Hypothesis Testing

@amity1415

Is there such a thing as home advantage in baseball league?

H_0: *There is no home field advantage*
H_a: *There is a field advantage*

Components of a Formal Hypothesis Test

Null Hypothesis: H_0

- The **null hypothesis** (denoted by H_0) is a statement that the value of a population parameter (such as proportion, mean, or standard deviation) is **equal to** some claimed value.
- We test the null hypothesis directly.
- Either reject H_0 or fail to reject H_0.

Alternative Hypothesis: H_1

- The **alternative hypothesis** (denoted by H_1 or H_a or H_A) is the statement that the parameter has a value that somehow differs from the null hypothesis.

- The symbolic form of the alternative hypothesis must use one of these symbols: $\neq$, $<$, $>$.

Note about Forming Your Own Claims (Hypotheses)

If you are conducting a study and want to use a hypothesis test to support your claim, the claim must be worded so that it becomes the alternative hypothesis.

Test Statistic - Formulas

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

Test statistic for proportions

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{\sigma}{\sqrt{n}}}$$

Test statistic for mean

Example: A survey of $n = 880$ randomly selected adult drivers showed that 56% (or $p = 0.56$) of those respondents admitted to running red lights. Find the value of the test statistic for the claim that the majority of all adult drivers admit to running red lights.

(In Section 8-3 we will see that there are assumptions that must be verified. For this example, assume that the required assumptions are satisfied and focus on finding the indicated test statistic.)

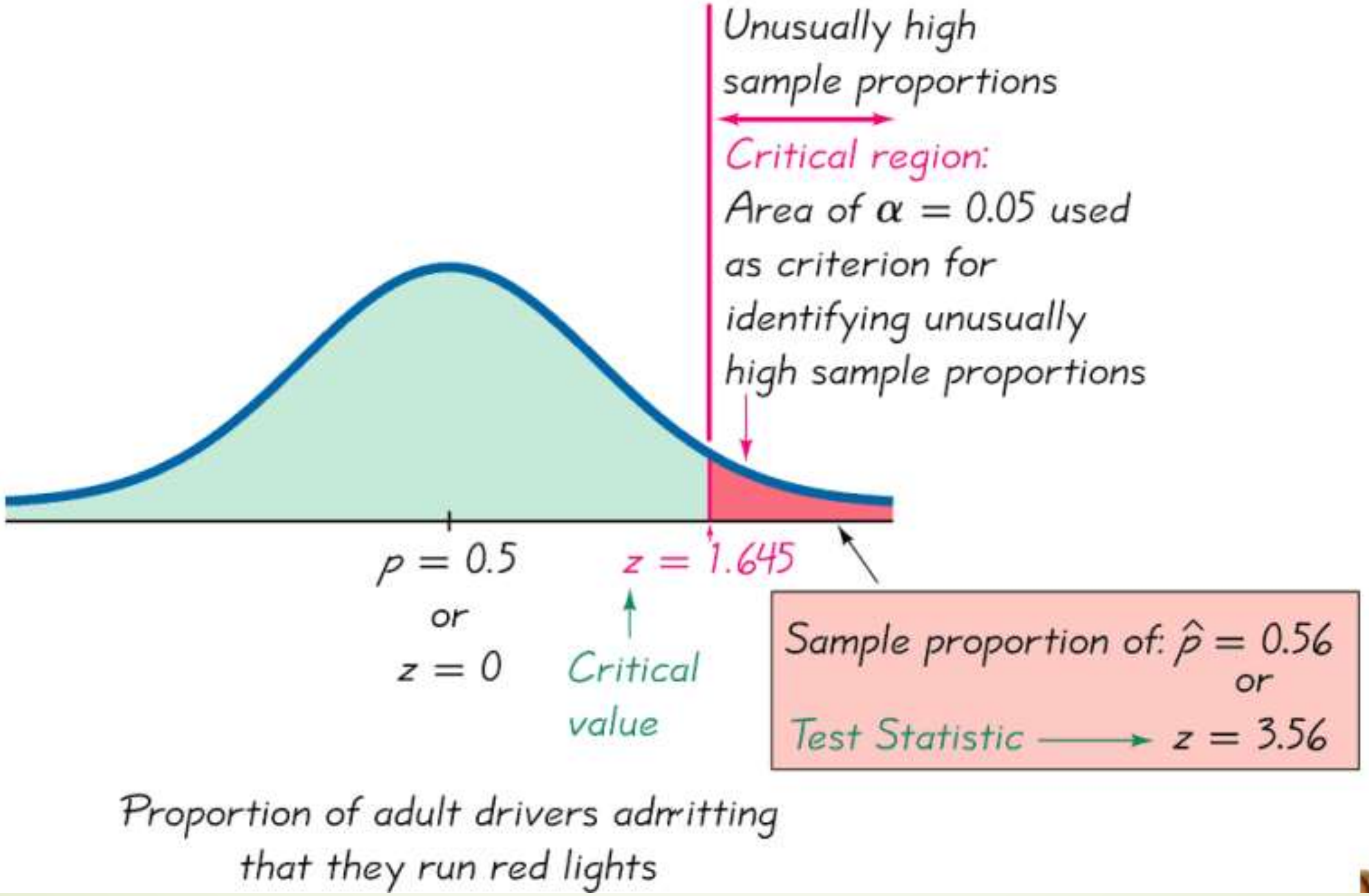
Solution: The preceding example showed that the given claim results in the following null and alternative hypotheses: H_0: $p = 0.5$ and H_1: $p > 0.5$. Because we work under the assumption that the null hypothesis is true with $p = 0.5$, we get the following test statistic:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.56 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{880}}} = 3.56$$

P-Value

The *P*-value (or *p*-value or probability value) is the probability of getting a value of the test statistic that is at least as extreme as the one representing the sample data, assuming that the null hypothesis is true. The null hypothesis is rejected if the *P*-value is very small, such as 0.05 or less.

Critical Region, Critical Value, Test Statistic



Critical Region

The critical region (or rejection region) is the set of all values of the test statistic that cause us to reject the null hypothesis. For example, see the red-shaded region in the previous figure.

Two-tailed, Right-tailed, Left-tailed Tests

The **tails** in a distribution are the extreme regions bounded by critical values.

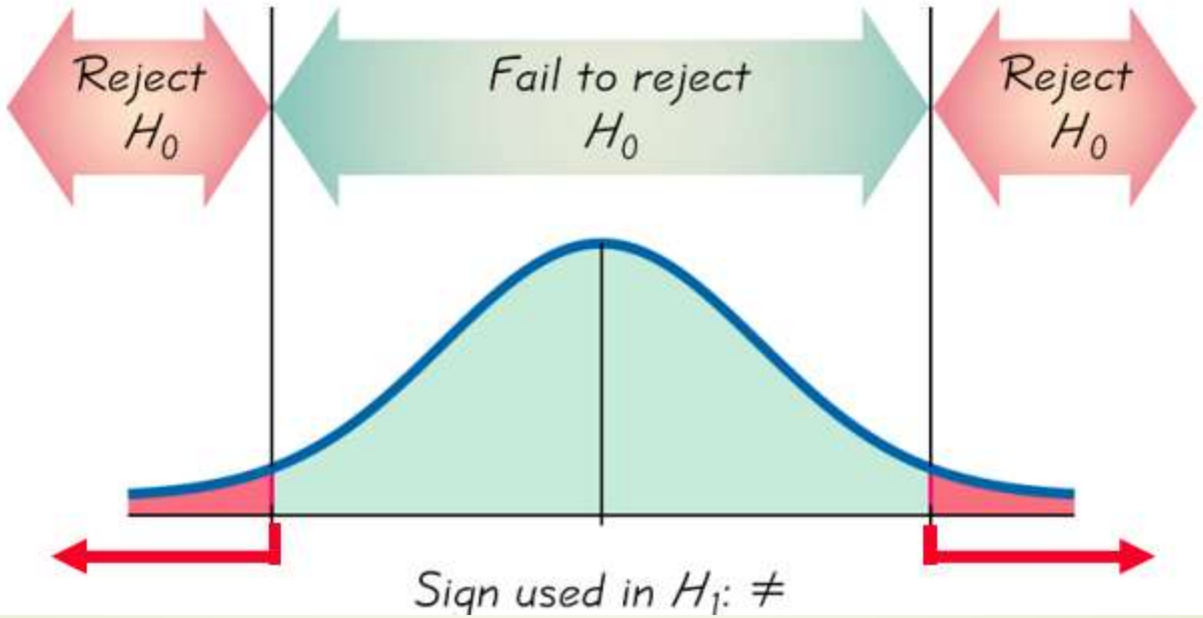
Two-tailed Test

H_0: =

H_1: ≠

Means less than or greater than

α is divided equally between the two tails of the critical region

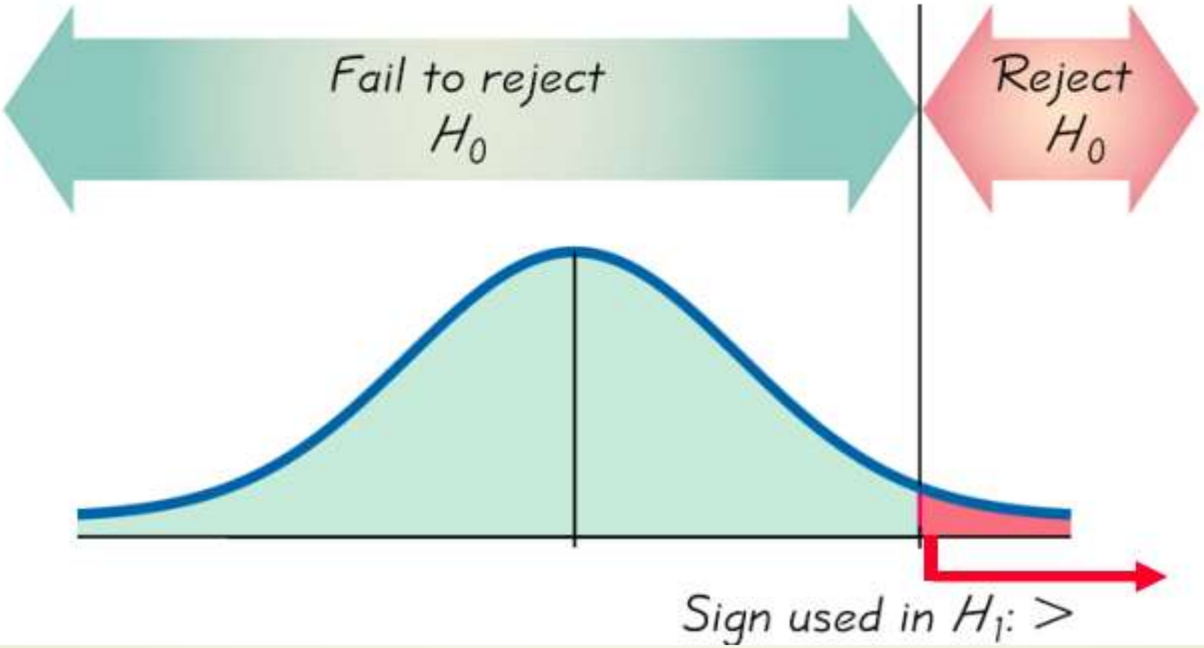


Right-tailed Test

H_0: =

H_1: >

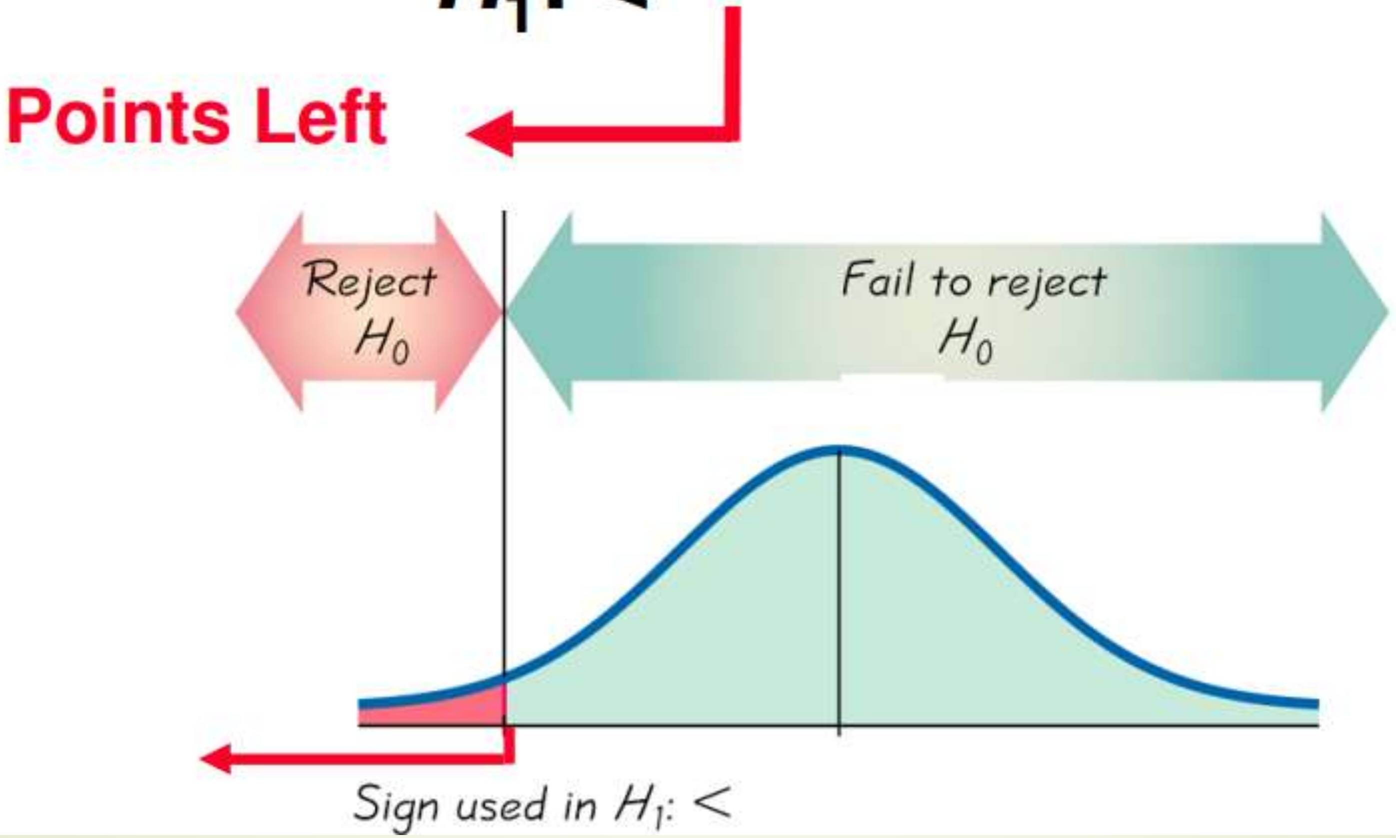
Points Right



Left-tailed Test

H_0: =

H_1: <



Conclusions in Hypothesis Testing

We always test the null hypothesis. The initial conclusion will always be one of the following:

1. Reject the null hypothesis.

2. Fail to reject the null hypothesis.

Decision Criterion

Traditional method:

Reject H_0 if the test statistic falls within the critical region.

Fail to reject H_0 if the test statistic does not fall within the critical region.

Decision Criterion - cont

P-value method:

Reject H_0 if the P-value $\leq \alpha$ (where α is the significance level, such as 0.05).

Fail to reject H_0 if the P-value $> \alpha$.

Decision Criterion - cont

P-value method:

Reject H_0 if the P-value $\leq \alpha$ (where α is the significance level, such as 0.05).

Fail to reject H_0 if the P-value $> \alpha$.

Steps to Hypothesis Testing:

* State the hypotheses and select an a   level.

* The null hypothesis, H0, always states that the treatment has no effect (no change, no difference.

* The a level establishes a criterion, or "cut-off", for making a decision about the null hypothesis.

* If the Sample Count is < 30, the formulas in the previous slide indicate T-Statistic.

* If the Sample Count is >30, the formulas in the previous slide gives us Z-Statistic.

Hypothesis Testing

Numerical Example:

- Assume we have a 20 observations of the weights of a product in grams. The mean of this sample is 8.059576
- The standard deviation is 0.3190233
- The question we want to answer : Is the mean statistically different from 7.82? (answer for a=0.01)

- In other words, *Step 1*: identify the hypothesis

Ho: μ=7.82 vs H1:μ different from 7.82

Hypothesis Testing

Numerical Example:

- *Step 2*: Calculate the statistic

$$T= \frac{\bar{x}-7.82}{s/\sqrt{n}} = \frac{8.059576-7.82}{0.3190233/\sqrt{20}} = 3.358424 \approx 3.4$$

- *Step 3*:Identify the distribution of t under null:

t distribution with 20-1=19 degrees of freedom

- Step 4:Calculate the p-value

Find P(t<3.4) using Tables for *Student -t* distribution

TABLE 9. THE t-DISTRIBUTION FUNCTION

$\nu =$	15	16	17	18	19	20	24	30	40	60	∞
t = 0·0	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000
·1	·5392	·5392	·5392	·5393	·5393	·5393	·5394	·5395	·5396	·5397	·5398
·2	·5779	·5780	·5781	·5781	·5782	·5782	·5784	·5786	·5788	·5789	·5793
·3	·6159	·6160	·6161	·6162	·6163	·6164	·6166	·6169	·6171	·6174	·6179
·4	·6526	·6528	·6529	·6531	·6532	·6533	·6537	·6540	·6544	·6547	·6554
0·5	0·6878	0·6881	0·6883	0·6884	0·6886	0·6887	0·6892	0·6896	0·6901	0·6905	0·6915
·6	·7213	·7215	·7218	·7220	·7222	·7224	·7229	·7235	·7241	·7246	·7257
·7	·7527	·7530	·7533	·7536	·7538	·7540	·7547	·7553	·7560	·7567	·7580
·8	·7819	·7823	·7826	·7829	·7832	·7834	·7842	·7850	·7858	·7866	·7881
·9	·8088	·8093	·8097	·8100	·8103	·8106	·8115	·8124	·8132	·8141	·8159
1·0	0·8334	0·8339	0·8343	0·8347	0·8351	0·8354	0·8364	0·8373	0·8383	0·8393	0·8413
·1	·8557	·8562	·8567	·8571	·8575	·8578	·8589	·8600	·8610	·8621	·8643
·2	·8756	·8762	·8767	·8772	·8776	·8779	·8791	·8802	·8814	·8826	·8849
·3	·8934	·8940	·8945	·8950	·8954	·8958	·8970	·8982	·8995	·9007	·9032
·4	·9091	·9097	·9103	·9107	·9112	·9116	·9128	·9141	·9154	·9167	·9192
1·5	0·9228	0·9235	0·9240	0·9245	0·9250	0·9254	0·9267	0·9280	0·9293	0·9306	0·9332
·6	·9348	·9354	·9360	·9365	·9370	·9374	·9387	·9400	·9413	·9426	·9452
·7	·9451	·9458	·9463	·9468	·9473	·9477	·9490	·9503	·9516	·9528	·9554
·8	·9540	·9546	·9552	·9557	·9561	·9565	·9578	·9590	·9603	·9616	·9641
·9	·9616	·9622	·9627	·9632	·9636	·9640	·9652	·9665	·9677	·9689	·9713
2·0	0·9680	0·9686	0·9691	0·9696	0·9700	0·9704	0·9715	0·9727	0·9738	0·9750	0·9772
·1	·9735	·9740	·9745	·9750	·9753	·9757	·9768	·9779	·9790	·9800	·9821
·2	·9781	·9786	·9790	·9794	·9798	·9801	·9812	·9822	·9832	·9842	·9861
·3	·9819	·9824	·9828	·9832	·9835	·9838	·9848	·9857	·9866	·9875	·9893
·4	·9851	·9855	·9859	·9863	·9866	·9869	·9877	·9886	·9894	·9902	·9918
2·5	0·9877	0·9882	0·9885	0·9888	0·9891	0·9894	0·9902	0·9909	0·9917	0·9924	0·9938
·6	·9900	·9903	·9907	·9910	·9912	·9914	·9921	·9928	·9935	·9941	·9953
·7	·9918	·9921	·9924	·9927	·9929	·9931	·9937	·9944	·9949	·9955	·9965
·8	·9933	·9936	·9938	·9941	·9943	·9945	·9950	·9956	·9961	·9966	·9974
·9	·9945	·9948	·9950	·9952	·9954	·9956	·9961	·9965	·9970	·9974	·9981
3·0	0·9955	0·9958	0·9960	0·9962	0·9963	0·9965	0·9969	0·9973	0·9977	0·9980	0·9987
·1	·9963	·9966	·9967	·9969	·9971	·9972	·9976	·9979	·9982	·9985	·9990
·2	·9970	·9972	·9974	·9975	·9976	·9978	·9981	·9984	·9987	·9989	·9993
·3	·9976	·9977	·9979	·9980	·9981	·9982	·9985	·9988	·9990	·9992	·9995
·4	·9980	·9982	·9983	·9984	·9985	·9986	·9988	·9990	·9992	·9994	·9997
3·5	0·9984	0·9985	0·9986	0·9987	0·9988	0·9989	0·9991	0·9993	0·9994	0·9996	0·9998
·6	·9987	·9988	·9989	·9990	·9990	·9991	·9993	·9994	·9996	·9997	·9998

Z- Score Table

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5	0.504	0.508	0.512	0.516	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5754
0.2	0.5793	0.5832	0.5871	0.591	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.648	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.67	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.695	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.719	0.7224
0.6	0.7258	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7518	0.7549
0.7	0.758	0.7612	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.791	0.7939	0.7967	0.7996	0.8023	0.8051	0.8079	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.834	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.877	0.879	0.881	0.883
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.898	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.937	0.9382	0.9394	0.9406	0.9418	0.943	0.9441
1.6	0.9452	0.9463	0.9474	0.9485	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.97	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.975	0.9756	0.9762	0.9767
2	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.983	0.9834	0.9838	0.9842	0.9846	0.985	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.989
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.992	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.994	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.996	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.997	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.998	0.998	0.9981
2.9	0.9981	0.9982	0.9983	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.999	0.999
3.1	0.999	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	1	1
3.9	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1
z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

Hypothesis Testing

Numerical Example:

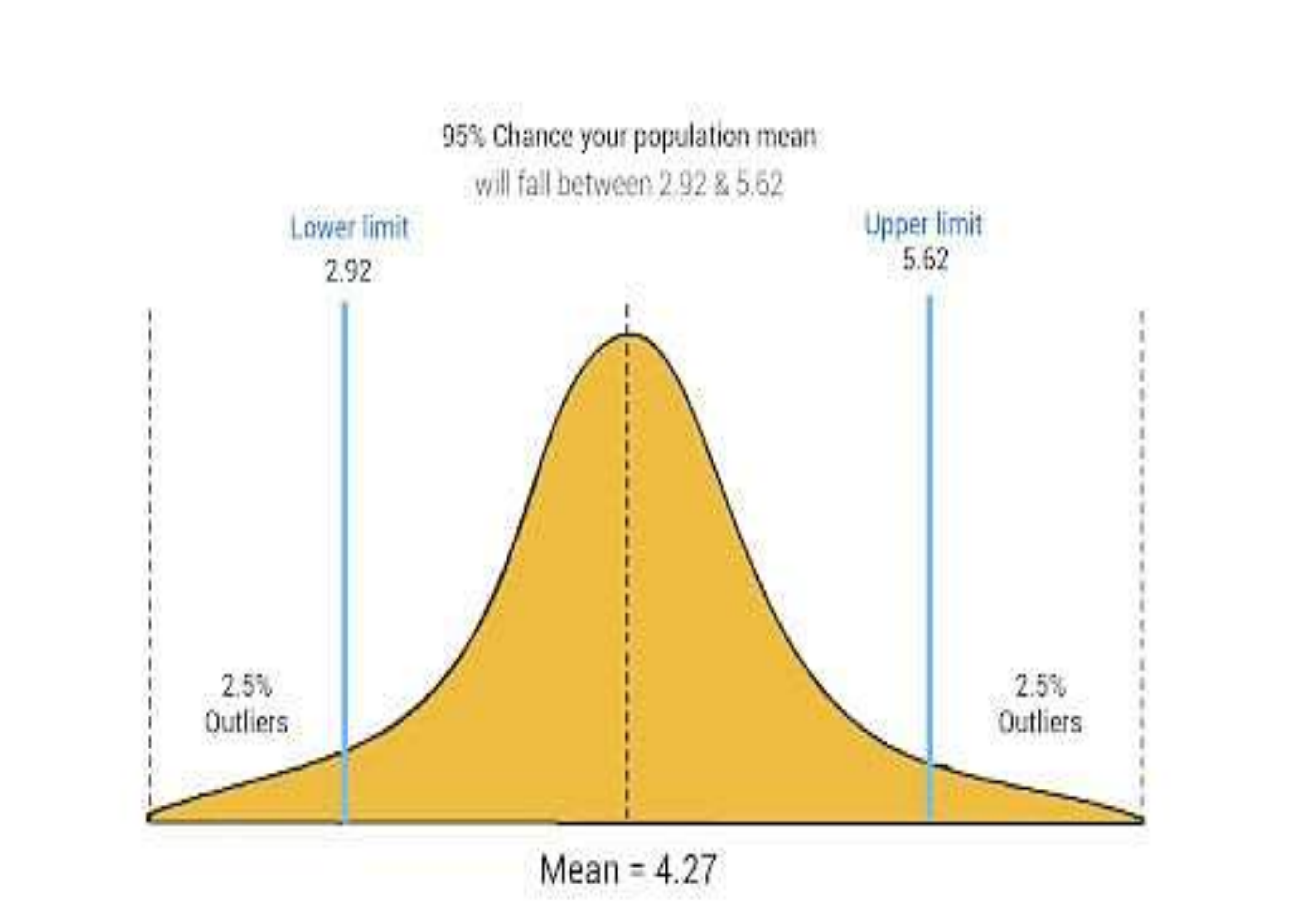
- From the Table we see that $P(t<3.4)=0.9985$
- The p-value is: $1-0.9985=0.0015$

$a=0.01>$ p-value $=0.0015$ so we reject Ho

That means that the mean seems to be significantly different from 7.82

What Is Confidence Interval?

A confidence interval shows the probability that a parameter will fall between a pair of values around the mean. Confidence intervals show the degree of uncertainty or certainty in a sampling method. They are constructed using confidence levels of 95% or 99%



What Does a 95% Confidence Interval Mean?

The 95% confidence interval is the range that you can be 95% confident that the similarly constructed intervals will contain the parameter being estimated.

The sample mean (center of the CI) will vary from sample to sample because of natural sampling variability.

Statisticians use confidence intervals to measure the uncertainty in a sample variable. The confidence is in the method, not in a particular CI. Approximately 95% of the intervals constructed would capture the true population mean if the sampling method was repeated many times.

Confidence Interval Formula:

The formula to find Confidence Interval is:

$$\bar{x} \pm Z \frac{s}{\sqrt{n}}$$

- X bar is the sample mean.
- Z is the number of standard deviations from the sample mean.
- S is the standard deviation in the sample.
- n is the size of the sample.

The value after the ± symbol is known as the margin of error.

Confidence Interval Example:

Question: In a tree, there are hundreds of mangoes. You randomly choose 40 mangoes with a mean of 80 and a standard deviation of 4.3. Determine that the mangoes are big enough.

Solution:

Mean = 80

Standard deviation = 4.3

Number of observations = 40

Take the confidence level as 95%. Therefore the value of Z = 1.9

Substituting the value in the formula, we get

$= 80 \pm 1.960 \times [\,4.3 / \sqrt{40}\,]$

$= 80 \pm 1.960 \times [\,4.3 / 6.32]$

$= 80 \pm 1.960 \times 0.6803$

$= 80 \pm 1.33$

The margin of error is 1.33

All the hundreds of mangoes are likely to be in the range of 78.67 and 81.33.