z=k]} \sigma(-\nu_k)^{\mathbb{1}[z > k]}$ (Likelihood)

- With a Gaussian Prior $p(x)$, the posterior $p(x|z) \propto p(x) * p(z|x)$ is non-Gaussian
- Bayesian updating is not efficient
 - Gibbs Sampler: sampling $x^{(i+1)} \sim p(x|z = z^{(i)})$, sampling $z^{(i+1)} \sim p(z|x = x^{(i+1)})$
 - Message Passing $m_{t \rightarrow (t+1)}(x_{t+1}) = \int \psi(x_t, y_t) \psi(x_t, z_{t+1}) \psi(x_t, x_{t+1}, z_{t+1}) m_{(t-1) \rightarrow t}(x_t) dx_t$

Polya-gamma augmentation

$$\frac{(e^\nu)^a}{(1 + e^\nu)^b} = 2^{-b} \int_0^\infty e^{\kappa\nu} e^{-\omega\nu^2/2} p_{\text{PG}}(\omega \mid b, 0) d\omega \quad (\kappa = a - \frac{b}{2}) \quad (2.1)$$

$$p(x_t | z_{t+1}) \propto \prod_{k=1}^{K-1} \frac{(e^{\nu_{t+1,k}})^{\mathbb{I}[z_{t+1}=k]}}{(1 + e^{\nu_{t+1,k}})^{\mathbb{I}[z_{t+1} \geq k]}} \quad (2.2)$$

$$p(x_t | z_{t+1}) = \int p(x_t, \omega_t | z_{t+1}) d\omega_t = \int \mathbf{p}(x_t | z_{t+1}, \omega_t) p(\omega_t) d\omega_t \quad (2.3)$$

$$\omega_{t,k} \mid x_t, z_{t+1} \sim \text{PG}(\mathbb{I}[z_{t+1} \geq k], \nu_{t+1,k}) \quad (2.4)$$

- $p(x_t | z_{t+1}, \omega_t)$ is Gaussian
- Message Passing $\psi(x_t, z_{t+1}, \omega_t) \propto \mathcal{N}(\nu_{t+1} | \Omega_t^{-1} \kappa_{t+1}, \Omega_t^{-1})$
- Thus instantiating these auxiliary variables in a Gibbs sampler enables efficient block updates



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- An item
- Another item
 - A subitem
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1. An item
2. Another item
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Example

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