LECTURE - 9

Defn:- A mixed strategy p_i is a randomization over i's pure strategies

$p_i(S_i)$ is the probability that p_i assigns to the pure strategy S_i

- $p_i(S_i)$ could be zero. (We need not involve all strategies in randomization)
  e.g. in RPS it could be $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$
- $p_i(S_i)$ could be one → So pure strategy is special case of mixed strategy.

Payoffs from Mixed Strategy

The expected payoff of the mixed strategy p_i is the weighted average of the expected payoffs of each of the pure strategies in the mix.

p \ q	a	b	
A	2, 1	0, 0	1/5
B	0, 0	1, 2	4/5
	1/2	1/2	

suppose $p = \left(\frac{1}{5}, \frac{4}{5}\right)$ → player 1 (A, B)

$q = \left(\frac{1}{2}, \frac{1}{2}\right)$ → player 2 (a, b)

What is p's expected pay off:-

1) Ask $E u_1(A, q) = \frac{1}{2}(2) + \frac{1}{2}(0) = 1$

$E u_1(B, q) = \frac{1}{2}(0) + \frac{1}{2}(1) = \frac{1}{2}$

$$E u_1(p, q) = \frac{1}{5}(1) + \frac{4}{5}\left(\frac{1}{2}\right) = \frac{6}{10} = \frac{3}{5}$$

$$E u_1(B, q) \leq E u_1(p, q) \leq E u_1(A, q)$$

- In general it will lie between payoffs of pure strategies

Lesson:- If a mixed strategy is a BR then each of the pure strategies in the mix must themselves be BR, so in particular, each must yield the same expected payoff.

Think about R, P, S mixed $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ is BR for $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ ~~but~~ but R/P/S are also BR for $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

Defn:- A mixed strategy profile $(p_1^*, p_2^*, \ldots, p_n^*)$ is a mixed strategy NE if for each player i, p_i^* is BR for p_{-i}^*

Lesson:- If $\underbrace{p_i^*(S_i)}_{}>0$ then S_i^* is also BR for p_{-i}^*

Means we are using S_i in our mix

Example:- Tennis

Venus & Serena Williams

↳ should she shoot towards left or right of Serena?

		S leans to l	r	
V shoots to	L	50, $\boxed{50}$	80, 20	(p)
	R	90, 10	20, $\boxed{80}$	(1-p)
		(q)	(1-q)	

% of time points wins of V & S

No ~~pure~~ strategy is ~~BR~~ strictly dominated

~~No NE~~

No pure ~~...~~ strategy NE

Let's find a mixed strategy NE

- Each persons ~~BR is~~ BR is ~~mixed~~ ~~..~~ for other person's ~~BR~~ mix.

Rough

V → L p, R 1-p

S → l q, r 1-q

~~V(L, Sq) =~~

$V(R, l) = 90$ → ~~BR~~

$V(L, r) = 80$

~~V(R)~~ ~~90q~~

$V(R,q) = 90q + (1-q)20$

$V(L,q) = 50q + (1-q)80$

~~p(90q + (1-q)10)~~

~~(1-p)[80q + (1-q)20]~~

~~90pq + 10p - 10pq =~~

~~80q - 80pq + 20 + 20pq~~

~~-20p - 20q~~

Rough

~~S(p, q)~~

~~St~~ $S(r, p) = 20p + 80(1-p)$

$S(l, p) = 50p + 10(1-p)$

$q(50p + 10(1-p)) + (1-q)(20p + 80(1-p))$

$= 50pq + 10q - 10pq + 20p - 20pq + 80 + 80pq - 80p - 80q$

$V(p,q) = p(50q + (1-q)80) + (1-p)(90q + (1-q)20)$

$= 50pq + 80p - 80pq + 90q - 90pq + 20 - 20p - 20q + 20pq$

$= -100pq + 60p + 70q + 20$

$(20q - 10)(p)$

~~V(p, q) = 20 + 20pq - 10p + 60q~~

~~∂V/∂p = 0,~~ ~~20q - 10 = 0~~ ~~q = 1/2~~

$\frac{\partial V(p,q)}{\partial p} = -100q + 60 = 0 \Rightarrow q = 6/10$

~~20 + 20pq - 30p + 60q~~

$q = 3/5$

$S(q,p) = 100pq - 70q - 60p + 80 = 0$

$100p - 70 = 0$

$p = 7/10$

$100q - 60 = 0$

$q = \frac{60}{100} = 3/5$

$V(p,q) =$

$-100pq + 60p + 70q + 20$

$-100q + 60 = 0$

$q = \frac{6}{10} = 3/5$

~~60~~ $70 - 100p = 0$

$p = 7/10$

Task:- To find Serena's NE mix $(q, 1-q)$ to look at Venus's ~~If~~ payoff

V's payoff against q:

$L \rightarrow 50q + 80(1-q)$

$R \rightarrow 90q + 20(1-q)$

both L & R are BRs

If Venus is mixing in this NE then the payoff to left and to right must be equal the both must be BR

So,

$$50q + 80(1-q) = 90q + 20(1-q)$$

$$60(1-q) = 40q$$

$$3(1-q) = 2q$$

$$3 = 5q$$

$$q = 3/5$$ (Serena's mix)

To find Venus's NE mix $(p, 1-p)$ use Serena's pay off

S's payoff $l \rightarrow 50p + 10(1-p)$

$r \rightarrow 20p + 80(1-p)$

Both l & r must be BR

$$50p + 10(1-p) = 20p + 80(1-p)$$

$$70(1-p) = 30p$$

$$7 - 7p = 3p$$

$$10p = 7$$

$$p = 7/10$$ (Venus mix)

NE :- [$\overset{V}{(0.7, 0.3)}$, $\overset{S}{(0.6, 0.4)}$]

If S leans to l more than 0.6 the V must always shoot to R
If " " " r " " 0.4 " " " " L

so, if S does not choses this exact mix then V's BR is a pure strategy
& vice versa

i.e. If V shoots more than 0.7 to L then S must always lean to l
" " " " " 0.3 " R " " " " " r

S gets a new coach & her payoff change :-

		S: l	S: r	
V	L	30, <u>70</u>	<u>80</u>, 20	p
	R	<u>90</u>, 10	20, <u>80</u>	1-p
		q	1-q	

Still ~~BR~~ BRs donot coincide
So no pure strategy ~~NE~~ NE

<u>Two effects</u> :-

(1) <u>Direct effect</u> ~~so~~ S should lean l more, so q should go up

(2) <u>Indirect / Strategic effect</u>
V ~~hits~~ knows that S has improved on l, so ~~she~~ she hits L less often ~~so~~ ~~so~~ thus S must reduce no. of times she leans to l, q goes down.

Two effects going in opposite direction

<u>Redo the calculation</u>

To find new q for S use V's payoff

$L \to 30q + 80(1-q)$

$R \to 90q + 20(1-q)$

$30q + 80(1-q) = 90q + 20(1-q)$

$\cancel{60}q = \cancel{60}(1-q)$

$q = \frac{1}{2} = 0.5$

<u>q goes down !!</u>

Strategic / Indirect effect dominated
So obviously V must be hitting L less often

To find new p for V use S's payoff

$l \to 70p + 10(1-p)$

$r \to 20p + 80(1-p)$

$70p + 10(1-p) = 20p + 80(1-p)$

$70(1-p) = 50p$

$1 - p = \frac{5p}{7}$

$\frac{12p}{7} = 1$

$p = \frac{7}{12} < \frac{7}{10}$

One change effects other which inturn effects ~~first~~ first one

<u>p goes down !!</u>