

$$y = mx + b$$

$$m = \text{slope}$$

$$b = \text{intercept}$$

Given matrices:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ and } y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

The equation in matrix form:

$$y = mx + b$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \end{bmatrix} \cdot \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} b + mx_1 \\ b + mx_2 \\ b + mx_3 \end{bmatrix}$$

$$XB = Y$$

$$(X^T X)^{-1} X^T XB = (X^T X)^{-1} X^T Y$$

$$B = (X^T X)^{-1} X^T Y$$

Where:

X^T represents the transpose of matrix X .
 X^{-1} represents the inverse of matrix X .

$$BIC_k = n \ln(SSE) + k \ln(x)$$

Where,

$k = \text{degree}$
 $SSE = \text{sum of squared errors}$
 $n = \text{number of data points}$

$$MSE = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2$$

Where,

$n = \text{number of data points}$
 $Y_i = \text{real values}$
 $\hat{Y}_i = \text{predicted values}$

You can rewrite MSE as

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - (m x_i + b))^2$$

$$\text{let, } f(m, b) = (y_i - (m x_i + b))^2$$

Partial derivatives,

$$\text{let, } g(m, b) = y_i - (m x_i + b)$$

$$\text{so, } f(m, b) = g(m, b)^2$$

$$\frac{\partial f}{\partial m} = 2 g(m, b) g'$$

$$\begin{aligned} \text{now, } g' &= \frac{\partial g}{\partial m} = 0 - (1 x_i + 0) \\ &= -x_i \end{aligned}$$

expanding,

$$\begin{aligned} \frac{\partial f}{\partial m} &= 2 (y_i - (m x_i + b))^2 \cdot (-x_i) \\ &= -2 x_i \cdot (y_i - (m x_i + b)) \end{aligned}$$

now,

$$\frac{\partial f}{\partial b} = 2 g(m, b) g'$$

$$\text{now, } g' = \frac{\partial f}{\partial b} = 0 - (0 \cdot 0 + 1) = -1$$

expanding,

$$\begin{aligned} \frac{\partial f}{\partial b} &= 2 \cdot (y_i - (m x_i + b))^2 \cdot -1 \\ &= -2 (y_i - (m x_i + b)) \end{aligned}$$

Final answer,

$$\frac{\partial}{\partial m} = \frac{2}{n} \sum_{i=1}^n -x_i (y_i - (m x_i + b))$$

$$\frac{\partial}{\partial b} = \frac{2}{n} \sum_{i=1}^n -(y_i - (m x_i + b))$$