Probability Basics Assignment

1. **A die is rolled. What is the probability of getting:**

   a) An even number

   (b) A number greater than 4

Answer: **Concept**

A standard die has 6 faces, numbered from 1 to 6. Each face has equal probability of appearing, so:

$$\text{Probability of any outcome} = \frac{1}{6}$$

Part (a): Probability of an Even Number

Even numbers on a die = 2, 4, 6 → Total favourable outcomes = 3 → Total possible outcomes = 6

$$\text{Probability} = \frac{3}{6} = 0.5 = 50\%$$

Part (b): Probability of a Number Greater Than 4

Numbers greater than 4 = 5, 6 → Total favourable outcomes = 2 → Total possible outcomes = 6

$$\text{Probability} = \frac{2}{6} = 0.3333 = 33.33\%$$

(Outcome)	(Even)	(Greater Than 4)
1	No	No
2	Yes	No
3	No	No
4	Yes	No
5	No	Yes
6	Yes	Yes
Probability (Fraction)	(3/6)	(2/6)
Probability (Decimal)	0.5	0.333333333
Probability (%)	50%	33.33%

Concept Overview

A standard die has **6 faces**, numbered from **1 to 6**. Each face has an **equal chance** of appearing when the die is rolled. So, the **total number of possible outcomes = 6**.

The formula for probability is:

$$\text{Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

2. **In a class of 50 students:**
   20 like Mathematics (M)
   15 like Science (S)
   5 like both subjects

What is the probability that a student chosen at random likes Mathematics or Science?

Answer: Concept overview

- ✓ Total students: 50, with equal chance of selection.
- ✓ Given counts: **Mathematics (M) = 20, Science (S) = 15, Both (M ∩ S) = 5.**
- ✓ Goal: Probability that a randomly chosen student likes Mathematics or Science, i.e., the union $M \cup S$.
- ✓ Union principle (inclusion–exclusion):
  - ❖ $\boldsymbol{|M \cup S| = |M| + |S| - |M \cap S| = 20 + 15 - 5 = 30}$
  - ❖ **Probability:**
    - ▪ $\boldsymbol{P(M \cup S) = \frac{|M \cup S|}{\text{Total}} = \frac{30}{50} = 0.6 = 60\%}$
- ✓ Final answer: The probability that a student likes Mathematics or Science is 0.6 (60%) or 30/50.

Step-by-step instructions

- ✓ **Enter counts:**
  - ❖ **Mathematics (M):** In B2 type 20.
  - ❖ **Science (S):** In B3 type 15.
  - ❖ **Both (M ∩ S):** In B4 type 5.
  - ❖ **Total Students:** In B5 type 50.
- ✓ **Calculate union (unique students who like M or S):**
  - ❖ **Math OR Science (M ∪ S) count:** In B6 type:
    - ▪ **=B2 + B3 − B4**
  - ❖ This returns 30.
- ✓ **Compute probability in decimal:**
  - ❖ **For M:** In C2 type:
    - ▪ **=B2 / B5**
  - ❖ **For S:** In C3 type:
    - ▪ **=B3 / B5**
  - ❖ **For Both:** In C4 type:
    - ▪ **=B4 / B5**
  - ❖ **For M ∪ S:** In C6 type:
    - ▪ **=B6 / B5**
  - ❖ Results should be 0.4, 0.3, 0.1, and 0.6 respectively.
- ✓ **Show percentage:**
  - ❖ **Link percentage to decimals:** In D2, D3, D4, D6 type:
    - ▪ **=C2, =C3, =C4, =C6**

- ❖ Select D2:D6 → Format as Percentage → set 2 decimal places.
- ❖ You'll see **40%, 30%, 10%, and 60%.**

✓ **Display fraction (count/total):**

- ❖ **For M:** In E2 type:
  - **=TEXT(B2,"#") & "/" & TEXT(B5,"#")**
- ❖ **For S:** In E3 type:
  - **=TEXT(B3,"#") & "/" & TEXT(B5,"#")**
- ❖ **For Both:** In E4 type:
  - **=TEXT(B4,"#") & "/" & TEXT(B5,"#")**
- ❖ **For M ∪ S:** In E6 type:
  - **=TEXT(B6,"#") & "/" & TEXT(B5,"#")**
- ❖ You'll see 20/50, 15/50, 5/50, and 30/50 (you can simplify to 2/5, 3/10, 1/10, 3/5 manually in Word if needed).

Clean summary row (optional)

✓ Add a final summary line below your table:

- ❖ **Result: P (M or S) = 0.6 (60%) = 30/50**

Category	Count	Probability (Decimal)	Probability (%)	Probability (Fraction)
Mathmatics (M)	20	0.4	40%	20/50
Science (S)	15	0.3	30%	15/50
Both (M ∩ S)	5	0.1	10%	5/50
Total Student	50			
Math OR Science (M ∪ S)	30	0.6	60%	30/50

3. A bag has 3 red and 2 blue balls. If one ball is drawn randomly and is red, what is the probability that the next ball is also red (without replacement)?

Answer: Concept explanation

Initial counts: Red = 3, Blue = 2, Total = 5.

✓ Given condition: The first ball drawn is red.

✓ Update after removing one red ball:

- ❖ Remaining red = 2
- ❖ Remaining blue = 2
- ❖ Remaining total = 4

✓ Required probability: Second ball is red, given the first ball was red.

$$P(\text{Second red | First red}) = \frac{\text{Remaining red}}{\text{Remaining total}} = \frac{2}{4} = 0.5 = 50\%$$

Step-by-step writing

1) **State the scenario: A bag contains 3 red balls and 2 blue balls (total 5). The first ball drawn is red.**

2) **Apply the condition (without replacement): Since one red ball is already removed, the new counts are:**

   ❖ Red balls left = 3 − 1 = 2

   ❖ Blue balls left = 2

   ❖ Total balls left = 4

3) **Set up the conditional probability: We need $P(\text{Second red} \mid \text{First red})$. With 2 red balls out of 4 remaining:**

$$P = \frac{2}{4} = 0.5$$

4) **Express in different formats:**

   ❖ Decimal: 0.5

   ❖ Percentage: 50%

   ❖ Fraction: $\frac{2}{4} = \frac{1}{2}$

5) **Final answer: The probability that the next ball is also red (given the first was red, without replacement) is 0.5or 50%.**

Label	Value
Initial Red (R)	3
Initial Blue (B)	2
Initial Total (T)	5
Remaining Red (after 1st)	2
Remaining Blue	2
Remaining Total	4
Probability (Decimal)	0.5
Probability (%)	50%
Probability (Fraction)	2/4

4. The population of a school is divided into 60% boys and 40% girls. If you want equal representation of both genders in the sample, which method should you use: Simple Random Sampling or Stratified Sampling? Why?

Answer:

1) Situation

✓ The school population is divided into:

   ❖ 60% Boys

   ❖ 40% Girls

✓ We want a sample where boys and girls are represented equally (50–50).

2) Sampling Methods Overview

a) Simple Random Sampling (SRS)

- Every student has an equal chance of being selected.
- The sample will reflect the population proportion (around 60% boys, 40% girls).
- Limitation: It does not guarantee equal representation of genders.

b) Stratified Sampling

- The population is divided into strata (groups) based on gender.
- From each stratum, a fixed number of students are selected.
- This ensures equal representation (e.g., 50% boys and 50% girls in the sample).
- Advantage: Guarantees balance even if the population itself is imbalanced.

3) Why Stratified Sampling is Correct Here

- The population is unequal (60/40).
- The requirement is equal representation (50/50).
- Stratified sampling allows us to control the sample size from each group.
- Simple random sampling would not achieve this balance.

4) Step-by-Step Process (Conceptual)

- ✓ Divide population into strata: Boys and Girls.
- ✓ Decide sample size: Suppose we want 100 students in the sample.
- ✓ Allocate equally:
  - 50 Boys
  - 50 Girls
- ✓ Select randomly within each stratum:
  - Use random numbers or lottery method to pick 50 boys and 50 girls.

Combine both groups: Final sample = 100 students with equal gender representation.

5) Conclusion

- ✓ Correct Method: Stratified Sampling
- ✓ Reason: It ensures equal representation of boys and girls in the sample, which Simple Random Sampling cannot guarantee.

5. The average height of 1000 students = 160 cm. A sample of 100 students shows an average height = 158 cm. Find the sampling error

Answer: Concept of Sampling Error

- ✓ Definition: Sampling error is the difference between the population mean and the sample mean.
- ✓ It occurs because the sample may not perfectly represent the population.
- ✓ Formula:

Sampling Error = Population Mean − Sample Mean

Step-by-Step Calculation

1) Population Mean (μ): 160 cm

2) Sample Mean ($\bar{x}$): 158 cm
3) Apply formula:
4) Sampling Error $= 160 - 158$
5) Result:

$$\text{Sampling Error} = 2 \text{ cm}$$

Interpretation

- ✓ The sample average is 2 cm lower than the population average.
- ✓ This difference of 2 cm represents the sampling error.
- ✓ It shows that the sample does not perfectly match the population, which is normal in statistics.

Final Answer

The sampling error is 2 cm.

6. The population mean salary is ₹50,000 with σ = ₹5,000. If we take a sample of 100 employees, what is the standard error of the mean (SEM)?

Answer: Concept of Standard Error

- ✓ Definition: The Standard Error of the Mean (SEM) measures how much the sample mean is expected to vary from the population mean.
- ✓ It is calculated using the formula:

$$SEM = \frac{\sigma}{\sqrt{n}}$$

where:

- ✓ σ = population standard deviation
- ✓ n = sample size

Step-by-Step Calculation

1. Identify values:
   - σ = 5,000
   - n = 100
2. Apply formula:

$$SEM = \frac{5000}{\sqrt{100}}$$

3. Simplify denominator:

$$\sqrt{100} = 10$$

4. Final calculation:

$$SEM = \frac{5000}{10} = 500$$

Interpretation

- ✓ The Standard Error of the Mean is ₹500.

- ✓ This means that the sample mean salary is expected to deviate from the population mean salary by about ₹500 on average.
- ✓ A larger sample size would reduce the SEM, making the sample mean closer to the population mean.

Final Answer

The Standard Error of the Mean (SEM) is ₹500.

7. In a group of 100 students:
40 like Cricket (C)
30 like Football (F)
10 like both Cricket and Football
Find the probability that a student likes at least one sport.

Answer: **Concept explanation**

- ✓ **Given counts:**
  - ❖ Cricket (C): 40
  - ❖ Football (F): 30
  - ❖ Both (C ∩ F): 10
  - ❖ Total: 100
- ✓ **Goal:** Probability that a randomly chosen student likes at least one sport, i.e., $C \cup F$.
- ✓ Inclusion–exclusion:

$$|C \cup F| = |C| + |F| - |C \cap F| = 40 + 30 - 10 = 60$$

$$P(C \cup F) = \frac{|C \cup F|}{\text{Total}} = \frac{60}{100} = 0.6 = 60\%$$

Step-by-step explanation for MS Word

1. **State the scenario:**
   - ❖ Total students: 100
   - ❖ Like Cricket (C): 40
   - ❖ Like Football (F): 30
   - ❖ Like both (C ∩ F): 10
2. **Identify the requirement:**
   - ❖ Find: $P(C \cup F)$, the probability that a student likes at least one of the two sports (Cricket or Football).
3. **Apply the union formula (inclusion–exclusion):**
   - ❖ Count of at least one:

$$|C \cup F| = |C| + |F| - |C \cap F| = 40 + 30 - 10 = 60$$

4. **Convert count to probability:**
   - ❖ Probability:

$$P(C \cup F) = \frac{60}{100} = 0.6$$

- Percentage: 60%
- **Fraction:** $\frac{60}{100} = \frac{3}{5}$

5. **Final answer:**
   - **A randomly selected student has a 0.6 (60%) probability of liking at least one sport.**

Category	Count	Decimal Probability	Percentage	Fraction
Cricket (C)	40	0.4	40%	40/100
Football (F)	30	0.3	30%	30/100
Both (C ∩ F)	10	0.1	10%	10/100
At least one (C ∪ F)	60	0.6	60%	60/100
Total Students	100			

8. From a deck of 52 cards, two cards are drawn without replacement. What is the probability that both are Aces?

Answer: Probability is a measure of how likely an event is to occur. It is calculated using the formula:

$$P(\text{Event}) = \frac{\textbf{Favorable outcomes}}{\textbf{Total possible outcomes}}$$

In this case, we are calculating the probability of drawing two Aces consecutively, without replacement.

Scenario and given values

- ✓ Total cards: 52
- ✓ Total aces: 4
- ✓ Sampling condition: Two draws without replacement
- ✓ Goal: Probability that both cards drawn are aces

Step-by-step explanation

1. First draw (probability of an Ace): There are 4 aces out of 52 cards.

$$P(\text{Ace on first}) = \frac{4}{52} = \frac{1}{13} \approx 0.0769231$$

2. Update counts after drawing an Ace: One ace is removed and one card is removed from the deck.
   - Remaining aces: 3
   - Remaining cards: 51

3. Second draw (probability of an Ace given the first was an Ace):

$$P(\text{Ace on second} \mid \text{Ace on first}) = \frac{3}{51} \approx 0.0588235$$

4. Multiply sequential probabilities (both aces):

$$P(\text{both aces}) = \frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652} = \frac{1}{221} \approx 0.0045249$$

5. Express in different formats:
    - Decimal: 0.0045249
    - Percentage: 0.45% (rounded)
    - Fraction: $\frac{1}{221}$

Alternative check (combinatorial method)

- Ways to choose 2 aces: $\binom{4}{2} = 6$
- Ways to choose any 2 cards: $\binom{52}{2} = 1326$
- Probability:

$$\frac{\binom{4}{2}}{\binom{52}{2}} = \frac{6}{1326} = \frac{1}{221} \approx 0.0045249\ (0.45\%)$$

Final answer

- ✓ **The probability that both cards drawn are aces (without replacement) is $\frac{1}{221} \approx 0.0045249$, which is about 0.45%.**

Category	Value
Total Cards	52
Total Aces	4
First draw: P(Ace)	0.0769231
Remaining Cards after 1st	51
Remaining Aces after 1st	3
Second draw: P(Ace 1st)	0.0588235
Final Probability (Decimal)	0.0045249
Final Probability (%)	0.45%
Final Probability (Fraction)	1/221

9. A factory produces bulbs with 2% defective rate. If 5 bulbs are chosen at random, what is the probability that all are non-defective?

Answer: What is probability in this context

- ✓ Probability measures the chance of an event occurring.
- ✓ Here, the event is “all 5 bulbs are non-defective.”

- ✓ If each bulb independently has a 0.98 chance to be good, the chance that all 5 are good is the product of those chances:

$$P(\text{all 5 good}) = 0.98 \times 0.98 \times 0.98 \times 0.98 \times 0.98 = (0.98)^5$$

- ✓ **Given:**
  - ❖ Defective rate (p): 2% = 0.02
  - ❖ Non-defective rate (q): $1 - p = 0.98$
  - ❖ Sample size (n): 5 bulbs chosen at random

- ✓ **Ask: What is the probability that all selected bulbs are non-defective? Also explain the reasoning.**

Step-by-step calculation

1) **Identify rates:**
   - ❖ Defective rate: $p = 0.02$
   - ❖ Non-defective rate: $q = 1 - p = 0.98$
2) **Model selection:**
   - Treat each bulb as an independent trial with success "non-defective" probability $q = 0.98$.
   - We want exactly 5 successes in 5 trials.
3) **Use the binomial formula for 0 defectives (or 5 non-defectives):**

$$P(X = 0) = \binom{5}{0}(0.02)^0(0.98)^5 = (0.98)^5$$

4) **Compute the value:**

$$(0.98)^5 \approx 0.9039208$$

- ✓ Decimal: 0.9039208
- ✓ Percentage: $0.9039208 \times 100\% \approx 90.392\%$

Interpretation

- ✓ There is about a 90.39% chance that all 5 randomly chosen bulbs are non-defective.
- ✓ High probability reflects the low defective rate and the independence assumption typical for large production runs.

Final answer

- ✓ Probability (decimal): $(0.98)^5 \approx 0.9039208$
- ✓ Probability (percentage): $\approx 90.392\%$
- ✓ Conclusion: The probability that all 5 bulbs are non-defective is $(0.98)^5$, which is approximately 90.39%.

Label	Value
Defective rate (p)	0.02
Non-defective rate (q)	0.98
Sample size (n)	5
Probability (decimal)	0.9039208
Probability (%)	90.392%
Binomial check (decimal)	0.9039208

10. Differentiate between discrete and continuous random variables with examples.

Answer: Introduction

In statistics and probability theory, a random variable is a variable whose value is subject to variations due to chance. Random variables are classified into two main types: Discrete and Continuous. Understanding their differences is essential for selecting appropriate statistical methods and interpreting data correctly.

Discrete Random Variable

Definition:

A discrete random variable can take only specific, countable values. These values are often whole numbers and arise from counting processes.

Characteristics:

- ✓ Values are finite or countably infinite.
- ✓ Gaps exist between possible values.
- ✓ Probabilities are assigned to individual values.

Examples:

- ✓ Number of students in a class (e.g., 25, 30, 32)
- ✓ Number of heads in 10 coin tosses (e.g., 0 to 10)
- ✓ Number of defective items in a batch
- ✓ Number of goals scored in a football match

Continuous Random Variable

Definition:

A continuous random variable can take any value within a given range, including fractions and decimals. These values arise from measurement processes.

Characteristics:

- ✓ Values are uncountably infinite within an interval.
- ✓ No gaps between values; they form a continuum.
- ✓ Probabilities are assigned to intervals, not exact values.

Examples:

- ✓ Height of students (e.g., 160.5 cm, 172.3 cm)
- ✓ Time taken to complete a task (e.g., 2.45 hours)
- ✓ Temperature in a city (e.g., 28.6°C)
- ✓ Weight of a packet (e.g., 1.25 kg)

Key Differences

Feature	Discrete Random Variable	Continuous Random Variable
Type of values	Countable (whole numbers)	Uncountable (fractions, decimals)
Based on	Counting	Measuring
Probability assigned to	Specific values	Ranges or intervals
Examples	Number of books, coin tosses	Height, weight, temperature

Conclusion

Discrete and continuous random variables differ in the nature of values they can take and how probabilities are assigned. Discrete variables deal with counts, while continuous variables deal with measurements. Recognizing the type of random variable helps in choosing the correct statistical tools and interpreting results accurately.