Day 2: Scalars, Vectors, Matrices & Tensors

In AI, everything starts with **numbers organized into structures**. Let's define them clearly.

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Scalar

A scalar is a single number:

$$x \in \mathbb{R}$$

Example in AI: Temperature = 30°C (just one value).

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Vector

A vector is an ordered list of numbers:

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \vec{x} \in \mathbb{R}^n$$

Example in AI: A word embedding of size 300

$$\vec{w} \in \mathbb{R}^{300}$$

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Matrix

A matrix is a rectangular array of numbers:

$$A \in \mathbb{R}^{m \times n}$$

Examples in AI:

- A grayscale image (28×28 pixels):

$$A \in \mathbb{R}^{28\times 28}$$

- A dataset with 10,000 rows and 50 features:

$$X \in \mathbb{R}^{10000\times 50}$$

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Tensor

A tensor is a generalization of matrices to higher dimensions.

- Rank 0 → Scalar
- Rank 1 → Vector
- Rank 2 → Matrix
- Rank 3+ → Tensor

Examples in AI:

- A color image ($28 \times 28 \times 3$):

$$T \in \mathbb{R}^{28\times 28\times 3}$$

- A batch of 64 such images:

$$T \in \mathbb{R}^{64\times 28\times 28\times 3}$$

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Why does this matter for AI?

- Input to every ML/DL model = vector, matrix, or tensor
- Neural networks:

$$z = Wx + b$$

  (Input vector × Weight matrix + Bias vector)
- CNNs process tensors of images
- Transformers process tensors of word embeddings

Everything in AI is just **numbers organized → transformed → optimized**.