INVERTER

- A device that converts DC power into AC power at desired output voltage and frequency is called an inverter.
- Output of inverter is A.C. But A.C. is easily available. So what is the use of inverter?. Answer to this Question is :-

(i) AC available is of fixed frequency & fixed voltage. But inverter can provide output AC of variable voltage and variable frequency. Inverter can be easily controlled.

(ii) We cannot store electricity directly in AC form. Electricity can be stored in DC form (DC battery). And this stored DC can be easily converted to usable AC form using Inverter.

Note :- Variable AC voltage output can be obtained using transformer (of suitable turn ratio) or by Variac. But using transformer, we cannot change the frequency.

Secondly, variable voltage & variable frequency AC output can be easily obtained using Cycloconverter.

- So why should we used inverters?

Cycloconverter has very good performance in lower frequency output (output frequency is lower than the input). But for higher output frequency, inverter has superior performance.

- Practical or industrial Applications of Inverter:-

(i) It is used in Variable Speed AC motor drive.

(ii) It is used in Induction heating.

(iii) It is used in Stand by power supplies (ex in aircraft). (59)

(iv) It is used in uninterruptible power supplies (UPS).

(v) It is used in reactive power compensation devices such as STATCOM.

- Classification of Inverters :-

(i) Classification on the basis of output:-

1 ϕ Inverter (produce 1 ϕ A.C output) or 3 ϕ inverter (produce 3 ϕ A.C. output).

(ii) Classification on the basis of Commutation used in Inverter:-

(a) Line Commutated Inverter (Rectifier circuit with firing angle greater than 90°, works as line commutated inverter).

* This type of inverter is used in HVDC transmission.

(b) Forced Commutated Inverter (Power electronic device such as thyristor used in this type of inverter, is forcefully turned OFF). In this unit, we will study only Forced Commutated Inverter.

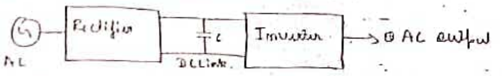
(c) Load Commutated Inverter (In these types of inverter, thyristor (used in inverter) commutated on its own due to the nature of load.). For ex. Series Inverter.

(iii) Classification on the basis of type of DC Source —

(a) Voltage Source Inverter (VSI) :- In VSI, input voltage is remain constant. Magnitude and waveshape of output load voltage is same for any kind of load. But magnitude and waveshape of output load current depends on load type & impedance.

(b) Current Source Inverter (CSI) :- In CSI, input current remain constant. Magnitude & waveshape of output load current is same for any kind of load. But magnitude & waveshape of output load voltage depends on load type & its impedance.

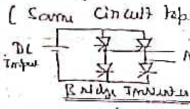
(c) Variable DC Link Inverter :-



In this, rectifier is controlled to obtain variable DC output. And this variable DC is given as input to the Inverter.

(iv) Classification on the basis of circuit topology :-

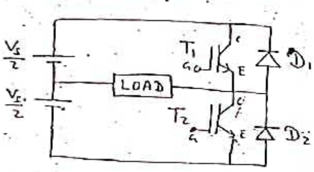
(a) Bridge Inverter. (Same circuit topology as that of Bridge rectifier).



(b) Resonant - pulse Inverter :- This type of circuit topology use LC resonant circuit. In this type of inverter, output voltage or current of the inverter is forced to pass through zero by an LC resonant circuit.

Note :- In our course, we will study Forced commutated VSI bridge type inverter. We will study both 1 ϕ & 3 ϕ inverter.

1 ϕ Bridge Inverter



- This is 1 ϕ Half Bridge Inverter
- It has 2 controlled switches (IGBT), T_1 & T_2
- It has 2 feed back diode D_1 & D_2

- 1 ϕ Half bridge inverter requires 3 terminal DC supply.
- We can achieve AC at the output terminal by proper switching of T_1 & T_2 in a fixed sequence.

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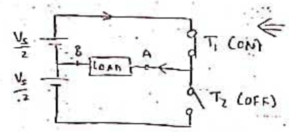
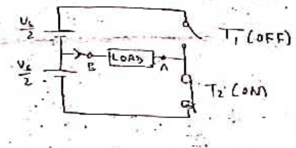


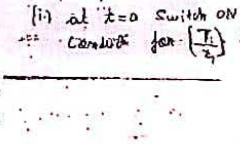
- Eq. Ckt when T_1 is ON & T_2 is OFF. In this condition, Load current flows from A to B (+ve load current direction).
- upper part of DC supply, delivers this (+)ve load current.



- Eq. Ckt when T_2 is ON & T_1 is OFF. In this condition, Load current flows from B to A (−ve load current direction).
- Lower part of DC supply, delivers this (−)ve load current.

- Thus by switching T_1 & T_2 alternately, direction of load current can be changed periodically. Hence AC can be achieved at the output from DC input.

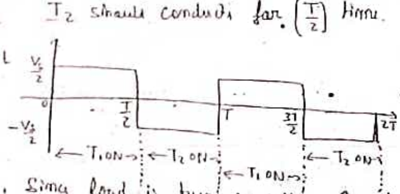
) 1 ϕ Half bridge Inverter with R LOAD only.



- T_1 & T_2 are IGBT, IGBT are simply used as a switch.
- When Gate pulse is present at the Gate terminal (G), IGBT conducts current from Collector (C) to Emitter (E).
- When Gate pulse is withdrawn, IGBT stops conducting. Current cannot flows from Emitter (E) to Collector (C).
- When AC of time period, T is to be obtained at the output, then following switching sequence should be followed.

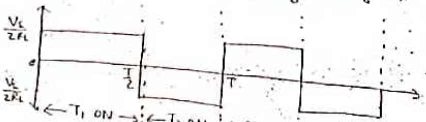
(i) at $t=0$ switch ON T_1 (T_2 should be OFF). T_1 should conduct for $\left(\frac{T}{2}\right)$ time.

(ii) At $t = \frac{T}{2}$, Switch OFF T_1 and Switch ON T_2.
T_2 should conduct for $\left(\frac{T}{2}\right)$ time.



Output Voltage of Inverter
(Voltage across Load resistor, R_L)

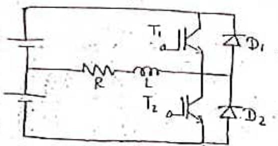
- Since load is pure resistive, so shape of load current waveform is similar to load voltage waveform.



Output LOAD current

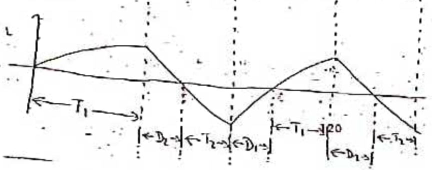
For pure R Load, feedback Diodes, D_1 & D_2 will not come into picture.

2) 1 φ Bridge (Half Bridge) Inverter with RL LOAD



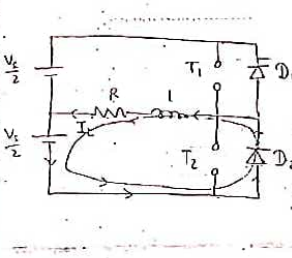
- At $t=0$, T_1 is switch ON & T_2 is OFF.
- At $t=\frac{T}{2}$, T_1 is switched OFF & T_2 is switched-ON.
- At $t = T$, T_1 is switched ON & T_2 is switched OFF.
- At $t = \frac{3T}{2}$, T_1 is switched OFF & T_2 is switched ON.

Output Load Voltage



Output Load Current

(61)



⇐ Eq. ckt at $t = \frac{T}{2}$

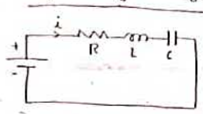
At $t = \frac{T}{2}$, T_1 is forcefully switched OFF & T_2 is given gate signal.

- But load is inductive, load current cannot change its direction instantly (in Zero-time).

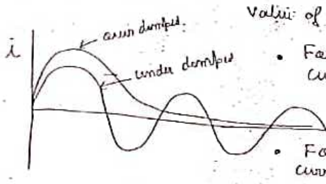
- At $t = \frac{T}{2}$ (T_1 is OFF), path of load current is
  $L \rightarrow R \rightarrow \frac{V_s}{2}$ (Lower arm) $\rightarrow D_2 \rightarrow L$
  ~~At t=~~ Load current will keep on flowing through D_2 till it decays exponentially to Zero.
- When the load current has decayed to zero; it will start growing exponentially through T_2.
  Path of reverse load current is :-
  $\frac{V_s}{2}$ (Lower arm) $\rightarrow R \rightarrow L \rightarrow T_2 \rightarrow \frac{V_s}{2}$ (Lower arm).
- Note :- whenever T_2 or D_2 is conducting, load voltage is $-\frac{V_s}{2}$ (Negative Load voltage).
  Verify this by applying KVL in lower mesh.
- Similarly, at $t = T$, T_2 is forcefully switched-OFF & T_1 is given gate signal.
  Since current through inductor cannot change instantly, therefore reverse load current ~~current~~ will finds its way through D_1.
  When load current becomes Zero, it will grow exponentially in (+ve) direction through T_1.
- Note :- whenever T_1 or D_1 is conducting; load voltage is $+\frac{V_s}{2}$ (Positive load voltage). Verify this by applying KVL in upper mesh.

- Diodes D_1 & D_2 ~~are~~ comes into conduction, when energy stores in inductor is to be transfered (or feed back) to the source. That's why D_1 & D_2 are called feed back diodes.

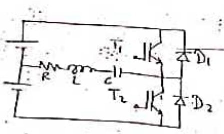
3) 1 ϕ Half bridge Inverter with RLC Load



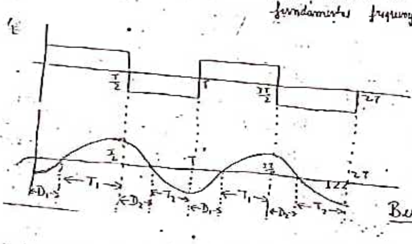
- This is a basic RLC circuit with DC excitation. Current response of this circuit can be under damped, or over damped depending on the value of R, L & C.



- For over damped response, current takes longer time to become zero ~~for~~
- For under damped response, current is becoming zero quickly, though the response is oscillatory.



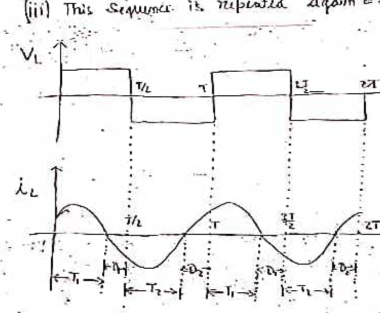
- RLC load circuit is under damped if capacitive reactance of the load is more than its inductive reactance at fundamental frequency.
- RLC load circuit is over damped if inductive reactance of the load is more than its capacitive reactance at " fundamental frequency.



Output Load Voltage

Output Load Current (Over damped Response) Behaving as Inductive Load

- Load current is exponentially damped sine wave. (62)
Since switching of T_1 & T_2 is taking place at fast rate and V_{load} is subjected to alternating voltage, therefore load current is seen varying sinusoidally.
- By switching T_1 & T_2 alternatively, AC is obtained at the output terminal.
(i) At $t = 0$, T_2 is switched OFF & T_1 is switched ON.
(ii) At $t = \frac{T}{2}$, T_1 is switched OFF & T_2 is switched ON.
(iii) This sequence is repeated again & again to achieve AC.



Output Load Voltage

Output Load Current (Under damped Response) Behaving as Capacitive Load, i_L leads the V_L.

- Note:- When current response is under damped, there is no need for forced commutation of T_1 or T_2.
For ex. Current through T_1 becomes zero before $t = \frac{T}{2}$. Therefore there is no need for forced commutation of T_1. Commutation takes place on its own due to the nature of load. This is called Load Commutation.
- When current response is overdamped, forced commutation of T_1 & T_2 is required.
For ex. At $t = \frac{T}{2}$, i_L is not zero. Therefore forced commutation of T_1 is required.

Summary of 1ϕ Half Bridge Inverter

Polarity of Load Voltage (V_L)	Polarity of Load Current (I_L)	Components Conducting (Path of Load Current)
$(+) v_L$	$(+) v_L$	T_1
$(+) v_L$	$(-) v_L$	D_1
$(-) v_L$	$(-) v_L$	T_2
$(-) v_L$	$(+) v_L$	D_2

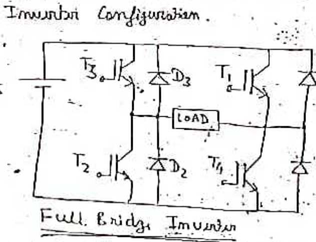
- Draw back of Half Bridge Inverter

(i) It require 3 terminal DC supply, which is difficult to realize practically.

(ii) 3 terminal DC Supply is under utilized. At any time, one half (either upper or lower half) supplies the power to the load.

(iii) Power electronic devices such as IGBT & feedback diodes are under utilized. At any time, load voltage is $\frac{V_s}{2}$ but voltage across any device in OFF state is V_s.

The above drawbacks can be overcome by using Full Bridge Inverter Configuration.



Full Bridge Inverter

- T_1, T_2, T_3 & T_4 are controlled switches such as IGBT
- D_1, D_2, D_3 & D_4 are feedback diodes.

⇒ For positive load current, T_1 & T_2 Conducts Simultaneously.

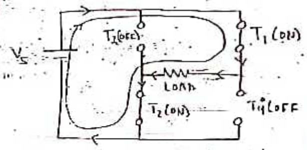
⇒ For negative load current, T_3 & T_4 Conducts Simultaneously.

- Following switching operation should be performed to obtain AC at the output terminal of Full Bridge Inverter. (63)

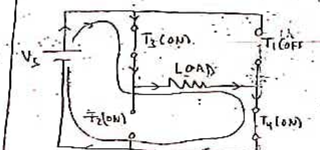
(i) At $t=0$, switch ON T_1 & T_2 simultaneously. T_3 & T_4 should be OFF

(ii) At $t = \frac{T}{2}$, T_1 & T_2 are turned OFF forcefully and T_3 & T_4 are turned ON.

(iii) The above sequence is repeated again and again to obtain AC of periodicity T.



Eq. Ckt for obtaining (+)ve load current. (T_1 & T_2 ON)



Eq. Ckt for obtaining (−)ve load current (T_3 & T_4 ON).

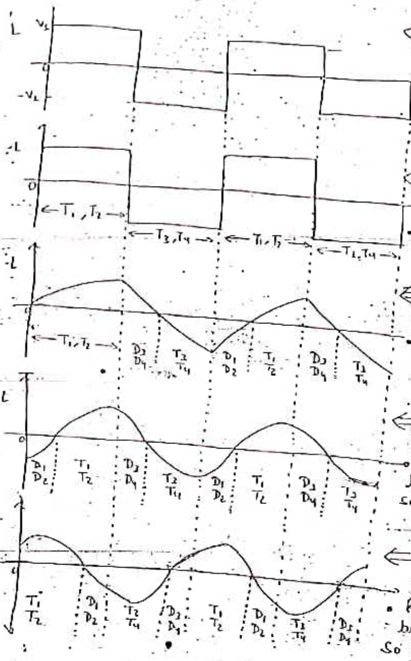
- Bridge Inverter (Half bridge & Full bridge) discussed so far are Voltage Source Inverter (VSI).

⇒ Load Voltage remains same for any kind of load. Here Load voltage is Square Wave for all types of load (R or RL) etc.

⇒ But magnitude and waveshape of load current depends on nature of load and its impedance.

- Wave shape of Load Current for Full Bridge Inverter:-

Shape of load current waveform for different type of load R, RL, RLC etc. is same as discussed in case of Half bridge inverter.



⇐ Output LOAD VOLTAGE

- V_L is Square wave. Magnitude & Shape of V_L is same for any kind of Load (R, RL, RLC etc) because inverter is VSI.

⇐ Output Load Current for Resistive or R Load.

- Since load is R only, load current waveform is similar to V_L

⇐ Output Load Current for R.L load

- i_L is exponentially growing & decaying. i_L lags behind V_L.

⇐ Output Load Current for RLC overdamped load.

- RLC overdamped load behaves as inductive load so i_L is sine wave and i_L lags behind V_L.

⇐ Output Load Current for RLC underdamped load.

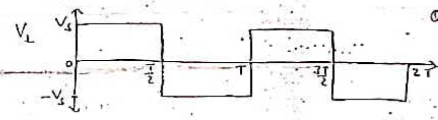
- RLC underdamped load behaves as capacitive load So i_L is sine wave &, i_L leads the V_L in phase.
- Forced Commutation is not required in RLC underdamped load

Summary of Full Bridge Inverter

Polarity of Load Voltage	Polarity of Load Current	Components Conducting
(+) v_L	(+) v_L	T_1 & T_2
(+) v_L	(−) v_L	D_1 & D_2
(−) v_L	(−) v_L	T_3 & T_4
(−) v_L	(+) v_L	D_3 & D_4

Fourier Analysis of Output Load voltage of VSI (64)

- Output Load voltage of Voltage Source Inverter (VSI) is Square Wave. It is not a pure sine wave. It contains harmonics.



Output Load voltage of VSI [Square Wave]

Let us do the fourier analysis of above square wave:

$$V_L = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos m\omega t + b_m \sin m\omega t)$$

Since the average value of above waveform is zero. Therefore $a_0 = 0$.

Also the above waveform has odd symmetry.

Therefore $a_m = 0$

For simplicity, periodicity of above waveform is taken as 2π i.e. $T = 2\pi$.

$$b_m = \frac{1}{\pi}\left[\int_0^{\pi} V_s \sin m\omega t \, d\omega t + \int_{\pi}^{2\pi} -V_s \sin m\omega t \, d\omega t\right]$$

$$= \frac{V_s}{\pi}\left[\left(-\frac{\cos m\omega t}{m}\right)_0^{\pi} - \left(-\frac{\cos m\omega t}{m}\right)_{\pi}^{2\pi}\right]$$

$$= \frac{V_s}{m\pi}\left[-\cos m\pi + 1 - (-1 + \cos m\pi)\right]$$

$$= \frac{V_s}{m\pi}\left[2 - 2\cos m\pi\right]$$

$$= \begin{cases} \frac{4V_s}{m\pi} & \text{for } m = 1, 3, 5, 7, \dots \\ 0 & \text{for } m = 2, 4, 6, \dots \end{cases}$$

$$\therefore V_L = \sum_{m=1,3,5,\dots}^{\infty} \frac{4V_s}{m\pi} \sin m\omega t$$

- V_L does not contain any even harmonics.
  V_L contains only odd harmonics.
- In VSI, load current depends on load impedance.
  In general for RLC load, load current is given by

$$i_L = \frac{V_L}{Z_L} = \sum_{n=1,3,5,\dots}^{\infty} \frac{4 V_s}{n\pi Z_{nL}} \sin(n\omega t - \phi_n)$$

where Z_{nL} is load impedance for n^{th} harmonics.
& ϕ_n is phase difference between n^{th} harmonic voltage and n^{th} harmonic current.

> Since reactance (X_L or X_C) is frequency dependent.

$$\therefore Z_{nL} = \sqrt{R^2 + \left(n\omega L - \frac{1}{n\omega C}\right)^2}$$

$$\phi_n = \tan^{-1}\left(\frac{n\omega L - \frac{1}{n\omega C}}{R}\right)$$

- Actual Load Power (Active Load Power) $= (i_L)^2 R_L$
  $= V_L\, i_L \cos\phi.$

where i_L & V_L are rms value of load current & load voltage respectively. & ϕ is the phase difference b/w V_L & i_L.

> Since load voltage & load contain harmonics

$\therefore$ Approximately Active Load Power $= (I_{01})^2 \cdot R_L$
$= V_{01}\, I_{01} \cos\phi_1$

where I_{01} is rms of fundamental load current; V_{01} is rms value of fundamental load voltage & ϕ_1 is phase difference between fundamental load current & fundamental load voltage.

$P_{01} = I_{01}^2 R_L = V_{01} I_{01} \cos\phi_1$ is the useful output power

- Power associated with harmonic current i.e. $I_{03}^2 R$ or $I_{05}^2 R$ etc. does no useful work and is dissipated as heat leading to rise in load temperature. (65)

Performance Parameters of Inverter

1. **Harmonic Factor of n^{th} harmonic** – It is the measure of individual harmonic contribution.

$$HF_n = \frac{V_{0n}}{V_{01}}$$

where V_{0n} is the rms value of the n^{th} harmonic component, V_{01} is the rms value of fundamental component.

2. **Total Harmonic Distortion** – It is the measure of closeness in shape between a waveform & its fundamental component.

$$THD = \frac{1}{V_{01}}\left(\sum_{n=2,3,\dots}^{\infty} V_{0n}^2\right)^{1/2}$$

3. **Distortion Factor** – It indicates the amount of harmonic that remains in a particular waveform after the harmonics of that waveform have been subjected to a second order attenuation (i.e. if a filter is used at the output of inverter).

$$DF_n = \frac{V_{0n}}{V_{01}\,(n)^2}$$

4. **Lowest order Harmonics** – It is that harmonic component whose frequency is nearest to the fundamental one, and its amplitude is greater than or equal to 3% of the fundamental component.

DC Supply Current

Let us assume that there is no losses in inverter. And load is inductive and switching frequency is high. Therefore load current is taken as sinusoidal.

Applying instantaneous power balance,

DC input power = AC output ~~power~~ power

$$V_s(t)\, i_s(t) = V_L(t)\, i_L(t)$$

$$V_s(t) = V_s \quad \rightarrow \text{because inverter is VSI}$$

$$\therefore\ i_s(t) = \frac{1}{V_s} \sqrt{2} V_{01} \sin \omega t \ \sqrt{2}\, I_o \sin(\omega t - \phi_1)$$

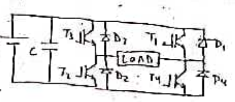
Instantaneous DC supply current is given by

$$i_s(t) = \frac{V_{01}}{V_s} I_o \cos\phi_1 - \frac{V_{01} I_o}{V_s} \cos(2\omega t - \phi_1)$$

where V_{01} is fundamental rms output voltage; I_o is rms load current & ϕ_1 is load impedance angle at fundamental frequency.

- DC supply current contain second order harmonic.
- Because of this second order harmonic & presence of source impedance, input voltage of inverter does not remain constant.

To maintain constant voltage at the input of inverter, capacitor is used at the input of inverter.

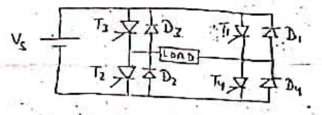


Bridge Inverter with capacitor at the input to maintain input voltage constant.

Thyristor Based Bridge Inverter

(66)

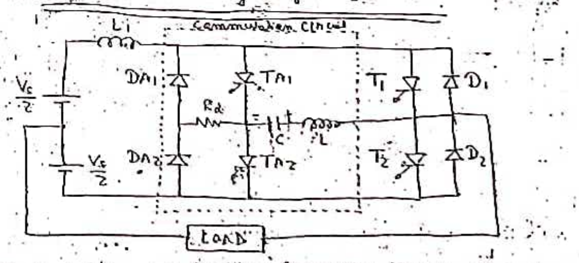
- For low power or medium power inverter, IGBT is used.
- But for higher power inverter, thyristors are more suitable.



This is thyristor based Bridge inverter.

- But when thyristors are used, commutation circuit for turning OFF of thyristor is a must. Because supply is DC and thyristor does not commutate on its own. Forced commutation of thyristor is required.
- In our course, we will study two types of force commutated inverter.
  (i) Modified McMurray Inverter
  (ii) Modified McMurray Bedford Inverter.

Modified McMurray Half Bridge Inverter

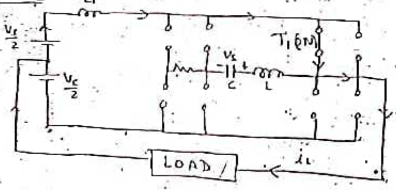


⇒ It is also called Auxiliary Commutated Inverter. This is because the auxiliary thyristor is turned ON for commutating the main thyristor.

⇒ T_1 & T_2 are Main Thyristors of Half Bridge Inverter.
⇒ D_1 & D_2 are Main feedback diodes of Half Bridge Inverter.
⇒ C & L are Capacitor & inductor for the commutation of main thyristors T_1 & T_2. L & C form a part of resonant circuit.
⇒ TA1 is auxiliary thyristor for turning OFF of T1.
⇒ TA2 is auxiliary thyristor for turning OFF of T2.
⇒ DA1 & DA2 are auxiliary diodes, for effective commutation of auxiliary thyristor.
⇒ R_d is damping resistor for fast commutation.
⇒ L_1 is inductor for limiting $\frac{di}{dt}$ to a safe value in thyristors. In addition to this, L_1 ~~keeps the~~ helps in keeping the input current almost constant.

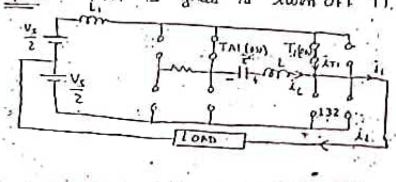
- Initial Condition of Circuit; T_1 is ON (Rest all thyristors are OFF). C is charged with V_s with right plate with positive polarity.

Step 1 – Only T_1 is ON (or Conducting)



- Now we want to turn OFF T1.
- To turn OFF T_1, we will fire TA1
- TA1 is auxiliary thyristor

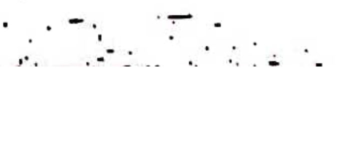
Step 2 – TA1 is fired to turn OFF T1.



- At this point of time, T_1 & TA1 both are conducting.

⇒ Since T_1 is still conducting, Load is subjected to constant voltage $\frac{V_s}{2}$. Therefore load current, i_L should remain constant.
⇒ With turning ON of TA1, another circuit containing L & C gets completed. Now capacitor start discharging. Path of capacitor current is:- $\frac{V_s}{2}$(upper) → L_1 → TA1 → C → L → LOAD → $\frac{V_s}{2}$(upper)
This circuit is resonant circuit. Capacitor current, i_c starts increasing.
⇒ Apply KCL at the junction of T_1, L & Load.
$i_{T_1} + i_c = i_L$ (i_{T_1} is current through T_1)
⇒ Since i_L is constant, i_c is increasing (due to resonant circuit) therefore i_{T_1} starts decreasing. When i_c becomes equal to i_L, i_{T_1} becomes zero and T_1 gets OFF.

STEP 3 D_1 is in antiparallel with T_1. When T_1 was conducting, D_1 was reverse biased because of 0.7 V drop across T_1.
⇒ When T_1 gets OFF, D_1 is no longer reverse biased.
Now capacitor current pass through D_1.
Path of excess capacitor current is
C → L → D_1 → TA1 → C
⇒ Since D_1 is conducting, load is subjected to same voltage $\frac{V_s}{2}$. Therefore load current is constant. Varying capacitor current is flowing through D_1.



- Since D_1 is conducting, it is reverse biasing T_1 (because of 0.7 V voltage drop across D_1).
- Reverse biasing T_1 is necessary for its effective commutation.

⇒ Diode D_1 carries current $i_D = i_c - i_L$.
⇒ When capacitor current i_c reaches its peak, capacitor voltage becomes zero.

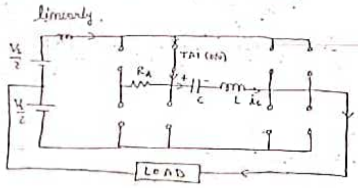
⇒ When i_c reaches its peak value, i_c starts decreasing and capacitor starts charging in reverse direction.

Step 4 :- When capacitor current i_c falls below load current i_L, diode D_1 current, i_{D1} becomes zero. D_1 will stop conducting.

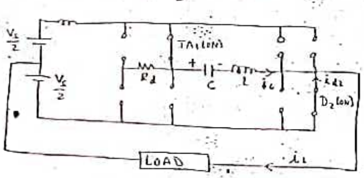
⇒ If we are assuming constant load current, i_L during commutation, path of this constant load current is –

$\frac{V_s}{2}$ (upper) → TA1 → C → L → LOAD → $\frac{V_s}{2}$ (upper)

⇒ Constant current flows through C. This current charges C linearly.



Step 5 :- When capacitor voltage becomes more than supply voltage V_s (i.e. $\frac{V_s}{2} + \frac{V_s}{2}$), diode D_2 becomes forward biased.



⇒ Apply KCL at the junction of L, D_2 & Load
$i_L = i_c + i_{d2}$

⇒ Since i_L is assumed constant during commutation process.

⇒ i_c is decreasing and ∴ i_{d2} should increase.

Path of diode D_2 current, i_{d2} :-

$\frac{V_s}{2}$ (lower) → D_2 → Load → $\frac{V_s}{2}$ (lower)

∴ Lower $\frac{V_s}{2}$ supply is not causing this i_{d2} to flow.

It is the energy stored in load inductance which is causing this i_{d2} to flow.

(58)

⇒ Path of capacitor current, i_c :-

$\frac{V_s}{2}$ (upper) → TA1 → C → L → Load → $\frac{V_s}{2}$ (upper)

Note :- Load current carries both i_c & i_{d2}. i.e. $i_L = i_c + i_{d2}$

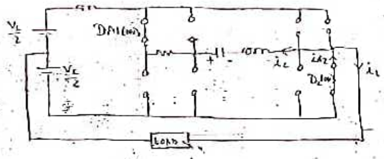
⇒ In this step (step 5), i_c is decreasing.
When i_c becomes zero, TA1 gets OFF.

Step 6 :- When capacitor current i_c becomes zero, TA1 gets OFF because i_c was flowing through TA1.

⇒ Since capacitor is over charged i.e. more than $(\frac{V_s}{2} + \frac{V_s}{2})$
Now capacitor will starts discharging in reverse direction.

⇒ As soon as TA1 gets OFF, reverse bias across DA1 (auxiliary diode 1) is over. DA1 was previously reverse biased because of 0.7V drop across conducting TA1.

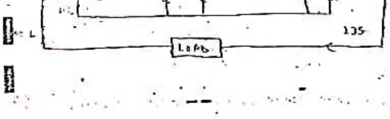
⇒ Capacitor voltage now forward bias DA1.
Reverse capacitor current flows through DA1
Path of this reverse capacitor current :-
C → R_L → DA1 → $\frac{V_s}{2}$ (upper) → $\frac{V_s}{2}$ (lower) → D_2 → L → C



- D_2 is still conducting.
- i_L is constant.
- Apply KCL at the junction of L, D_2 & Load.
  $i_{d2} = i_L + i_c$
- i_{d2} is more than i_L.

Step 7 :- Reverse capacitor current flow will decrease the capacitor voltage to V_s.

⇒ When capacitor voltage becomes equal to V_s, Aux. diode DA1 becomes reverse biased & gets OFF.



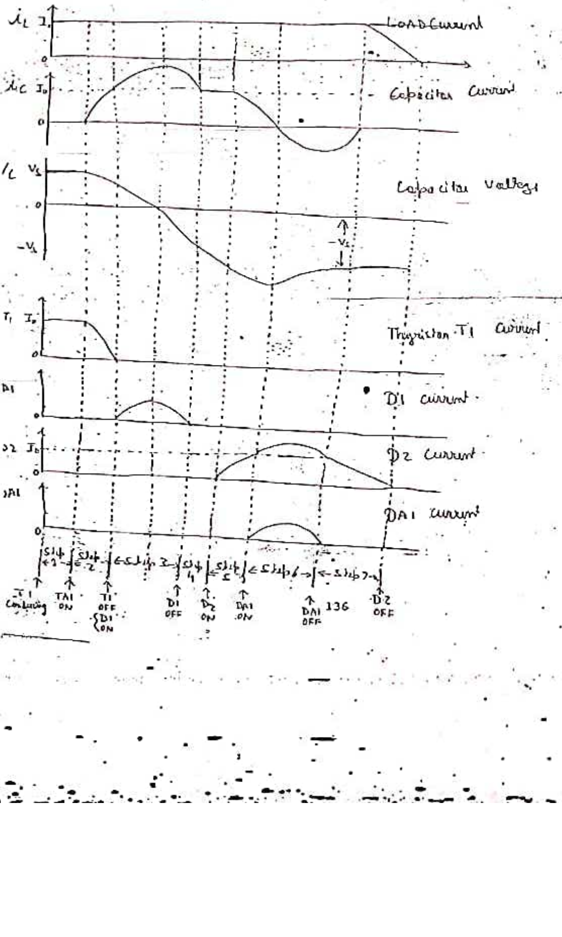
- At this point of time, only D_2 is ON.
- Since DA1 gets OFF, there is no path for capacitor current to flow.
- Load current flows through D_2.
  Path of load current is :-
  $\frac{V_s}{2}$ (lower) → D_2 → Load → $\frac{V_s}{2}$ (lower)

- This load current does not flow due to lower $\frac{V_s}{2}$ supply. This flows due to energy stored in load inductance. & This load current will slowly decay to zero.

⇒ When load current becomes zero, current through D_2 will also become zero. So D_2 gets OFF. Now main thyristor T_2 can be fired.

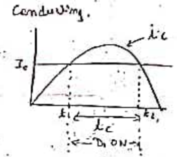
⇒ Commutation process of main thyristor (T_1) & auxillary thyristor (TA1) is over. This completes the commutation process.

⇒ Now T_2 conducts. To turn OFF T_2, TA2 is fired and whole commutation process repeats again.



Designing of Commutation Circuit for Modified McMurray Inverter (69)

- For succesful commutation of main thyristor T_1, it should remain reverse biased for time equal to thyristor turn OFF time, t_q.
- Main thyristor T1 remain reverse bias as long as Diode D1 is conducting.



- i_c is capacitor current. i_c is flowing in LC resonant circuit.
- I_o is constant load current.
- D_1 conducts for $t_c = t_2 - t_1$

- By solving resonant circuit, ~~peak value of~~ i_c can be found as

$$i_c = V_s \sqrt{\frac{C}{L}} \sin \omega_o t \qquad \text{where } \omega_o = \frac{1}{\sqrt{LC}}$$

At $t = t_1$, $i_c = I_o$ ⇒ $I_o = I_{cp} \sin \omega_o t_1$

$$\therefore t_1 = \frac{1}{\omega_o} \sin^{-1}\left(\frac{I_o}{I_{cp}}\right) \qquad \text{where } I_{cp} = V_s \sqrt{\frac{C}{L}}$$

At $t = t_2$: $i_c = I_o$ (Again)

$$\therefore t_2 = \frac{1}{\omega_o}\left[\pi - \sin^{-1}\left(\frac{I_o}{I_{cp}}\right)\right]$$

$$t_c = t_2 - t_1 = \frac{1}{\omega_o}\left[\pi - 2\sin^{-1}\left(\frac{I_o}{I_{cp}}\right)\right]$$

⇒ For optimum value of C, $\frac{I_{cp}}{I_o} = 1.5$.

⇒ Design is carried out on the basis of worst operating condition which consists of minimum supply voltage, V_{smin} & maximum load current, I_{omax}.

$$\therefore C = \frac{1.5\, t_c\, I_{omax}}{1.682\, V_{min}}$$

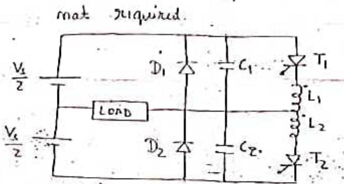
$$L = \frac{t_c\, V_{min}}{1.682 \times 1.5\, I_{omax}}$$

⇒ For succesful commutation, t_c must be greater than thyristor turn OFF time, t_q.

Modified McMurray-Bedford 1φ Half Bridge Inverter

- Modified McMurray Bedford Half Bridge Inverter, uses less number of thyristors & diodes as compared to modified McMurray half bridge inverter.

⇒ In this, auxiliary thyristors & auxiliary diodes are not required.



- T_1 & T_2 are main thyristors. If one thyristor is turned ON, the other conducting thyristor gets turn OFF.

⇒ This is Complementary Commutation. It is therefore called Complementary Commutated inverter.

⇒ Since it is a complementary commutation, therefore it is faster.

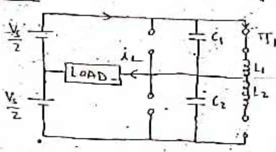
⇒ D_1 & D_2 are feedback diodes.

⇒ C_1 & C_2 are Capacitor. They are the part of Commutation Ckt.

⇒ L_1 & L_2 are magnetically coupled inductors. They are also the part of Commutation Ckt. $\boxed{\text{Note :- } C_1 = C_2 \ \& \ L_1 = L_2}$

Various steps in Commutation process are :-

Step 1 :- ~~As~~ T_1 is initially conducting.



- Initially Circuit is in steady State. Therefore constant Load Current is flowing.

⇒ Path of constant load current is:- $\frac{V_s}{2}$ (upper) → T_1 → L_1 → Load → $\frac{V_s}{2}$ (upper)

⇒ Since initially Ckt is in steady State, no current is flowing in capacitors C_1 & C_2.

⇒ Since current through L_1 is constant (not changing), therefore there will be no emf induced across L_1.

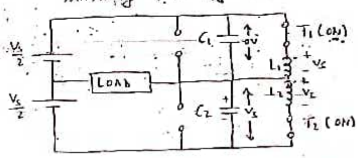
⇒ Since Capacitor C_1 is in parallel with series combination of T_1 & L_1. Voltage across T_1 is Zero because it is conducting. And there is no emf induced across L_1. Therefore Voltage across C_1 is Zero. (Initially)

⇒ Series Combination of C_1 & C_2 is in parallel with Supply i.e. $\left(\frac{V_s}{2} + \frac{V_s}{2}\right)$. (70)

Initially when T_1 is ON, Voltage across C_1 is Zero.
Therefore, initially when T_1 is ON, Voltage across C_2 is V_s. $\left(= \frac{V_s}{2} + \frac{V_s}{2}\right)$

Step 2 :- T_2 is forward biased because of Capacitor C_2 Voltage, V_s.

⇒ Now T_2 is fired (turned ON), to turn OFF initially conducting T_1.



- When T_2 is turned ON, whole Capacitor C_2 voltage equal to V_s is applied across L_2.

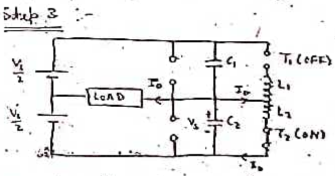
⇒ Instantly, very large emf equal to V_s gets induced across L_2.

⇒ Since L_1 & L_2 are magnetically coupled. Therefore V_s volt emf gets induced across L_1 also.

⇒ Now apply KVL in the outer most loop.
[Loop ≡ $\frac{V_s}{2}$ (upper) → T_1 → L_1 → L_2 → T_2 → $\frac{V_s}{2}$ (Lower)]

Therefore voltage across T_1 will be $-V_s$.
This reverse voltage across T_1, turn OFF T_1 almost instantly.

Step 3 :-



- Since T_1 gets OFF on turning ON T_2 instantly, Current through L_1 becomes Zero instantly.
- Acc. to constant flux linkage theorem, flux linked with closed circuit cannot change instantly.

⇒ Since Current through L_1 changes instantly, flux will also change instantly. But this is against constant flux linkage theorem.

Since L_1 & L_2 are magnetically coupled, to keep the flux linkage constant, L_2 should carry the same current as carried by L_1 (when T_1 was ON).

⇒ When T_2 is turned ON, following changes will take place:

(i) T_1 gets OFF instantly.

(ii) L_1 current changes instantly from full load current, I_0 to zero

(iii) L_2 current changes instantly from zero to full load current, I_0.

⇒ Apply KVL in a loop containing :- $\left(\frac{V_s}{2}(\text{upper}) \to C_1 \to C_2 \to \frac{V_s}{2}(\text{Lower})\right)$

Suppose capacitor C_1 carries i_{c1} current & capacitor C_2 carries i_{c2} current. (Both C_1 & C_2 carries current)

$$\frac{V_s}{2} + \frac{V_s}{2} - \left(\frac{1}{C_1}\int_0^t i_{c1}\,dt + V_{c1}(0)\right) + \left(\frac{1}{C_2}\int_0^t i_{c2}\,dt + (-V_{c2}(0))\right) = 0$$

Since at any given time, $V_{c1} + V_{c2} = V_s$.

where V_{c1} is voltage across C_1 & V_{c2} is voltage across C_2.

$$\therefore \cancel{V_s} - \cancel{(V_{c1}+V_{c2})} - \frac{1}{C}\int_0^t i_{c1}\,dt + \frac{1}{C}\int_0^t i_{c2}\,dt = 0$$

$$\therefore i_{c1} = i_{c2}$$

Hence whenever both capacitors C_1 & C_2 carries current, their currents will be equal i.e $i_{c1} = i_{c2}$.

⇒ Apply KCL at the common junction of C_1, C_2, load & L_2 (At the instant when T_2 is turned ON).

$$i_{c1} + i_{c2} = \underset{\substack{\uparrow \\ i_L}}{I_0} + \underset{\substack{\uparrow \\ i_{L2}}}{I_0}$$

Since $i_{c1} = i_{c2}$ $\therefore$ $i_{c1} = i_{c2} = I_0$

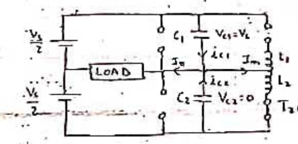
⇒ Note :- Capacitor C_1 is charging & capacitor C_2 is discharging, during this time

Step 4 :- Now components C_2, L_2 & T_2 forms a resonant circuit. When the resonant current reaches its peak, capacitor C_2 voltage (which was initially V_s) becomes zero. After this, capacitor C_2 voltage will tends to become negative. This negative C_2 voltage forward bias D_2.

⇒ Now D_2 will start conducting.

⇒ As long as D_2 is conducting, no current through capacitor C_2 will flow. (D_2 short circuit C_2). (71)

Energy stored in inductor L_2 will flow through D_2.



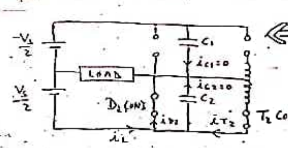
⇐ Eq. Ckt when voltage across C_2 is zero i.e $V_{c2} = 0$

- At this instant, voltage across C_1 is V_s i.e. $V_{c1} = V_s$.
- At this instant, peak resonant current I_m is flowing through L_2.

⇒ Apply KCL at the common junction of C_1, C_2, Load & L_2 (At the instant when $V_{c2} = 0$)

$$i_{c1} + i_{c2} = I_0 + I_m$$ (I_m is peak resonant current)

Since $i_{c1} = i_{c2}$ $\therefore$ $i_{c1} = i_{c2} = \frac{I_0 + I_m}{2}$



⇐ Eq. Ckt. when voltage across C_2 tends to become negative. i.e $V_{c2} \to (-)ve$

- At this instant, D_2 gets ON.
- At this instant, $i_{c2} = 0$ because D_2 short circuit C_2.
- At this instant, $i_{c1} = 0$ because at any time $i_{c1} = i_{c2}$.
- At this instant, $i_{D2} = I_0 + I_m$.

⇒ Apply KCL at the common junction of $\frac{V_s}{2}$(lower), D_2 & T_2 (At the instant when D_2 is ON).

$$\underset{\substack{\uparrow \\ D_2\ \text{current}}}{i_{D2}} = \underset{\substack{\uparrow \\ \text{Load current}}}{i_L} + \underset{\substack{\uparrow \\ T_2\ \text{current}}}{i_{T2}}$$

⇒ Since the energy stored in inductor L_2 is released through D_2. When all the stored energy in L_2 is released, L_2 current becomes zero (Because Energy $= \frac{1}{2} L_2 I^2$)

⇒ When L_2 current becomes zero, T_2 gets OFF because T_2 is in series with L_2.

Step 5 :- When thyristor T_2 current become zero i.e $i_{T_2} = 0$.

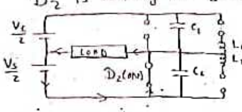
D_2 carries only the load current.

$\Rightarrow$ Path of load current is :- $D_2 \rightarrow$ Load $\rightarrow \frac{V_s}{2}$ (Lower) $\rightarrow D_2$

Note :- This load current is flowing against $\frac{V_s}{2}$ (Lower)

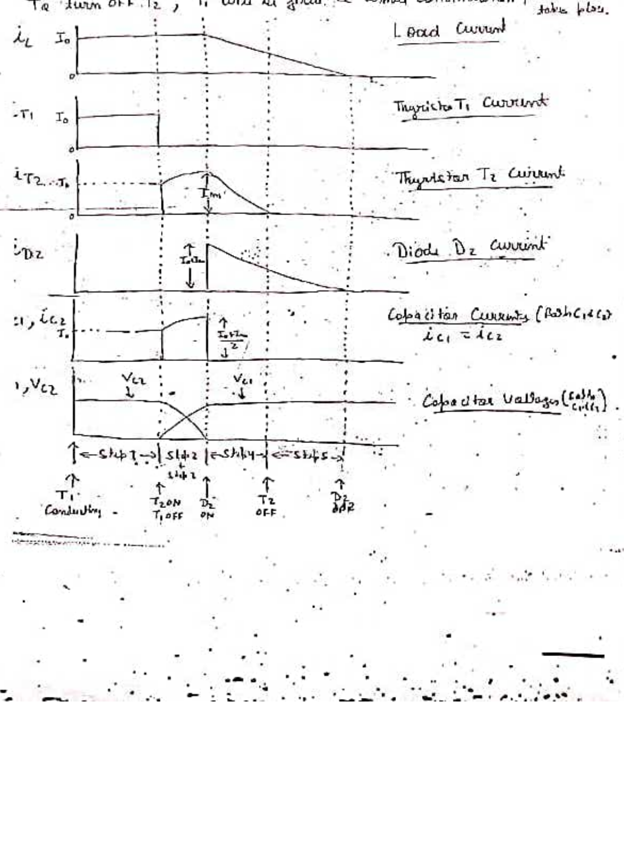
This current is flowing due to energy stored in inductive load.

D_2 is acting as feed back diode.



- $i_L = i_{D_2}$ $\rightarrow$ At this interval of time.
- i_{D_2} becomes zero, D_2 stop conducting.
- Since D_2 is not conducting, now T_2 can be fired because T_2 is no longer reverse biased by conducting D_2.

- Now T_2 will conducts and will carry the negative load current. To turn OFF T_2, T_1 will be fired & whole commutation process will take place.



Designing of Commutation Circuit for Modified McMurray Bed ford Inverter (72)

- Main Thyristor remains reverse bias for $\left(\frac{T_0}{4}\right)$ time

where T_0 is time period of oscillation of resonant circuit.

$$T_0 = \frac{1}{f_0} = \frac{2\pi}{\omega_0} = 2\pi\sqrt{LC}$$

- Therefore thyristor turn OFF time, t_q must be less than $\left(\frac{T_0}{4}\right)$.

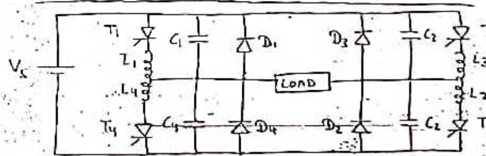
- Optimum value of L & C are given by

$$L = 2.35 \frac{V_s t_q}{I_{om}}$$

$$C = 2.35 \frac{I_{om} t_q}{V_s}$$

(where t_q is thyristor turn OFF time & I_{om} is max. load current to be commutated.)

Modified Mc Murray Bed Ford Full Bridge Inverter



- Working is similar to Half Bridge Inverter
- T_1 & T_2 always conducts in pair (simultaneously)
- When T_1 & T_2 are ON, path of forward load current is :

$V_s \rightarrow T_1 \rightarrow L_1 \rightarrow$ LOAD $\rightarrow L_2 \rightarrow T_2 \rightarrow V_s$

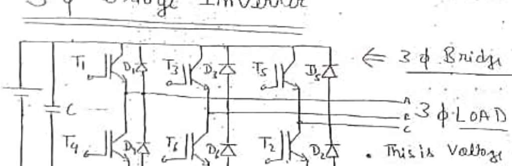
$V_{C_1} = 0$; $V_{C_2} = 0$; $V_{C_3} = V_s$; $V_{C_4} = V_s$ (when T_1 & T_2 are ON)

- To turn OFF T_1 & T_2, thyristor T_3 & T_4 are fired. And the whole commutation process take place.
- Under steady state, when T_3 & T_4 are ON, path of reverse current is

$V_s \rightarrow T_3 \rightarrow L_3 \rightarrow$ Load $\rightarrow L_4 \rightarrow T_4 \rightarrow V_s$

$V_{C_1} = V_s$; $V_{C_2} = V_s$; $V_{C_3} = 0$; $V_{C_4} = 0$ (when T_3 & T_4 are ON)

3 φ Bridge Inverter



⇐ 3 φ Bridge Inverter

- This is Voltage Source Inverter (VSI).

- It has 6 Controlled Switches such as Thyristor, IGBT etc. In the above circuit, there are 6 IGBT namely $T_1, T_2, T_3, T_4, T_5, T_6$.
- Across each Controlled switch (such as IGBT), a diode is connected across it in antiparallel. This diode is called feedback diode. Function of feedback diode is to allow the flow of current through it when the load is reactive (Inductive or Capacitive) in nature. They feedback energy stored in load to the source. In above circuit, there are 6 feedback diodes namely $D_1, D_2, D_3, D_4, D_5, D_6$.
- In practical, a large Capacitor is connected at the input terminal across the DC supply to keep the input DC Voltage constant. Load (if reactive) fed back harmonic a.c current to the DC source. This harmonic a.c. current produces varying DC voltage at the input of inverter ~~to~~ due to source resistance. Capacitor suppresses this harmonic a.c. current.
- 6 IGBTs are gated (switch ON or OFF) at regular intervals in proper sequence so that a 3 φ A.C. Voltage is synthesized at the output terminal of Inverter.

There are two possible patterns of gating the IGBTs :-

(i) Pattern 1 is called 180° Conduction Mode.

(ii) Pattern 2 is called 120° Conduction Mode.

3 φ 180° Conduction Mode Inverter

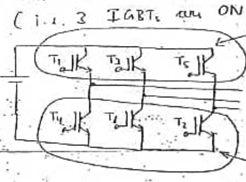
73

- In this mode, each IGBT conducts for 180° in a complete 360° AC cycle.
- Since there are 6 IGBTs; 6 gating pulses are generated by the Control System.

⇒ 6 gating pulses are generated at the regular intervals of 60° $\left(= \dfrac{360° \leftarrow \text{1 complete AC cycle}}{6 \leftarrow \text{No. of IGBTs}} \right)$

⇒ whenever a gate pulse is given to IGBT, it gets ON (or starts conducting). whenever a gate pulse is removed from IGBT, it gets OFF. Continuous gate pulse is required by IGBT to keep it in conduction.

- In this mode, 3 IGBTs are conducting simultaneously (i.e. 3 IGBTs are ON at any instant of time.)



UPPER GROUP

- In this mode, one IGBT must conduct from upper Group & two IGBTs must conduct from Lower Group.

OR

⇒ 2 IGBTs from upper Group & 1 IGBT from lower group.

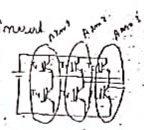
LOWER GROUP

- Suppose initially T_4, T_5, T_6 are conducting (ON). Then Conduction Sequence of IGBT for 1 complete cycle will be:

$\boxed{T_4, T_5, T_6}$ (Conducting) $\xrightarrow[T_1 \text{ ON}]{T_4 \text{ OFF}}$ After 60° $\boxed{T_5, T_6, T_1}$ (Conducting) $\xrightarrow[T_2 \text{ ON}]{T_5 \text{ OFF}}$ After 60° $\boxed{T_6, T_1, T_2}$ (Conducting) $\xrightarrow[T_3 \text{ ON}]{T_6 \text{ OFF}}$ After 60° $\boxed{T_1, T_2, T_3}$ (Conducting)

$\boxed{T_1, T_2, T_3}$ $\xrightarrow[T_4 \text{ ON}]{T_1 \text{ OFF}}$ $\boxed{T_2, T_3, T_4}$ (Conducting) $\xrightarrow[T_5 \text{ ON}]{T_2 \text{ OFF}}$ After 60° $\boxed{T_3, T_4, T_5}$ (Conducting) $\xrightarrow[T_6 \text{ ON}]{T_3 \text{ OFF}}$ After 60° $\boxed{T_4, T_5, T_6}$ (Conducting)

<u>Note</u> :- There are 3 arms in 3 ϕ Bridge inverter.
(i) Arm 1 has T_1 & T_4 connected in series.
(ii) Arm 2 has T_2 & T_6 connected in series.
(iii) Arm 3 has T_5 & T_2 connected in series.

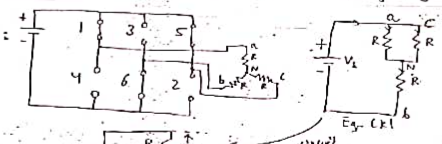


<u>Warning</u> :- <u>No</u> two IGBTs of same arm must conduct simultaneously, otherwise it will lead to short circuit of DC source.

⇒ For analysis, we will take star connected 3 ϕ pure resistive load (for simplicity).

<u>Assumption</u> :- when the current is going through any phase of load towards the load neutral, we will take the voltage of that phase to be positive and vice versa. <u>Note</u> :- 3 ϕ load is Balanced.

<u>Case I</u> : when T_5, T_6, T_1 are conducting together —

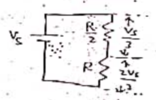


Eq. Ckt

$V_a = \frac{V_s}{2}$

$V_b = -\frac{2V_s}{2}$

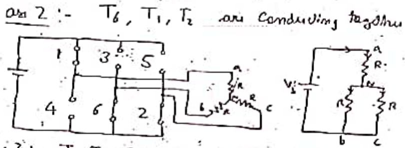
$V_c = \frac{V_s}{2}$



Voltage across $\frac{R}{2} = \frac{V_s}{2}$

Voltage across $R = \frac{2V_s}{2}$

<u>Case 2</u> :- T_6, T_1, T_2 are conducting together

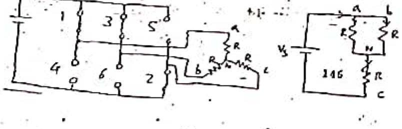


$V_a = \frac{2V_s}{2}$

$V_b = -\frac{V_s}{2}$

$V_c = -\frac{V_s}{2}$

<u>Case 3</u> :- T_1, T_2, T_3 are conducting together



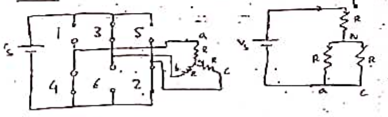
$V_a = \frac{V_s}{2}$

$V_b = \frac{V_s}{2}$

$V_c = -\frac{2V_s}{2}$

(74)

<u>Case 4</u> :- T_2, T_3, T_4 are conducting together

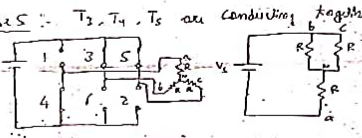


$V_a = -\frac{V_s}{2}$

$V_b = \frac{2V_s}{2}$

$V_c = -\frac{V_s}{2}$

<u>Case 5</u> :- T_3, T_4, T_5 are conducting together

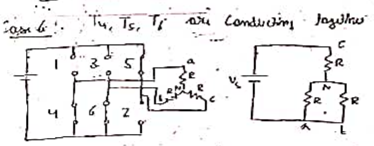


$V_a = -\frac{2V_s}{2}$

$V_b = \frac{V_s}{2}$

$V_c = \frac{V_s}{2}$

<u>Case 6</u> :- T_4, T_5, T_6 are conducting together

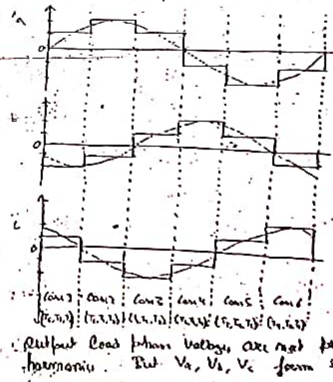


$V_a = -\frac{V_s}{2}$

$V_b = -\frac{V_s}{2}$

$V_c = \frac{2V_s}{2}$

- The above 6 cases, constitute one complete AC cycle of 360°. Now plot phase voltages for each case.



<u>Voltage across Load phase A</u>

<u>Voltage across Load phase B</u>

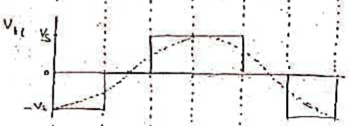
<u>Voltage across Load phase C</u>

Output load phase voltages are not perfect sine wave. They contain harmonics. But V_a, V_b, V_c form system of balanced 3 ϕ load voltages.

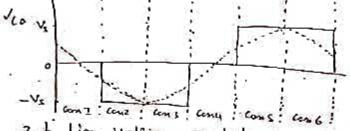
Note :- All the 3 output phase voltages has a peak magnitude of $\frac{2V_s}{3}$ and they are phase shifted from one another by 120°.

- Similarly line to line voltages can also be plotted using phase voltages such as line voltage $V_{ab} = V_a - V_b$.
  Also $V_{bc} = V_b - V_c$ & $V_{ca} = V_c - V_a$.

Line voltage, $V_{ab} = V_a - V_b$



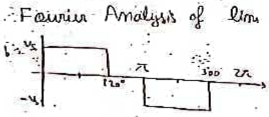
Line voltage, $V_{bc} = V_b - V_c$



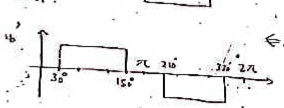
Line Voltage, $V_{ca} = V_c - V_a$

3 ϕ Line voltages are balanced and contains harmonics.

Fourier Analysis of line to line voltage, V_{ab}



Actual line to line voltage, V_{ab}



⇐ Shifted line to line voltage, V_{ab} by 30° (delayed by 30°)

Note :- Shifting of V_{ab} is done because fourier analysis of shifted waveform is much easier.

$$V_{ab}' = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos m\omega t + b_m \sin m\omega t)$$

⇒ $a_0 = 0$ because average value of V_{ab}' is Zero

⇒ $a_m = 0$ because V_{ab}' has odd wave symmetry.

(75)

$$b_m = \frac{1}{\pi}\int_0^{2\pi} V_{ab}' \sin m\omega t \, d\omega t$$

$$= \frac{1}{\pi}\left[\int_{30°}^{150°} V_s \sin m\omega t \, d\omega t + \int_{210°}^{330°} (-V_s) \sin m\omega t \, d\omega t\right]$$

$$= \frac{V_s}{\pi}\left[\left(-\frac{\cos m\omega t}{m}\right)_{\frac{\pi}{6}}^{\frac{5\pi}{6}} - \left(-\frac{\cos m\omega t}{m}\right)_{\frac{7\pi}{6}}^{\frac{11\pi}{6}}\right]$$

$$= \frac{V_s}{m\pi}\left[-\cos\frac{m5\pi}{6} + \cos\frac{m\pi}{6} + \cos\frac{m11\pi}{6} - \cos\frac{m7\pi}{6}\right]$$

$$= \frac{V_s}{m\pi}\left[-\cos\left(\frac{m5\pi}{6}\right) + \cos\left(\frac{m\pi}{6}\right) + \cos\left(2\pi m - \frac{m\pi}{6}\right) - \cos\left(2\pi m - \frac{m5\pi}{6}\right)\right]$$

$$= \frac{V_s}{m\pi}\left[-\cos\frac{m5\pi}{6} + \cos\frac{m\pi}{6} + \cos\frac{m\pi}{6} - \cos\frac{m5\pi}{6}\right]$$

$$= \frac{2V_s}{m\pi}\left[\cos\frac{m\pi}{6} - \cos\frac{m5\pi}{6}\right] = \frac{2V_s}{m\pi}\left[2\sin\left(\frac{m\pi}{2}\right)\sin\left(\frac{m\pi}{3}\right)\right]$$

$$= \frac{4V_s}{m\pi}\sin\left(\frac{m\pi}{2}\right)\sin\left(\frac{m\pi}{3}\right)$$

$$\therefore V_{ab}' = \sum_{m=1}^{\infty}\frac{4V_s}{m\pi}\sin\left(\frac{m\pi}{2}\right)\sin\left(\frac{m\pi}{3}\right)\sin m\omega t$$

$$V_{ab} = \sum_{m=1}^{\infty}\frac{4V_s}{m\pi}\sin\left(\frac{m\pi}{2}\right)\sin\left(\frac{m\pi}{3}\right)\sin m\left(\omega t + \frac{\pi}{6}\right)$$

⇐ This is actual V_{ab}. It leads the V_{ab}' by $\frac{\pi}{6}$.

⇒ Note :- V_{ab} does not contain even harmonics because for even value of m, sin value of $\sin\left(\frac{m\pi}{2}\right)$ is Zero.
Similarly, V_{ab} does not contain triplen harmonic (3, 6, 9 etc). because for m = multiple of 3, value of $\sin\left(\frac{m\pi}{3}\right)$ is Zero.

$$\text{Similarly, } V_{bc} = \sum_{m=1}^{\infty}\frac{4V_s}{m\pi}\sin\left(\frac{m\pi}{2}\right)\sin\left(\frac{m\pi}{3}\right)\sin m\left(\omega t + \frac{\pi}{6} - \frac{2\pi}{3}\right)$$

$$V_{ca} = \sum_{m=1}^{\infty}\frac{4V_s}{m\pi}\sin\left(\frac{m\pi}{2}\right)\sin\left(\frac{m\pi}{3}\right)\sin m\left(\omega t + \frac{\pi}{6} - \frac{4\pi}{3}\right)$$

- T_1 & T_4 are connected in series across DC supply (T_1 & T_4 are IGBTs connected in same arm). T_1 conducts for first 180°. And then T_4 conducts for next 180°. There are chances that both T_1 & T_4 conducts together because there is no time in between turning OFF of T_1 & turning ON of T_4. So there is a possibility of short circuit of DC supply.

3ϕ 120° Conduction Mode Inverter

- In this mode, each IGBT conducts for 120° in a complete 360° AC cycle.
- 6 gating pulses are generated at the regular intervals of 60°.
- In this mode, 2 IGBTs are conducting simultaneously. One is conducting from the upper group and one from the lower group.
- Suppose initially T_5, T_6 are conducting (ON).

Then conduction sequence of IGBT for 1 complete cycle will be :-

$$\boxed{T_5, T_6} \xrightarrow[T_1\ ON]{T_5\ OFF} \boxed{T_6, T_1} \xrightarrow[T_2\ ON]{T_6\ OFF} \boxed{T_1, T_2} \xrightarrow[T_3\ ON]{T_1\ OFF} \boxed{T_2, T_3} \xrightarrow[T_4\ ON]{T_2\ OFF} \boxed{T_3, T_4} \xrightarrow[T_5\ ON]{T_3\ OFF} \boxed{T_4, T_5} \xrightarrow[T_6\ ON]{T_4\ OFF} \boxed{T_5, T_6}$$

Conducting pair

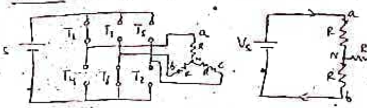
- Consider IGBTs connected in series in the same arm, say T_1 & T_4. T_1 conducts for 120°. There is a gap of 60° between turning OFF of T_1 and turning ON of T_4. T_4 also conducts for 120°. Again there is a gap of 60° between turning OFF of T_4 and turning ON of T_1. Due to gap of 60°, there is a very rare chance of short circuit of supply due to simultaneous conduction of IGBTs of same arm.
- For understanding the working, we will take star connected 3ϕ pure resistive load (for simplicity).

Note :- 3ϕ load is balanced.

<u>Assumption</u> :- When the current is going through any phase of load, toward the neutral, we will take the voltage of that phase to be positive & vice versa.

(76)

<u>Case 1</u> :- When T_6 & T_1 are conducting together

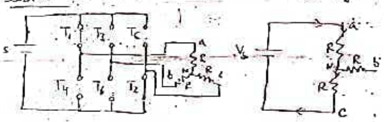


$V_a = \frac{V_s}{2}$

$V_b = -\frac{V_s}{2}$

$V_c = 0$ (No current in phase c)

<u>Case 2</u> :- When T_1 & T_2 are conducting together

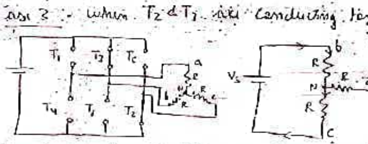


$V_a = \frac{V_s}{2}$

$V_b = 0$ (No current in phase b)

$V_c = -\frac{V_s}{2}$

<u>Case 3</u> :- When T_2 & T_3 are conducting together

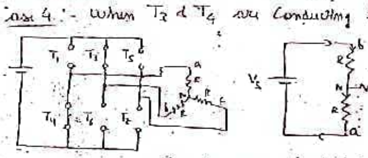


$V_a = 0$

$V_b = \frac{V_s}{2}$

$V_c = -\frac{V_s}{2}$

<u>Case 4</u> :- When T_3 & T_4 are conducting together

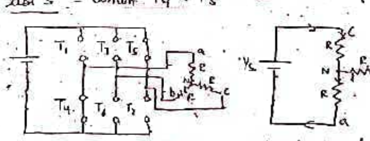


$V_a = -\frac{V_s}{2}$

$V_b = \frac{V_s}{2}$

$V_c = 0$

<u>Case 5</u> :- When T_4 & T_5 are conducting together

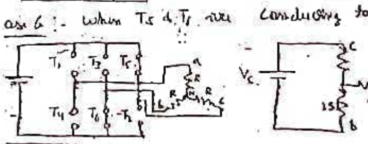


$V_a = -\frac{V_s}{2}$

$V_b = 0$

$V_c = \frac{V_s}{2}$

<u>Case 6</u> :- When T_5 & T_6 are conducting together

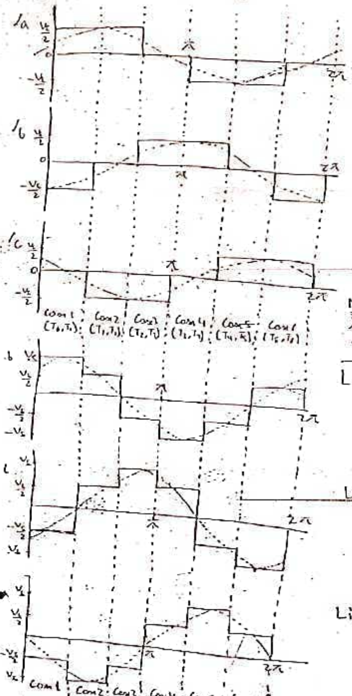


$V_a = 0$

$V_b = -\frac{V_s}{2}$

$V_c = \frac{V_s}{2}$

- 6 Cases discussed above, Constitute one complete AC cycle of 360°. Now plot phase Voltages for each Case.



Voltage across Load phase A

Voltage across Load phase B

Voltage across Load phase C

Note:- All the 3 phase Voltages V_a, V_b & V_c has a peak magnitude of $\frac{V_s}{2}$ and they are phase shifted from one another by 120°.

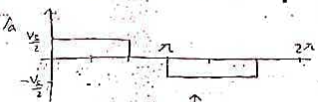
Line Voltage, $V_{ab} = V_a - V_b$

Line Voltage, $V_{bc} = V_b - V_c$

Line Voltage, $V_{ca} = V_c - V_a$

- Note:- Output load phase Voltages are not perfect Sine wave. They contain harmonics. But V_a, V_b, V_c form System of balanced 3φ load voltage. Similarly, line voltages V_{ab}, V_{bc} & V_{ca} are balanced 3φ voltages.

- Fourier Analysis of phase Voltage V_a (77)



⇐ Phase a Voltage V_a (120° Conduction Mode)

Note:- This waveform is similar to line voltage V_{ab} discussed in 180° Conduction Mode Inverter (Except for the peak magnitude.)

$$\therefore V_a = \sum_{n=1}^{\infty} \frac{2V_s}{n\pi} \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{n\pi}{3}\right) \sin n\left(\omega t + \frac{\pi}{6}\right)$$

$$V_b = \sum_{n=1}^{\infty} \frac{2V_s}{n\pi} \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{n\pi}{3}\right) \sin n\left(\omega t - \frac{\pi}{2}\right)$$

$$V_c = \sum_{n=1}^{\infty} \frac{2V_s}{n\pi} \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{n\pi}{3}\right) \sin n\left(\omega t - \frac{7\pi}{6}\right)$$

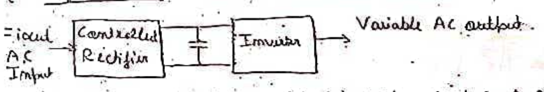
Note:- Phase voltages does not contain even harmonics because $\sin\left(\frac{n\pi}{2}\right)$ is Zero for n = even.

Phase voltages does not contain triplen harmonics because $\sin\left(\frac{n\pi}{3}\right)$ is Zero for n = multiple of 3.

Methods of Voltage Control in Inverter

- Output Voltage of Inverter can be varied by following three ways:-

(i) By varying the input DC voltage of Inverter.



⇒ In this, Controlled rectifier is used. Output of Controlled rectifier is given as Input to the Inverter. By Controlling the rectifier, Input DC voltage of inverter can be varied & hence variable AC voltage output can be obtained from Inverter.

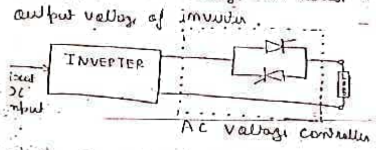
Disadvantage of this method of voltage control :-

(i) Extra Converter is needed in the form of Rectifier. This extra Converter (Rectifier) increases the Cost of the inverter System. Extra filter circuitry is also needed to filter out ripples in output DC of rectifier. Extra Converter increases the losses in the inverter system.

ii) At low DC input voltage at the inverter, there is a chance of Commutation failure (if thyristors are used in inverter). At low DC input voltage, Commutating Capacitor (part of forced Commutation circuitry) are charged to low voltage. This decreases the Circuit turn OFF time of thyristor.

Note :- Circuit turn OFF time is the time for which thyristor remains reverse biased.

Method 2 of voltage Control :- (ii) By Varying the output voltage of Inverter :- AC voltage controller is used to vary the output voltage of inverter.



AC Voltage controller

- AC voltage Controller uses two thyristors in antiparallel. Firing angle of these thyristors are varied to Control the output Load voltage.

Note :- Input DC voltage of inverter is Fixed.

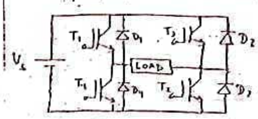
Disadvantage of this method :- Higher Harmonic content in the Load voltage, particularly when load voltage is low (at high value of firing angle).

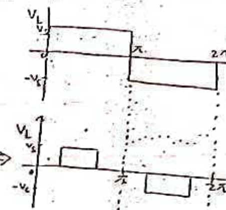
Method 3 of Voltage Control :- (iii) By PWM Control of Inverter :-

⇒ PWM stands for Pulse Width Modulation.

⇒ In this method, output voltage of inverter can be varied (or controlled) by varying the ON & OFF time of switching device such as IGBT or thyristor.

⇒ Output voltage of inverter is Controlled by varying the Conduction time of IGBT or SCR. (78)





Output Voltage (Without PWM Control)

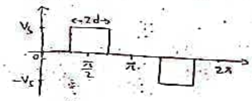
PWM Controlled output ⇒

Output Voltage (With PWM Control)

⇒ Without PWM Control :- output voltage is (+) V_s from $\omega t = 0$ to π ; & output voltage is (−) V_s from $\omega t = \pi$ to 2π.

⇒ With PWM Control :- output voltage is Zero & (+) V_s from $\omega t = 0$ to π ; & output voltage is Zero & (−) V_s from $\omega t = \pi$ to 2π.

⇒ By varying the duration of output Pulse, rms value of output AC voltage of inverter Can be varied.



⇒ Pulse width = 2d (in each half)

⇒ Note :- Pulse of width 2d is located Symmetrically about $\frac{\pi}{2}$.

⇒ RMS value of above wave form (PWM Controlled output voltage)

$$= \sqrt{\frac{2}{2\pi}\int_{\frac{\pi}{2}-d}^{\frac{\pi}{2}+d} V_s^2 \, d\omega t} = \sqrt{\frac{V_s^2}{\pi}\left(\frac{\pi}{2}+d-\frac{\pi}{2}+d\right)}$$

$$= V_s\sqrt{\frac{2d}{\pi}}$$ ⇐ RMS value of output voltage depends on pulse width.

- Advantage of PWM Control :-

(i) Simple method of voltage control. It does not require any extra component (such as rectifier or AC voltage Controller).

(ii) Selective lower order harmonics can be reduced or eliminated using PWM technique.

Disadvantage of PWM technique

(i) Increased Control Complexity.

(ii) Increased Switching power losses in inverter if multiple pulse width modulation is used.

(iii) Power electronic device used in PWM control inverter should be capable of working under high switching frequency (i.e. device should have low turn ON & turn OFF time.).

Different PWM techniques & Methods of Harmonic Reduction

. Undesirable effects of Harmonics are :-

(i) Pulsating torque produced by motor due to harmonics

(ii) Extra heat loss in motor (or any other appliance) due to harmonics. Harmonics increase the rms value of current and hence increases $i^2 r$ losses. But useful power is due to fundamental component of

(iii) Undesirable resonance problem (or oscillation problem) can occur due to certain specific harmonic frequency.

(iv) Harmonics increases interference in the Communication network.

. Different types of PWM techniques are :-

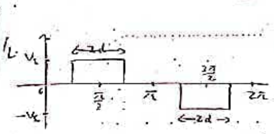
(i) Single Pulse Width Modulation.

(ii) Multiple Pulse Width Modulation.

(iii) Sinusoidal Pulse Width Modulation.

Single Pulse Width Modulation

In this, there is only one pulse per half cycle and the width of the pulse is varied to control the inverter output voltage (and/or to reduce harmonics).



→ Output Voltage of Single PWM Inverter. (There is Single Pulse per half cycle.)

Fourier Analysis of output voltage of Single PWM Inverter.

$$V_L = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos m\omega t + b_m \sin m\omega t)$$

⇒ $a_0 = 0$ — because average value of waveform is zero over complete cycle.

⇒ $a_m = 0$ because waveform has odd wave symmetry.

$$\therefore b_m = \frac{1}{\pi}\left[\int_{\frac{\pi}{2}-d}^{\frac{\pi}{2}+d} V_s \sin m\omega t \, d\omega t + \int_{\frac{3\pi}{2}-d}^{\frac{3\pi}{2}+d} -V_s \sin m\omega t \, d\omega t\right]$$

$$b_m = \frac{V_s}{\pi}\left[\left(-\frac{\cos m\omega t}{m}\right)_{\frac{\pi}{2}-d}^{\frac{\pi}{2}+d} - \left(-\frac{\cos m\omega t}{m}\right)_{\frac{3\pi}{2}-d}^{\frac{3\pi}{2}+d}\right]$$

$$= \frac{V_s}{m\pi}\left[-\cos m\left(\frac{\pi}{2}+d\right) + \cos m\left(\frac{\pi}{2}-d\right) + \cos m\left(\frac{3\pi}{2}+d\right) - \cos m\left(\frac{3\pi}{2}-d\right)\right]$$

$$= \frac{V_s}{m\pi}\left[\begin{matrix} -\cancel{\cos \frac{m\pi}{2} \cos md} + \sin \frac{m\pi}{2} \sin md + \cancel{\cos \frac{m\pi}{2} \cos md} + \sin \frac{m\pi}{2} \sin md \\ \cancel{\cos \frac{m3\pi}{2} \cos md} - \sin \frac{m3\pi}{2} \sin md - \cancel{\cos \frac{m3\pi}{2} \cos md} - \sin \frac{m3\pi}{2} \sin md \end{matrix}\right]$$

$$= \frac{V_s}{m\pi}\left[2 \sin \frac{m\pi}{2} \sin md - 2 \sin \frac{m3\pi}{2} \sin md\right]$$

$$= \begin{cases} 0 & , m = \text{even} \\ \frac{4V_s}{m\pi} \sin\left(\frac{m\pi}{2}\right) \sin(md) & , m = \text{odd} \end{cases}$$

$$\therefore V_L = \sum_{m=1,3,5,7\ldots}^{\infty} \frac{4V_s}{m\pi} \sin\left(\frac{m\pi}{2}\right) \sin(md) \sin m\omega t$$

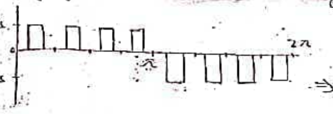
⇒ m^{th} harmonic can be eliminated from the output voltage waveform if $d = \frac{\pi}{m}$. (For ex. 3^{rd} harmonic can be eliminated if $d = \frac{\pi}{3}$).

⇒ Selective lower order harmonics such as 3^{rd} or 5^{th} can be removed by adjusting the width of pulse.

Disadvantage of Single PWM :- To obtain low output voltage from inverter, width of pulse is small.
At small wise pulse width, magnitude of harmonics become comparable to the fundamental.
Therefore harmonic contents increase at lower output voltage.

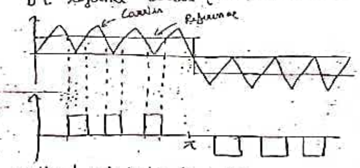
Multiple Pulse Width Modulation

- In this, there are several equidistant pulses per half cycle.



Output Voltage of Multiple PWM Inverter. (Multiple pulse per half cycle)
⇒ In this, there are 4 pulses per half cycle.

- Way to obtain Multiple PWM :- It is obtain by Compairing high frequency triangular wave with DC reference wave (Square reference).

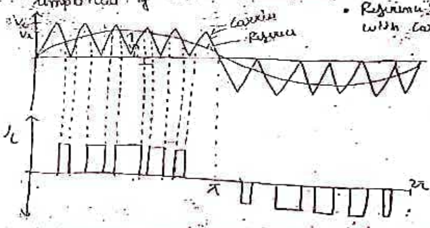


- When the reference become more than carrier, Gate pulses are generate by control system for IGBT.
- Width of each pulse is same.
- Frequency of carrier wave decides number of pulses per half.
- Frequency of Reference Square wave decides the frequency of output voltage.
- Magnitude of Reference Square wave, decides the width of pulse and hence magnitude of output voltage.
- By using Multiple Pulse Width Modulation, lower order harmonics can be reduced but magnitude of higher

order harmonic goes up. (80)
Higher order harmonic can be easily filter out.
Note :- There is a problem in filtering lower order harmonic because large filters are required for filtering lower order harmonic.

Sinusoidal Pulse Width Modulation

- In multiple PWM, pulse width is equal for all the pulses.
- In Sinusoidal PWM, there are multiple pulses per half cycle. And width of each pulse is variable in proportion to the amplitude of Sine wave.



- Reference Sine wave is compare with carrier triangular wave.
- When reference wave exceeds the carrier wave, gate pulses are generated.

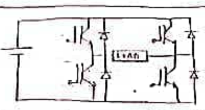
If there are p, pulses per half cycle, then dominant harmonic will be $(2p-1)$. i.e. it reduces (or eliminates) all harmonic less than or equal to $(2p-1)$.

- Frequency of reference Sine wave decides the frequency of output voltage.
- Magnitude of reference Sine wave decides the magnitude of output voltage.
- Frequency of carrier wave decides the number of pulses per half.
- Modulation Index = $\dfrac{\text{Peak Magnitude of Reference Sine wave}}{\text{Peak Magnitude of Triangular Carrier wave}}$

Modulation Index is kept less than 1.

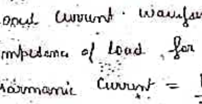
- If Modulation Index is more than 1, then it is called over modulation. In that case, lower order harmonics will appear in output voltage waveform. This is because pulse width is no longer sine function.
- Sinusoidal PWM is very popular & it widely used in Inverter control.

Q



For 1φ Full Bridge Inverter, Input DC supply, $V_s = 220V$, Load is RLC where $R = 10\Omega$, $L = 31.5\ mH$ & $C = 112\ \mu F$. Output load voltage frequency is 60Hz.
Harmonic Components greater than 9th Harmonic are neglected.
Calculate the following :-
(i) Rms of fundamental load voltage ; (ii) Rms of fundamental load current,
(iii) Power consumed by the load (Both actual Power & fundamental Power)
(iv) Average value of current supplied by DC Supply.

al Load Voltage waveform is Square wave



Load voltage waveform contains harmonics

$$V_L = \sum_{m=1,3,5\ldots}^{\infty} \frac{4V_s}{m\pi} \sin m\omega t$$

- Load current waveform is damped sine wave & contains harmonics.
Impedance of load for harmonic current is dependent on harmonic frequency.

$$\text{Harmonic Current} = \frac{\text{Harmonic Voltage}}{\text{Impedance @ Harmonic Frequency}}$$

Harmonic frequency	R	X_{Lm}	X_{Cm}	Z_m	V_m (rms)	I_m (rms)
Fundamental (60Hz)	10Ω	11.869Ω	23.68Ω	15.475Ω	$V_1 = 198.19V$	12.81A ← I_1
3rd Harmonic (180Hz)	10Ω	35.607Ω	7.894Ω	29.462Ω	$V_3 = 66.06V$	$I_3 = 2.242A$
5th Harmonic (300Hz)	10Ω	59.345Ω	4.736Ω	55.51Ω	$V_5 = 39.62V$	$I_5 = 0.713A$
7th Harmonic (420Hz)	10Ω	83.083Ω	3.382Ω	80.32Ω	$V_7 = 28.31V$	$I_7 = 0.352A$
9th Harmonic (540Hz)	10Ω	106.82Ω	2.63Ω	104.66Ω	$V_9 = 22.02V$	$I_9 = 0.210A$

As :- $X_{Lm} = 2\pi f m L$ where $m = 1, 3, 5, 7 \& 9$

$X_{Cm} = \frac{1}{2\pi f m C}$

$$Z_m = \sqrt{R^2 + (X_{Lm} - X_{Cm})^2}$$ (81)

$V_m = \frac{4V_s}{m\pi\sqrt{2}}$ ← Rms value of Harmonic load voltage

$I_m = \frac{V_m}{Z_m}$ ← Rms value of Harmonic load current.

(i) Rms value of fundamental load voltage $= \frac{4 \times 220}{(1)\pi\sqrt{2}} = 198.19V$

(ii) Rms value of fundamental load current $= \frac{198.19}{15.475} = 12.81A$

Rms value of total load current (Fundamental + upto 9th Harmonics)

$$I_L = \sqrt{I_1^2 + I_3^2 + I_5^2 + I_7^2 + I_9^2}$$
$$= \sqrt{(12.81)^2 + (2.242)^2 + (0.713)^2 + (0.352)^2 + (0.210)^2}$$
$$= 13.03A$$

where I_1 is fundamental (rms) load current ; I_3, I_5, I_7, I_9 are rms harmonic load current.

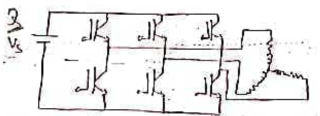
(iii) Actual Power consumed by load $= (I_L)^2 R_L = (13.03)^2 \times 10$
$= 1697.8W$

Power consumed by load (only due to fundamental current) $= (I_1)^2 R_L$
$= (12.81)^2 \times 10 = 1640.4W$

Note :- Power consumed by load is mainly due to fundamental component of current. This power (due to fundamental) gets converted to useful power (like mechanical output of motor).
⇒ Power due to harmonic current does not involve in useful work. It only raises the temperature of load.

(iv) If Inverter is loss less, then
Input power supplied by DC source = Output Power of inverter
$220 \times I_s = 1697.8$
$I_s = 7.71A$

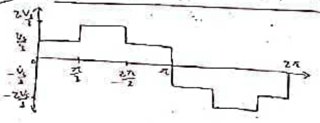
I_s is average current supplied by DC supply.



3φ Full Bridge inverter is operated in 180° Conduction Mode. Input DC Supply; $V_s = 220V$. Load is RL with $R = 5\Omega$, $L = 23mH$. Output load voltage frequency is 60 Hz. Load is Star Connected. Harmonic Components greater than 17^{th} Harmonic are neglected. Calculate the following :-

i) Rms value of line to line output voltage; (ii) Rms value of output phase voltage.
iii) Rms value of fundamental line to line output voltage; (iv) Rms value of fundamental output phase voltage; (v) Power Consumed by the load; (vi) Harmonic Factor & Distortion factor for Lowest order Harmonic

Sol: Inverter is operated in 180° Conduction Mode:

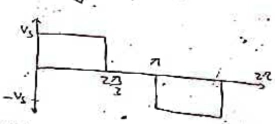


Output phase Voltage.

Rms value of output phase voltage $= \sqrt{\frac{1}{\pi}\left[\int_0^{\frac{\pi}{3}}\left(\frac{V_s}{3}\right)^2 d\omega t + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}\left(\frac{2V_s}{3}\right)^2 d\omega t + \int_{\frac{2\pi}{3}}^{\pi}\left(\frac{V_s}{3}\right)^2 d\omega t\right]}$

$= \sqrt{\frac{V_s^2}{\pi}\left[\frac{\pi}{3}\times\frac{1}{9} + \frac{\pi}{3}\times\frac{4}{9} + \frac{\pi}{3}\frac{1}{9}\right]}$

$= V_s\sqrt{\frac{6}{27}} = 220\times\sqrt{\frac{6}{27}} = 103.70 V$



Output line to line voltage.

i) Rms value of output line to line voltage =

$\sqrt{\frac{1}{\pi}\int_0^{\frac{2\pi}{3}}(V_s)^2\, d\omega t} = V_s\sqrt{\frac{2}{3}} = 220\times\sqrt{\frac{2}{3}} = 179.62V$

(iii) Rms value of fundamental output line to line voltage (82)

$V_{Line-Line} = \sum_{m=1,3,5}^{\infty} \frac{4V_s}{m\pi}\sin\frac{m\pi}{3}\sin m\left(\omega t + \frac{\pi}{6}\right)$

For fundamental, $m = 1$

$V_{L1}\,(rms) = \frac{4\times V_s}{m\pi\sqrt{2}}\sin\frac{m\pi}{3} = \frac{4\times 220}{(1\,\pi\times\sqrt{2})}\sin\frac{\pi}{3} = 171.64V.$

(iv) Rms value of fundamental output phase voltage $= \frac{V_{L1}}{\sqrt{3}} = \frac{171.64}{1.732} = 99.09$

$Z_m = \sqrt{R^2 + (2\pi f m L)^2} \rightarrow$ Impedance for m^{th} harmonic

$V_{pm} = \frac{4V_s}{\sqrt{3}\,m\pi\sqrt{2}}\sin\left(\frac{m\pi}{3}\right) \rightarrow V_{phase}\,(rms)$ for m^{th} harmonic

$I_{pm} = \frac{V_{pm}}{Z_m} \rightarrow I_{phase}\,(rms)$ for m^{th} harmonic

Harmonic Frequency	R	$X_{Lm}(\Omega)$	$Z_m(\Omega)$	V_{pm} (rms)	I_{pm} (rms)
Fundamental (60Hz)	5Ω	8.671	10.009	$V_{1p} = 99.09V$	$I_{1p} = 9.909A$
5^{th} Harmonic (300Hz)	5Ω	43.35	43.63	$V_{5p} = 19.82V$	$I_{5p} = 0.454A$
7^{th} Harmonic (420Hz)	5Ω	60.69	60.89	$V_{7p} = 14.15V$	$I_{7p} = 0.232A$
11^{th} Harmonic (660Hz)	5Ω	95.38	95.51	$V_{11p} = 9.009V$	$I_{11p} = 0.094A$
13^{th} Harmonic (780Hz)	5Ω	112.72	112.83	$V_{13p} = 7.62V$	$I_{13p} = 0.0675$
17^{th} Harmonic (1020Hz)	5Ω	147.40	147.48	$V_{17p} = 5.82V$	$I_{17p} = 0.0395$

Note :- Line Voltage does not contain triplen Harmonics ($3^{rd}, 9^{th}, 15^{th}$).

Rms value of phase Current $= \sqrt{(9.909)^2 + (0.454)^2 + (0.232)^2 + (0.094)^2 + (0.0675)^2 + (0.0395)^2}$

$I_{Lp} = 9.922A$

v) 3φ power Consumed by Load $= 3(I_{Lp})^2R = 3\times(9.922)^2\times 5$

$= 1476.69\,W.$

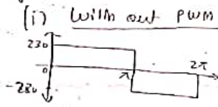
vi) Lowest order Harmonic is 5^{th}.

$HF = \frac{V_{L5}}{V_{L1}}$ ← Line to line 5^{th} Harmonic / ← Line to line Fundamental $= \frac{1}{5} = 0.2 = 20\%$

$DF = \frac{V_{L5}}{V_{L1}(5)^2} = \frac{1}{5\times 5^2} = \frac{1}{125} = 0.008 = 0.8\%$ (Harmonic gets reduced by 2nd order Filter.)

Q Single PWM technique is used to control 1ϕ Full bridge inverter. Input DC Supply is 230V. Pulse width for half cycle is 90°. Find the rms value of actual Load voltage, fundamental of load voltage, 3rd & 5th harmonic load voltage. Compare the result with for without PWM control.

Sol (i) With out PWM Control



Actual load voltage (rms) = 230V.

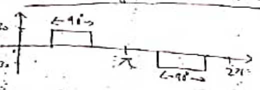
Rms of harmonic load voltage $= \frac{4 V_s}{n \pi \sqrt{2}}$

Fundamental load voltage $= \frac{4 \times 230}{3.14 \times 1.414} = 207.209$

3rd Harmonic Load voltage $= \frac{4 \times 230}{3 \times 3.14 \times 1.414} = 69.06V$

5th Harmonic Load voltage $= \frac{4 \times 230}{5 \times 3.14 \times 1.414} = 41.44V$

ii) With Single PWM Control



$2d = 90° \Rightarrow d = 45°$

Rms of actual load voltage = $230\sqrt{\frac{1}{\pi} \times \frac{\pi}{2}} = 162.63V$

$V_L(rms) = 162.63V$

Rms of harmonic load voltage $= \frac{4 V_s}{n \pi \sqrt{2}} \sin\left(\frac{n\pi}{2}\right) \sin(nd)$

Fundamental Load voltage $= \frac{4 \times 230}{1 \times \pi \times \sqrt{2}} (1) \sin 45° = 146.51V$

3rd Harmonic Load voltage $= \frac{4 \times 230}{3 \times \pi \sqrt{2}} (-1) \sin 135° = 48.83V$

5th Harmonic Load voltage $= \frac{4 \times 230}{5 \times \pi \times \sqrt{2}} (1) \sin 225 = 29.302V$

Note:- There is a reduction in harmonic contents using Single PWM technique.

AC VOLTAGE CONTROLLER

(23)

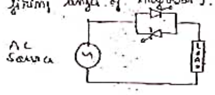
- Applications requiring Variable AC Voltage:-

(i) Industrial Heating → To obtain variable heat output.
(ii) Lighting Control → To obtain variable luminous intensity.
(iii) Motor Drive → To Control the speed of AC motor.
(iv) AC electromagnets Control (in relays).

- Methods to obtain Variable AC voltage:-

(i) Through Auto transformer
(ii) By Tap changing Transformer.
(iii) By Thyristor based AC voltage Controller.

- Thyristor based AC voltage Controller:- If a thyristor switch is connected between AC supply and load, the power flow can be controlled by varying the rms value of AC voltage applied to the load (Voltage can be varied by controlling the firing angle of thyristor).



← Thyristor Based AC voltage Controller

- Two thyristors in antiparallel are inserted between Source & Load.
- Two thyristors are in antiparallel, so that AC can flow into Load.

- Advantage of Thyristor based AC voltage Controller:-

(i) Flexible Voltage Control :- Control is easy.
(ii) Precise voltage control is possible:- Stepless Control.
(iii) Wide range of voltage control is possible.
(iv) This type of Controller is Cheap & Compact in size.

- Disadvantage of Thyristor based AC voltage Controller:-

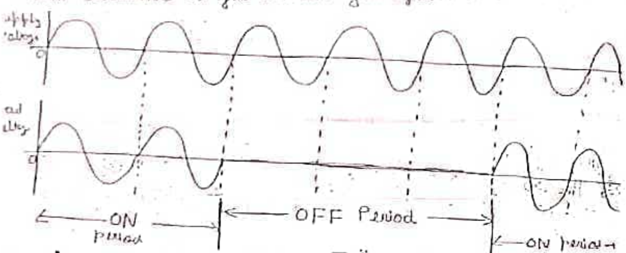
(i) It introduces objectionable harmonics in the supply current & load voltage; particularly at low output voltage levels.
(ii) Load voltage greater than supply voltage is not achievable through this type of Controller. (This can be achieved by step up transformer.)

- Types of thyristor based AC voltage Controller:-

(i) ON-OFF Control; (ii) Phase Control; (iii) Sequence Control.

ON - OFF Control

- ON - OFF Control is also called Integral Cycle Control
- In ON-OFF control, thyristor switches connect the load to the AC source for a few cycles of input voltage and then disconnect it for another few cycles.



Thyristor Switch is ON for 2 cycles. It is OFF for next 3 cycles. Again it is ON for 2 cycles & so on.

- Firing angle of both the two thyristors (in antiparallel) is Zero (i.e. $\alpha=0$). Thyristor is simply used as switch. $\alpha=0$ during ON time.
- Gate pulses are withdrawn during OFF time.
- Applications of ON-OFF Control :

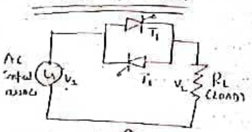
(i) Speed Control of motor where load has high mechanical inertia (i.e. no variation in speed is observed or noticed if motor is disconnected from supply for few cycles).

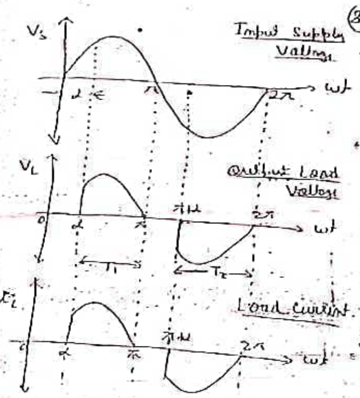
(ii) Industrial heating where thermal time constant is high. (i.e. No variation in temperature is observed if heater is disconnected from supply for few cycles).

- Disadvantage of ON-OFF control :- It cannot be used for application with low mechanical time constant, / or low thermal time constant.
- Advantage of ON-OFF control :- It introduces less harmonics into the supply system.

Phase Control

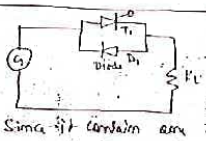
(34)





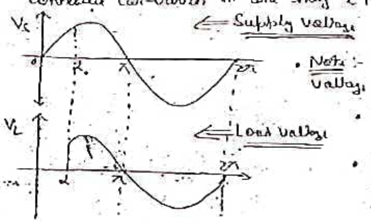
- This is Full wave or Bidirectional AC voltage Controller.
- T_1 is forward biased between 0 to π.
- T_1 is fired at $\omega t=\alpha$.
- T_1 conducts from (α to π).
- T_2 is forward biased between π to 2π.
- T_2 is fired at $\omega t=\pi+\alpha$
- T_2 conducts from ($\pi+\alpha$) to 2π.

- Thus, Rms value of Load voltage is dependent on firing angle, α. Hence load voltage can be varied by varying α.



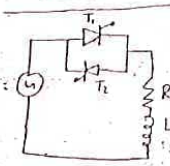
This is a Half wave or Unidirectional AC voltage Controller

- It uses one thyristor (T_1) & one diode (D_1)
- Since it contains one thyristor only, thus, the voltage can be controlled (or varied) in one half (positive Half).

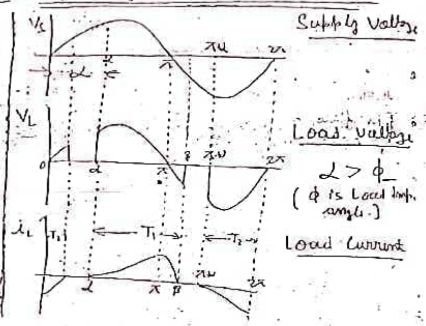


- Note :- Here average value of load voltage is not Zero.
  - Therefore, supply current contains DC component.
  - This DC component will lead to transformer saturation problem (if used).

- Note :- If 2 thyristors are used in antiparallel (in AC voltage controller) then gate triggering source for two thyristors must be isolated from one another because otherwise the two cathode would be connected together & the two thyristors would be out of circuit.
- TRIAC can be used in place of 2 Thyristors (2 antiparallel thyristors).



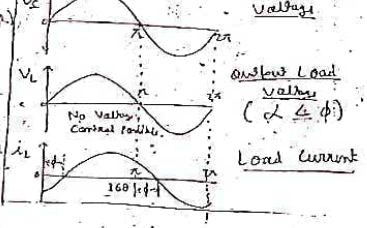
⇐ AC Voltage Controller with RL Load



At $\omega t = \pi$, $i_L \neq 0$, therefore T_1 keeps on conducting beyond π. T_1 will conduct till $\omega t = \beta$.

If ϕ is Load impedance angle of RL Load, then firing angle α should be more than ϕ, to achieve voltage control.

- If α is less than load impedance angle ϕ, then voltage control is not possible.



⇒ T_1 is conducting from (α to π). Since, load is inductive, T_1 will keep on conducting beyond $\omega t = \pi$ till $\omega t = \pi + \phi$ where ϕ is load impedance angle.
If α is less than ϕ, then T_2 will not get ON at $\omega t = \pi + \alpha$ because T_1 is still conducting. T_2 is reverse biased by on state drop of T_1.

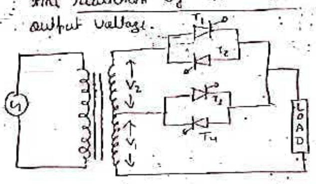
(85)

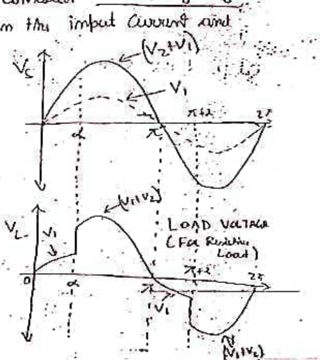
⇒ Therefore T_2 will not go ON till $\omega t = \pi + \phi$.
⇒ Hence firing angle α should be greater than Load impedance angle ϕ to achieve the Load voltage Control.
⇒ Control range of firing angle is $\phi < \alpha < 180°$

Sequence Control

- Disadvantage of Phase Control :- Load voltage & Supply current contain large harmonic contents.

⇒ Sequence Control of AC voltage controller is employed for the reduction of harmonics in the input current and output voltage.





Note :- V_1 is voltage across lower half of Secondary (Rms value)
V_2 is voltage across upper half of Secondary (Rms value).

⇒ From $\omega t = 0$ to α, T_2 conducts & Load voltage is V_1.
⇒ ~~From $\omega t = \alpha$ to π, T_1 is~~ At $\omega t = \alpha$, T_1 is fired.
At $\omega t = \alpha$, T_3 gets reverse bias & turns OFF.
⇒ From $\omega t = \alpha$ to π, T_1 conducts & Load voltage is (V_1+V_2).
⇒ At $\omega t = \pi$, T_4 is fired. T_4 Conducts from $\omega t = \pi$ to $\pi+\alpha$.
⇒ At $\omega t = \pi + \alpha$, T_2 is fired. This reverse bias T_4 & turns it OFF. From $\omega t = \pi$ to $\pi+\alpha$, Load voltage is V_1.
⇒ From $\omega t = \pi+\alpha$ to 2π, T_2 conducts & load voltage is (V_1+V_2)

- When $\alpha = 0$, rms of load voltage is $(V_1 + V_2)$

  When $\alpha = 180$, rms of load voltage is V_1.

  Hence by varying α, rms of load voltage is controlled from V_1 to $(V_1 + V_2)$

Q. ON-OFF Control is applied to a heater with high thermal time constant. Input A.C supply is 120V, 60Hz. Thyristor switch is on for 25 cycles & is OFF for 75 cycles. Resistance of heater is 10Ω. Find the following:-
(i) rms output load voltage ; (ii) Power Consumed by load ;
(iii) Input power factor ; (iv) rms value of current through thyristor.

Sol (i) Rms output load voltage :-

$$V_L = \sqrt{\frac{1}{2\pi(25+75)}\left[\int_0^{2\pi}(\sqrt{2}\,120\sin\omega t)^2\,d\omega t + \int_{2\pi}(\sqrt{2}\,120\sin\omega t)^2\,d\omega t + \ldots \ldots \text{upto 25 cycles}\right]}$$

Periodicity

$$= \sqrt{\frac{25}{2\pi(25+75)}\int_0^{2\pi}(\sqrt{2}\,120\sin\omega t)^2\,d\omega t}$$

$$= \sqrt{\frac{25\times 2\times(120)^2}{2\pi\times 100}\int_0^{2\pi}\left(\frac{1-\cos 2\omega t}{2}\right)dt} = \sqrt{\frac{25\times 2\times(120)^2}{2\pi\times 100\times 2}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{2\pi}}$$

$$= 120\sqrt{\frac{25\times 2\pi\times 2}{100\times 2\pi\times 2}} = 120\sqrt{0.25} = 120\times 0.5$$

$= 60\,V$ (rms)

Rms value of load current $= \dfrac{V_L(\text{rms})}{R_L} = \dfrac{60}{10} = 6\,A$

(ii) Power consumed by load $= I_L^2 R = (6)^2\times 10 = 360\,W$

(iii) Input VA supplied by source $= V_S \cdot I_S$

$= 120\times 6 = 720\,W$

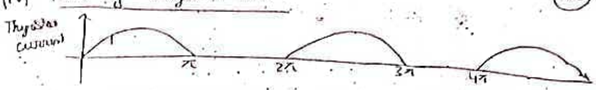
(Because supply current is equal to rms of load current)

Input power factor $= \dfrac{\text{Power consumed (or True Power)}}{\text{Input VA (or Apparent Power)}}$

$= \dfrac{360}{720} = 0.5$

(86)

(iv) Rms of thyristor current



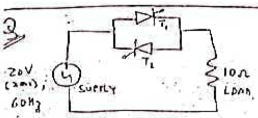
$$i_{th} = \sqrt{\frac{1}{2\pi(25+75)}\left[\int_0^{\pi}\left(\frac{\sqrt{2}\,120}{10}\sin\omega t\right)^2 d\omega t + \int_{2\pi}^{3\pi}\left(\frac{\sqrt{2}\,120\sin\omega t}{10}\right)^2 d\omega t + \ldots \text{upto 25 cycles}\right]}$$

$$= \sqrt{\frac{25\times 2\times(12)^2}{2\pi(100)}\int_0^{\pi}\sin^2\omega t\,d\omega t}$$

$$= \sqrt{\frac{25\times 2\times(12)^2}{2\pi\times 100\times 2}\left(\omega t - \frac{\sin 2\omega t}{2}\right)_0^{\pi}}$$

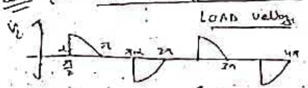
$$= \sqrt{\frac{25\times 2\times(12)^2\times\pi}{2\pi\times 100\times 2}} = 12\sqrt{\frac{25}{200}} = 4.24\,A$$

Q.



A.C Voltage Controller is used in Phase Control mode with firing angle of both thyristor equal to $\frac{\pi}{2}$.
Find the following:-
(i) Rms output load voltage ; (ii) Input power factor ; (iii) Rms value of thyristor current.

Sol (i) Rms value of Load voltage



$$V_L = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{\pi}(\sqrt{2}\,120\sin\omega t)^2\,d\omega t + \int_{\pi+\alpha}^{2\pi}(\sqrt{2}\,120\sin\omega t)^2\,d\omega t\right]}$$

$$V_L = \sqrt{\frac{2\times 2\times(120)^2}{2\pi}\int_{\alpha}^{\pi}\sin^2\omega t\,d\omega t} = \sqrt{\frac{2\times 2\times(120)^2}{2\pi\times 2}\int_{\alpha}^{\pi}(1-\cos 2\omega t)\,d\omega t}$$

$$= \sqrt{\frac{2\times 2\times(120)^2}{2\pi\times 2}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\pi/2}^{\pi}} = \sqrt{\frac{2\times 2\times(120)^2}{2\pi\times 2}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]}$$

$$= \frac{120}{\sqrt{2}} = 84.85\,V$$

Rms value of load current $= \frac{V_L(rms)}{R_L} = \frac{84.85}{10} = 8.48A$

Since Supply current = Load current = 8.48A

Input volt amp (App. Power) $= 120 \times 8.48 = 1017.6$ VA

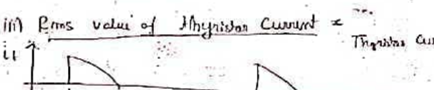
Power Consumed by Load $= I_L^2 \times R_L = (8.48)^2 \times 10 = 720$ W

ii) Input power factor = $\frac{\text{Power Consumed by Load (or True Power)}}{\text{Input Volt-ampere (or Apparent Power)}}$

$= \frac{720}{1017.6} = 0.707$ (Lag)

iii) Rms value of thyristor current =

Thyristor Current of T_1

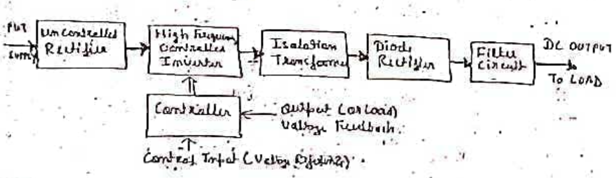


$$i_T = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi}\left(\frac{\sqrt{2}\,120}{10}\sin\omega t\right)^2 d\omega t} = \sqrt{\frac{2\times(12)^2}{2\pi}\int_{\frac{\pi}{2}}^{\pi}\sin^2\omega t\, d\omega t}$$

$$= \sqrt{\frac{2\times(12)^2}{2\pi\times 2}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi}} = 12\sqrt{\frac{1}{4}} = 6A$$

Switched Mode Power Supply (SMPS)

- SMPS is DC power Supply.
- Application of SMPS :- It provides Smooth & Control DC Supply to Sensitive electronic Circuitry.
- General Block diagram of SMPS :-



⇒ Uncontrolled Rectifier :- Convert AC into fixed value DC. It is usually a diode bridge rectifier. (87)

Note :- Output of uncontrolled rectifier (or even Controlled rectifier) can be feed to the Load directly (electronic circuitry). But output voltage of rectifier Contain high ripple Content. High ripple Content is due to low input AC frequency (50 or 60 Hz). Large & Bulky filters are required to filter out ripple from DC voltage. This increases the Size & weight of DC power Supply.

⇒ High Frequency Controlled Inverter :- Convert DC into high frequency AC (in KHz). Note :- Here DC is output of uncontrolled rectifier.

* Output of inverter is of very high frequency. High frequency AC reduces the Size & weight of Filter Circuitry.

* Voltage & frequency of inverter output can be Controlled by PWM technique.

⇒ Isolation Transformer :- Provides isolation between Supply and Load. It is usually Centre tapped at Secondary. Centre tap secondary is necessary for diode rectifier.

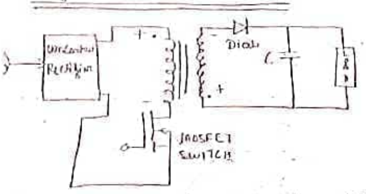
⇒ Diode Rectifier :- Convert AC (output of inverter) into DC.

⇒ Filter Circuitry :- Filter out ripples present in DC output of diode rectifier. Since ripple content is low due to high frequency AC input to diode rectifier. So Hence small filter is sufficient. Filter is usually LC.

- SMPS is Called So because power semiconductor device in inverter is Switched ON & Switch OFF at very high frequency to obtain high frequency AC output from inverter.
- Usually MOSFET is used in SMPS because MOSFET has high Switching frequency (high switching speed).
- Types of SMPS :

(i) Fly back SMPS ; (ii) Push-Pull SMPS ; (iii) Half Bridge Converter SMPS ; (iv) Full Bridge Converter SMPS.

Fly back Converter SMPS



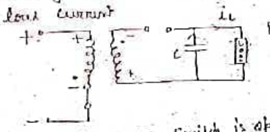
- This type of SMPS is quite different from general SMPS.
- This is because it does not use high frequency inverter in intermediate stage.

Sequence of operation of this type of SMPS.

(i) When MOSFET switch is close, (+)ve voltage is applied across primary of transformer

(ii) In transient stage, current through primary is varying. Therefore an emf is induced in secondary

(iii) When MOSFET switch is ~~after~~ closed, emf across secondary is such that it keeps the diode reverse biased. Therefore no current flows from secondary to load. During this period, capacitor current provides the load current



Equivalent Ckt when MOSFET switch is close.

(iv) ~~When~~ Now MOSFET switch is opened. Emf will get induced in secondary in reverse polarity to maintain the flux in transformer constant during opening of MOSFET switch. This reverse voltage across secondary forward bias the diode; Now secondary current flows into capacitor & load.

(v) Capacitor get charged by the energy stored in the transformer.

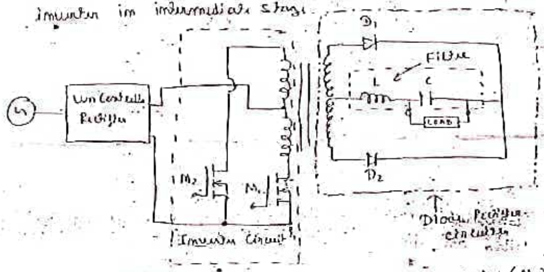
(vi) MOSFET switch is operated at very high frequency to achieve smooth DC at the load (output).

∴ Fly back Converter SMPS is used for low power application (below 500 W).

Push Pull Converter SMPS

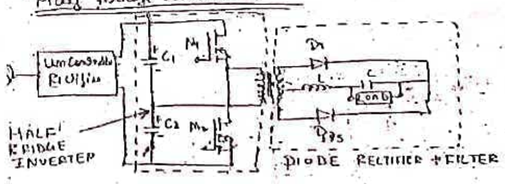
(83)

- It is normal (or general) SMPS. It uses high frequency inverter in intermediate stage.



- When MOSFET switch (M_1) is close & MOSFET switch (M_2) is open, then primary current flows through lower half of primary. This induces (+)ve polarity voltage across secondary.
- When M_1 is open & M_2 is closed, then primary current flows through upper half of primary. This induces (−)ve polarity voltage across secondary.
- MOSFET switches are operated at high frequency to achieve high frequency AC at the secondary. This is rectified by Diode Rectifier.
- When MOSFET switch is open, open circuit voltage across MOSFET is twice the supply voltage. Therefore, this type of SMPS is suitable for low voltage application only.

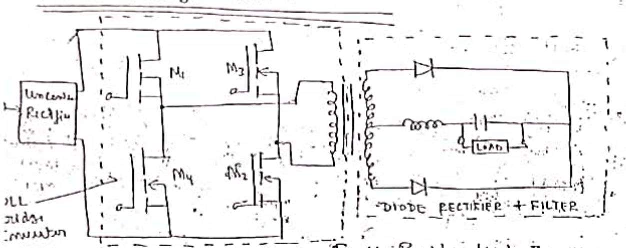
Half Bridge Converter SMPS



- This is normal (or general) SMPS.
- It uses Half Bridge Inverter as an intermediate stage.
- MOSFET switch M_1 is closed & M_2 is opened, to obtain (+)ve voltage at the secondary of transformer
- MOSFET switch M_2 is closed & M_1 is opened, to obtain (−)ve voltage at the secondary of transformer
- M_1 & M_2 (MOSFET switches) are operated at very high switching frequency, to obtain high frequency AC output at secondary.
- Secondary AC voltage (High frequency AC) is converted to DC by Diode Rectifier.

⇒ When any MOSFET switch is open, voltage across it is equal to supply voltage. Therefore it is preferred for High voltage & medium power application.

Full - Bridge Converter SMPS



- This type of SMPS uses Full Bridge High frequency, operated Inverter as an intermediate stage.
- MOSFET switches M_1 & M_2 are operated simultaneously. MOSFET switches M_3 & M_4 are operated simultaneously.
- (M_1 & M_2) are closed & (M_3 & M_4) are opened to obtain (+)ve voltage at secondary
- (M_1 & M_2) are opened & (M_3 & M_4) are closed to obtain (−)ve voltage at secondary.

(87)

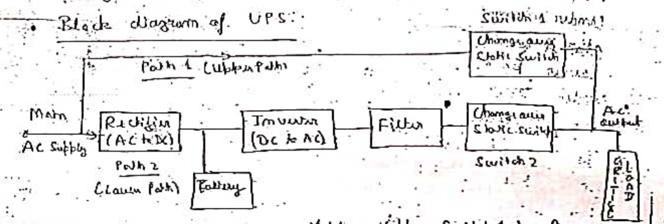
- MOSFET switches are operated at very high switching frequency, to obtain high frequency AC output at secondary.
- High Frequency AC output at the transformer secondary is converted to DC by Diode Rectifier.

⇒ Open circuit voltage across each MOSFET is equal to supply voltage.

⇒ This SMPS operates with minimum voltage & current stress on the MOSFETs. Therefore it is most suitable for high power application above 750 W.

Uninterruptible Power Supply (UPS)

- Output of UPS is AC. Therefore it is a AC power supply.
- It is used when continuity of AC power supply is essential.
- UPS provides continuous power to critical loads such as computer load, loads in Hospitals, industrial process control etc. to maintain the continuity of supply in case of power outages (or Power cut).
- Block diagram of UPS:



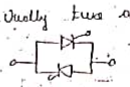
⇒ When Main Power is available, upper switch 1 is closed and lower switch 2 is open. And the power flows through upper path 1.

⇒ When Main Power is available, AC power also flows through path 2 (Rectifier to Battery). This charges the Battery of UPS.

⇒ When Main Power is not available, & automatic changeover switch 2 (in Lower path) gets ON.
Now power flows through lower path 2.

⇒ Since Battery is charged. This DC voltage of Battery is converted to AC by the inverter.
Inverter output is filtered (to filter out Harmonics) and is given to the Load.

Note :- Changeover Static Switch is thyristor based.
It is actually two antiparallel connected thyristor.



← Change over static switch.
(It can conduct current in both directions due to antiparallel thyristor connection.)

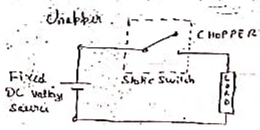
- What is on line UPS?

- UPS discussed so far, is OFF line UPS.
- In on line UPS, under normal condition (when Main power supply is available) Changeover switch 1 is OFF (upper path is not conducting).
  Under normal condition (when Main power supply is available) changeover Switch 2 is ON. And power flows through lower path 2.
- Under every condition (Main available / or not available), power flows to the load through inverter.
- In case of fault in inverter, automatic changeover switches are operated. Upper change over switch 1 is turned ON.
  Lower change over switch 2 is turned OFF.
  And power flows flow from directly AC Mains to load directly through upper path 1.
- In OFF line UPS, there is a momentary interruption in supply to the load during operation of changeover switch.
- In ON line UPS, there is no interruption in supply to the load during power outage (or Power cut).

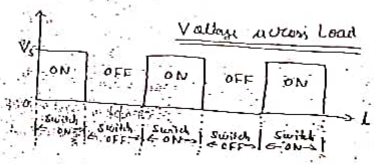
UNIT-3

CHOPPER

- In this unit, we will study about DC to DC Converter.
- Chopper is a high speed static switch, which is used in a DC to DC Converter.
- DC to DC Converter, converts a fixed voltage DC source into a variable voltage DC source. And this variable DC voltage is applied to the load.
- Variable DC voltage across load is obtained from fixed DC source, by placing a high speed switch between the Fixed DC source and the load. This high speed switch (Static Switch) is called Chopper.



- When the static switch is closed, DC supply voltage is applied to the load. When the switch is open, the load is disconnected from supply.



- By varying the ON time and OFF time of the switch, average output voltage across load can be controlled.

- Chopper (Static switch) is a power electronics device.
  When high power is involved, a thyristor or GTO is used as a chopper. Whereas in medium power application, a MOSFET or a IGBT is the most suitable. For low power application, Power BJT can be used as chopper.

- Application of chopper circuit (or DC to DC Converter) :-

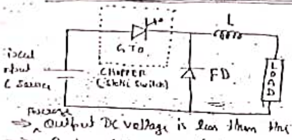
(i). DC to DC Converter is widely used for traction motor control in electric automobiles (i.e. speed control & braking of DC motor).
(ii) It is also used in trolley cars, marine hoists, forklift trucks and mine haulers.

- **Advantages of chopper circuit in controlling DC motor.**

(i) Regenerative braking of DC motor is easily achievable using chopper circuit. This Regenerative braking results in energy saving for transportation system with frequent stops.

(ii) It provides smooth acceleration control, high efficiency and fast dynamic response of DC motor.

Step Down Chopper Circuit



This is Step down chopper circuit. It is used to step down the average value of DC voltage available at the output load terminal.

⇒ Output DC voltage is less than the input DC voltage.

⇒ Role of inductor 'L' – To keep the load current continuous.

When there is no inductor and or even no load inductance, load current becomes zero instantly when chopper switch is turned OFF.

⇒ Role of Freewheeling Diode (FD) – When there is a inductance in the load (or intentionally a inductor is used to keep load current continuous), this Free wheeling Diode (FD) provide the path for energy stored in inductor (when chopper switch is OFF). Hence FD provides path for continuous inductive load current when chopper switch is OFF.

⇒ Here, GTO is used as a chopper switch.

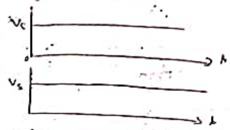
Case 1 :- When Chopper Switch (GTO) remain ON continuously all the time.

(i) No chopping action takes place. Average output load voltage is fixed (equal to input supply voltage).

(ii) Inductor 'L' ~~remains short~~ acts as short circuit in steady state, because supply is DC and chopper is ON all the time.

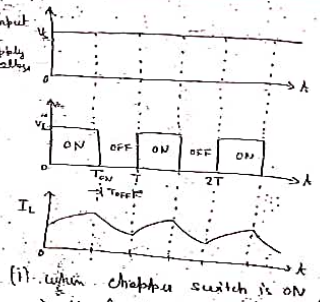
(iii) Freewheeling Diode (FD) will not conduct at any point of time. Because chopper switch is ON all the time. (91)

Path of Load Current is :- DC Source → GTO (chopper) → L → LOAD → DC Source.



(iv) Average Load voltage is V_s (= equal to Fixed input voltage).

Case II :- When Chopper Switch is operated ON and OFF repeatedly at very ~~ns~~ high speed.



(i) When chopper switch is ON, instantaneous load voltage is V_s (= Input Supply voltage). Load current rises exponentially due to inductance in circuit.

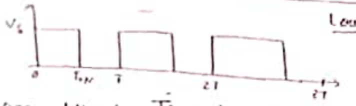
Path of Load current :- DC Supply → GTO (chopper) → L → Load → DC Supply.

(ii) When chopper switch is OFF, instantaneous load voltage is zero. Now load current flows through Freewheeling diode (FD).

Path of Load current :- Load → FD → L → Load.

This load current will decay exponentially with time due to inductance.

- Average value of output load voltage



Assumption :- Time period of output load voltage is constant (= T) throughout.

Average load voltage, $V_L = \frac{1}{T}\left[\int_0^{T_{ON}} V_s \, dt + \int_{T_{ON}}^{T} 0 \, dt\right]$

$$V_L = \frac{V_s}{T}[T_{ON} - 0] = V_s\left(\frac{T_{ON}}{T}\right)$$

where T_{ON} is the time duration for which the chopper is on.
T is the time period of load voltage.

⇒ Note $\left(\frac{1}{T}\right)$ is also called chopping frequency.

⇒ Duty cycle = $\frac{T_{ON}}{T}$

∴ Average Load voltage = Input supply voltage × Duty cycle.
Average value of load voltage can be controlled by varying the duty cycle.

Methods to control average load voltage, V_L

$$V_L = V_s\left(\frac{T_{ON}}{T}\right)$$

⇒ Method 1 :- Keep 'T' (chopping period) constant and vary 'T_{ON}' (ON time of chopper).
Hence variable load voltage, V_L is obtained by varying T_{ON}.
This method is called constant frequency (chopping frequency) operation or Pulse width Modulation Control or Time ratio control.

⇒ Method 2 :- Keep 'T_{ON}' constant and vary T.
Hence variable average load voltage, V_L is obtained by varying T.

This method is called Variable frequency (chopping frequency) operation or Frequency Modulation. (97)

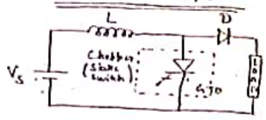
⇒ Constant frequency operation (Method 1) is most preferred method of load voltage control.

⇒ Disadvantage of Variable Frequency operation (Method 2).

(i) Since load time period of load voltage is variable, therefore this method of control would generate harmonics at unpredictable frequencies. Hence filter design would be difficult.

(ii) For fixed 'T_{ON}' and large 'T', T_{OFF} (OFF time of chopper) will be large. Large T_{OFF} can make the load current discontinuous, which is undesirable.

Step Up Chopper



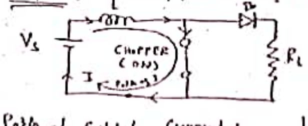
⇐ This is step up chopper. It is used to step up the average value of DC voltage available at the output load terminal.

⇒ Average output DC voltage is more than the input DC voltage.

- Working of Step Up chopper

For simplicity assume load to be resistive.

Step 1 :- Chopper switch is turned ON



- Path of supply current :-
$V_s \rightarrow L \rightarrow$ chopper switch $\rightarrow V_s$

- When chopper switch is turned ON (or closed), current supplied by the source will flow through chopper switch only. This is because current follows least resistance path. Load R_L will be bypassed by the chopper because chopper acts as short circuit when it is ON.

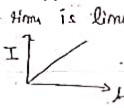
- Voltage across $L = V_s$ (when chopper is ON)

emf induced across $L = L\frac{di}{dt} = V_s$

Since V_s & L are constant; therefore $\frac{di}{dt}$ should be constant.

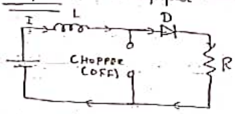
Hence variation of current w.r.t time is linear.

Current is increasing linearly.



- Note :- Chopper switch is remain ON for same time.

Step 2 :- Chopper switch is turned OFF (opened)



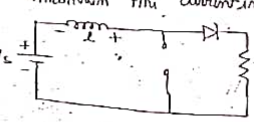
- Path of supply current :-

$V_s \rightarrow L \rightarrow D \rightarrow R_L \rightarrow V_s$

- When chopper was ON, internal resistance of the circuit was due to L only.

When chopper was turned OFF, resistance of the circuit is due to R & L.

- Now resistance of circuit has increased. So current should decrease.
- But due to presence of inductor 'L', current cannot decrease instantly. So an emf will get induced in L to maintain the current in circuit.



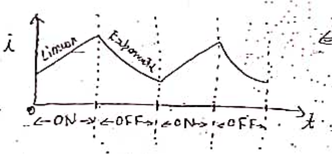
- Voltage across load, R_L will be

$V_L = V_s + e$

$V_L = V_s + L\frac{di}{dt}$

- Note that load voltage is more than V_s.
- Load voltage get stepped up.
- Note current will decrease exponentially due to R & L.

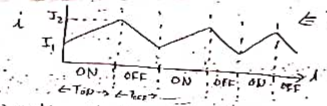
93



- ⇐ Inductor current or current supplied by source

- Average stepped up output (or load) voltage :-

Assumption :- Assume linear current variation



⇐ Inductor current (Linear variation)

(i) When chopper is ON for T_{ON} period

Voltage across $L = V_s = L\frac{\Delta I}{\Delta t}$ (Because current is varying linearly)

$$V_s = L\frac{(I_2 - I_1)}{(T_{ON} - 0)} = L\frac{(I_2 - I_1)}{T_{ON}} \rightarrow \text{I}$$

(ii) When chopper is OFF for T_{OFF} period

$$\text{Load voltage} = V_s + L\frac{\Delta I}{\Delta t}$$

$$V_L = V_s + L\frac{(I_2 - I_1)}{-(T_{OFF} - 0)}$$

Put value of $[L(I_2 - I_1)]$ from eq. I

$$V_L = V_s + \frac{V_s T_{ON}}{T_{OFF}}$$

$$= V_s\left(1 + \frac{T_{ON}}{T_{OFF}}\right)$$

$$= V_s\left(\frac{T_{OFF} + T_{ON}}{T_{OFF}}\right)$$

$$V_L = V_s\left(\frac{T}{T - T_{ON}}\right) = V_s\left(\frac{1}{1 - \delta}\right)$$

where $T = T_{ON} + T_{OFF}$ & δ is duty cycle

$\delta = \frac{T_{ON}}{T}$ $\Rightarrow$ Duty cycle.

Note :- If a large capacitor is connected across load, the output voltage will be continuous. Otherwise output voltage will be discontinuous i.e. becomes zero (during T_{ON} period).

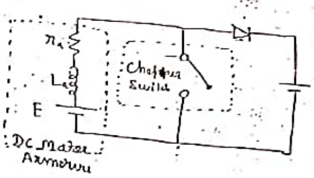
- Application of Step up Chopper

(i) Principle of Step up chopper can be employed for the regenerative braking of DC motor even at lower speed.

$\Rightarrow$ Normally regenerative braking in DC motor takes place when back emf (E) exceeds the supply voltage (V_s)

$\Rightarrow$ $E > V_s$ takes place in hoist application when load is decending down (during this, speed becomes very high).

$\Rightarrow$ $E > V_s$ for regenerative braking can be achieved at low motor speed with the help of Step up chopper.

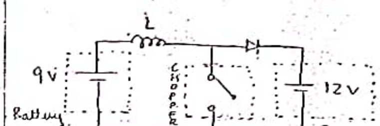


By Chopping action $E > V_s$ can be achieved. Hence regenerative braking is possible at any speed.

ii) Battery of higher emf can be charged from a source of lower voltage.

For ex. 12 V battery can be charged from a source of 9 V.

(94)

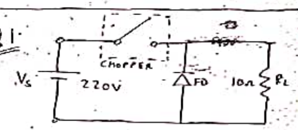


- By chopping action, power can flow from 9V battery to 12 battery

$$\Rightarrow \underbrace{9\,V}_{emf} + L\frac{di}{dt} = \underbrace{12\,V}_{emf}$$

$\Rightarrow$ without chopping action, power cannot flow from 9V battery to 12V battery.

Q1.



For a step down chopper circuit,

Input DC Supply, $V_s = 220V$

Load Resistance, $R_L = 10\Omega$

Chopping frequency is 1 KHz. Duty cycle is 50% (or 0.5)

On state voltage drop across Chopper switch is 2V.

Find the following

(i) Average output (load) voltage

(ii) Rms output (load) voltage.

(iii) Rms value of fundamental component of output harmonic voltage.

Sol (i)

Load (output) Voltage.

$T = \frac{1}{\text{Chopping frequency}}$; $T_{ON} = T \times$ Duty cycle

$$\text{Average output voltage} = \frac{1}{T}\left[\int_0^{T_{ON}} V_L\,dt + \int_{T_{ON}}^{T} V_L\,dt\right]$$

$$= \frac{1}{T}\left[\int_0^{T_{ON}} (V_s - \text{on state voltage drop across chopper})\,dt + \int_{T_{ON}}^{T} (0)\,dt\right]$$

Average output voltage $= \frac{1}{T}\left[\int_0^{T_{ON}} (220-2)\, dt\right]$

$= \frac{218}{T}\left[T_{ON} - 0\right] = 218 \times \frac{T_{ON}}{T}$

$= 218 \times$ duty cycle $= 218 \times 0.5 = 109\ V$

i) Rms value of output voltage $= \sqrt{\frac{1}{T}\left[\int_0^{T_{ON}} (220-2)^2\, d\omega t + \int_{T_{ON}}^{T} (0)^2\, d\omega t\right]}$

$= \sqrt{\frac{(218)^2}{T}(T_{ON}-0)} = 218\sqrt{\frac{T_{ON}}{T}}$

$= 218\sqrt{\text{Duty cycle}} = 218\sqrt{0.5} = 154.149\ V$

(ii)

ii) 

Load voltage

Since load voltage is non sinusoidal, it contains harmonics.

Magnitude of fundamental as well as harmonic can be found from its fourier analysis.

$V_L = a_0 + \sum_{m=1}^{\infty} (a_m \cos m\omega t + b_m \sin m\omega t)$

$a_0 =$ average value of $V_L = V_o \times$ duty cycle.

$a_m = \frac{1}{T/2}\int_0^T V_L \cos m\omega t\, d\omega t = \frac{2}{T}\int_0^{T_{ON}} V_o \cos m\omega t\, d\omega t$

$= \frac{2V_o}{T}\left[\frac{\sin m\omega t}{m}\right]_0^{T_{ON}} = \frac{2V_o}{mT}\sin m T_{ON}$

$= \frac{2V_o}{mT}\sin m\delta T$ (where δ is duty cycle $= \frac{T_{ON}}{T}$)

$b_m = \frac{1}{T/2}\int_0^T V_L \sin m\omega t\, d\omega t = \frac{2}{T}\int_0^{T_{ON}} V_o \sin m\omega t\, d\omega t$

$= \frac{2V_o}{T}\left[\frac{-\cos m\omega t}{m}\right]_0^{T_{ON}} = \frac{2V_o}{mT}\left[-\cos m T_{ON} + 1\right]$

$= \frac{2V_o}{mT}\left[1 - \cos m\delta T\right]$ 188

For fundamental component of output voltage, put $m = 1$ (95)

$\therefore$ Fundamental component $= a_1 \cos \omega t + b_1 \sin \omega t$

$= \sqrt{a_1^2 + b_1^2}\ \sin(\omega t + \phi)$

where $\phi = \tan^{-1}\left(\frac{a_1}{b_1}\right)$

Rms of Fundamental $= \frac{\sqrt{a_1^2 + b_1^2}}{\sqrt{2}}$ ← peak value

$= \frac{1}{\sqrt{2}}\sqrt{\left(\frac{2V_o}{mT}\right)^2 \sin^2 m\delta T + \left(\frac{2V_o}{mT}\right)^2 (1 - \cos m\delta T)^2}$

$= \frac{2V_o}{\sqrt{2}\, mT}\sqrt{\sin^2 m\delta T + 1 + \cos^2 m\delta T - 2\cos m\delta T}$

$= \frac{2V_o}{\sqrt{2}\, mT}\sqrt{1 + 1 - 2\cos m\delta T} = \frac{2V_o}{\sqrt{2}\, mT}\sqrt{2 - 2\cos m\delta T}$

$= \frac{2V_o}{\sqrt{2}\, mT}\sqrt{2 \times 2\sin^2\left(\frac{m\delta T}{2}\right)} = \frac{2V_o}{\sqrt{2}\, mT} \cdot 2 \cdot \sin\left(\frac{m\delta T}{2}\right)$

Put $T = 2\pi$ (Time period of fundamental sine component)

$= \frac{2(220-2)}{\sqrt{2}(1)\,2\pi} \cdot 2 \cdot \sin\left(\frac{(1)\,2\pi}{2 \times 2}\right)$

$= \frac{2 \times 218}{\sqrt{2}\,\pi} = 98.149\ V$

$= 98.144\ V$ (← rms of fundamental output load voltage)

Q 

For a step down chopper, Input DC voltage, $V_s = 220V$

Load Resistance, $R_L = 5\Omega$

Inductance, $L = 7.5mH$

Chopping frequency is 1kHz. Duty cycle is 50% (0.5).

Calculate the following:

i) Maximum and Minimum value of load current.

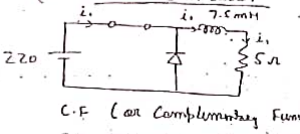
ii) Average and rms value of load current.

iii) Average and rms value of chopper current.

189

Sol $\Rightarrow$ Apply KVL in the chopper circuit when chopper switch

(i) is ON (or closed)



Loop $\equiv V_s \rightarrow L \Rightarrow R_L \Rightarrow V_s$

$$220 = i_1 R + L\frac{di_1}{dt}$$

$i_1 = C.F + P.I$ (By classical Approach)

C.F (or Complementary Function) $= A\, e^{-\frac{R}{L}t}$ (Transient response)

P.I. (Particular Integral) $= \frac{V_s}{R}$ (Steady state response).

$$i_1 = A\, e^{-\frac{R}{L}t} + \frac{V_s}{R}$$

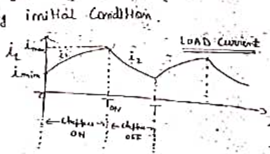
At $t=0$, $i_1 = i_{min}$ (See graph of load current).

Find arbitrary constant, A using initial condition.

$$i_{min} = A\, e^{-\frac{R}{L}(0)} + \frac{V_s}{R}$$

$$\therefore A = i_{min} - \frac{V_s}{R}$$

$$\therefore i_1 = \left(i_{min} - \frac{V_s}{R}\right) e^{-\frac{R}{L}t} + \frac{V_s}{R}$$



At $t = T_{ON}$, $i_1 = i_{max}$

$$i_{max} = \left(i_{min} - \frac{V_s}{R}\right) e^{-\frac{R}{L}T_{ON}} + \frac{V_s}{R}$$

$$T = \frac{1}{\text{Chopping frequency}} = \frac{1}{1000} = 0.001$$

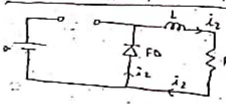
$$\frac{T_{ON}}{T} = \text{Duty cycle} = 0.5 \Rightarrow T_{ON} = 0.001 \times 0.5 = 0.0005$$

$$\therefore i_{max} = \left(i_{min} - \frac{220}{5}\right) e^{-\frac{5 \times 0.0005}{7.5 \times 10^{-3}}} + \frac{220}{5}$$

$$i_{max} = (i_{min} - 44)(0.7167) + 44$$

$$i_{max} = 12.47 + 0.7167\, i_{min} \rightarrow \text{I}$$

Now Apply KVL in chopper circuit when chopper switch is OFF (open)



Loop $\equiv L \rightarrow R \rightarrow FD \rightarrow L$

$$0 = i_2 R + L\frac{di_2}{dt'}$$

$$i_2 = B\, e^{-\frac{R}{L}t'}$$

(96)

Note :- $t' = t - T_{ON}$

$t' = 0^+$ at $t = T_{ON}$

At $t'=0$, $i_2 = i_{max}$.

Find arbitrary constant, B using initial condition.

$$i_{max} = B\, e^{-\frac{R}{L}(0)} \Rightarrow B = i_{max}$$

$$\therefore i_2 = i_{max}\, e^{-\frac{R}{L}t'}$$

At $t' = T_{OFF}$ (or $t = T$), $i_2 = i_{min}$

$$i_{min} = i_{max}\, e^{-\frac{R}{L}T_{OFF}} = i_{max}\, e^{-\frac{R}{L}(T - T_{ON})}$$

$$\therefore i_{min} = i_{max}\, e^{-\frac{5(0.001 - 0.0005)}{7.5 \times 10^{-3}}}$$

$$i_{min} = 0.7167\, i_{max} \rightarrow \text{II}$$

(iii)

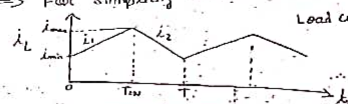
Solve equation I & II for i_{min} & i_{max}

$i_{max} = 25.605$ A $\rightarrow$ Maximum value of load current

$i_{min} = 18.35$ A $\rightarrow$ Minimum value of load current.

(ii) $\Rightarrow$ For simplicity assume linear variation of load current.

Load current (Linear variation)



$$\text{Average value of load current} = \frac{i_{max} + i_{min}}{2} = \frac{25.60 + 18.35}{2}$$

$$= 21.97 \text{ A}$$

$\Rightarrow$ Linear equation of i_1 :- Find eq. of line using 2 point form.

Two points are $(0, i_{min})$ & (T_{ON}, i_{max}), with $x_1 = 0$, $y_1 = i_{min}$, $x_2 = T_{ON}$, $y_2 = i_{max}$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{i_1 - y_1}{t - x_1} \Rightarrow i_1 = 14500t + 18.35$$

$\Rightarrow$ Linear equation of i_2 :- Two points are (T_{ON}, i_{max}) & (T, i_{min}).

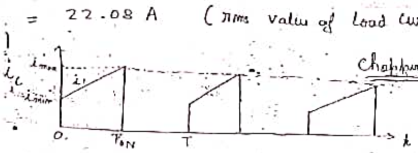
$$\therefore i_2 = -14500t + 32.85$$

Rms value of load current $= \sqrt{\frac{1}{T}\left[\int_0^{T_{ON}} (i_1)^2\, dt + \int_{T_{ON}}^{T} (i_2)^2\, dt\right]}$

$$= \sqrt{\frac{1}{0.001}\left[\int_0^{0.0005} (14500t + 18.35)^2\, dt + \int_{0.0005}^{0.001} (-14500t + 32.85)^2\, dt\right]}$$

$$= \sqrt{1000\left[\left(\frac{(14500)^2 t^3}{3} + (18.35)^2 t + \frac{532150\, t^2}{2}\right)_0^{0.0005} + \left(\frac{(14500)^2 t^3}{3} + (32.85)^2 t - \frac{952650\, t^2}{2}\right)_{0.0005}^{0.001}\right]}$$

$= 22.08$ A (rms value of load current).



Chopper Switch Current

Average value of chopper current $= \frac{1}{T}\left[\int_0^{T_{ON}} (i_1)\, dt + \int_{T_{ON}}^{T} (0)\, dt\right]$

$$= \frac{1}{0.001}\left[\int_0^{0.0005} (14500t + 18.35)\, dt + \int_{0.0005}^{0.001} (0)\, dt\right]$$

$$= 1000\left[\frac{14500\, t^2}{2} + 18.35t\right]_0^{0.0005} = 10.98 \text{ A}$$

Rms value of chopper current $= \sqrt{\frac{1}{T}\left[\int_0^{T_{ON}} (i_1)^2\, dt + \int_{T_{ON}}^{T} (0)^2\, dt\right]}$

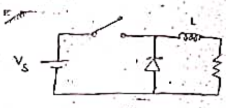
$$= \sqrt{\frac{1}{0.001}\left[\int_0^{0.0005} (14500t + 18.35)^2\, dt\right]}$$

$$= \sqrt{\frac{1}{0.001}\left[0.24382\right]} = 15.60 \text{ A}$$

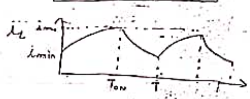
Q. Show that peak to peak ripple current in the load current is maximum when duty cycle is 50%.

Sol. Load current pulsates between i_{max} and i_{min}.

$\therefore$ Peak to peak ripple current $= (i_{max} - i_{min})$



Consider this step down chopper circuit (97)



Apply KVL in chopper circuit when chopper switch is ON.

$$V_s = i_1 R + L\frac{di_1}{dt}$$

After solving this eq. by classical approach, we get

$$i_1 = \left(i_{min} - \frac{V_s}{R}\right) e^{-\frac{R}{L}t} + \frac{V_s}{R}$$

At $t = T_{ON}$, $i_1 = i_{max} = \frac{V_s}{R} + \left(i_{min} - \frac{V_s}{R}\right) e^{-\frac{R}{L}T_{ON}}$

$$\therefore i_{max} = \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}T_{ON}}\right) + i_{min}\, e^{-\frac{R}{L}T_{ON}} \rightarrow \text{(I)}$$

Similarly, Apply KVL in circuit when chopper switch in OFF.

$$\therefore 0 = i_2 R + L\frac{di_2}{dt}$$

Solution of i_2 by classical Approach :- $i_2 = i_{max}\, e^{-\frac{R}{L}t'}$

At $t' = T_{OFF}$, $i_2 = i_{min} = i_{max}\, e^{-\frac{R}{L}T_{OFF}}$

$$\therefore i_{min} = i_{max}\, e^{-\frac{R}{L}(T - T_{ON})} \rightarrow \text{(II)}$$

Solve Equations I & II.

For simplicity let $e^{-\frac{R}{L}T} = x$ & $e^{-\frac{R}{L}T_{ON}} = y$.

$$\therefore i_{max} = \frac{V_s}{R}(1 - y) + i_{min}\, y \rightarrow \text{Eq (I)}$$

$$i_{min} = i_{max}\frac{x}{y} \rightarrow \text{Eq (II)}$$

Put value of i_{min} in I

$$\therefore i_{max} = \frac{V_s}{R}\frac{(1-y)}{(1-x)}$$

Put value of i_{max} in II

$$i_{min} = \frac{V_s}{R}\frac{(1-y)}{(1-x)}\frac{x}{y}$$

I = Peak to Peak ripple current $= i_{max} - i_{min} = \frac{V_s}{R}\frac{(1-y)}{(1-x)}\frac{(y-x)}{y}$

We know that Duty cycle, $\delta = \frac{T_{ON}}{T}$

& $y = e^{-\frac{R}{L}T_{ON}} = e^{-\frac{R}{L}T\delta} = x^{\delta}$

Put $y = x^{\delta}$ in ΔI equation

$$\Delta I = \frac{V_s}{R}\left[\frac{1-x^{\delta}}{1-x}\right]\left[\frac{x^{\delta}-x}{x^{\delta}}\right] = \frac{V_s}{R(1-x)}\left[1 - x x^{-\delta} - x^{\delta} + x\right]$$

Now put original value i.e $x = e^{-\frac{R}{L}T}$

$$\Delta I = \frac{V_s}{R(1-e^{-\frac{R}{L}T})}\left[1 - e^{-\frac{R}{L}T} e^{\frac{R}{L}T\delta} - e^{-\frac{R}{L}T\delta} + e^{-\frac{R}{L}T}\right]$$

For Maximum value of ΔI ∴ $\frac{d\Delta I}{d\delta} = 0$

$$\frac{d\Delta I}{d\delta} = \frac{-V_s \cdot e^{-\frac{R}{L}T}}{R(1-e^{-\frac{R}{L}T})} e^{\frac{R}{L}T\delta}\frac{RT}{L} - \frac{V_s}{R(1-e^{-\frac{R}{L}T})}\left(-\frac{RT}{L}\right)e^{-\frac{R}{L}T\delta} = 0$$

$$-e^{-\frac{R}{L}T} e^{\frac{R}{L}T\delta} + e^{-\frac{R}{L}T\delta} = 0$$

$$e^{-\frac{R}{L}T\delta} = e^{-\frac{R}{L}T(1-\delta)}$$

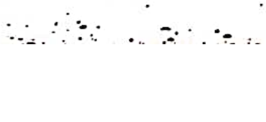
$$-\frac{R}{L}T\delta = -\frac{R}{L}T(1-\delta)$$

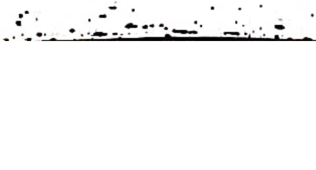
$$\delta = 0.5 = 50\%$$

∴ Peak to Peak ripple current will be maximum when duty cycle is 50%.

Hence Proved

1) A DC load of rating 110 V, 700 W is fed through a Step down chopper. Chopping frequency is 400 Hz. Input DC source voltage is 230 V. The load is operated at rated condition. Find the Critical value of inductance to be inserted in series with the load to maintain the continuity of load current.





⇒ Critical inductance is that value of inductance for which the output current falls to zero at $t = T$. (93)

Average value of output (or load current), $I_{av} = \frac{1}{T}\left[\int_0^{T_{ON}} i_1\,dt + \int_{T_{ON}}^{T} i_2\,dt\right]$

⇒ Eq. of i_1 using two point form : Two points are $(0,0)$ & (T_{ON}, I_{max})

$$\therefore\ i_1 = \frac{I_{max}}{T_{ON}} t$$

⇒ Eq. of i_2 using two point form : Two points are (T_{ON}, I_{max}) & $(T, 0)$

$$i_2 = -\frac{I_{max}}{T_{OFF}}(t - T_{ON}) + I_{max}$$

$$I_{AV} = \frac{1}{T}\left[\int_0^{T_{ON}}\left(\frac{I_{max}}{T_{ON}}\right)t\,dt + \int_{T_{ON}}^{T}\left(-\frac{I_{max}}{T_{OFF}}t + \frac{I_{max}}{T_{OFF}}T_{ON} + I_{max}\right)dt\right]$$

$$= \frac{1}{T}\left[\left(\frac{I_{max}}{T_{ON}}\frac{t^2}{2}\right)_0^{T_{ON}} + \left(-\frac{I_{max}}{T_{OFF}}\frac{t^2}{2} + \frac{I_{max}}{T_{OFF}}T_{ON}t + I_{max}t\right)_{T_{ON}}^{T}\right]$$

$$= \frac{1}{T}\left[\frac{I_{max}}{2T_{ON}}(T_{ON})^2 + \left(-\frac{I_{max}}{T_{OFF}}\frac{(T)^2}{2} + \frac{I_{max}}{T_{OFF}}T_{ON}T + I_{max}T\right) - \left(-\frac{I_{max}}{T_{OFF}}\frac{(T_{ON})^2}{2} + \frac{I_{max}}{T_{OFF}}T_{ON}T_{ON} + I_{max}T_{ON}\right)\right]$$

$$I_{AV} = \frac{I_{max}}{2} \quad \text{(After Simplification)}$$

⇒ Now, when chopper switch is ON ;-

$$V_s = V_o + L\frac{di}{dt} \quad \text{(KVL equation)}$$

If current variation is assume linear.

$$V_s = V_o + L\left(\frac{I_{max}-0}{T_{ON}-0}\right) \qquad \left(\Rightarrow \frac{di}{dt} = \frac{\Delta i}{\Delta t} = \frac{I_{max}-0}{T_{ON}-0}\right)$$

$$L = \frac{(V_s - V_o)T_{ON}}{I_{max}}$$

$$= \frac{V_s}{V_s}\frac{(V_s - V_o)T_{ON}.T}{2I_{av}\ T} \qquad (\text{Put } I_{max} = 2I_{av})$$

$$= \frac{(V_s - V_o)V_o}{2fV_sI_{av}}$$

195 $\left(\because \frac{1}{T} = f \ \&\ V_o = V_s\frac{T_{ON}}{T}\right)$

$$L = \frac{(V_s - V_0)\, V_0 \cdot V_0}{2 f V_s I_{av} \; V_0} \quad \left(\text{Multiply Numerator \& Denominator by } V_0\right)$$

$$L = \frac{(V_0)^2 (V_s - V_0)}{2 f V_s P_0} \quad \left(\because P_0 = V_0 I_{av}\right)$$

This is the general expression for critical inductance.

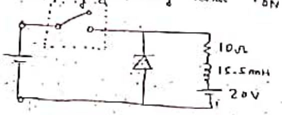
For $V_0 = 110\,V$; $P_0 = 700\,W$; $V_s = 220\,V$; $f = 400\,Hz$

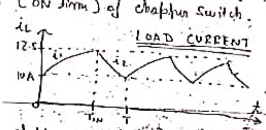
$$L = \frac{(220 - 110)(110)^2}{2 \times 400 \times 220 \times 700} = 0.01127\,H$$

$L = 11.27\,mH$

Q. A DC chopper feeds an RLE load. The source voltage of the chopper is 220 V. The load comprises a resistance of 10 Ω in series with an inductance of 15.5 mH and back emf of 20 V. The chopper is controlled to operate between 10 and 12.5 A in the load circuit. Determine the value of the chopping frequency and T_{ON} (ON time) of chopper switch.

Sol





⇒ Apply KVL in the circuit when chopper switch is ON.

$$220 = i_1 10 + (15.5 \times 10^{-3}) \frac{d i_1}{dt} + 20$$

Solving the above differential equation by classical method

$$i_1 = A e^{-\frac{10}{15.5 \times 10^{-3}} t} + \frac{220 - 20}{10} \quad (i_1 = C.F + P.I)$$

CF ⇒ Transient response

P.I ⇒ Steady state response

$$i_1 = A e^{-645.16 t} + 20$$

Find A using initial condition (At $t = 0$, $i_1 = 10A$)

$10 = A + 20$

$A = -10$

$$i_1 = -10 e^{-645.16 t} + 20$$

At $t = T_{ON}$, $i_1 = 12.5A$

$$12.5 = -10 \cdot e^{-645.16 T_{ON}} + 20$$

$e^{-645.16 T_{ON}} = 0.75$

Taking natural log both side

$-645.16\, T_{ON} = \ln(0.75)$

$-645.16\, T_{ON} = -0.2876$

$T_{ON} = 0.000445\, s = 445\,\mu s$

⇒ Apply KVL in the circuit when chopper switch is OFF

$$0 = i_2 10 + (15.5 \times 10^{-3}) \frac{d i_2}{dt} + 20$$

Solving above differential equation by classical method

$$i_2 = B e^{-\frac{10}{15.5 \times 10^{-3}} t'} + \left(-\frac{20}{10}\right)$$

Find A using initial condition (At $t' = 0$, $i_2 = 12.5\,A$)

$12.5 = B - 2$

$B = 14.5$

$$i_2 = 14.5\, e^{-645.16 t'} - 2$$

At $t' = T_{OFF}$, $i_2 = 10A$

$\therefore 10 = 14.5\, e^{-645.16 T_{OFF}} - 2$

$12 = 14.5\, e^{-645.16 T_{OFF}}$

$e^{-645.16 T_{OFF}} = 0.8275$

Taking natural log both side

$-645.16\, T_{OFF} = \ln(0.8275)$

$-645.16\, T_{OFF} = -0.18924$

$T_{OFF} = 0.000293\, s = 293\,\mu s$

Note :- This example illustrates the CURRENT LIMIT CONTROL of chopper.
In this control strategy, switching on and off of the chopper switch is so controlled that the load current varies between the specified minimum and maximum value.
In this method, load current is measured and fed to the comparator and accordingly chopper switch is turned on/off.

⇒ Chopping Frequency $= \dfrac{1}{T_{ON} + T_{OFF}} = \dfrac{1}{0.000445 + 0.000293} = 1354.47\,Hz$

T_{ON} (ON Time of chopper switch) $= 445\,\mu s$

Classification of Chopper Circuit

- Chopper circuit can be classify on the basis of "Quadrant operation" i.e. in how many quadrants of V–I plane it can operate

⇐ 4 Quadrants of V–I plane

V_0 = Load Voltage

I_0 = Load Current

- Chopper Circuit Can operate in single Quadrant, or two Quadrants or four Quadrants, by appropriate arrangement of Semiconductor devices.
- Classification of chopper circuit :-

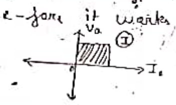
(i) Type A or class A chopper :- Works only in I Quadrant.
(ii) Type B or class B chopper :- Works only in II Quadrant.
(iii) Type C or class C chopper :- Works both in I & II Quadrants. (It is also known as Two Quadrant type A chopper).
(iv) Type D or class D chopper :- Works both in I & IV Quadrants, ~~(v) Type~~ (It is also known as Two Quadrant type B chopper).
(v) Type E or class E chopper :- Works in all the 4 Quadrants. (It is a 4 Quadrant chopper).

(1) Types A or Class A chopper

⇐ Type A chopper

- It is a normal step down Chopper (discussed earlier).

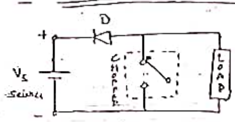
- Load current is always positive
- Load voltage can never be negative due to presence of free wheeling diode across load.
- Power always flows from Source to load.
- Since load voltage and load current both are positive. Therefore it works in (I) quadrant.



- If load is DC Motor, then only Forward Motoring is possible through Type A chopper.

(100)

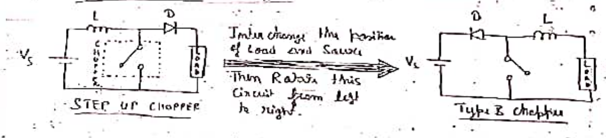
Type B or class B chopper



⇐ This is a type B chopper

- In this type of chopper circuit, load must be active i.e. it must have a source of emf.

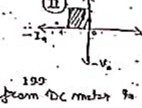
- Load can be a DC Motor or a battery. If load is battery, then an external inductance must be added in series with the battery.
- In this type of chopper circuit, power flows from load to the source.
- Type B chopper is similar to "Step up Chopper".



- Operation of Type B chopper is exactly ~~the same~~ similar to step up chopper.

(i) Chopper switch is operated (ON-OFF operation)
(ii) When combine voltage across load and inductor exceeds the supply voltage (V_s), diode (D) gets forward biased and power flows from load to source.

- Load current is always negative (Because power flows from load to source)
- Diode (D) prevents the flow of power from Source to load.
- Load voltage can never be negative.
- Since load current is negative & load voltage is positive, therefore it operates in (II) Quadrant.



- Type B chopper is used in the regenerative braking of DC Motor (when power flows from DC motor to source).

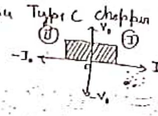
- Type B chopper can never be used alone. It should be used along with Type A chopper.

Type C chopper

- Type C chopper is the combination of Type A & Type B chopper

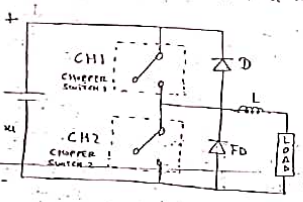
$\boxed{A + B = C}$

- Type A chopper works in Ist Quadrant.
  Type B chopper works in IInd Quadrant.
  Hence Type C chopper works both in Ist & IInd Quadrant.



- Type C chopper works in 2 Quadrants.

- Load voltage can never be negative.
- Load current can have both polarities (-ve & (+ve)

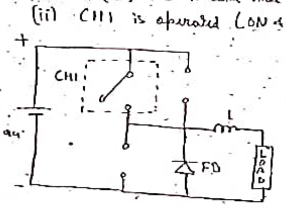


⟸ Type C chopper

- Note :- If load is DC Motor, then there is no need to add inductor (L) in series with load.
- If Load is battery, then L is required for IInd Quadrant operation.

- Type C chopper in Ist Quadrant

For Ist Quadrant operation :- (i) CH2 is always open ∴ Ignore diode (D) as it will not conduct.
(ii) CH1 is operated (ON & OFF operation)

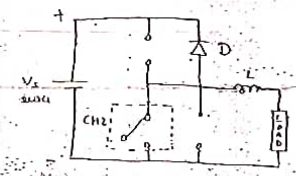


⟸ This circuit is similar to Type A chopper

- Operation will only be in Ist Quadrant.
- Power flows from Source to load.
- FD is free wheeling diode.
  FD will conduct when CH1 is OFF (or OPEN)

- Type C chopper in IInd Quadrant (101)

For IInd Quadrant operation :- (i) CH1 is alway open (or OFF)
Ignore free wheeling diode (FD) as it will not conduct.
(ii) CH2 is operated (ON & OFF operation)



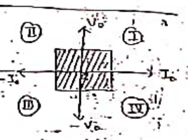
⟸ This circuit is similar to Type B chopper.

- Operation will only be in IInd Quadrant.
- Power flows from Load to Source (Load should be active like DC Motor).

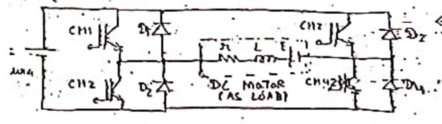
⟹ CH2 is operated (ON & OFF). When voltage across (L + Load) exceeds V_s, diode D becomes forward biased & power flows from load to source.

- If Load is DC Motor, both forward motoring and regenerative braking is possible through Type C chopper. This is because it works in two Quadrants (I & II).

Type E chopper



- Type E chopper works in all 4 Quadrants.
- It is a multi Quadrant chopper.
- If Load is DC Motor, then it will be called 4 Quadrant DC motor drive.
  Following operations of DC Motor are possible :-
  (i) Forward Motoring (in Ist Quadrant)
  (ii) Forward regenerative braking (in IInd Quadrant)
  (iii) Reverse Motoring (in IIIrd Quadrant)
  (iv) Reverse regenerative braking (in IVth Quadrant)

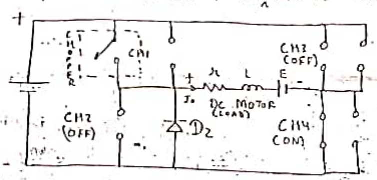


⟸ Type E chopper

- Here IGBTs are used as a chopper switch.

- Operation of Type E Chopper in Ist Quadrant (Forward Motoring)

(i) CH2 & CH3 are always open (OFF)
(ii) CH4 is always closed (ON)
(iii) CH1 is operated as chopper switch (ON & OFF operation).
(iv) D2 acts as free wheeling diode when CH1 is open.
(v) Ignore D_1, D_3, D_4 as they will not conduct.

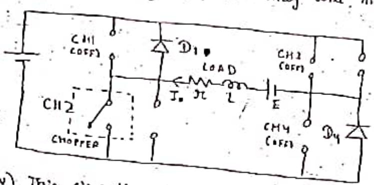


- When CH1 is on, path of load current is :- $V_s \rightarrow$ CH1 $\rightarrow$ LOAD $\rightarrow$ CH4 $\rightarrow V_s$.
- When CH1 is OFF, path of load current is :- Load $\rightarrow$ CH4 $\rightarrow$ D2 $\rightarrow$ Load (Free wheeling path)

(vi) This circuit acts as step down chopper in Ist Quadrant (Both load voltage & load current are positive).
(vii) When load is DC motor, it will operate in Forward Motoring mode.

- Operation of Type E chopper in IInd Quadrant (Forward Regenerative Braking).

(i) CH1, CH3, CH4 are always open (OFF)
(ii) CH2 is operated as chopper switch (ON & OFF operation).
(iii) Ignore D_2 & D_3 as they will not conduct.



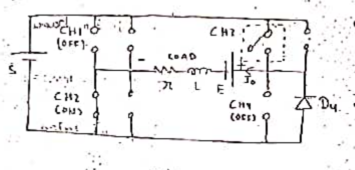
- When CH2 is on, path of load current is :- Load $\rightarrow$ CH2 $\rightarrow$ D4 $\rightarrow$ Load
- When CH2 is OFF, path of load current is :- Load $\rightarrow$ D1 $\rightarrow V_s \rightarrow D_4 \rightarrow$ Load.

v) This circuit acts as step up chopper. It work in IInd Quadrant (Load voltage is (+)ve & Load current is (-)ve.)
vi) When load is DC motor, it will operate in Forward regenerative braking mode. During regenerative braking, power flows from DC motor to source and motor will slow down due to braking torque.

(Note :- Torque = $K\phi I_a$ $\rightarrow$ in DC motor.
Since load current is negative therefore negative braking torque is produced.)

- Operation of Type E chopper in IIIrd Quadrant (Reverse Motoring)

(i) CH1 & CH4 are always open (OFF).
(ii) CH2 is always closed (ON)
(iii) CH3 is operated as chopper switch (ON & OFF operation).
(iv) D4 acts as free wheeling diode when CH3 is open.
(v) Ignore diode D_1, D_2 & D_3 as they will not conduct.



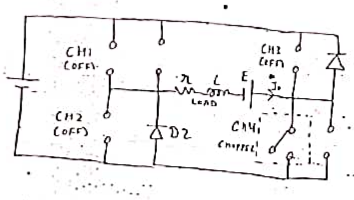
- When CH3 is ON, path of load current is :- $V_s \rightarrow$ CH3 $\rightarrow$ LOAD $\rightarrow$ CH2 $\rightarrow V_s$
- When CH3 is OFF, path of load current is :- Load $\rightarrow$ CH2 $\rightarrow$ D4 $\rightarrow$ Load (Free wheeling path)

(vi) This circuit acts as step down chopper in IIIrd Quadrant (Both load voltage & load current are negative).
(vii) When load is DC motor, it will operate in reverse motoring mode (Motor will run in opposite direction).

- Operation of Type E chopper in IVth Quadrant (Reverse Regenerative braking)

(i) CH1, CH2, CH3 are always open (OFF)
(ii) CH4 is operated as chopper switch (ON & OFF operation)
(iii) Ignore diodes D_1 & D_4 as they will not conduct.
iv) This circuit acts as step up chopper. It works in IVth Quadrant (Load voltage is (-)ve & load current is (+)ve.
v) When load is DC motor, it will operate in reverse regenerative braking mode, During this, motor will decelerate due to

braking (negative) torque and power flow from DC motor to source.



- When CH4 is on, path of load current is :- Load → CH4 → D2 → Load
- When CH4 is OFF, path of load current is :- Load → D3 → V_s → D2 → Load

Note :- Load in the case (of Type E chopper) is separately excited DC motor.

High Power Chopper Circuit

- Chopper is basically a high speed power semi conductor based static switch.
- In low power DC to DC Converter, IGBT or Power MOSFET or Power BJT is used as a chopper switch. When these devices are used as chopper switch, no extra forced commutation circuitry is required.
- In high power DC to DC Converter, thyristor is used as a chopper switch. This is because thyristor has higher voltage and current rating. Since input is DC, therefore thyristor requires extra forced commutation circuitry (for turn OFF).
- In this unit, we will study only two types of thyristor based chopper circuits.

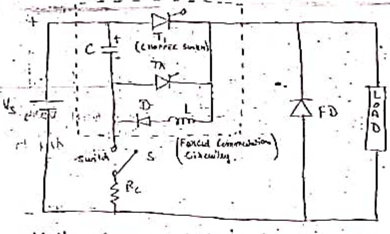
(i) Voltage Commutated chopper

(ii) Current Commutated chopper.

Voltage - Commutated Chopper

(103)

- In this type of chopper, Conducting thyristor is Commutated by the application of pulse of large reverse voltage across it. This reverse voltage is usually applied by switching a charged capacitor.
- This is also known as parallel voltage commutated chopper or impulse commutated chopper or classical chopper.



⟸ Voltage Commutated Chopper,

- This is a normal step down (Type A) chopper.
- Thyristor T_1 is used as chopper switch.
- This type of chopper has extra forced commutation circuitry.

- Voltage Commutated chopper is not a new topic. Its working has already been discussed in Commutation technique of a normal thyristor (Voltage Commutation).
- Switch 'S' is for charging the capacitor with correct polarity. Initially for the first time, S is closed and C charges through charging resistor (R_c) with upper plate (+)ve. Once the C is charged with correct polarity, switch (S) can be left open.
- T_1 is the main thyristor (or chopper switch). T_1 is turned OFF by turning ON auxiliary thyristor (TA).
- Various Steps in the working of Voltage Commutated Chopper.

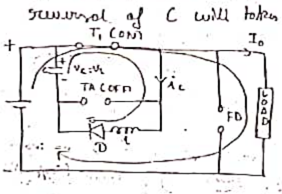
X- Initial condition of the Circuit :- C should be charged with upper plate positive. Magnitude of capacitor voltage is equal to supply voltage (V_s).

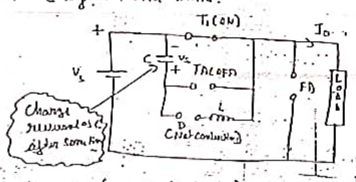
Assumption :- Load current is constant throughout the operation of chopper.

Step 1 :- Main thyristor T_1 (Chopper Switch) is turned ON.
with the turning ON of T_1 two Circuits get closed.

⇒ Circuit 1 is the main circuit Containing V_S, T_1 & Load

⇒ Circuit 2 is the resonant circuit Containing C, T_1, L & D.
Since circuit 2 is the resonant circuit therefore charge reversal of C will take place after some time.



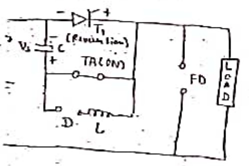


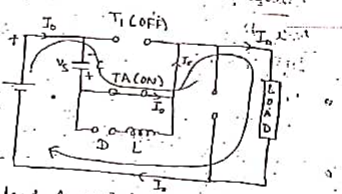
⇒ Note Capacitor current i_c in circuit 2 (resonant circuit) will vary as half sinusoidally and becomes zero after some time due to presence of unidirectional devices (like T_1 & D).

Step 2 :- Keep T_1 ON for some time. (depending on duty cycle of chopper).
Path of load current is :- $V_S \to T_1 \to Load \to V_S$.

Step 3 :- Turn ON TA (Auxiliary Thyristor) to turn OFF main thyristor T_1 (Chopper switch).

⇒ When TA is turned ON, Capacitor voltage will reverse bias T_1. This will turn OFF T_1 instantly.





⇒ After T_1 gets OFF, path of load current is :-
$V_c \to C \to TA \to Load \to V_S$
Since load current is assumed constant. Therefore Constant current is flowing through Capacitor (Path of load current is through C).

⇒ $i_C = C \frac{dV_c}{dt}$ (Basic equation of capacitor) (104)

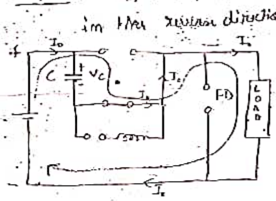
Since i_C is constant & C is also constant. Therefore $\frac{dV_c}{dt}$ should also be constant.

$\frac{dV_c}{dt}$ (i.e slope of V_c) is constant if V_c is varying linearly.

Therefore Capacitor will charge linearly towards reverse polarity

⇒ Note :- At the instant when T1 got OFF. Voltage across load was $2V_S$ (Apply KVL in the loop ≡ $V_S \to C \to TA \to Load \to V_S$)

Step 4 :- In the previous step (step 3), C was getting charged in the reverse direction.

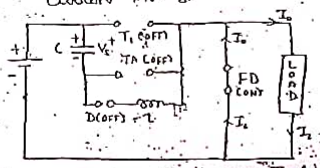


⇒ When $V_C = V_S$, Voltage across load will be zero
(verify by applying KVL in the loop
Loop ≡ $V_s \to C \to TA \to Load \to V_S$)

⇒ When V_C becomes slightly more than V_S, load voltage becomes negative.

⇒ As soon as Load voltage becomes slightly negative, Free wheeling diode (FD) gets forward biased & Starts conducting.

⇒ When FD is conducting, path of load current is :-
Load → FD → Load
Current through TA becomes zero & it gets off.

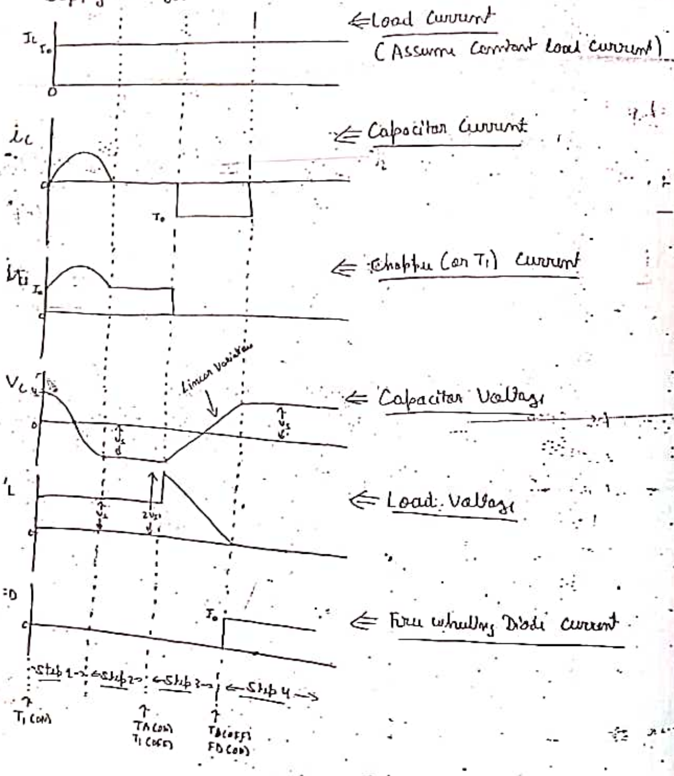


- New commutation process is complete. Capacitor is again charged with correct polarity for next cycle.

Step 5 :- Again Main thyristor T_1 (Chopper Switch) is turned ON. T_1 is turned off by turning ON TA. And the whole process repeats.

- **Drawback of Voltage Commutated Chopper :-**

Load voltage at once rises to twice of supply voltage ($2V_s$) at the instant when main thyristor (T_1) gets OFF.
Since free wheeling diode is connected in antiparallel with load. Therefore free wheeling diode is subjected to take the supply voltage.

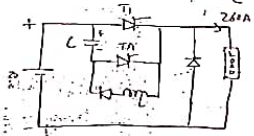


Designing Voltage Commutated Chopper (105)

- Designing of Voltage commutated chopper deals with the determination of value of commutating components C & L.
- Consider an example :-

Q. Input Source Voltage = 220 V
Average Load current = 260 A (Assume Constant)
Turn OFF time for main thyristor (Chopper switch) = 18 μs
Peak current through main thyristor is limited to 1.8 times the constant load current.
Design Voltage Commutated chopper which will drive the given load and will satisfy the above condition.

Sol. Main thyristor should remain reverse biased for atleast 18 μs for successful turn OFF.
Taking a safety factor of 2, thyristor should remain reverse bias for (18×2) μs (i.e. 36 μs).



- When TA is fired, T_1 (main thyristor) gets OFF instantly due to reverse voltage of capacitor.
- When TA is fired, C is charged with voltage equal to 220V.
- When only TA is conducting, constant load current is flowing through capacitor. This constant capacitor current discharges C linearly (see waveform of V_c).
- For successful commutation, C should discharge from 220V to 0V in 36 μsec.

$$i_c = C\frac{dv}{dt} \quad \text{(Basic eq. of C)}$$

$$i_c = \text{Constant load current} = 260A$$

$$\frac{dv}{dt} = \frac{\Delta V}{\Delta t} = \frac{0-(-220)}{36\ \mu sec} \quad \left(\text{Because C is charging/discharging linearly}\right)$$

$$\therefore C = \frac{260 \times 36 \times 10^{-6}}{220} = 42.54\ \mu F.$$

When T_1 (Main thyristor or chopper switch) is turned ON, current flowing through T_1 is sum of Load current and Capacitor current (due to resonant circuit)

$$i_{T_1} = i_L + i_c$$

- Since i_c is due to resonant circuit formed by $T_1 \to L \to D \to C \to T_1$

Peak Resonant current $= \underset{(\text{Peak value})}{i_c} = V_s\sqrt{\frac{C}{L}}$ (See concept of resonant circuit on Page No. 26)

It is given that T_1 can handle only 1.8 times the load current.

$$\therefore \text{Peak } i_{T_1} = 1.8 \times 260 = \underset{\overset{\uparrow}{i_L}}{260} + \underset{\overset{\uparrow}{\text{Peak } i_c}}{V_s\sqrt{\frac{C}{L}}}$$

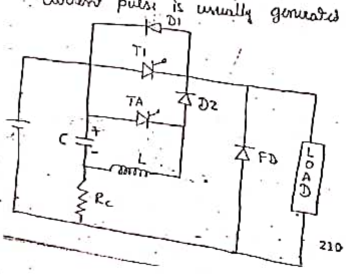
$$\therefore 0.8 \times 260 = 220\sqrt{\frac{42.54 \times 10^{-6}}{L}}$$

$$\therefore L = \frac{42.54 \times 10^{-6}}{0.8938} = 47.59\ \mu H.$$

$$\therefore \boxed{C = 42.54\ \mu F \ \& \ L = 47.59\ \mu H}$$ ⟸ Designed parameter of Voltage Commutated Chopper.

Current Commutated Chopper

- In this type of Chopper, Conducting thyristor (or Chopper switch) is Commutated by application of external current pulse (of magnitude greater than load current) in the reverse direction of conducting thyristor.
- Current pulse is usually generated by an initially charged capacitor.



⟸ Current Commutated Chopper

- It is a normal step down chopper (Type A) with extra force commutation circuitry.

- T1 is the main thyristor and it is acting as the chopper switch. (106)
- Main thyristor T_1 is turned OFF by firing the auxiliary thyristor TA.

⟹ Note :- It is not a new topic. It is a step down chopper with forced commutation circuitry for the thyristor based chopper switch. Current Commutation used here has already been studied earlier in the Commutation techniques of thyristor.

- R_C is the charging resistor for charging the capacitor with the correct polarity.

Note :- Rc is so large that it can be treated as open circuit during the commutation process.

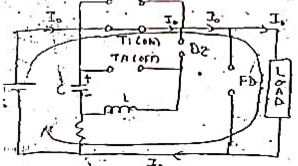
- Steps involve in the operation of Current Commutated Chopper.

⟹ Initial Condition of circuit :- Capacitor is charged with Supply voltage (V_s) with upper plate (+)ve.

⟹ Assumption :- Load current is Constant throughout the chopper operation.

Step 1 :- Main thyristor T_1 is turned ON and it will remain ON for some time (depending on duty cycle of Chopper).

Path of load current will be :- $V_s \to T_1 \to \text{Load} \to V_s$

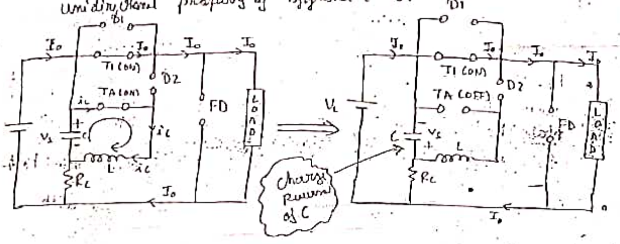


Step 2 :- Main thyristor T_1 is turned OFF by turning ON TA (auxiliary thyristor).

⇒ With the turning ON of TA, loop containing [C → TA → L → C] gets closed. This forms a resonant circuit.

⇒ Therefore charge reversal of Capacitor will take place after some time.

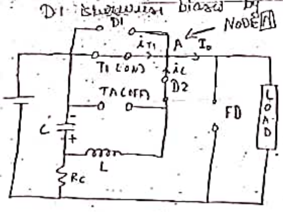
⇒ In this resonant circuit, Capacitor current will vary in half sinusoidally manner. It will become zero due to unidirectional property of Thyristor (TA).



Step.3 :- Due to charge reversal of C (in the Step 2), now Capacitor current will flow through the path:-

$C \to L \to D2 \to Load \to V_s \to C$ [Resonant Circuit]

⇒ Note :- Capacitor current will not flow through D1 because D1 ~~is reverse biased by on state voltage drop of T1~~



Apply KCL at node A

$$i_{T1} + i_c = I_o$$

T1 current, Capacitor current, Constant Load current

⇒ I_o is Constant Load current

⇒ i_c is current of resonant circuit, formed by L & C.

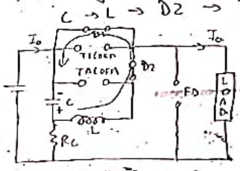
⇒ Since i_c (resonant circuit current) is increasing, therefore i_{T1} (T1 current) should decrease to maintain I_o Constant.

∴ At $i_c = I_o$, i_{T1} becomes zero and it gets turn OFF.

Step.4 :- Since main thyristor T1 is OFF, now diode D1 is no longer a reverse biased (by on state voltage drop of T1). Now D1 will conduct. (107)

⇒ Path of excess Capacitor current (more than I_o) is

$C \to L \to D2 \to D1 \to C$



⇒ Conduction of D1 is necessary because on state voltage drop of D1 will reverse bias T1

⇒ Reverse bias of T1 is necessary for successful commutation of T1.

⇒ Diode D1 will stop conducting when i_c becomes equal to I_o.

Step 5 :- When D1 gets OFF, path of Capacitor Current is

$C \to L \to D2 \to Load \to V_s \to C$

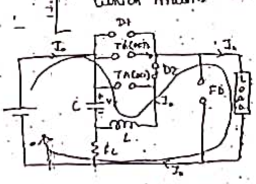
⇒ Capacitor current = Load current (I_o) (I_o is assumed Constant)

Therefore Capacitor current is Constant.

⇒ Due to Constant Capacitor current, Capacitor voltage will vary linearly.

[$i_c = C\frac{dv_c}{dt}$ ⇒ If i_c is constant then $\frac{dV_c}{dt}$ should be constant.

which means linear variation of V_c.]



⇒ Note :- Charge reversal of C has already been taken place. (Due to resonant circuit)

⇒ Since constant current is flowing through L, so there will be no emf induced across L.

⇒ Load Voltage = $V_s - V_c$ (verify by applying KVL)

Supply Voltage, Capacitor Voltage

∴ When V_c becomes equal to V_s, load voltage becomes zero.

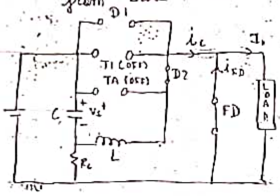
Step 6 :- when V_c becomes more than V_c , Load Voltage becomes negative

⇒ Negative load voltage ~~reverse~~ forward bias the free wheeling diode (FD) connected in antiparallel with Load.

⇒ FD starts conducting.

⇒ As soon as FD starts conducting, FD does not carries full load current $[I_o]$ due to the inductance (L) in the circuit.

⇒ Inductor current (or capacitor current) will drop from I_o to zero. Simultaneously FD current increases from zero to I_o.



⇒ When FD starts carrying full load current (I_o), commutation process completes

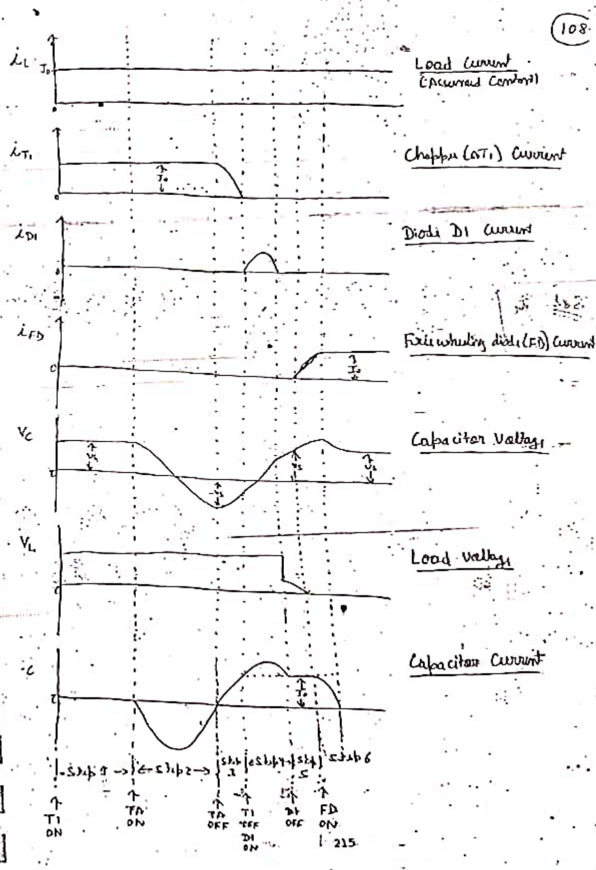
⇒ Since capacitor is overcharged (V_c is more than V_s), & C will discharge through R_c.

After some time, V_c will become equal to V_s.

⇒ C is ready for next commutation cycle (i.e. it is charged with correct polarity)

Step 7 :- Again T_1 will be fired after some times depending on duty cycle of chopper.

T1 is turned OFF by turning ON TA, and the whole process repeats.



Designing Current Commutated Chopper

- Designing of current commutated chopper deals with the determination of value of commutating components C & L.

Q. Design a current commutated chopper, if :

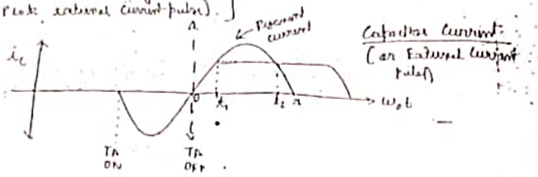
Input DC voltage source = 230 V.

Constant load current = 200 A.

Turn off time of main thyristor, T_1 = 30 µs.

[Peak commutating current (Peak external current pulse).] is twice of Load current.

Sol



Resonant current = $V_s \sqrt{\frac{C}{L}} \sin \omega_0 t$ (External current pulse)

Peak resonant current = $2 \times I_o = 2 \times 200$ (Given in Question)

$$\therefore 400 = V_s \sqrt{\frac{C}{L}}$$

$$400 = 230 \sqrt{\frac{C}{L}}$$

$$\therefore C = 3.0245\, L \rightarrow \text{(I)}$$

⇒ At $\omega_0 t_1$, resonant current = I_o = 200 A

$$\therefore 200 = V_s \sqrt{\frac{C}{L}} \sin \omega_0 t_1$$

$$200 = 400 \sin \omega_0 t_1$$

$$\omega_0 t_1 = 30^\circ$$

⇒ At $\omega_0 t_2$, resonant current is again equal to I_o (= 200 A)

$$\therefore \cancel{\omega_0 t} \; \pi - \omega_0 t_2 = 30^\circ$$

$$\therefore \omega_0 (t_2 - t_1) = 180^\circ - (30^\circ + 30^\circ) = 120^\circ = 2.094 \text{ (in radian)}$$

(109)

⇒ From t_1 to t_2, diode D_1 (antiparallel with main thyristor, T_1) conducts.

⇒ Conduction of D1 reverse bias T1 (Reverse bias by on state voltage drop across D1)

⇒ For successful commutation of main thyristor T1, it should remain reverse biased for atleast 30 µs. Taking safety factor of 2, T1 should remain reverse bias for 2×30 (= 60 µs.)

$$\therefore \omega_0 (t_1 - t_2) = 2.094$$

$$\omega_0 \times 60 \times 10^{-6} = 2.094$$

$$\omega_0 = 34900$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 34900$$

$$\therefore LC = \frac{1}{121801 \times 10^4} \rightarrow \text{(II)}$$

Solving eq I & II, we get

$$\therefore L = \sqrt{\frac{10^{-4}}{3.0245 \times 121801}} = 16.47\ \mu H.$$

$$C = 3.0245 \times 16.47 \times 10^{-6} = 49.81\ \mu F$$

$$\therefore \boxed{L = 16.47\ \mu H \ \& \ C = 49.81\ \mu F}$$ ⇐ Design parameter of current commutated chopper.