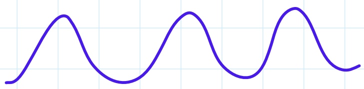
TS - Class 4

26 March 2024 20:56

Agenda → ACF (Auto Corel^n fn) & PACF → Partial Auto corelation fn)

→ ARIMA models → AR, → MA, → ARIMA, → SARIMA, → SARIMAX

$$\boxed{y_t - y_{t-m}}$$



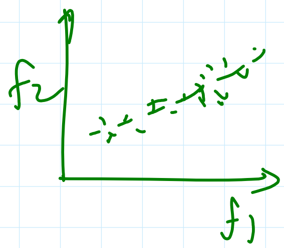
$m = 12$

stock → $(m = 3$ months)

Corelation & Covariance

→ $r = +1$ to -1

f_1	f_2
2	5
3	6
5	7
8	9
9	10



$r = +1$ (there is Linear relt^n)

$r = -1$ (f_1 ↑ then f_2 will ↓)

($r =$ f_1 & f_2 = 0.8)

(temp in °C) | (Ice cream Sales)

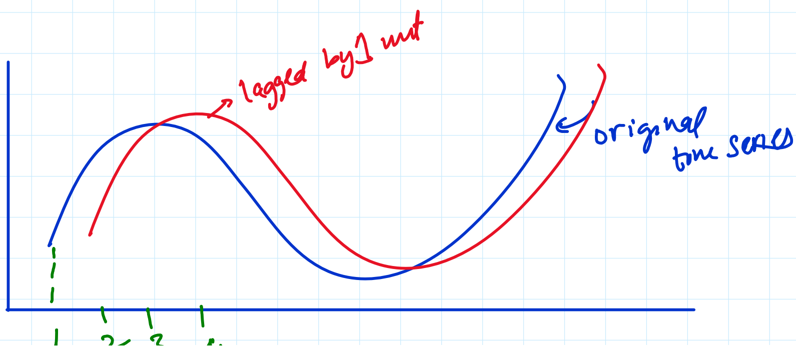
self

Auto Corelation → & (±1)

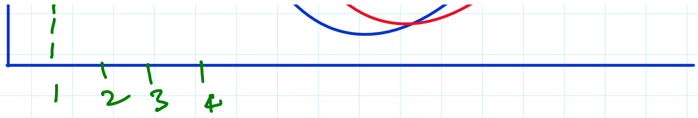
→ optimal value of m = 12 / 3 / ---
(which is the best m)

New Time series

y_t	y_{t-1}
t_1 2	NA
t_2 3	→ 2
t_3 8	→ 3
9	→ 8



t2	
t3 8	3
9	8
10	9
15	10



$$\rho = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{SD(x)\ SD(y)} =$$

ACF(1)

put here

$$\rho_{y_t \& y_{t-2}} = 0.7$$

** ACF(2)

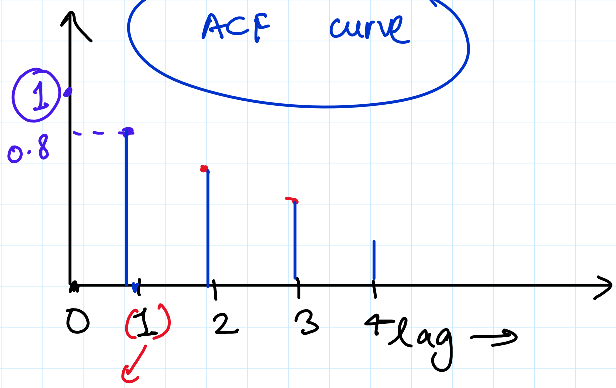
y_t	y_{t-2}
—	—
—	—
—	—

ACF curve

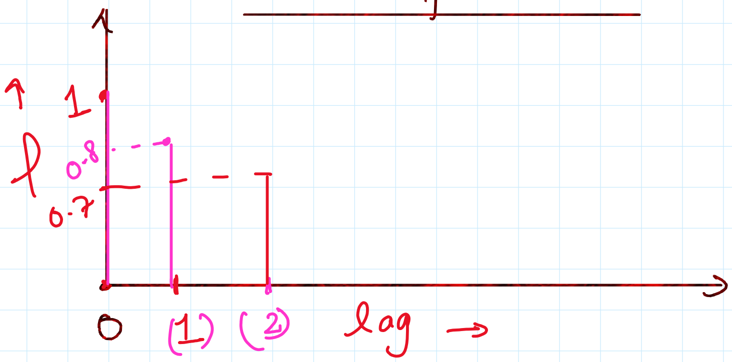
y(t)

y(t)	y(t-2)
—	—
—	—
—	—
—	—

ACF curve



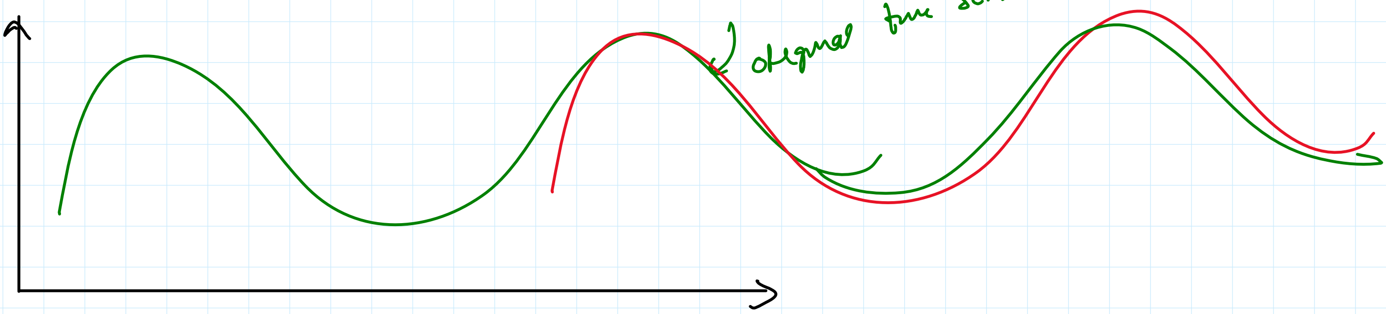
ACF curve from scratch



y(t)	y(t-2)
2	NA
3	NA
5	2
8	3
9	5
10	8
11	9

= 0.8

Q why ρ at y^t and y^{t-m} so high?



→ That's how you find the optimum value of "m"

→ Business Sense → 3/4

PACF → Partial Auto correlation fn

The difference is that: **All intermediate/indirect correlations are removed**.

ACF $y(t)$ and $y^{12}(t)$

y & y^{12} → $\rho_{y \,\&\, y^{12}}$

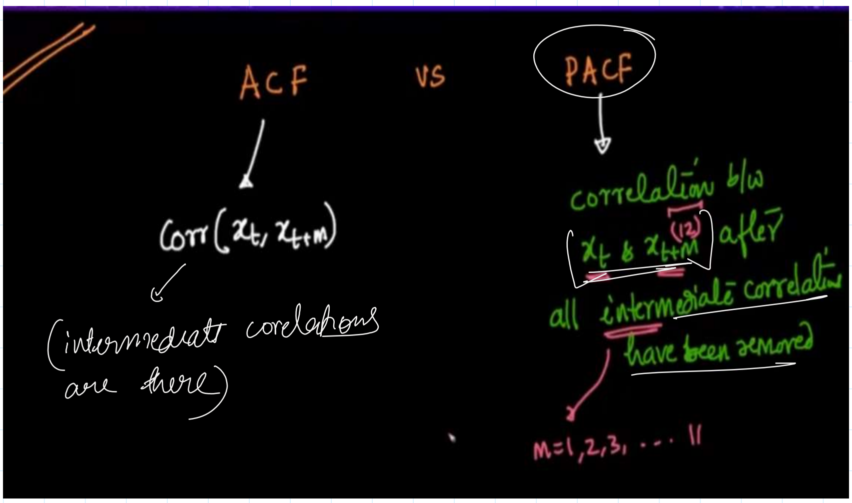
$*(y^1 - - y^{11})$ also considered

$\rho_{yy^1 + yy^2 + \cdots yy^{11} + yy^{12}}$

$\rho_{y \text{ and } y^{12}}$

ACF $\rho_{yy^{12}} = \rho_{yy^1 + yy^2 + yy^3 + \cdots yy^{12}}$

PACF $\rho_{yy12} = \rho_{yy12} = ACF_{yy12} - [yy^2 + yy^3 + \cdots yy^{11}]$



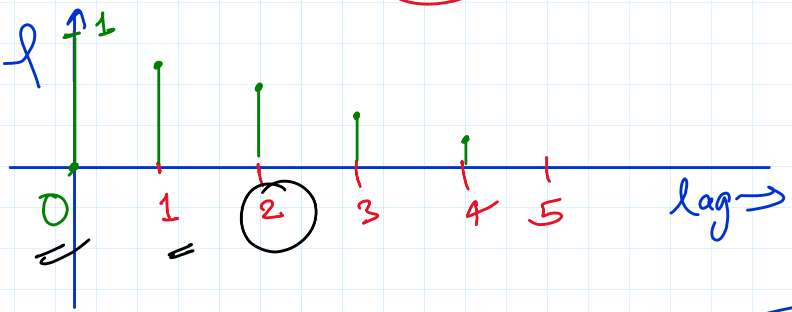
$$fn(x, y, t, w) \longrightarrow * \left(\frac{\partial f}{\partial w}\right)$$

* (PACF significance)

The partial correlation for each lag is the **unique correlation** between the two observations after the intermediate correlations have been removed.

ACF

ACF & PACF

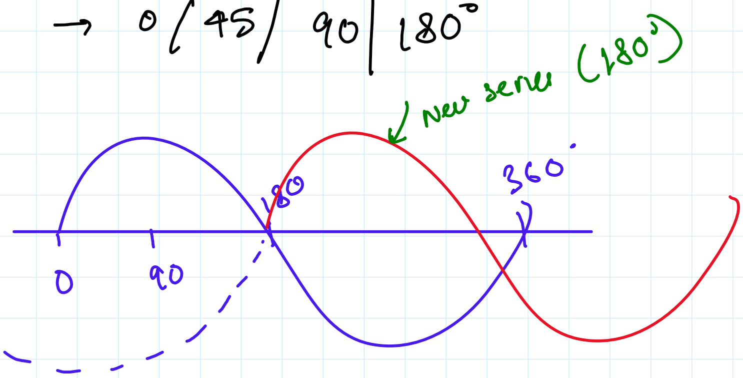


$$\underbrace{\text{Pure corr}\,(y_t \text{ and } y_{t-2})}_{\text{PACF } y_t \,\&\, y_{t-2}} = \text{ACF}(y_t, y_{t-2}) - [\text{corr}(y_t, y_t) + \text{cor}(y_t, y_{t-1})]$$

Quiz

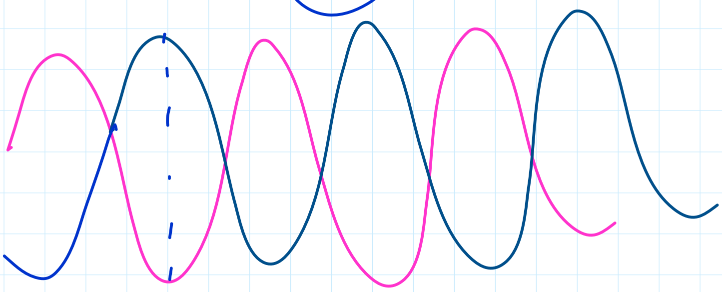
If we have a sine wave, plotted in degrees, with time period 360 degree, and 1 lag represents 1 degree, at what lag will the autocorrelation be minimum?

→ 0 / 45 / 90 / 180°



Jan 2023 → Jan 2022

m/2



At m/2

ACF at m → maxima

m/2 → local minima

US data

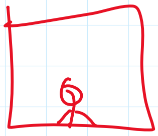
Corelation Vs Causation

Ice Cream sales	Skin Burns (city)
—	—
—	—
—	—
—	—
—	—
—	—

High corelation b/w them

** $\rho = 0.81$

Ques Can I say that ice-cream is causing skin burn?



*(They are corelated but one does not cause other)

** Sun → eat ice cream ↑

→ skin burns ↑

confounding variable

{ (A) corelated to (B)
But A is not causing B }

Causation → when one causes other

Cause & Effect

200 people → severe headache
(Randomly selecting 2 groups)

group 1 100 μ_1 = 80 units

Test group

100

→ Crocin Advance 500

100 $\mu_{new} = 5$

group 2 100 μ_2 = 80 units

Control group

→ Toffee (Nothing)

100 $\mu_{new} = 79$

(#) (Crocin is CAUSING headache to drop)

End Goal → Forecasting

still → corelation is easy

(SUN)

US data

Today Record high → {Skin Burns}

SUN

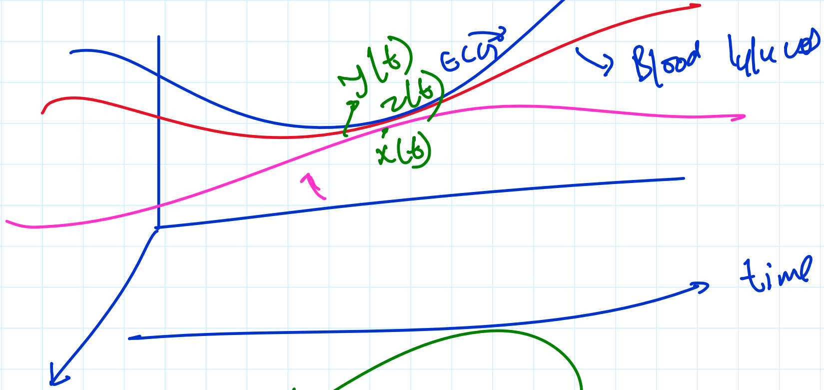
corr →

Correlations are useful for forecasting, even when there is no causal relationship between the two variables.

For example,
- It is possible to forecast if it will rain in the afternoon by observing the number of cyclists on the road in the morning.
- When there are fewer cyclists than usual, it is more likely to rain later in the day.

[Using corelation to my advantage
ACF PACF]

At (22 : 32 pm) 7 min 15 – 20 mins



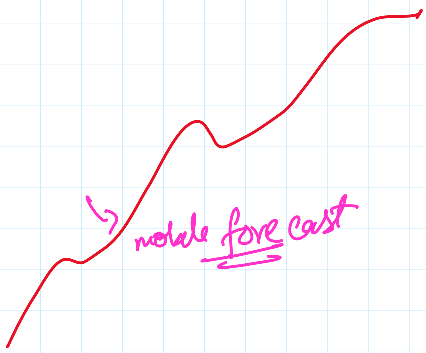
(x ⊥ y)
x(Pairwise independent)

(MVT) = UniVar + Stats (R.V)

Forecast →

ML [x_1 x_2 — — x_p] | Profit Y

TS → (only have mobile Sales column)



Corelation & Causation

Corelation & Causation

Temp ✗

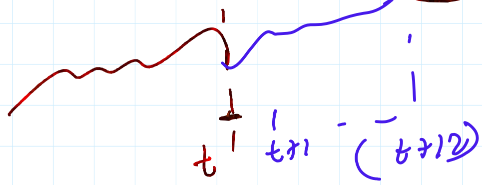
→ [ice cream corelates Skin burns] → Prediction

ARIMA → (SARIMAX)

(p, d, q)

AR(p)   MA(q)

```
from ts.sta-- import SARIMAX
model = SARIMA(p,d,q)
model.forecast(12)
```

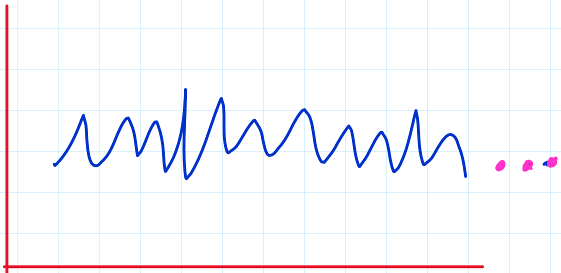


Note : Your series need to be stationary for these models to work

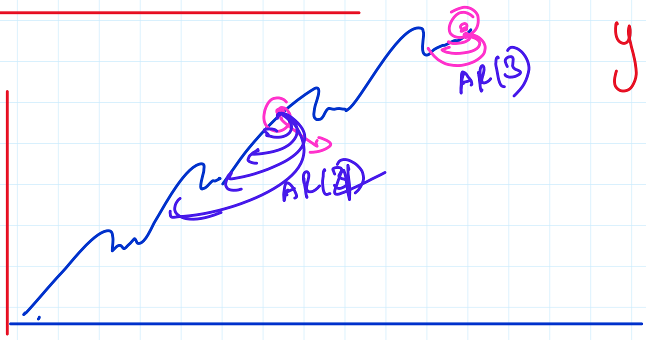
ARIMA → (Auto Regressive Integrated Moving Average) (ARIMA)

ARIMA → (1) AR(p), (2) MA(q) ⊕ ARMA(p, q) ⊕ ARIMA (Automation)

AR model → Auto Regressive model (stationary series → No Trend, No seasonality)



Idea (Model)



$y(t) = fn$(some previous time stamps)

$$\left[y_t = \phi_1 y_{t-1} + c \right]$$

** { AR(1), AR(3) }

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + c$$

$$\hat{y}_t = c + \phi y_{t-1} + \phi_2 y_{t-2} + \cdots \phi_p y_{t-p} = AR(p)$$

MLRM

Q. What does the math look like?

F → LR

Essentially, we're converting our forecasting problem to Linear Regression.

We're saying that,

Future value $\hat{y}_t$ = LinearRegression(Past p values)

Recall the Linear Regression lecture, this converts our problem into following form

AR(P) $\left[\hat{y}_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} \ldots + \alpha_p y_{t-p} + \epsilon_t \right.$

where

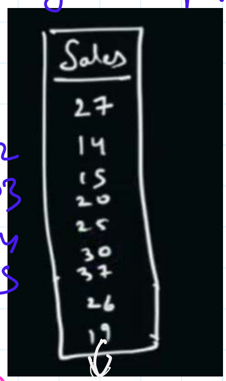
- α_0: intercept (parameter)
- ϵ_t: error term
- $\alpha_1, \alpha_2, \ldots, \alpha_p$: weights (parameters)
- p: No of past values to be considered (hyperparameter)

(α → parameter)

Hence, this is also known as the **AR(p) model**

Toy Example!

D1 D2 D3 D4 D5



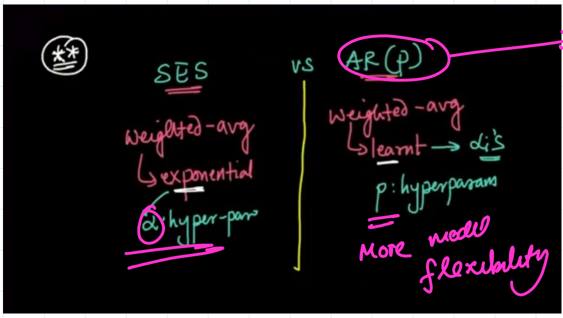
$$* \left[D_t \Rightarrow \phi_1(19) + c \right]$$

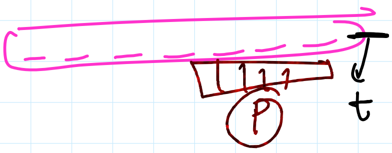
SES What is the difference ?

Recall the Simple Exponential Model. The formulation there was also similar to the one we have for AR(p) model:

$$\hat{y}_{t+1} = \alpha y_t + \alpha(1-\alpha) y_{t-1} + \alpha(1-\alpha)^2 \hat{y}_{t-2} + \ldots$$

$$(\alpha = 0.6)$$

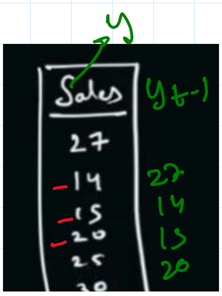


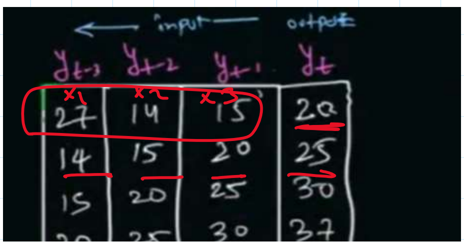


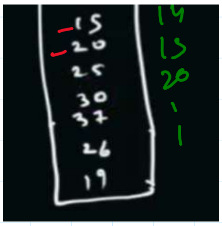
$$(\phi_1 \; - - \; \phi_p)$$

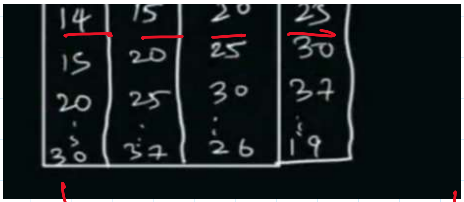
(*) How the Training looks like

y









*(G Rad Descent)

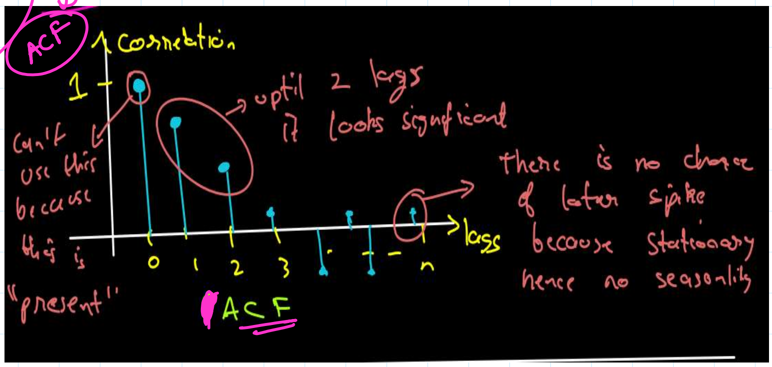
set i will find ϕ_i'(s)

How to choose the numbers of "p" (no of lagged variables)

AR(p) $\hat{y}_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \dots \alpha_p y_{t-p}$

$\rho\uparrow$ $\rho\uparrow$ $\rho\uparrow$

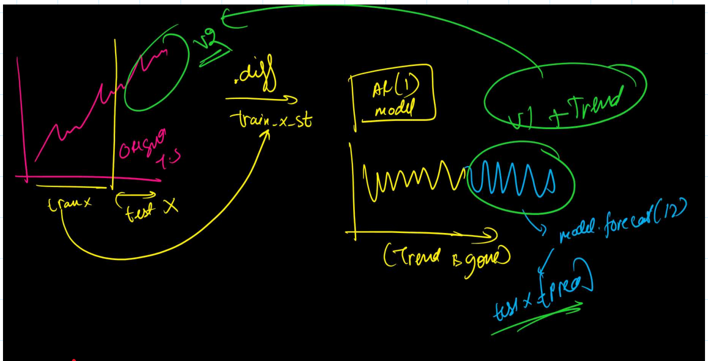
PACF



y_t → high corel^n y_{t-1} and y_{t-2}

$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2}$ (AR(2) eq^n)

Code



From start

test_x

t = original

100 120 135 140 150

20 15 5 10

Detrend

19 14 6 11

fitting AR(1) model

19 14 6 11

Cum-sum

integration

119	119 + 14 = 134	134 + 6 = 140	140 + 11 = 151