Class 1 & Class 2 - 10 Minute summary

Time Series Data

A **time series** is a sequence of data points collected or recorded at successive time intervals. The key characteristics of time series data include:

- **Temporal Ordering**: Data points in a time series have a natural temporal ordering.
- **Trends**: Long-term movement in the data over time, which could be upwards, downwards, or stationary.
- **Seasonality**: Patterns that repeat over a known, fixed period such as hours, days, months, seasons, or years.
- **Cyclical Patterns**: Fluctuations occurring at irregular intervals beyond seasonal effects.
- **Noise/Random Variation**: Random variation present in all time series data.

Handling Missing Values

Missing data can lead to significant biases or inaccuracies in time series analysis. Methods to handle missing values include:

- **Mean/Median Imputation**: Replacing missing values with the overall mean or median of the data.
- **Zero Imputation**: Filling missing values with zeros, which is context-dependent.
- **Interpolation**: Estimating missing values using other data points. Linear interpolation is common, where missing values are filled based on a line connecting surrounding data points.

Anomalies/Outliers

Anomalies are data points that deviate significantly from the rest of the dataset. They can be due to errors or genuinely unusual but significant events. Methods to handle anomalies include:

- **Capping/Truncation**: Using quantile information to cap values at a certain threshold to minimize the impact of extreme values.
- **Transformation**: Applying transformations (e.g., log transformation) to reduce the impact of outliers.

Trends and Seasonality

- **Trend**: The underlying direction in which a time series is moving. It can be estimated using moving averages or regression techniques.
- **Seasonality**: Repeating patterns or cycles in a time series data within a fixed period. It can be identified and measured through techniques like autocorrelation analysis or Fourier transforms.

Time Series Decomposition

Decomposing a time series means separating it into its basic components:

1. **Trend Component (`T`)**: The long-term progression of the series. Mathematically, a trend can be estimated as a moving average:
$$T_t = \frac{1}{n} \sum_{i=-k}^{k} y_{t+i}$$
where `n = 2k + 1` for a centered moving average.
2. **Seasonal Component (`S`)**: The repeating short-term cycle in the series. It can be estimated by averaging the series over the same periods in different cycles.
3. **Residual Component (`R`)**: The random variation left after trend and seasonality have been removed. It's what remains when you remove the trend and seasonal components from the original data:
$$R_t = Y_t - T_t - S_t$$

These components can be recombined in an additive model (`Y_t = T_t + S_t + R_t`) or a multiplicative model (`Y_t = T_t \times S_t \times R_t`), depending on the nature of the series.

Forecasting Methods

Several simple forecasting methods include:

1. **Mean Forecast**: Uses the average of the entire series to forecast future values.
2. **Naive Approach**: Assumes future values will equal the most recent observation.

3. **Seasonal Naive Approach**: Forecasts are set to the last observed value from the same season.
4. **Drift Method**: A linear trend is assumed between the first and last observed values, and this trend is projected into the future.

Mathematical Concepts in Forecasting

- **Mean Absolute Percentage Error (MAPE)** is a common metric used to assess the accuracy of forecasts:

$$\text{MAPE} = \frac{100\%}{n} \sum_{t=1}^{n} \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right|$$

where `Y_t` is the actual value and `\hat{Y}_t` is the forecasted value at time `t`.

By understanding and applying these concepts, analysts can create models that accurately predict future events based on historical data, allowing businesses and organizations to make more informed decisions.

Simple Exponential Smoothening

Simple Exponential Smoothing (SES) is a fundamental forecasting technique used in time series analysis, particularly suited for univariate data without clear trends or seasonality. The essence of SES is to generate forecasts using weighted averages of past observations, where the weights decrease exponentially for older data points. This method emphasizes more recent observations, assuming they are more relevant to the future.

What is Smoothing?

Smoothing in time series analysis refers to techniques used to remove noise and fluctuations from data to reveal underlying patterns, trends, or to make the data more digestible. Smoothing helps in highlighting the broader, more significant movements over time, making the data easier to analyze and forecast.

Why Do We Use Smoothing?

1. **Noise Reduction**: To filter out random variability or noise from the data, providing a clearer view of the underlying signal or trend.

2. **Data Analysis**: To analyze the more significant trends and patterns by overlooking minor fluctuations.
3. **Forecasting**: To generate more accurate forecasts by using the smoothed values, which represent the most relevant information.

Simple Exponential Smoothing (SES) - Detailed Analysis

In SES, forecasts are calculated using a weighted average of past observations, where the weights decrease exponentially as the observations get older. The formula for SES is given by:

$$\hat{y}_{t+1} = \alpha y_t + \alpha(1-\alpha)y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \cdots$$

- $\hat{y}_{t+1}$ is the forecast for the next period.
- y_t is the actual observation at time t.
- α is the smoothing parameter, also known as the smoothing constant, between 0 and 1.

The parameter α determines the rate at which the weights decrease. A higher α gives more weight to recent observations, making the series more responsive to changes. A lower α gives more weight to older observations, making the series smoother but less responsive to recent changes.

Mathematical Formulation

The SES can also be formulated as a recursive relationship:

$$\hat{y}_{t+1} = \alpha y_t + (1-\alpha)\hat{y}_t$$

This equation shows that the forecast for the next period is a weighted average of the current observation and the current forecast.

Selection of Smoothing Constant α

Choosing the right α is crucial for the model's performance. It can be selected using various methods, including:

- **Trial and Error**: Trying different values and selecting the one that minimizes a chosen error criterion.
- **Optimization**: Using techniques like gradient descent to find the value of α that minimizes the forecasting error.
- **Cross-Validation**: Splitting the time series into training and validation sets and choosing α that performs best on the validation set.

Limitations

- **Non-Trend Data**: SES is not suitable for data with trends or seasonal patterns as it does not account for these components.
- **Single Observation**: The method heavily relies on the most recent observation, which can be problematic if the latest data point is an outlier.

In summary, Simple Exponential Smoothing is a foundational tool in time series forecasting, particularly useful for its simplicity and effectiveness in certain contexts. Understanding and applying SES appropriately allows for better decision-making in various fields such as finance, economics, and inventory management.

Toy Example

Month	Sales (units)
January	120
February	130
March	125
April	140
May	135
June	150

Step 1: Initialize the Forecast

The initial forecast ($\hat{y}_1$) is often set to the first actual observation, so $\hat{y}_1 = 120$ units.

Step 2: Choose a Smoothing Constant (α)

For this example, let's choose $\alpha = 0.2$. This value means we're putting 20% weight on the most recent observation and 80% on the past forecast.

Step 3: Apply SES to Forecast the Next Month

Using the SES formula, we calculate the forecast for February ($\hat{y}_2$) as follows:

$$\hat{y}_2 = \alpha y_1 + (1 - \alpha)\hat{y}_1$$
$$\hat{y}_2 = 0.2 \times 120 + 0.8 \times 120 = 120$$

Since our initial forecast for January was 120 and we have no prior forecast to adjust, our forecast for February remains 120.

Step 4: Continue the Forecasting Process

We continue the process for the subsequent months:

- Forecast for March ($\hat{y}_3$):
  $\hat{y}_3 = 0.2 \times 130 + 0.8 \times 120 = 122$
- Forecast for April ($\hat{y}_4$):
  $\hat{y}_4 = 0.2 \times 125 + 0.8 \times 122 = 122.6$
- Forecast for May ($\hat{y}_5$):
  $\hat{y}_5 = 0.2 \times 140 + 0.8 \times 122.6 = 126.08$
- Forecast for June ($\hat{y}_6$):
  $\hat{y}_6 = 0.2 \times 135 + 0.8 \times 126.08 = 12$[illegible]$4$

Step 5: Forecast for July

Finally, we forecast sales for July ($\hat{y}_7$) based on the most recent actual sales in June:

$$\hat{y}_7 = 0.2 \times 150 + 0.8 \times 127.664 = 132.1312$$

Summary

According to Simple Exponential Smoothing with $\alpha = 0.2$, our forecast for July's sales is approximately 132 units. This example illustrates how SES gives more weight to recent observations while still considering past forecasts, resulting in a smoothed forecast that's responsive to changes but not overly so.

Keep in mind, this is a simplified example. In practice, you might use software like Excel, Python, or R to compute these values, especially with larger datasets. Also, the choice of α can significantly impact the forecast, and methods like cross-validation might be used to select an optimal α value.

To extend our Simple Exponential Smoothing (SES) example and forecast beyond July, let's continue with the forecast for August and September to illustrate how the forecast becomes a straight line, especially when using constant smoothing factor �α and when there's no new actual sales data to update the forecast.

Recall, our last forecast for July (�^7y^7) was approximately 132.1312 units, based on �=0.2α=0.2 and the most recent actual sales data from June (150 units).

Forecast for August ($\hat{y}_8$)

Without new actual sales data for July, we use the last forecast ($\hat{y}_7$) as our most recent "observation" to forecast August:

$$\hat{y}_8 = \alpha y_7 + (1-\alpha)\hat{y}_7$$
$$\hat{y}_8 = 0.2 \times 132.1312 + 0.8 \times 132.1312 = 132.1312$$

Forecast for September ($\hat{y}_9$)

Similarly, for September, without actual sales data for August, the forecast again uses $\hat{y}_8$:

$$\hat{y}_9 = \alpha y_8 + (1-\alpha)\hat{y}_8$$
$$\hat{y}_9 = 0.2 \times 132.1312 + 0.8 \times 132.1312 = 132.1312$$

Explanation of the Straight Line Forecast

The forecasts for August and September are the same as for July, creating a flat or straight line when plotted. This outcome is a characteristic feature of Simple Exponential Smoothing when applied in the absence of new data points. The method essentially "carries forward" the last forecast into the future indefinitely, as each new forecast is a weighted average of the previous forecast and the most recent actual data point, which, in this case, remains unchanged.