AJHSMEs are the Pre-1999 version of the AMC8 and will be treated like an AMC8.
Some solutions may have small parts of in LaTeX (Typesetting system)

Category: Number Theory

1990 AJHSME (Problem 4) (Correct)

To determine which digits could not be the unit's digit (one's digit) of the square of a whole number, we can analyze the unit's digits of perfect squares.

The unit's digits of perfect squares can only be 0, 1, 4, 5, 6, or 9. These digits are obtained by squaring the digits 0-9 individually:

0^2 = 0

1^2 = 1

2^2 = 4

3^2 = 9

4^2 = 6

5^2 = 5

6^2 = 6

7^2 = 9

8^2 = 4

9^2 = 1

From the above results, we can see that the unit's digit of a perfect square can never be 2, 3, 7, or 8. Therefore, these digits could not be the unit's digit of the square of a whole number.

Difficulty: Easy

2014 AMC8 (Problem 8) (Correct)

To find the missing digit A in the 3-digit number representing the amount paid by the Middle School Math Club members, we need to consider the divisibility rules for integers.

If the sum of the digits of a number is divisible by 3, then the number itself is divisible by 3.

Let's analyze the sum of the digits in the number 1A2:

$1 + A + 2 = 3 + A$

For the number to be divisible by 3, the sum of the digits must also be divisible by 3. Therefore, we need to find a digit A that satisfies the following equation:

$3 + A \equiv 0 \pmod{3}$

To find the possible values for A, we can try different numbers and check if the sum of the digits is divisible by 3:

A = 0: 3 + 0 = 3 (not divisible by 3)

A = 1: 3 + 1 = 4 (not divisible by 3)

A = 2: 3 + 2 = 5 (not divisible by 3)

A = 3: 3 + 3 = 6 (divisible by 3)

Therefore, the missing digit A in the 3-digit number 1A2 is 3.

Difficulty: Easy

2023 AMC8 (Problem 10) (Correct)

Let's calculate the remaining portion of the pie step by step.

Harold ate 1/4 of the pie, which means there was 1 - 1/4 = 3/4 of the pie left for his friends.

The moose came and ate 1/3 of what Harold left behind. So, the moose ate 1/3 * (3/4) = 1/4 of the original pie.

After the moose ate its share, there was 3/4 - 1/4 = 2/4 = 1/2 of the original pie remaining.

Then, the porcupine came and ate 1/3 of what the moose left behind. Therefore, the porcupine ate 1/3 * (1/2) = 1/6 of the original pie.

The remaining portion after the porcupine left is 1/2 - 1/6 = 3/6 - 1/6 = 2/6 = 1/3 of the original pie.

Therefore, after the porcupine left, 1/3 of the original pie still remained.

Difficulty: Easy

2013 AMC8 (Problem 2) (Correct)

If the half-pound package is being sold for $3 at a 50% discount, it means that the regular price for a half-pound package would be $3 * 2 = $6.

Since a full pound is equivalent to two half-pound packages, the regular price for a full pound of fish would be $6 * 2 = $12.

Therefore, the regular price for a full pound of fish is $12.

Difficulty: Easy

2001 AMC8 (Problem 6) (Correct)

If there are six trees equally spaced along the road, it means that the distance between each consecutive pair of trees is the same.

Let's denote the distance between each pair of trees as "d". Therefore, the distance between the first tree and the fourth tree is 3d, as there are three gaps between the first and fourth trees.

Given that the distance between the first and fourth trees is 60 feet (3d = 60), we can solve for the value of "d":

3d = 60,

d = 60 / 3,

d = 20.

So, the distance between each consecutive pair of trees is 20 feet.

Now, to find the distance between the first and last trees, we need to consider the number of gaps between them. Since there are five gaps between six trees, the distance between the first and last trees would be (5 * d):

Distance between first and last trees = 5 * d = 5 * 20 = 100 feet.

Therefore, the distance in feet between the first and last trees is 100 feet.

Difficulty: Easy

1999 AMC8 (Problem 13) (Correct)

To find the average age of the adults in the computer science camp, we need to consider the average age of all camp members and the average age of the girls and boys separately.

The total number of camp members is 40. We can determine the total age of all camp members by multiplying the average age by the total number of members:

Total age of all camp members = Average age of all camp members * Total number of members = 17 * 40 = 680

Next, we need to find the total age of the girls and boys separately. There are 20 girls with an average age of 15, so the total age of the girls is:

Total age of girls = Average age of girls * Total number of girls = 15 * 20 = 300

Similarly, there are 15 boys with an average age of 16, so the total age of the boys is:

Total age of boys = Average age of boys * Total number of boys = 16 * 15 = 240

To find the total age of the adults, we subtract the total age of the girls and boys from the total age of all camp members:

Total age of adults = Total age of all camp members - Total age of girls - Total age of boys = 680 - 300 - 240 = 140

Since there are 5 adults, we can find the average age of the adults by dividing the total age of the adults by the number of adults:

Average age of adults = Total age of adults / Number of adults = 140 / 5 = 28

Therefore, the average age of the adults in the computer science camp is 28 years.

Difficulty: Medium

2017 AMC8 (Problem 14) (Correct)

Let's assume Chloe and Zoe each had to solve a total of 100 problems in their homework assignment.

Chloe solved half of the problems alone, which is 50 problems. Since she had correct answers to only 80% of the problems she solved alone, the number of problems she solved correctly is 0.8 * 50 = 40 problems.

The other half of the problems, which is also 50 problems, were solved together by Chloe and Zoe. Chloe's overall percentage of correct answers is 88%, which means she got 88% of the problems correct out of the total 100 problems. Since she already solved 40 problems correctly alone, the number of problems they solved correctly together is 88 - 40 = 48 problems.

Now let's focus on Zoe. She solved half of the problems alone, which is also 50 problems. Zoe had correct answers to 90% of the problems she solved alone, so the number of problems she solved correctly alone is 0.9 * 50 = 45 problems.

When Chloe and Zoe solved the remaining 50 problems together, they got 48 problems correct.

To find Zoe's overall percentage of correct answers, we need to calculate the total number of problems she solved correctly out of 100.

Total number of problems solved correctly by Zoe = Problems solved correctly alone + Problems solved correctly together = 45 + 48 = 93 problems

Zoe's overall percentage of correct answers = (Total number of problems solved correctly by Zoe / Total number of problems) * 100 = (93 / 100) * 100 = 93%

Therefore, Zoe's overall percentage of correct answers is 93%.

Difficulty: Medium

2018 AMC 10A (Problem 2) (Correct)

Let's denote the amount of soda Jacqueline has as x.

According to the given information, Liliane has 50% more soda than Jacqueline, which means Liliane has $x + 0.5x = 1.5x$ amount of soda.

Alice, on the other hand, has 25% more soda than Jacqueline, which means Alice has $x + 0.25x = 1.25x$ amount of soda.

To find the relationship between the amounts of soda that Liliane and Alice have, we can compare their quantities.

Liliane has $1.5x$ amount of soda, and Alice has $1.25x$ amount of soda.

To determine the difference, we subtract the amount of soda Alice has from the amount of soda Liliane has:

$1.5x - 1.25x = 0.25x$

The difference is $0.25x$.

To find the percentage difference, we can express this difference as a percentage of what Alice has:

Percentage difference = $\frac{0.25x}{1.25x} \times 100\% = 20\%$

Therefore, Liliane has 20% more soda than Alice.

The answer is (A) 20%

Difficulty: Medium

2011 AMC 10A (Problem 5) (Incorrect)

To find the average number of minutes run per day by the students in third, fourth, and fifth grades, we need to consider the number of students in each grade level and their respective running times.

Let's denote the number of fifth graders as x. Since there are twice as many fourth graders as fifth graders, the number of fourth graders would be 2x. Similarly, there are twice as many third graders as fourth graders, so the number of third graders would be 2(2x) = 4x.

The total number of students in all three grades would be:

x (number of fifth graders) + 2x (number of fourth graders) + 4x (number of third graders) = 7x

Now, we can calculate the total time spent running by all students:

Total running time = (number of fifth graders * time per fifth grader) + (number of fourth graders * time per fourth grader) + (number of third graders * time per third grader) = x * 10 + 2x * 15 + 4x * 12 = 10x + 30x + 48x = 88x

To find the average number of minutes run per day, we divide the total running time by the total number of students:

Average running time = Total running time / Total number of students = 88x / 7x = 88 / 7 = 12

Therefore, the average number of minutes run per day by the students in the third, fourth, and fifth grades is 12 minutes.

Difficulty: Medium

2015 AMC 10B (Problem 3) (Incorrect)

Let's denote the unknown number as "x".

According to the given information, one integer (which we'll call the "first integer") has been written down twice, and another integer (which we'll call the "second integer") has been written down three times.

We know that the sum of the five numbers is 100. Since one of the numbers is 28, the sum of the remaining four numbers is 100 - 28 = 72.

Now, let's consider the first integer, which has been written down twice. Since it appears twice, its contribution to the sum is 2x.

Similarly, the second integer, which has been written down three times, contributes 3x to the sum.

Therefore, the sum of the first integer (2x), the second integer (3x), and the remaining four numbers (72) is 2x + 3x + 72.

We can set up an equation based on this information:

2x + 3x + 72 = 100.

Combining like terms:

5x + 72 = 100.

Subtracting 72 from both sides:

5x = 28.

Dividing both sides by 5:

x = 28 / 5 = 5.6.

Therefore, the other number is 5.6

Difficulty: Medium

1990 AJHSME (Problem 21) (Correct)

To find the first number in the list, we need to work backwards by dividing the last three numbers by their preceding numbers. Let's denote the first number as A, and the list of numbers can be represented as:

A, B, C, D, E, 16, 64, 1024

According to the given pattern, each new number in the list is the product of the two previous numbers. Therefore, we can set up the following equations:

B = A * A C = B * A D = C * B E = D * C

To find A, we divide E by D:

A = E / D

Using the values provided, we can calculate the first number:

A = 16 / 64 A = 1/4 A = 0.25

Therefore, the first number in the list is 0.25.

Difficulty: Hard

2003 AMC 10A (Problem 10) (Correct)

The sum of the first n even counting numbers can be calculated using the formula: n * (n + 1), where n is the number of terms.

Similarly, the sum of the first n odd counting numbers can be calculated using the same formula: n * n.

In this case, we are looking for the difference between the sum of the first 2003 even counting numbers and the sum of the first 2003 odd counting numbers.

For the even counting numbers: n = 2003 Sum of even numbers = 2003 * (2003 + 1)

For the odd counting numbers: n = 2003 Sum of odd numbers = 2003 * 2003

To find the difference, we subtract the sum of the odd numbers from the sum of the even numbers:

Difference = (2003 * (2003 + 1)) - (2003 * 2003)

Simplifying this expression:

Difference = 2003 * (2003 + 1 - 2003) Difference = 2003 * 1 Difference = 2003

Therefore, the difference between the sum of the first 2003 even counting numbers and the sum of the first 2003 odd counting numbers is 2003.

Difficulty: Hard

2014 AMC8 (Problem 21) (Correct With Answer Choices)

With Answer Choices:

To determine the possible values of the digit C, we need to consider the divisibility rule for 3.

A number is divisible by 3 if the sum of its digits is divisible by 3.

Let's analyze the given numbers:

1. The 7-digit number 74A52B1 is a multiple of 3. Sum of the digits: 7 + 4 + A + 5 + 2 + B + 1 = 19 + A + B. Since this number is divisible by 3, 19 + A + B must be divisible by 3. The only possible values for A and B that make 19 + A + B divisible by 3 are A = 1 and B = 5. Therefore, C can take any value from the given options.
2. The 7-digit number 326AB4C is a multiple of 3. Sum of the digits: 3 + 2 + 6 + A + B + 4 + C = 15 + A + B + C. Since this number is divisible by 3, 15 + A + B + C must be divisible by 3. To determine the possible values of C, we can subtract 15 + A + B from each of the given options and check if the result is divisible by 3.

Checking the options: A) 15 + 1 + 5 = 21 -> Divisible by 3.

B) 15 + 1 + 5 + 2 = 23 -> Not divisible by 3.

C) 15 + 1 + 5 + 3 = 24 -> Not divisible by 3.

D) 15 + 1 + 5 + 5 = 26 -> Not divisible by 3.

E) 15 + 1 + 5 + 8 = 29 -> Not divisible by 3.

Therefore, the only possible value for the digit C is 1.

Hence, the answer is A (1).

Without Answer Choices:

To determine the possible value of the digit C in the numbers 74A52B1 and 326AB4C, we need to consider the divisibility rule for 3.

For a number to be divisible by 3, the sum of its digits must be divisible by 3.

Let's analyze the sum of the digits in the number 74A52B1:

7 + 4 + A + 5 + 2 + B + 1

The sum of these digits must be divisible by 3. To find the possible values of A and B, we need to consider the digits 0-9 and check which values make the sum divisible by 3.

For the number 326AB4C:

3 + 2 + 6 + A + B + 4 + C

Again, the sum of these digits must be divisible by 3. We will find the possible values of A, B, and C by checking which combinations satisfy the divisibility rule.

Considering both numbers together, we need to find a value for C that makes the sum of the digits in both numbers divisible by 3.

Here are the possible values for the digit C that make the numbers divisible by 3:

C = 0: The sum remains unchanged (unknown if it is divisible by 3).

C = 3: The sum becomes 3 + 2 + 6 + A + B + 4 + 3 = 18 + A + B (divisible by 3).

C = 6: The sum becomes 3 + 2 + 6 + A + B + 4 + 6 = 21 + A + B (divisible by 3).

C = 9: The sum becomes 3 + 2 + 6 + A + B + 4 + 9 = 24 + A + B (divisible by 3).

Therefore, the possible values for the digit C that make both numbers divisible by 3 are C = 3, 6, and 9.

Difficulty: Hard

2017 AMC8 (Problem 24) (Incorrect)

To determine the number of days during the next year when Mrs. Sanders did not receive a phone call from any of her grandchildren, we need to consider the least common multiple (LCM) of 3, 4, and 5.

The LCM of 3, 4, and 5 is the smallest positive integer that is divisible by all three numbers. In this case, it represents the number of days until the grandchildren's phone call patterns align again.

To find the LCM of 3, 4, and 5, we can start by listing the multiples of each number until we find a common multiple:

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, ... Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ... Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, ...

From the lists, we can see that the smallest common multiple is 60. Therefore, the grandchildren's phone call patterns will align every 60 days.

Now, to calculate the number of days during the next year when Mrs. Sanders did not receive a phone call, we need to subtract the number of days with phone calls from the total number of days in a year (365 days).

Number of days without a phone call = Total number of days in a year - Number of days with a phone call = 365 - (365 ÷ 60) * 60 = 365 - 6 * 60 = 365 - 360 = 5

Therefore, Mrs. Sanders did not receive a phone call from any of her grandchildren on 5 days during the next year.

Difficulty: Hard

2002 AMC 12A (Problem 2) (Incorrect)

Let's denote the unknown number as "x".

According to the given information, Cindy incorrectly subtracted 9 from the number and then divided the result by 3, resulting in an answer of 43.

This can be represented as: $(x - 9) / 3 = 43$.

To find out what Cindy's answer would have been if she worked the problem correctly, we need to reverse the steps she took.

First, let's undo the division by 3 by multiplying both sides of the equation by 3:

$(x - 9) = 43 * 3$.

Simplifying the right side of the equation:

$(x - 9) = 129$.

Next, let's undo the subtraction of 9 by adding 9 to both sides of the equation:

$(x - 9) + 9 = 129 + 9$.

Simplifying both sides:

$x = 138$.

Therefore, if Cindy had worked the problem correctly, her answer would have been 138.

Difficulty: Hard

Category: Geometry

2018 AMC8 (Problem 9) (Incorrect)

To find the number of tiles Bob will use, we need to calculate the area of the room and then divide it by the area covered by each tile.

The area of the living room is given as 12 feet by 16 feet, which gives us a total area of 12 * 16 = 192 square feet.

The border along the edges of the room will have a width of one tile, so it will have dimensions of 10 feet by 14 feet (subtracting 2 feet from each side of the room). This gives us an area of 10 * 14 = 140 square feet.

The remaining area to be covered with two-foot by two-foot tiles is the difference between the total area and the border area: 192 - 140 = 52 square feet.

Now, let's calculate the number of two-foot by two-foot tiles needed to cover the remaining area. Each tile has an area of 2 * 2 = 4 square feet.

The number of tiles required is given by dividing the remaining area by the area covered by each tile: 52 / 4 = 13 tiles.

Since Bob will also use the border tiles, the total number of tiles he will use is 13 (for the remaining area) + 140 (for the border) = 153 tiles.

Therefore, Bob will use 153 tiles to tile his living room.

Difficulty: Easy

2020 AMC8 (Problem 9) (Incorrect)

To determine the number of small pieces that will have icing on exactly two sides, we need to analyze the structure of the larger cube and how it is divided into smaller cubes.

The larger cube has dimensions of 4 x 4 x 4 inches, and it is cut into 64 smaller cubes, each measuring 1 x 1 x 1 inch.

Let's examine the edges of the larger cube. Each edge of the cube has 4 smaller cubes arranged in a row. Out of these 4 smaller cubes, only the middle two cubes will have icing on exactly two sides: the top and one side.

There are 4 edges on the larger cube, so there will be a total of 4 x 2 = 8 smaller cubes that have icing on exactly two sides along the edges.

Next, let's look at the corners of the larger cube. Each corner is formed by 3 edges meeting at a point. In each corner, there will be one smaller cube that has icing on exactly two sides: the top and two adjacent sides.

There are 8 corners on the larger cube, so there will be a total of 8 x 1 = 8 smaller cubes that have icing on exactly two sides at the corners.

Therefore, the total number of small pieces that will have icing on exactly two sides is 8 (from the edges) + 8 (from the corners) = 16.

Hence, there will be 16 small pieces that have icing on exactly two sides.

Difficulty: Easy

2019 AMC8 (Problem 4) (Incorrect)

To find the area of a rhombus, we can use the formula:

Area = (1/2) * (product of diagonals)

In this case, we are given the length of one diagonal (24 meters), but we need to find the length of the other diagonal.

In a rhombus, the diagonals bisect each other at right angles, and they divide the rhombus into four congruent right triangles.

Let's use the Pythagorean theorem to find the length of the other diagonal. In one of the right triangles formed by the diagonals, the hypotenuse is the given diagonal of length 24 meters, and the legs are half the lengths of the sides of the rhombus.

Using the Pythagorean theorem, we have:

(1/2 * side length)^2 + (1/2 * side length)^2 = 24^2

(1/4 * side length^2) + (1/4 * side length^2) = 576

(1/2 * side length^2) = 576

Now we can solve for the side length:

side length^2 = (576 * 2)

side length^2 = 1152

side length ≈ √1152

side length ≈ 33.941

The perimeter of the rhombus is given as 52 meters, and since a rhombus has four congruent sides, each side length is 52 / 4 = 13 meters.

Now that we know the side length, we can find the length of the other diagonal:

other diagonal = 2 * (1/2 * side length) = side length = 13 meters

Now we have both diagonals:

Diagonal 1 = 24 meters Diagonal 2 = 13 meters

Let's calculate the area:

Area = (1/2) * (Diagonal 1) * (Diagonal 2) = (1/2) * 24 * 13 = 312 square meters

Therefore, the area of the rhombus is 312 square meters.

Difficulty: Easy

2012 AMC8 (Problem 6) (Correct)

To find the area of the border, we need to calculate the difference between the area of the outer rectangle (frame) and the area of the inner rectangle (photograph).

The outer rectangle has dimensions of (8 + 22*) inches by (10* + 22) inches. Adding 2 inches to each side accounts for the two-inch border on all sides of the photograph. Therefore, the dimensions of the outer rectangle are 12 inches by 14 inches.

The inner rectangle (photograph) has dimensions of 8 inches by 10 inches.

The area of the outer rectangle (frame) is given by:

Area of outer rectangle = Length * Width = 12 inches * 14 inches = 168 square inches.

The area of the inner rectangle (photograph) is given by:

Area of inner rectangle = Length * Width = 8 inches * 10 inches = 80 square inches.

To find the area of the border, we subtract the area of the inner rectangle from the area of the outer rectangle:

Area of border = Area of outer rectangle - Area of inner rectangle = 168 square inches - 80 square inches = 88 square inches.

Therefore, the area of the border is 88 square inches.

Difficulty: Easy

2006 AMC8 (Problem 7) (Incorrect)

To compare the circles in terms of their radii, we need to examine the given information and make the necessary calculations.

Circle X: Radius = π

Circle Y: Circumference = 2πr Given: Circumference = 8π Using the formula: 2πr = 8π Simplifying, we find: r = 4

Circle Z: Area = πr^2 Given: Area = 9π Using the formula: πr^2 = 9π Simplifying, we find: r^2 = 9 Taking the square root, we find: r = 3

Comparing the radii of the circles, we have: Circle X: Radius = π Circle Z: Radius = 3 Circle Y: Radius = 4

Listing the circles in order from smallest to largest radius:

1. Circle X with a radius of π
2. Circle Z with a radius of 3
3. Circle Y with a radius of 4

Therefore, the circles in order from smallest to largest radius are Circle X, Circle Z, and Circle Y.

Difficulty: Easy

2015 AMC8 (Problem 12) (Incorrect)

A cube has 12 edges in total. To determine the number of pairs of parallel edges, we need to identify the edges that are parallel to each other.

A cube has three sets of parallel edges: one set of parallel edges on the top face, one set on the bottom face, and one set connecting corresponding vertices on opposite faces.

On the top face, there are four parallel edges: AB, BC, CD, and DA. On the bottom face, there are also four parallel edges: EF, FG, GH, and HE. Connecting corresponding vertices on opposite faces, there are four more pairs of parallel edges: AE and BF, BG and CF, CH and DG, and DH and EG.

So, in total, there are 4 + 4 + 4 = 12 pairs of parallel edges in a cube.

Therefore, a cube has 12 pairs of parallel edges.

Difficulty: Medium

2000 AMC 8 (Problem 13) (Correct)

Since we are given that ∠ACT = ∠ATC, we can conclude that triangle ACT is an isosceles triangle.

In an isosceles triangle, the base angles (the angles opposite the equal sides) are congruent. Therefore, we have ∠ACT = ∠ATC.

Since ∠ACT = ∠ATC, we know that each of these angles measures 36 degrees.

Now, let's consider the angle ∠CRT.

Since TR bisects ∠ATC, we can split ∠ATC into two equal angles. Let's call the point of intersection between TR and AC as P.

So, we have ∠RTP = ∠PTC.

The sum of the angles in a triangle is 180 degrees. Therefore, ∠ATC + ∠PTC + ∠PTC = 180 degrees.

Since ∠ATC = 36 degrees, we can substitute the known values into the equation:

36 + ∠PTC + ∠PTC = 180

2∠PTC + 36 = 180

2∠PTC = 180 - 36

2∠PTC = 144

∠PTC = 144/2

∠PTC = 72

So, ∠CRT measures 72 degrees.

Difficulty: Medium

2007 AMC8 (Problem 14) (Correct)

To find the length of one of the congruent sides of the isosceles triangle, we need to use the formula for the area of a triangle.

The formula for the area of a triangle is given by:

Area = (base * height) / 2.

In this case, we are given that the base of the triangle is 24 and the area is 60. Plugging these values into the formula, we get:

60 = (24 * height) / 2.

To solve for the height, we can multiply both sides of the equation by 2 and then divide by 24:

2 * 60 = 24 * height.

120 = 24 * height.

Now, we can divide both sides of the equation by 24 to isolate the height:

120 / 24 = height.

5 = height.

So, the height of the triangle is 5. Since the triangle is isosceles, the height bisects the base and forms a right triangle with half of the base and one of the congruent sides.

Let's denote the length of one of the congruent sides as x. By using the Pythagorean theorem in the right triangle formed by the height, half of the base, and one of the congruent sides, we have:

x^2 = height^2 + (base/2)^2.

Plugging in the values we know:

x^2 = 5^2 + (24/2)^2.

x^2 = 25 + 12^2.

x^2 = 25 + 144.

x^2 = 169.

Taking the square root of both sides, we get:

x = √169.

x = 13.

Therefore, the length of one of the congruent sides of the isosceles triangle is 13 units.

Difficulty: Medium

2012 AMC8 (Problem 17) (Incorrect)

Let's assume the side length of the original square is "n" units.

We are given that 10 squares are cut from the original square, and at least 8 of them have an area of 1 square unit.

If 8 squares have an area of 1 square unit, then the combined area of those squares is 8 square units.

The remaining 2 squares must have a combined area of (n^2 - 8) square units.

Since all the squares have integer side lengths, we can express the combined area of the remaining 2 squares as the sum of two perfect squares.

Therefore, (n^2 - 8) must be expressible as the sum of two perfect squares.

The smallest possible value of n that satisfies this condition is 5.

For n = 5, we have (n^2 - 8) = 17 = 1^2 + 4^2.

Thus, the smallest possible value of the length of the side of the original square is 5 units.

Difficulty: Medium

2014 AMC8 (Problem 19) (Incorrect)

The larger 3-inch cube is made up of 27 smaller cubes, arranged in a 3x3x3 pattern. Among these smaller cubes, 21 are colored red and 6 are colored white.

To minimize the white surface area showing on the larger cube, we want to position the white cubes in the interior, surrounded by red cubes. In this way, only the faces of the white cubes that are not touching other cubes will be visible.

There are 6 white cubes in total, and each cube has 6 faces. However, the faces of the white cubes that are in contact with other cubes will not be visible. Each white cube has 3 faces that are not touching other cubes and will be visible.

Therefore, the total white surface area showing is 6 cubes * 3 visible faces = 18 square inches.

The total surface area of the larger cube is 6 faces * (3 inches * 3 inches) = 54 square inches.

The fraction of the surface area that is white is given by:

Fraction of white surface area = (white surface area) / (total surface area) = 18 square inches / 54 square inches = 1/3

Therefore, the fraction of the surface area that is white is 1/3.

Difficulty: Medium

2013 AMC8 (Problem 22) (Incorrect)

To calculate the total number of toothpicks used in the grid, we need to consider the number of horizontal toothpicks and vertical toothpicks.

For a grid that is 60 toothpicks long and 32 toothpicks wide, there will be 61 vertical toothpicks (one on each end and 60 in between) and 33 horizontal toothpicks (one on each end and 32 in between).

The total number of toothpicks used is the sum of the vertical toothpicks and horizontal toothpicks:

Total number of toothpicks = Number of vertical toothpicks + Number of horizontal toothpicks

Total number of toothpicks = 61 + 33

Total number of toothpicks = 94

Therefore, there are 94 toothpicks used altogether in the grid.

Difficulty: Hard

2018 AMC8 (Problem 23) (Incorrect)

To find the probability that at least one of the sides of the triangle is also a side of the octagon, we can use complementary probability.

The total number of ways to choose 3 vertices from the octagon is given by the combination formula: C(8, 3) = 56.

Now, let's consider the cases where none of the sides of the triangle are sides of the octagon. In order for this to happen, we need to choose 3 vertices that are not adjacent (since adjacent vertices form sides of the octagon).

We have 8 vertices to choose from, and if we choose one vertex, we cannot choose the adjacent vertices. So, for the first vertex, we have 8 choices, for the second vertex we have 4 choices (since we cannot choose the adjacent vertices), and for the third vertex we have 3 choices (excluding the two adjacent vertices).

Therefore, the number of ways to choose 3 vertices such that none of the sides of the triangle are sides of the octagon is 8 * 4 * 3 = 96.

Now, we can find the probability of this event occurring:

P(no side of triangle is a side of the octagon) = 96 / 56.

To find the probability that at least one of the sides of the triangle is also a side of the octagon, we subtract this probability from 1:

P(at least one side of triangle is a side of the octagon) = 1 - P(no side of triangle is a side of the octagon).

P(at least one side of triangle is a side of the octagon) = 1 - 96 / 56.

Simplifying this expression:

P(at least one side of triangle is a side of the octagon) = (56 - 96) / 56.

P(at least one side of triangle is a side of the octagon) = -40 / 56.

Since probability cannot be negative, the probability is 0.

Therefore, the probability that at least one of the sides of the triangle is also a side of the octagon is 0.

Difficulty: Hard

2000 AMC8 (Problem 22) (Incorrect)

Let's calculate the surface area of the original cube and the new solid formed after gluing the smaller cube on top.

The surface area of a cube is given by the formula: 6 times the edge length squared.

For the original cube with edge length 2, the surface area is:

6 * (2^2) = 6 * 4 = 24.

Now, let's calculate the surface area of the new solid formed after gluing the smaller cube on top.

The new solid will have the same surface area as the original cube, except for the top face that is covered by the smaller cube.

The surface area of the smaller cube is 1 times 1 = 1.

So, the increase in surface area from the original cube to the new solid is equal to the surface area of the smaller cube, which is 1.

To calculate the percent increase in surface area, we need to find the percentage of the increase relative to the original surface area.

Percent increase = (increase in surface area / original surface area) * 100.

Percent increase = (1 / 24) * 100 ≈ 4.17%.

Therefore, the percent increase in surface area from the original cube to the new solid formed is closest to 4.17%.

Difficulty: Hard

2000 AMC8 (Problem 25) (Incorrect)

To find the area of the triangle formed by connecting point A and the midpoints of BC and CD, we can use the fact that the midpoints of BC and CD divide those sides into two equal segments.

Let's denote the midpoint of BC as M and the midpoint of CD as N.

Since M is the midpoint of BC, we can say that BM = MC and MN is parallel to AD.

Similarly, since N is the midpoint of CD, we can say that CN = ND and NA is parallel to BD.

Now, the triangle formed by connecting point A, M, and N is a triangle with bases AD and BD and height MN.

The area of a triangle can be calculated using the formula: Area = (base * height) / 2.

In this case, the base of the triangle is AD and BD, and the height is MN.

Since AD is the same as BC (opposite sides of a rectangle are equal), we can say that the base of the triangle is BC.

Therefore, the area of the triangle formed by connecting point A, M, and N is:

Area = (BC * MN) / 2

Since the area of rectangle ABCD is given as 72, we can find the length of BC using the formula: Area = length * width.

72 = BC * AD

Now, since the diagonals of a rectangle bisect each other, we can say that BC is half the length of the diagonal AC.

Let's denote the length of AC as x.

BC = x/2

Substituting this into the equation 72 = BC * AD, we get:

72 = (x/2) * AD

Simplifying, we find:

AD = (2 * 72) / x

Now, let's consider the triangle formed by connecting point A, M, and N.

Since MN is parallel to AD, we can say that MN is half the length of AD.

MN = (1/2) * AD

Substituting the value of AD we found earlier, we get:

MN = (1/2) * [(2 * 72) / x] = 72 / x

Finally, substituting the values of BC and MN into the area formula, we find:

Area = (BC * MN) / 2 = [(x/2) * (72/x)] / 2 = 36/2 = 18

Therefore, the area of the triangle formed by connecting point A and the midpoints of BC and CD is 18.

Difficulty: Hard

2005 AMC 8 (Problem 25) (Correct)

Let's denote the radius of the circle as r.

The area of the square is given by the formula: side length squared.

In this case, the side length of the square is 2, so the area of the square is $2^2 = 4$.

The area of a circle is given by the formula: π times the radius squared.

In this case, the area of the circle is πr^2.

The total area of the regions that are inside the circle and outside the square is given by:

$\pi r^2 - 4$.

The total area of the regions that are outside the circle and inside the square is given by:

$4 - \pi r^2$.

According to the given condition, these two areas are equal:

$\pi r^2 - 4 = 4 - \pi r^2$.

Let's simplify and solve this equation:

$\pi r^2 + \pi r^2 = 8$.

$2\pi r^2 = 8$.

Dividing both sides by 2:

$\pi r^2 = 4$.

Dividing both sides by π:

$r^2 = 4/\pi$.

Taking the square root of both sides:

$r = \sqrt{4/\pi}$.

Simplifying this expression:

$r = 2/\sqrt{\pi}$.

So, the radius of the circle is $2/\sqrt{\pi}$.

Difficulty: Hard

Category: Algebra

2009 AMC8 (Problem 1) (Correct)

Let's work through the information provided step by step to determine the number of apples Bridget bought.

1. Bridget gave half of the apples to Ann. Let's denote the total number of apples Bridget bought as "x". After giving half to Ann, Bridget has x/2 apples remaining.
2. Bridget gave 3 apples to Cassie. After giving 3 apples to Cassie, Bridget has (x/2) - 3 apples remaining.
3. Bridget kept 4 apples for herself. After keeping 4 apples for herself, Bridget has ((x/2) - 3) - 4 apples remaining.

According to the information provided, the number of apples remaining after these distributions should be zero, since Bridget has given away all the apples:

((x/2) - 3) - 4 = 0.

Simplifying the equation:

(x/2) - 3 - 4 = 0.

(x/2) - 7 = 0.

(x/2) = 7.

Multiplying both sides of the equation by 2:

x = 14.

Therefore, Bridget bought 14 apples.

Difficulty: Easy

2004 AMC8 (Problem 9) (Correct)

To find the average of the last three numbers, we can use the concept of weighted averages.

The average of the five numbers in the list is given as 54, which means the sum of all five numbers is 5 * 54 = 270.

The average of the first two numbers is given as 48, which means the sum of the first two numbers is 2 * 48 = 96.

To find the sum of the last three numbers, we can subtract the sum of the first two numbers from the sum of all five numbers:

Sum of the last three numbers = Sum of all five numbers - Sum of the first two numbers = 270 - 96 = 174.

Since we want to find the average of the last three numbers, we divide the sum of the last three numbers by 3:

Average of the last three numbers = Sum of the last three numbers / 3 = 174 / 3 ≈ 58.

Therefore, the average of the last three numbers is approximately 58.

Difficulty: Easy

2010 AMC8 (Problem 8) (Incorrect)

To determine the time during which Emily can see Emerson, we need to calculate the time it takes for Emily to pass Emerson and for him to be 1/2 mile behind her.

Let's first calculate the time it takes for Emily to pass Emerson: The relative speed between Emily and Emerson is 12 mph - 8 mph = 4 mph. Since the distance between them is 1/2 mile, the time it takes for Emily to pass Emerson is (1/2) mile / 4 mph = 1/8 hour.

Next, let's calculate the time it takes for Emerson to be 1/2 mile behind Emily: Since Emily is riding at 12 mph and Emerson is skating at 8 mph, the relative speed between them is 12 mph - 8 mph = 4 mph. The time it takes for Emerson to be 1/2 mile behind Emily is (1/2) mile / 4 mph = 1/8 hour.

Since the time for Emily to pass Emerson and for Emerson to be 1/2 mile behind her is the same (1/8 hour), Emily can see Emerson for 1/8 hour.

To convert this time to minutes, we multiply by 60: 1/8 hour * 60 minutes/hour = 7.5 minutes.

Therefore, Emily can see Emerson for 7.5 minutes.

Difficulty: Easy

2011 AMC8 (Problem 1) (Correct)

To calculate the change Margie received, we need to determine the total cost of the apples and subtract it from the amount she paid.

Margie bought 3 apples at a cost of 50 cents per apple, so the total cost is 3 * 50 cents = 150 cents.

To convert the cost to dollars, we divide by 100 since there are 100 cents in a dollar:

150 cents / 100 = 1.50 dollars.

Margie paid with a 5-dollar bill, so the change she received is:

Change = Amount paid - Total cost = $5 - $1.50 = $3.50.

Therefore, Margie received $3.50 in change.

Difficulty: Easy

2012 AMC 8 (Problem 9) (Correct)

Let's denote the number of two-legged birds as "b" and the number of four-legged mammals as "m".

Each bird has 2 legs, so the total number of legs contributed by the two-legged birds is 2b. Each mammal has 4 legs, so the total number of legs contributed by the four-legged mammals is 4m.

According to the information provided, the total number of heads is 200 and the total number of legs is 522.

We can set up a system of equations based on the given information:

Equation 1: b + m = 200 (total number of heads) Equation 2: 2b + 4m = 522 (total number of legs)

To solve this system of equations, we can use substitution or elimination.

Let's solve the system using substitution. We'll solve Equation 1 for b and substitute it into Equation 2:

b = 200 - m (from Equation 1)

Substituting into Equation 2:

2(200 - m) + 4m = 522

400 - 2m + 4m = 522

2m = 522 - 400

2m = 122

m = 122 / 2

m = 61

Now that we have the value of m, we can substitute it back into Equation 1 to find the value of b:

b + 61 = 200

b = 200 - 61

b = 139

Therefore, according to the given information, there were 139 two-legged birds among the animals Margie counted at the zoo.

Difficulty: Easy

2004 AMC 8 (Problem 12) (Incorrect)

Let's break down the usage of Niki's cell phone to determine how much battery life is remaining.

Niki has used her phone for 60 minutes, which is equivalent to 1 hour. During this time, the battery life decreased by 1 hour.

Since the phone has been on for a total of 9 hours, but Niki used it for only 1 hour, the remaining 8 hours were spent with the phone on but not in use.

In this case, when the phone is on but not in use, the battery lasts for 24 hours. Therefore, the remaining 8 hours will not significantly affect the battery life.

Hence, the battery will last for the usual 24 hours when the phone is left on but not in use.

Therefore, the battery will last for an additional 24 hours from the current point.

Difficulty: Medium

2015 AMC8 (Problem 20) (Incorrect)

Let's denote the number of pairs of $1 socks as "x", the number of pairs of $3 socks as "y", and the number of pairs of $4 socks as "z".

From the given information, we can set up a system of equations based on the number of pairs and their total cost:

Equation 1: $x + y + z = 12$ (total number of pairs) Equation 2: $1x + 3y + 4z = 24$ (total cost in dollars)

To solve this system of equations, we can use trial and error or substitution.

Let's start by considering the possible values for x, y, and z.

Since we know that Ralph bought at least one pair of each type of sock, the minimum values for x, y, and z are 1.

We can begin by assuming $x = 1$ and then check if there is a valid solution for y and z that satisfies both equations.

Plugging in $x = 1$ into Equation 1, we get:

$1 + y + z = 12$ $y + z = 11$

Now, plugging in $x = 1$ into Equation 2, we get:

$1(1) + 3y + 4z = 24$ $1 + 3y + 4z = 24$ $3y + 4z = 23$

By trying different values for y and z, we can see if there is a solution that satisfies both equations.

If we let $y = 5$ and $z = 6$, we have:

$y + z = 11$ (satisfies Equation 1) $3y + 4z = 23$ (satisfies Equation 2)

Therefore, when $x = 1$, $y = 5$, and $z = 6$, we have a valid solution.

So, Ralph bought 1 pair of $1 socks, 5 pairs of $3 socks, and 6 pairs of $4 socks.

Difficulty: Medium

2007 AMC 8 (Problem 19) (Incorrect)

To find the difference of the squares of two consecutive positive integers, we can choose two consecutive integers, square each of them, and calculate the difference.

Let's denote the two consecutive integers as "n" and "n+1". The sum of these two integers is less than 100, so we have the inequality:

$n + (n+1) < 100$

Simplifying this inequality, we get:

$2n + 1 < 100$

$2n < 99$

$n < 49.5$

Since n is a positive integer, the largest possible value for n is 49.

Now, let's calculate the difference of the squares for n and n+1.

Difference of squares = $(n+1)^2 - n^2 = (n^2 + 2n + 1) - n^2 = 2n + 1$

For n = 49, the difference of the squares is:

$2n + 1 = 2(49) + 1 = 98 + 1 = 99$

Therefore, the difference of the squares could be 99.

Difficulty: Medium

2009 AMC 8 (Problem 15) (Incorrect)

To determine the greatest number of servings of hot chocolate Jordan can make, we need to identify the ingredient that he has the least amount of relative to the recipe's requirement.

Let's compare the available ingredients to the recipe's requirements:

- Jordan has 5 squares of chocolate, and the recipe requires 2 squares. Therefore, he has enough chocolate for at least 2 servings of hot chocolate.
- Jordan has 2 cups of sugar, and the recipe requires 1/4 cup. Since Jordan has more than enough sugar, it will not limit the number of servings he can make.
- Jordan has 7 cups of milk, and the recipe requires 4 cups. Therefore, he has enough milk for at least 4 servings of hot chocolate.
- Jordan has lots of water, which means he has enough water for any number of servings he can make.

Based on the ingredient that Jordan has the least amount of, which is the 2 squares of chocolate, he can make a maximum of 2 servings of hot chocolate.

Therefore, the greatest number of servings of hot chocolate Jordan can make is 2.

Difficulty: Medium

AMC 8, 2010, Problem 15

Let's first calculate the total number of gumdrops in the jar. We know that 30% are blue, 20% are brown, 15% are red, 10% are yellow, and 30 gumdrops are green.

If we let the total number of gumdrops be x, we can write the following equations based on the given percentages:

Blue gumdrops: 0.30x Brown gumdrops: 0.20x Red gumdrops: 0.15x Yellow gumdrops: 0.10x Green gumdrops: 30

The total percentage of gumdrops accounted for is:

0.30 + 0.20 + 0.15 + 0.10 = 0.75

So, the remaining percentage for green gumdrops is:

1 - 0.75 = 0.25

Since 30 gumdrops are green, we can set up the following equation:

0.25x = 30

Solving for x:

x = 30 / 0.25 x = 120

Now, we can calculate the new number of blue gumdrops after replacing half of them with brown gumdrops:

New blue gumdrops: 0.50 * 0.30x = 0.15x

Since we know that 0.15x represents the number of blue gumdrops after the replacement, we can subtract this value from the total number of gumdrops to find the number of brown gumdrops:

Brown gumdrops: 0.20x - 0.15x = 0.05x

Plugging in the value of x:

Brown gumdrops: 0.05 * 120 = 6

Therefore, after replacing half of the blue gumdrops with brown gumdrops, there will be 6 brown gumdrops.

Difficulty: Medium

2017 AMC8 (Problem 23) (Incorrect)

Let's denote the number of minutes it takes Linda to travel one mile on the first day as "x" minutes.

On the second day, her speed decreases so that the number of minutes to travel one mile increases by 5 minutes. Therefore, on the second day, it takes her (x + 5) minutes to travel one mile.

Similarly, on the third day, it takes her (x + 10) minutes to travel one mile, and on the fourth day, it takes her (x + 15) minutes to travel one mile.

We know that each day, her distance traveled was an integer number of miles. Since her speed remained constant within each day, the number of minutes it took to travel one mile must also be a divisor of 60.

To find the possible values of "x", let's consider the divisors of 60. The divisors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60.

Out of these divisors, let's find the values of "x" that satisfy the conditions for all four days.

If "x" is 1, it would result in (1 + 5 + 10 + 15) = 31 minutes for the total time, which is not possible since each day has 60 minutes.

If "x" is 2, it would result in (2 + 7 + 12 + 17) = 38 minutes for the total time, which is also not possible since each day has 60 minutes.

If "x" is 3, it would result in (3 + 8 + 13 + 18) = 42 minutes for the total time, which is still less than 60 minutes.

Continuing this pattern, we find that "x" cannot be 4, 5, 6, or 10 since the total time would still be less than 60 minutes.

If "x" is 12, it would result in (12 + 17 + 22 + 27) = 78 minutes for the total time, which is more than 60 minutes.

Therefore, the only possible value for "x" is 15. With "x" equal to 15, the total time for the four trips is (15 + 20 + 25 + 30) = 90 minutes, which is equal to 60 minutes (1 hour) multiplied by the number of trips (4).

Since Linda traveled for one hour each day, the total number of miles for the four trips is 1 mile per hour * 4 hours = 4 miles.

Therefore, the total number of miles for the four trips is 4 miles.

Difficulty: Hard

2014 AMC8 (Problem 24) (Incorrect)

To determine the maximum possible median number of cans of soda bought per customer, we need to distribute the 252 cans of soda in a way that maximizes the median.

Let's consider a scenario where the first 50 customers each buy 1 can of soda. In this case, we have 50 cans distributed among the first 50 customers.

Now, we need to distribute the remaining 202 cans among the remaining 50 customers while maximizing the median. To do this, we can give each of the remaining 50 customers 4 cans of soda, resulting in a total of 200 cans.

The remaining 2 cans can be given to any 2 of the remaining 50 customers, which will not affect the median.

In this scenario, the median number of cans of soda bought per customer is 4, as half of the customers have bought 4 or more cans, and the other half have bought fewer than 4 cans.

Therefore, the maximum possible median number of cans of soda bought per customer on that day is 4.

Difficulty: Hard

2010 AMC8 (Problem 24) (Correct)

To determine the correct ordering of the three numbers, let's compare them using their exponential forms:

10^8 = 100,000,000 5^12 = 244,140,625 2^24 = 16,777,216

Comparing the numbers, we can see that:

16,777,216 < 100,000,000 < 244,140,625

Therefore, the correct ordering of the three numbers is:

2^24 < 10^8 < 5^12

Difficulty: Hard

2009 AMC8 (Problem 23) (Correct)

Let's denote the number of boys in the class as "b" and the number of girls as "g". We are given that there were two more boys than girls, so we can express this as:

b = g + 2

Mrs. Awesome gave each boy as many jelly beans as there were boys in the class, which means each boy received b jelly beans. Similarly, each girl received g jelly beans.

The total number of jelly beans given out can be calculated as:

Total jelly beans = (b * number of boys) + (g * number of girls)

Substituting the values, we have:

400 = (b * b) + (g * g)

From the given information, we know that there were six jelly beans left, so we can modify the equation:

400 - 6 = (b * b) + (g * g)

394 = (b * b) + (g * g)

We also know that b = g + 2, so we can substitute this expression in the equation:

394 = ((g + 2) * (g + 2)) + (g * g)

394 = g^2 + 4g + 4 + g^2

394 = 2g^2 + 4g + 4

Rearranging the terms:

2g^2 + 4g - 390 = 0

Dividing the equation by 2:

g^2 + 2g - 195 = 0

Now we can solve this quadratic equation. Factoring or using the quadratic formula, we find:

(g + 15)(g - 13) = 0

This gives us two possible solutions: g = -15 or g = 13.

Since we are dealing with the number of students in the class, we discard the negative solution.

Therefore, the number of girls in the class (g) is 13.

Substituting this value back into the equation b = g + 2, we find:

b = 13 + 2 b = 15

Thus, there are 13 girls and 15 boys in the class, for a total of 13 + 15 = 28 students.

Difficulty: Hard

2009 AMC8 (Problem 21) (Incorrect)

To find the value of A/B, we need to determine the relationship between the average of Andy's row sums and the average of Bethany's column sums.

The total sum of all the numbers in the rectangular array can be calculated by multiplying the number of rows (40) by the number of columns (75). Since all the numbers are greater than zero, the sum will be greater than zero as well.

Now, let's consider the average of Andy's row sums. The average of a set of numbers can be calculated by dividing the total sum of the numbers by the number of values.

In this case, the total sum of Andy's row sums is equal to the total sum of all the numbers in the rectangular array. Therefore, the average of Andy's row sums is equal to the average of all the numbers in the array.

Similarly, the average of Bethany's column sums is also equal to the average of all the numbers in the array.

Since both averages are calculated based on the same set of numbers, A/B would be equal to 1.

Difficulty: Hard

Category: Combinatorics

2006 AMC8 (Problem 4) (Correct)

To determine the final direction of the spinner, we need to consider the total number of revolutions and the direction of each move.

Chenille first moves the spinner clockwise 2 1/4 revolutions. This means the spinner completes 2 full revolutions and then rotates an additional 1/4 of a revolution in the clockwise direction.

Next, Chenille moves the spinner counterclockwise 3 3/4 revolutions. This means the spinner completes 3 full revolutions and then rotates an additional 3/4 of a revolution in the counterclockwise direction.

To find the net effect of the two moves, we need to subtract the counterclockwise rotations from the clockwise rotations:

2 1/4 - 3 3/4 = (8/4 + 1/4) - (12/4 + 3/4) = 9/4 - 15/4 = -6/4 = -3/2

The result -3/2 represents a counterclockwise rotation of 3/2 revolutions.

Since the initial direction was west, a counterclockwise rotation corresponds to moving in the opposite direction.

Therefore, the final direction of the spinner after the two moves is east.

Difficulty: Easy

2007 AMC8 (Problem 5) (Incorrect)

To determine the number of weeks Chandler will need to buy the mountain bike, we need to calculate the total amount of money he has and divide it by the amount he earns per week.

The total amount of money Chandler has is the sum of the money he receives from his grandparents, aunt, cousin, and the money he earns from his paper route.

Grandparents' gift: $50 Aunt's gift: $35 Cousin's gift: $15 Money earned per week: $16

Total money: $50 + $35 + $15 + $16 = $116

Now, we can divide the total money by the amount Chandler earns per week to find the number of weeks:

Number of weeks = Total money / Money earned per week Number of weeks = $116 / $16 ≈ 7.25

Since Chandler cannot have a fraction of a week, we need to round up the number of weeks to the nearest whole number.

Therefore, Chandler will need approximately 8 weeks to be able to buy the mountain bike.

Difficulty: Easy

2010 AMC8 (Problem 8) (Correct)

To find the different possible values for the sum of the numbers on the chips drawn from each bag, we need to consider all the possible combinations.

In Bag A, there are three chips labeled 1, 3, and 5. In Bag B, there are three chips labeled 2, 4, and 6.

When we draw one chip from each bag, we can calculate all the possible sums:

1 + 2 = 3 1 + 4 = 5 1 + 6 = 7 3 + 2 = 5 3 + 4 = 7 3 + 6 = 9 5 + 2 = 7 5 + 4 = 9 5 + 6 = 11

From these combinations, we can see that there are five different possible values for the sum of the two numbers on the chips: 3, 5, 7, 9, and 11.

Therefore, there are five different values possible for the sum of the two numbers on the chips.

Difficulty: Easy

2017 AMC8 (Problem 10) (Incorrect)

To find the probability that 4 is the largest value selected, we need to determine the total number of possible outcomes and the number of favorable outcomes.

Total number of possible outcomes: Since three cards are selected without replacement from a box containing five cards, the total number of possible outcomes can be calculated using the concept of combinations. We can use the formula C(n, r) to calculate the number of combinations. In this case, we have C(5, 3) = 10 possible outcomes.

Number of favorable outcomes: To have 4 as the largest value selected, we need to consider two cases:

1. The three cards selected are 1, 2, and 4.
2. The three cards selected are 1, 3, and 4.

For case 1, there is only one way to select the cards (1, 2, 4). For case 2, there is also only one way to select the cards (1, 3, 4).

Therefore, the number of favorable outcomes is 2.

Probability: The probability is calculated by dividing the number of favorable outcomes by the total number of possible outcomes:

Probability = Favorable outcomes / Total outcomes Probability = 2 / 10 Probability = 1/5

So, the probability that 4 is the largest value selected is 1/5 or 0.2.

Difficulty: Easy

2012 AMC 8 (Problem 10) (Incorrect)

To determine the number of 4-digit numbers greater than 1000 that can be formed using the four digits of 2012, we need to consider the placement of the digits.

We have the digits 2, 0, 1, and 2. Since the number must be greater than 1000, the thousands place can only be filled with the digit 2.

For the hundreds place, we have three choices (0, 1, or 2) since we can use any of the remaining three digits.

For the tens place, we have two choices (0 or 1) since we have used one of the digits for the hundreds place.

For the units place, we have one choice (the remaining digit).

Therefore, the total number of 4-digit numbers greater than 1000 that can be formed using the digits of 2012 is:

3 (choices for the hundreds place) × 2 (choices for the tens place) × 1 (choice for the units place) = 6

So, there are 6 such 4-digit numbers.

Difficulty: Easy

2008 AMC8 (Problem 17) (Correct)

To find the difference between the largest and smallest possible areas of the rectangles, we need to consider the possible dimensions of the rectangles with a perimeter of 50 units.

Let's denote the length and width of a rectangle as "l" and "w", respectively.

Since the perimeter of a rectangle is given by the formula: perimeter = 2l + 2w, we can write the equation as:

$2l + 2w = 50$

Dividing both sides of the equation by 2, we get:

$l + w = 25$

To find the largest and smallest possible areas, we need to determine the maximum and minimum values of the product $l \times w$ under the constraint that $l + w = 25$.

By applying the AM-GM inequality (Arithmetic Mean-Geometric Mean Inequality), we know that the maximum value of $l \times w$ occurs when $l = w$. Similarly, the minimum value occurs when the difference between l and w is the greatest.

Since l and w are both integers, we need to consider the pairs of integers whose sum is 25:

(1, 24) -> area = 1 × 24 = 24 (2, 23) -> area = 2 × 23 = 46 (3, 22) -> area = 3 × 22 = 66 ... (12, 13) -> area = 12 × 13 = 156

Therefore, the smallest possible area is 24, and the largest possible area is 156.

The difference between the largest and smallest possible areas is:

156 - 24 = 132

Hence, the difference between the largest and smallest possible areas of the rectangles is 132 square units.

Difficulty: Medium

2004 AMC8 (Problem 14) (Incorrect)

Let's analyze the given statements:

I. Bill is the oldest. II. Amy is not the oldest. III. Celine is not the youngest.

Based on these statements, we can deduce the following:

- If Bill is the oldest (Statement I), then Amy cannot be the oldest (Statement II) because only one statement is true.
- If Amy is not the oldest (Statement II), then Celine must be the oldest because we know Bill cannot be the oldest.
- If Celine is not the youngest (Statement III), then Amy must be the youngest because Bill is not the oldest and Celine is not the youngest.

Therefore, the ranking from oldest to youngest is: Celine > Bill > Amy

Difficulty: Medium

2003 AMC 8 (Problem 16) (Incorrect)

To determine the number of possible seating arrangements, we need to consider the different options for assigning individuals to the seats in the car.

Since Bonnie and Carlo are the only ones who can drive, one of them must occupy the driver's seat. There are two choices for the driver's seat: Bonnie or Carlo.

Once the driver's seat is assigned, there is one choice remaining for the front passenger seat.

For the two back passenger seats, there are two remaining individuals: Ali and Dianna. The order in which they occupy the back seats does not matter, as both seats are identical.

Therefore, the number of possible seating arrangements can be calculated as:

Number of possible seating arrangements = Number of choices for the driver's seat × Number of choices for the front passenger seat × Number of ways to arrange the back passengers

Number of possible seating arrangements = 2 (Bonnie or Carlo for the driver's seat) × 1 (remaining choice for the front passenger seat) × 2 (number of ways to arrange Ali and Dianna in the back seats)

Number of possible seating arrangements = 2 × 1 × 2 = 4

Thus, there are 4 possible seating arrangements.

Difficulty: Medium

2009 AMC 8 (Problem 13) (Correct)

To determine the probability that a three-digit integer containing the digits 1, 3, and 5 is divisible by 5, we need to count the favorable outcomes and the total possible outcomes.

Total possible outcomes: The total number of three-digit integers that can be formed using the digits 1, 3, and 5 is 3! = 3 × 2 × 1 = 6. This is because we have three choices for the hundreds digit, two choices for the tens digit (since we cannot repeat digits), and one choice for the units digit.

Favorable outcomes: To be divisible by 5, the units digit of the three-digit integer must be either 5 or 0.

Since we are given that the integer contains the digits 1, 3, and 5, the units digit can only be 5.

Now, we need to determine the placement of the remaining digits (1 and 3). The hundreds and tens digits can be filled with the remaining two digits in any order.

Therefore, the number of favorable outcomes is 2! = 2 × 1 = 2.

Probability: The probability is calculated by dividing the number of favorable outcomes by the total number of possible outcomes:

Probability = Favorable outcomes / Total outcomes Probability = 2 / 6 Probability = 1/3

So, the probability that the three-digit integer is divisible by 5 is 1/3.

Difficulty: Medium

2002 AMC 8 (Problem 18) (Correct)

Let's calculate the total skating time for the first 5 days and the next 3 days:

Skating time for the first 5 days: 1 hour 15 minutes/day × 5 days = 6 hours 15 minutes. Skating time for the next 3 days: 1 hour 30 minutes/day × 3 days = 4 hours 30 minutes.

To find the total skating time for the entire period, we add the skating time for the first 5 days and the next 3 days:

Total skating time = Skating time for the first 5 days + Skating time for the next 3 days Total skating time = 6 hours 15 minutes + 4 hours 30 minutes = 10 hours 45 minutes.

Now, let's determine how long Gage would have to skate on the ninth day to average 85 minutes of skating each day for the entire time.

We can set up the equation: (10 hours 45 minutes + Skating time on the ninth day) / 9 days = 85 minutes/day.

To solve for the skating time on the ninth day, we can multiply both sides of the equation by 9:

10 hours 45 minutes + Skating time on the ninth day = 9 days × 85 minutes/day Skating time on the ninth day = 9 days × 85 minutes/day - 10 hours 45 minutes.

Converting 10 hours 45 minutes to minutes: 10 hours × 60 minutes/hour + 45 minutes = 600 minutes + 45 minutes = 645 minutes.

Substituting the values into the equation: Skating time on the ninth day = 9 days × 85 minutes/day - 645 minutes.

Calculating the skating time on the ninth day: Skating time on the ninth day = 765 minutes - 645 minutes = 120 minutes.

Therefore, Gage would need to skate for 120 minutes on the ninth day in order to average 85 minutes of skating each day for the entire time.

Difficulty: Medium

2001 AMC 8 (Problem 22) (ncorrect)

To determine which score is not possible, we need to analyze the scoring system and find any limitations or restrictions.

On a twenty-question test, each correct answer is worth 5 points, each unanswered question is worth 1 point, and each incorrect answer is worth 0 points.

Let's consider the possible scores:

1. If a student answers all 20 questions correctly, they would earn 20 * 5 = 100 points.
2. If a student leaves all 20 questions unanswered, they would earn 20 * 1 = 20 points.
3. If a student answers all 20 questions incorrectly, they would earn 20 * 0 = 0 points.

Now, let's analyze the given answer choices:

(A) 90 points: This is a possible score if a student answers 18 questions correctly and leaves 2 questions unanswered.

(B) 91 points: This is a possible score if a student answers 17 questions correctly, leaves 2 questions unanswered, and answers 1 question incorrectly.

(C) 92 points: This is a possible score if a student answers 18 questions correctly, leaves 1 question unanswered, and answers 1 question incorrectly.

(D) 95 points: This is a possible score if a student answers 19 questions correctly and leaves 1 question unanswered.

(E) 97 points: This is a possible score if a student answers 19 questions correctly and leaves 1 question unanswered.

Therefore, the score that is NOT possible is 92 points (Option C).

Difficulty: Hard

2019 AMC8 (Problem 25) (Incorrect)

To find the number of ways Alice can share her 24 apples with Becky and Chris, we need to consider the distribution of the apples while ensuring that each person receives at least 2 apples.

Let's analyze the possible scenarios:

1. Alice gives 2 apples to Becky and 2 apples to Chris. This leaves her with 24 - 2 - 2 = 20 apples to distribute further.
    - The remaining 20 apples can be distributed among Becky and Chris in various ways, including the possibility of one person receiving all the remaining apples.
    - We can calculate the number of ways to distribute the remaining 20 apples using a stars and bars method. There are 20 + 2 - 1 = 21 positions (stars and bars) to distribute the remaining apples.
    - Therefore, there are C(21, 1) = 21 ways to distribute the remaining apples.
2. Alice gives 2 apples to Becky and 3 apples to Chris. This leaves her with 24 - 2 - 3 = 19 apples to distribute further.
    - The remaining 19 apples can be distributed among Becky and Chris in various ways, including the possibility of one person receiving all the remaining apples.
    - We can calculate the number of ways to distribute the remaining 19 apples using a stars and bars method. There are 19 + 2 - 1 = 20 positions (stars and bars) to distribute the remaining apples.
    - Therefore, there are C(20, 1) = 20 ways to distribute the remaining apples.
3. Alice gives 3 apples to Becky and 2 apples to Chris. This leaves her with 24 - 3 - 2 = 19 apples to distribute further.
    - The remaining 19 apples can be distributed among Becky and Chris in various ways, including the possibility of one person receiving all the remaining apples.
    - We can calculate the number of ways to distribute the remaining 19 apples using a stars and bars method. There are 19 + 2 - 1 = 20 positions (stars and bars) to distribute the remaining apples.
    - Therefore, there are C(20, 1) = 20 ways to distribute the remaining apples.
4. Alice gives 3 apples to Becky and 3 apples to Chris. This leaves her with 24 - 3 - 3 = 18 apples to distribute further.
    - The remaining 18 apples can be distributed among Becky and Chris in various ways, including the possibility of one person receiving all the remaining apples.
    - We can calculate the number of ways to distribute the remaining 18 apples using a stars and bars method. There are 18 + 2 - 1 = 19 positions (stars and bars) to distribute the remaining apples.
    - Therefore, there are C(19, 1) = 19 ways to distribute the remaining apples.

Adding up the possibilities from all the scenarios, the total number of ways Alice can share her 24 apples with Becky and Chris is:

21 + 20 + 20 + 19 = 80 ways.

Therefore, Alice can share her 24 apples with Becky and Chris in 80 different ways.

Difficulty: Hard

2016 AMC8 (Problem 21) (Correct)

To determine the probability that the 3 red chips are drawn before both green chips are drawn, we need to consider the different possible outcomes.

Let's analyze the possibilities:

1. Red-Red-Red: The 3 red chips are drawn consecutively. The probability of this outcome is (3/5) * (2/4) * (1/3) = 1/10.
2. Red-Red-Green: Two red chips are drawn first, followed by one green chip. The probability of this outcome is (3/5) * (2/4) * (2/3) = 1/5.
3. Red-Green-Red: One red chip is drawn first, followed by one green chip, and then another red chip. The probability of this outcome is (3/5) * (2/4) * (1/3) = 1/10.
4. Green-Red-Red: Two red chips are drawn after one green chip. The probability of this outcome is (2/5) * (3/4) * (2/3) = 1/5.
5. Green-Green-Red: Both green chips are drawn before any red chip. The probability of this outcome is (2/5) * (1/4) * (3/3) = 1/10.

Since we are interested in the probability that the 3 reds are drawn (scenario 1), we need to sum up the probabilities of all the outcomes where the 3 reds are drawn:

P(3 reds) = 1/10 + 1/5 + 1/10 = 1/10 + 2/10 + 1/10 = 4/10 = 2/5.

Therefore, the probability that the 3 red chips are drawn before both green chips are drawn is 2/5.

Difficulty: Hard

2015 AMC8 (Problem 23) (Incorrect)

Tom has twelve slips of paper which he wants to put into five cups labeled A, B, C, D, E. He wants the sum of the numbers on the slips in each cup to be an integer. Furthermore, he wants the five integers to be consecutive and increasing from A to E. The numbers on the papers are $2,2,2,2.5,2.5,3,3,3,3,3.5,4$ and 4.5. If a slip with 2 goes into cup E and a slip with 3 goes into cup B, then the slip with3.5must go into what cup?

Difficulty: Hard

2012 AMC8 (Problem 22) (Correct)

To determine the number of possible values of the median of set R, we need to consider the possible arrangements of the remaining three distinct integers.

Given that six elements of set R are 2, 3, 4, 6, 9, and 14, we can arrange these six elements in different ways to determine the possible values of the median.

Let's consider the possible scenarios:

1. If the median is one of the six given elements (2, 3, 4, 6, 9, or 14), there is only one possibility for the arrangement of the remaining three integers. This is because the given elements are fixed, and the median value is already determined.
2. If the median is an integer between the given elements, we can choose any three distinct integers from the remaining available integers. The number of ways to choose three distinct integers from the remaining three integers is C(3, 3) = 1.

Therefore, the total number of possible values of the median of set R is 6 (from the given elements) + 1 (from the case where the median is between the given elements) = 7.

Therefore, there are 7 possible values for the median of set R.

Difficulty: Hard

Category: Computation

2006 AMC8 (Problem 1) (Correct)

To find Mindy's total purchase amount, we can add up the three purchase amounts and round to the nearest dollar.

$1.98 + $5.04 + $9.89 = $16.91

Rounding $16.91 to the nearest dollar gives us $17.

Therefore, Mindy's total purchase amount, to the nearest dollar, is $17.

Difficulty: Easy

1990 AJHSME (Problem 2) (Correct)

To determine which digit of 0.12345, when changed to 9, gives the largest number, we need to compare the resulting numbers after changing each digit to 9.

If we change the digit in the thousandths place (0.12"3"45) to 9, we get 0.12945, which is larger than 0.12345.

If we change the digit in the hundredths place (0.1"2"345) to 9, we get 0.19345, which is larger than 0.12345.

If we change the digit in the tenths place (0."1"2345) to 9, we get 0.92345, which is larger than 0.12345.

If we change the digit in the ones place (0.12"3"45) to 9, we get 0.12945, which is larger than 0.12345.

If we change the digit in the tens place (0.123"4"5) to 9, we get 0.92345, which is larger than 0.12345.

Based on the comparisons, changing the digit in the tenths place to 9 gives the largest number, 0.92345.

Therefore, changing the digit in the tenths place (1) to 9 gives the largest number.

Difficulty: Easy

2022 AMC8 (Problem 7) (Correct)

To calculate the time it would take to download a 4.2-megabyte song at a download speed of 56 kilobits per second, we need to convert the file size and download speed to the same units.

First, we convert the file size of 4.2 megabytes to kilobits: 4.2 megabytes = 4.2 * 8000 kilobits = 33600 kilobits.

Now, we can calculate the download time by dividing the file size by the download speed: Download time = File size / Download speed = 33600 kilobits / 56 kilobits per second.

Canceling out the kilobits, we are left with: Download time = 33600 / 56 seconds.

To convert the time from seconds to minutes, we divide by 60: Download time = (33600 / 56) / 60 minutes.

Simplifying the expression, we have: Download time = 600 / 60 minutes = 10 minutes.

Therefore, at a download speed of 56 kilobits per second, it would have taken approximately 10 minutes to download a 4.2-megabyte song.

Difficulty: Easy

2023 AMC8 (Problem 1) (Correct)

To calculate the value of the expression (8 x 4 + 2) - (8 + 4 x 2), we follow the order of operations, which is often remembered using the acronym PEMDAS (Parentheses, Exponents, Multiplication and Division from left to right, Addition and Subtraction from left to right).

Let's break down the expression step by step:

(8 x 4 + 2) - (8 + 4 x 2) = (32 + 2) - (8 + 4 x 2) = 34 - (8 + 4 x 2) = 34 - (8 + 8) = 34 - 16 = 18

Therefore, the value of the expression (8 x 4 + 2) - (8 + 4 x 2) is 18.

Difficulty: Easy

1985 AJHSME (Problem 1) (Correct)

To simplify the expression (3 * 5)/(9 * 11) * (7 * 9 * 11)/(3 * 5 * 7), we can cancel out the common factors in the numerator and denominator.

The expression can be simplified as follows:

(3 * 5)/(9 * 11) * (7 * 9 * 11)/(3 * 5 * 7) = (3/3) * (5/5) * (7/7) * (9/9) * (11/11) = 1 * 1 * 1 * 1 * 1 = 1

Therefore, the simplified value of the expression is 1.

Difficulty: Easy

2008 AMC8 (Problem 12) (Incorrect)

To determine on which bounce the ball will not rise to a height of 0.5 meters, we need to examine the pattern of the ball's bounces.

Given that the ball is dropped from a height of 3 meters and on its first bounce it rises to a height of 2 meters, we can observe the following bounces:

1st bounce: 2 meters 2nd bounce: 2/3 * 2 meters = 4/3 meters 3rd bounce: 2/3 * (4/3) meters = 8/9 meters 4th bounce: 2/3 * (8/9) meters = 16/27 meters 5th bounce: 2/3 * (16/27) meters = 32/81 meters ...

From the pattern, we can see that the height of the ball after each bounce follows a geometric sequence with a common ratio of 2/3.

To find the bounce where the ball does not rise to a height of 0.5 meters, we can set up an equation and solve for the number of bounces.

Let H represent the height of the ball after each bounce. H = 2 * (2/3)^(n-1)

We want to find the bounce where H is less than or equal to 0.5 meters: $2 * (2/3)^{(n-1)} \le 0.5$

Dividing both sides of the inequality by 2: $(2/3)^{(n-1)} \le 0.25$

Taking the logarithm (base 2/3) of both sides: $(n-1) \le \log(2/3)\ (0.25)$

Using logarithmic properties, we can rewrite the inequality as: $(n-1) \le \log(2/3)\ (1/4)$

Evaluating the logarithm: $(n-1) \le \log(1/3)\ (1/4)$

Simplifying the inequality: $(n-1) \le \log(1/3)\ (-1/2)$

Using logarithmic properties to simplify further: $(n-1) \le -\log(3)\ (1/2)$

Since the logarithm of a number less than 1 is negative, we can rewrite the inequality as: $(n-1) \le \log(3)\ (1/2)$

Using logarithmic properties: $(n-1) \le \log(3)\ (1) - \log(3)\ (2)$

Approximating the logarithms to decimal values: $(n-1) \le 0.4771 - 0.3010$ $(n-1) \le 0.1761$

Adding 1 to both sides of the inequality: $n \le 1.1761$

Since the number of bounces must be a positive integer, we round up to the nearest whole number: $n \le 2$

Therefore, on the 2nd bounce, the ball will not rise to a height of 0.5 meters.

Note: In this case, the ball reaches a height of 0.5926 meters on the 2nd bounce, which is greater than 0.5 meters.

Difficulty: Medium

2012 AMC 10B (Problem 5) (Incorrect)

Let's assume the cost of Anna's dinner without tax and tip is x dollars.

The sales tax on meals is 10%, so the tax amount is $0.10 * x = 0.1x$ dollars.

Anna leaves a 15% tip on the pre-tax amount, so the tip amount is $0.15 * x = 0.15x$ dollars.

The total cost of dinner, including tax and tip, is given as 27.50 dollars, so we can set up the equation:

$x + 0.1x + 0.15x = 27.50$

Combining like terms:

0.25x = 27.50

Dividing both sides of the equation by 0.25:

x = 27.50 / 0.25

x = 110

Therefore, the cost of Anna's dinner without tax or tip is $110.

Difficulty: Medium

2011 AMC 10B (Problem 19) (Incorrect)

To simplify the expression (2+4+6)/(1+3+5)-(1+3+5)/(2+4+6), we follow the order of operations, which is often remembered using the acronym PEMDAS (Parentheses, Exponents, Multiplication and Division from left to right, Addition and Subtraction from left to right).

Let's break down the expression step by step:

(2+4+6)/(1+3+5)-(1+3+5)/(2+4+6)

First, we perform the addition inside the parentheses: 12/9 - 9/12

Next, we simplify each fraction by finding the least common denominator (LCD), which is 36: (12/9) * (4/4) - (9/12) * (3/3)

This gives us: 48/36 - 27/36

Now, we can subtract the fractions: (48 - 27) / 36

This simplifies to: 21/36

Finally, we can simplify the fraction by dividing both the numerator and denominator by their greatest common divisor (GCD), which is 3: (21/3) / (36/3)

This gives us: 7/12

Therefore, the value of the expression (2+4+6)/(1+3+5)-(1+3+5)/(2+4+6) is 7/12.

Difficulty: Medium

2004 AMC 10A (Problem 1) (Correct)

To find out how many dollars each of you will need to raise, divide the total amount that needs to be raised ($1500) by the number of people (you and five friends).

$1500 / 6 = $250

Therefore, each of you will need to raise $250.

Difficulty: Medium

2013 AMC 10A (Problem 2) (Incorrect)

To determine how many times Alice must fill her 1/4 cup measuring cup to get 2 1/2 cups of sugar, we can set up a proportion:

(1/4) / (x) = (2 1/2) / (1)

First, we convert the mixed number 2 1/2 to an improper fraction:

2 1/2 = (2 * 2 + 1) / 2 = 5/2

Now, we can rewrite the proportion:

(1/4) / (x) = (5/2) / (1)

To solve for x, we can cross multiply:

1 * (1/4) = x * (5/2)

Multiplying both sides:

1/4 = (5x) / 2

To isolate x, we can multiply both sides by 2:

(2 * 1/4) = 5x

1/2 = 5x

Now, we can solve for x by dividing both sides by 5:

(1/2) / 5 = x

1/10 = x

Therefore, Alice must fill her 1/4 cup measuring cup 1/10 or 0.1 times to get the correct amount of sugar, which is equivalent to filling it once.

Difficulty: Medium

2022 AMC 12A (Problem 1) (Correct)

To find the value of the given expression, let's start from the innermost fraction and work our way outward:

13+13+133+3+3111

Let's simplify the expression inside the parentheses:

13+13+13=13+1103=13+3103+3+3111=3+31011=3+1031

Now, we can add the fractions inside the parentheses:

13+310=13310=10333+1031=10331=3310

Now, let's substitute this value back into the original expression:

3+13+13+13=3+11033=3+33103+3+3+3111=3+33101=3+1033

To add these fractions, we need to find a common denominator, which is 10:

3+3310=3010+3310=63103+1033=1030+1033=1063

Therefore, the value of the given expression is 63101063, which is equivalent to 6.3.

Difficulty: Hard

2022 AMC 12A (Problem 8) (Correct)

To find the value of the given infinite product, let's denote it as PP:
P=103 · 1033 · 10333…P=310 · 3310 · 33310

…

We can observe a pattern in the exponents of the radicals. The index of each radical is increasing by a factor of 3 as we go further. So, we can rewrite PP as follows:
P=103 · 10133 · 10193…P=310

⋅31031⋅31091

…

Now, let's consider the exponent of 10 in each term of the product: 1331, 132321, 133331, 134341, and so on.

We can notice that the exponents form a geometric progression with a common ratio of 1331. To determine the sum of this infinite geometric progression, we'll use the formula for the sum of an infinite geometric series: S=a1−r,S=1−ra, where aa is the first term and rr is the common ratio.

In our case, a=13a=31 and r=13r=31, so the sum of the exponents is:
S=131−13=1323=12.S=1−3131=3231=21.

Therefore, the infinite product PP converges to the real number 10 raised to the power of 1221:
P=1012=10.P=1021=10

.

Hence, the value of the given infinite product is sqrt10

Difficulty: Hard

2022 AMC 12B (Problem 1)

To find the value of (1◊(2◊3))−((1◊2)◊3)(1◊(2◊3))−((1◊2)◊3), let's first evaluate the expressions within the parentheses.

(2◊3)=∣2−3∣=∣−1∣=1(2◊3)=∣2−3∣=∣−1∣=1

(1◊2)=∣1−2∣=∣−1∣=1(1◊2)=∣1−2∣=∣−1∣=1

Now, we substitute these values back into the main expression:

(1◊(2◊3))−((1◊2)◊3)=(1◊1)−(1◊3)(1◊(2◊3))−((1◊2)◊3)=(1◊1)−(1◊3)

=∣1−1∣−∣1−3∣=∣1−1∣−∣1−3∣

=∣0∣−∣−2∣=∣0∣−∣−2∣

=0−2=0−2

=−2=−2

Therefore, the value of (1◊(2◊3))−((1◊2)◊3)(1◊(2◊3))−((1◊2)◊3) is −2.

Difficulty: Hard

2021 Fall AMC 12A (Problem 1)

To find the remainder when a number expressed in base-nine, $27{,}006{,}000{,}052_{\text{nine}}$, is divided by 5, we can convert the number to base-ten and then perform the division.

Let's convert the base-nine number to base-ten:

$27{,}006{,}000{,}052_{\text{nine}} = 2 \cdot 9^{10} + 7 \cdot 9^9 + 6 \cdot 9^6 + 5 \cdot 9^2 + 2$

Now, we evaluate this expression in base-ten:

$2 \cdot 9^{10} + 7 \cdot 9^9 + 6 \cdot 9^6 + 5 \cdot 9^2 + 2 = 2 \cdot (9^{10}) + 7 \cdot (9^9) + 6 \cdot (9^6) + 5 \cdot (9^2) + 2$

Calculating the powers of 9:

$9^{10} = 348{,}678{,}440{,}100$ $9^9 = 387{,}420{,}489$ $9^6 = 531{,}441$ $9^2 = 81$

Substituting these values into the expression:

$2 \cdot 348{,}678{,}440{,}100 + 7 \cdot 387{,}420{,}489 + 6 \cdot 531{,}441 + 5 \cdot 81 + 2$

Simplifying:

$697{,}356{,}880{,}200 + 2{,}711{,}942{,}423 + 3{,}188{,}646 + 405 + 2$

Now, perform the addition:

$700{,}257{,}717{,}276$

To find the remainder when this base-ten number is divided by 5, we can simply calculate the remainder after division:

$700{,}257{,}717{,}276 \bmod 5 = 1$

Therefore, the remainder when N is divided by 5 is 1.

Difficulty: Hard

2021 Spring AMC 12A (Problem 9)

We can see that each term in the given expression follows the pattern of $a^2 - b^2$, which can be factored as $(a - b)(a + b)$. In this case, we have $a = 3$ and $b = 2$. Therefore, we can rewrite the expression as follows:

(2+3)(22+32)(24+34)(28+38)(216+316)(232+332)(264+364)(2+3)(22+32)(24+34)(28+38)(216+316)(232+332)(264+364)
=(3−2)(3+2)(22+32)(24+34)(28+38)(216+316)(232+332)(264+364)=(3−2)(3+2)(22+32)(24+34)(28+38)(216+316)(232+332)(264+364)
=(32−22)(22+32)(24+34)(28+38)(216+316)(232+332)(264+364)=(32−22)(22+32)(24+34)(28+38)(216+316)(232+332)(264+364)
=(34−24)(24+34)(28+38)(216+316)(232+332)(264+364)=(34−24)(24+34)(28+38)(216+316)(232+332)(264+364)
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=(332−232)(232+332)(264+364)=(332−232)(232+332)(264+364)
=(364−264)(264+364)=(364−264)(264+364) =364−264=364−264

Therefore, the given expression is equivalent to $3^{64} - 2^{64}$.

Difficulty: Hard