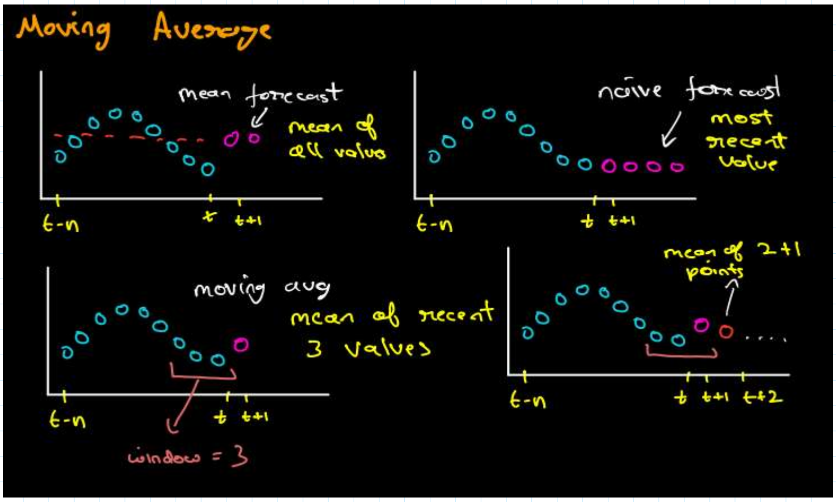
Live TS Class 3

23 March 2024 20:59



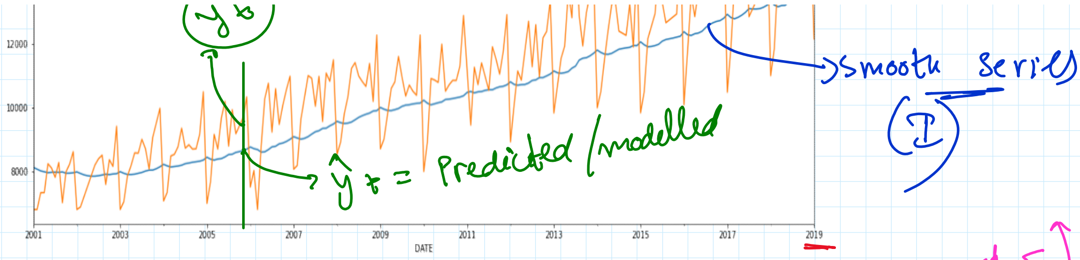
Forecast → last few observ^n

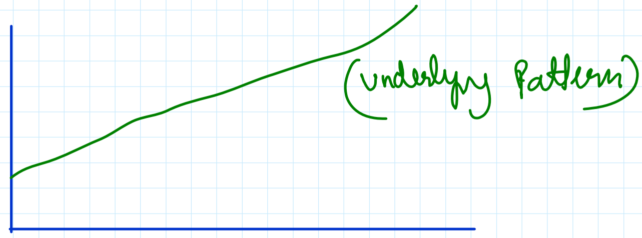
SES → (memory of entire T-S) + (more weight to recent forecast)

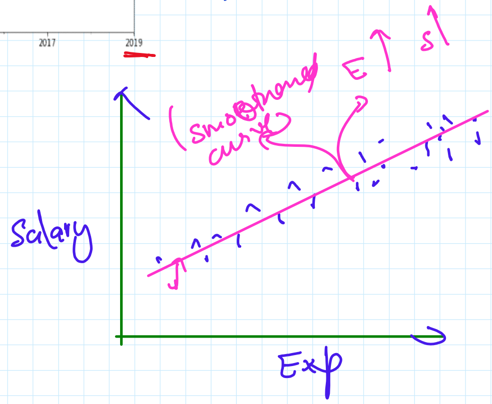
Instead of ignoring the past values completely, let's assign them a small weight.
The major weight would still be assigned to the recent most entry.

Model

(1) (Forecast)







Modelling

$$\boxed{\hat{y}_{t+1} = \alpha y_t + \alpha(1-\alpha) y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \alpha(1-\alpha)^3 y_{t-3} + \cdots}$$

(Claim) $*$ (I)

$$\hat{y}_{t+1} = \alpha y_t + (1-\alpha)\hat{y}_{t-1}$$

$$\left[\, y_t \;\middle|\; \hat{y}_t \,\right]$$

$$\hat{y}_{t+1} = \alpha y_t + (1-\alpha)\left[\alpha y_{t-1} + (1-\alpha)\hat{y}_{t-2}\right]$$

$$\hat{y}_{t+1} = \alpha y_t + \alpha(1-\alpha) y_{t-1} + (1-\alpha)^2 \hat{y}_{t-2}$$

Simple

$$\hat{y}_{t+1} = \underbrace{\alpha y_t}_{\text{Recent obs}} + (1-\alpha)\hat{y}_{t-1}$$

Toy Example ($\alpha = 0.2$)

	y_t	$\hat{y}_t = (S_t)$
W1	20	→ 20
W2	18	→ $\alpha(18) + (1-\alpha)20 = 19.6$
W3	21	→ $\alpha(21) + (1-\alpha)19.6 = 20$
Today → W4	22	→ $\alpha(22) + (1-\alpha)20$
W5		
W6		
⋮		

$\hat{y}_2 \leftarrow$ W2

$\hat{y}_t$ (smoothened) $= \alpha y_t + (1-\alpha)\hat{y}_{t-1}$

Recent obs

$$\hat{y}_t = \alpha y_t + (1-\alpha)\hat{y}_{t-1}$$

	y_t	S_t
w1	20 →	20
w2	18 →	19.6
w3	21 →	$\alpha(21) + (1-\alpha)\,19.6 = 20$
w4	22 →	$\alpha(22) + (1-\alpha)\,20 = 20.4$
Today →		
w5		
w6		

$$\hat{y}_t = S_t = \alpha y_t + (1-\alpha) S_{t-1}$$

Real value

smoothened at prev step

$$F_t = \alpha(y_t) + (1-\alpha) S_{t-1} \qquad \left[F_5 = \alpha(y_5) + (1-\alpha) S_4\right]$$

Dont have data

$$F_5 = \alpha(20.4) + (1-\alpha)\,20.4$$

$$F_5 = 20.4$$

→ Trend

→ Seasonality

$$\hat{y}_{t+h} = \underbrace{\alpha \hat{y}_t + \alpha(1-\alpha) y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \cdots}$$

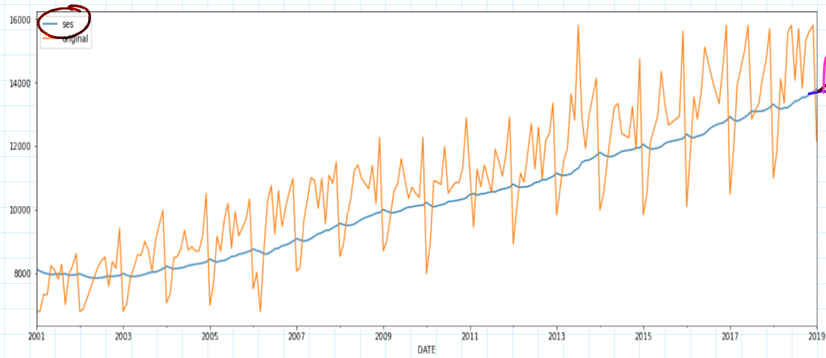
$h = 5$

$$\hat{y}_{t+5} = \alpha \hat{y}_t + \cdots$$

$$\hat{y}_{t+100} = \alpha \hat{y}_t + \cdots$$

TES = SES + Trend(·) + Seas(·)

DES = SES + Trend



SES

SES → No Trend
→ No seasonality

** DES

** TES

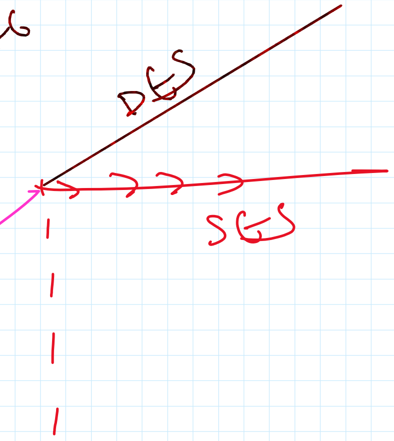
Trend is there

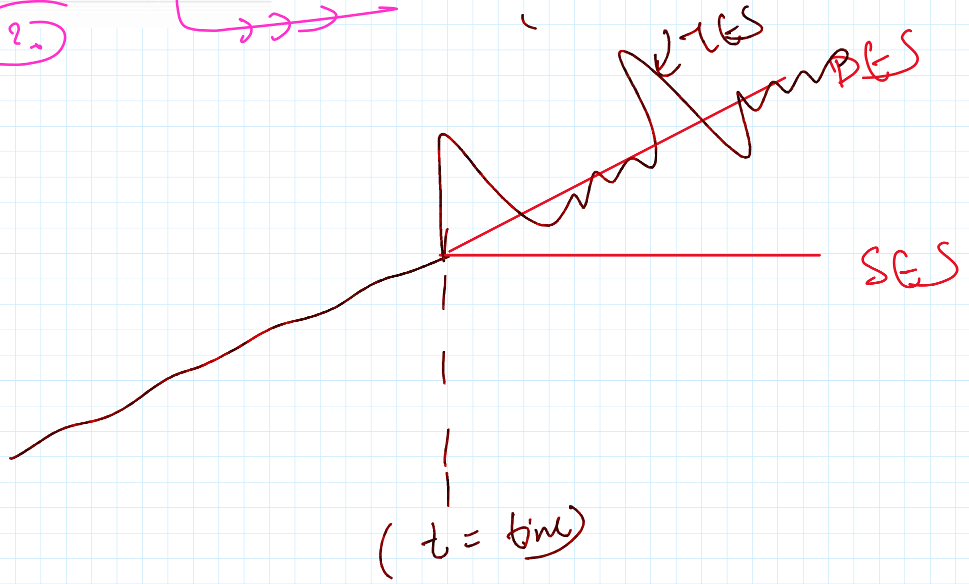
DES — Trend is there

TES — Trend is there + Seaso is also there

Week	Actual Sales (y_t)	Level (L) (a, S_t)	Trend (b)	Seaso (c)	TES ($a+bt+c$)
1	20	-			
2	22	20.5			
3	24	21.5			
4	28	23.0			
5	26	24.5			
6	30	26.5			
7	32	28.8			

?





SES

$$\boxed{S_t = \alpha y_t + (1-\alpha) S_{t-1}}$$

$S_1 = y_1$

$S_2 = \alpha y_2 + (1-\alpha) S_1$

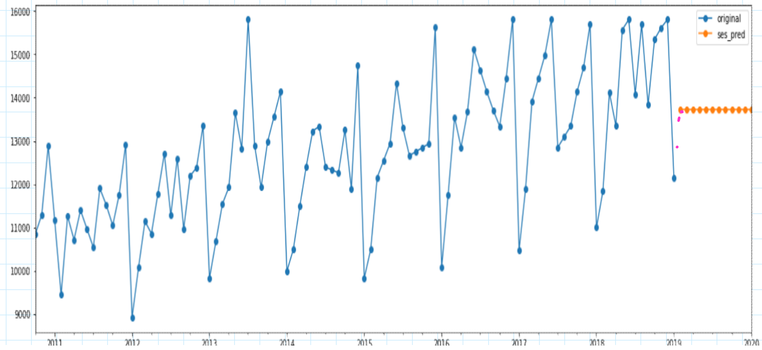
$S_3 = \alpha y_3 + (1-\alpha) S_2$

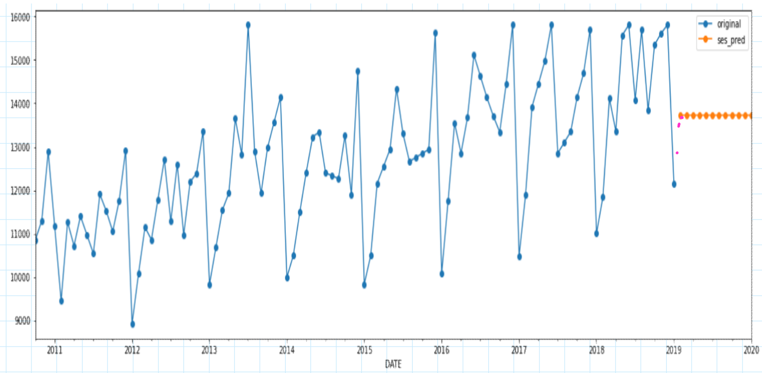
$S_4 = \alpha y_4 + (1-\alpha) S_3$

$\alpha S_3 + (1-\alpha) S_3$

$S_4 = S_3$

$S_5 = S_4$





$S_4 = \alpha y_4 + (1-\alpha) S_3$

$\alpha S_3 + (1-\alpha) S_3$

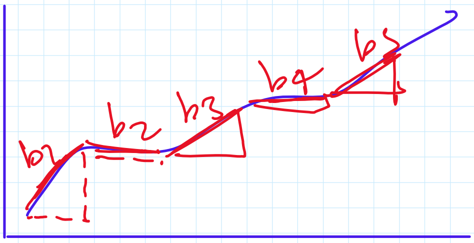
$S_4 = S_3$

$S_5 = S_4$

Double Exponential Smoothening (Holts method)

→ ✗ seasonality

→ ✓ only Trend

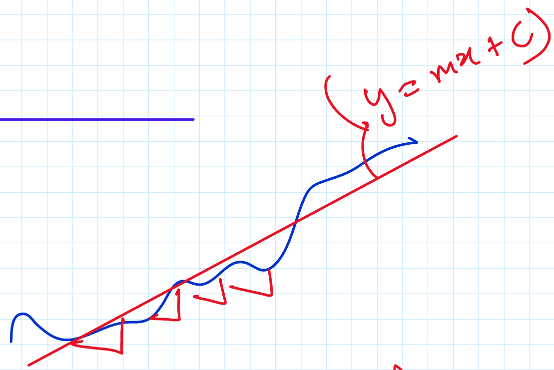


(Trend) → (b_t)

capture level

SES ⇒ $(S_t) = (l_t)$

$$\begin{cases} DES = S_t + h b_t \\ \hat{y}_{t+h} = l_t + h b_t \end{cases}$$



(trend = slope)

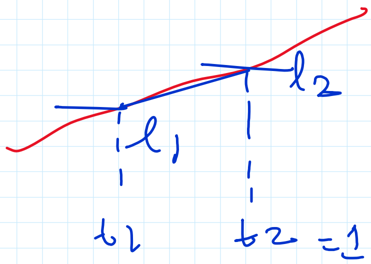
* $(b_t \approx \text{slope})$

$\beta \to [0, 1]$

$$\boxed{b_t = \beta \underbrace{(l_t - l_{t-1})}_{\text{trend}(t)} + (1-\beta) \underbrace{b_{t-1}}_{\text{trend } t-1}}$$

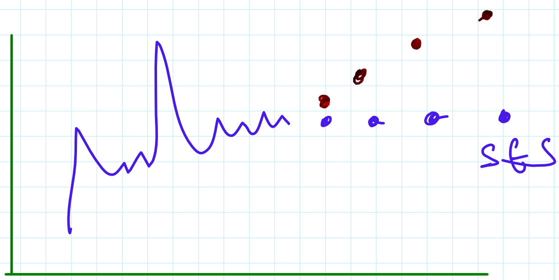
Trend

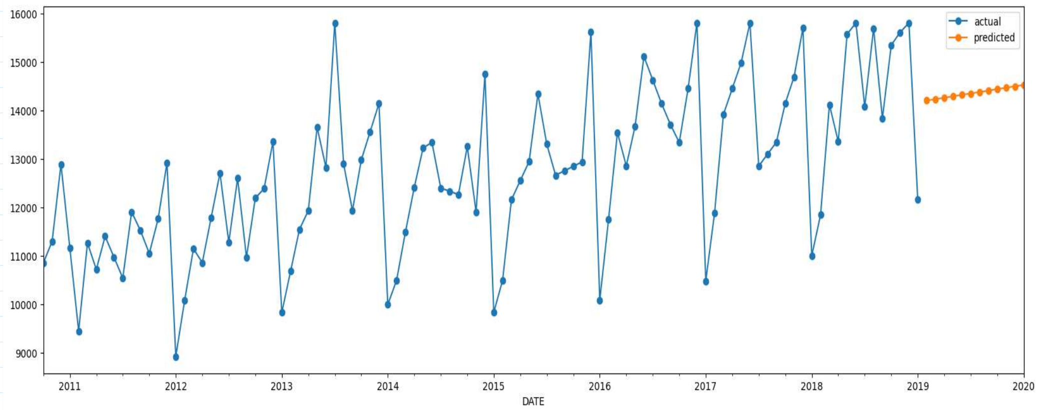
$$\boxed{b_t = \beta (\text{trend})_t + (1-\beta)\, \text{trend}_{t-1}}$$



(trend) $b_2 = \left(\frac{l_2 - l_1}{t_2 - t_1}\right) \Rightarrow (l_2 - l_1)$

$DES = SES + \underbrace{b_t}_{\text{(growth)}}$



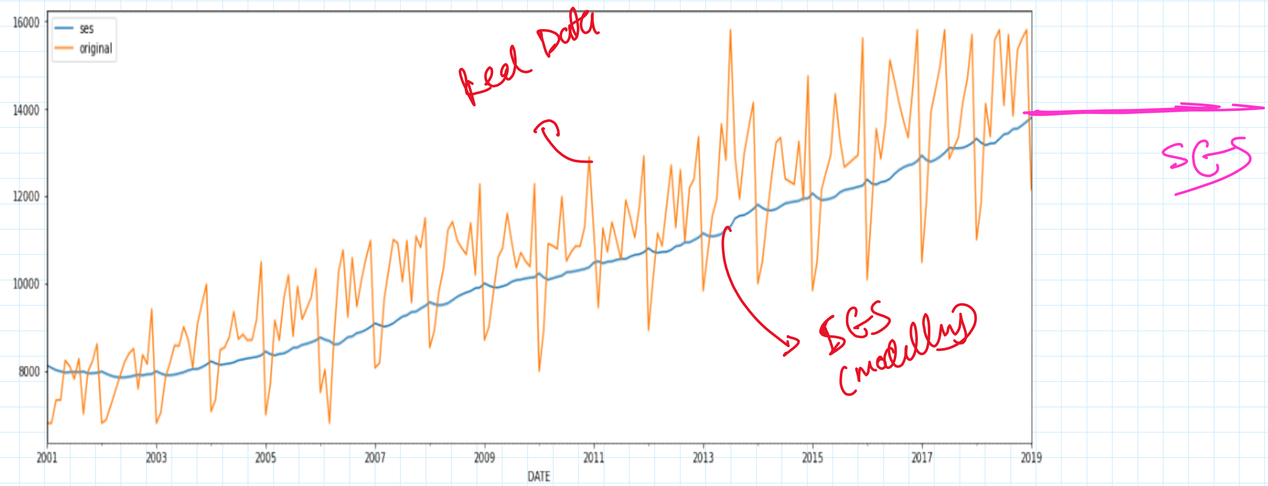


TES * → *(Trend + Seasonality)

$\hat{y}_{t+h} =$

h=1 → h=2 → h=3 $\hat{y}_{t+3}$

\# ⇒ ** 22:17 pm → 5min break



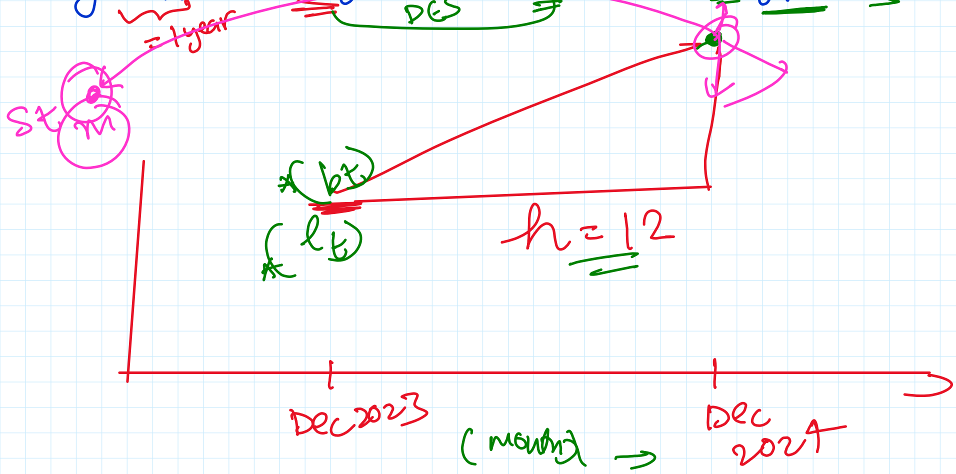
Triple Exponential Smoothing

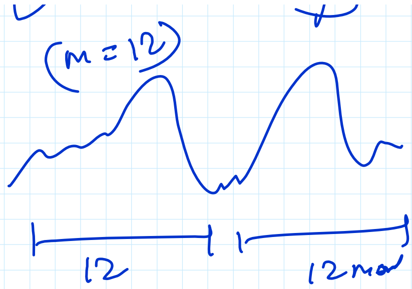
$\hat{y}_{t+h} = l_t + h b_t + S_{t+h-m}$ [m → seasonality] (m=12)

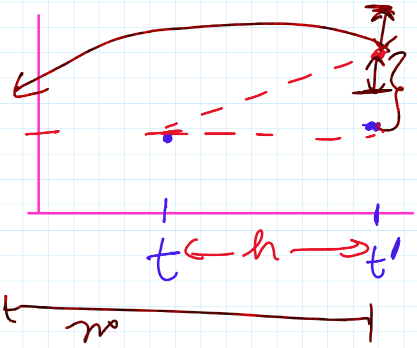
= 1 year

DES

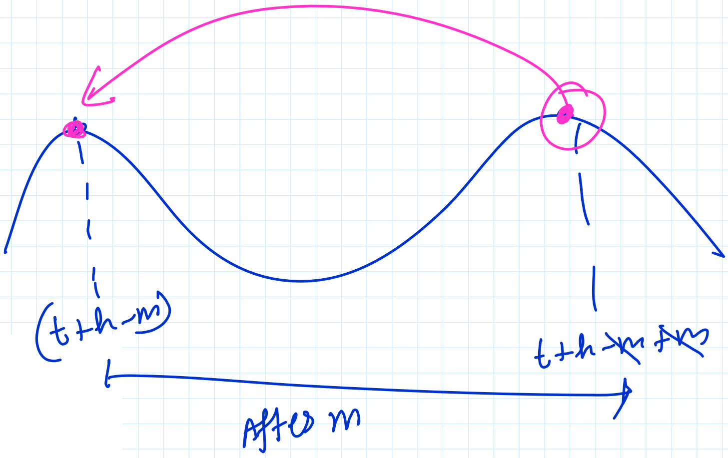
$y_{t+h} = l_t + h b_t + s_{t+h-m}$







(t+h −m)



Don't memorise

Q. How does this change the math formulation?

Upon incorporating the seasonality, our equation becomes,

$$\hat{y}_{t+h} = l_t + hb_t + s_{t+h-m}$$

where m -> frequency of the seasonality, i.e., the number of seasons in a year.

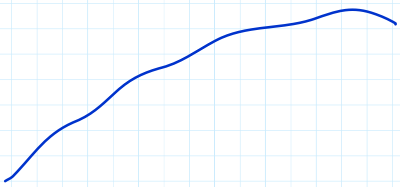
For example, for quarterly data m=4, and for monthly data m=12.

Here,

$$l_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1})$$
$$b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1}$$
$$s_t = \gamma(y_t - l_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m}$$

Holt winters

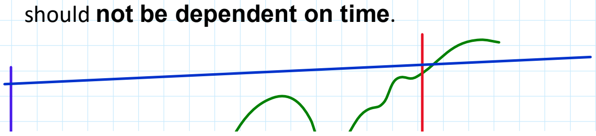
Stationarity →



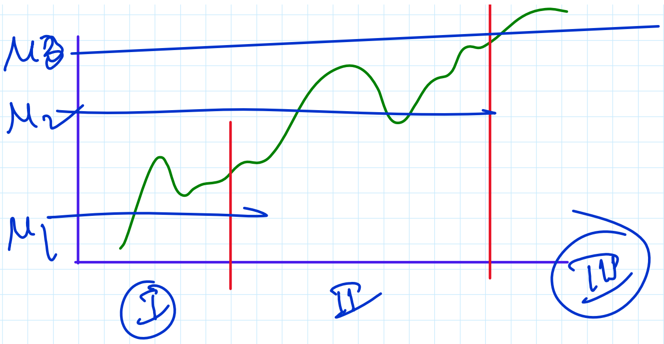
(mean, variance, amp) should not vary with time

Being stationary means that, parameters (like mean, variance, amplitude, frequency) of the models should **not be dependent on time**.

MB



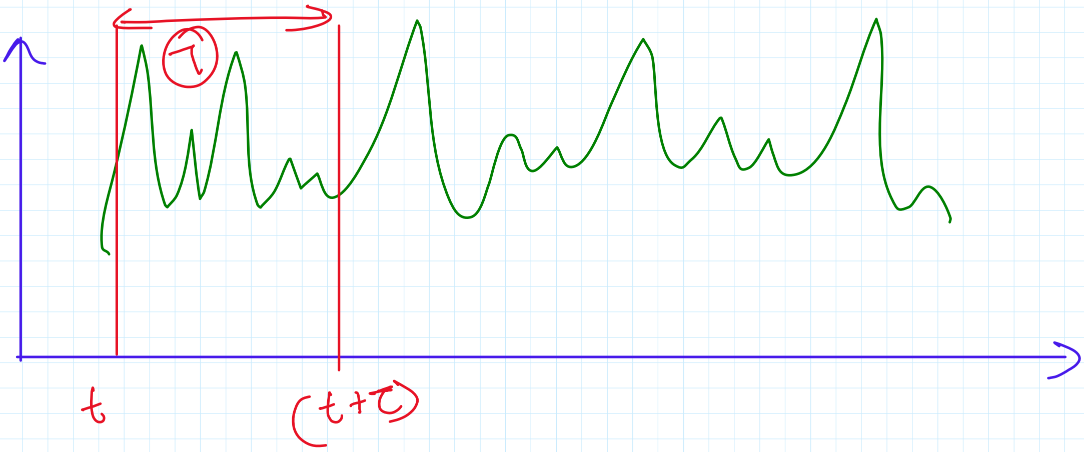
Trend / seasonality is NOT



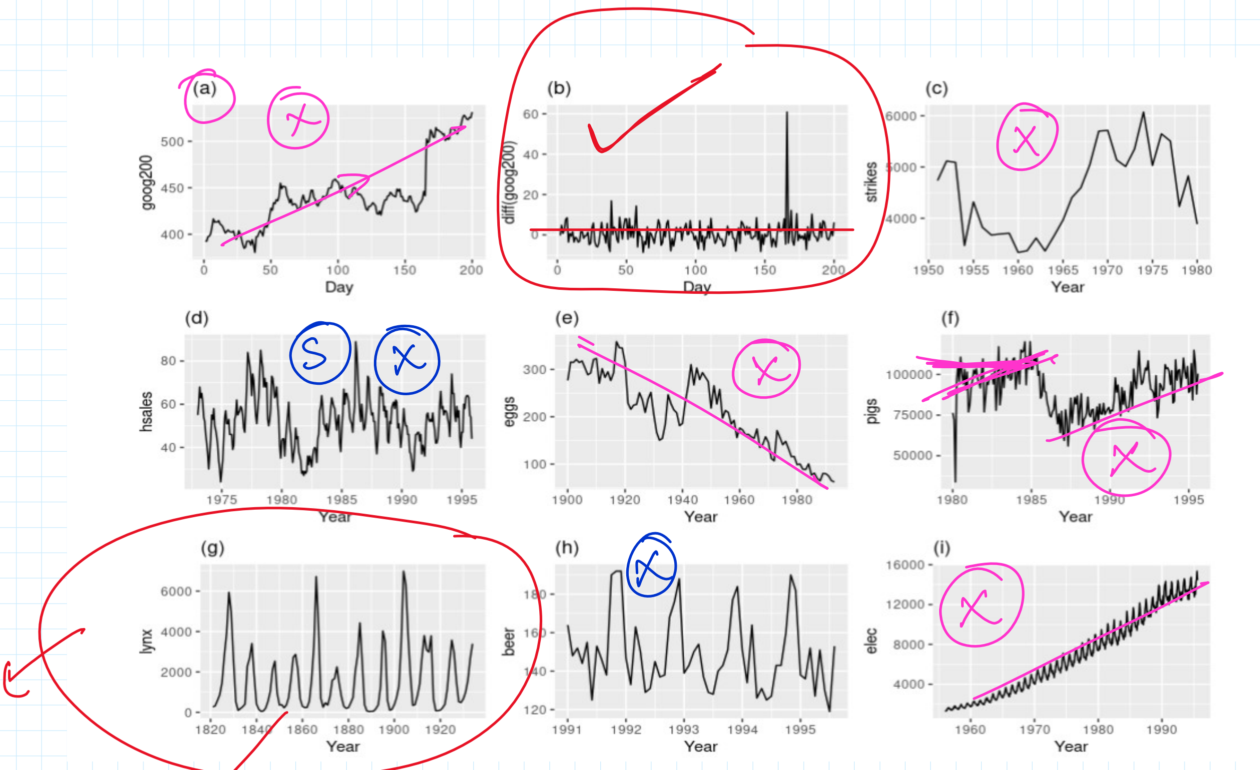
* {Trend / seasonality is NOT stationary}

Assumption →

Def^n



Distrib^n of T is independent of (τ)



(can't say) >> Stationary

[/] → (fn) → Yes/No w some confidency

(4) [ ] → (fn) → [t]/ some constant

(Augmented Dickey fuller Test)

H_0: Sequence is not Stationary

H_a:

* P (value)

$y(t) =$ Trend + Season + error

$y(t)$ − Trend − Seasonality = error → could be Stationary

De-Trend

$y(t) = \underbrace{m x_t + c}_{\text{(straight line)}} + s(t) + r(t)$

$y'(t) =$ const $+ s'(t) + r'(t)$ → (free of trend)

$mx^2 + cx + d$

$y'(t) = 2mx + c$

$y''(t) = 2m$ → free of trend

Differencing {≈ to differentiation}

week	Y_t sales	$Y[t-1]$	$y(t) - y(t-1)$
1	10	NA	NA
2	15	10	5
3	18	15	3
4	19	18	1
5	25	19	6
6	30	25	5
		30	NA

This resulting $y(t) - y(t-1)$ is free of trend

effectively derivative

Differentiation (Trend ⇒ $|l_t - l_{t-1}|$

Differentiation

Trend l_t @ $\Rightarrow \left(\frac{l_t - l_{t-1}}{1}\right)$

$$y(t) = (mx + c) + (S(t)) + \text{error}$$

$$y'(t) = (\text{const}) + s'(t) + err'(t)$$

$$\left(y' = \frac{dy}{dt}\right)$$

$$\boxed{y' = \frac{dy}{dt} = \lim_{\Delta t \to 0} \frac{y(t+\Delta t) - y(t)}{\Delta t}}$$

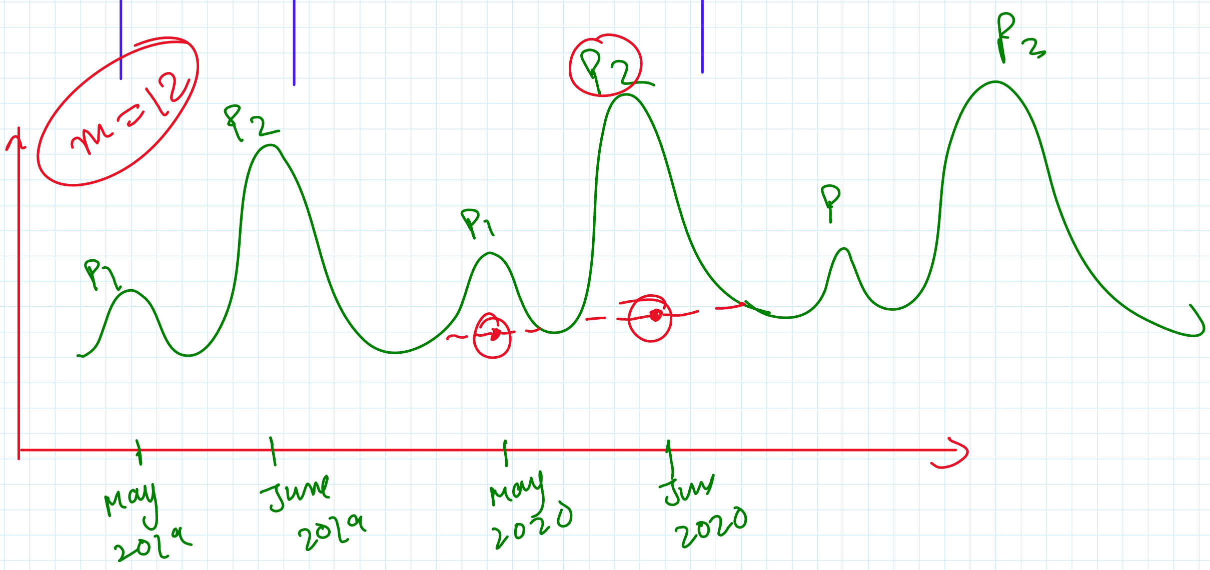
$(\Delta t = 1)$

$$\frac{dy}{dt} \approx \frac{y(t+1) - y(t)}{1}$$

De-Seasonlising (Removing seasonality) {m-differencing}

m

$y(t)$	$y(t-m)$	$\Delta y = y(t) - y(t-m)$

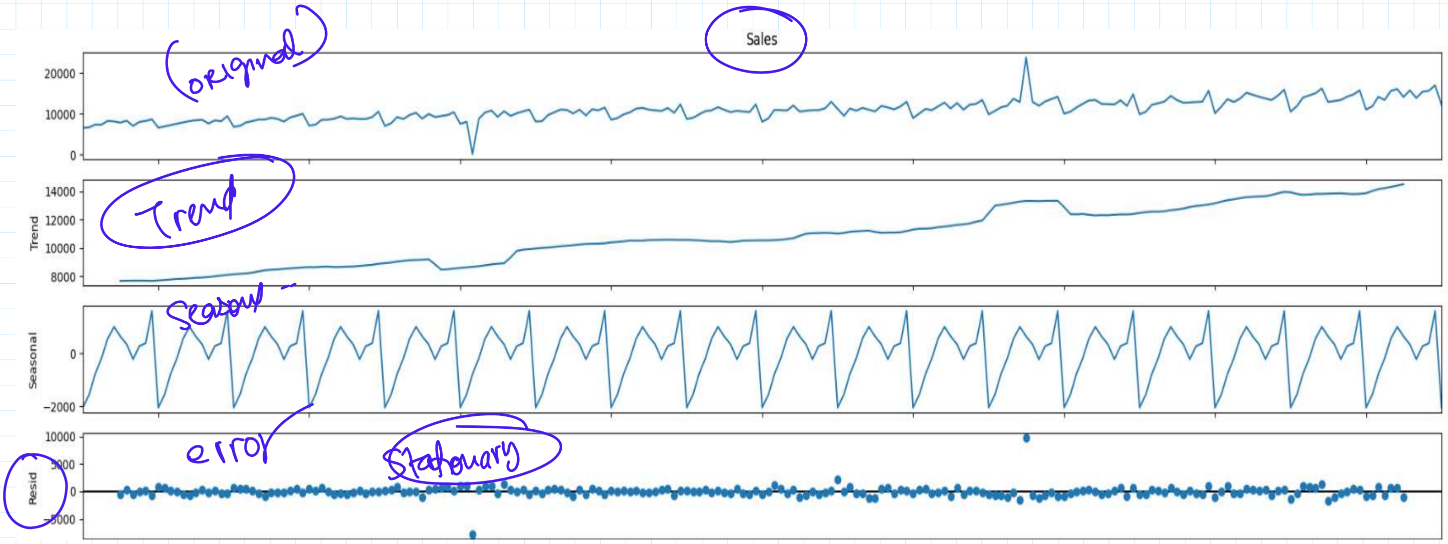


2019 202 202 202

week	yt	y(t-5)
1	10	
2	12	
3	18	
4	19	
5	15	
6	12	10
7	20	12
8	25	18
9		
10		
11		
12		
13		
14		
15		

m=5

df.shift(12)



Next class

How to find m = ?
→ ACF
→ PACF

5 } → 10 mints
5 mint Summary

github →

github → 5 minute Summary
→ Interview Ques (only from the Today's class)

df.diff(1).diff(1)

$*(y = \underline{t} + s + \text{error})$

{detrend} = df.diff(1)

de-seas-~~trend~~ = detrend.diff(1)

→ t+s