QUANTUM ENTANGLEMENT

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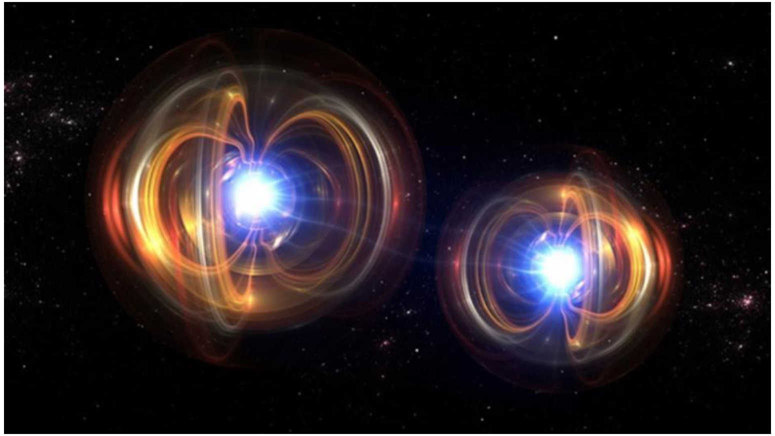
Qiskit

Definition of Entanglement

A property in which a pair of particles is generated in such a way that the individual quantum states of each are indefinite until measured and the act of measuring one determines the result of measuring the other, even when at a distance from each other.



Let us understand Entanglement

- When two objects are entangled they share a defined relationship!
- That is, when you know one object's state, you know the other as well.
- Entanglement occurs between two or more qubits.

Basic:

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Right Indexing

$$|q_1 q_0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

- The first number in each ket corresponds to q_1
- The second number in each ket corresponds to q_0.

Let us dive in with examples!

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State 1:

$$|q_1 q_0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

Now make a measurement on q_0 and we will get |0>

Q:But what about q_1, does the above measurement tell us anything about q_1?

NO! q_1 is still in a superposition, it is not affected by the measurement on q_0.

State 2:

$$|q_1 q_0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Now make a measurement on q_0 and we will get |0>

Q:But what about q_1, does the above measurement tell us anything about q_1?

YES! If q_0 is measured to be zero, then q_1 has to be zero.

Entangled vs Non Entangled

Entangled States

Cannot be factorized i.e. cannot be written in the form of two separate qubits.

Measuring on one qubit gives you information on the other.

Non Entangled States

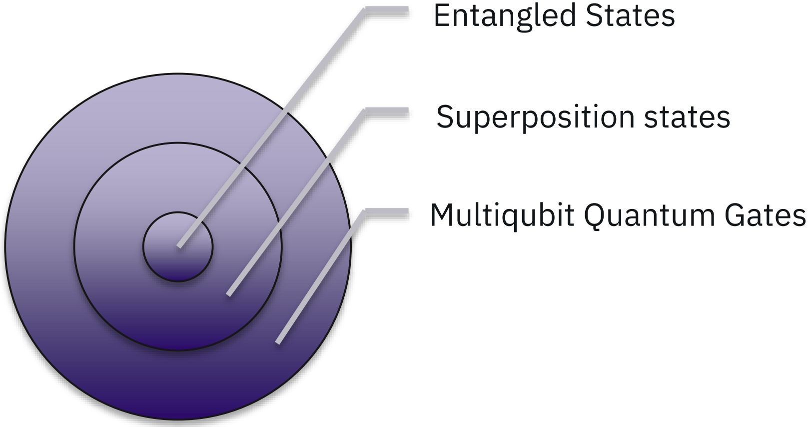
Can be factorized i.e. can be written in the form of two separate qubits.

Measurement on two qubits are independent.

Key Concept Alert!

All entangled states are multi-qubit superposition states.

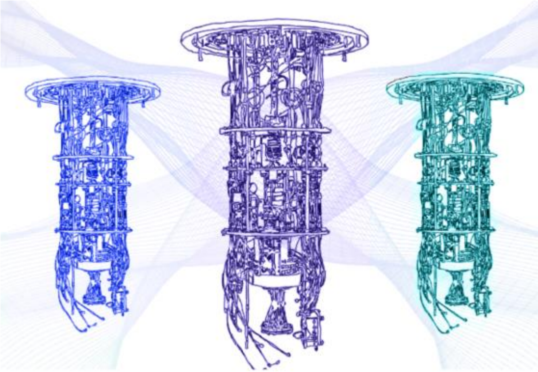
But all multi-qubit superposition states are not entangled: only non-factorizable superposition states are entangled.



Exponential scaling of information:

The exponential scaling of bits required to represent qubits only happens because of entanglement.

Entanglement makes *quantum advantage* possible.



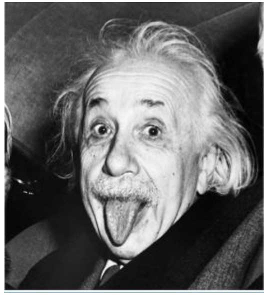
Key concept Alert:

With entanglement, we need **2^n classical bits** to represent n qubits.

Without entanglement, we need **2*n classical bits** to represent n qubits.

Spooky action at a distance!

One of the foundational insights of Einstein's Theory of Relativity is that the **speed of light is a fundamental speed limit in the universe.**



$$E = mc^2$$

No information can be transmitted faster than this speed.

But what does entanglement say? Two objects could be correlated so that even if they were separated by **millions of miles**, if you changed one of them, the other one was affected too!

To Einstein, This instantaneous change was a **violation of Relativity.**
Thus he concluded that quantum mechanics must be incomplete.

What next? We have John Bell to the rescue!

Qiskit

Einstein’s argument was theoretical!

John Bell moved from **theory to experiment.**

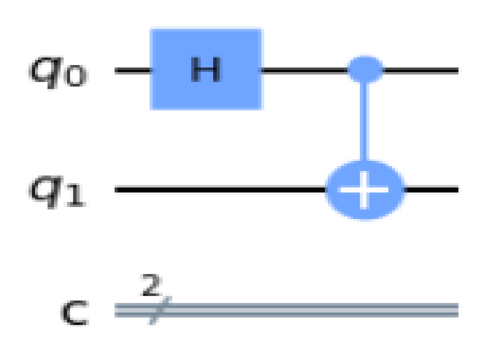
In 1960’s he showed that we could do an experiment to determine if QM was right or did it have any hidden information that QM missed.

So what did the experiments say?

The experiment is called the **Bell Test.**

We’ve been doing the experiment proposed by Bell with increasing accuracy for over 40 years!
Every experiment shows that **Quantum Mechanics was right**, and Einstein was wrong.

A must to create an entangled state



Truth table of the Bell States

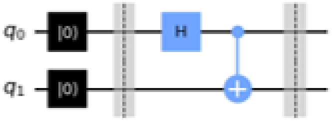
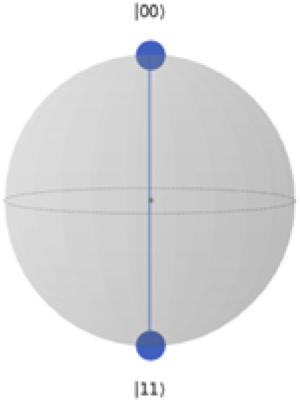
Input	Output
00	$\frac{1}{\sqrt{2}}(\lvert 00\rangle + \lvert 11\rangle)$
01	$\frac{1}{\sqrt{2}}(\lvert 01\rangle + \lvert 10\rangle)$
10	$\frac{1}{\sqrt{2}}(\lvert 00\rangle - \lvert 11\rangle)$
11	$\frac{1}{\sqrt{2}}(\lvert 01\rangle - \lvert 10\rangle)$

Visualizations

Qiskit

- ✓ Circuit Diagram
- ✓ Histogram Representation
- ✓ State Vector Representation
- ✓ Qsphere Representation
- ✓ Unitary Representation
- ✓ Bloch Sphere Representation
- ✓ Plot state city
- ✓ Plot state hinton
- ✓ Plot state paulivec

Visualizations – Bell State 1

<table>
<tr><th>State</th><th>Circuit Diagram</th><th>State vector</th><th>Q-sphere</th><th>Histogram</th></tr>
<tr>
<td>|00></td>
<td>
</td>
<td>[0.707+0j, 0+0j, 0+0j, 0.707+0j]</td>
<td>
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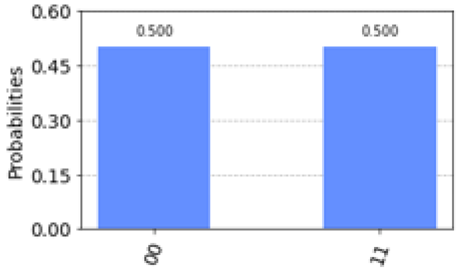
<td>
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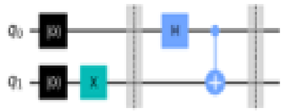
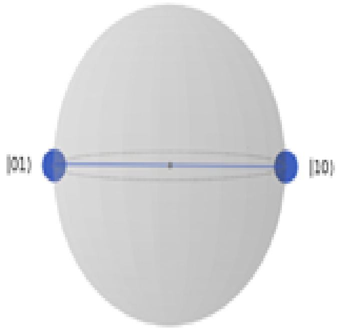
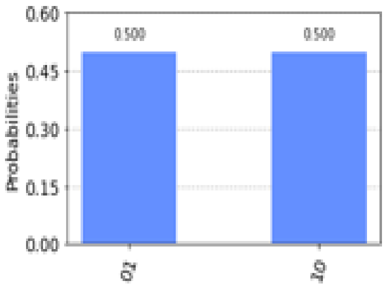
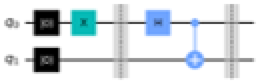
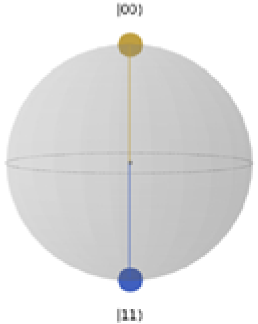
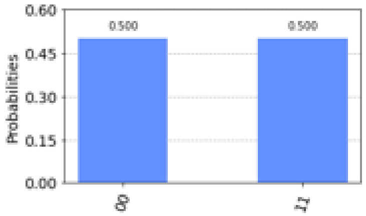
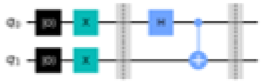
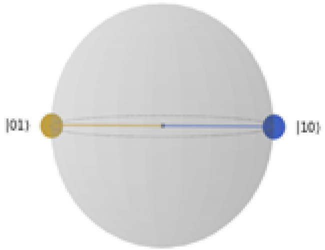
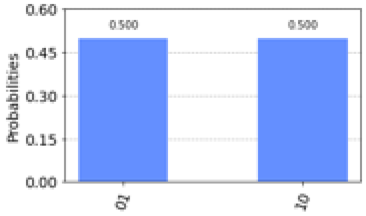
Visualizations – Bell State 2

|01>		[0+0j, 0.707+0j, 0.707+0j, 0+0j]		

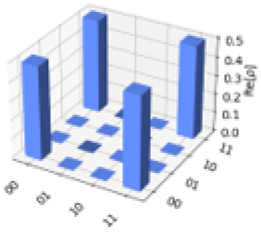
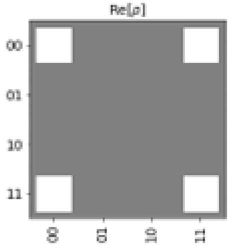
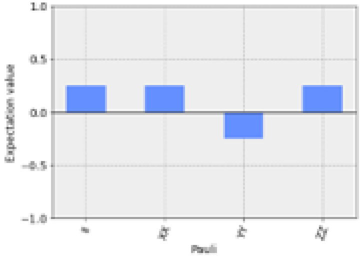
Visualizations – Bell State 3

|10>		[0.707+0j, 0+0j, 0+0j, -0.707+0j]		

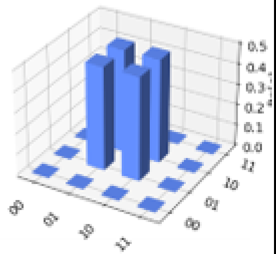
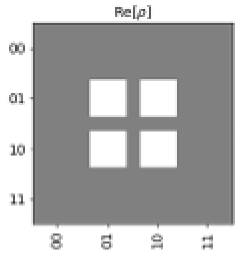
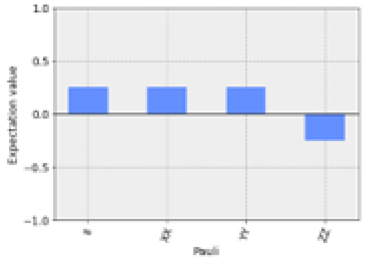
Visualizations – Bell State 4

|11>		[0+0j, -0.707+0j, 0.707+0j, 0+0j]		

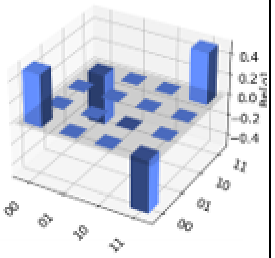
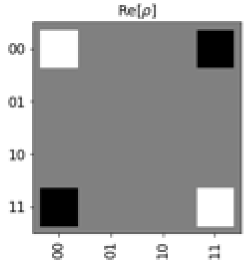
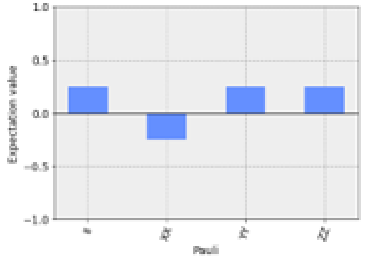
Visualizations – Bell State 1

State	Output	plot_state_city	plot_state_hinton	plot_state_paulivec
|00>	$1/\sqrt{2}$ (|00> + |11>)			

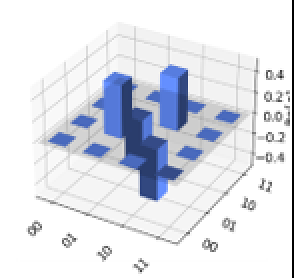
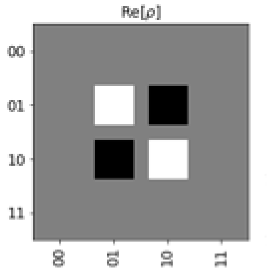
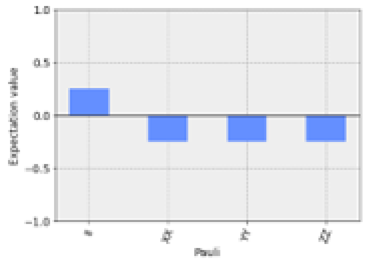
Visualizations – Bell State 2

$\|01\rangle$	$1/\sqrt{2}\,(\|01\rangle + \|10\rangle)$			

Visualizations – Bell State 3

$\|10\rangle$	$1/\sqrt{2}\,(\|00\rangle - \|11\rangle)$			

Visualizations – Bell State 4

$\|11\rangle$	$1/\sqrt{2}(\|01\rangle - \|10\rangle)$			

Application: Super dense Coding

Super dense coding is about efficient information transfer

Why is this important?

In the year 2020, the average rate at which data was generated in the world was-600 Gbps

And the global average internet speed was approximately 30 Mbps, which is almost 20,000 times slower.

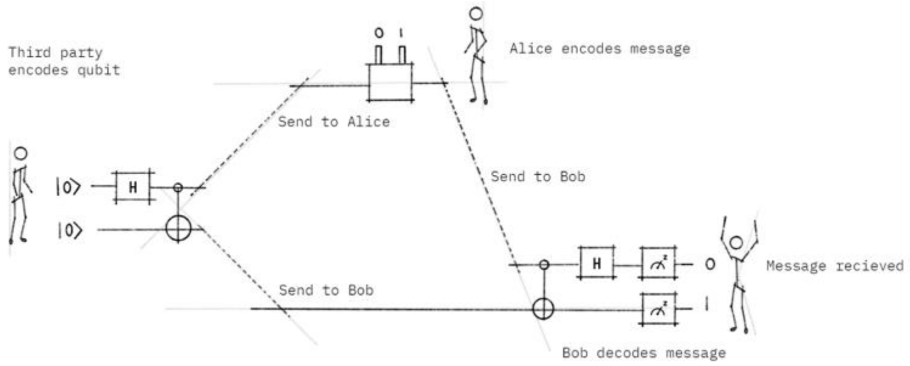
And in the upcoming years we are going to generate more and more data and it becomes increasingly difficult for information transfer speeds to keep up!

Entanglement at action!

In super dense coding, we can use entanglement to communicate the same information by sending half as many qubits as classical bits.

That is, by sending 1 qubit , we will transfer 2 bits of information.

Using the same technique, we could have halve the required speed for information transfer!



In a nutshell…

- Objects which are entangled share a defined relationship
- The measurement on one qubit affects the other qubit
- Entangled states cannot be factorized
- Exponential Scaling of information
- Bell States
- Visualizations
- Superdense Coding

References

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For any queries…

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Thank You