# Advanced Image Processing

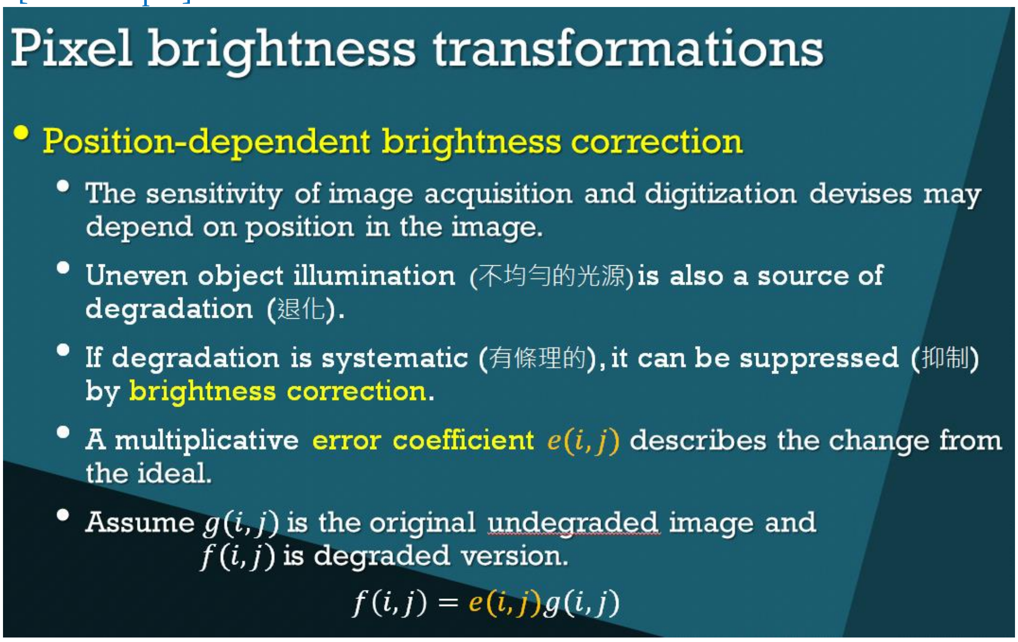
Final Exam.

Student No.: ____________________ Name: ____________________ 109/12

## 一、是非題(T/F test) (30%)

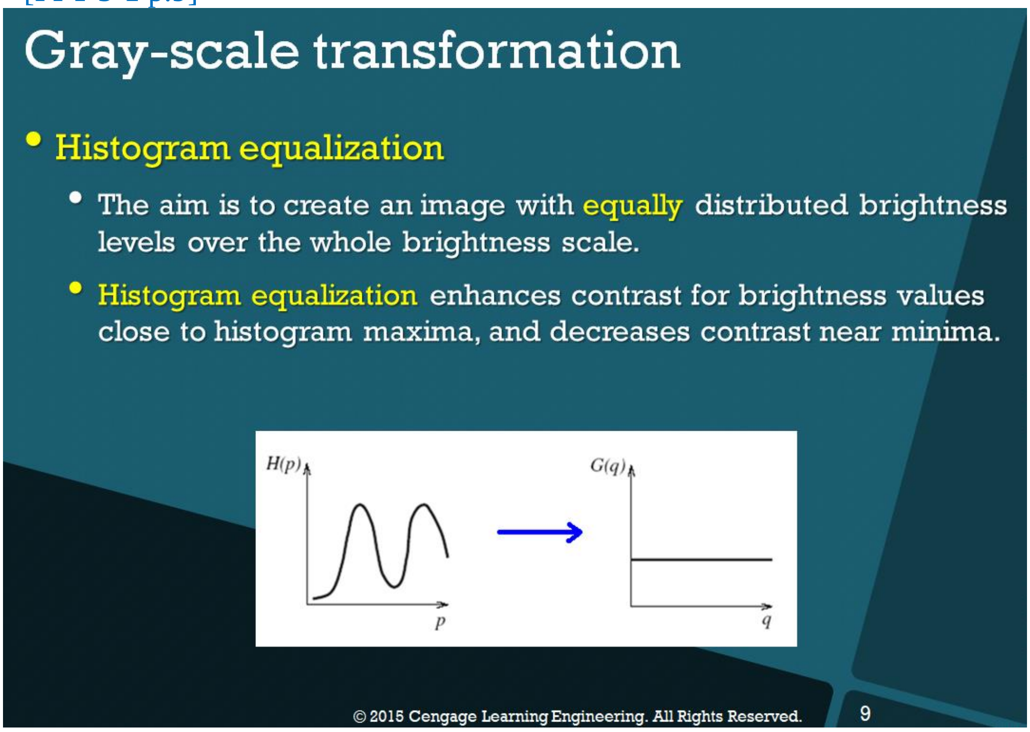
( O ) 1. If degradation is systematic (有條理的), it can be suppressed (抑制) by brightness correction. The brightness correction method can be used only if the image degradation process is stable.

[PPT 5-1 p.4]



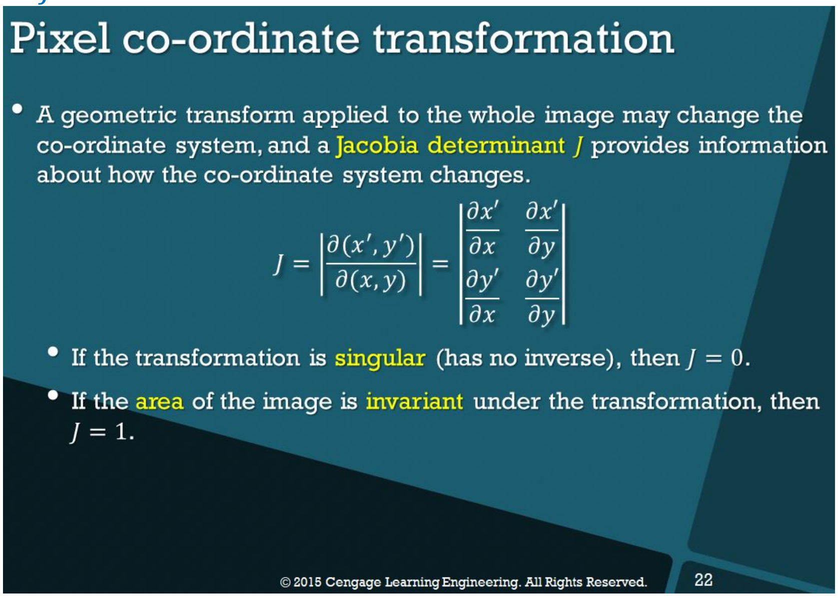

( O ) 2. The aim of histogram equalization is to create an image with equally distributed brightness levels over the whole brightness scale.

- [PPT 5-1 p.9]



( O ) 3. If the area of the image is invariant under a geometric transformation, then its Jacobia determinant J = 1.

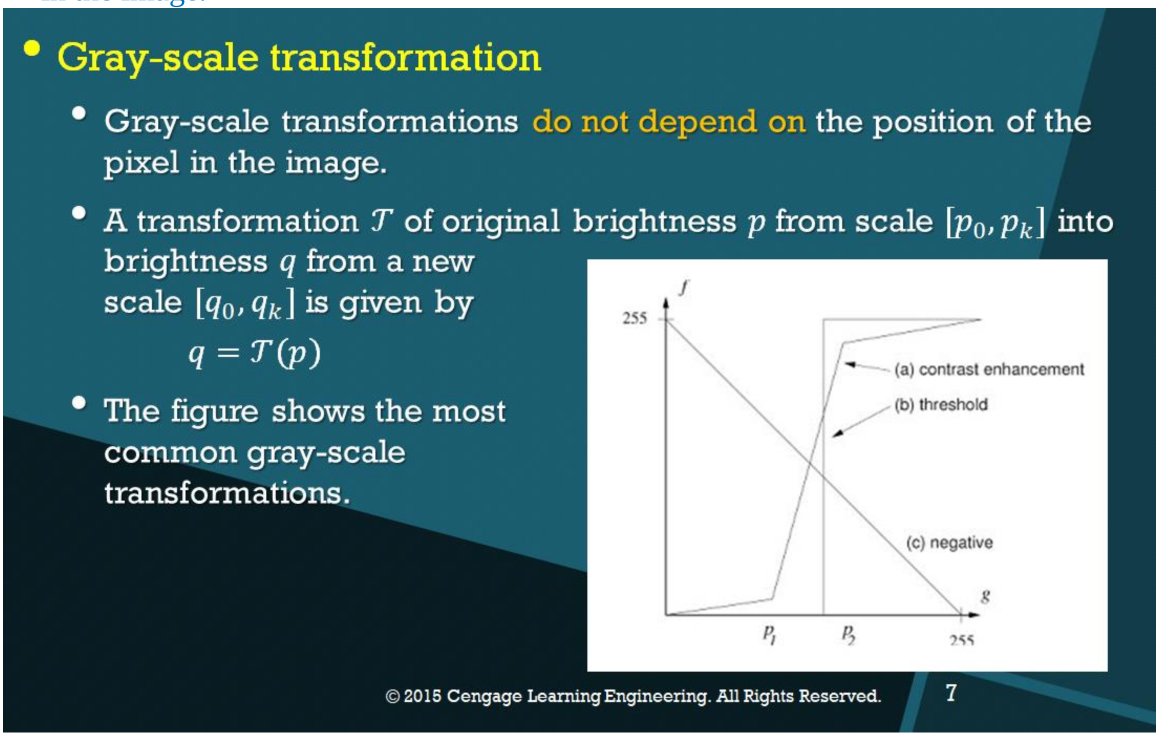
- [PPT 5-1 p.22] If the area of the image is invariant under the transformation, then $J = 1$.




- [補充]

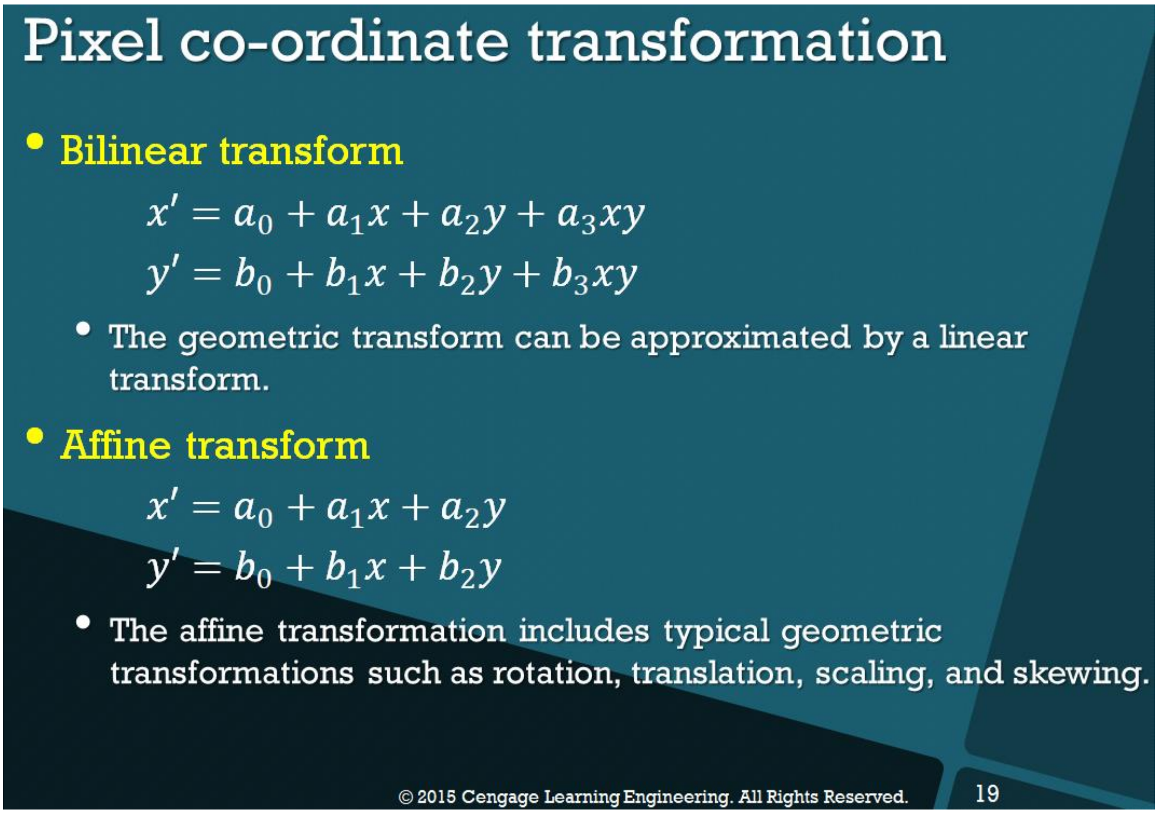

( X ) 4. Gray-scale transformations (do not) depend on the position of the pixel in the image.

- [PPT 5-1 p.7] Gray-scale transformations do not depend on the position of the pixel in the image.



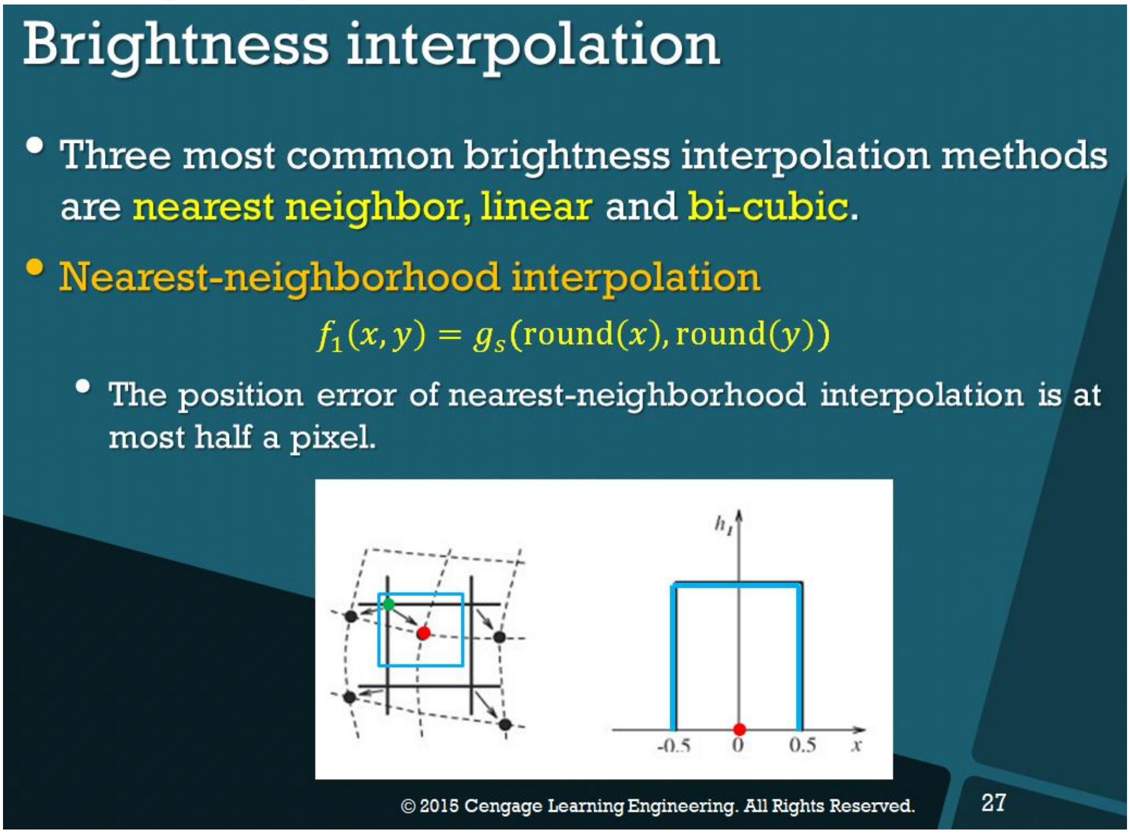
( X ) 5. The bilinear transformation includes typical geometric transformations such as rotation and translation, but ~~not~~ scaling.

- [PPT 5-1 p.19]

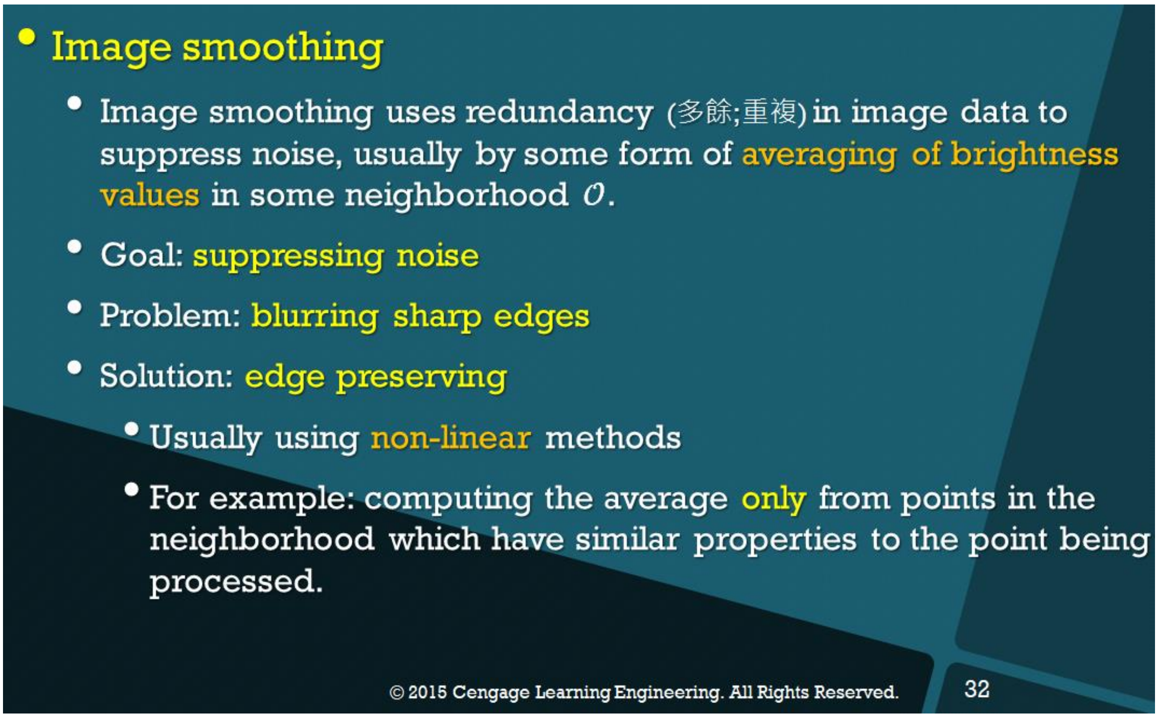



( X ) 6. The position error of nearest-neighborhood brightness interpolation is at ~~least~~ (most) half a pixel.

- [PPT 5-1 p.25] interpolation(插值)

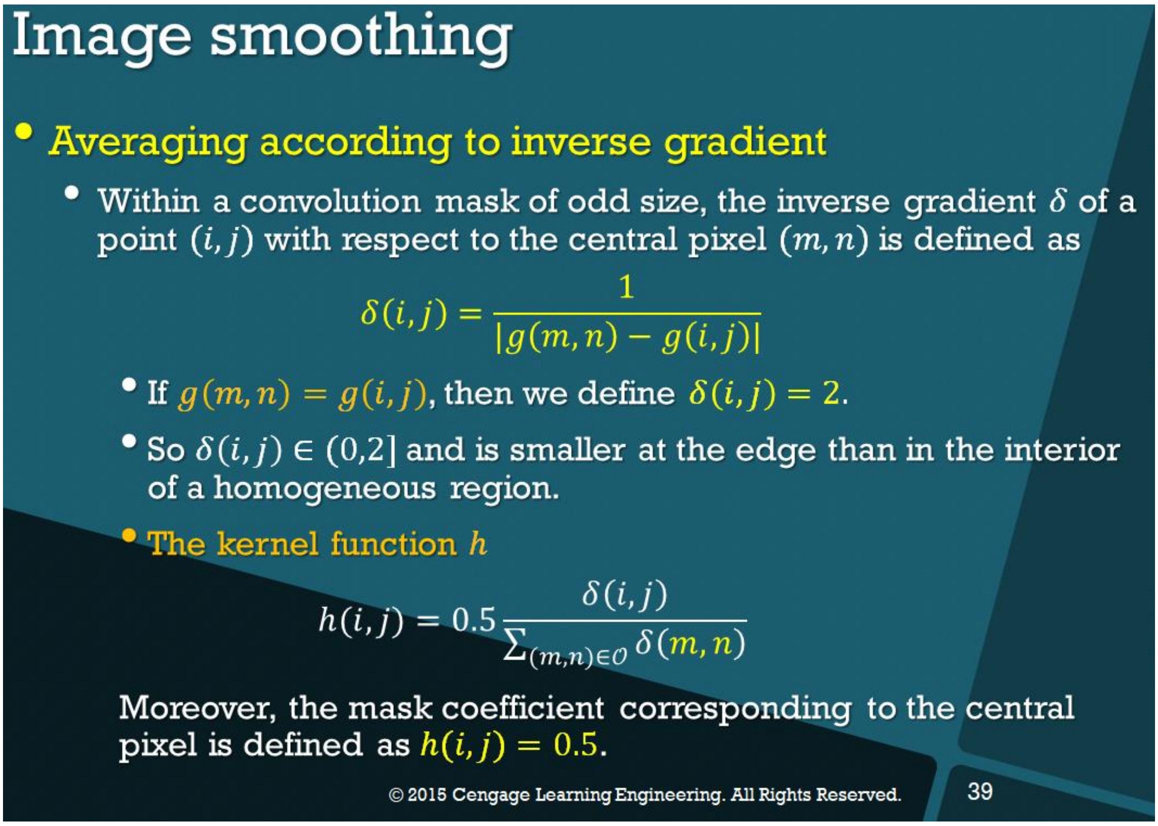


( O ) 7. Image smoothing uses redundancy in image data to suppress noise, usually by some form of averaging of brightness values in some neighborhood.



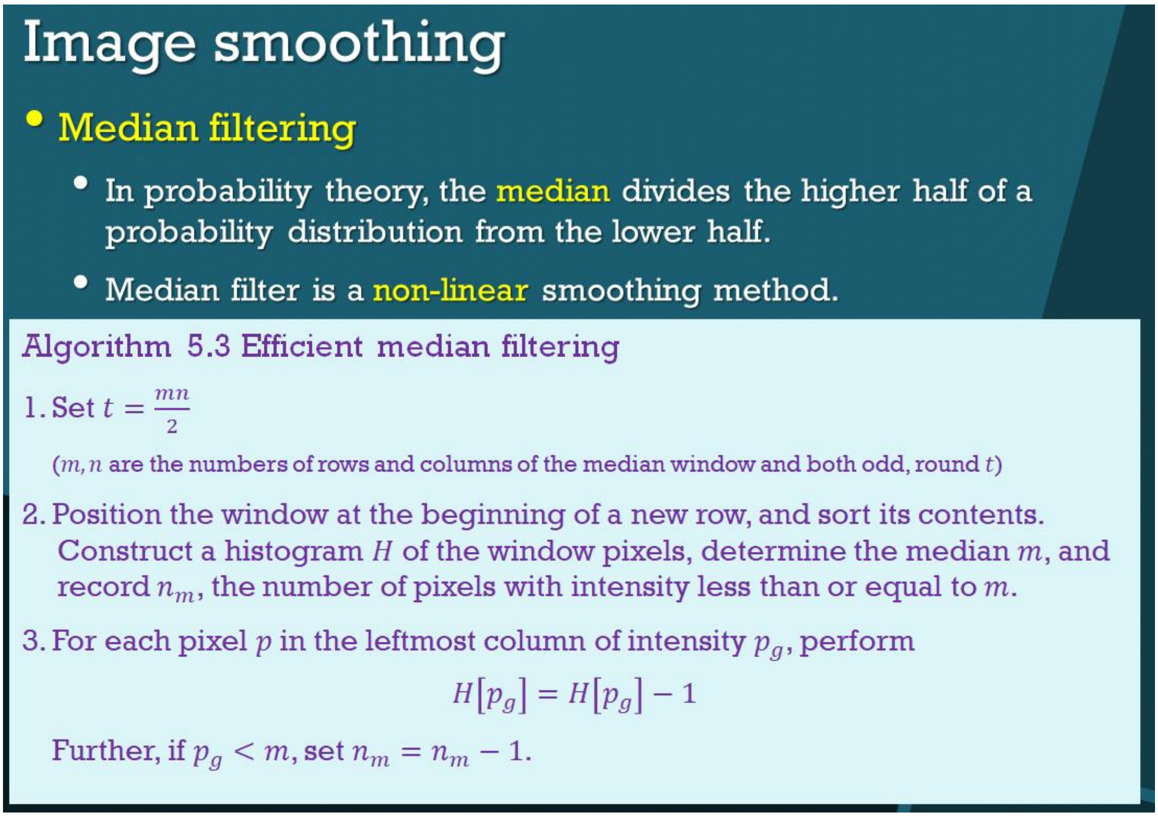

?( X ) 8. "Averaging according to inverse gradient" is a (non-)linear method that avoids edge blurring.

[PPT 5-1 p.32]



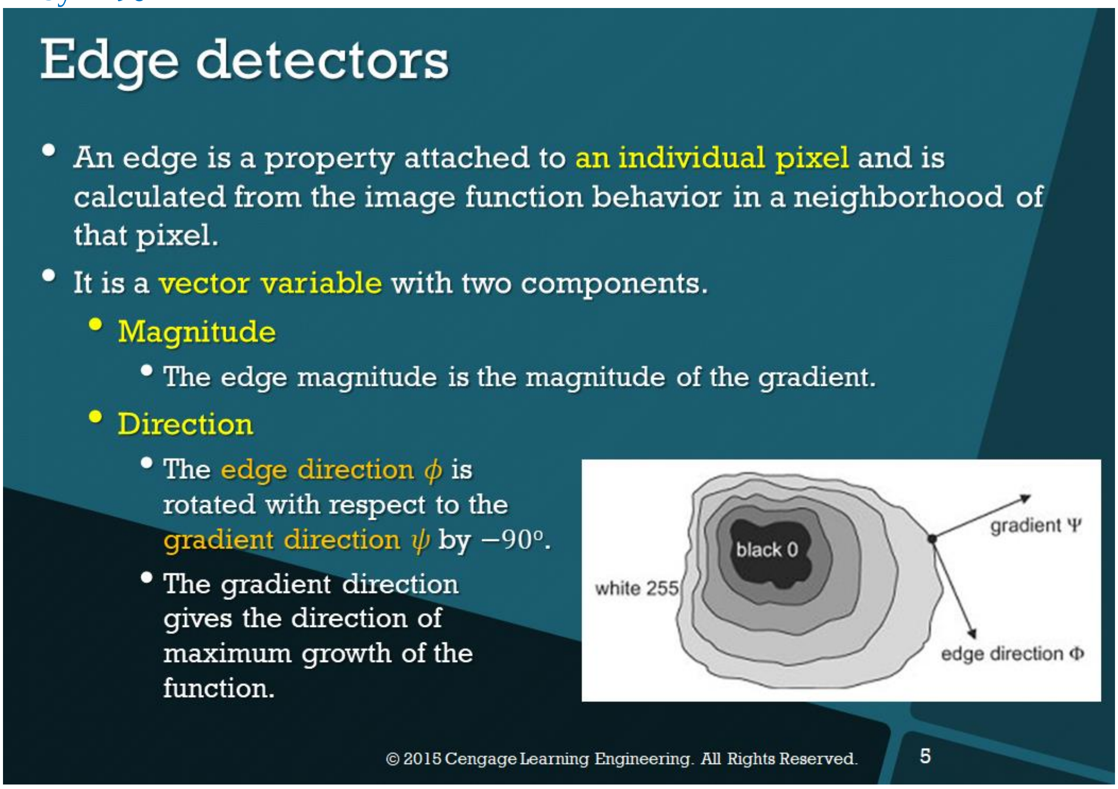
( O ) 9. Median filter is a nonlinear smoothing method.

- [PPT 5-1 p.43] Median filtering:

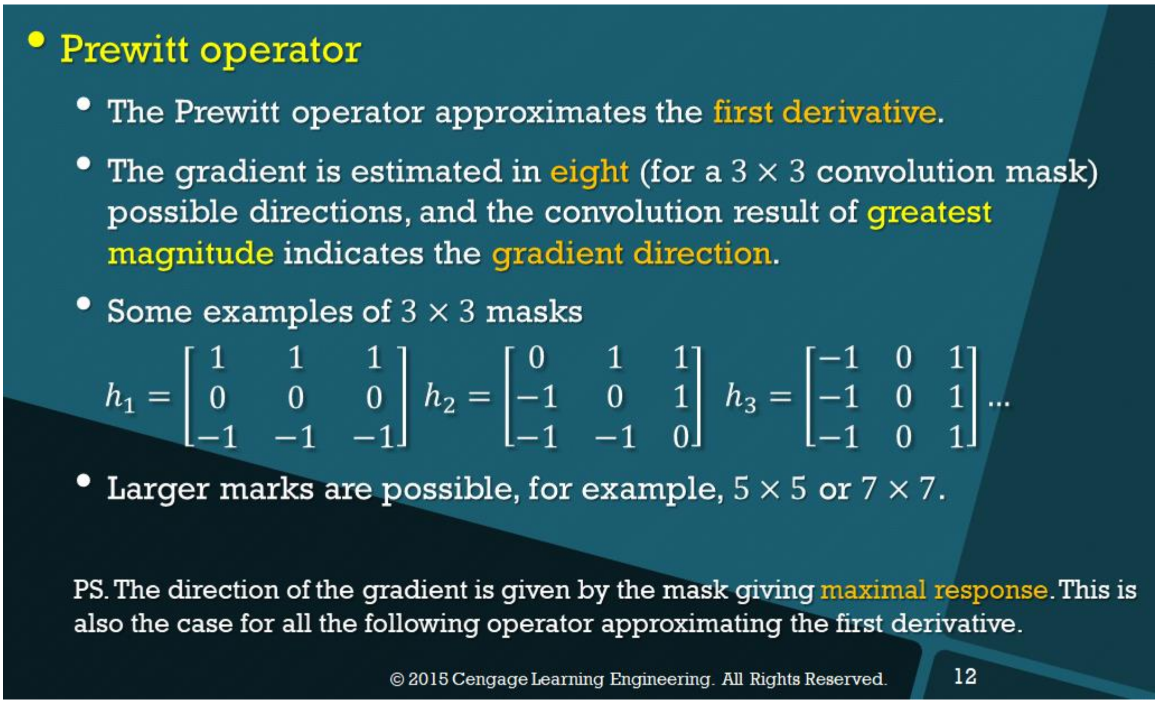



( O ) 10. The edge direction is rotated with respect to the gradient direction by $-90^{\circ}$.

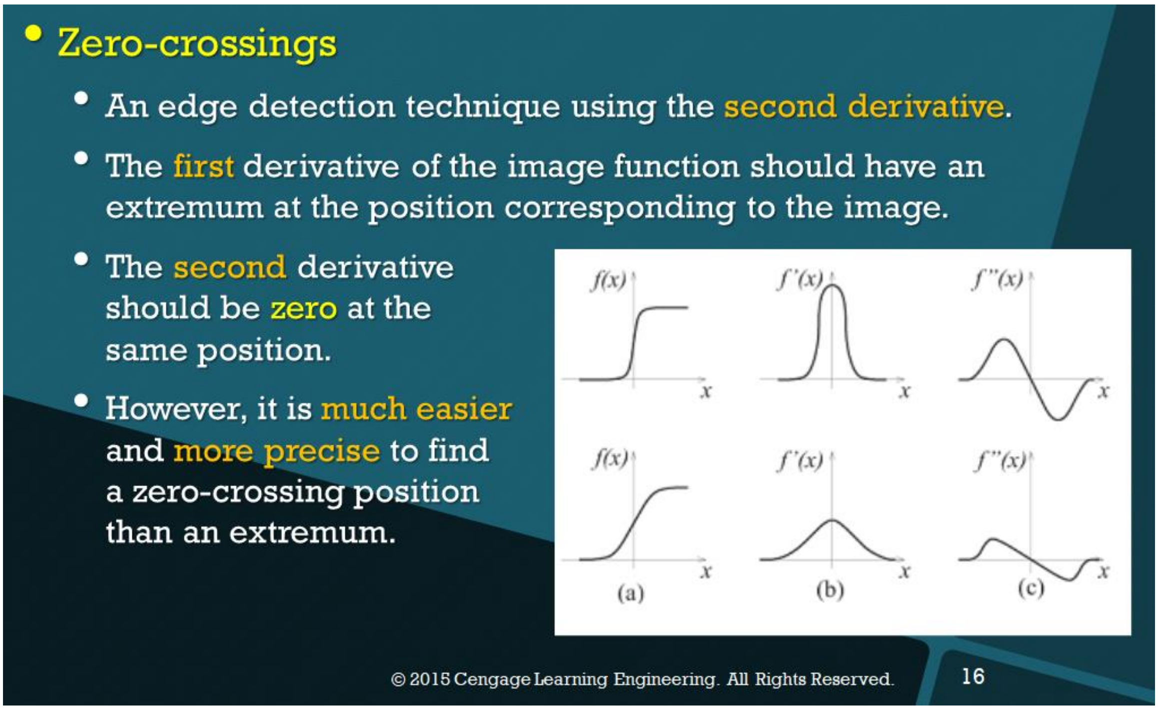
[PPT 5-2 p.5] The edge direction $\phi$ is rotated with respect to the gradient direction $\psi$ by $-90$



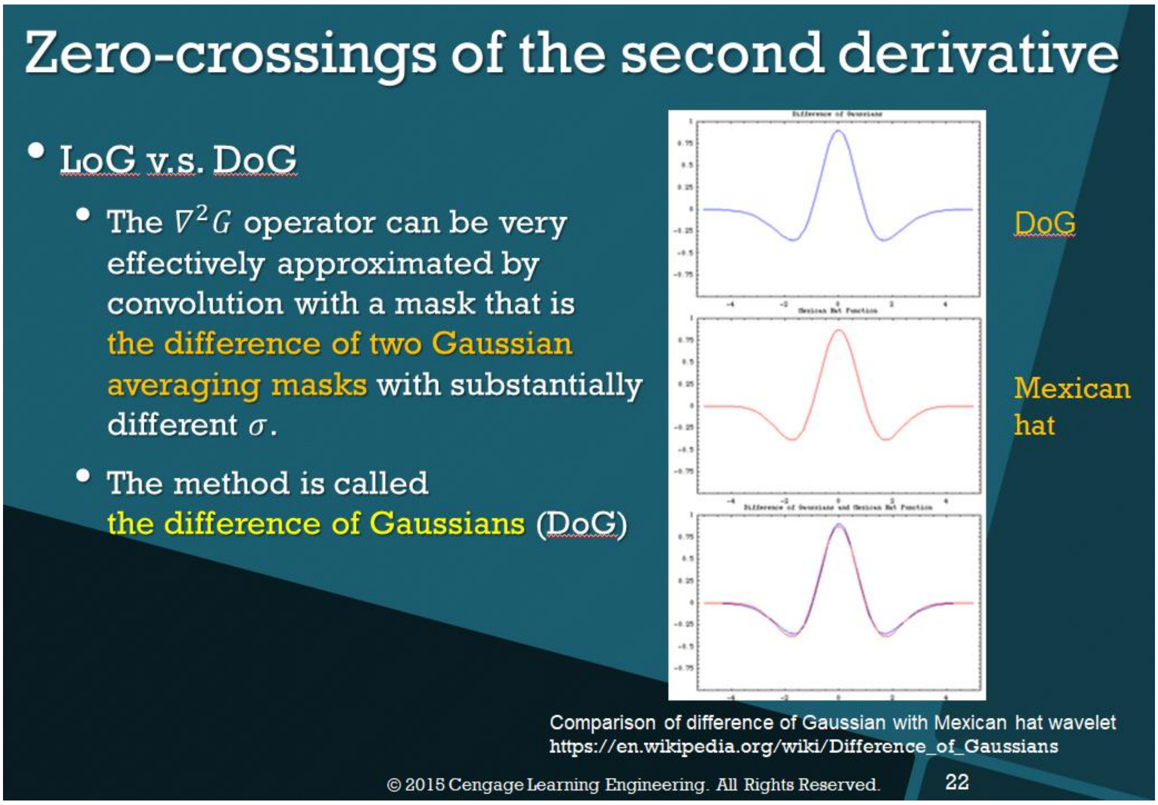
( O ) 11. The Prewitt operator approximates the first derivative. The gradient is estimated in eight (for a $3\times3$ convolution mask) possible directions, and the convolution result of largest magnitude indicates the gradient direction.



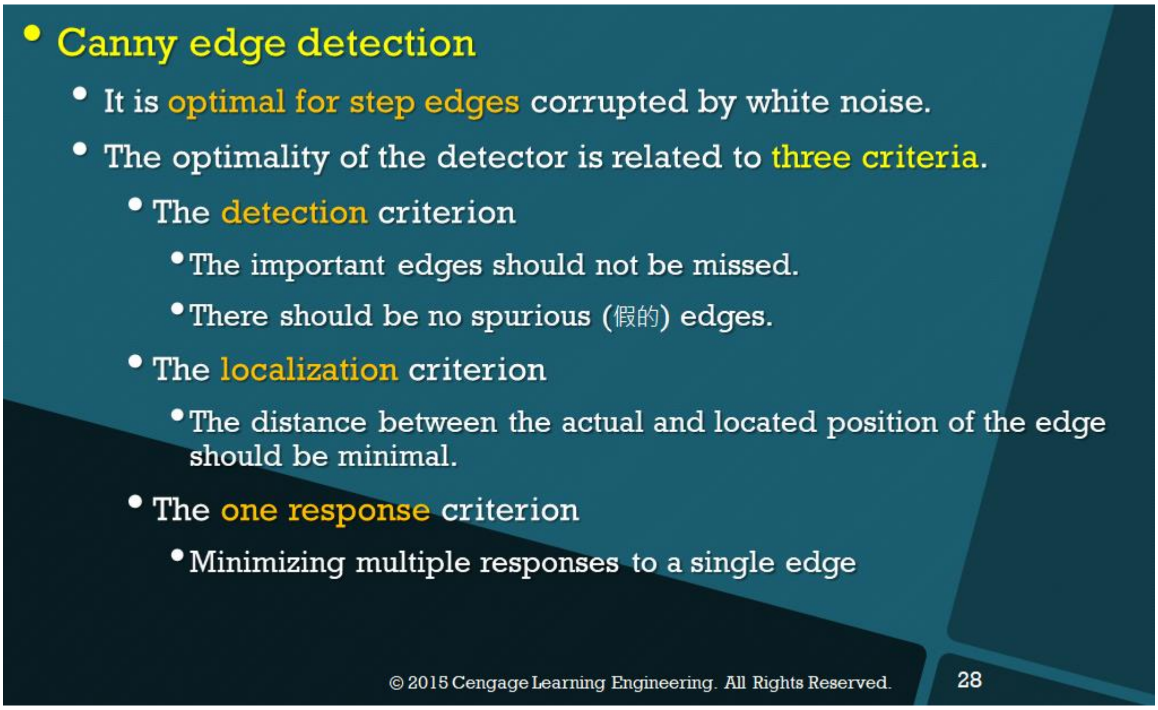

( X ) 12. Zero-crossings is an edge detection technique using the ~~first~~ (second) derivative.



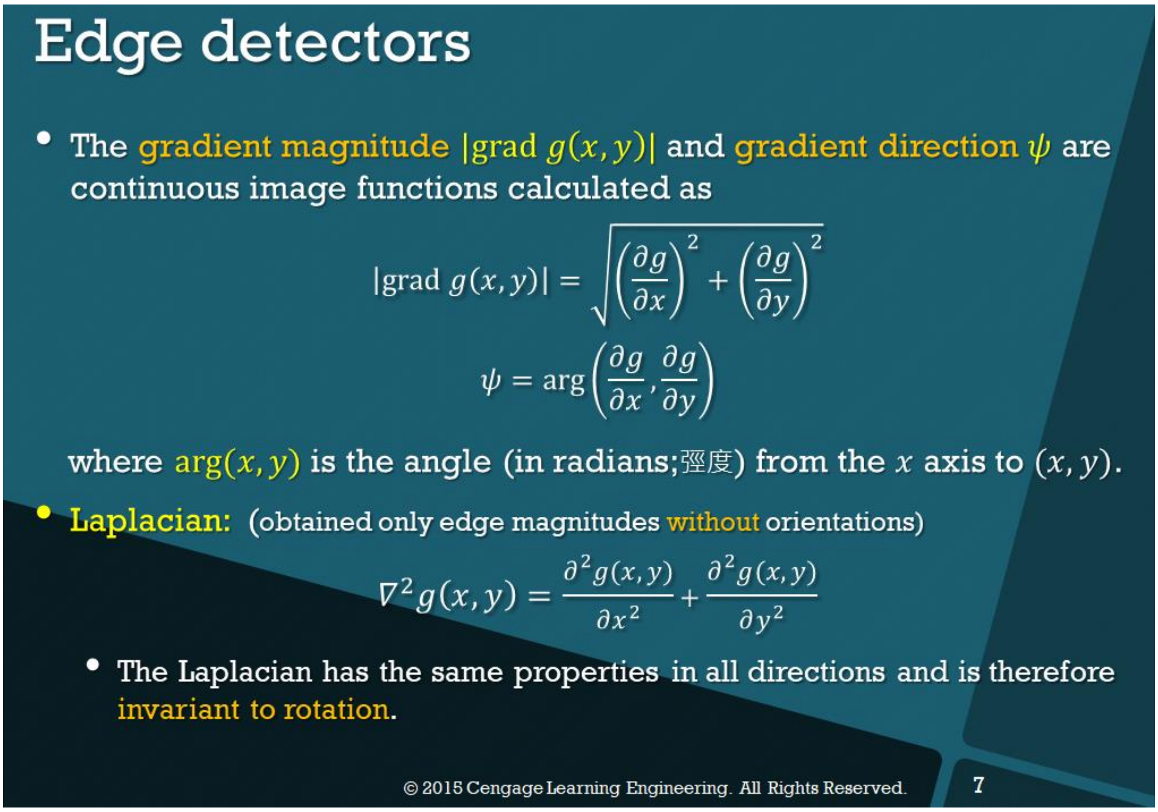

( X ) 13. The definition of ~~Laplacian of Gaussian (LoG)~~ (the difference of Gaussians (DoG)) operator is the difference of two Gaussian averaging masks with substantially different variance.



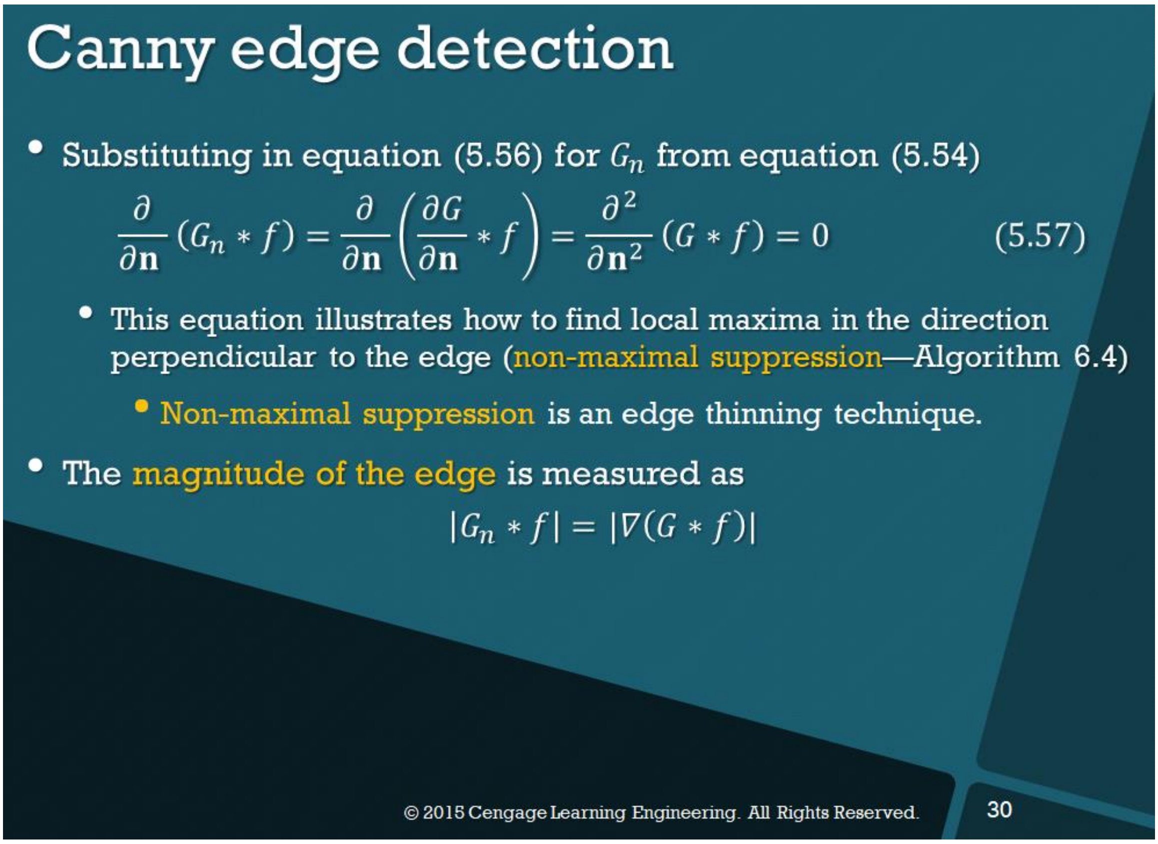

( X ) 14. The ~~one response~~ (detection) criterion of edge detector indicates that the important edges should not be missed.



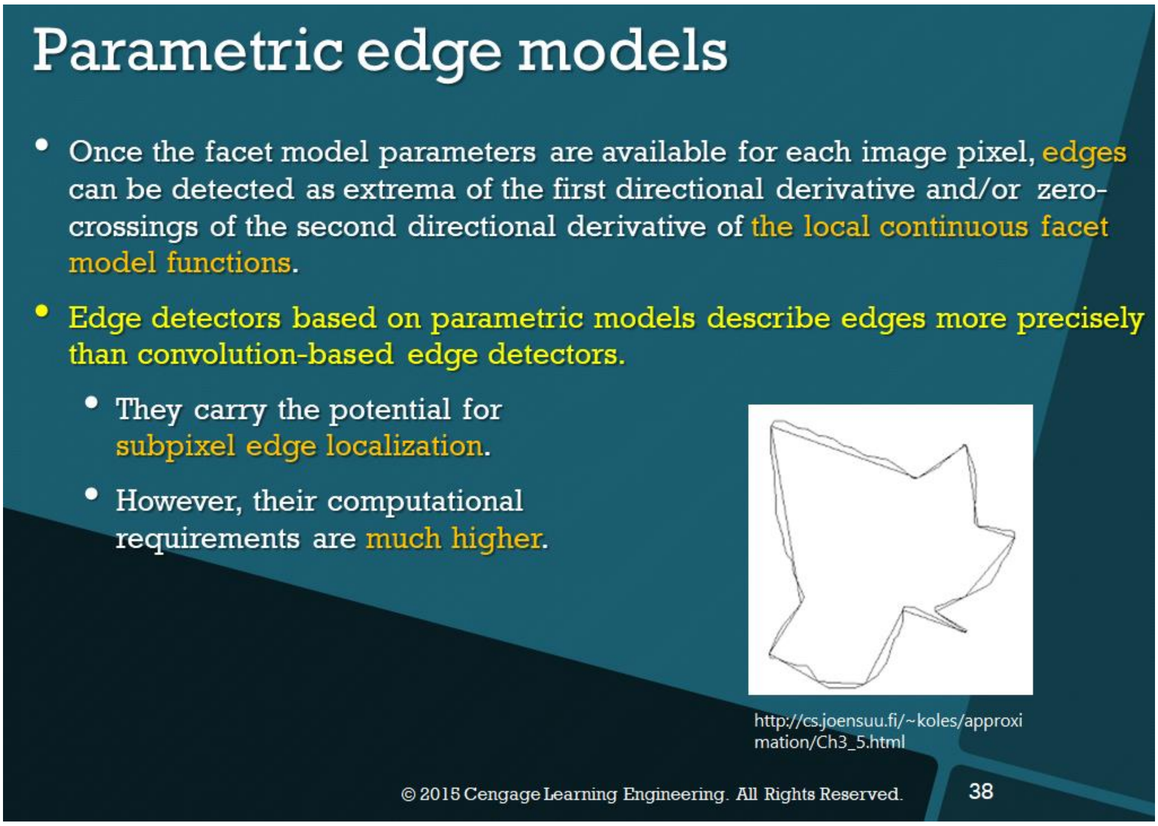

( O ) 15. The Laplacian operator for edge detection has the same properties in all directions and is therefore invariant to rotation.



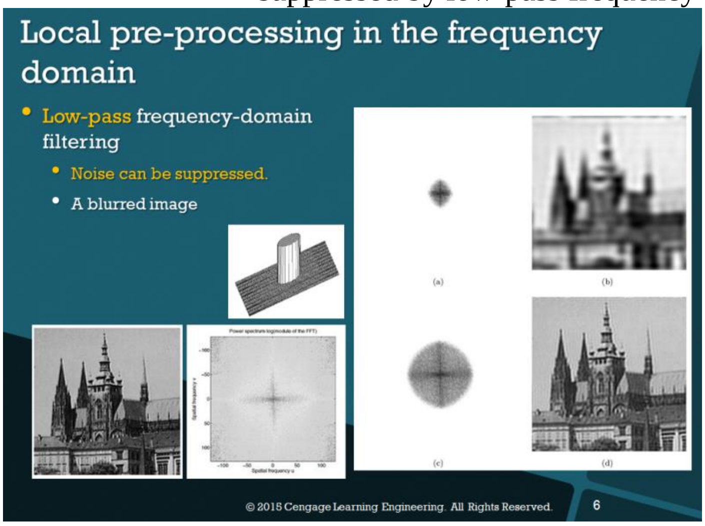

( O ) 16. Non-maximal suppression is an edge thinning technique.

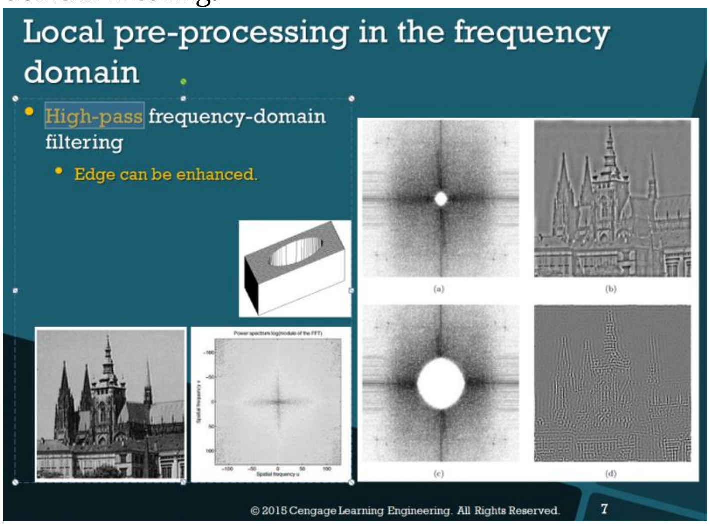


( O ) 17. Edge detectors based on parametric models describe edges more precisely than convolution-based edge detectors. However, their computational requirements are much higher.

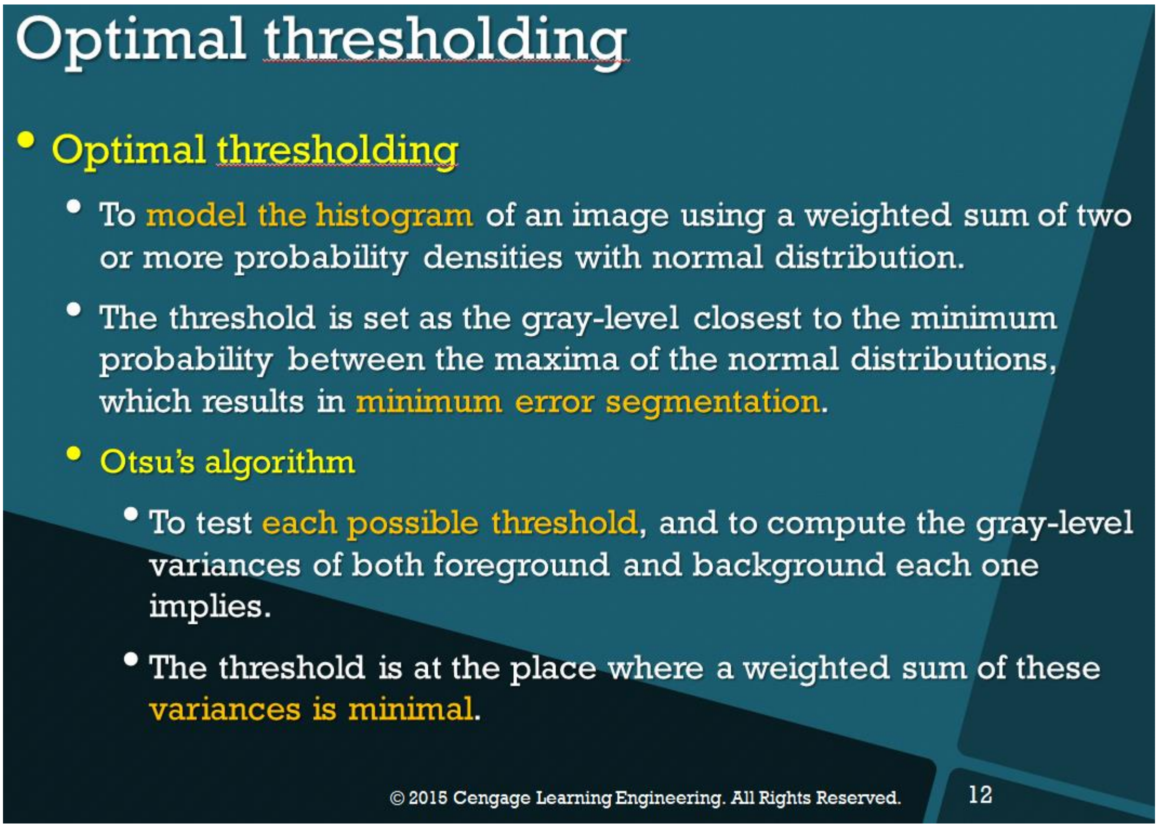



( O ) 18. Edge can be enhanced by high-pass frequency-domain filtering and noise can be suppressed by low-pass frequency-domain filtering.

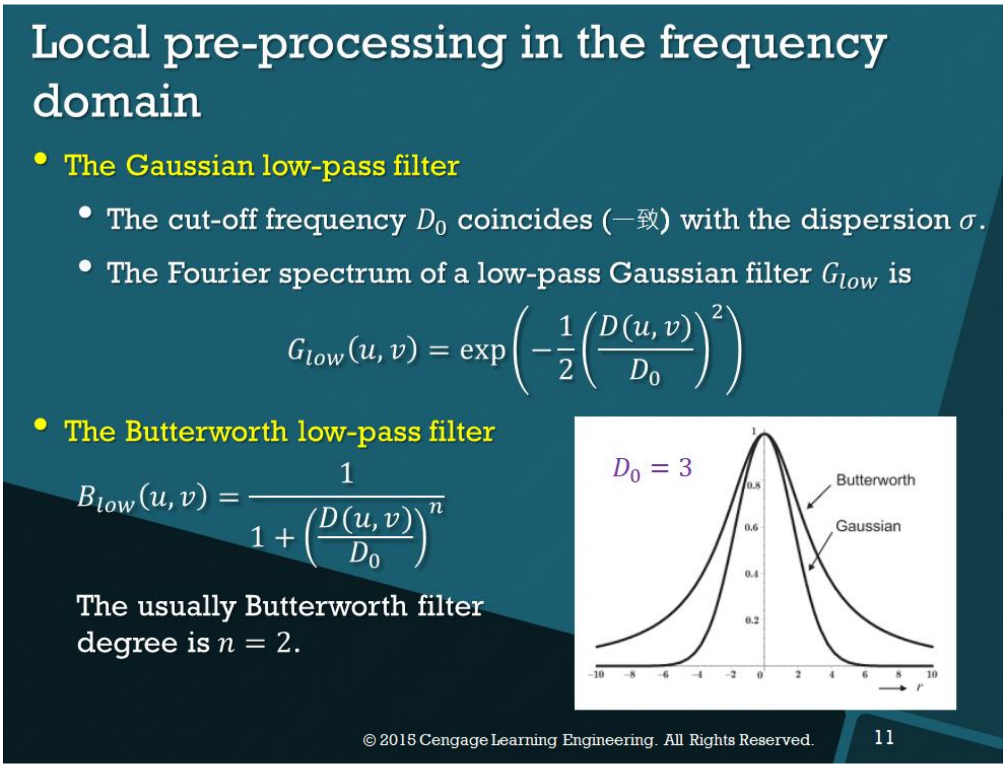


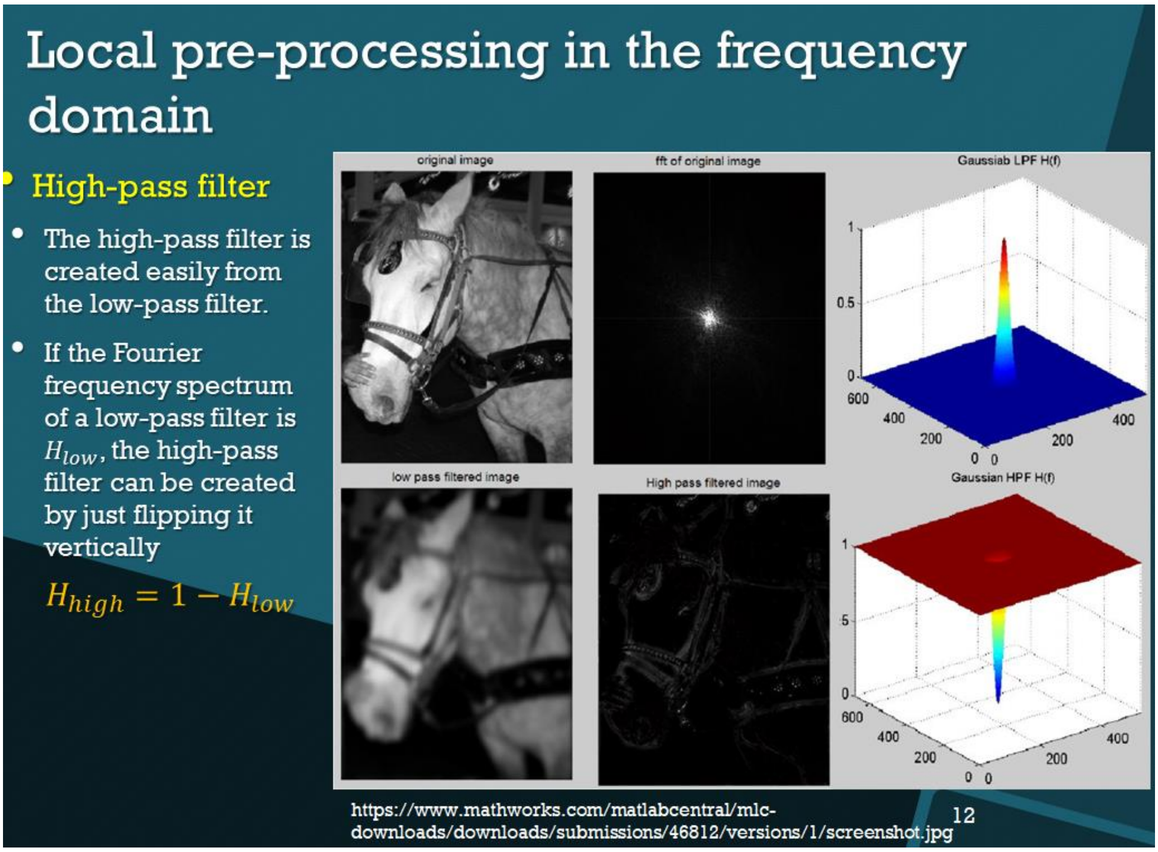
( O ) 19. Otsu's algorithm is one kind of optimal thresholding.



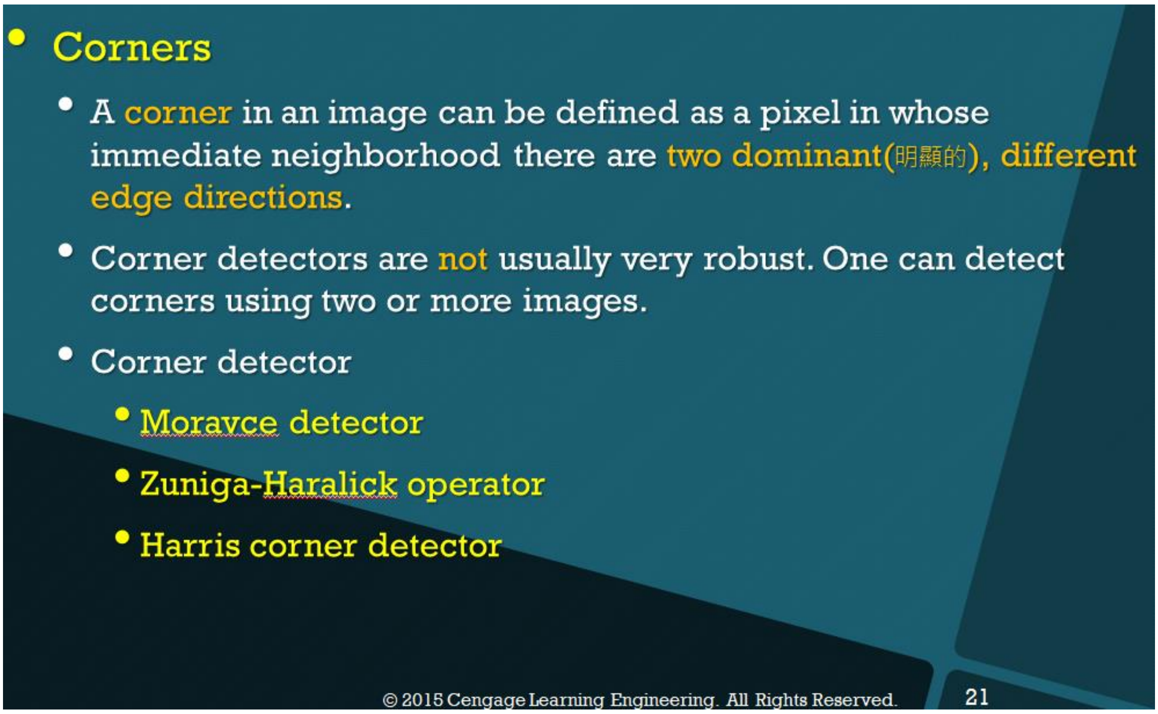

( X ) 20. Let $D_0$ be the cut-off frequency coincides with the dispersion $\sigma$, $D(u,v)$ be an isotropic (等向的) filter (for example, $D(u,v)=\sqrt{u^2+v^2}$ ), then the Fourier spectrum of the filter $G(u,v)=\exp\left(-\frac{1}{2}\left(\frac{D(u,v)}{D_0}\right)^2\right)$ is a ~~high~~(low)-pass Gaussian filter.



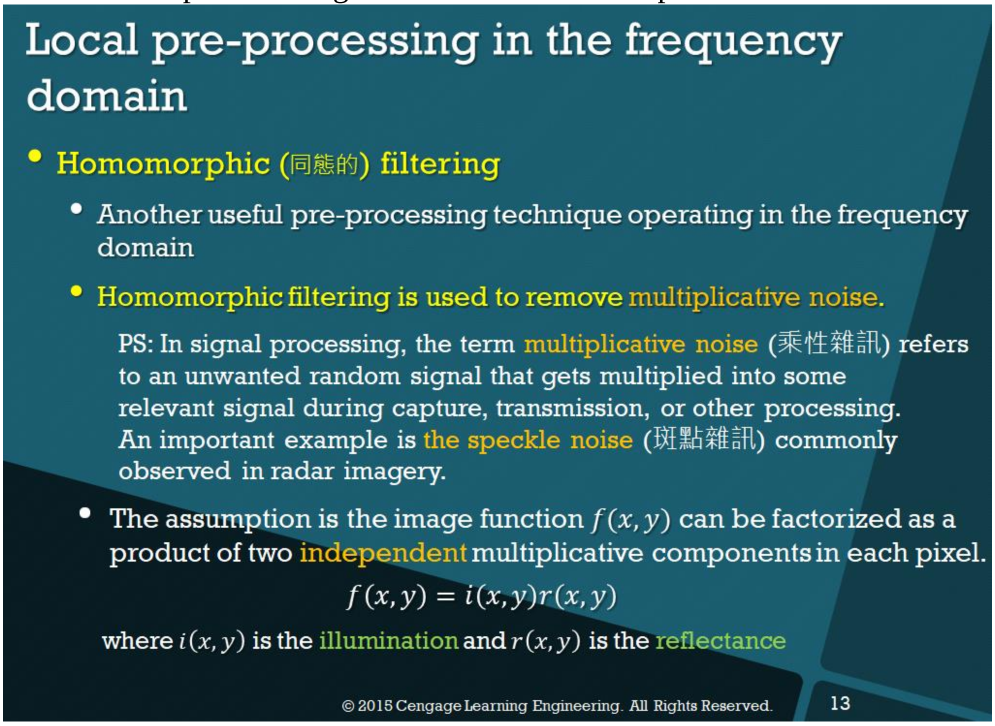
( X ) 21. The high-pass filter is created easily from the low-pass filter. If the Fourier frequency spectrum of a low-pass filter is $H_{low}$, the high-pass filter can be created by just flipping it vertically $\cancel{H_{high} = 1/H_{low}.}$ $(H_{high} = 1 - H_{low})$



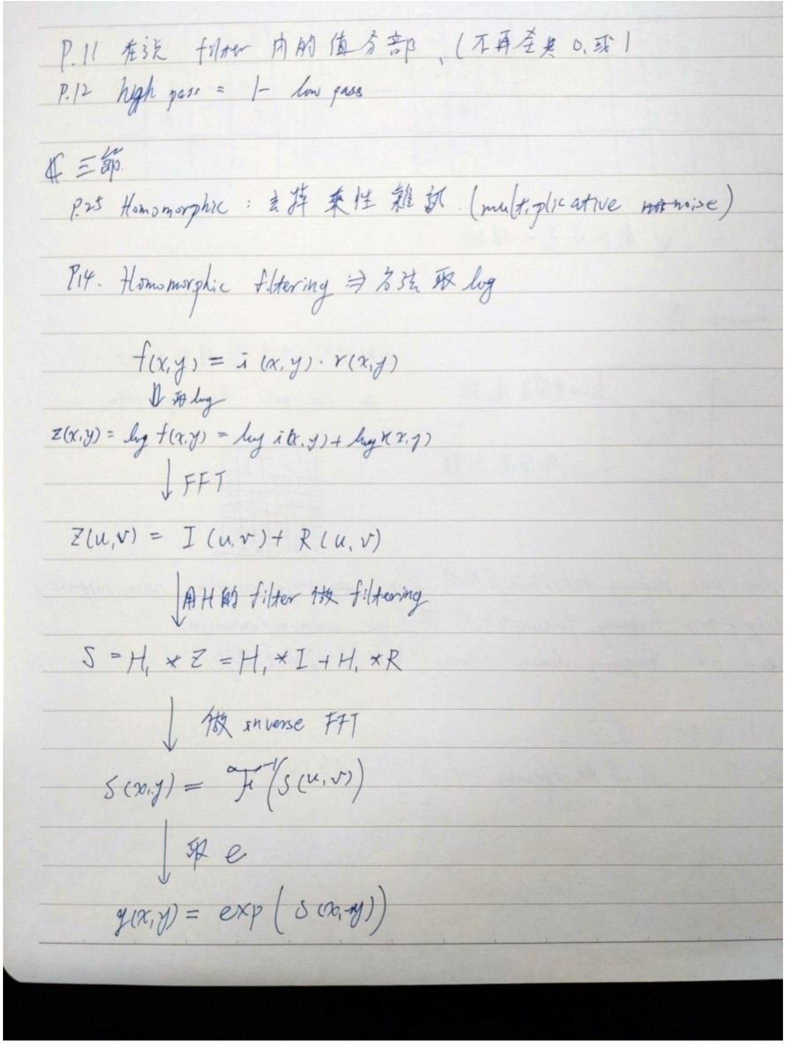

( O ) 22. A corner in an image can be defined as a pixel in whose immediate neighborhood there are two dominant, different edge directions.



( O ) 23. Homomorphic filtering is used to remove multiplicative noise.




[補充]:Homomorphic filtering 的作法



P.11 在說 filter 内的值分部，(不再是 0 或 1

P.12 high pass = 1 − low pass

第三節

P.25 Homomorphic：去掉乘性雜訊 (multiplicative ~~noi~~ noise)

P.14. Homomorphic filtering ⇒ 方法 取 log

$$f(x,y) = i(x,y) \cdot r(x,y)$$

$\Downarrow$ 取 log

$$z(x,y) = \log f(x,y) = \log i(x,y) + \log r(x,y)$$

$\downarrow$ FFT

$$Z(u,v) = I(u,v) + R(u,v)$$

$\downarrow$ 用 H 的 filter 做 filtering

$$S = H_1 * Z = H_1 * I + H_1 * R$$

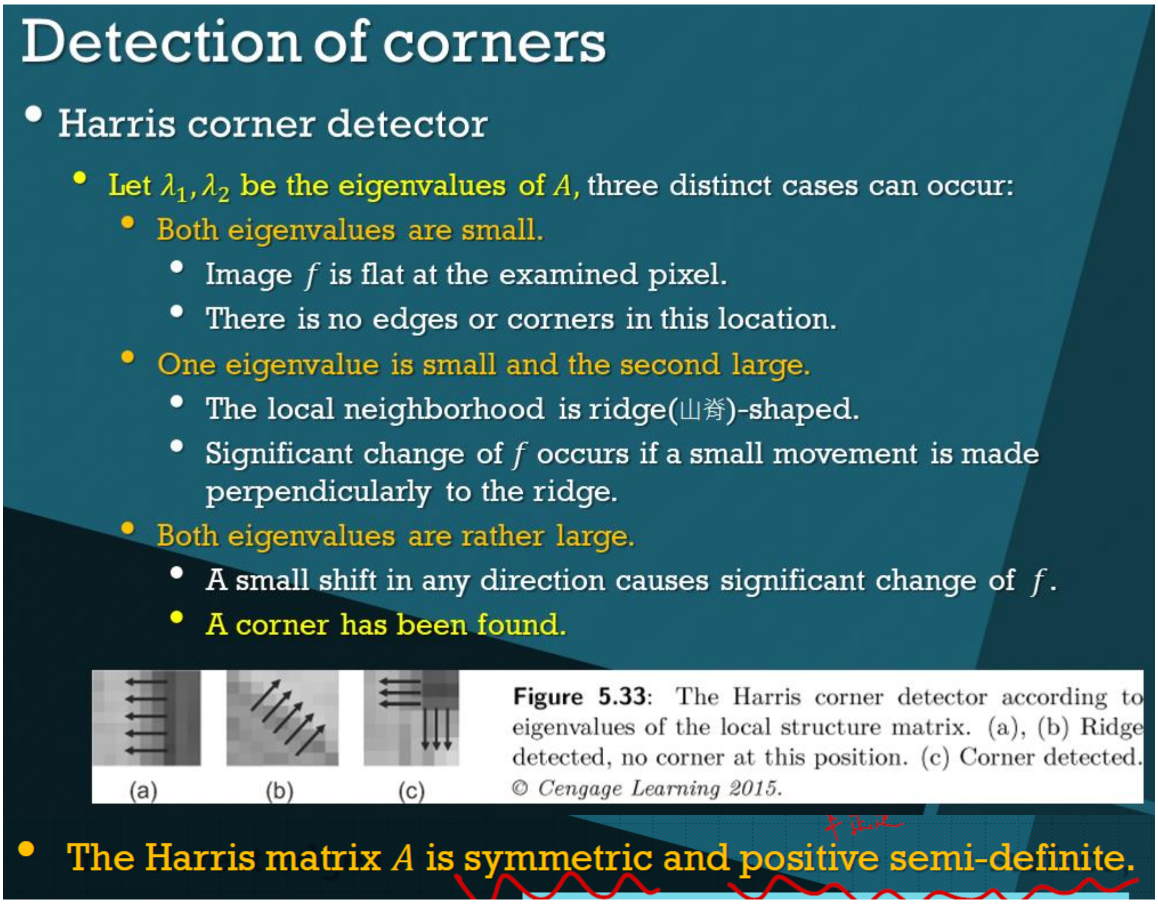
$\downarrow$ 做 inverse FFT

$$s(x,y) = \mathcal{F}^{-1}(S(u,v))$$

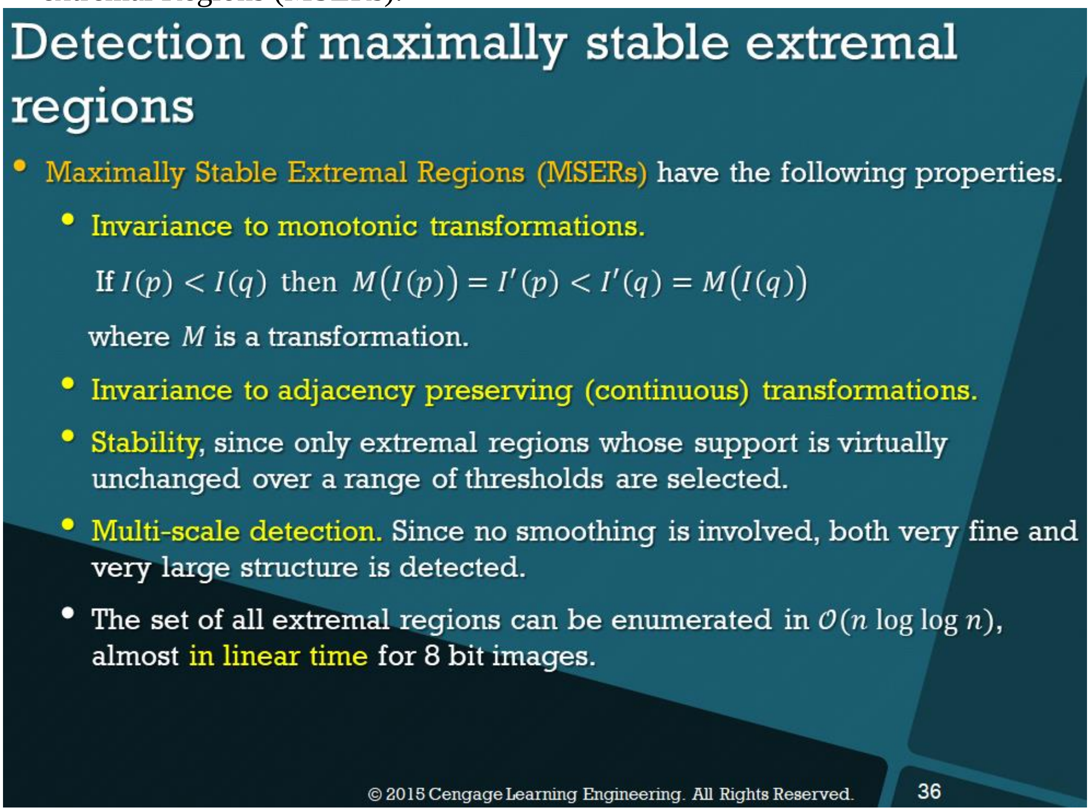
$\downarrow$ 取 e

$$g(x,y) = \exp(s(x,y))$$

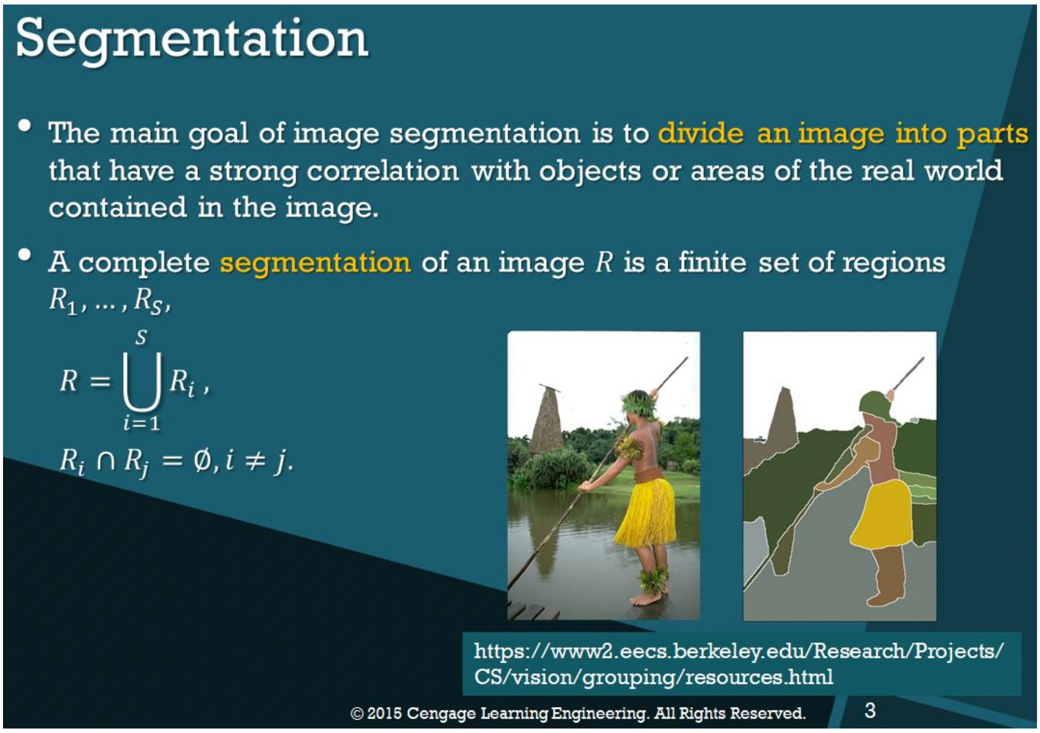
( X ) 24. In Harris corner detector, let $\lambda_1$ and $\lambda_2$ be the eigenvalues of the Harris matrix A, a corner has been found if both eigenvalues are ~~small~~.(large)



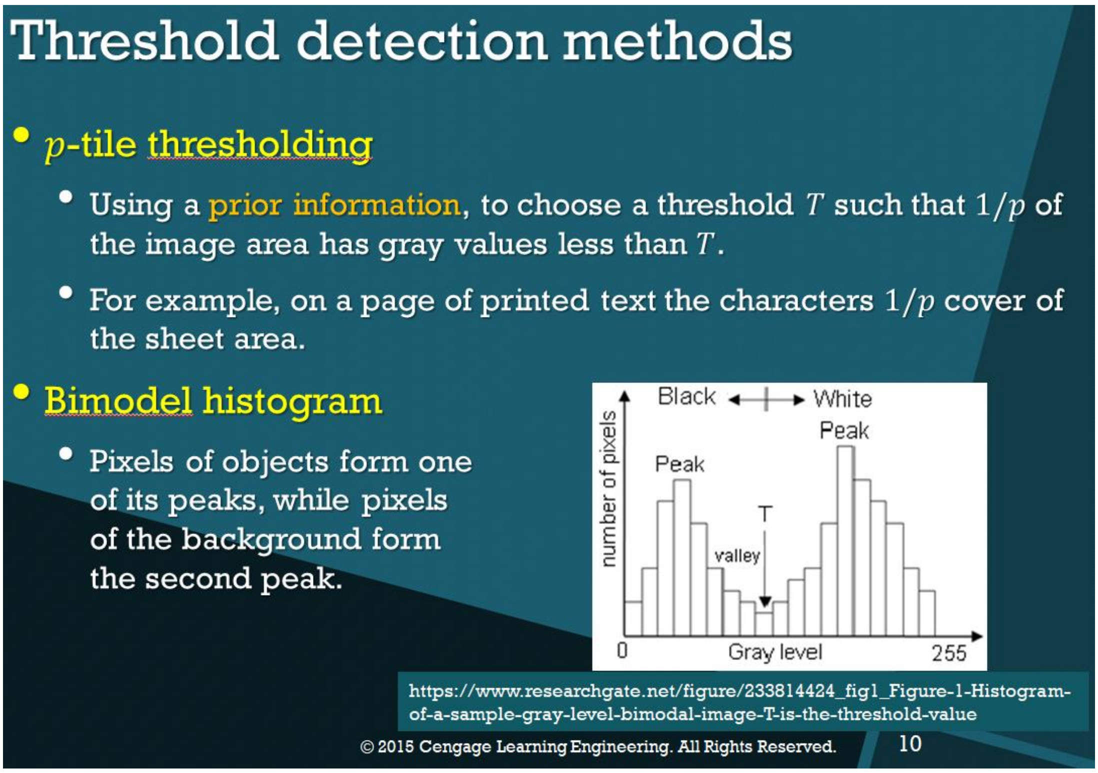

( O ) 25. Invariance to monotonic transformations is one property of maximally stable extremal Regions (MSERs).



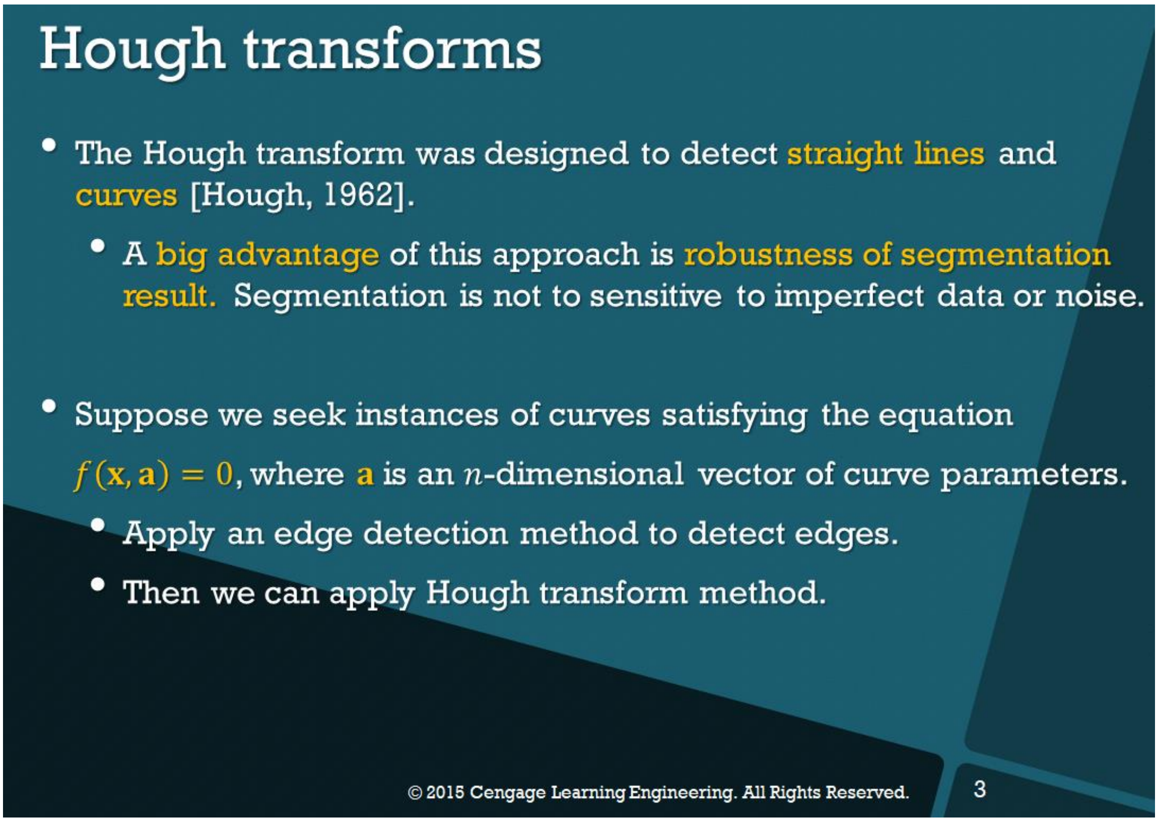
( O ) 26. A complete segmentation of an image $R$ is a finite set of regions $R_1, R_2, \ldots, R_S$, where $R = \bigcup_{i=1}^{S} R_i$, and $R_i \cap R_j = \emptyset,\ i \neq j$.



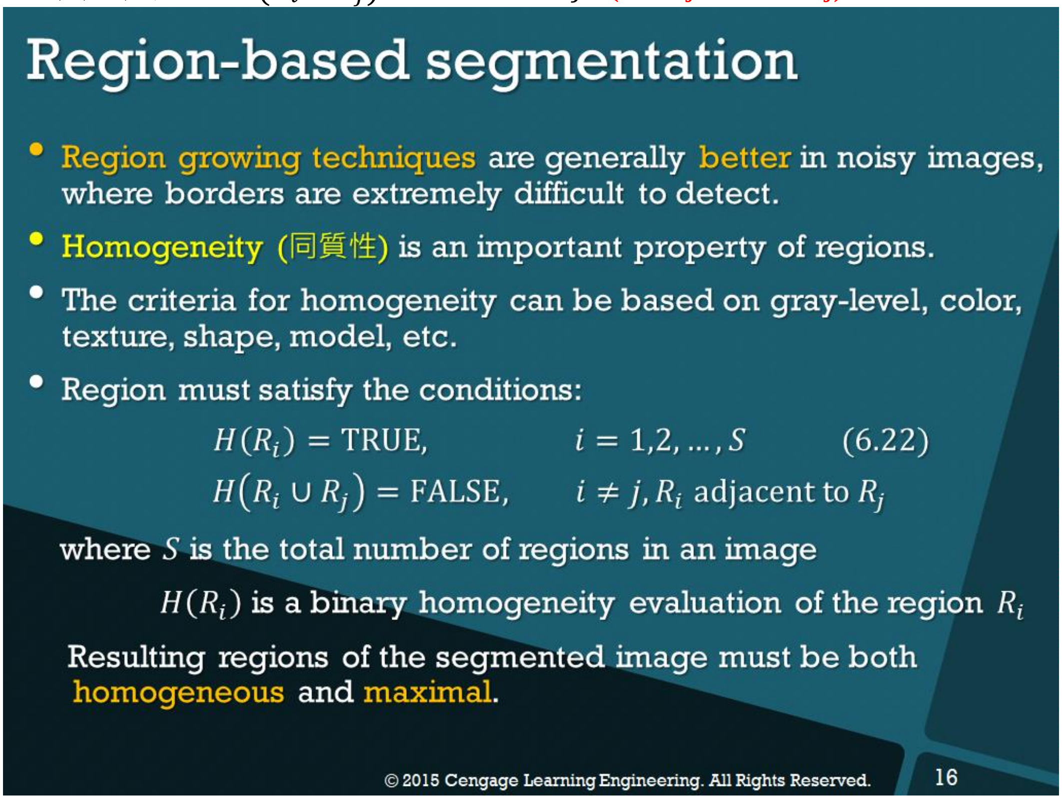

( O ) 27. $p$-tile(瓦)thresholding method uses a prior information to choose a threshold $T$ such that $1/p$ of the image area has gray values less than $T$.



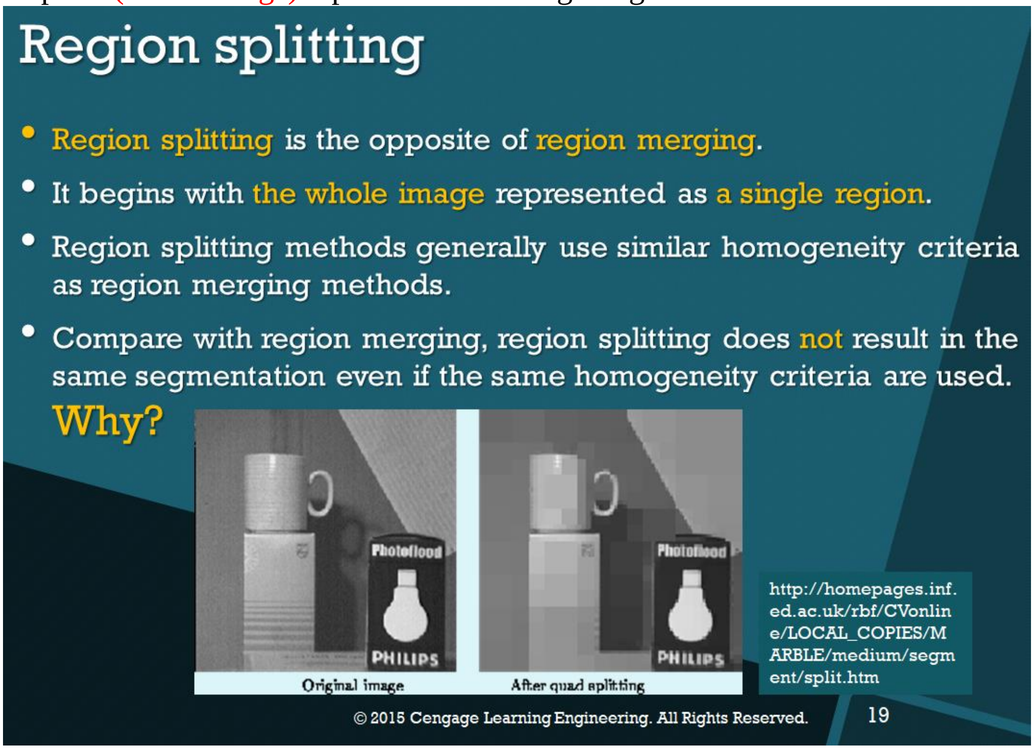
( X ) 28. The Hough transform was designed to detect straight lines and curves. However, this approach is (not) very sensitive to imperfect data or noise.




( X ) 29. Homogeneity (同質性) is an important property of regions. Let $S$ be the total number of regions in an image, and $H(R_i)$ be a binary homogeneity evaluation of the region $R_i$, the criteria for homogeneity can be defined as $H(R_i) = \mathrm{TRUE}, i = 1,2, \ldots, S$, and $H(R_i \cup R_j) = \mathrm{FALSE}, i \neq j$. ($R$i adjacent to $R$j)



( X ) 30. Region splitting is the opposite of region merging. Region splitting begins with ~~each pixel~~ (whole image) represented as a single region.




## 二、簡答證明題(assay question)

1. (2%) Here shows an edge detection mask, what is the value of *a*?

$$h = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & a & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

ANS: -12

2. (2%) Here shows a smoothing mask, what is the value of *b*?

$$h = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & b & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

ANS:4

3. (2%) What kind of objects can be detected by the following masks?

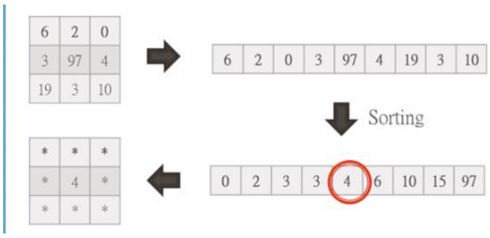
$$h = \begin{bmatrix} -1 & -1 & -1 \\ 2 & 2 & 2 \\ -1 & -1 & -1 \end{bmatrix}$$

ANS: 水平線

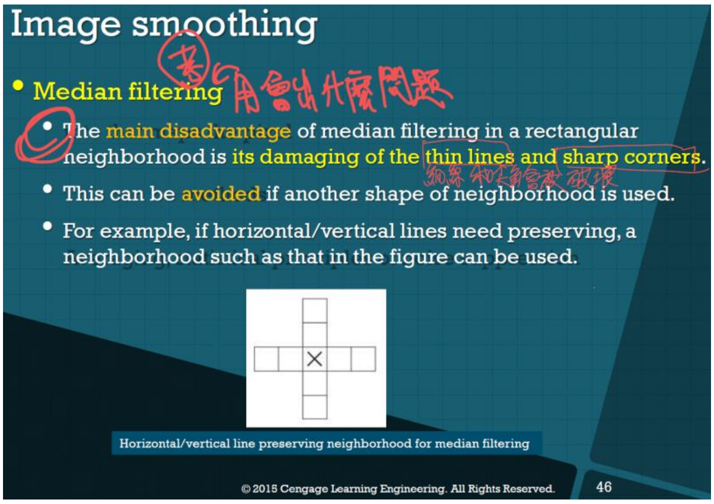
4. (6%) (a) How to remove the salt-and-pepper noises?
(b) How to remove the periodic (周期性的) noises?
(c) How to remove Gaussian noises?

ANS:

(a) 用 Median Filter 非常適合去除影像中較為極端的雜訊




[補充]




(b) 先轉換到 frequency domain 做 filtering 後 再轉換回 spatial domain

(c) 可以用(b)的方法 也可以用 averaging 的 filter(ex: $\frac{1}{10}\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 1\end{bmatrix}$ *這樣的filter*

5. (3%) Compare with region merging, region splitting does not result in the same segmentation even if the same homogeneity criteria are used. Why?

ANS: region merging 跟 region splitting 可能會因為執行順序不一樣而有不一樣的結果。比如:一張圖片[20 15 10 5] 用 15 為分界。在 merging 時會分成 [15 15 15] [5]，這時要再分裂就會還是[15 15 15] [5]，回不去了。

6. (2%) Hough transform is a very powerful technique for detection. What is the main problem of Hough transform?

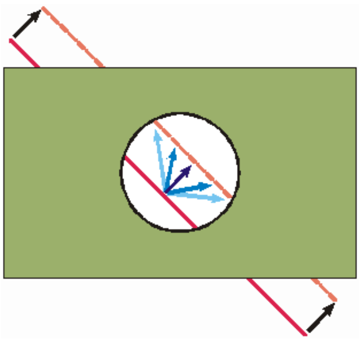
ANS: Hough 變換算法的特點導致其時間複雜度和空間複雜度都很高，並且在檢測過程中只能確定直線方向，丟失了線段的長度信息。

7. (4%) Corners is one kind of interest points for the correspondence problem.
   (a) (2%) What is a corner? (please give the definition of a corner)
   (b) (2%) Corners serves better than edges (lines) when the correspondence problem is to be solved because of the aperture problem (孔徑問題). What is the aperture problem?

   ANS:

   (a) A corner in an image can be defined as a pixel in whose immediate neighborhood there are two dominant(明顯的), different edge directions.

   (b) 當只能小範圍的觀察直線時，會無法判斷直線的移動方向。如下圖你會不知道是往哪個方向移動。



8. (2%) The following image is one example of corrupted (篡改;塗改) image. How to remove this kind of noise?

apply image averaging only to pixels in that interval

9. (4%) What is the meanings of the difference of Gaussian? Can you give an application which can use the convolution masks of the difference of Gaussian?

ANS:

(a) The $\nabla$^2 $G$ operator can be very effectively approximated by convolution with a mask that is the difference of two Gaussian averaging masks with substantially different $\sigma$.

The method is called
the difference of Gaussians (DoG)

(b) 可以用在 Zero-crossing 時做輔助

10. (4%) Zero-crossings is an edge detection technique. What is zero-crossings? What is the main advantages of zero-crossings?

ANS:

(a) An edge detection technique using the second derivative.
(b) it is much easier and more precise to find a zero-crossing position than an extremum.

11. (2%) Three most common brightness interpolation methods are nearest neighbor, linear and bi-cubic. Please explain the nearest neighbor interpolation method.

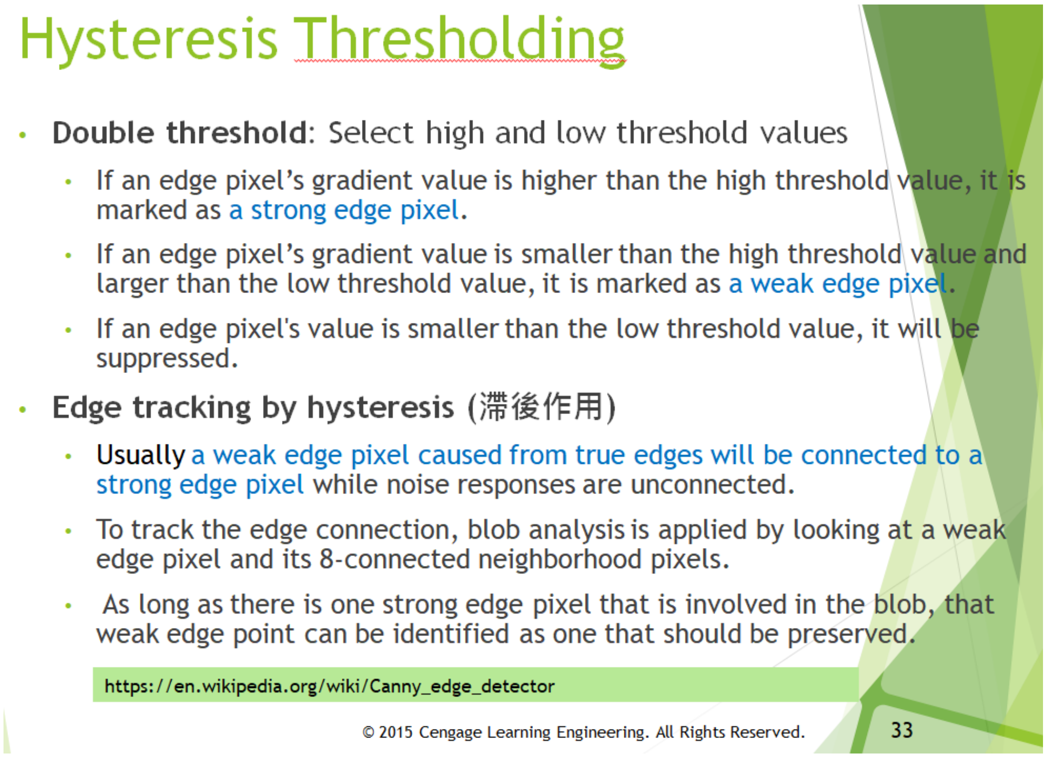
ANS:

$$f_1(x, y) = g_s(\mathrm{round}(x), \mathrm{round}(y))$$

12. (3%) Please explain what the hysteresis (滯後作用) approach used by Canny edge detection is.

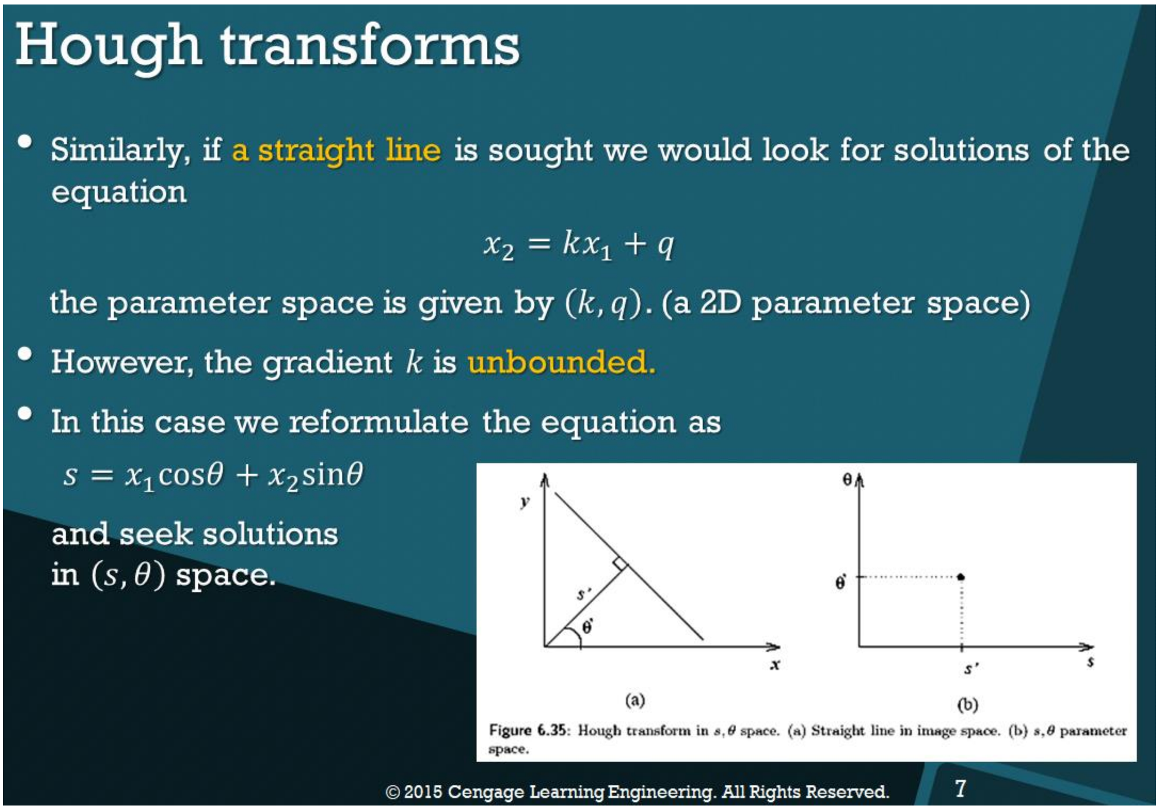
ANS:

Usually a weak edge pixel caused from true edges will be connected to a strong edge pixel while noise responses are unconnected



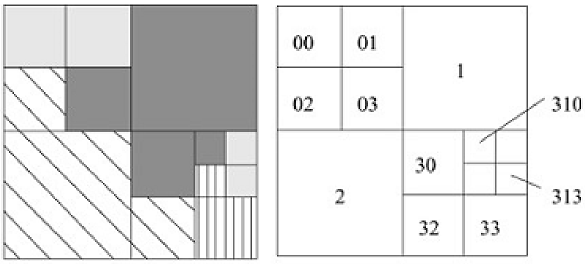

13. (2%) A straight line can be represented by the equation $x_2 = kx_1 + q$ with two the parameters $k$ and $q$. Why this equation is not suitable to represent a straight line in Hough transforms?

ANS: 因為 k 與 q 值為固定，所以可以產生無限多種可能的直線



14. (4%) (a) (2%) Please draw the corresponding quadtree of the following image.

(b) (2%) What is the **drawback** of segmentation quadtree?

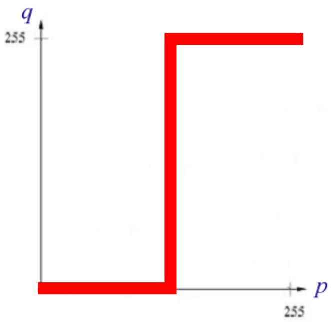



(a)

(b) the square-region shape assumption

15. (2%) A transformation $T$ of original brightness $p$ from scale $[p_0, p_k]$ into brightness $q$ from a new scale $[q_0, q_k]$ is given by $q = T(p)$. Given an original image as Figure (a), can you draw the transformation $T$ if the output image is (b)?


(a) original image



Function $T$


(b) output image

16. (4%) (1) (2%) Figure (a) shows the original image, and Figure (b) shows the filtered image. What kind of filters is used in the frequency domain? Low-pass filters? High-pass filters? Band-pass filters? Or homomorphic filtering? ANS:___Band-pass filters

(2) (2%) If the filtered image is shown in Figure (c), then what kind of the filters is used in the frequency domain? ANS:____________low-pass filters


(a)


(b)


(c)

17. (4%) Here shows Otsu's threshold detection algorithm, please fill in the blanks.

**Algorithm 6.2 Otsu's threshold detection**

1. For an image $I$ on $G$ gray levels, compute the gray-level histogram $H(0), H(1), \ldots, H(G\text{-}1)$. Normalize the histogram by dividing through by the number of pixels in $I$ –the histogram now represents the probability of each gray level.

2. For each possible threshold $t = 0,1,\ldots,G-2$, partition the histogram into background $B$ (gray-levels less than or equal to $t$), and foreground $F$ (gray-levels more than $t$).

3. Compute $\sigma_B(t)$, $\sigma_F(t)$, the variance of the background and foreground gray-levels. Compute the probability of a pixel being background

$w_B(t) = \sum_{j=0}^{t} H(j)$ and $w_F(t)$ similarly. Set

$\sigma(t) =$ ______ $\sigma(t) = w_B(t)\sigma_B(t) + w_F(t)\sigma_F(t)$ ______

and select as threshold $\hat{t} =$ ______ $\arg\min_t(\sigma(t))$ ______.

18. (3%) Here shows a histogram equalization algorithm, please fill in the blanks.

**Algorithm 5.1 Histogram equalization**

1. For an $N \times M$ image of $G$ gray-levels, initialize an array $H$ of length $G$ to 0.
2. From the image histogram: Scan every pixel $p$ – if it has intensity $g_p$, perform

   $H[g_p] =$ ______ $H[g_p] + 1$ ______

   Then let $g_{\min}$ be the minimum $g$ for which $H[g] > 0$.
3. Form the cumulative image histogram $H_c$:

   $H_c[0] = H[0]$

   $H_c[g] =$ ______ $H_c[g-1] + H[g]$ ______, $g = 1,2,\ldots,G-1$

   Let $H_{\min} = H_c[g_{\min}]$.
4. Set

   $$T[g] = round\left(\frac{H_c[g] - H_{\min}}{MN - H_{\min}}(G-1)\right)$$
5. Rescan the image and write an output image with gray-levels $g_q$, setting

   $g_q =$ ______ $T[g_p]$ ______

19. (4%) Given a 2D Gaussian filter

$$G(x, y) = e^{-(x^2+y^2)/2\sigma^2}$$

where $\sigma$ is the standard deviation. Please derive $\frac{\partial^2 G}{\partial x^2}$, and the Laplacian of Gaussian $\nabla^2 G$.

( PS. $\nabla^2 G = \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2}$ )

$$G(x,y) = e^{-(x^2+y^2)/2\sigma^2}$$

$$\frac{\partial G}{\partial x} = -\frac{2x}{2\sigma^2} \cdot e^{-\frac{(x^2+y^2)}{2\sigma^2}} = -\frac{x}{\sigma^2} \times e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

$$\frac{\partial^2 G}{\partial x^2} = -\frac{1}{\sigma^2} \times e^{-\frac{(x^2+y^2)}{2\sigma^2}} + + \frac{x}{\sigma^2} \cdot \left[ + \frac{x}{\sigma^2} \times e^{\frac{-(x^2+y^2)}{2\sigma^2}} \right]$$

$$= \frac{1}{\sigma^2} \times e^{-\frac{(x^2+y^2)}{2\sigma^2}} \cdot \left( \frac{x^2}{\sigma^2} - 1 \right)$$

$$\frac{\partial G}{\partial y} = -\frac{y}{\sigma^2} \times e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

$$\frac{\partial^2 G}{\partial y^2} = \frac{1}{\sigma^2} \times e^{-\frac{(x^2+y^2)}{2\sigma^2}} \times \left( \frac{y^2}{\sigma^2} - 1 \right)$$

$$\nabla^2 G = \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2}$$

$$= \left[ \frac{1}{\sigma^2} \times e^{-\frac{(x^2+y^2)}{2\sigma^2}} \cdot \left( \frac{x^2}{\sigma^2} - 1 \right) \right] + \left[ \frac{1}{\sigma^2} \times e^{-\frac{(x^2+y^2)}{2\sigma^2}} \cdot \left( \frac{y^2}{\sigma^2} - 1 \right) \right]$$

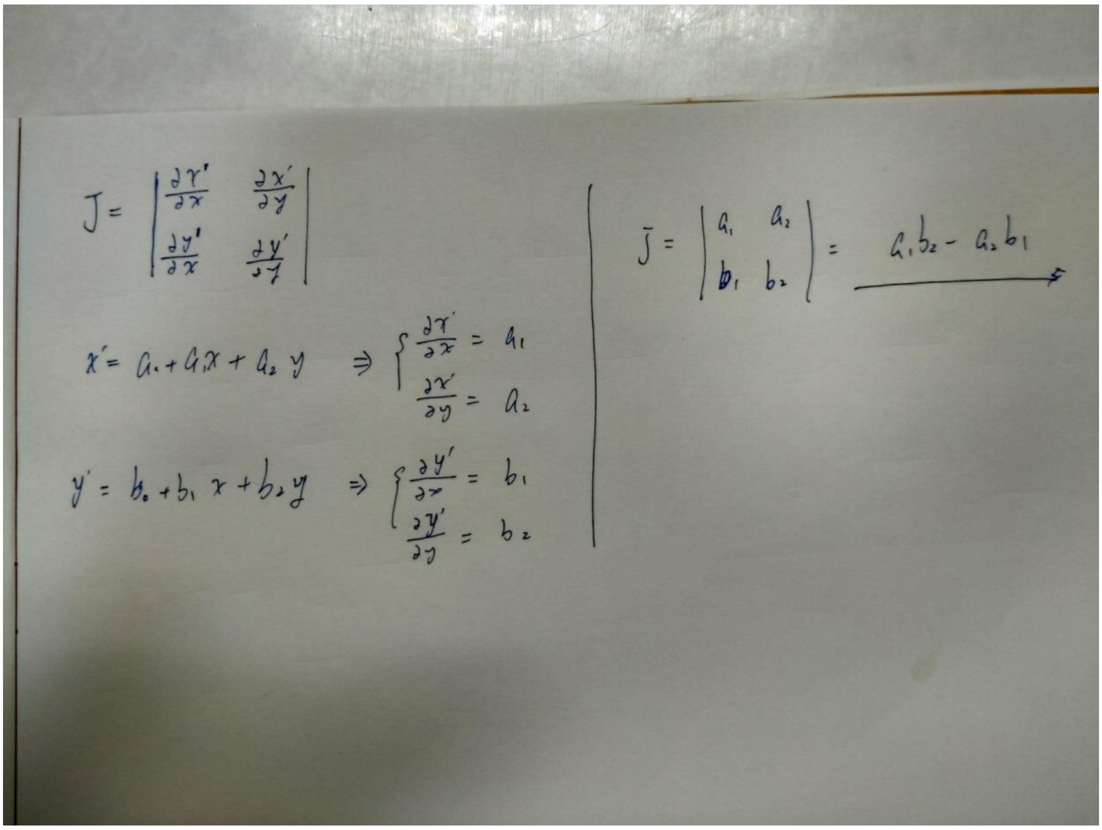
$$= \frac{1}{\sigma^2} \times e^{-\frac{(x^2+y^2)}{2\sigma^2}} \times \left( \frac{x^2+y^2}{\sigma^2} - 2 \right)$$

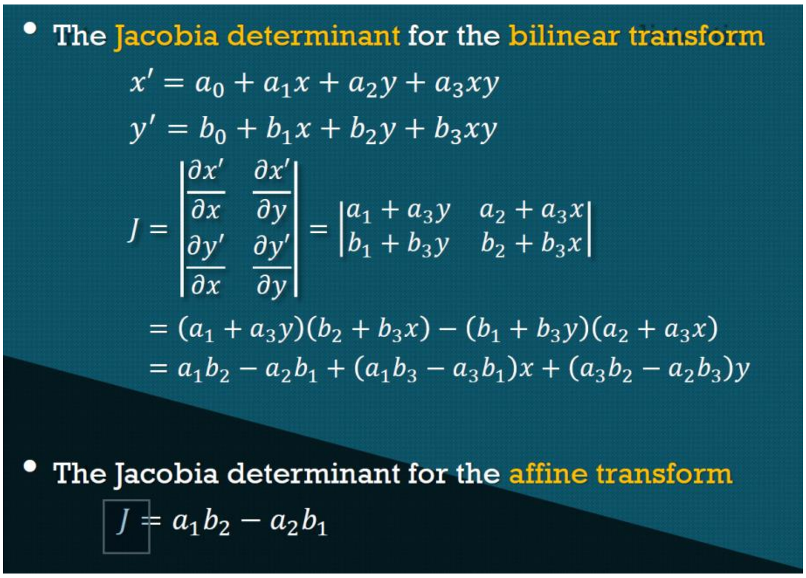
20. (2%) A geometric transform is a vector function $T$ that maps the pixel $(x, y)$ to a new position $(x', y')$. $T = (T_x, T_y)$, where $x' = T_x(x, y), y' = T_y(x, y)$. The geometric transform can be approximated by an affine transform.

$$x' = a_0 + a_1 x + a_2 y;\ y' = b_0 + b_1 x + b_2 y$$

A Jacobia determinant $J$ can provides information about how the co-ordinate system changes. $J = \left|\frac{\partial(x', y')}{\partial(x, y)}\right|$

Please show the Jacobia determinant for the transform.



21. (6%) Given an inner boundary tracing algorithm as follows.

**Algorithm: Inner boundary tracing in 8-connectivity**

1. Search the image from top left until a pixel $P_0$ of a new region is found; this has the minimum column value of all pixels of that region having the minimum row value. $P_0$ is a starting pixel of the region border.

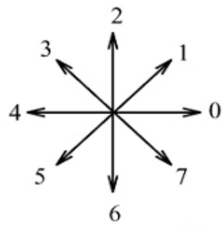
   Define a variable $dir$ which stores the direction of the previous move along the border from the previous border element to the current border elements. Assign $dir = 7$ if the border is detected in 8-connectivity.

2. Search the $3 \times 3$ neighborhood of the current pixel in an anti-clockwise direction, beginning the neighborhood search in the pixel positioned in the direction

   $(dir + 7) \bmod 8$ if $dir$ is even. (8-connectivity)

   $(dir + 6) \bmod 8$ if $dir$ is odd. (8-connectivity)

   The first pixel found with the same value as the current pixel is a new boundary element $P_n$. Update the $dir$ value.




3. If the current boundary element $P_n$ is equal to the second border element $P_1$, and if the previous border element $P_{n-1}$ is equal to $P_0$, stop. Otherwise repeat step 2.
4. Pixels $P_0, P_1, ..., P_{n-2}$ are now the detected inner border.

**Questions:**

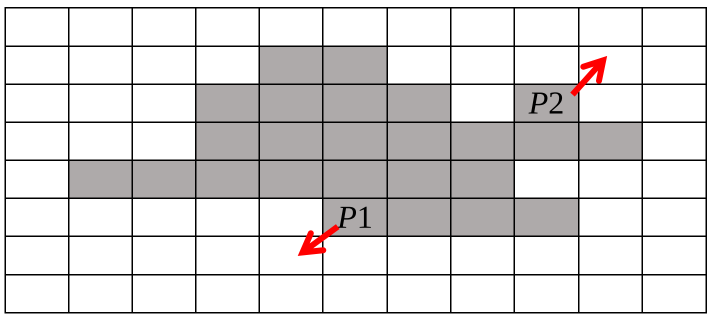
(1) (2%) What are the $dir$ values at $P1$ and $P2$?

ANS:

P1:的 dir = 7

P2:的  dir = 3

(2) (4%) What is the starting search pixel after $P1$ and $P2$? Please mark them in the image.




P1:下一個: 7 為奇數=> (7+6)%8=5

P2:下一個: 3 為奇數=> (3+6)%8=1

22. (3%) Here define four kind of inner boundary pixels of a region $R$:

If $Q$ denote outside the region $R$, then pixel $P \in R$ is

a LEFT pixel of $R$ if $P_4(P) \in Q$

a RIGHT pixel of $R$ if $P_0(P) \in Q$

an UPPER pixel of $R$ if $P_2(P) \in Q$

a LOWER pixel of $R$ if $P_6(P) \in Q$

| | | |
|---|---|---|
| 3 | 2 | 1 |
| 4 | $P$ | 0 |
| 5 | 6 | 7 |

For example, $P_4(P)$ denotes the pixel immediately to the left of pixel $P$.

Let $LEFT(R), \mathrm{RIGHT}(R), \mathrm{UPPER}(R), \mathrm{LOWER}(R)$ represent the corresponding subsets of $R$.

How to define the extended boundary EB?

$\mathrm{EB} = \{P: P \in LEFT(R)\} \cup \{P: P \in \mathrm{UPPER}(R)\}$

$\cup$ {____P0____ $: P \in \mathrm{RIGHT}(R)\}$ $\cup$ {____P7____ $: P \in \mathrm{RIGHT}(R)\}$

$\cup$ {____P6____ $: P \in \mathrm{LOWER}(R)\}$