

Insights from leptohadronic modelling of the brightest blazar flare

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ABSTRACT

The blazar 3C 454.3 experienced a major flare in November 2010 making it the brightest γ -ray source in the sky of the *Fermi*-LAT. We obtain seven daily consecutive spectral-energy distributions (SEDs) of the flare in the infra-red, optical, ultra-violet, X-ray and γ -ray bands with publicly available data. We simulate the physical conditions in the blazar and show that the observed SEDs are well reproduced in the framework of a “standing feature” where the position of the emitting region is almost stationary, located beyond the outer radius of the broad-line region and into which fresh blobs of relativistically moving magnetized plasma are continuously injected. Meanwhile, a model with a single “moving blob” does not describe the data well. We obtain a robust upper limit to the amount of high-energy protons in the jet of 3C 454.3 from the electromagnetic SED. We construct a neutrino light curve of 3C 454.3 and estimate the expected neutrino yield at energies ≥ 100 TeV for 3C 454.3 to be up to $6 \times 10^{-3} \nu_\mu$ per year. Finally, we extrapolate our model findings to the light curves of all *Fermi*-LAT flat-spectrum radio quasars. We find that next-generation neutrino telescopes are expected to detect approximately one multimessenger ($\gamma + \nu_\mu$) flare per year from bright blazars with neutrino peak energy in the hundreds TeV – hundreds PeV energy range and show that the electromagnetic flare peak can precede the neutrino arrival by months to years.

Key words: galaxies: active – galaxies: jets – quasars: individual: 3C 454.3 – neutrinos – methods: data analysis – methods: numerical

1 INTRODUCTION

Blazars – bright active galactic nuclei (AGNs) with their relativistic jets pointing at small angles towards Earth – dominate the γ -ray sky visible by the *Fermi* Large Area Telescope (*Fermi*-LAT) (Atwood et al. 2009) in the region far from the galactic plane (Abdollahi et al. 2020). Since the evidence of the association of the *IceCube* neutrino event IceCube-170922A with the electromagnetic flare of blazar TXS 0506+056 in 2017 (IceCube Collaboration et al. 2018), blazars became objects of intensive theoretical studies as potential sources of the astrophysical neutrinos (e.g., Keivani et al. 2018; Ansoldi et al. 2018; Murase et al. 2018; Sahakyan 2018; Gao et al. 2019; Oikonomou et al. 2019; Cerruti et al. 2019; Reimer et al. 2019; Rodrigues et al. 2019; Liu et al. 2019; Petropoulou et al. 2020; Viale et al. 2024; Robinson & Böttcher 2024). Multiple searches for correlations between the *IceCube* neutrinos and different electromagnetic counterparts among blazars resulted in the conclusion that blazars contribute to the total *IceCube* astrophysical neutrino flux at the level of around $\lesssim 1 - 30\%$ (for a review, see Troitsky 2021, table 4 and references therein; Abbasi et al. 2023b; Troitsky 2024).

Although some individual blazars were suggested as candidates for the sources of astrophysical neutrinos – besides TXS 0506+056, there are PKS 0735+178 by Sahakyan et al. (2023); Prince et al. (2024); Acharyya et al. (2023); PKS 1424+240 by Padovani et al. (2022); PKS 1502+106 by Rodrigues et al. (2021); Oikonomou et al. (2021) – there is at the moment no firmly established paradigm on

what particular properties of blazars make them the most efficient neutrino emitters, or, in other words, what subclass of the blazars should contribute the most to the astrophysical neutrino flux. Traditionally blazars are divided into the subclasses of flat-spectrum radio quasars (FSRQs) and BL Lacertae-like objects (BL Lacs), where the former are characterised by the presence of strong line emission from the broad-line region (BLR) and higher accretion rate, while the latter lack the presence of the BLR and have a lower accretion rate (Blandford et al. 2019). FSRQs hosting strong photon fields serving as a target for photopion production may be promising candidates for the sources of high-energy neutrinos (Murase et al. 2014; Rodrigues et al. 2018).

FSRQs are sources with high flaring activity (Ackermann et al. 2011; Abdollahi et al. 2017; Yoshida et al. 2023) with enhanced observable electromagnetic intensity due to the increase of the luminosity of charged particles in the source and/or the apparent Doppler factor which leads to the amplified neutrino production. Depending on the details of a particular model flares may dominantly contribute to the diffuse neutrino flux (Murase et al. 2018). Besides, the atmospheric background is limited by the time window of a flare.

Among FSRQs in the *Fermi*-LAT catalogue 4FGL, 3C 454.3 is the brightest one (Abdollahi et al. 2020). In November 2010, it flared in the broad electromagnetic spectrum (from radio waves to γ rays) and became, for some time, the brightest object in the *Fermi*-LAT sky (Abdo et al. 2011). Nalewajko (2013) ranked that flare with the peak around MJD 55520 as the brightest blazar flare (defined by the flux at the peak). Observations with different instruments covered it very well. Hence, we choose this source as the study object.

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In this work, we develop a model which describes the observational data during the flare and calculate the expected neutrino yield observable by *IceCube* from 3C 454.3. After that, we extrapolate the obtained results on the whole population of FSRQs observed by the *Fermi*-LAT to estimate their contribution to the *IceCube* neutrino flux at energies ≥ 100 TeV within the framework of our single-zone leptohadronic model.

Using publicly-available data from *SMARTS*, *Swift*-UVOT, *Swift*-XRT, and *Fermi*-LAT, we obtain seven multi-wavelength quasi-simultaneous one-day averaged SEDs observed during the flare period. We describe the multi-wavelength data analysis in Sect. 2. We model (Sect. 3) the observable flare SEDs with the publicly-available AM³ program (Klinger et al. 2023, 2024) within three different frameworks: 1) assuming a standard leptonic model where the observed electromagnetic emission comes from relativistic electrons losing energy via synchrotron and inverse-Compton (IC) radiation in compact blobs passing through a quasi-stationary feature in the jet into which relativistic plasma is injected with large bulk Lorentz factor (“standing-feature” model, Sects. 3.1, 4.1); 2) utilizing the AM³’s capabilities of the time-dependent modelling, we attempt to model the whole flare in framework of a “moving-blob” model where the whole radiation comes from a single blob co-moving with the bulk motion of the relativistic jet plasma (“moving-blob” model, Sects. 3.2, 4.2); 3) assuming that in addition to electrons, protons might be co-accelerated in the jet with the same efficiency, we calculate the maximum-allowed by the data proton energy density u_p^{UL} within a leptohadronic “standing-feature” model (Sec. 3.3, 4.3), providing our method to estimate the expected *IceCube* neutrino yield in Sect. 3.4 with corresponding results in Sect. 4.4. Our method to extrapolate the results for 3C 454.3 on the whole population of *Fermi*-LAT FSRQs is presented in Sect. 3.5 and the results of this are presented in Sect. 4.5. We discuss our findings in Sect. 5. In Sect. 6, we conclude that it is not possible to describe the seven days of the flare with a single “moving-blob” model while a “standing-feature” model does it successfully. Moreover, we find that multi-wavelength flares of *Fermi*-LAT FSRQs similar to the studied 3C 454.3 flare contribute to the neutrino flux at energies ≥ 100 TeV at the level of $\sim 0.5\%$ under the assumption that their emission can be described within a single-zone “standing-feature” leptohadronic model with the scaling between the observed integral neutrino and γ -ray fluxes inferred from the 3C 454.3 modelling for MJD 55517–55524.

2 DATA ANALYSIS

2.1 *Fermi*-LAT – γ rays

For our object of interest, 3C 454.3, we select the *Fermi*-LAT data in the energy range $100 \text{ MeV} \leq E \leq 1 \text{ TeV}$ and in time intervals corresponding to one day within the period MJD 55517–55524. The analysis is performed with the standard `fermipy` (Wood et al. 2017) package for the binned likelihood analysis. During the *Fermi*-LAT data analysis of the 15° ROI (region of interest) centred at the position of 3C 454.3 we apply a cut on the zenith angle value $\theta_z \leq 90^\circ$ in order to remove contamination from the Earth’s limb. We use the P8R3_SOURCE_V3 instrument response function (`evclass = 128`) with detected γ rays both in FRONT and BACK of the *Fermi*-LAT (`evtype = 3`)¹. We divide the data into six bins per decade in energy,

and the angular binning is 0.1° per pixel. To describe the observed γ -ray emission we construct a model containing all sources from the 4th *Fermi*-LAT source catalogue 4FGL (Abdollahi et al. 2020) located within 15° from the centre of the ROI², the model of the diffuse galactic background `gll_iem_v07`, and the model of the isotropic γ -ray background `iso_P8R3_SOURCE_V3_v1`³. The normalizations of the diffuse backgrounds, 3C 454.3 and other sources within 7.5° from the centre of the ROI are left free during the initial fitting of the ROI with all spectral shapes fixed according to the catalogue; both normalizations and spectral shapes of other sources outside 7.5° from the ROI centre are fixed to their catalogue values. After the initial fit, the spectral shape of 3C 454.3 is freed and the fit is repeated. The γ -ray SED of 3C 454.3 is then obtained using the `fermipy.GTAnalysis.sed` method with the default parameter values⁴.

2.2 *Swift*-XRT – X rays

The *Swift*-XRT data analysis is performed with `ISIS` (Houck & Denicola 2000) and `swiftnull.ukssdc/xrt_prods`⁵ (Evans et al. 2009) which rely on HEASOFT, version 6.32.1 (Nasa High Energy Astrophysics Science Archive Research Center (HEASARC) 2014; Blackburn 1995). To retrieve the *Swift*-XRT SED quasi-simultaneous to the *Fermi*-LAT one-day-averaged SED, we download the data from all the *Swift*-XRT pointings to 3C 454.3 within the given one-day-long period. We analyse the photon counts within the energy range $[0.3; 10.0]$ keV, requiring the number of channels per energy bin to be ≥ 64 and the signal-to-noise ratio in each bin ≥ 5.0 .

To better constrain the column density along the line of sight (LoS), we first obtain the SED averaged over the whole seven-day period MJD 55517–55524. The observed spectrum is modelled as a broken power law accounting for the photoelectric absorption along the LoS. The solar abundance is adopted from Wilms et al. (2000) and the photoionization cross-section table $\sigma_{\text{photoel}}(E_i)$ is from Verner et al. (1996). The initial guess for the hydrogen column density κ is set to $\kappa = 6.81 \times 10^{20} \text{ cm}^{-2}$ according to the average value calculated with the use of the web tool⁶ within the 0.1° -cone with the centre at the position of 3C 454.3 on H I maps obtained by HI4PI Collaboration et al. (2016), although the column density is left free along with the normalization, break energy, and the two spectral indices of the broken-power-law X-ray spectrum. Using `fit`, `plot_unfold`, and `write_plot` `ISIS` functions we extract both the observed SED _{o} (E_i) and the deabsorbed SED _{d} (E_i) from the best-fit model and take into account the uncertainty related to the deabsorption correction in the following manner:

$$k_a(E_i) = \frac{\text{SED}_o(E_i)}{\text{SED}_d(E_i)} = \exp(-\kappa\sigma_{\text{photoel}}(E_i)), \quad (1)$$

$$\varsigma_d(E_i) = \sqrt{\left(\frac{\varsigma_o(E_i)}{k_a(E_i)}\right)^2 + \left(\sigma_{\text{photoel}}(E_i)\text{SED}_d(E_i)\varsigma_\kappa\right)^2}, \quad (2)$$

² 3C 454.3 is present itself in the 4FGL with the name 4FGL J2253.9+1609.

³ <https://fermi.gsfc.nasa.gov/ssc/data/access/lat/BackgroundModels.html>

⁴ <https://fermipy.readthedocs.io/en/latest/advanced/sed.html#id3>

⁵ https://www.swift.ac.uk/user_objects/API/

⁶ <https://heasarc.gsfc.nasa.gov/cgi-bin/Tools/w3nh/w3nh.pl?>

¹ https://fermi.gsfc.nasa.gov/ssc/data/analysis/documentation/Cicerone/Cicerone_LAT_IRFs/IRF_overview.html

$$\sigma_{\text{photoel}}(E_i) = -\frac{\ln k_a(E_i)}{\kappa}, \quad (3)$$

where $\zeta_d(E_i)$ is the final uncertainty of the deabsorbed SED after taking into consideration the uncertainty of the hydrogen column density ζ_κ , and $\zeta_o(E_i)$ is the statistical uncertainty of the observed SED _{o} (E_i) in the i th energy bin E_i . ζ_κ is estimated using conf_loop ISIS function calculating 68% confidence interval for each of the free parameters after having obtained the best-fit with the absorbed broken-power-law model. After obtaining $\kappa = 7.7^{+1.8}_{-2.4} \times 10^{20} \text{ cm}^{-2}$ for MJD 55517–55524, we perform the same fit as described in this paragraph for each of the seven one-day-averaged consecutive SEDs allowing κ to vary only within the 68% confidence interval obtained above.

2.3 Swift-UVOT, SMARTS, and Steward – UV, optical, IR

2.3.1 Swift-UVOT

The Swift-UVOT SEDs are obtained with swift-uvot-analysis-tools⁷ program providing the values of the observed SED averaged over a given day within the time interval MJD 55517–55524.

2.3.2 SMARTS and Steward Observatory

For each considered period, when available, we use the data from the SMARTS program from optical/IR observations of *Fermi* blazars (Bonning et al. 2012) provided as photometric measurements in B, V, R, J, and K bands at the corresponding web page⁸ and the data from the Steward observatory (Smith et al. 2009) provided as photometric measurements in R and V bands⁹. The conversion of apparent Vega magnitudes into the observed SEDs was done using the zero-flux points from table A2 by Bessell et al. (1998).

2.3.3 Deredenning

The deredenning for IR, optical and UV data was performed using extinction program (Barbary 2016) assuming the extinction law of Fitzpatrick (1999), with the mean value of $E(B-V)$ extracted from IRSA (2022) using astroquery.ipac.irsa.irsa_dust module (Ginsburg et al. 2024; Ginsburg et al. 2019) with the assumed value of $R_V = 3.1$.

2.4 Radio

The one-zone model used in the current study invokes compact blobs. In such compact regions, the radio waves get absorbed due to the synchrotron self-absorption (SSA). Thus, the observed radio signal must originate from larger-scale parts of the jet. We use the archival data from NASA/IPAC Extragalactic Database (NASA/IPAC Extragalactic Database 2019) as upper limits when comparing our model SEDs to the observations of 3C 454.3.

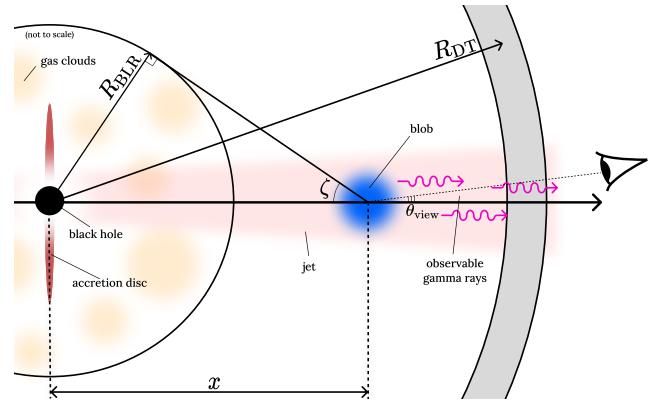


Figure 1. Geometrical scheme of the emitting region in the jet of 3C 454.3 (figure adapted from Dzhatdoev et al. 2022).

2.5 Assumed systematic uncertainties

From the comparison of measured magnitudes by SMARTS and Steward for the same object for the same night, we found a typical value of ≈ 0.08 for the relative uncertainty of the SED (see Appendix A). We add in quadrature 8% relative systematic uncertainty to the statistical uncertainty of all IR, optical and UV data points. Moreover, both *Fermi*-LAT and *Swift*-XRT have some uncertainty in the estimations of their effective areas resulting in the systematic uncertainty of the measured SEDs. We assume 3% relative systematic uncertainty, which can be expected for both *Fermi*-LAT¹⁰ and *Swift*-XRT (Moretti et al. 2006, sect. 5), adding it in quadrature to all the uncertainties of all the analysed SEDs (including IR, optical, and UV data points).

The obtained SEDs have been compared to the SEDs available in the Markarian Multiwavelength Data Center (MMDC)¹¹ (Sahakyan et al. 2024). The comparison showed a good agreement except for the low-energy part of the *Swift*-XRT spectrum where our data points lie somewhat higher than those from the MMDC. The disagreement is at a level consistent with the expected discrepancy due to different treatments of the photoelectric absorption (Giommi et al. 2021). The MMDC was not available when the data analysis for this work was performed.

3 MODELLING

We assume that the observed multi-wavelength spectral-energy distribution (SED = $E^2 dN/dE = \nu F_\nu = EF(E)$) can be modelled as the radiation emitted by a population of relativistic electrons (including positrons which we do not distinguish from electrons unless explicitly stated otherwise) trapped in the turbulent magnetic field of a spherical plasmoid (blob) located in the jet at a distance x from the supermassive black hole (SMBH) where the plasma flows with bulk Lorentz factor Γ with a small angle θ_{view} to the LoS (see Fig. 1).

For modelling the observable SED, we use the time-dependent program for the simulation of radiative processes AM³ (Klinger et al. 2023, 2024), although other solutions are also available (see, e.g., the review of programs simulating radiative processes by Nigro & Tramacere 2022). AM³ is a versatile program for time-dependent mod-

⁷ <https://github.com/KarlenS/swift-uvot-analysis-tools>

⁸ <http://www.astro.yale.edu/smarts/glast/tables/3C454.tab>

⁹ <https://james.as.arizona.edu/~psmith/Fermi/DATA/Objects/3c454.3.html>

¹⁰ https://fermi.gsfc.nasa.gov/ssc/data/analysis/scitools/Aeff_Systematics.html

¹¹ <https://mmdc.am/>

Variable	Explanation	Value
Geometry and conditions in the blob		
B' [G]	Root-mean-square magnetic field strength in the blob	f
R'_b [cm]	Blob radius	f
x_s [cm]	Blob launching distance from the SMBH ^{<i>m</i>}	f^m
x_f [cm]	Blob final distance from the SMBH ^{<i>m</i>}	f^m
x [cm]	Effective (current ^{<i>m</i>}) blob distance from the SMBH	$f(d^m)$, see Eq. 32 ^{<i>m</i>}
θ_{view} [$^\circ$]	Viewing angle w.r.t. the LoS	0.4
β	Blob speed divided by the speed of light c	d
Γ	Blob Lorentz factor	$f(d^m)$
δ	Blob Doppler factor	d , see Eq. (5)
z	Cosmological redshift	0.859
d_L [cm]	Luminosity distance	$d, 1.74 \times 10^{28}$
Injected electrons		
E_e^{\min} [eV]	Minimum electron injection energy	f
E_e^{\max} [eV]	Maximum electron injection energy	f
s_e	Electron injection power-law spectral index	f
L'_e [erg/s]	Injected electron luminosity	f
u'_e [erg/cm ³]	Injected electron energy density	d
η_e^{esc}	Escape factor for electrons w.r.t. photons	5×10^3
t'_e^{esc} [s]	Electron escape timescale	d , see Eq. (13)
Injected protons (for the leptohadronic model)		
E_p^{\min} [eV]	Minimum proton injection energy	2×10^9
E_p^{\max} [eV]	Energy of the exponential cutoff of the proton injection spectrum	d , see Eq. (38)
s_p	Proton injection power-law spectral index	2.0
u_p^{UL} [erg/cm ³]	Upper limit on the proton energy density	d , see Eq. (39)
η_p^{esc}	Escape factor for protons w.r.t. photons	5×10^3
t_p^{esc} [s]	Proton escape timescale	d , same as t'_e^{esc}
Accretion disc (AD)		
M_{SMBH} [M_\odot]	Assumed SMBH mass ^{<i>h</i>}	1.7×10^9
η_{acc}	Accretion efficiency	1/12
L_{AD} [erg/s]	Disc luminosity	3×10^{46} , \approx median value of those from Table 2
R_g [cm]	SMBH gravitational radius	$d, 5 \times 10^{14}$
$R_{\text{AD,in}}$ [R_g]	Inner radius of the accretion disc	3
$R_{\text{AD,out}}$ [R_g]	Outer radius of the accretion disc	10^3
Broad-line region (BLR)		
R_{BLR} [cm]	BLR radius	$d, 5.48 \times 10^{17}$, see Eq. (8)
C_{BLR}	BLR continuum reprocessing factor	0.01
ξ_{BLR}	BLR covering factor for the lines	0.10
Dust torus (DT)		
R_{DT} [cm]	Dust torus radius	$d, 1.37 \times 10^{19}$, see Eq. (9)
T_{DT} [K]	Dust torus temperature	370
C_{DT}	Dust torus covering factor	0.30
Time-related variables		
t'_e^{esc} [s]	Photon escape timescale	d , see Eq. (10)
t'_e^{gyro} [s]	Gyroperiod of an elementary charge in the blob	d , see Eq. (11)
t'_e^{Bohm} [s]	Bohm diffusion timescale	d , see Eq. (12)
t'_{obs} [s]	Total time of observations in the observer's rest frame ^{<i>m</i>}	d^m
t_{tot} [s]	Total time of the blob motion in the host galaxy rest frame ^{<i>m</i>}	d^m , see Eq. (27)
t'_{shift} [s]	Time delay of the observations w.r.t. the blob launch ^{<i>m</i>}	d^m
t'_{sim} [s]	Total simulation time in the blob rest frame	d , see Eq. (4) or Eq. (28) ^{<i>m</i>}
$\Delta t'/t'_{\text{sim}}$	ΔM^3 relative time step	0.003

Table 1. Definitions of variables mentioned throughout the text. Primed quantities ['] denote variables measured in the blob rest frame. Unprimed variables are either invariant or in the host galaxy rest frame. A superscript ^{*o*} denotes variables measured in the Earth rest frame at the present cosmological epoch. A letter d denotes variables whose values are derived from the other variables, and a letter f denotes variables that are free during the optimization. A superscript ^{*m*} denotes variables used in the model with a single “moving blob”. A superscript ^{*h*} denotes the average value of those reported by Ghisellini et al. (2010); Nalewajko et al. (2019); Titarchuk et al. (2020).

elling of various radiative processes (both leptonic and hadronic) including the synchrotron and inverse-Compton (IC) processes (for leptonic models), and the photopion and Bethe-Heitler (BH) processes (for lepto-hadronic models) with the subsequent cooling of the charged particles. Throughout the paper, we use the variables with their notations explained in Table 1. We assume the fiducial model of Gilmore et al. (2012) for the calculation of the absorption of γ rays on the extragalactic background light (EBL) on their way from 3C 454.3 to Earth and the flat Λ CDM Universe with the Hubble parameter $H_0 = 67.74 \text{ km/(s Mpc)}$ and the normalized matter density $\Omega_m = 0.3075$ (Planck Collaboration et al. 2016) resulting in the luminosity distance to 3C 454.3 $d_L = 1.74 \times 10^{28} \text{ cm}$ given its redshift $z = 0.859$ (Jackson & Browne 1991).

Using the time-dependent solver AM³, we model the observed SEDs of the MJD period 55517–55524 in two approaches: 1) assuming that the radiation comes from a region located at a particular distance x from the SMBH which does not change throughout the modelled period, i.e. the emission region is attributed to a stationary feature in the jet – hereafter the “standing-feature” model; 2) alternatively, assuming that the radiation comes from a single blob which moves along with the bulk motion of the jet plasma – hereafter the “moving-blob” model. We perform parameter optimization utilizing iMINUIT (Dembinski et al. 2021; James & Roos 1975) and emcee (Foreman-Mackey et al. 2013) as well as develop routines to estimate values of some model parameters as described in the following section.

3.1 Description of the “standing-feature” model

By the “standing feature” we mean a quasi-stationary region in the jet in which injection of charged particles and their subsequent cooling are responsible for most of the observed radiation during the flare. The distance x between the “standing feature” and the SMBH is kept constant throughout each day of observations. For each of the seven one-day averaged datasets, we obtain a fit with a “standing-feature” model, with eight free parameters:

- (i) injected electron luminosity L'_e ;
- (ii) spectral index s_e of the electron power-law injection intensity;
- (iii) minimum electron injection energy E'_e^{\min} ;
- (iv) maximum electron injection energy E'_e^{\max} ;
- (v) blob radius R'_b ;
- (vi) root-mean-square strength of the magnetic field B' ;
- (vii) Lorentz factor of the blob Γ ;
- (viii) dissipation radius x .

The characteristic timescale over which we average the observed SED is $t_{\text{av}}^o = 1 \text{ d}$. Therefore, the corresponding time interval in the blob rest frame,

$$t'_{\text{sim}} = \frac{\delta}{1+z} t_{\text{av}}^o, \quad (4)$$

serves as the total time of the calculation of the particle yields after which we stop the AM³ simulation and extract the particle yields from its last time step, where the apparent Doppler factor is defined as

$$\delta = [\Gamma(1 - \beta \cos \theta_{\text{view}})]^{-1}, \quad (5)$$

$\beta = \sqrt{1 - \Gamma^{-2}}$, and $\theta_{\text{view}} = 0.4^\circ$, is the angle between the jet direction and the LoS (the assumption is based on the radio data by, e.g., Kovalev et al. 2020; Pushkarev et al. 2017; Jorstad et al. 2013). We assume that the injection of electrons stops after t'_{sim} and – since their energy-loss timescale is for most relevant energies much less than t'_{sim} – their emission stops shortly after that.

Following Rodrigues et al. (2024), we model all radiative processes

in the blob rest frame having transformed external photon fields into this frame¹². The obtained number densities $n'(E')$, differential per volume and per energy, are transformed into the observer frame SED νF_ν as (see, e.g., Jiménez-Fernández & van Eerten (2021), eq. 1)

$$\nu F_\nu = E'^2 n'(E') \frac{\delta^4}{4\pi(1+z)d_L^2} \frac{V'}{t'_\gamma^{\text{esc}}}, \quad (6)$$

where d_L is the cosmological luminosity distance, $V' = 4\pi R_b'^3/3$ is the blob volume, t'_γ^{esc} is the photon escape timescale.

The emission enhancement factor of δ^4 is applicable in the case of the “standing feature”¹³ as long as the ratio N_b of the duration of emission of each blob t_{tot} and the time interval between two blobs passing through the “standing feature” $t_{\text{btw b}}$ is (Sikora et al. 1997, appendix B)

$$N_b = \frac{t_{\text{tot}}}{t_{\text{btw b}}} = \Gamma \delta. \quad (7)$$

If, in addition, $t_{\text{btw b}} = t_{\text{av}}^o$, the observer throughout the time period t_{av}^o will detect photons only from a single “active” blob passing through the “standing feature”. If $t_{\text{btw b}} < t_{\text{av}}^o$ (and N_b still equals $\Gamma \delta$) the observed emission throughout t_{av}^o is the emission averaged over the superposition of emissions of several identical blobs. If $t_{\text{btw b}} \gg t_{\text{av}}^o$ then all the observed emission comes from a single moving blob. The latter case is considered in Sects. 3.2, 4.2. As explained by Sikora et al. (1997, appendix B), Eq. (6) works both for the “moving blob” and the “standing feature” (for the latter, if Eq. (7) is satisfied).

In the following subsections, we describe the model parameters in greater detail and present some (semi-)analytical estimates for initial guesses for some of the parameters.

3.1.1 External photon fields

Our model includes the following sources of the photon fields external to the moving blob (see also Fig. 1):

Accretion disc: Direct radiation of the accretion disc (AD) in the model of Shakura & Sunyaev (1976) is calculated according to Ghisellini & Tavecchio (2009, sect. 3.1) and Ehrlert et al. (2024, appendix C1). On various estimates of the 3C 454.3 AD luminosity in literature, see Table 2. We adopt $L_{\text{AD}} = 3 \times 10^{46} \text{ erg/s}$ which is approximately the median value of those given in Table 2.

Broad line region: Radiation intercepted and reprocessed by the BLR modelled as the superposition of the H I Ly α and He II Ly α lines and thermal continuum according to the examples in the AM³ documentation (Klinger et al. 2023, 2024; Rodrigues et al. 2024) with the continuum re-emitting $C_{\text{BLR}} = 0.01$ of the AD luminosity, where C_{BLR} is the BLR continuum reprocessing factor. In addition, the BLR lines reprocess $\xi_{\text{BLR}} = 0.10$ of the AD luminosity where ξ_{BLR} is the BLR covering factor (Baldwin & Netzer 1978). We calculate the BLR outer radius using the standard relation (Kaspi et al. 2007; Ghisellini & Tavecchio 2009)

$$R_{\text{BLR}} = 1.0 \times 10^{17} \text{ cm} \left(\frac{L_{\text{AD}}}{10^{45} \text{ erg/s}} \right)^{\frac{1}{2}}, \quad (8)$$

resulting in $R_{\text{BLR}} = 5.48 \times 10^{17} \text{ cm}$ in our case.

¹² Though, an alternative sequence of frame transformations is possible (e.g., Khangulyan et al. 2018).

¹³ Sikora et al. (1997) use the term “steady pattern”.

L_{AD} , erg/s	Reference	Note
1.00×10^{46}	Finke & Dermer (2010)	From their assumed ratio $l_{\text{Edd}} = L_{\text{AD}}/L_{\text{Eddington}} = 0.04$ and SMBH mass $M_{\text{SMBH}} = 2 \times 10^9 M_{\odot}$
2.51×10^{46}	Rajguru & Chatterjee (2024)	From power-law + black-body fitting of the IR and optical photometric SED
3.0×10^{46}	This work	Assumed value
3.0×10^{46}	Ghisellini et al. (2010)	From multi-wavelength SED fitting with a single-zone leptonic model
3.31×10^{46}	Sbarato et al. (2012); Xiong & Zhang (2014)	Multiplying their BLR luminosity by 10
3.33×10^{46}	Pian et al. (2005)	Multiplying their BLR luminosity by 10
4.47×10^{46}	Cao & Jiang (1999); Xiong & Zhang (2014)	Multiplying their BLR luminosity by 10
4.79×10^{46}	Liu et al. (2006)	Multiplying their BLR luminosity by 10
6.75×10^{46}	Bonnoli et al. (2011)	Their assumed value

Table 2. Various values of the 3C 454.3 AD luminosity L_{AD} present in the literature.

Dust Torus: Radiation intercepted and reprocessed by the dust torus (DT) region is modelled as a thermal continuum with temperature $T_{\text{DT}} = 370$ K and covering factor $C_{\text{DT}} = 0.30$ according to the examples in the AM³ documentation (Klinger et al. 2023, 2024; Rodrigues et al. 2024). The DT outer radius is assumed to follow (Blažejowski et al. 2000; Sikora et al. 2002; Ghisellini & Tavecchio 2009)

$$R_{\text{DT}} = 2.5 \times 10^{18} \text{ cm} \left(\frac{L_{\text{AD}}}{10^{45} \text{ erg/s}} \right)^{\frac{1}{2}} \quad (9)$$

resulting in $R_{\text{DT}} = 1.37 \times 10^{19}$ cm.

The dependence of the SED and energy density of the BLR and DT photon fields in the blob rest frame on parameters x and Γ is modelled according to Ghisellini & Tavecchio (2009, sect. 3.3, 3.4) as done by Klinger et al. (2024); Rodrigues et al. (2024).

3.1.2 Escape timescales

For a leptonic setup, the only particles involved in the simulation are photons and electrons. The timescale for photons escaping from the blob is (see, e.g., Jiménez-Fernández & van Eerten 2021, eq. 2):

$$t_{\gamma}'^{\text{esc}} = \frac{3}{4} \frac{R'_b}{c} = 3 \times 10^5 \text{ s} \times \left(\frac{R'_b}{10^{16} \text{ cm}} \right). \quad (10)$$

In principle, electrons can also be assumed to escape with the same timescale as photons (e.g., Rodrigues et al. 2024, so-called stream-like escape). However, the period of gyration of the electrons equals

$$t_e'^{\text{gyro}} = 7 \times 10^{-4} \text{ s} \times \left(\frac{E'}{1 \text{ GeV}} \right) \left(\frac{B'}{1 \text{ G}} \right)^{-1}, \quad (11)$$

thus, $t_e'^{\text{gyro}} \ll t_{\gamma}'^{\text{esc}}$, i.e. electrons are confined in the magnetic field and are in the diffusion regime which can be characterised, e.g., by Bohm diffusion:

$$t_e'^{\text{Bohm}} = 1 \times 10^{15} \text{ s} \times \left(\frac{R'_b}{10^{16} \text{ cm}} \right)^2 \left(\frac{B'}{1 \text{ G}} \right) \left(\frac{E'}{1 \text{ GeV}} \right)^{-1}. \quad (12)$$

As Rodrigues et al. (2018) pointed out, the true electron escape timescale $t_e'^{\text{esc}}$ should be somewhere in between $t_{\gamma}'^{\text{esc}}$ and $t_e'^{\text{Bohm}}$. However, when modelling the flaring behaviour, it is important rather to compare $t_e'^{\text{esc}}$ and the total time of the simulation t_{sim}' . We assume $t_e'^{\text{esc}} \gg t_{\text{sim}}'$, and we can neglect the dependence of the electron escape timescale on their energy assuming just a large enough factor $\eta_e \sim 5 \times 10^3$ by which the electron escape timescale is greater than the photon one:

$$t_e'^{\text{esc}} = \eta_e t_{\gamma}'^{\text{esc}}. \quad (13)$$

3.1.3 Lorentz and Doppler factors

Jorstad et al. (2013) observed the flare of 3C 454.3 in radio waves in the period of MJD 55518–55568 using the very-long baseline interferometry (VLBI) and obtained the estimation of the Lorentz factor $\Gamma = 26 \pm 3$. Based on these observations, we use $\Gamma = 25$ as our initial guess and the corresponding Doppler factor δ calculated with Eq. (5).

3.1.4 Blob radius

Meyer et al. (2019) investigated the same flare of 3C 454.3 analysing the *Fermi*-LAT data. In their fit of the flare light curve, they estimated the observed flare decay time $\tau_{\text{decay}}^o = 2.6 \pm 1.0$ h, see their table 5. Assuming that this time is related to the size of the emitting region, we can estimate the blob radius as

$$R'_b = \frac{c \tau_{\text{decay}}^o}{1+z} \delta. \quad (14)$$

For $\Gamma = 25$, $\delta = 50$, $z = 0.859$, we obtain $R'_b = 7.5 \times 10^{15}$ cm, which we can use as an initial guess for the blob radius.

3.1.5 Location of the γ -ray emitting region

In this subsection, we obtain an analytical estimate of x to use as an initial guess before the iMINUIT optimization. The distance x of the region emitting the observed radiation from the SMBH establishes the energy at which the high-energy γ rays will be absorbed by the photons of the BLR photon field.

First, for a blob with the Lorentz factor of $\Gamma = 25$ and redshift $z = 0.859$, we calculate the $\gamma\gamma$ interaction rate $r'_{\gamma\gamma}(E'_{\gamma}, x)$ on the photon field presented in Sect. 3.1.1 for 100 positions of the blob in between $x = R_{\text{BLR}}$ and $x = 2R_{\text{BLR}}$ following the prescriptions of Kachelrieß et al. (2012) and their equations (5), (6). Then for each value of x , we integrate the obtained interaction rate $r'_{\gamma\gamma}(E'_{\gamma}, x)$ along the LoS to obtain the value of the photon absorption optical depth $\tau_{\gamma\gamma}$ for γ rays with the observable energy E_{γ}^o emitted at distance x from the SMBH:

$$\tau_{\gamma\gamma}(E_{\gamma}^o, x) = \int_x^{+\infty} r'_{\gamma\gamma}(E'_{\gamma}, y) dy, \quad (15)$$

where $E_{\gamma}^o = \delta E'_{\gamma}/(1+z)$. The obtained optical depth for ten values of x is plotted in Fig. 2. The sharp rise of the optical depth happens due to the absorption of γ rays on the He II Ly α line (40.8 eV) and the second rise – due to the beginning of pair production on the H I Ly α line (10.2 eV). From Fig. 2, one can see that for $x \lesssim 1.7R_{\text{BLR}}$ the optical depth on the He II Ly α line is greater than unity.

To obtain an analytical estimate of x , we derive analytically the

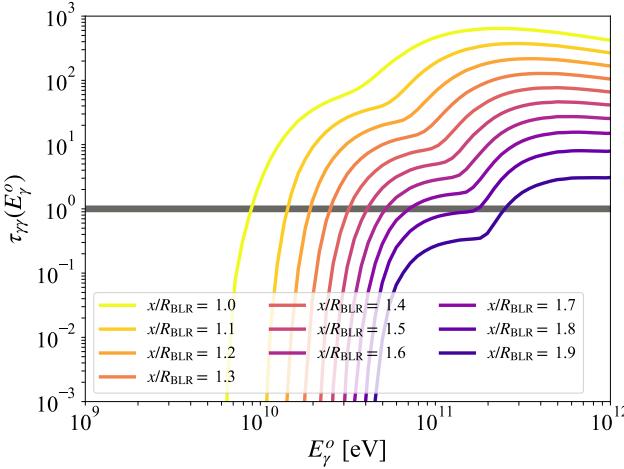


Figure 2. The optical depth of $\gamma\gamma$ pair production for γ rays with the observed (at Earth) energy E_γ^o emitted in a blob with $\Gamma = 25$ and $z = 0.859$ at a particular distance x from the SMBH as indicated in the legend.

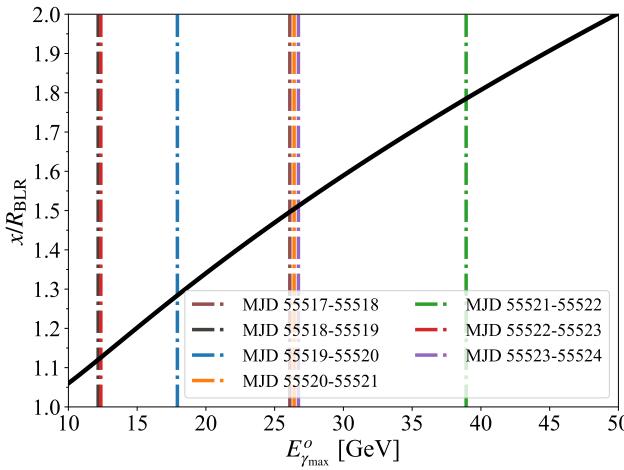


Figure 3. The dependence of the dissipation radius x on the maximum observed (on Earth) γ -ray energy assuming it corresponds to pair production at the threshold on photons from the $He\text{ II Ly }\alpha$ line in the blob with Lorentz factor $\Gamma = 25$ at redshift $z = 0.859$. The dashed-dotted vertical lines show the central energies of the last observed *Fermi*-LAT bins during one-day periods as indicated in the legend (closely plotted lines correspond to the same energy but slightly shifted for better visibility).

equation linking x with the maximum energy $E_{\gamma_{\max}}^o$ among the observed γ -ray energies assuming it corresponds to the threshold of pair production on $He\text{ II Ly }\alpha$ line. In the blob frame a γ -ray with energy E'_γ collides with a photon with energy E'_{γ_2} and creates an electron-positron pair at the threshold (Breit & Wheeler 1934; Nikishov 1962; Gould & Schréder 1967)

$$E'_\gamma E'_{\gamma_2} = \frac{2m_e^2 c^4}{1 - \cos \zeta'}, \quad (16)$$

where ζ' is the angle between the momenta of the photons. The photon energy ϵ is set in the host galaxy rest frame and related to the energy in the blob frame E'_{γ_2} as (Dermer & Schlickeiser 1993; Blandford & Levinson 1995; Dermer & Schlickeiser 2002)

$$E'_{\gamma_2} = \epsilon [\Gamma(1 - \beta \cos \zeta)] = \frac{\epsilon}{\Gamma(1 + \beta \cos \zeta')}, \quad (17)$$

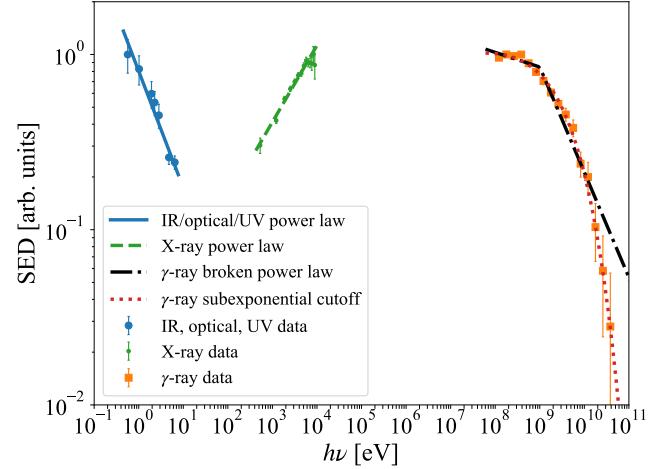


Figure 4. Phenomenological fits to the synchrotron, X-ray, and γ -ray parts of the SED averaged over 55517–55524. Data for each energy segment are normalized by the maximum value of the SED in the energy range. See Table 3 for the details of the fits.

Parameter	Value
IR/optical/UV power law	
Γ_γ	2.67 ± 0.07
χ^2_{red}	0.77
X-ray power law	
Γ_γ	1.56 ± 0.02
χ^2_{red}	1.17
γ -ray broken power law	
Γ_γ (before the break)	2.1
Γ_γ (after the break)	2.6
χ^2_{red}	2.83
γ -ray subexponential cutoff	
γ_1	1.92
γ_b	0.51
χ^2_{red}	0.89

Table 3. Details of the fits to the normalised SEDs shown in Fig. 4. Γ_γ is the power-law slope of the photon intensity $dN_\gamma/dE_\gamma \propto E_\gamma^{-\Gamma_\gamma}$, $E_\gamma \equiv h\nu$. Parameters γ_1 and γ_b are from the parameters of the subexponential function described in Eq. (21)

where the cosines in the two frames are related as

$$\cos \zeta' = \frac{\cos \zeta - \beta}{1 - \beta \cos \zeta}. \quad (18)$$

Assuming the blob is located at $x > R_{BLR}$ and, following Klinger et al. (2023); Rodrigues et al. (2024),¹⁴ that the photons coming from the largest angle between the jet axis and the edge of the BLR sphere contribute the most to the BLR photon field observed in the blob (see Fig. 1), and the γ -ray colliding with the photon propagates along the jet axis at zero angle,

$$\sin \zeta = \frac{R_{BLR}}{x}. \quad (19)$$

Combining Eqs. (16), (17), (18), and (19) and taking into account the redshift z and the Doppler factor δ , we obtain the estimate of x given

¹⁴ https://am3.readthedocs.io/en/latest/examples/blazar_detailed_example.html

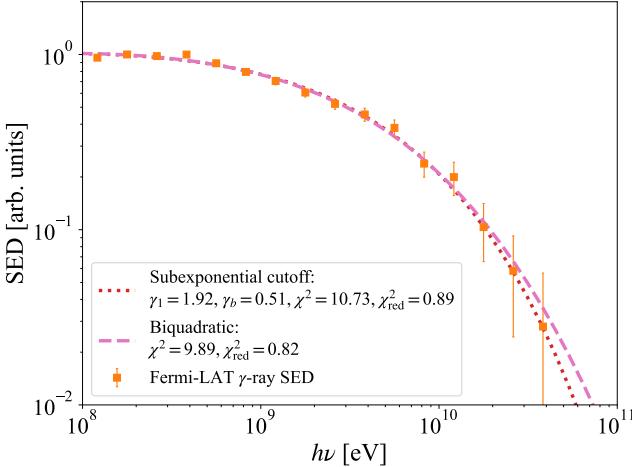


Figure 5. The results of the fits to the normalised *Fermi*-LAT SED averaged over MJD 55517–55524 by subexponential cutoff (Eq. 21) and biquadratic function (Eq. 22).

the energy $E_{\gamma_{\max}}^o$ of the γ -ray observed on Earth corresponding to the threshold of pair-production on the line with energy ϵ :

$$x = R_{\text{BLR}} \left[1 - \left(1 - \frac{2m_e^2 c^4}{\epsilon E_{\gamma_{\max}}^o} \frac{\delta}{\Gamma(1+z)(1+\beta)} \right)^2 \right]^{-\frac{1}{2}}. \quad (20)$$

We show as the black solid curve the graph of function (20) in Fig. 3 for $\Gamma = 25$, and $\epsilon = 40.8$ eV corresponding to the $He\text{ II Ly }\alpha$ line. In Fig. 3, we superimpose vertical lines showing the central energies of the last bins observed with *Fermi*-LAT in each of the seven days of MJD 55517–55524. From Fig. 3, we take $x = 1.5R_{\text{BLR}}$ as our initial guess for the dissipation radius.

3.1.6 Magnetic field and parameters of the electron injection spectrum

In this subsection, we infer initial guesses for the root-mean-square magnetic field strength B' , the power-law slope of the injected electron spectrum s_e and the maximum electron injection energy $E_e'^{\max}$. To do so, we use the SED averaged over the seven-day period of MJD 55517–55524. To focus on the shapes of the parts of the observed SED, we present it normalized by the maximum values as data points with error bars in each energy band (IR/optical/UV photons, X rays, and γ rays) in Fig. 4.

Using iMINUIT, we obtain functional forms fitting the observed bands of the SED presented in Fig. 4 as curves. For the first two bands, we obtain good fits with simple power laws with the parameter values and their errors presented in Table 3 along with the values of $\chi_{\text{red}}^2 = \chi^2/\text{ndf}$, where ndf stands for the number of degrees of freedom in a particular fit.

For the *Fermi*-LAT data, it is clear that they cannot be described by a single power law (see also, e.g., Rajput et al. 2019; Sahakyan 2021). We failed to obtain a good fit with a broken power law (BPL) with

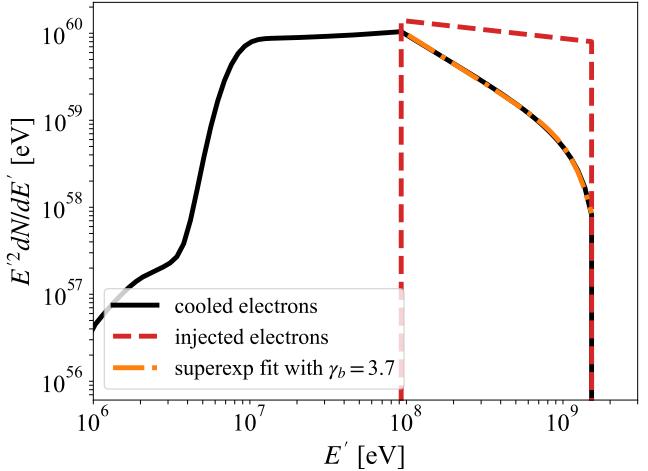


Figure 6. The SEDs of the injected and cooled electrons and the fit with Eq. (21) to the cooled electron SED.

$\chi_{\text{red}}^2 = 2.83$. On the contrary, a fit with a subexponential cutoff¹⁵

$$\text{SED}(E) = E^2 N_0 \left(\frac{E}{E_0} \right)^{-\gamma_1} \exp \left(- \left(\frac{E}{E_{\text{cut}}} \right)^{\gamma_b} \right) \quad (21)$$

describes the *Fermi*-LAT range quite well ($\chi_{\text{red}}^2 = 0.89$). We interpret this as evidence for the curvature of the spectrum reported previously by Abdo et al. (2011); Pacciani et al. (2014); Sahakyan (2021). Indeed, in the 4FGL catalogue itself, the spectral model for 3C 454.3 is chosen to have a subexponential cutoff (Abdollahi et al. 2020) providing the best description of the eight-year data. Besides, Stern & Poutanen (2014) from the analysis of almost five years of *Fermi*-LAT data, found that the SED of 3C 454.3 is best described by a biquadratic function

$$\text{SED}(E) = F_0 \exp \left(-A \ln^2 \left(E/E_{\text{peak}} \right) - B \ln^4 \left(E/E_{\text{peak}} \right) \right) \quad (22)$$

with $A \approx 0$. We argue that:

(i) SEDs described by Eqs. (21) and (22) can be practically indistinguishable from each other under certain values of the parameters (see Fig. 5);

(ii) SEDs described by Eqs. (21) or (22) can be a result of the IC scattering of cooled electrons with small maximum injection energies (Lefa et al. 2012; Costamante et al. 2018) – see below and Figs. 5 and 6.

If we assume that photons with maximum observed energy $E_{\gamma_{\max}}^o$ are emitted as a result of the IC scattering of electrons at the end of their spectrum, then we can estimate the maximum electron injection energy as

$$E_e'^{\max} = \frac{1+z}{\delta} E_{\gamma_{\max}}^o, \quad (23)$$

so $E_e'^{\max} \approx 1.5 \times 10^9$ eV for $E_{\gamma_{\max}}^o \approx 4 \times 10^{10}$ eV.

From the broken power-law fit to the *Fermi*-LAT data in Fig. 4, we can assume the power-law slope of the *cooled* electron spectrum

¹⁵ The *Fermi*-LAT Collaboration uses the term “superexponential cutoff” https://fermi.gsfc.nasa.gov/ssc/data/analysis/scitools/source_models.html, but in our case $\gamma_b < 1$, so we use the term “subexponential” instead.

$dN_e/dE_e \propto E^{-P_e}$ to be (Blumenthal & Gould 1970; Finke & Dermer 2010; Ghisellini et al. 2020)

$$p_e = 2\Gamma_\gamma - 1, \quad (24)$$

so the slope of the *injected* electron spectrum $dN_e/dE_e \propto E^{-s_e}$ will be

$$s_e = p_e - 1 = 2\Gamma_\gamma - 2, \quad (25)$$

i.e. $s_e \approx 2.2$ for $\Gamma_\gamma \approx 2.1$ (see Table 3).

Assuming that the UV photons at the energy $E_{\text{UV}}^0 \approx 4$ eV are exclusively produced by the cooled electrons at the cutoff $E_e'^{\max}$, we can estimate the root-mean-square magnetic field strength as (e.g., Jiménez-Fernández & van Eerten 2021, eq. 55)

$$B' \approx \frac{4\pi m_e^3 c^5 E_{\text{UV}}^0}{h q_e \delta (E_e'^{\max})^2}, \quad (26)$$

which gives $B' \approx 1$ G in our case.

Finally, as initial guess we assume $E_e'^{\min} = 10^8$ eV and $L'_e = 2 \times 10^{43}$ erg/s.

With the model parameters’ values from our initial guess, we run the AM^3 program with the total simulation time according to Eq. (4). The injected and cooled electron SED at the end of the simulation are shown in Fig. 6 along with the fit of the functional form of Eq. (21) to the high-energy part of the cooled electron SED resulting in excellent agreement with the superexponential cutoff with $\gamma_b = 3.7$. The *superexponential* cutoff of the *cooled* electron SED in Fig. 6 results in the *subexponential* cutoff of the γ -ray SED with $\gamma_b = 0.51$ (see Fig. 5), which is in agreement with the expected value from Lefa et al. (2012) for a case in between the synchrotron ($\gamma_b = 3.7/(3.7+4) = 0.48$) and Planckian ($\gamma_b = 3.7/(3.7+2) = 0.65$) photons serving as the target photon fields (Lefa et al. 2012, table 1) for the IC in the Thomson regime.

The results of the “standing-feature” leptonic model are presented in Sect. 4.1.

3.2 Description of the “moving-blob” model

We assume that the blob is launched at the distance x_s from the SMBH with a Lorentz factor Γ and an angle θ_{view} between the blob velocity and the LoS. At the end of the simulation, the blob reaches distance x_f from the SMBH. The total simulation time is t'_{sim} in the blob rest frame, t_{tot} in the object’s host galaxy rest frame, and t'_{tot} in the observer’s rest frame (equal the time between the receipt by the observer of the first photon emitted from the position x_s and the last photon emitted from the position x_f). These are related as follows (see, e.g., Kumar & Zhang 2015, sect. 2.1):

$$t_{\text{tot}} = \frac{x_f - x_s}{\beta c}, \quad (27)$$

$$t'_{\text{sim}} = \frac{t_{\text{tot}}}{\Gamma}, \quad (28)$$

$$t'_{\text{tot}} = (1+z) \frac{t'_{\text{sim}}}{\delta} = (1+z) \frac{t_{\text{tot}}}{\Gamma \delta} = (1+z)(1-\beta \cos \theta_{\text{view}}) t_{\text{tot}}. \quad (29)$$

If observations start not exactly at the moment when the first photon emitted by the blob from x_s has reached the Earth, but with some time shift t'_{shift} between the moment of the beginning of the observations and the moment of the first photon reaching the Earth (albeit not registered due to the delayed observations), then

$$t'_{\text{tot}} = t'_{\text{shift}} + t'_{\text{obs}}, \quad (30)$$

where t'_{obs} is the total duration of the observations in the Earth frame. Fitting the model SED to the data, we keep t'_{shift} , x_s , x_f as free parameters, whereas θ_{view} is fixed, and t'_{obs} is also fixed equal to the actual duration of observations over which the observed SED is averaged (7 d if not stated otherwise). Thus, β , hence Γ and δ as well, become dependent variables following

$$\beta = \left(\frac{c(t'_{\text{obs}} + t'_{\text{shift}})}{(1+z)(x_f - x_s)} + \cos \theta_{\text{view}} \right)^{-1}. \quad (31)$$

In the AM^3 simulation, we set the number of time steps equal to $N_s = 100$. At each time step the current position of the blob is updated according to the equation (see Eqs. 27 and 28)

$$x(t') = x_s + \beta c \Gamma t'. \quad (32)$$

All the external parameters dependent on x are updated at the beginning of each step. The photon fields are calculated the same way it is described in Sect. 3.1.1. The escape timescales are calculated the same way as in Sect. 3.1.2. The snapshot of the model SED($E; t'_k$) is stored at each k th time step. Then, for each j th day of the flare (from first to seventh), following Eqs. (29) and (30), we calculate the model SED averaged over all those times t'_k in the blob frame which satisfy the condition

$$j-1 \leq \frac{t'_k (1+z)/\delta - t'_{\text{shift}}}{1 \text{ d}} \leq j, \quad (33)$$

where $k = 1, 2, \dots, N_s$. We assume the last observation period stops exactly at the moment when the photon emitted from the blob position at x_f arrives at the Earth, i.e. there is no time shift between the end of the observations and the end of the simulation.

We assume that electrons are injected into the blob at a rate

$$\left(\frac{d\dot{N}'_e}{dE'_e} \right)_{\text{inj}} = \begin{cases} \dot{N}'_e \left(\frac{E'_e}{E'_{\text{ref}}} \right)^{-s_e}, & E'_e \leq E'_e \leq E'_e^{\max}, \\ 0, & \text{otherwise,} \end{cases} \quad (34)$$

where $\dot{N}'_e \equiv dN'_e/dt'$, normalization factor \dot{N}'_e is adjusted corresponding to the value of the electron injection luminosity L'_e serving as a free parameter along with the minimum injection energy E'_e^{\min} , the spectral index s_e , and the maximum injection energy E'_e^{\max} , whereas the reference energy E'_{ref} is fixed. We allow the injection electron spectrum to vary over time following

$$\dot{N}'_e(t') = \dot{N}'_e(0) \left(\frac{x(t')}{x_{e_0}} \right)^{\alpha_e \Theta(t' - t'_{e_0})}, \quad (35)$$

where Θ denotes the Heaviside step function, t'_{e_0} is the time when the injection starts changing, α_e is a free parameter, x_{e_0} is the position of the blob when $t' = t'_{e_0}$. We also allow the blob radius to vary over time according to a similar equation

$$R'_b(t') = R'_b(0) \left(\frac{x(t')}{x_{b_0}} \right)^{\alpha_b \Theta(t' - t'_{b_0})}, \quad (36)$$

where t'_{b_0} denotes the time when the blob radius starts changing, α_b is a free parameter, x_{b_0} is the position of the blob when $t' = t'_{b_0}$. In total, the “moving-blob” model has 12 free parameters: $L'_e(0)$, s_e , E'_e^{\min} , E'_e^{\max} , B' , x_s , x_f , $R'_b(0)$, t'_{e_0} , t'_{b_0} , α_e , and α_b . The transformation of the SED from the blob frame into the observer frame for a single blob propagating along the jet is described by the same Eq. (6) as for the “standing-feature” model satisfying Eq. (7) as shown by Sikora et al. (1997, appendix B).

The results of the “moving-blob” leptonic model are presented in Sect. 4.2.

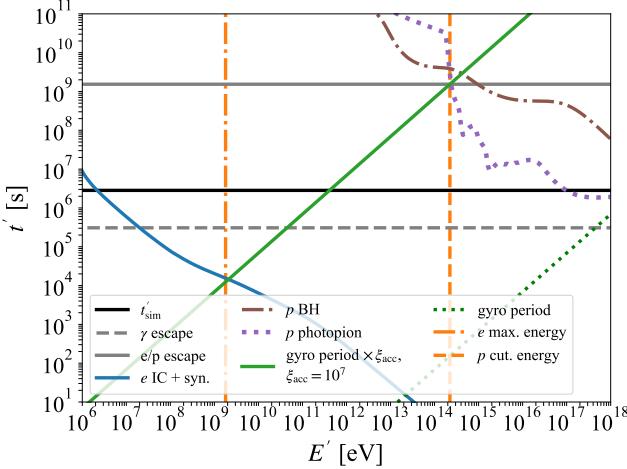


Figure 7. The characteristic energy-loss timescales for the “standing-feature” lepto hadronic model for the averaged dataset MJD 55517–55524 with t'_sim corresponding to one day in the Earth rest frame.

3.3 Description of the lepto hadronic model

maximum contribution of protons to the observed SED allowed by the data assuming the hadronic contribution in the model serves as a subdominant component of the model SED, in addition to the already obtained leptonic fit.

For the best fit of the “standing-feature” leptonic model averaged over MJD 55517–55524, we show the characteristic timescales in the blob frame in Fig. 7. To minimize the number of free parameters, we assume that the protons are co-accelerated with electrons in a process with an acceleration timescale

$$t_p'^{\text{acc}}(E'_p) = \xi_{\text{acc}} t_p'^{\text{gyro}}, \quad (37)$$

where $t_p'^{\text{gyro}} = t_e'^{\text{gyro}}$ is defined in Eq. (11). We assume for simplicity that ξ_{acc} is the same for both electrons and protons (see Appendix D for an investigation of other values of ξ_{acc} for protons). To estimate the value of ξ_{acc} , we look at the interception of the vertical line corresponding to the maximum electron injection energy and the curve of the total electron energy-loss timescale (see Fig. 7) and plot the dependence of the gyroperiod on the energy multiplied by the corresponding value which is found to be $\xi_{\text{acc}} = 10^7$. This value is the benchmark value for the “standing-feature” lepto hadronic model.

To determine the maximum proton energy, we assume that the acceleration started long before the flare¹⁶ and look at the interception of the line of $t_p'^{\text{acc}}(E'_p)$ with the proton escape timescale (and energy-loss timescale due to photopion production having almost the same value at this energy, see Fig. 7):

$$t_p'^{\text{acc}}(E'_p^{\text{max}}) = t_p'^{\text{esc}}, \quad (38)$$

where $t_p'^{\text{esc}} = t_e'^{\text{esc}}$ is defined in Eq. (13). This results in $E'_p^{\text{max}} \approx 2 \times 10^{14} \text{ eV}$ for MJD 55517–55524, where E'_p^{max} is taken as the energy of the exponential cutoff of the proton spectrum with the minimal proton energy fixed at $E'_p^{\text{min}} = 2 \times 10^9 \text{ eV}$, and their power-law spectral index fixed at $s_p = 2.0$.

From Fig. 7, one can see that within the simulation timescale corresponding to one day in the observer’s frame, most of the electrons

¹⁶ Otherwise, the maximum proton energy would be determined by the interception of $t_p'^{\text{acc}}$ with the simulation timescale t'_sim and would be a few GeV.

effectively cool and a continuous injection of freshly accelerated electrons is required to maintain the blob radiation. On the other hand, protons with energies up to 10^{16} eV have an energy-loss timescale much greater than the simulation timescale. Thus, given the maximum energy of protons is $E'_p^{\text{max}} \approx 2 \times 10^{14} \text{ eV}$, no continuous proton injection is required, and the initially predetermined steady-state proton SED remains almost unchanged throughout the simulation.

We found that for the whole period of MJD 55517–55524, if we set the steady-state proton SED with energy density in the blob frame equal to the energy density of the injected electrons $u'_p = u'_e$, this results in approximately the same value of the χ^2 as for the purely leptonic model. We use $u'_p = u'_e$ as a starting point. Further in a loop, we increase the normalization of the proton SED by 5% each step until the Bayes factor (Kass & Raftery 1995)

$$BF = \exp\left(-\frac{\chi^2(u'_p) - \chi^2_{\text{leptonic}}}{2}\right) \quad (39)$$

drops to $BF = 10^{-2}$ ($\approx 3\sigma$ Gaussian equivalent). We denote the corresponding value of the proton energy density u'_p^{UL} as the upper limit (UL). The obtained ratio between the UL proton energy density and injected electron energy density is

$$\rho_{p/e} \equiv \frac{u'_p^{\text{UL}}}{u'_e} \approx 1.3 \times 10^2. \quad (40)$$

The results obtained with the “standing-feature” lepto hadronic model are presented in Sect. 4.3. Since the leptonic model within the “moving-blob” approach resulted in a poor description of the data (see Sect. 4.2), we do not develop its lepto hadronic counterpart.

3.4 Method of estimating the expected muon neutrino yield from 3C 454.3 in IceCube

In this section, we describe our procedure for calculating the *IceCube* muon neutrino yield from 3C 454.3 for any given moment of time assuming a lepto hadronic model with the constant factor $\rho_{p/e}$ as the one obtained in Eq. (40) for MJD 55517–55524.

(i) We select four other additional time periods of the 3C 454.3 observations when quasi-simultaneous data from X-ray and IR/optical/UV telescopes (among *Swift*-UVOT, *SMARTS* and *Steward Observatory*) are available. The selected periods are MJD 54700–54701; 54827–54833; 55180–55181; 55471–55472 and cover different values of observed γ -ray fluxes from 3C 454.3.

(ii) For each of the additionally selected periods, we repeat the data analysis as described in Sect. 2 and fitting within the “standing-feature” leptonic model as described in Sect. 3.1.

(iii) For each of the additionally selected periods, having obtained the best-fit values of the “standing-feature” leptonic model’s parameters, we turn on hadronic processes injecting protons with the energy density $\rho_{p/e} = 1.3 \times 10^2$ times greater than the injected electron energy density for the period under consideration. The energy of the exponential cutoff of the proton injection spectrum is defined by the interception of the acceleration timescale with $\xi_{\text{acc}} = 10^7$ and escape timescale (see Eq. 38).

(iv) The obtained with AM³ observable neutrino SED _{ν} (E_ν^o) = $E_\nu^o{}^2 dN_\nu^o / dE_\nu^o$ for each of the five considered periods is used to calculate the integral neutrino particle flux Φ_ν^o for observable neutrino energies $E_\nu^o \geq 100 \text{ GeV}$.

(v) We obtained a good linear fit in the form of $\Phi_\nu^o = a\Phi_\gamma^o$, where a is a constant, and Φ_γ^o is the integral model photon flux for observable γ -ray energies $E_\gamma^o \geq 100 \text{ MeV}$.

(vi) Assuming that the 3C 454.3 neutrino SED obtained for MJD 55517–55524 can be used (after scaling to the corresponding value of Φ_γ^o) for each time bin t_i of a light curve from the *Fermi*-LAT light curve repository (LCR) curated by Abdollahi et al. (2023), we calculate the neutrino SED for a given time bin t_i as follows:

$$\text{SED}_\nu(E_\nu^o; t_i) = \frac{\Phi_\gamma^o(t_i) \text{SED}_\nu(E_\nu^o; t_0)}{\Phi_\gamma^o(t_0)}, \quad (41)$$

where t_0 represents the interval of MJD 55517–55524. If at time t_i in the LCR there is only an UL on the flux value (the source is not detected with the test statistic $TS > 4$), we assume $\Phi_\gamma^o(t_i) = 0$ for that time bin.

(vii) For each time bin t_i , muon (+ anti-muon) *IceCube* neutrino expected daily rate \dot{N}_ν^{UL} for observable neutrinos with energies $E_\nu^o \geq 100$ TeV is calculated using $\text{SED}_\nu(E_\nu^o, t_i)$ and the *IceCube* muon neutrino effective area $A_{\text{eff}}(E_\nu^o)$ at the declination of 3C 454.3 16.15° (IceCube Collaboration 2021; IceCube Collaboration et al. 2021; Aartsen et al. 2020). We use the threshold of $E_\nu^o \geq 100$ TeV since approximately at these energies the *IceCube* collaboration issues alerts (Abbasi et al. 2023a) reporting neutrino-like events with the probability of the signal being of astrophysical origin ≥ 0.3 (BRONZE events) or ≥ 0.5 (GOLD events) and the expected background counts become low (Aartsen et al. 2016, fig. 18). When converting all-flavour neutrino (and anti-neutrino) intensity into the muon one we use the factor of 1/3 due to neutrino oscillations in vacuum.

(viii) Having obtained the light curve of the *IceCube* muon neutrino yield $\dot{N}_\nu^{\text{UL}}(t_i)$, we average it over the whole range of the *Fermi*-LAT observation time to obtain the mean yearly expected yield.

The obtained *IceCube* neutrino light curve and expected mean neutrino yield are presented in Sect. 4.4.

3.5 Method of estimating the contribution of *Fermi*-LAT FSRQs to the *IceCube* neutrino flux

This section describes our method to roughly estimate the expected *IceCube* muon neutrino yield at energies ≥ 100 TeV from all *Fermi*-LAT FSRQs within the framework of the single-zone model used to describe the brightest blazar flare. We use the fourth *Fermi*-LAT source catalogue 4FGL, version g11_psc_v35.fit (Abdollahi et al. 2020) and select all objects with flags fsrc or FSRQ. For each of the FSRQs, we extract its light curve $\Phi_\gamma^o(t_i)$ from the LCR or use the yearly-binned light curve from the catalogue if the object is not present in the LCR. Then we repeat steps (vi), (vii), and (viii) described in the previous section using light curves $\Phi_\gamma^o(t_i)$ of the objects of interest instead of the light curve of 3C 454.3 and the *IceCube* effective area at the declination corresponding to the object of interest but keeping the shape of the neutrino SED and $v - \gamma$ integral flux proportionality constant a from that of 3C 454.3. This method can give only a rough estimate of the expected neutrino yields from FSRQs assuming their neutrino SED shape follows the one obtained for 3C 454.3 for MJD 55517–55524 and it does not take into account that for different objects the dissipation radius x , Doppler factor δ , proton-to-electron energy density ratio $\rho_{p/e}$, and acceleration parameter ξ_{acc} can vary as well as the eight parameters of the leptonic model and parameters defining the photon fields.

To estimate the neutrino flux observed by *IceCube* at energies $E_\nu^o \geq 100$ TeV, we sum up all signalnesses of all the *IceCube* alert tracks¹⁷ from the public *IceCat-1* catalogue (Abbasi et al. 2023a).

¹⁷ excluding those giving a signal in the *IceTop* detector

This results in ≈ 12 “pure” astrophysical muon neutrinos observed by *IceCube* at energies $E_\nu^o \geq 100$ TeV.

The estimated contribution of the *Fermi*-LAT FSRQs to the *IceCube* neutrino flux is presented in Sect. 4.5.

4 RESULTS

4.1 Results of the “standing-feature” leptonic model

The individual observed SEDs from each of the days of MJD 55517–55524 and the best-fit SED model curves are shown in Fig. 8 along with the fit to the whole-seven-day averaged dataset (Fig. 8h) performed in the same manner as for the one-day-long SEDs with t'_{sim} from Eq. (4) calculated for $t'_{\text{av}} = 1$ d to keep this parameter fixed. The best-fit values of the parameters for all seven days are shown in Fig. 9. In coloured shades, we show 1σ (darker) and 2σ (lighter) uncertainties derived with emcee (Foreman-Mackey et al. 2013) from the Monte-Carlo Markov Chain (MCMC) fitting for the seven-day-averaged period (see Appendix B) while the MCMC median values are shown as black solid horizontal lines.

From Figs. 8–9 we conclude the following:

(i) The “standing-feature” pure leptonic model provides a good fit to all one-day-averaged SEDs as well as the seven-day-averaged SED with the exception of MJD 55518–55519 (Fig. 8b): due to the under-fluctuation in the second to last *Fermi*-LAT bin, the quality of the fit worsens. If one were simply to remove that bin and recalculate the chi-squared (without re-optimization), the reduced chi-squared value would become $\chi^2_{\text{red}} = 2.16$ and a new optimization could improve this value even further. Thus, we argue that for all seven days of the flare, the “standing-feature” pure leptonic model provides a good description of the observed SEDs.

(ii) The IR, optical, and UV data are well described with the synchrotron emission from the relativistic electrons in the magnetic field with value not far from our initial guess (see Eq. 26) and fluctuations within $\approx 3\sigma$ from the median MCMC value for the seven-day-averaged SED.

(iii) The radio data cannot be described with our model due to the compactness of the blob leading to synchrotron self-absorption between $10^{-3} - 10^{-2}$ eV ($\sim 10^{11} - 10^{12}$ Hz). This implies that an extended emission zone must exist to account for the enhanced radio emission during the flare (Jorstad et al. 2013).

(iv) The contribution of the direct AD radiation to the optical and UV SED is subdominant.

(v) The X-ray data are well described as the synchrotron self-Compton (SSC) emission of electrons from the part of the electron spectrum below the cooling break (see Fig. 6) where their cooled spectral index is $p_e \approx 2$ resulting in an X-ray spectral index of $\Gamma_\gamma \approx 3/2$ (see Eq. 24) which is close to the value obtained from fitting with a functional form (see Table 3).

(vi) The γ -ray data are well described by IC emission of electrons with energies above the cooling break. The IC process uses the BLR photons as target photons and is in the Thomson regime. The *Swift*-XRT and *Fermi*-LAT data are connected with emission from the electron IC on the AD radiation coming from behind the blob in the host galaxy rest frame.

(vii) Among the eight free parameters, it is s_e which experiences the most significant change from one day to another during the flare. We argue that this is mostly driven by fluctuations in the highest-energy part of the *Fermi*-LAT SED due to low photon statistics within one day.

(viii) While the electron injection luminosity does not seem to

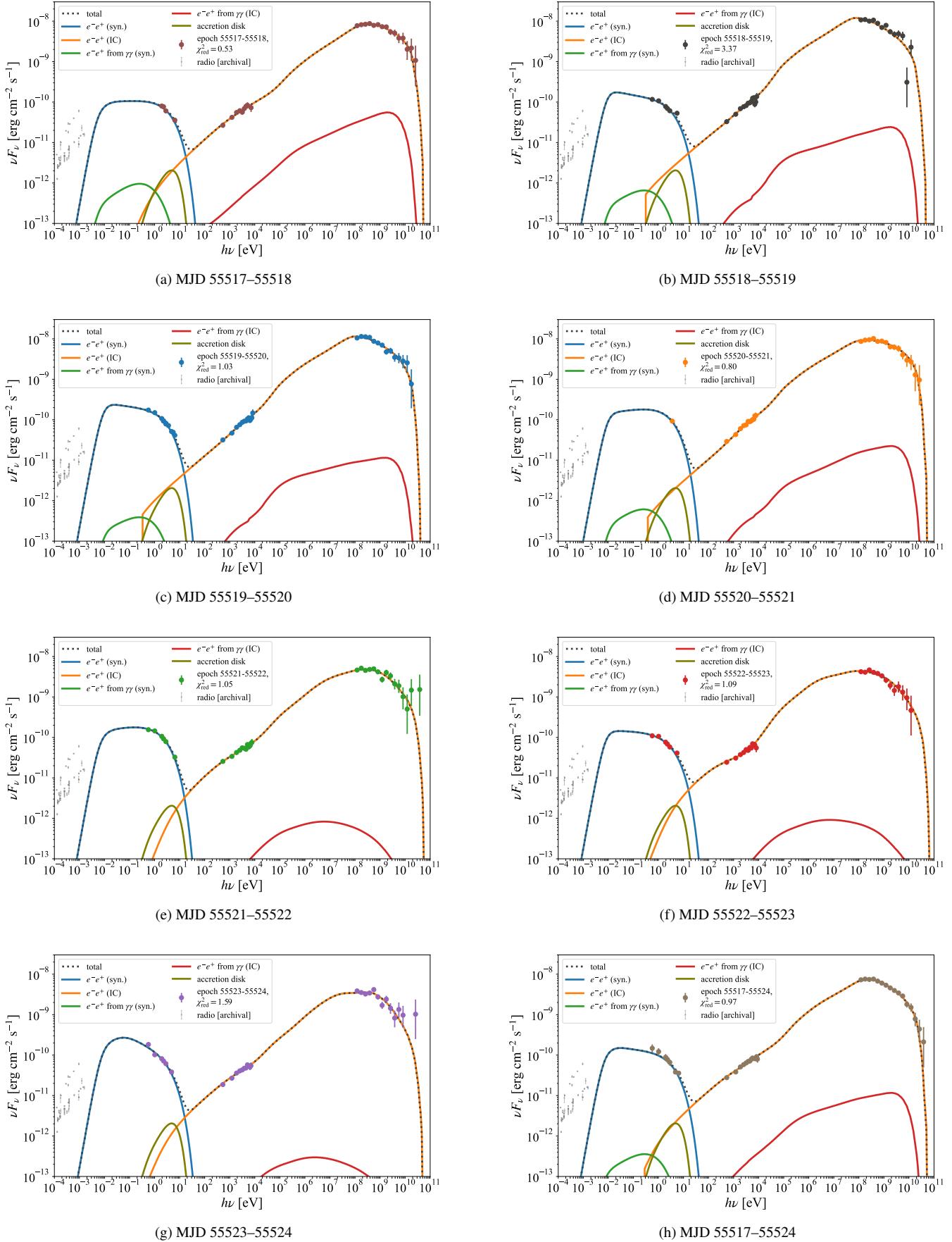


Figure 8. SEDs averaged over 1-day periods (a to g) and SED averaged over the whole period of MJD 55517–55524 (h).

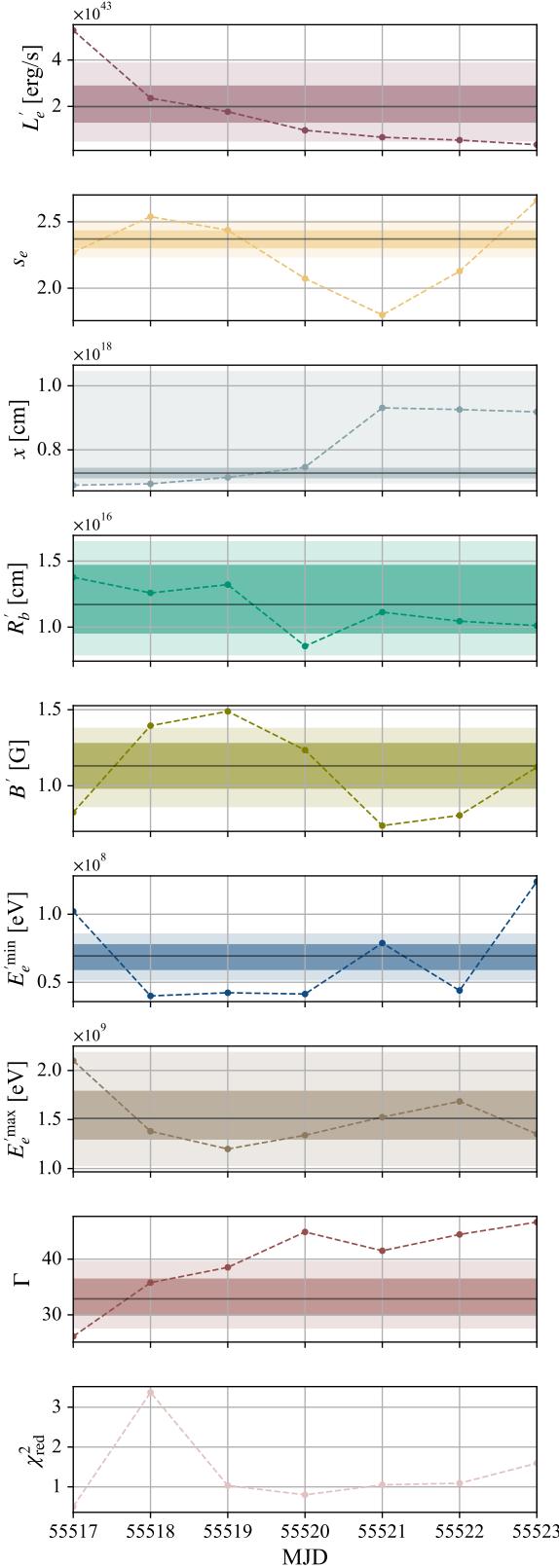


Figure 9. Best-fit values of the free parameters obtained from the fit of the “standing-feature” model to one-day-long quasi-simultaneous SEDs observed during MJD 55517–55524 along with the reduced chi-squared value (circles). In coloured shades, 1σ (darker) and 2σ (lighter) uncertainties are shown, while the MCMC median values are shown as black solid horizontal lines from the MCMC fitting performed for the seven-day-averaged dataset (see Appendix B).

change significantly within MJD 55517–55524, the Lorentz factor does change and shows an increasing trend throughout the flare. This can be explained as follows. During the last three days of the flare, the *Fermi*-LAT observes higher values of the γ -ray SED in the two last energy bins which implies that x must increase in order to avoid $\gamma\gamma$ absorption. Increasing x when it is greater than R_{BLR} leads to a significant drop in the density of the photon field. To compensate for this drop the Lorentz factor Γ should be increased accordingly.

(ix) The asymmetry of the MCMC uncertainty of x for MJD 55517–55524 is driven by the lower limit on x due to $\gamma\gamma$ absorption (see Eq. 20).

(x) The maximum electron injection energy slightly varies but stays within the 2σ uncertainty of the MCMC value for MJD 55517–55524 and is close to the value estimated with Eq. (23).

(xi) The obtained fit with a low value of the maximum electron injection energy $E_e^{\max} \sim 10^9$ eV results in significant curvature of the observed γ -ray spectrum (see Sect. 3.1.6). In Fig. 7, we can see that this maximum injection energy corresponds to the electron acceleration timescale $t_e^{\text{acc}} = \xi_{\text{acc}} t_e^{\text{gyro}}$ with $\xi_{\text{acc}} \sim 10^7$, where the gyroperiod t_e^{gyro} is defined in Eq. (11). A model with a similar value of ξ_{acc} was suggested by Inoue & Takahara (1996) to explain early observations of another FSRQ: 3C 279. They interpreted $\xi_{\text{acc}} \sim 10^7$ as a sign that the ratio of the energy of the background magnetic field to the energy of the magnetic turbulence is as large.

(xii) The blob size stays within the 2σ MCMC uncertainty of the value obtained for the seven-day-averaged dataset. The inferred variability timescale (see Eq. 14) is approximately from 2 h to 4.5 h. These values are close to the values of 2.6 ± 1.0 h obtained by Meyer et al. (2019).

(xiii) For all of the days of the flare the condition $R'_b \Gamma / x \leq 1$ that the blob cross-section does not exceed the cross-section of the jet in the conical approximation is satisfied.

4.2 Results of the “moving-blob” leptonic model

Using **iMINUIT**, we fit the SEDs of the “moving-blob” model to the data from seven one-day-averaged observed SEDs using first the **SIMPLEX** algorithm and **MIGRAD**. The SEDs resulting from the fit are shown in Fig. 10. The total quality of the obtained fit for the union of the seven SEDs is rather poor: $\chi^2_{\text{red}} \approx 6$. While the model manages to describe the changes in the X -ray energy range, it fails to do so in the IR-optical-UV range and in the higher-energy part of the *Fermi*-LAT spectrum. The latter is mainly due to the movement of the blob far away from the BLR photon field resulting in the decrease of the target photon field energy (see Eq. 17) and density which leads to the under-prediction of the number of the γ -ray photons at higher energies at later days of the flare. Moreover, to compensate for the decrease of the external photon density, L'_e is increased accordingly. This would overshoot the X-ray data which are explained by SCC with the flux proportional to L'_e^2 . To mitigate overshooting of the X-ray data, the fitter increases the initial blob radius $R'_b(0)$ up to 6×10^{16} cm to decrease the synchrotron photon density in the blob frame (decreasing as $R_b'^{-3}$) which provides a good description of the X-ray data but is in tension with the values expected from the typical flare decay timescale of a few hours (see Sect. 3.1.4).

The failure to describe the flare in the framework of the “moving-blob” model implies that the observed emission during the flare originates in a quasi-stationary “standing feature” in the jet which does not move along with the jet plasma bulk motion and into which multiple blobs are injected as was modelled in Sect. 3.1. We discuss

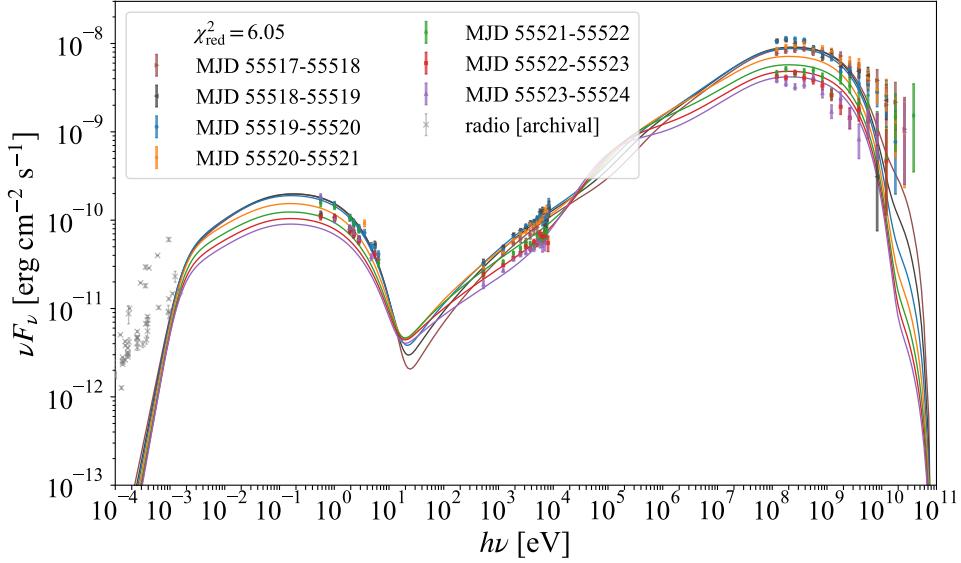


Figure 10. “Moving-blob” leptonic model SEDs averaged over one day each from the fit to the seven observed one-day-averaged SEDs for MJD 55517–55524.

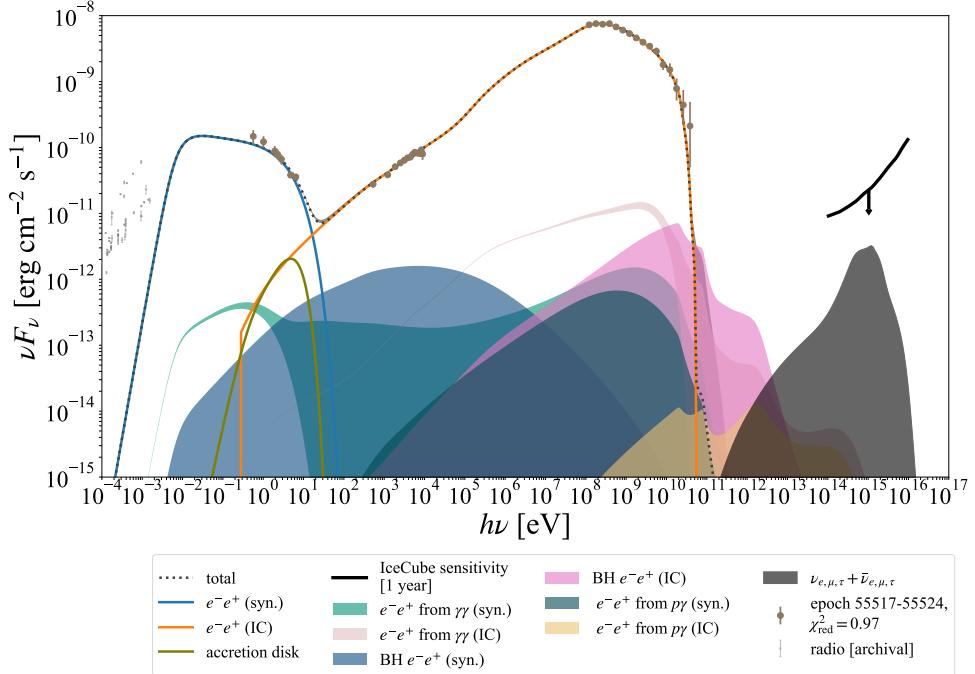


Figure 11. The photon and neutrino SED with model curves for the lepto hadronic “standing-feature” model with a fit to the observed SED averaged over MJD 55517–5524 with proton acceleration parameter $\xi_{\text{acc}} = 10^7$. Shaded regions correspond to the spread of proton luminosities $0 \leq u'_p \leq u'^{\text{UL}}$, where u'^{UL} is the UL corresponding to the Bayes factor with respect to the best-fit leptonic model $BF = 10^{-2}$ ($\approx 3\sigma$ Gaussian equivalent). The black solid curve with a downward arrow shows the *IceCube* one-year sensitivity (see Eq. 42) for the UL of 2.44 muon neutrinos between $E_\nu^{\text{o min}} = 10^{14}$ eV and $E_\nu^{\text{o max}} = 10^{16}$ eV.

possible interpretations of the prevalence of the “standing feature” in Sect. 5.2.

4.3 Results of the lepto hadronic model

The plot with the lepto hadronic “standing-feature” model fitting the observed SED for MJD 55517–55524 is shown in Fig. 11. In the lepto hadronic model, all relevant processes responsible for the production of secondaries are considered: BH pair production, decay of

particles produced in $p\gamma$ collisions, $\gamma\gamma$ absorption with the subsequent synchrotron and IC emission of the e^+e^- pairs produced in these processes. Photon and all-flavour neutrino SEDs produced as a result of these processes are shown in Fig. 11 as colourful shades (see the legend) with their upper boundary corresponding to the UL proton energy density $u'^{\text{UL}} = \rho_p/e u'_e$ with $\rho_p/e = 1.3 \times 10^2$ and the lower boundary corresponding to the purely leptonic model with $u'_p = 0$. Proton and muon synchrotron and IC emissions are completely negligible. As in the case of TXS 0506+056 (e.g., Keivani

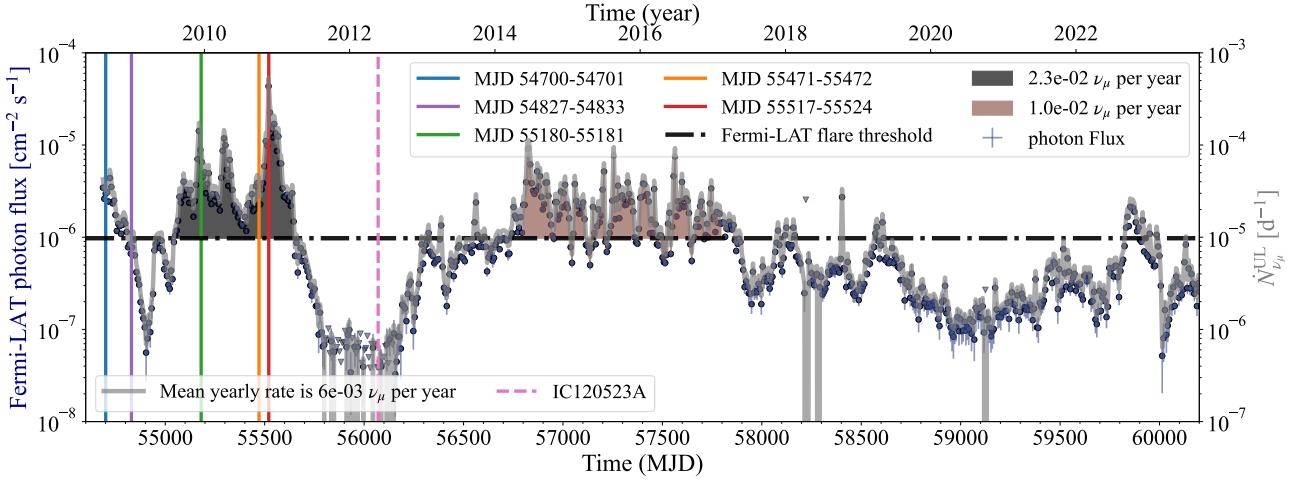


Figure 12. The light curve of 3C 454.3 based on the *Fermi*-LAT ≥ 100 MeV photon flux (blue data points with error bars for detections if the source-detection test statistic $TS \geq 4$ and triangles for ULs otherwise, left ordinate axis) from the *Fermi*-LAT LCR (Abdollahi et al. 2023), and the corresponding *IceCube* daily muon neutrino yield for energies $E_\nu^o \geq 100$ TeV (grey curve, right ordinate axis). The *Fermi*-LAT flare threshold is defined as the 68% percentile of the distribution of the *Fermi*-LAT weekly-binned photon fluxes. The mean yearly *IceCube* muon neutrino rate for $E_\nu^o \geq 100$ TeV is shown in the legend along with the rates averaged over the periods of outbursts highlighted with grey and brown shaded regions. The *IceCube* muon neutrino alert IC120523A is shown as the dashed vertical curve.

et al. 2018), it is X-ray data which drive the constraints on $\rho_{p/e}$ where at higher values of $\rho_{p/e}$ the synchrotron emission from BH electrons starts overshooting the observed X-ray SED.

The obtained proton energy density $u_p^{\text{UL}} = \rho_{p/e} u_e' = 1.1 \times 10^2$ erg/cm³ corresponds to apparent jet power of $P_p^{\text{UL}} = \pi c \beta \Gamma^2 R_b'^2 u_p^{\text{UL}} = 1.6 \times 10^{48}$ erg/s while the Eddington luminosity is $L_{\text{Edd}} = 2.1 \times 10^{47}$ erg/s. The fact that the apparent proton jet power exceeds the Eddington luminosity by an order of magnitude implies that these parameters cannot be at this level for a long time if we do not assume the possibility of super-Eddington accretion. Such high power is mainly attributed to a high value of the Lorentz factor $\Gamma = 32$ during the flare and is lower during quieter periods of activity (see Appendix C). Alternatively, the value of the proton-to-electron-energy density can be lower than the UL $\rho_{p/e} = 1.3 \times 10^2$ or the SMBH mass is higher than we assumed. The derived value of $\rho_{p/e}$ is, however, two orders of magnitude lower than those obtained in some models for TXS 0506+056 (e.g., Cerruti et al. 2019) and of the same order of magnitude as those obtained by Keivani et al. (2018, model LMBB2b).

We superimpose the *IceCube* sensitivity in Fig. 11 obtained as follows. For the neutrino energy range from $E_\nu^{\text{min}} = 10^{14}$ eV to $E_\nu^{\text{max}} = 10^{16}$ eV, $i = 0, 1, \dots, N_{\text{bins}}$, we calculate the factor

$$m_i = \frac{N_{\text{max}}^{\text{90\% CL}}}{N_{\text{bins}}} \times \left[\int_{E_{i-1}^o}^{E_i^o} \frac{1}{3} \frac{dN_\nu^o}{dE_\nu^o} A_{\text{eff}}(E_\nu^o; \delta_{3C454.3}) dE_\nu^o \times \Delta T \right]^{-1} \quad (42)$$

by which we multiply the upper boundary of the neutrino SED presented in Fig. 11 assessed at the intermediate energy bins E_i^o between E_ν^{min} and E_ν^{max} with the number of bins $N_{\text{bins}} = 30 \times \lg(E_\nu^{\text{max}}/E_\nu^{\text{min}})$. $A_{\text{eff}}(E_\nu^o; \delta_{3C454.3})$ is the *IceCube* muon neutrino effective area at the declination of 3C 454.3 $\delta_{3C454.3} = 16.15^\circ$ (IceCube Collaboration 2021; IceCube Collaboration et al. 2021; Aartsen et al. 2020). The number $N_{\text{max}}^{\text{90\% CL}} = 2.44$ (muon neutrinos per whole energy range from E_ν^{min} to E_ν^{max}) is the UL at 90% confidence level from Feldman & Cousins (1998, table XII) in case of zero background and zero observed events. The time period consid-

ered is $\Delta T = 1$ yr. The factor of 1/3 accounts for neutrino oscillations in vacuum. Thus, the black solid curve with a downward arrow in Fig. 11 shows a one-year *IceCube* sensitivity [90% confidence UL] for the 3C 454.3 neutrino spectrum presented as the black shaded region in Fig. 11.

4.4 *IceCube* expected neutrino yield and light curve of 3C 454.3

The obtained *IceCube* neutrino light curve superimposed on the *Fermi*-LAT one for 3C 454.3 is presented in Fig. 12. The four other periods used to obtain the $\nu - \gamma$ relation between integral fluxes are indicated with vertical lines in Fig. 12, their SEDs and best-fit parameters are presented in Appendix C. The average *IceCube* neutrino yield for 3C 454.3 for $E_\nu^o \geq 100$ TeV is $\sim 6 \times 10^{-3}$ muon neutrinos per year.

4.4.1 On the *IceCube* neutrino alert IC120523A associated with 3C 454.3

In *IceCat-1* (Abbasi et al. 2023a), there is one muon neutrino event with estimated energy $E_\nu^o = 168$ TeV and signalness (probability of the neutrino being astrophysical) 0.49 positionally associated with 3C 454.3 albeit not contemporaneous with the studied flare. Taking the mean rate of 6×10^{-3} muon neutrinos per year from our estimations for 3C 454.3 and the duration of *IceCube* observations of ≈ 10 years, we obtain the Poisson probability of detecting one or more muon neutrinos with energies ≥ 100 TeV from 3C 454.3 to be $(1 - \exp[-6 \times 10^{-3} \times 10]) \approx 0.06$. The obtained value indicates that on a long-period average, our model agrees with the single positional association of a muon neutrino with 3C 454.3, especially in light of the possible Eddington bias for cosmic neutrino sources (Strotjohann et al. 2019). The fact that the neutrino event was on MJD 56070.6 during the global minimum of the γ -ray activity (see Fig. 12) may indicate (assuming the neutrino is not atmospheric) that the neutrino emission in blazars is lagging behind the leptonic electromagnetic emission (e.g., due to slowness of photopion processes,

see Fig. 7). If we assume the protons continue to lose their energy under the same conditions as during MJD 55517–55524, then 99% of the neutrino fluence would be emitted in less than 450 days after the peak of the flare, while the alert came 551 days after the flare which makes the association of this neutrino with the flare improbable. However, if the Lorentz factor of the emitting region drops to $\Gamma \approx 10$ (typical for more quiescent states, see Appendix C), 99% of the neutrino fluence would be emitted in ~ 1300 days which covers well the neutrino-flare lag of 551 days. On the other hand, the expected neutrino yield from a source with a lower Lorentz factor is lower. If there is a time lag between neutrino and γ -ray emission, it may explain why 3C 454.3 was among the three brightest neutrino candidate sources in the search performed by Abbasi et al. (2023b) with γ -ray average flux weights (in agreement with our long-term average estimate) but was not among the best ten candidates in their search with monthly flux weights since they did not take into account a possible $\nu - \gamma$ time lag. The detailed investigation of the properties of delayed neutrino emission is beyond the scope of the paper.

4.5 Contribution of the 4FGL FSRQs to the *IceCube* neutrino flux at energies $E_\nu^o \geq 100$ TeV

The estimated average neutrino yield in *IceCube* from all 4FGL FSRQs is $\sim 6 \times 10^{-2}$ muon neutrinos per year given the linear relation between the ν and γ -ray integral fluxes and Eq. (41) for linking neutrino SEDs and *Fermi*-LAT light curves from the LCR. Given ~ 12 “pure” muon neutrinos per year observed by *IceCube*, it implies the 4FGL FSRQs contribute at the level up to $\sim 0.5\%$ of the *IceCat-1* yearly muon neutrino rate at energies $E_\nu^o \geq 100$ TeV.

4.5.1 The case of TXS 0506+056

Although TXS 0506+056 is not classified as an FSRQ in the 4FGL catalogue (Abdollahi et al. 2020), it still may belong to this class as pointed out by Padovani et al. (2019) showing that this source may host BLR and DT external photon fields and a model similar to the one used for 3C 454.3 may describe the geometry of this source. We apply the procedure described in Sects. 3.4 and 3.5 to TXS 0506+056. This results in $\sim 4 \times 10^{-4}$ *IceCube* muon neutrinos per year on long-term average and in $\sim 10^{-3}$ muon neutrinos per year within ± 180 days around the *IceCube*-170922A arrival time (at energies $E_\nu^o \geq 100$ TeV) which lies within the range of different model predictions of Keivani et al. (2018, table 9) who performed dedicated modelling for this source specifically with various parametrization of the external photon field.

4.5.2 Expected neutrino yields in other neutrino telescopes

Other neutrino telescopes under construction, such as the Pacific Ocean Neutrino Experiment (*P-ONE*, Agostini et al. 2020), the Baikal Gigaton Volume Detector (*Baikal-GVD*, Avrorin et al. 2011), and the Cubic Kilometre Neutrino Telescope (*KM3NeT*, Adrian-Martinez et al. 2016) will have the expected yearly muon neutrino rate from *Fermi*-LAT FSRQs comparable to the one in *IceCube*. Their combined statistics with *IceCube* will result in the detection of a muon neutrino with energy ≥ 100 TeV from *Fermi*-LAT FSRQs every 4–5 years. The projected Tropical Deep-sea Neutrino Telescope (*TRIDENT*, Ye et al. 2023) with its effective area a factor of ~ 10 larger than that of *IceCube* will detect on average ~ 1 muon neutrino with energy ≥ 100 TeV from flares in *Fermi*-LAT FSRQs every

other year. A similar detection rate can be expected from the proposed *IceCube-Gen2* (Ishihara et al. 2023). Therefore, in our model, next-generation neutrino telescopes are expected to detect approximately one multimessenger $(\gamma + \nu_\mu)$ flare per year from *Fermi*-LAT FSRQs.

5 DISCUSSION

5.1 Location of the γ -ray emitting region beyond R_{BLR}

The location of the dissipation radius beyond the outer radius of the BLR but not very far from it is in line with recent studies (see, however, Appendix E): Zheng et al. (2017) within leptonic models for 36 FSRQs obtained fits indicating the location of the emitting zone beyond the outer radius of the BLR; Costamante et al. (2018), after analyzing around seven years of *Fermi*-LAT data, did not find features in the γ -ray spectrum indicating presence of strong $\gamma\gamma$ absorption concluding that “ γ -rays seem either produced outside the BLR most of the time, or the BLR is ~ 100 times larger than given by reverberation mapping”; Meyer et al. (2019) in a similar *Fermi*-LAT data analysis of a longer observation period concluded that the emitting region should be located beyond $1.36R_{\text{BLR}}$ although their BLR model is somewhat different from the one in our work. Wang et al. (2022) suggested that multi-wavelength flares (contemporaneous optical, X -, and γ -ray) should occur at intermediate distances from the SMBH as in the case for the flare of 3C 454.3 we considered.

5.2 “Standing feature” vs. “moving blob”

The fact that the dissipation radius x does not change much during the modelling and the model with a “moving blob” fails to describe the seven consecutive SEDs can be interpreted in favour of a model like the turbulent, extreme multi-zone (TEMZ) model by Marscher (2014) or other models invoking standing or slowly moving bright features in blazar jets like standing shocks (Marscher & Gear 1985; Daly & Marscher 1988) or reconnection layers (Sironi et al. 2016; Petropoulou et al. 2016). The latter model can also describe the day-to-day change in the parameters of the leptonic model stemming from an injection of a new plasmoid with parameters different from each previous one since the generation of new plasmoids is expected to be stochastic. Thus, works focusing rather on the statistical properties of blazar variability as a stochastic process might be applicable (Ding et al. 2024; Fagin et al. 2024; Thiersen et al. 2024).

The stochastic nature of blazar variability may hinder attempts to model the variability with regular changes of one-zone model parameters and/or invoking of a “moving blob” as we tried to do in Sect. 3.2. Attempts of Diltz & Böttcher (2016); Diltz (2016) in changing some of the parameters with its profile obeying a functional form of a rapid increase with an exponential decay did not result in a good description of the data. Finke (2016) concluded that he cannot describe the 3C 454.3 variability with a blob moving from $x < R_{\text{BLR}}$ to $x > R_{\text{BLR}}$ as well as with a model of a blob changing its radius (Finke 2024). On the other hand, models invoking a single moving and expanding blob may successfully explain the time lag between radio and γ -ray light curves (Boula & Mastichiadis 2022; Tramacere et al. 2022; Zacharias 2023) observed in many blazars (e.g., Kramarenko et al. 2022).

5.3 Contribution of *Fermi*-LAT FSRQs to the neutrino flux

Neutrinos with observable energies E_ν^o generally come from protons with observable energies $E_p^o \sim 20E_\nu^o$ (for detailed studies of the E_p^o - E_ν^o relation see [Roulet & Vissani 2021](#)). Given $E_p^o = \delta E'_p / (1+z)$, Lorentz factor $\Gamma \approx 25$, and the redshift $z = 0.859$ for 3C 454.3, we can get that the proton energy in the blob frame roughly equals the observed neutrino energy $E'_p \sim E_\nu^o$. Thus, if the neutrino SED peaks at energies between $10^{14} - 10^{16}$ eV (as in Fig. 11), it is produced by the protons with energies $E'_p \lesssim 10^{16}$ eV in the blob frame. In our leptohadronic model, for protons at these energies, as can be seen in Fig. 7, the timescale of photopion losses is much greater than the timescale of the flare implying that the $p\gamma$ process in the blob is optically thin $\tau_{p\gamma} \ll 1$.

If one wants to increase $\tau_{p\gamma}$, the most efficient way to do so is to move the production zone inside the BLR since beyond R_{BLR} the BLR photon field density falls very rapidly with increasing x according to [Ghisellini & Tavecchio \(2009\)](#) and the apparent energies of photons decrease according to Eq. (17). By moving the dissipation radius x into the BLR one can increase the expected neutrino flux by a factor of $\sim 7 \times 10^1$ (on average) but this makes it impossible to describe the observed γ -rays due to severe $\gamma\gamma$ absorption as elaborated in Sect. 3.1.5 (see also [Murase et al. 2016, 2020](#)).

Protons can have an acceleration timescale different from the one of electrons. By means of a simple scan (see Appendix D), we obtained that for $\xi_{acc} \sim 5 \times 10^4$ neutrino production reaches maximum and is ~ 3 times more efficient than for the case of $\xi_{acc} \sim 10^7$. Thus, for a model with a separate neutrino production zone inside the BLR and optimal acceleration timescale, one can increase the neutrino yield by a factor of $\sim 2 \times 10^2$ raising the potential contribution of 4FGL FSRQs to the *IceCat-1* neutrinos up to 100%. This is just a rough estimate and does not take into account possible changes in other model parameters. The consideration of a model with several emission zones is beyond the scope of the current paper.

5.4 A possible delay between neutrino emission and electromagnetic flares in blazars

For neutrino SEDs peaking at a few hundred TeV – a few PeV, the energy-loss time scale of proton photopion production exceeds the timescale of a typical multi-wavelength flare by almost three orders of magnitude. Depending on the Lorentz factor, dissipation radius and parameters of the external photon field, if the conditions for $p\gamma$ interactions hold for sufficiently long, the time window within which a significant neutrino fluence is expected can be as wide as $\sim 10^3$ d implying the possibility of a substantial months-to-years lag between a potential neutrino detection and the corresponding multi-wavelength flare. This circumstance can be accounted for in future searches of correlations between high-energy neutrinos and their electromagnetic counterparts.

6 CONCLUSIONS

We studied in detail the brightest *Fermi*-LAT blazar flare in FSRQ 3C 454.3 that occurred in November 2010 having obtained seven consecutive multi-wavelength SEDs of each day of the flare around its peak (MJD 55517–55524). By modelling the flare SEDs with AM³, we showed that the seven consecutive SEDs cannot be described with the synchrotron and IC emission from a single blob moving down the jet whereas the “standing-feature” model in which multiple blobs are passing through a quasi-stationary feature located beyond the outer

radius of the BLR describes the data well. A pure leptonic model is, in principle, sufficient to describe the seven observed consecutive SEDs.

If protons are co-accelerated with electrons, the X-ray data constrain the proton energy density to be at maximum $\rho_p/e \approx 1.3 \times 10^2$ times greater than the electron energy density. By applying the same approach to four other periods of activity of 3C 454.3 we determined that the integral neutrino and γ -ray fluxes are approximately linearly proportional. Applying the obtained $\nu - \gamma$ scaling to the whole period of *Fermi*-LAT observations using the light curve from the *Fermi* LCR we constructed a neutrino light curve of 3C 454.3. We estimate that on average 3C 454.3 produces 6×10^{-3} *IceCube* muon neutrinos per year at energies $E_\nu^o \geq 100$ TeV in the framework of the single-zone “standing-feature” leptohadronic model. The proton maximum energy is not directly constrained by the electromagnetic flaring SED of 3C 454.3. Under more optimistic assumptions about the maximum proton energy (i.e. larger values E_p^{\max} , lower values ξ_{acc}), the maximum neutrino yield is similar or a factor of a few lower than in our benchmark model but the peak neutrino energy is approximately 100 PeV, consistent with the energy of the recently discovered >100 PeV neutrino KM3-230213A ([The KM3NeT Collaboration 2025](#)).

Extrapolation of the 3C 454.3 model to the whole list of *Fermi*-LAT 4FGL FSRQs shows that they can account for up to $\sim 0.5\%$ of the *IceCat-1* neutrino alerts at energies $E_\nu^o \geq 100$ TeV implying that FSRQs during multi-wavelength flares are not the main source of ≥ 100 TeV neutrinos observed by *IceCube*. The total expected yield from *Fermi*-LAT FSRQs in the planned *TRIDENT* and *IceCube-Gen2* neutrino telescopes will be about $\dot{N}_{\text{yr}}(E_\nu^o \geq 100 \text{ TeV}) \sim 1 \times (\rho_p/e/10^2)$ muon neutrinos per year. Next-generation neutrino telescopes are thus expected to detect approximately one multimesenger ($\gamma + \nu_\mu$) flare per year from FSRQs. We have shown that the electromagnetic flare peak can precede the neutrino arrival by months to years and that the Lorentz factor of the jet plays a crucial role in determining this time delay.

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Roos 1975), `snakemake` (Mölder et al. 2021), `pyLCR` (Abdollahi et al. 2023)¹⁸, `numba` (Lam et al. 2015; Lam et al. 2022), and `tqdm` (da Costa-Luis et al. 2023). This research has made use of NASA’s Astrophysics Data System and Pathfinder (Iyer et al. 2024). Software citation information aggregated using [The Software Citation Station](#) (Wagg & Broekgaarden 2024a; Wagg & Broekgaarden 2024b).

DATA AVAILABILITY

All data used for obtaining the observed SEDs are publicly available at corresponding web resources of the instruments cited in Sect. 2. The `AM3` program for modelling the observable SEDs is publicly-available (Klinger et al. 2023)¹⁹. Our codes for producing the figures will be published upon acceptance of this manuscript.

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¹⁸ <https://github.com/dankocevski/pyLCR>

¹⁹ <https://gitlab.desy.de/am3/am3>

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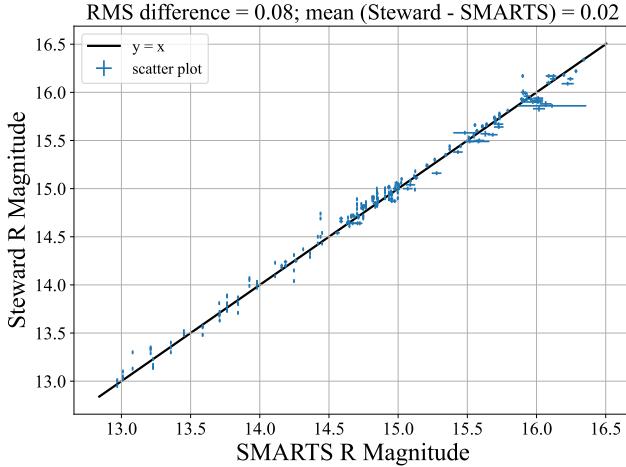


Figure A1. Scatter plot of 3C 454.3 Vega apparent magnitudes measured by *Steward Observatory* and *SMARTS* within same nights in the R band.

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APPENDIX A: ON THE SYSTEMATIC UNCERTAINTY OF THE UV, OPTICAL, AND IR SPECTRAL-ENERGY DISTRIBUTIONS

In this appendix, we compare *Steward* and *SMARTS* apparent magnitudes of 3C 454.3 measured within the same nights over the whole history of their joint observations in the R and V bands. The corresponding scatter plots are shown in Figs. A1 and A2 respectively. The root-mean-square difference (i.e. the square root of the mean squared difference of the magnitude) and the mean difference between *Steward* and *SMARTS* magnitudes are displayed in the titles of Figs. A1 and A2. There is a systematic shift in the values of magnitudes between Steward and SMARTS (0.02 – 0.04) which is subdominant to the root-mean-square scatter (0.08 – 0.09). An investigation of the reasons behind these discrepancies is beyond the scope of the paper. We take into account the observed discrepancy as an additional 8% relative systematic uncertainty added quadratically to all IR, UV, and optical SEDs as stated in Sect. 2.5.

APPENDIX B: CORNER PLOT FOR THE MCMC FIT TO THE SEVEN-DAY AVERAGED SED (MJD 55517–55524)

The corner plot is shown in Fig. B1. The values of the parameters and their mutual correlations are in line with the qualitative description of the results presented in Sect. 4.1.

APPENDIX C: SPECTRAL-ENERGY DISTRIBUTIONS FOR FOUR OTHER PERIODS

Spectral-energy distributions for four other periods indicated with colourful vertical lines in Fig. 12 are shown in Fig. C1. Table C1 presents the best-fit values of the parameters of the leptonic model for these four periods and MJD 55517–55524 as well.

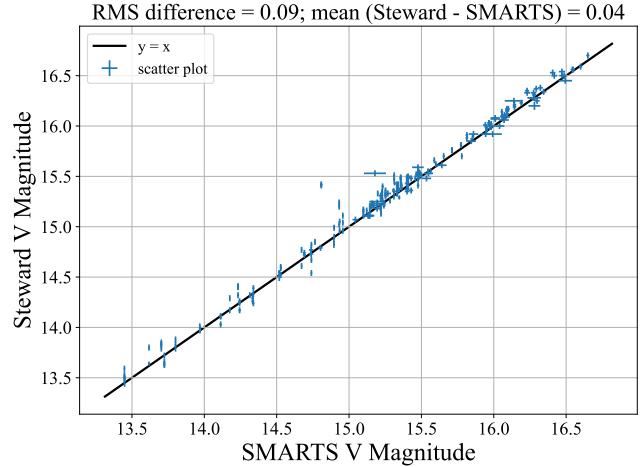


Figure A2. Scatter plot of 3C 454.3 Vega apparent magnitudes measured by *Steward Observatory* and *SMARTS* within same nights in the V band.

APPENDIX D: HOW DO VARIOUS VALUES OF PROTON ACCELERATION EFFICIENCY AFFECT THE EXPECTED NEUTRINO YIELD?

To investigate the impact of our choice of the acceleration parameter $\xi_{\text{acc}} = 10^7$ for protons, the same as for electrons (obtained from fitting the electromagnetic SED), we show how the *IceCube* muon neutrino daily yield from 3C 454.3 changes if we assume a different value of ξ_{acc} for protons in Fig. D1 (left axis, black circles). For each value of ξ_{acc} , we determine the maximum proton energy by finding the intersection of the proton acceleration timescale t_p^{acc} (Eq. 37) with the proton energy-loss timescale (dominated by the photopion losses, see the purple dotted curve in Fig. 7). Then we repeat the procedure for finding $\rho_{p/e}$ (Eq. 40) as was described in Sect. 3.3 until the Bayes factor (Eq. 39) drops to 10^{-2} . The corresponding values of $\rho_{p/e}$ are shown in the same plot in Fig. D1 (right axis, blue squares). Since at a fixed proton energy density, SEDs of secondary photons increase with increasing proton maximum energy (i.e. decreasing ξ_{acc}), the overshooting of the X-ray data happens at a lower normalization of the proton SED. Hence, $\rho_{p/e}$ decreases when ξ_{acc} decreases. In Figs. D2 and D3, we show photon and neutrino SEDs for $\xi_{\text{acc}} = 5 \times 10^4$ and $\xi_{\text{acc}} = 10$ respectively. One can see that the peak energy of the neutrino SED shifts to higher values with decreasing ξ_{acc} (compare also to Fig. 11). The value of $\xi_{\text{acc}} = 5 \times 10^4$ is an optimal value in terms of maximizing neutrino yield. For $\xi_{\text{acc}} \leq 10^3$, $\gamma\gamma$ -initiated cascades start to dominate over BH-initiated cascades and $\rho_{p/e}$ must be significantly decreased not to overshoot the X-ray data while the energy of the neutrino SED continues shifting to higher values resulting in the lower expected *IceCube* neutrino yield. Values $\xi_{\text{acc}} \leq 10^3$ are required to produce neutrinos with energies similar to the energy of the recently discovered >100 PeV neutrino KM3-230213A (The KM3NeT Collaboration 2025).

APPENDIX E: ON THE DISCREPANCY OF THE LOCATION OF THE γ -RAY EMITTING REGION WITH SOME WORKS

In contrast to the results of Bonnoli et al. (2011); Hu et al. (2015), who modelled SEDs averaged over the days around the flare MJD 55517–55524 (the closest date is MJD 55527) or the results of Sahakyan (2021) for the period we are studying in this work, we could not

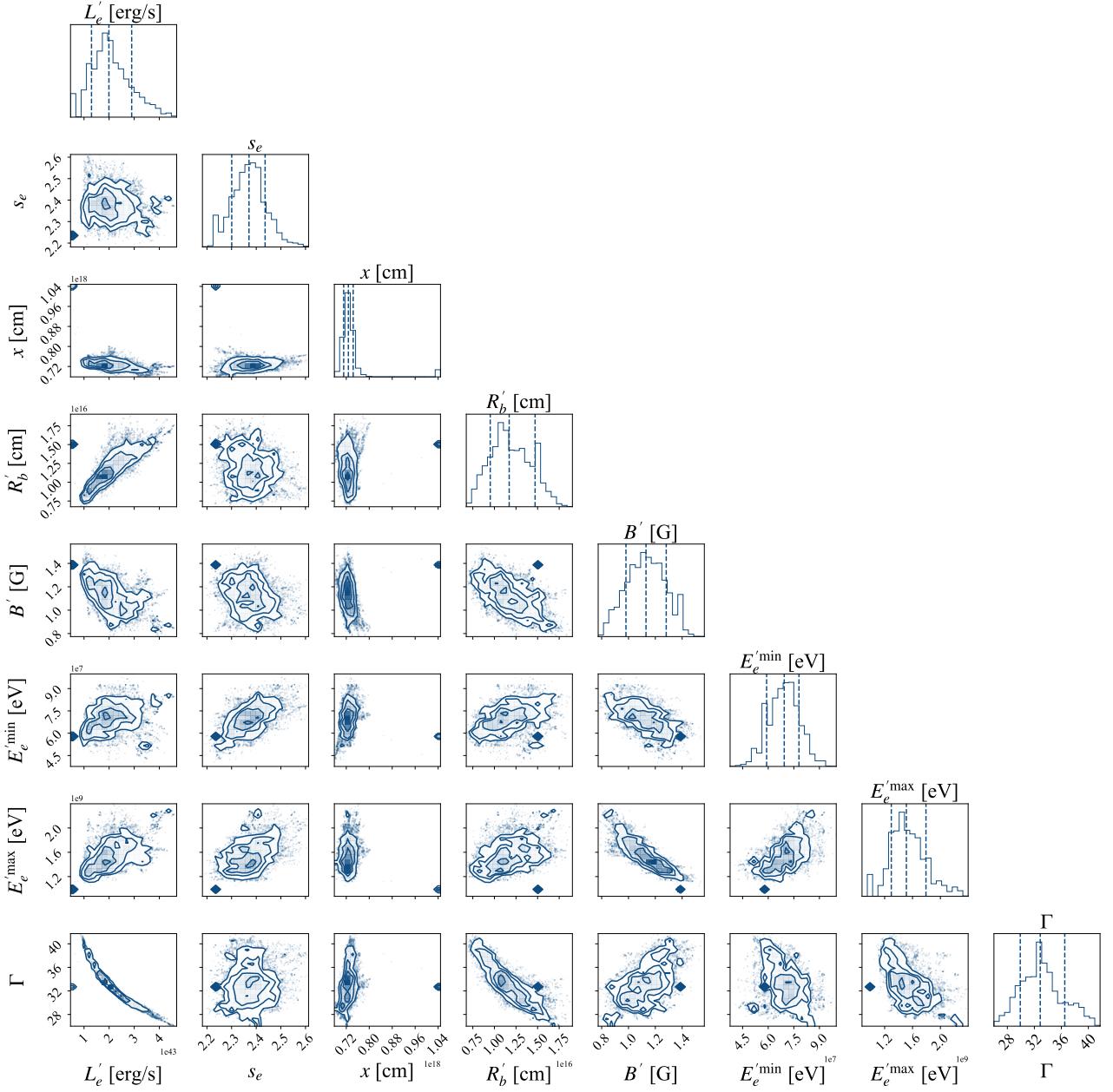


Figure B1. The corner plot (Foreman-Mackey 2016) obtained from the MCMC fitting (Foreman-Mackey et al. 2013) of the seven-day-averaged SED (MJD 55517–55524) with a “standing-feature” leptonic model. t'_{sim} here is the same as for the one-day-averaged SEDs with $t'_{\text{av}} = 1$ d (see Eq. 4). The corner plot is obtained from an MCMC containing 6,000 steps per walker with $3 \times 8 = 24$ walkers after combining chains from all the walkers (discarding 1,000 initial steps of each walker as a burn-in) and applying thinning of 10. Dashed vertical lines in the one-dimensional marginal histograms represent 16%, 50%, and 84% percentiles of the posterior distributions. Contour lines in the 2D marginal histograms represent 11.8%, 39.3%, 67.5%, and 86.4% 2D containment regions.

obtain a fit with the dissipation radius $x < R_{\text{BLR}}$ lying within the BLR due to the $\gamma\gamma$ absorption as explained in Sect. 3.1.5. The different conclusions about the value of x might be explained by different assumptions about the BLR size and/or the disc luminosity (see Table 2) resulting in non-zero but less-than-unity optical depth for the pair-production process at the threshold. Zhou et al. (2024) pointed out that the uncertainty of the BLR size can be of the order of half a magnitude, Abolmasov & Poutanen (2017) demonstrated that the uncertainty in the geometry of the BLR can result in significant

uncertainty in the density and angular distribution of the BLR photon field in the blob frame, and Hopkins (2024) even argued that the Shakura-Sunayev disc (Shakura & Sunyaev 1976) might be a poor model for the accretion flows around the SMBH. These uncertainties may be the reason for different conclusions about the location x of the emission region for the 3C 454.3 flare of MJD 55517–55524. Our assumption $L_{\text{AD}} = 3 \times 10^{46}$ erg/s is close to the estimations obtained from the observations of the broad lines of the BLR by Sbarro et al. (2012); Pian et al. (2005) in the standard assumption

Parameter	MJD 54700–54701	MJD 54827–54833	MJD 55180–55181	MJD 55471–55472	MJD 55517–55524
L'_e [erg/s]	6.5×10^{43}	2.3×10^{43}	1.5×10^{43}	8.2×10^{42}	2.2×10^{43}
s_e	2.7	2.4	3.3	2.6	2.4
x [cm]	7.3×10^{17}	7.3×10^{17}	7.3×10^{17}	6.6×10^{17}	7.3×10^{17}
R'_b	2.9×10^{16}	1.8×10^{16}	1.5×10^{16}	5.8×10^{15}	1.2×10^{16}
B' [G]	1.0	9.6×10^{-1}	1.7	2.2	1.0
E_e^{\min} [eV]	2.2×10^8	1.7×10^8	2.1×10^8	1.1×10^8	8.3×10^7
E_e^{\max} [eV]	3.6×10^9	6.9×10^9	2.8×10^9	4.2×10^9	1.8×10^9
Γ	13	11	22	19	32
χ^2_{red}	1.85	2.13	1.38	0.79	0.97

Table C1. Values of the “standing-feature” model parameters providing the best fit to the observed SEDs in the corresponding periods. In all cases t'_{sim} is derived from Eq. (4) for $t'_{\text{av}} = 1$ d.

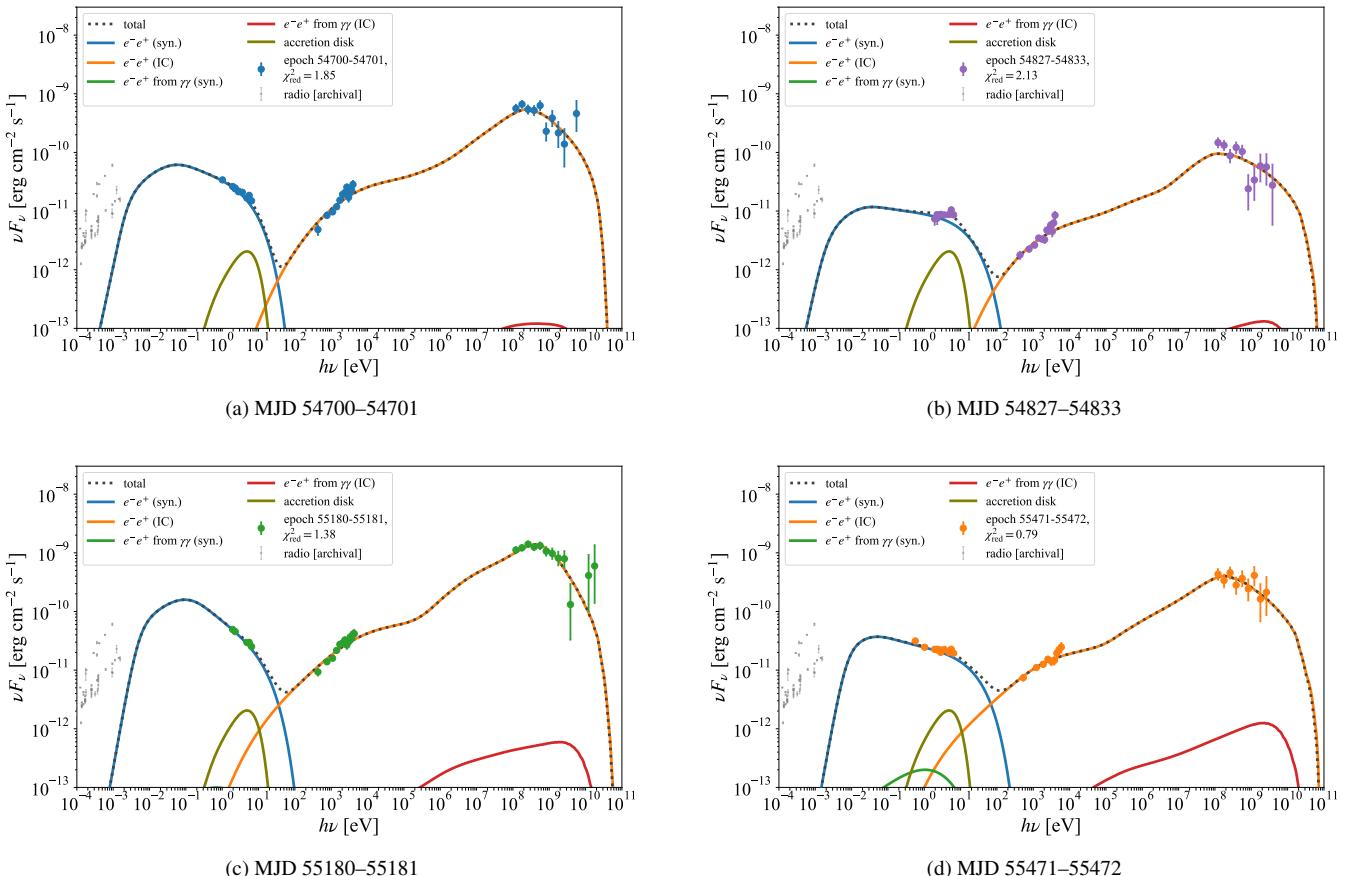


Figure C1. SEDs averaged over MJD 54700–54701 (a), MJD 54827–54833 (b), MJD 55180–55181 (c), MJD 55471–55472 (d) fitted with the “standing-feature” leptonic model in the same manner as SEDs in Fig 8. The best-fit values of the model parameters are shown in Table C1.

of the BLR covering factor $\xi_{\text{BLR}} = 0.10$ (although this value can vary, e.g., Naddaf et al. 2021; Naddaf & Czerny 2024), and the assumption about the BLR radius follows the standard phenomenological relation $R_{\text{BLR}} \propto \sqrt{L_{\text{AD}}}$ (Ghisellini & Tavecchio 2009; Kaspi et al. 2007). A systematic study of the influence of the uncertainties of the BLR and AD parameters on the estimation of x is beyond the scope of the current work.

This paper has been typeset from a TeX/LaTeX file prepared by the author.

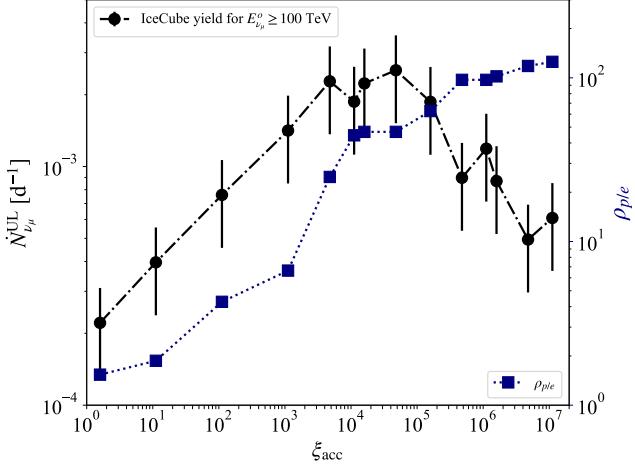


Figure D1. *IceCube* daily muon neutrino yield from 3C 454.3 averaged over MJD 55517–55524 for different values of the proton acceleration parameter ξ_{acc} (black circles, left ordinate axis) along with the UL on the proton-to-electron energy density ratio $\rho_{p/e}$ as a function of ξ_{acc} (blue squares, right ordinate axis). The error bars for neutrino yields represent the numerical uncertainty $\sim 40\%$ of normalization of spectra of secondaries in hadronic processes (Cerruti et al. 2024).

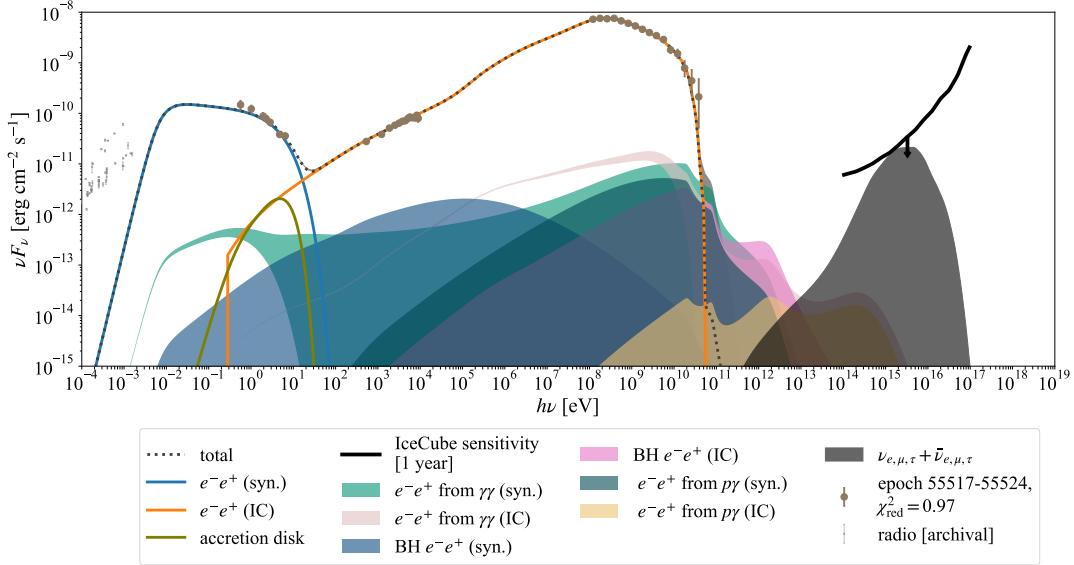


Figure D2. The photon and neutrino SED with model curves for the “standing-feature” leptohadronic model with a fit to the observed SED averaged over MJD 55517–55524 for proton acceleration parameter $\xi_{\text{acc}} = 5 \times 10^4$. Shaded regions correspond to the spread of proton luminosities $0 \leq u'_P \leq u_P^{\text{UL}}$, where u_P^{UL} is the UL corresponding to the Bayes factor with respect to the best-fit leptonic model $BF = 10^{-2}$ ($\approx 3\sigma$ Gaussian equivalent). The black solid curve with a downward arrow shows the *IceCube* one-year sensitivity (see Eq. 42) for the UL of 2.44 muon neutrinos between $E_\nu^{O\min} = 10^{14}$ eV and $E_\nu^{O\max} = 10^{17}$ eV.

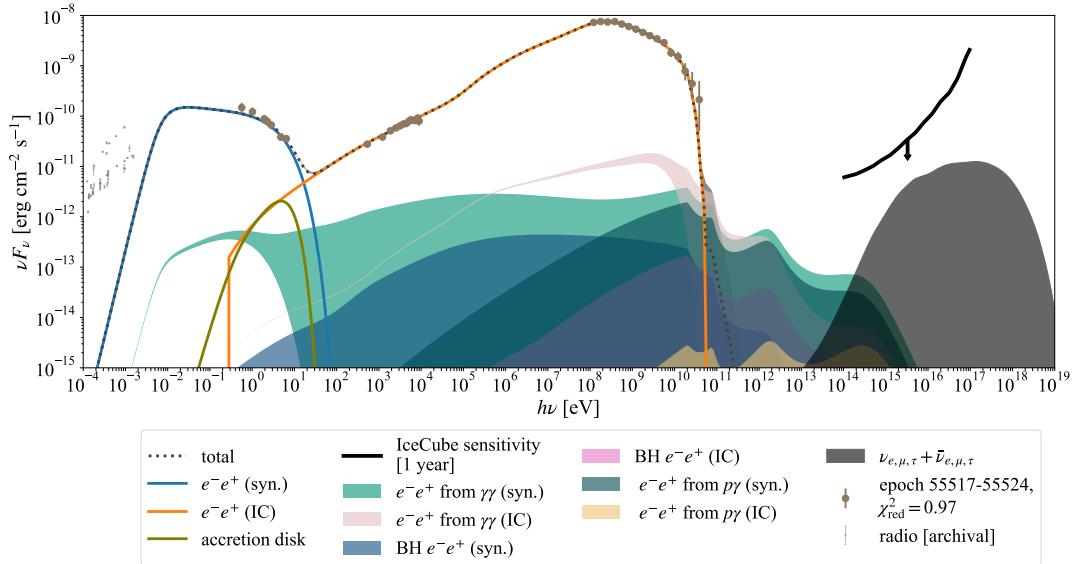


Figure D3. Same as Fig. D2 but for $\xi_{\text{acc}} = 10$.