Applied Data Science with Python (Session-5)

Saturday, 20 September 2025   7:23 PM

Linear algebra

→ Scalar and vectors. ✓
→ Vector Operation ✓
→ Norm of a Vector. ✓
→ Matrix and Matrix operation. ✓
→ Determinant & Identity matrix. ✓
→ Inverse of matrix, Eigen Values & Eigen Vector.

① Scalar and Vector.

↓ Magnitude.

Eg:- Distance, speed

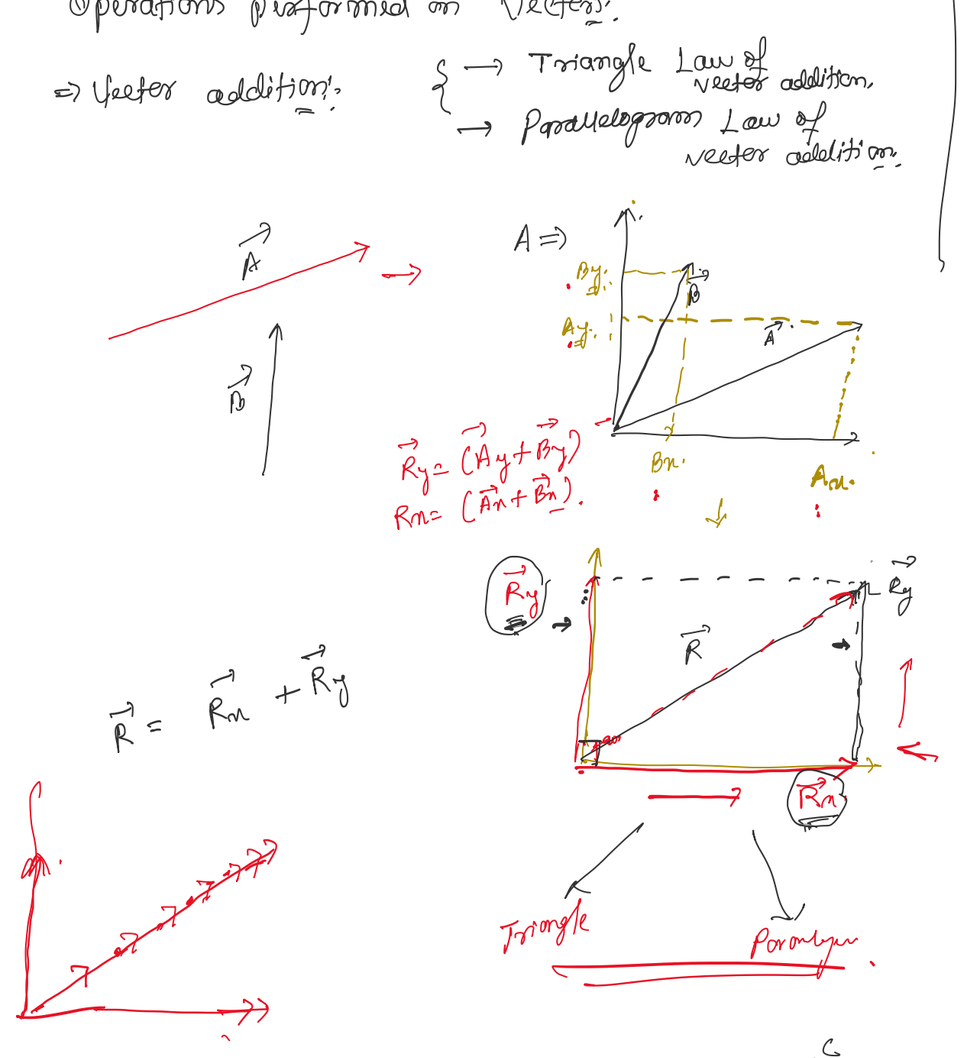
Scalar. ← Time.

↳ Magnitude
→ Direction.

Eg: Displacement. velocity. acceleration.

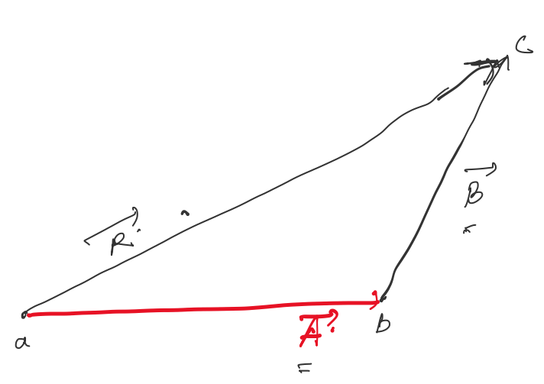
Operations performed on Vectors.

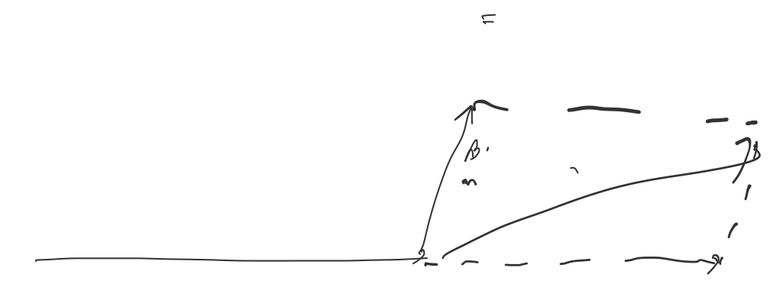
⇒ Vector addition. { → Triangle Law of vector addition. → Parallelogram Law of vector addition.



A
B
A ⇒
By
Ay
B
A
$R_y = (\vec{A_y} + \vec{B_y})$
$R_x = (\vec{A_x} + \vec{B_x})$
Bx
Ax
Ry
R
Rx
Triangle
Parallelogram

$\vec{R} = \vec{R_x} + \vec{R_y}$

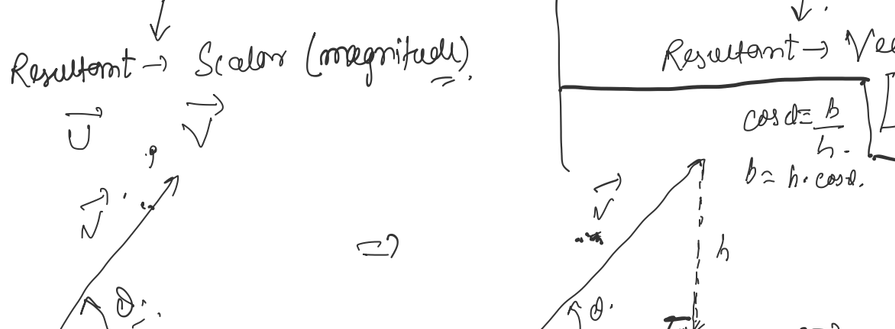




Vector operation:- Multiplication.

Scalar Product (Dot product)
• 2 vectors multiplies
↓
Resultant → Scalar (magnitude).
$\vec{U}$, $\vec{V}$

Vector Product. (Cross product)
• 2 vectors multiplies.
↓
Resultant → Vector → magnitude, direction.
$\cos\theta = \frac{b}{h}$
$b = h \cdot \cos\theta$



$\vec{u} = \vec{x_1} + \vec{y_1} + \vec{z_1}$

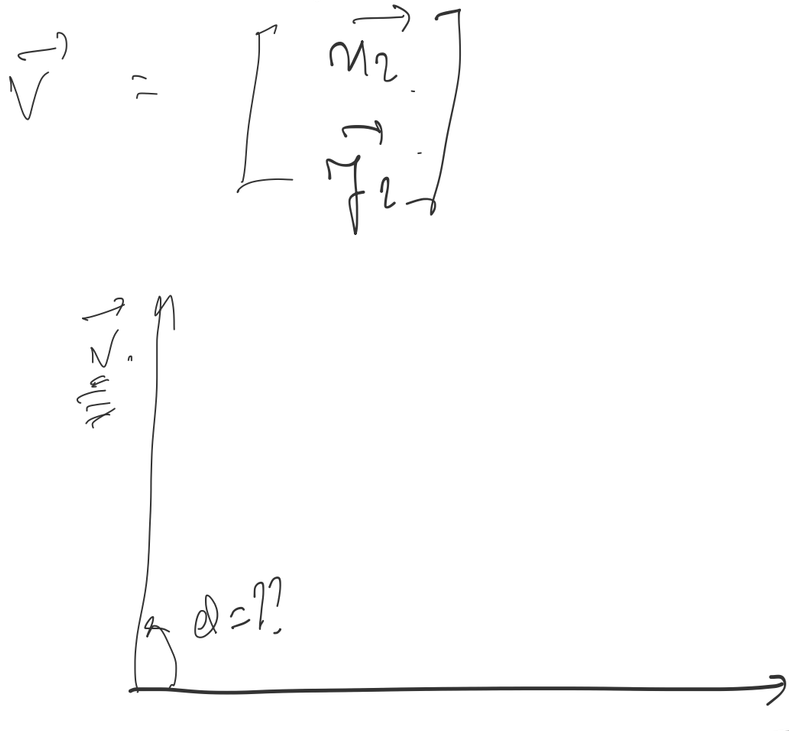
$\vec{v} = \vec{x_2} + \vec{y_2} + \vec{z_2}$

$u \cdot v = x_1 \cdot x_2 + y_1 y_2 + z_1 z_2$

$u \cdot v = |\vec{u}||\vec{v}| \cos\theta$

$\vec{u} = \begin{bmatrix} \vec{x_1} \\ \vec{y_1} \end{bmatrix}$ $u \cdot v = x_1 \cdot x_2 + y_1 \cdot y_2$

$\vec{V} = \begin{bmatrix} \vec{x_2} \\ \vec{y_2} \end{bmatrix}$



$u \cdot v = u \cdot v\cos\theta$
$u \cdot v\cos 90$
$u \cdot 0$

$|\vec{u}||\vec{v}| = v\cos\theta$
$\vec{u} \cdot \vec{v} = 0$

Eigen value & Eigen vector

no change in direction
→ change in magnitude [λ and V]
→ shape.

$A\vec{V} = \vec{z} = \lambda\vec{V}$
A → transformation matrix.
λ = eigen value.
V = eigen vector.

$Av = \lambda v$.
or, $A = \lambda$.
$(A - \lambda I) = 0$

$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

Solve and find, related eigen value and eigen vector.

① Eigenvalues

$(\det(A - \lambda I) = 0$. $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}$

$\det(A-\lambda I) = (2-\lambda)^2 - 1$.

Solving for 0.

$(2-\lambda)^2 - 1 = 0$.

$\lambda^2 - 4\lambda + 3 = 0$.

$(\lambda - 1)(\lambda - 3)$ $\lambda_1 = 3$. $\lambda_2 = 1$.

② Solve for eigen vector.

For, $\lambda_1 = 3$.

$|A - \lambda I| \Rightarrow |A - 3I| = \begin{bmatrix} 2-3 & 1 \\ 1 & 2-3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \Rightarrow -x + y = 0$
$x = y$

Eigen vector, $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$Av_1 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 3 \\ 3 \end{bmatrix} \Rightarrow 3 \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \lambda_1 v_1$

$\lambda_2 = 1$.

$A - I = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow x + y = 0$.
$x = -y$.

Eigen vector $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Check, v_2.

$Av_2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} +1 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \lambda_2 v_2$

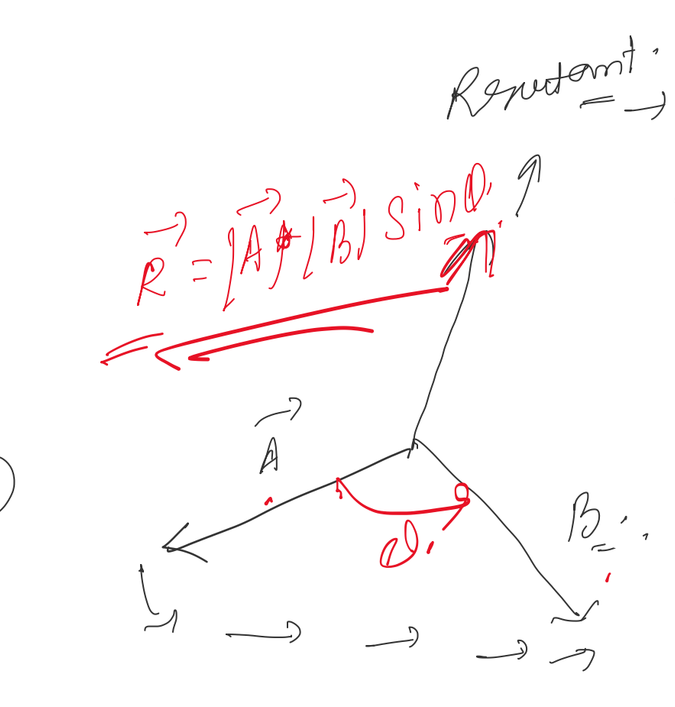
Normal of a vector.

$\vec{V} = \begin{bmatrix} \vec{v_1} \\ \vec{v_2} \\ \vec{v_3} \end{bmatrix} \begin{matrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{matrix}$ $|\vec{V}| = v_1^2 + v_2^2 + v_3^2$

$\vec{V} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ $|\vec{V}| = \sqrt{2^2 + 3^2 + 4^2}$
$= \sqrt{4 + 9 + 16}$
$= \sqrt{29}$

Resultant → magnitude & direction.

$\vec{R} = |\vec{A}||\vec{B}| \sin\theta$



$\vec{A} \times \vec{B}$

$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

$\Rightarrow \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

now $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$(-1)^2 (b_3 \cdot a_2 - b_2 \cdot a_3)\hat{i}$

$(-1)^3 (b_3 \cdot a_1 - b_1 a_3)\hat{j}$

$(-1)^4 (a_1 b_2 - b_1 a_2)\hat{k}$

$\vec{R} =$

$\Rightarrow (a_2 b_3 - a_3 b_2)\hat{i}$
$+ (a_3 b_1 - a_1 b_3)\hat{j}$
$+ (a_1 b_2 - a_2 b_1)\hat{k}$

$\vec{R} = |\vec{A}||\vec{B}| \sin\theta \hat{k}$

Matrix.

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 7 & 8 \\ 2 & 3 & 5 \\ 1 & 6 & 2 \end{bmatrix}$

A + B = Element wise addition

$\begin{bmatrix} 5 & 9 & 11 \\ 6 & 8 & 11 \\ 8 & 14 & 11 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + B = \begin{bmatrix} 2 & 2 & 2 \end{bmatrix}$
↓ Broadcasting

$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + B = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

$\Rightarrow A + B = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$

Matrix multiplication

$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 5 & 7 \\ 2 & 7 & 9 \end{bmatrix}$ (r_1, c_1) (3×3) $B = \begin{bmatrix} 7 & 4 & 2 \\ 6 & 8 & 1 \\ 3 & 7 & 8 \end{bmatrix}$ (r_2, c_2) (3×3)

A × B ⇒ Rule:- Check if two matrices are multipliable??
$(c_1 = r_2)$ no. of cols. of 1st matrix (A) = no. of rows of 2nd matrix (B).

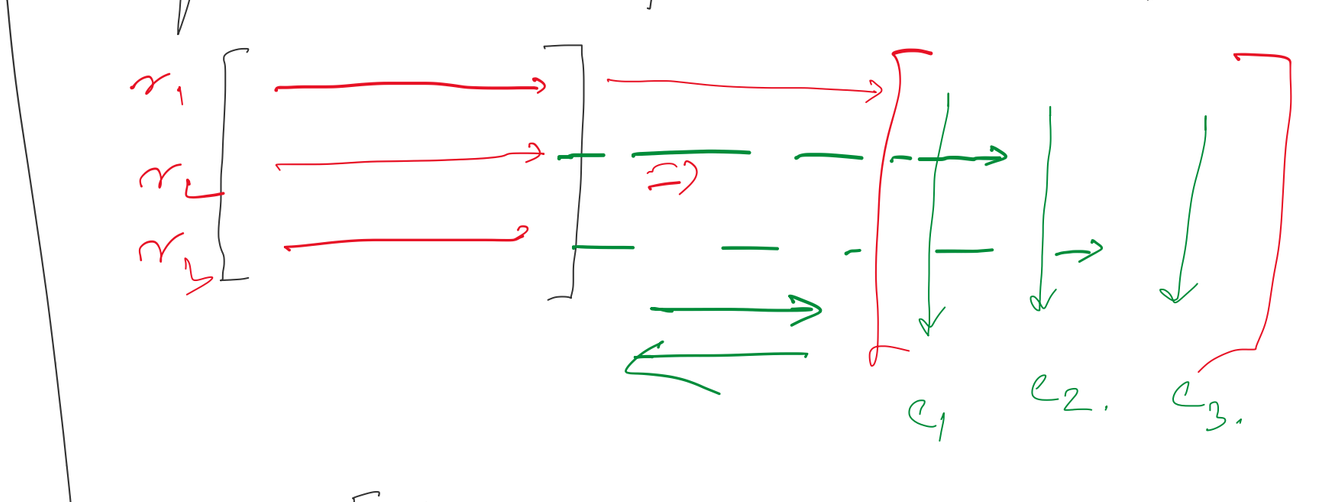
Resultant matrix ⇒ (r_1, c_2)
$(3, 3)$.

$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 5 & 7 \\ 2 & 7 & 9 \end{bmatrix} * \begin{bmatrix} 7 & 4 & 2 \\ 6 & 8 & 1 \\ 3 & 7 & 8 \end{bmatrix}$

$\begin{bmatrix} \underset{a_{11}}{7+12+6} & \underset{a_{12}}{4+16+14} & \underset{a_{13}}{2+2+16} \\ \underset{a_{21}}{14+30+21} & \underset{a_{22}}{8+40+49} & \underset{a_{23}}{4+5+56} \\ \underset{a_{31}}{14+42+27} & \underset{a_{32}}{8+56+63} & \underset{a_{33}}{4+7+72} \end{bmatrix}$

$= \begin{bmatrix} 25 & 34 & 20 \\ 65 & 97 & 65 \\ 83 & 127 & 83 \end{bmatrix}$

Transpose of a matrix.



$X = \begin{bmatrix} 1 & 9 & -6 \\ 5 & 3 & -7 \end{bmatrix}$

$X^T = \begin{bmatrix} 1 & 5 \\ 9 & 3 \\ -6 & -7 \end{bmatrix}$

Determinant of a matrix.
→ Same implementation as cross product.

$\vec{A} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 1 \\ 1 & 7 & 2 \end{bmatrix}$

Det. A / |A|

$= (-1)^{i+j}\, 1 * \begin{vmatrix} 4 & 1 \\ 7 & 2 \end{vmatrix} \quad (-1)^{i+j} -2 * \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} + 3 * \begin{vmatrix} 3 & 4 \\ 1 & 7 \end{vmatrix}$

$= (8 - 7) \quad - 2 * (6 - 1) + 3(21 - 4)$

$= 1 \quad - 10 \quad + 51$

$|A| = 42$

If det. of matrix ≠ 0 then only a matrix is invertible.

$A * A^{-1} = I$

Identity Matrix. $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ✓

Inverse of a matrix.

$A^{-1} = \frac{1}{\det \cdot A} * \operatorname{adj}(A)$.

Adj. Adjucate of A = [Cofactor Matrix of A]^T.

Cofactor Matrix = $(-1)^{i+j} *$ Minor of ele$_{ij}$.

$A = \begin{bmatrix} 6 & 4 & 3 \\ 9 & 2 & 5 \\ 1 & 7 & 8 \end{bmatrix}$ ← $|A| = -119$

$C_{11} = (-1)^2 * \begin{vmatrix} 2 & 5 \\ 7 & 8 \end{vmatrix}$ $C_{21} = -\begin{vmatrix} 4 & 3 \\ 7 & 8 \end{vmatrix}$ $C_{31} = \begin{vmatrix} 4 & 3 \\ 2 & 5 \end{vmatrix}$

$C_{12} = (-1)^3 * \begin{vmatrix} 9 & 5 \\ 1 & 8 \end{vmatrix}$ $C_{22} = \begin{vmatrix} 6 & 3 \\ 1 & 8 \end{vmatrix}$ $C_{32} = \begin{vmatrix} 6 & 3 \\ 9 & 5 \end{vmatrix}$

$C_{13} = (-1)^4 * \begin{vmatrix} 9 & 2 \\ 1 & 7 \end{vmatrix}$ $C_{23} = -\begin{vmatrix} 6 & 4 \\ 1 & 7 \end{vmatrix}$ $C_{33} = \begin{vmatrix} 6 & 4 \\ 9 & 2 \end{vmatrix}$

Cofactor matrix. $\begin{bmatrix} -19 & -67 & 61 \\ -11 & 45 & -38 \\ 14 & -3 & -24 \end{bmatrix}$.

Adjucate. A = (Cofactor matrix)^T.

$= \begin{bmatrix} -19 & -11 & 14 \\ -67 & 45 & -3 \\ 61 & -38 & -24 \end{bmatrix}$.

$A^{-1} = -\frac{1}{199} \begin{bmatrix} -19 & -11 & 14 \\ -67 & 45 & -3 \\ 61 & -38 & -24 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} +0.095 & +0.055 & -0.07 \\ .33 & -0.24 & +.015 \\ -0.30 & 0.19 & 0.012 \end{bmatrix}$

$A * A^{-1} = I. = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Use Cases in Data Science :-

ⓐ Dot product → Similarity Score and projection.
↳ feature importance in ML.

ⓑ Cross product.
→ 3D Graphics in Data Visualisation
→ Simulation in Data Science.

② Inverse of a matrix.

ⓐ Solving linear equation.
$Ax = b$.   , if A is invertible
$x = A^{-1}b$.

ⓑ Dimentionality reduction.
→ Inverse of a matrix.
→ Eigen value − vector.

ⓒ Regularization in ML.
→ Ridge regression.
(100) → Of1 − − Of2 − − Of100.