

# Assignment 9

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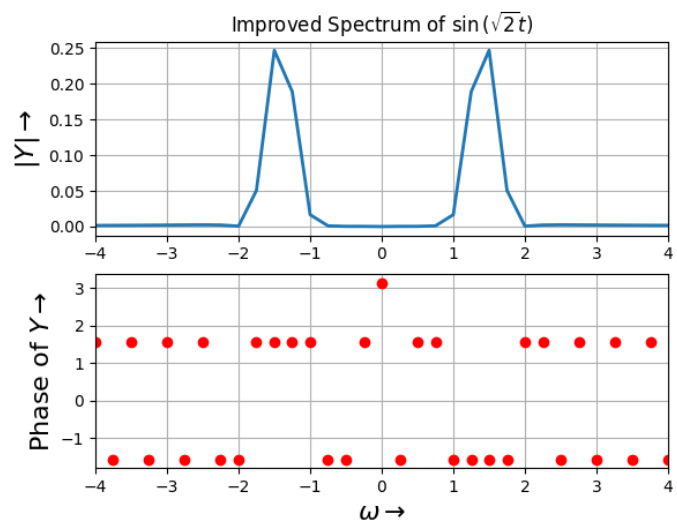
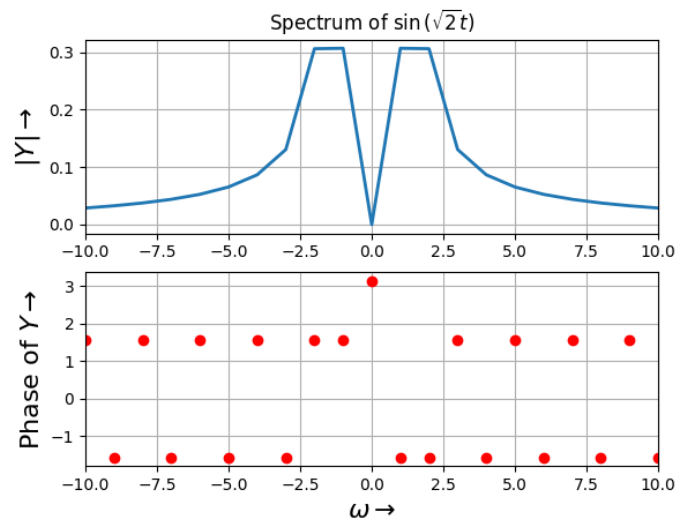
## 1 Introduction

In this assignment, we continue our analysis of signals using Fourier Transforms. This time, we focus on finding transforms of non periodic functions. These functions have a discontinuity when periodically extended. The discontinuity causes fourier components in frequencies other than the sinusoids frequency which decay as  $\frac{1}{\omega}$ , due to Gibbs phenomenon. We resolve this problem using a hamming window in the case of this assignment. We use this windowed transform to analyse signals known to contain a sinusoid of unknown frequencies and extract its phase and frequency. We also perform a sliding DFT on a chirped signal and plot the results

## 2 worked example

The worked example of assignment are given below: spectrum of  $\sin(\sqrt{2}t)$  is given below

Improved spectrum of  $\sin(\sqrt{2}t)$  is given below



### 3 Assignment questions

#### 3.1 Helper Functions

Given below is a helper function that plots and returns the DFT of an arbitrary function.

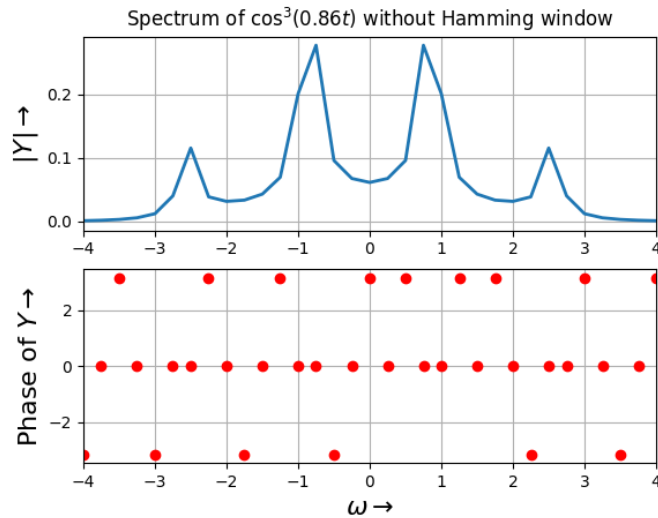
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```
def spectrum_plot(fig_no,w,Y,xlimit ,Title ,ylabel1 ,ylabel2 ,Xlabel ,Grid=True):  
    figure(fig_no)  
    subplot(2,1,1)  
    plot(w,abs(Y),lw=2)  
    xlim([-xlimit ,xlimit])  
    ylabel(ylabel1 ,size=16)  
    title(Title)  
    grid(Grid)  
    subplot(2,1,2)  
    plot(w,angle(Y), 'ro' ,lw=2)  
    xlim([-xlimit ,xlimit])  
    ylabel(ylabel2 ,size=16)  
    xlabel(Xlabel ,size=16)  
    grid(Grid)
```

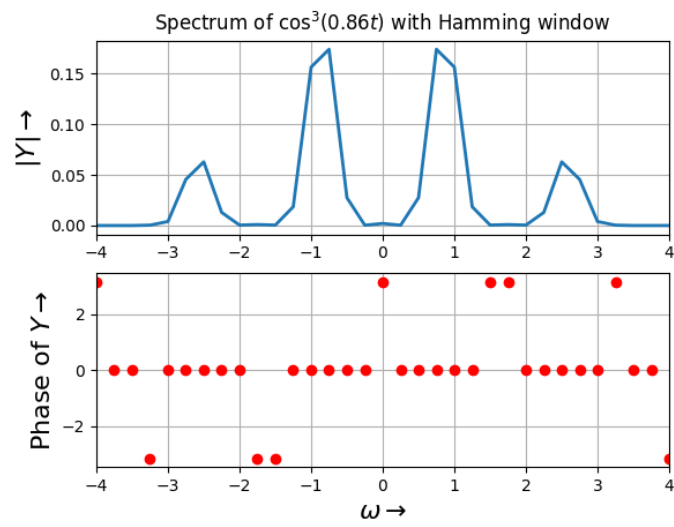
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#### 3.2 Question 2

In this question, we shall plot the FFT of  $\cos^3(0.86t)$  The FFT without the hamming Window:



The FFT with the hamming Window:



We notice that a lot of the energy is stored in frequencies that aren't a part of the signal. After windowing, these frequencies are attenuated and hence the peaks are sharper in the windowed function. It is still not an impulse because the convolution with the Fourier transform of the windowed function smears out the peak

### 3.3 Question 3

We need to estimate  $\omega$  and  $\delta$  for a signal  $\cos(\omega t + \delta)$  for 128 samples between  $[-\pi, \pi)$ . We estimate omega using a weighted average. We have to extract the digital spectrum of the signal and find the two peaks at  $\pm\omega_0$ , and estimate  $\omega$  and  $\delta$ .

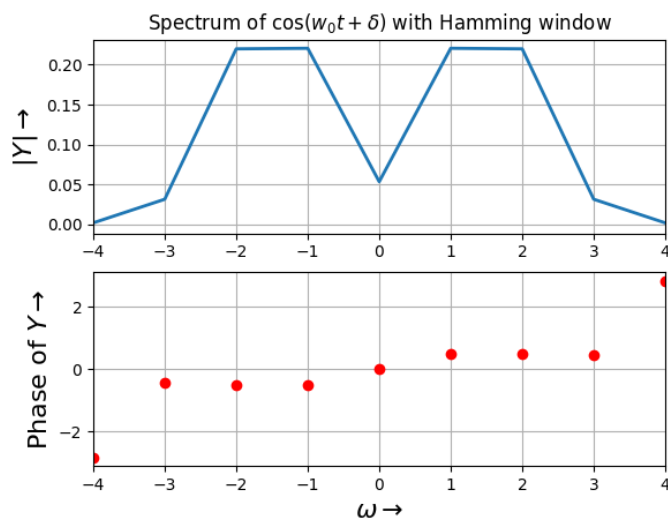


Figure 1: Fourier transform of  $\cos(1.5t + 0.5)$

We estimate omega by performing a Mean average of  $\omega$  over the magnitude of  $|Y(j\omega)|$ .

For delta we consider a widow on each half of  $\omega$  (split into positive and negative values) and extract their mean slope. The intuition behind this is that, a circular shift in the time domain of a sequence results in the linear phase of the spectra.

### 3.4 Question 4

We repeat the exact same process as question 3 but with noise added to the original signal.

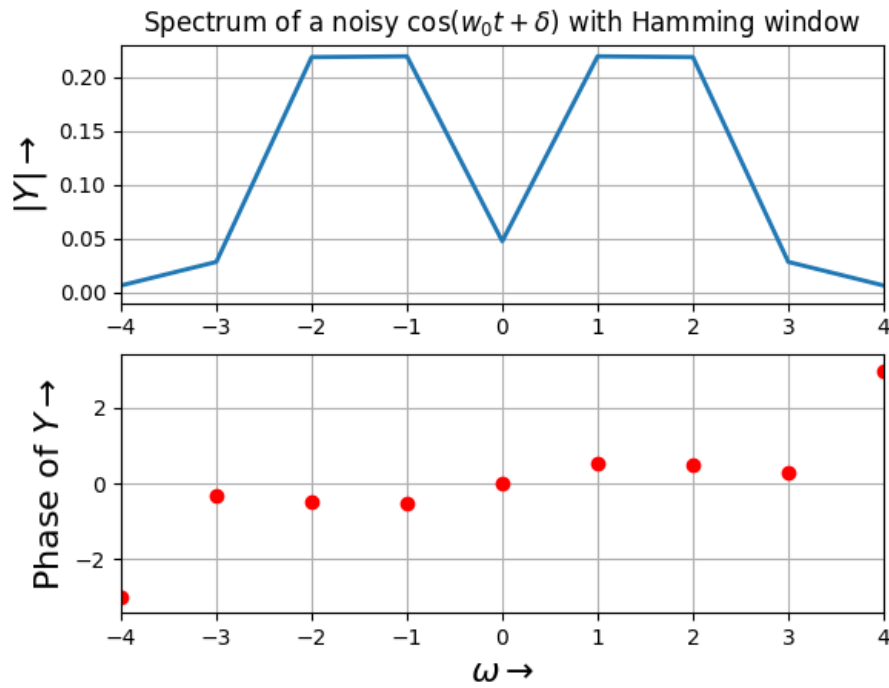


Figure 2: Fourier transform of noise +  $\cos(1.5t + 0.5)$

For true value of  $\omega$  and  $\delta = 1.5$  and  $0.5$  respectively I got :

omega = 1.4730276250507859  
delta = 0.5018760117245951 in no noise case  
omega = 2.161405845557151  
delta = 0.495061685242815 in the noisy case

### 3.5 Question 5

In this question we analyze a chirp signal which is an FM signal where frequency is directly proportional to time. A chirp signal we shall consider is given by

$$f(t) = \cos(16t(1.5 + \frac{t}{2\pi})) \quad (1)$$

The FFT of the chirp is given by: We note that the frequency response is spread between 5-50 rad/s. A large section of this range appears due to Gibbs phenomenon. On windowing, only frequencies between 16 and 32 rad/s remain.

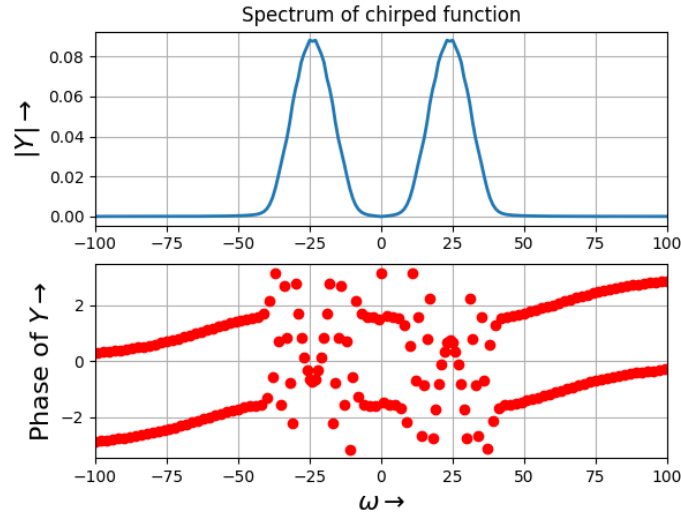


Figure 3: Chirp function fourier transform, windowed

### 3.6 Question 6

For the same chirped signal, we break the 1024 vector into pieces that are 64 samples wide. Extract the DFT of each and store as a column in a 2D array. Then plot the array as a surface plot to show how the frequency of the signal varies with time. This is new. So far we worked either in time or in frequency. But this is a “time- frequency” plot, where we get localized DFTs and show how the spectrum evolves in time. We do this for both phase and magnitude. Let us explore their surface plots.

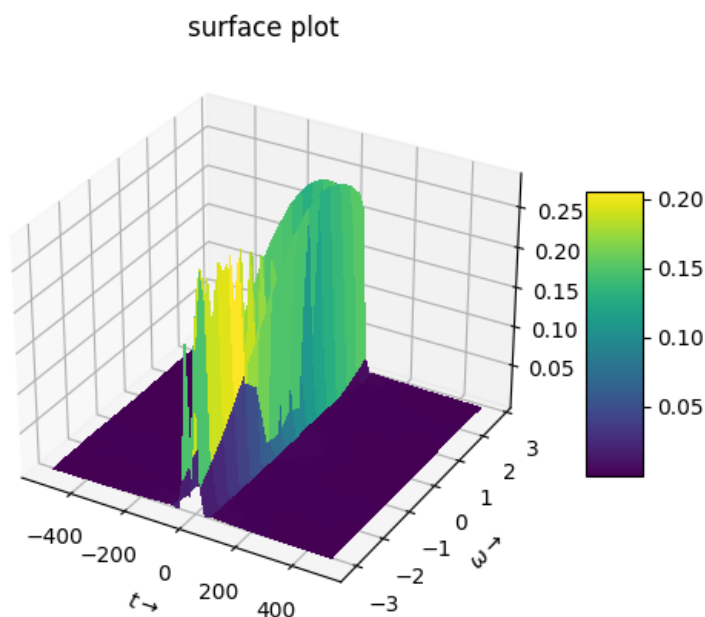


Figure 4: Chopped Chirp function, —Fourier transform—

## 4 Conclusion

In this assignment we have covered the requirement of windowing in the case of non-periodic series in DFT's. In particular this is to mitigate the effect of Gibbs phenomena owing to the discontinuous nature of the series  $\hat{x}[n]$  realised by a discrete fourier transform.

The last question addresses the time varying spectra for a chirped signal, where we plot fourier spectra for different time slices of a signal. We noted the case of sparse number of slices and hence took more closely spaced slices.



The general properties of a fourier spectra for a chirped signal are observable in the time avrying plots , ie..., existence of two peaks (slow growth), vanishing of chirp effects in case of a windowed transform, and a phase plot that periodically varies with reduced phase near maximum values.