LECTURE-10

S

		l	r
V	L	50, 50	80, 20
	R	90, 10	20, 80

$p^* = (0.7)$, $(1-p^*) = 0.3$ Check p^* is BR to q^*

$q^* = (0.6)$, $(1-q^*) = 0.4$

V payoff $L \to 50(0.6) + 80(0.4) = 0.62$

$R \to 90(0.6) + 20(0.4) = 0.62$

V payoff $p^* = (0.7)(0.62) + (0.3)(0.62)$

$= 0.62$

If p^* is not BR of V then there should be deviation which makes her strictly better off

p^* is as good as L

" " " " " R

deviating L ~~to~~ R does not increase payoff

– V has no strictly profitable pure strategy deviation

– V " " " mixed " "

$\because p(0.62) + (1-p)(0.62) = 0.62$

any mix will yield 0.62

Since there is no pure strategy deviation profitable so there won't be any mixed strategy deviation that is profitable.

Lesson:– We only have to check strictly profitable pure strategy deviations

Dating Game (cont.)

Boy

	AP	REP	
AP	2, 1	0, 0	p
REP	0, 0	1, 2	1-p
	q	1-q	

AP → Apple Picking
REP → Yale Rep

Pure Strategies:– NE (AP, AP)
(REP, REP)

Battle of the Sexes

Find mixed NE in the game

To find NE q, use G's payoff

AP → $2q + 0(1-q)$

REP → $0q + 1(1-q)$

Must be equal

$2q = 1-q$

$q = 1/3$ $1-q = 2/3$

Boy assigning 2/3rd to REP which his more favourite

To find $\overset{NE}{p}$, use B's payoff:-

$$AP \rightarrow 1p + 0(1-p)$$
$$REP \rightarrow 0p + 2(1-p)$$
$$\cancel{B}\ p = 2-2p$$
$$3p = 2$$
$$p = 2/3 \qquad 1-p = 1/3$$

Girl assigning 2/3rd to AP which is her more favourite

Check that $p = 2/3$ is BR for G:-

$$AP \rightarrow 2(1/3) + 0(2/3) = 2/3$$
$$REP \rightarrow 0(1/3) + 1(2/3) = 2/3$$
$$\tfrac{2}{3}p + (1-p)\tfrac{2}{3} = \tfrac{2}{3}$$

$$NE: \Big[\underset{G}{\big(\overset{AP}{\tfrac{2}{3}}, \overset{REP}{\tfrac{1}{3}}\big)}, \underset{B}{\big(\overset{AP}{\tfrac{1}{3}}, \overset{REP}{\tfrac{2}{3}}\big)}\Big] \xrightarrow{\text{payoff}} \Big[\big(\tfrac{2}{3}\big), \big(\tfrac{2}{3}\big)\Big]$$

~~End up~~ Payoffs are bad & ∵ they don't meet many times

$$\text{Prob. of meeting :- } \overset{AP}{\big(\underset{G}{\tfrac{2}{3}}\big)\big(\underset{B}{\tfrac{1}{3}}\big)} + \overset{RE}{\big(\underset{G}{\tfrac{1}{3}}\big)\big(\underset{B}{\tfrac{2}{3}}\big)} = \tfrac{4}{9}$$

Tax Paying

Tax Payer

Auditor	H (q)	C ($1-q$)	
A	2, $\boxed{0}$	4, −10	p
N	4, 0	0, $\boxed{4}$	$1-p$

H = Honestly
C = Cheat
A = Audit
N = Not audit

So, no pure strategy NE

Find NE

Auditor:- (Payoffs)

$$A: 2q + 4(1-q)$$
$$N: 4q + 0(1-q)$$
$$2q + 4(1-q) = 4q$$
$$4 = 6q$$
$$q = 2/3$$

2/3rd of the tax payers paying taxes honestly

NE

$$\Big[\underbrace{\big(\overset{A}{\tfrac{2}{7}}, \overset{N}{\tfrac{5}{7}}\big)}_{\text{Auditor}}, \underbrace{\big(\overset{H}{\tfrac{2}{3}}, \overset{C}{\tfrac{1}{3}}\big)}_{\text{Tax Payer}}\Big]$$

We can think it not as randomization but proportion of the payers paying taxes

Tax Payer:- (Payoffs)

$$H: 0p + (1-p)0$$
$$C: -10p + 4(1-p)$$
$$0 = 4 - 14p$$
$$p = 2/7$$

Auditors audit 2/7th times.

Change payoffs

- Policy: Let's raise the fine to -20

Will this ~~fund~~ tax compliance go up/down?

		T: H	T: C	
Audit	A	2,0	4,−20	p
	N	4,0	0,4	$1-p$
		q	$1-q$	

Auditor:

A: $2q + 4(1-q)$

N: $4q + 0(1-q)$

$2q + 4 - 4q = 4q$

$q = 2/3$

In equilibrium tax compliance has not changed!!

~~Column~~ ~~Row~~ Column player's (Tax payer) mix depends on row player's (auditor's) payoff which didn't change.

Taxpayer:

H: $0p + (1-p)0$

C: $-20p + 4(1-p)$

$0 = 4 - 24p \Rightarrow p = 1/6$

Audit rate has gone down

→ If we increase this then also q remains same but p increases

⇒ Rich people who have higher payoff ~~when~~ on cheating & not getting audited will have same compliance rate but will be audited more.

- To get higher compliance rate change payoffs to auditor.

3 different interpretations of mix:-

1. Randomization
2. Beliefs
3. Proportion

LECTURE - 11

Evolution & Game Theory

1. Influence of GT on bio

Animal behaviour

strategies → genes

payoffs → genetic fitness