A New Class of Cosmologically viable $f(R)$ models

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Background

1. Alternative gravity models - to resolve problems of Cosmology?
2. ΛCDM considered only 'viable' Cosmology despite unknown "Dark" entities
3. $f(R)$ theories are generally studied to be fit as the possible candidates for either dark energy, dark matter or both - but their efficacy is judged based on fit with ΛCDM only

Motivation

- No one definitive $f(R)$ model that possibly satisfies all the required criteria to be an alternative to ΛCDM model.
- Their viability is always judged based on it's ability to reproduce scale factor evolution as predicted by ΛCDM model.
- idea! - "Designer" approach to $f(R)$ with alternative models of scale factor evolution!
- To explore the possible new viable models assuming the universe is evolving with linear scale factor (at least during matter domination).

Linearly Coasting Universe?

- A linearly coasting $a(t)$ is as good a fit to Supernovae data as ΛCDM[4]
- Doesn't need inflation & "Dark" stuff e.g. empty open Universe (Milne)
- $a \propto t$ - pioneering work done by Daksh Lohiya et al.[1]
- Recent proposals include $R_h = ct$ model[3]

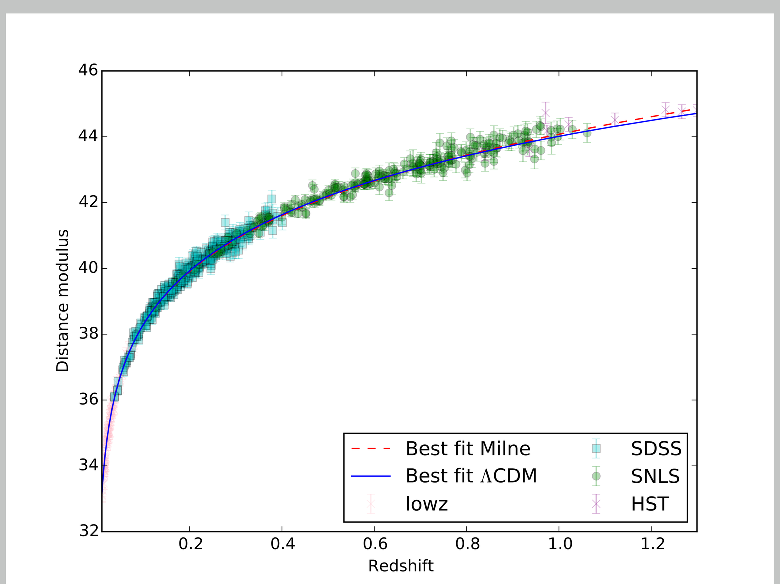


Figure 1: Linearly Coasting vs. ΛCDM[4]

Basic $f(R)$ Theory

In the metric formalism of $f(R)$ theory. We write the action as

$$S = \frac{1}{2\chi}\int d^4x\sqrt{-g}f(R) + S_{matter} \quad (1)$$

Here $\chi = 8\pi G$ where 'G' is universal gravitational constant. Extremizing the action w.r.t $\delta g^{\alpha\beta}$ to write $\delta S = 0$

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - (\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)f'(R) = \chi T_{\mu\nu} \quad (2)$$

Starting with the FLRW metric element

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-\kappa r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \quad (3)$$

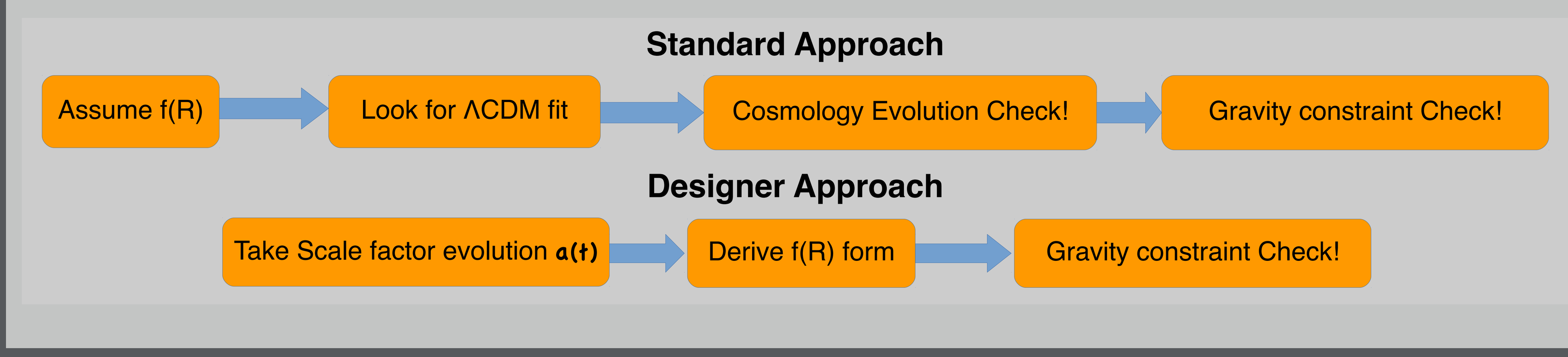
for a generic $f(R)$ model, we get modified Friedmann equations as

$$H^2 + \frac{\kappa}{a^2} = \frac{1}{3f'(R)}\left[\chi\rho + \frac{Rf'(R) - f(R)}{2} - 3Hf''(R)\dot{R}\right] \quad (4)$$

and

$$2\dot{H} + 3H^2 + \frac{\kappa}{a^2} = -\frac{1}{f'(R)}\left[\chi P + 2H\dot{R}f''(R) + \frac{f(R) - Rf'(R)}{2} + \ddot{R}f''(R) + \dot{R}^2f'''(R)\right] \quad (5)$$

'Designer' Approach



$f(R)$ of a Linearly Coasting Universe

For a linearly coasting scale factor we have

$$a(t) \propto t \implies a(t) = t/t_0 \implies H(t) = \frac{1}{t} \quad \& \quad H_0 = \frac{1}{t_0} \quad (6)$$

For FLRW metric Ricci scalar is

$$R = 6(2H^2 + \dot{H} + \kappa/a^2) \quad (7)$$

For $\kappa = 0$: Assuming a flat Universe, we have

$$R = 6(2H^2 + \dot{H}) = \frac{6}{t^2} = 6H^2 \implies H = \sqrt{R/6} \quad (8)$$

This leads us to

$$R^2f'' - f/2 + \chi\rho = 0 \quad (9)$$

$$2R^3f''' + R^2f'' - Rf' + \frac{3}{2}f + 3\chi P = 0 \quad (10)$$

Assuming the equation of state $w = P/\rho$, we get a 2^{nd} order differential equation

$$f'' - \frac{1}{2R^2}f + \frac{\alpha R^{\frac{3}{2}(1+w)}}{R^2} = 0 \quad (11)$$

As one can quickly identify, the numerator in the last term is nothing but the matter/radiation density

$$\rho = \frac{\rho_0 a_0^3}{a^3} = \frac{\rho_0 t_0^3}{t^3} \implies \rho = \alpha R^{\frac{3}{2}(1+w)} \quad (12)$$

where $\alpha = \chi\rho_0 t_0^3 = \frac{8\pi G\rho_0}{H_0^3}$

Solving (11) we get a possibly viable form for $f(R)$

$$f(R) = -\frac{4\alpha}{1 + 3w(4+3w)}R^{\frac{3}{2}(1+w)} + C_1R^{(\sqrt{3}+1)/2} + C_2R^{(-\sqrt{3}+1)/2} \quad (13)$$

- This form is a potentially viable $f(R)$ with constants C_1, C_2 and ρ_0(and hence α) need to be constrained using observational data.
- Since we are primarily interested in late-time acceleration as observed by supernovae data we can assume the case of matter dominated era $P_{matter} = 0$ making $w = 0$ in (13) giving us

$$f(R) = -4\alpha R^{\frac{3}{2}} + C_1R^{(\sqrt{3}+1)/2} + C_2R^{(-\sqrt{3}+1)/2} \quad (14)$$

Conclusions & Future Work

- Assuming a linearly coasting scale factor, we derived a potentially new 'viable' forms of $f(R)$.
- While some forms may look familar, they need to be re-evaluated in the light of linear coasting.
- Constraining these $f(R)$ models with Cosmological observations
  - linear growth rate of structures
  - gravitational weak lensing
  - CMB and structure formation theories
  - weak field limit from the solar system tests
  - gravitational wave observations
- These areas are to be explored in the subsequent work(s).

References

[1] Gehlaut, S., Mukherjee, A., Mahajan, S., and Lohiya, D.
A "freely coasting" universe.
arXiv preprint astro-ph/0209209 (2002).

[2] Kumar, R.
A new class of cosmologically 'viable' $f(R)$ models.
arXiv preprint arXiv:1611.03728 (2016).

[3] Melia, F., and Shevchuk, A.
The $R_h = ct$ universe.
Monthly Notices of the Royal Astronomical Society 419, 3 (2012), 2579–2586.

[4] Nielsen, J. T., Guffanti, A., and Sarkar, S.
Marginal evidence for cosmic acceleration from type ia supernovae.
arXiv preprint arXiv:1506.01354 (2015).