電腦視覺與應用
Computer Vision and Applications

Lecture08-3D reconstruction

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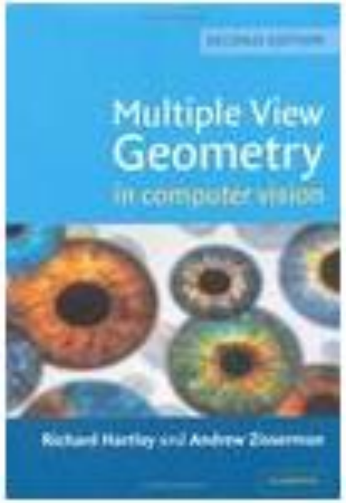
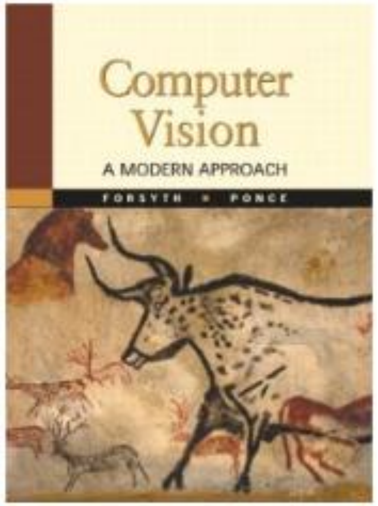
3D reconstruction

- Either from calibrated images
- Or from uncalibrated images

3D reconstruction

- Lecture Reference at:
  - Multiple View Geometry in Computer Vision, (*Chapter10 and 12)
  - Computer Vision A Modern Approach, Chapter 13



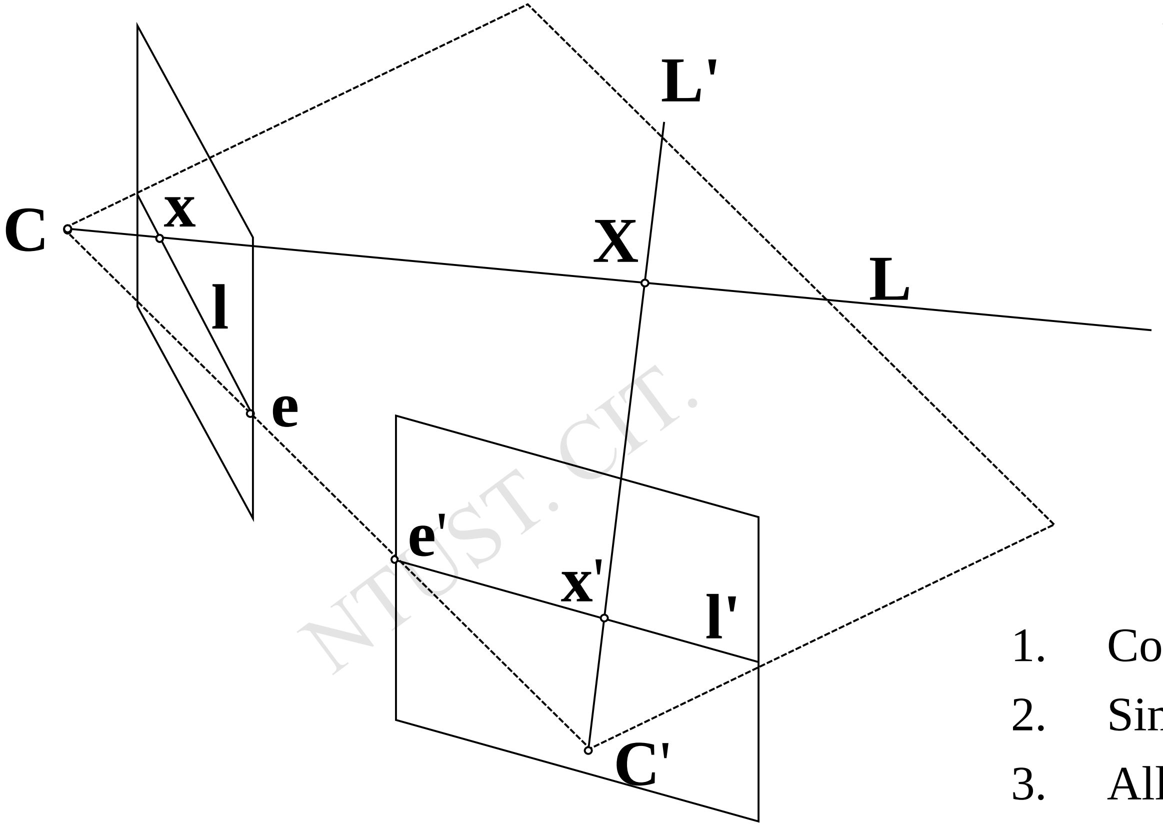


3D reconstruction

- The questions?
  - Correspondence geometry: Given an image point x in the first image, how does this constrain the position of the corresponding point x' in the second image?
  - Camera geometry (motion): Given a set of corresponding image points $\{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i\}$, $i=1,\ldots,n$, what are the cameras **P** and **P'** for the two views?
  - Scene geometry (structure): Given corresponding image points $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$ and cameras **P**, **P'**, what is the position of **X** in space?

3D reconstruction

- Epipolar geometry



$$\underbrace{\mathbf{x}'^{\mathrm{T}}\overbrace{\mathbf{F}\mathbf{x}}^{\mathbf{l}'}}_{\mathbf{l}^{\mathrm{T}}} = 0$$

1. Computable from corresponding points
2. Simplifies matching
3. Allows to detect outliers
4. Related to calibration

3D reconstruction of cameras and structure

- Reconstruction problem:

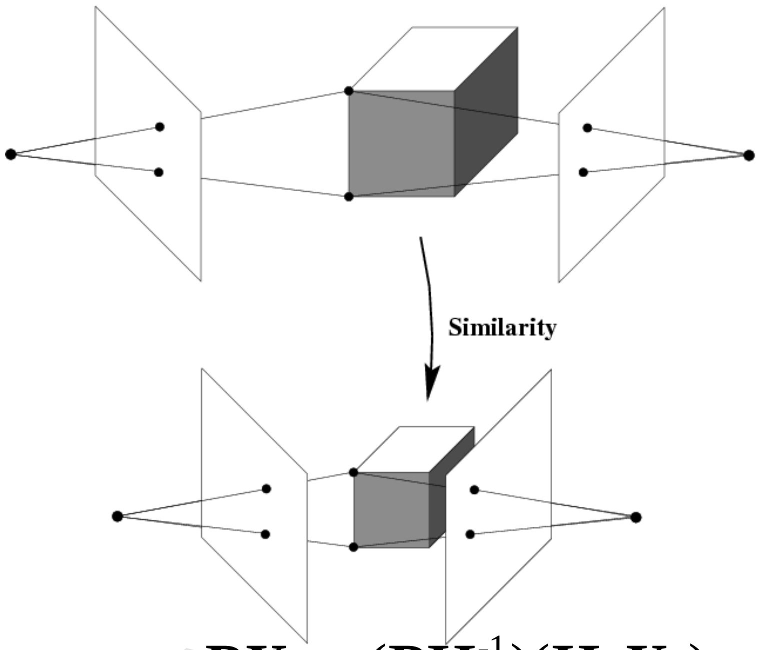
Given $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$, compute $\mathbf{P}$, $\mathbf{P}'$ and $\mathbf{X}_i$

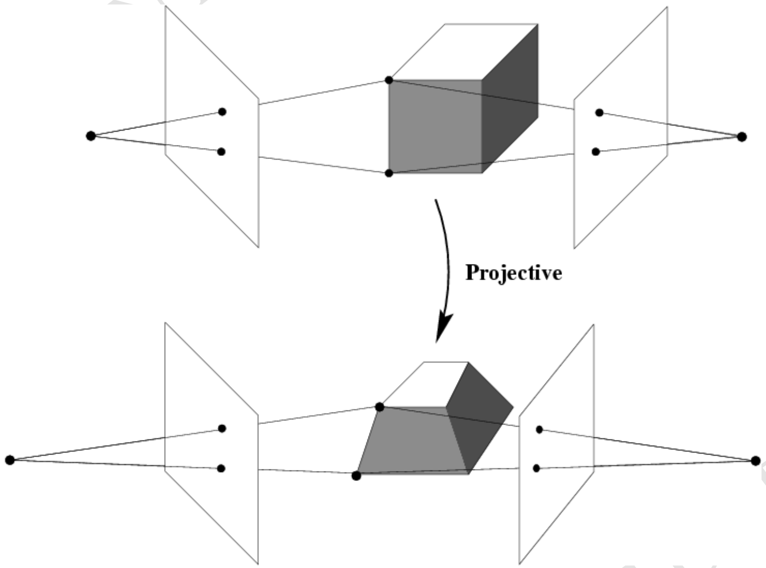
$$\mathbf{x}_i = \mathbf{P}\mathbf{X}_i \quad \text{and} \quad \mathbf{x}_i' = \mathbf{P}'\mathbf{X}_i \quad \text{for all } i$$

without additional constraints possible up to projective ambiguity

3D reconstruction of cameras and structure

- Reconstruction ambiguity (projective ambi.)





$$\mathbf{x}_i = \mathbf{P}\mathbf{X}_i = (\mathbf{P}\mathbf{H}_S^{-1})\underbrace{(\mathbf{H}_S\mathbf{X}_i)}$$

$$\mathbf{x}_i = \mathbf{P}\mathbf{X}_i = (\mathbf{P}\mathbf{H}_P^{-1})\underbrace{(\mathbf{H}_P\mathbf{X}_i)}$$

One solution which is projected to $\mathbf{x}_i$, as well.

Outline of 3D reconstruction (from uncalibrated images)

- Compute F from correspondences
- Compute camera matrices from F
- Compute 3D point for each pair of corresponding points

computation of **F**

use $\mathbf{x}'_i\mathbf{F}\mathbf{x}_i=0$ equations, linear in coeff. F

computation of camera matrices

use $\mathbf{P}=[\mathbf{I}\,|\,0]\quad \mathbf{P}'=[[\mathbf{e}']_\times\mathbf{F}+\mathbf{e}'\mathbf{v}^T\,|\,\lambda\mathbf{e}']$ → General formula

or $\mathbf{P}=[\mathbf{I}\,|\,0]\quad \mathbf{P}'=[[\mathbf{e}']_\times\mathbf{F}\,|\,\mathbf{e}']$

detail at

Q. Luong and T. Vieville, "Canonical representations for the geometries of multiple projective views," *Computer vision and image understanding*, vol. 64, no. 2, pp. 193-229, 1996.

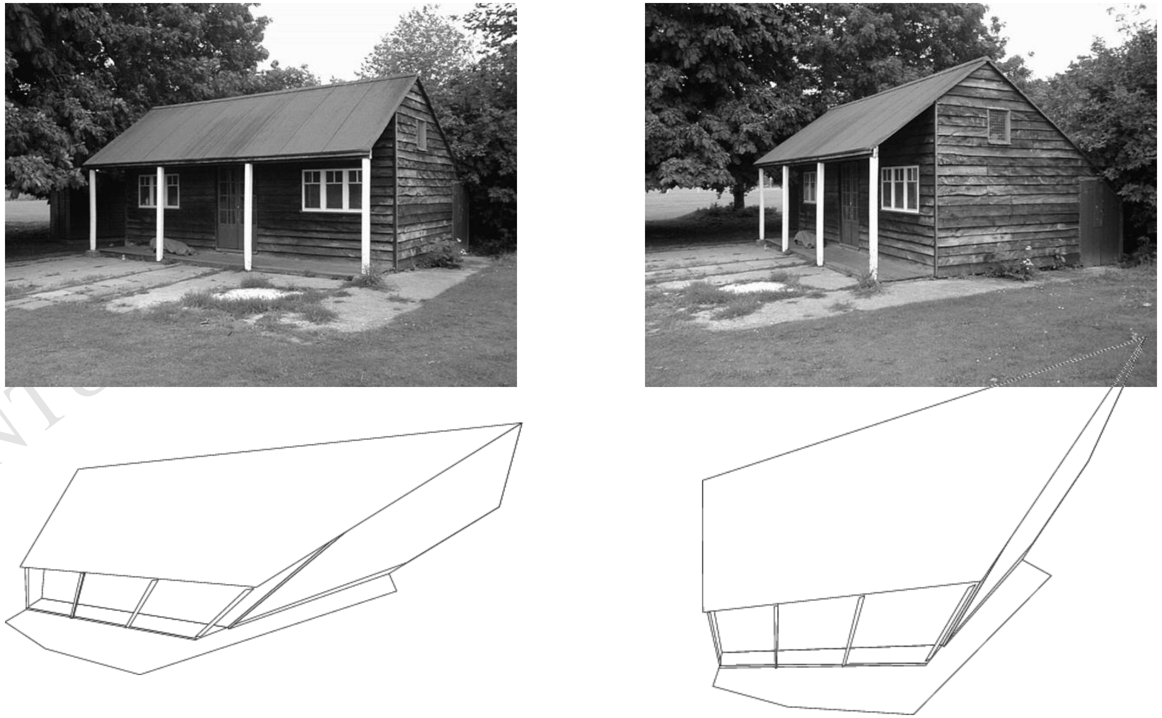
triangulation

compute intersection of two back-projected rays

[from Hartley04 sec. 10.1]

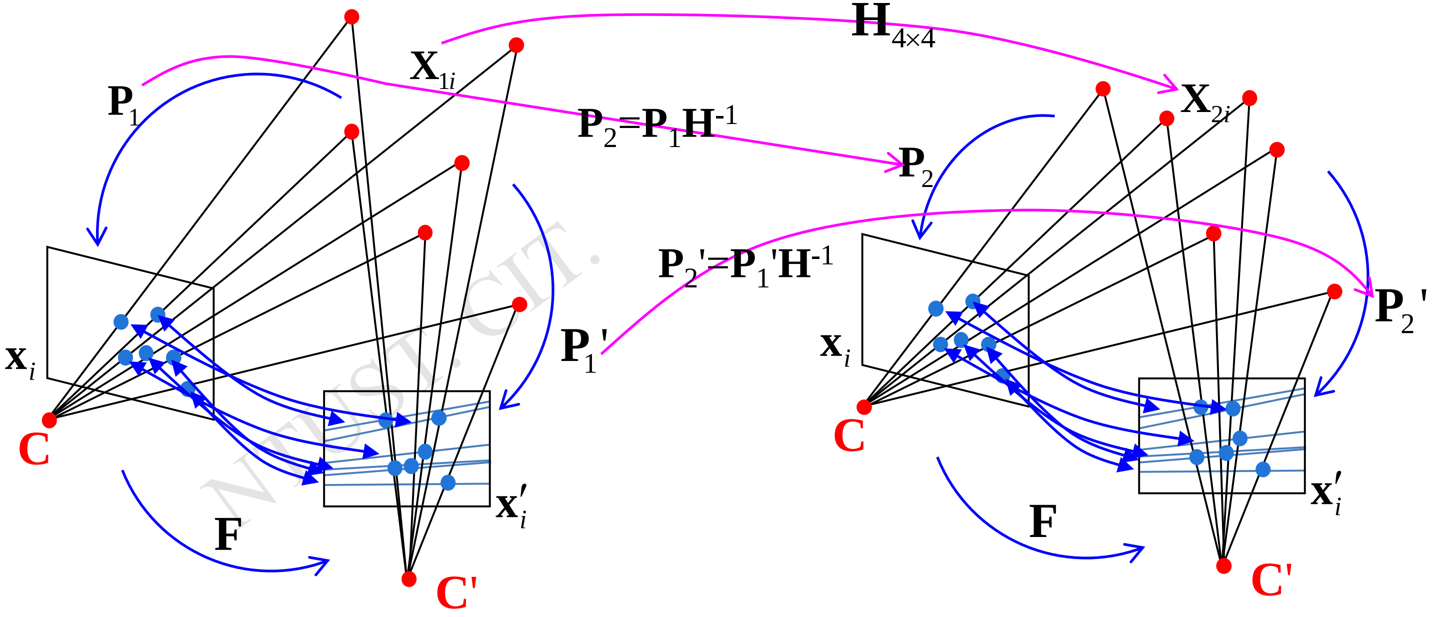
Projective reconstruction

- The reconstruction required no information about camera matrices, or information about the scene geometry. The fundamental matrix **F** is computed from point correspondences between the iamges, camera matrices are retrived from **F**, and then 3D points are computed by triangulation from triangulation from the correspondences.



Theorem for the projective reconstruction

Suppose that $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$ is a set of correspondences between points in two images and that the fundamental matrix $\mathbf{F}$ is uniquely determined by the condition $\mathbf{x}'^{\mathrm{T}}_i \mathbf{F} \mathbf{x}_i = 0$ for all i. Let $(\mathbf{P}_1, \mathbf{P}_1', \{\mathbf{X}_{1i}\})$ and $(\mathbf{P}_2, \mathbf{P}_2', \{\mathbf{X}_{2i}\})$ be two reconstructions of the correspondences $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$. Then there exists a non-singular matrix $\mathbf{H}$ such that $\mathbf{P}_2 = \mathbf{P}_1 \mathbf{H}^{-1}$, $\mathbf{P}_2' = \mathbf{P}_1' \mathbf{H}^{-1}$ and $\mathbf{X}_2 = \mathbf{H}\mathbf{X}_1$ for all i, except for those i such $\mathbf{F}\mathbf{x}_i = \mathbf{x}_i'\mathbf{F} = 0$



$\mathbf{P}_2 = \mathbf{P}_1 \mathbf{H}^{-1}$

$\mathbf{P}_2' = \mathbf{P}_1' \mathbf{H}^{-1}$

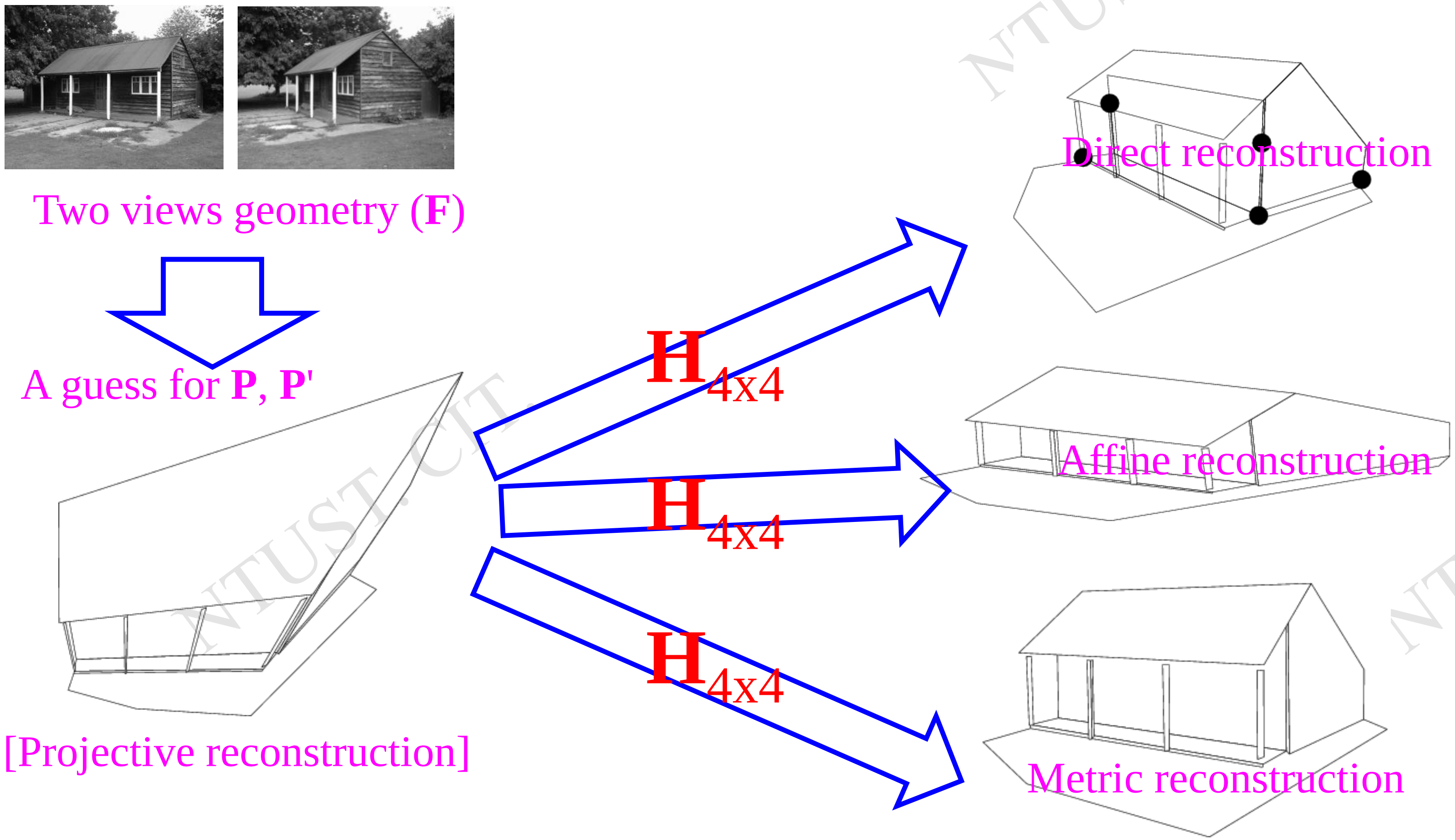
$\mathbf{X}_2 = \mathbf{H}\mathbf{X}_1$

key result:
allows reconstruction from pair of uncalibrated images

Theorem 10.1 [Hartley04]

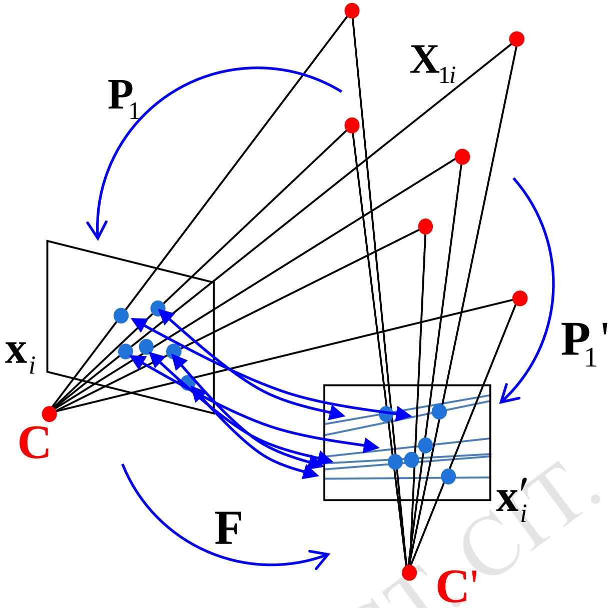
3D reconstruction for projective geometry

- In practice

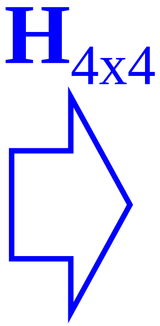


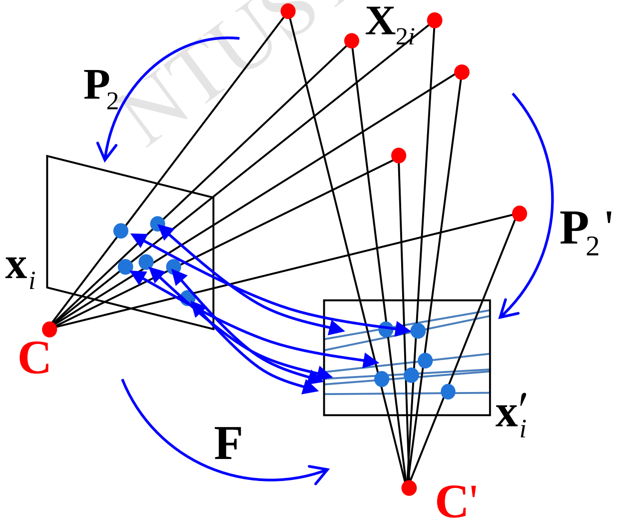
Theorem for the projective reconstruction

- In practice



[Projective reconstruction]





Affine reconstruction
or Metric reconstruction

Step-1: solve epipolar geometry (**F**)
Step-2: guess **P** & **P'** for intitalization, then determine **X**.
Step-3: determine **H**, then, finding new **P** & **P'** & **X**.

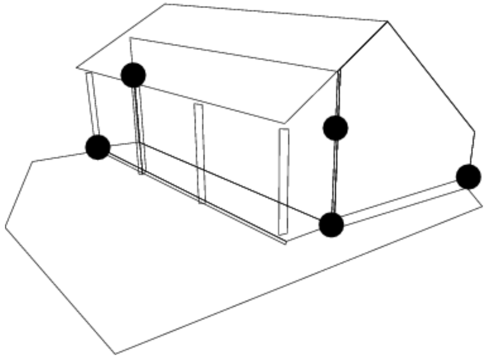
$\mathbf{P}_2 = \mathbf{P}_1\mathbf{H}^{-1}$

$\mathbf{P}_2' = \mathbf{P}_1'\mathbf{H}^{-1}$

$\mathbf{X}_2 = \mathbf{H}\mathbf{X}_1$

Direct reconstruction using ground truth

- use control points $\mathbf{X}_{Ei}$ with know coordinates, to go from projective to metric



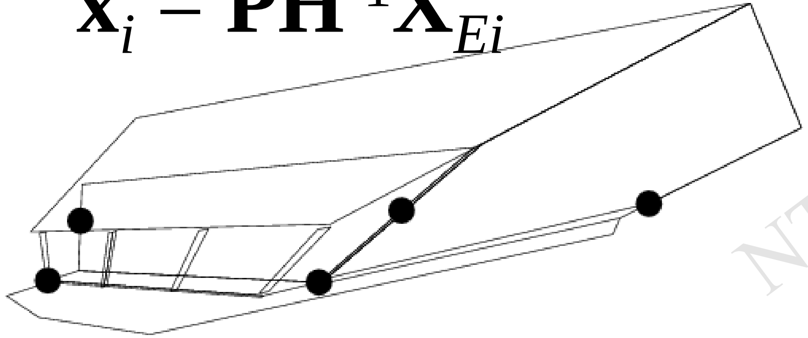
$$\mathbf{X}_{Ei} = \mathbf{H}\mathbf{X}_{\mathrm{i}}$$

Homography (4x4)

3D points for an estimated projective reconstruction.

Ground truth
(measurement in real environment)

$$\mathbf{x}_i = \mathbf{P}\mathbf{H}^{-1}\mathbf{X}_{Ei}$$

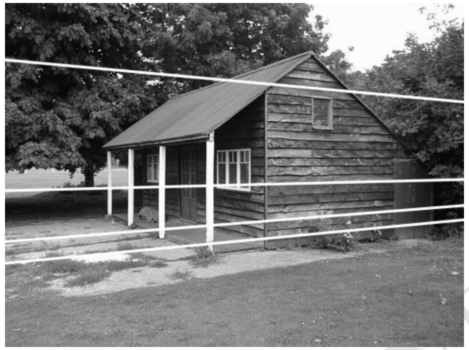
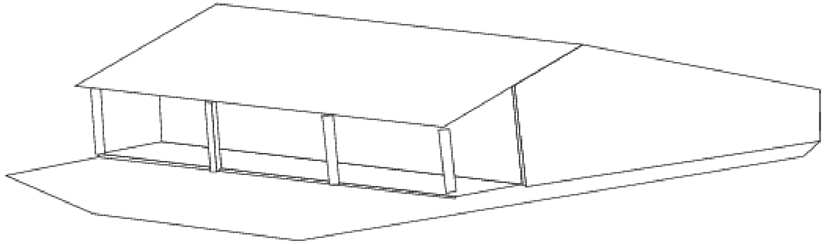
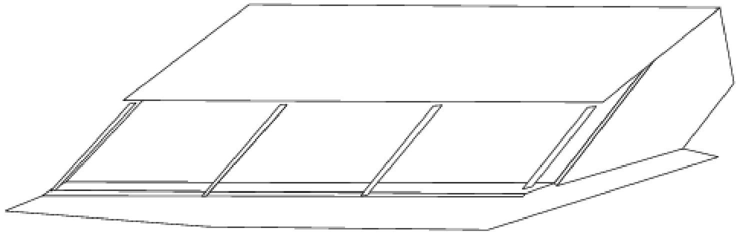


Note! do NOT select degenerated cases,
Ex. 4 points on a plane, 3 points on a line.

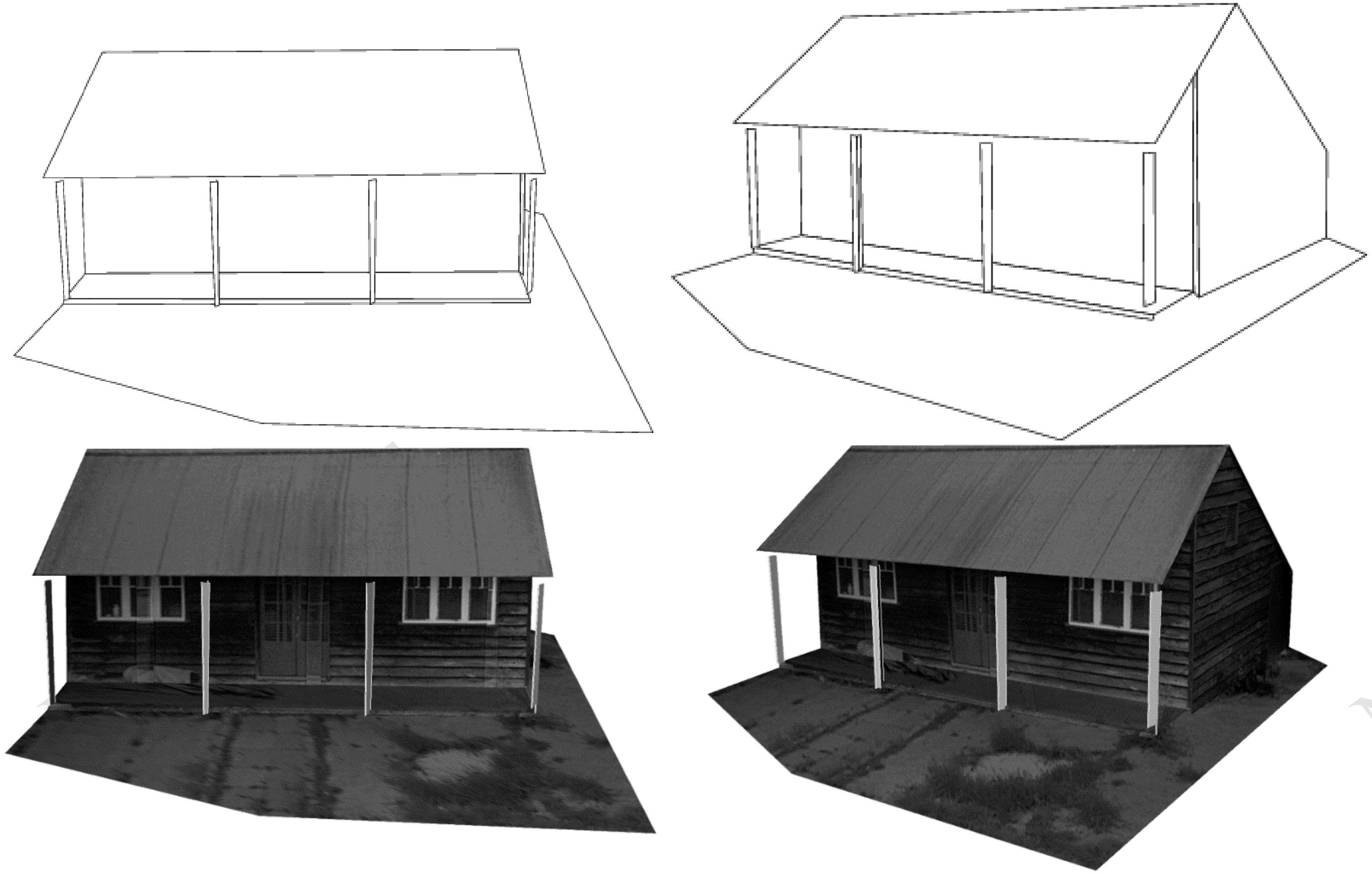
Affine reconstruction

- There are 3 sets of parallel lines in the scene, each set with a different direction. These 3 sets enable the position of the plnae at infinity to be computeed in the projective reconstruction. Note that parallel scene lines are parallel in reconstruction, but lines are NOT perpendicular in the reconstruction.

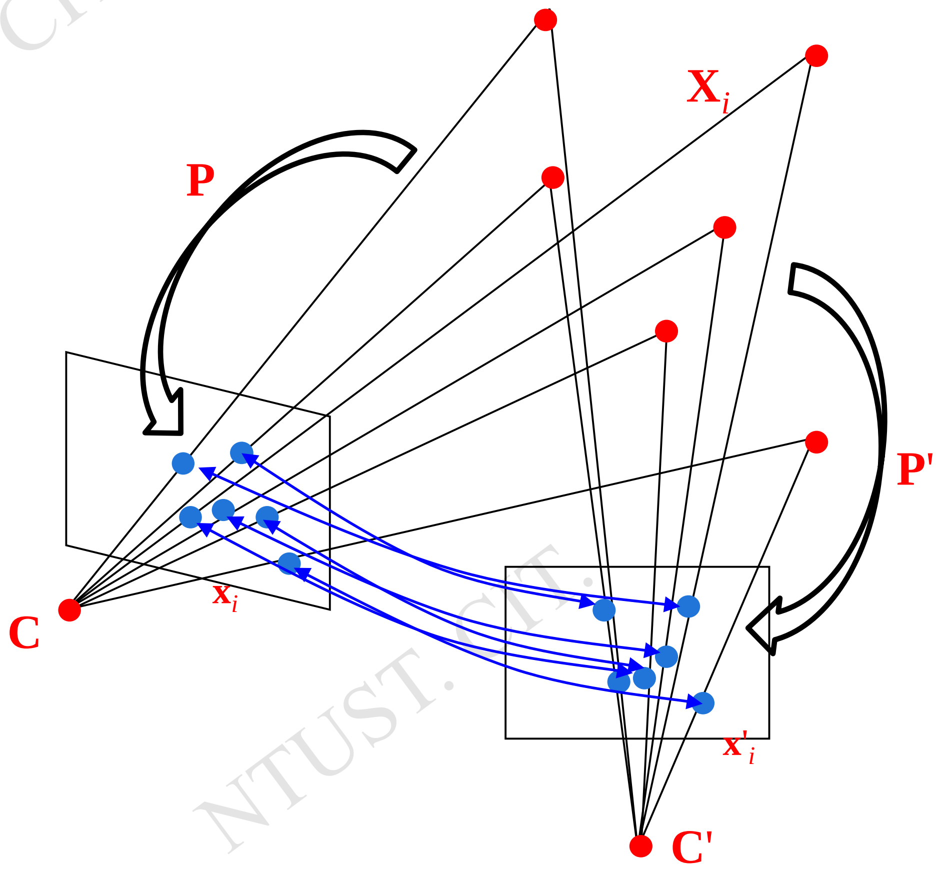


Metric reconstruction



3D estimation Direct Triangulation method



- Condtion-1

P, P', x_i, x'_i : known

X : unknown (to be solved)

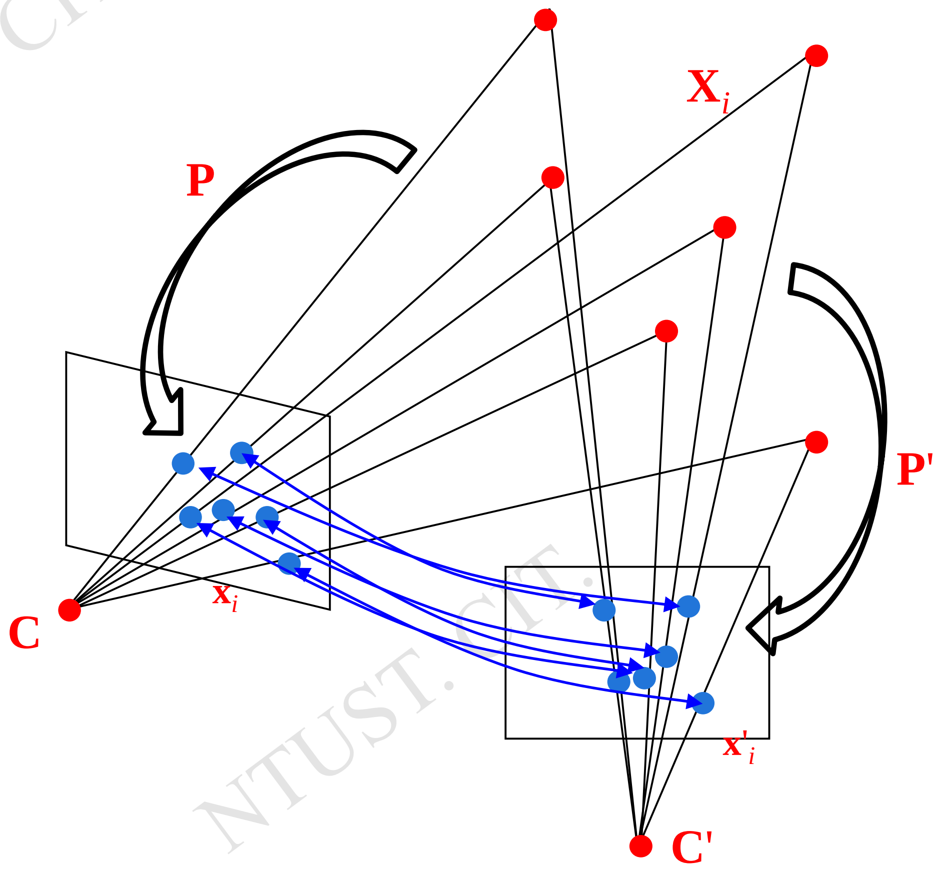
- Condtion-2

x_i, x'_i, X: known

P, P': unknown (to be solved)

→become a calibration problem

3D estimation Direct Triangulation method



- Condtion-3**(direct reconstruction)**

$\mathbf{x}_i$, $\mathbf{x}'_i$: known
5 points $\mathbf{X}$: known
P, P': unknown (to be solved)

- Condtion-4**(affine reconstruction)**

$\mathbf{x}_i$, $\mathbf{x}'_i$, other cues: known
P, P': unknown (to be solved)

- Condtion-5**(metric reconstruction)**

$\mathbf{x}_i$, $\mathbf{x}'_i$, **K**: known
X, P, P': unknown (to be solved)

Objective (for uncalibrated situation)

Given two uncalibrated images compute ($\mathbf{P}_M$,$\mathbf{P}'_M$,{$\mathbf{X}_{Mi}$})
(i.e. within similarity of original scene and cameras)

Algorithm [Textbook: Hartley04, Algorithm 10.1]

(i) Compute projective reconstruction ($\mathbf{P}$, $\mathbf{P}'$,{$\mathbf{X}_i$})
- (a) Compute $\mathbf{F}$ from $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$
- (b) Compute $\mathbf{P}$, $\mathbf{P}'$ from $\mathbf{F}$
- (c) Triangulate $\mathbf{X}_i$ from $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$

(ii) Rectify reconstruction from projective to metric

(a) **Direct method**: compute $\mathbf{H}$ from control points →(condition-3 in the previous slide)

$$\mathbf{P}_M = \mathbf{P}\mathbf{H}^{-1}, \quad \mathbf{P}_M' = \mathbf{P}'\mathbf{H}^{-1}, \quad \mathbf{X}_{Mi} = \mathbf{H}\mathbf{X}_i$$

Stratified method:

(a) **Affine reconstruction**: →(condition-4 in the previous slide)

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} \,|\, 0 \\ \boldsymbol{\pi}_\infty^{\mathrm{T}} \end{bmatrix}$$

(b) **Metric reconstruction**: compute IAC $\boldsymbol{\omega}$ →(condition-5 in the previous slide)

$$\mathbf{H} = \begin{bmatrix} \mathbf{A}^{-1} & 0 \\ 0 & 1 \end{bmatrix} \qquad \mathbf{A}\mathbf{A}^{\mathrm{T}} = (\mathbf{M}^{\mathrm{T}}\boldsymbol{\omega}\mathbf{M})^{-1}$$

3D reconstruction

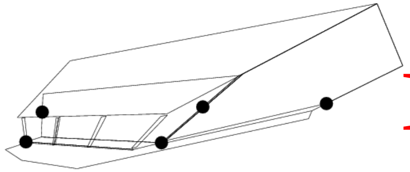
Image information provided	View relations and projective objects	3-space objects	reconstruction ambiguity
point correspondences	**F**		projective
point correspondences including vanishing points	**F**, $\mathbf{H}_\infty$	$\mathbf{p}_\infty$	affine
Points correspondences and internal camera calibration	**F**, $\mathbf{H}_\infty$ ω, ω'	$\mathbf{p}_\infty$ Ω_∞	metric

The two-view relations, image entities, and their 3-space conterpart for various classes of reconstruction ambiguity.

3D reconstruction for projective geometry

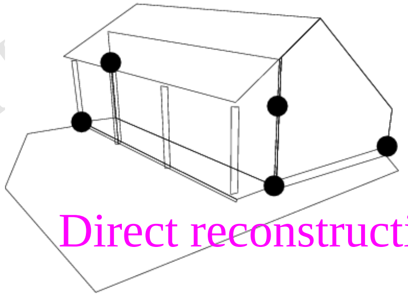
- Direction reconstruction


Two views geometry (**F**)

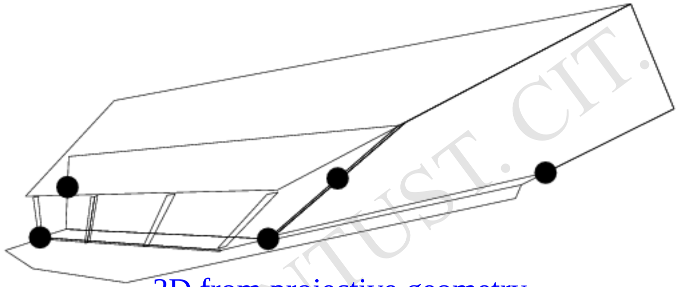


A guess for **P**, **P'**
[Projective reconstruction]

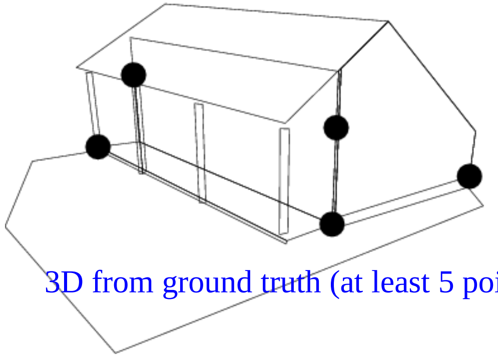
$\mathbf{H}_{4x4}$



Direct reconstruction



3D from projective geometry
(at least 5 points)



3D from ground truth (at least 5 points)

$$\begin{bmatrix} X_1' \\ X_2' \\ X_3' \\ X_4' \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

$$\boxed{\begin{array}{l} \tilde{\mathrm{H}}_1^{\mathrm{T}} = [H_{11} \quad H_{12} \quad H_{13} \quad H_{14}] \\ \tilde{\mathrm{H}}_2^{\mathrm{T}} = [H_{21} \quad H_{22} \quad H_{23} \quad H_{24}] \\ \tilde{\mathrm{H}}_3^{\mathrm{T}} = [H_{31} \quad H_{32} \quad H_{33} \quad H_{34}] \\ \tilde{\mathrm{H}}_4^{\mathrm{T}} = [H_{41} \quad H_{42} \quad H_{43} \quad H_{44}] \\ \mathbf{X}^{\mathrm{T}} = [X \quad Y \quad Z \quad 1] \end{array}}$$

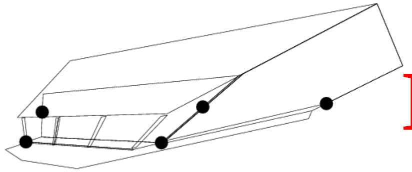
$$\begin{bmatrix} \mathbf{X}^{\mathrm{T}} & 0^{\mathrm{T}} & 0^{\mathrm{T}} & -X'\mathbf{X}^{\mathrm{T}} \\ 0^{\mathrm{T}} & \mathbf{X}^{\mathrm{T}} & 0^{\mathrm{T}} & -Y'\mathbf{X}^{\mathrm{T}} \\ 0^{\mathrm{T}} & 0^{\mathrm{T}} & \mathbf{X}^{\mathrm{T}} & -Z'\mathbf{X}^{\mathrm{T}} \end{bmatrix}_{3\times16} \begin{bmatrix} \tilde{\mathrm{H}}_1 \\ \tilde{\mathrm{H}}_2 \\ \tilde{\mathrm{H}}_3 \\ \tilde{\mathrm{H}}_4 \end{bmatrix}_{16\times1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3\times1}$$

Select at least 5 correspondences→computer **H**
(please review chapter "Projective 3D geometry" for detail)

3D reconstruction for projective geometry

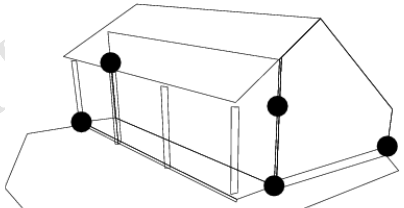
- Direction reconstruction

Two views geometry (**F**)



A guess for **P**, **P'**
[Projective reconstruction]

$\mathbf{H}_{4x4}$



Direct reconstruction

Once having **H**:

$\boxed{\begin{array}{l} \mathbf{P}_2 = \mathbf{P}_1 \mathbf{H}^{-1} \\ \mathbf{P}_2' = \mathbf{P}_1' \mathbf{H}^{-1} \\ \mathbf{X}_2 = \mathbf{H}\mathbf{X}_1 \end{array}}$

Either

Apply homography **H** to all old 3D points ($\mathbf{X}_{1i}$), then gel all new 3D points ($\mathbf{X}_{2i}$)

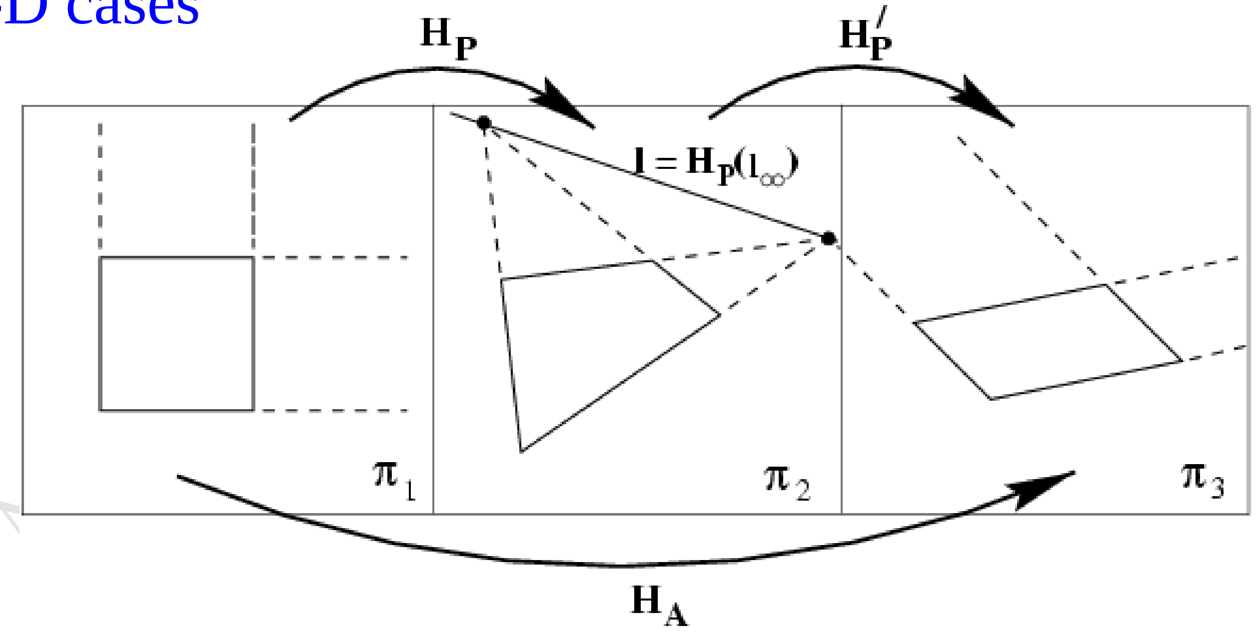
Or

Compute new **P** & **P'**, and use triangulation (back-projection) from all 2D correspondences, then generate new 3D points ($\mathbf{X}_{2i}$)

3D reconstruction for projective geometry

- Affine reconstruction
  - Projective transformation, then affine trans..

2-D cases



3D reconstruction for projective geometry

- Affine reconstruction
  - Projective transformation, then affine trans..

$$(\mathbf{P}, \mathbf{P}', \{\mathbf{X}_i\})$$

$$\boldsymbol{\pi}_\infty = (A, B, C, D)^{\mathrm{T}} \rightarrow (0,0,0,1)^{\mathrm{T}}$$

$$\mathbf{H}^{-\mathrm{T}}\boldsymbol{\pi}_\infty = (0,0,0,1)^{\mathrm{T}}$$ →to find a $\boldsymbol{\pi}_\infty$ under this constraint, but how?

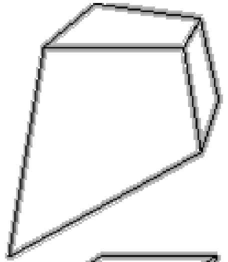
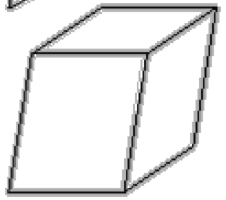
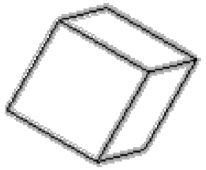
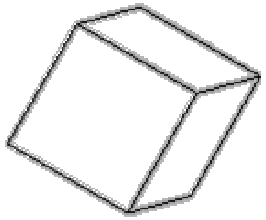
$$\mathbf{H} = \begin{bmatrix} \mathbf{I} \mid 0 \\ \boldsymbol{\pi}_\infty^{\mathrm{T}} \end{bmatrix}$$

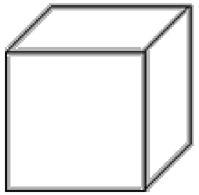
(if determinent ≠0)

A desired solution if $\boldsymbol{\pi}_\infty$ can be determined, the mapped 3D points will be an affine-mapping

3D reconstruction for projective geometry

- Hierarchy of transformations

Projective 15dof	$\begin{bmatrix} A & t \\ v^T & v \end{bmatrix}$		Intersection and tangency
Affine 12dof	$\begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix}$		Parallellism of planes, Volume ratios, centroids, **The plane at infinity π_∞**
Similarity 7dof	$\begin{bmatrix} sR & t \\ 0^T & 1 \end{bmatrix}$		**The absolute conic Ω_∞**
Euclidean 6dof	$\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}$		Volume



3D reconstruction for projective geometry

- Affine reconstruction
  - Projective transformation, then affine trans..

How to determine $\boldsymbol{\pi}_\infty$?

Here are various cases:

$$\mathbf{H}^{-T}\underbrace{\boldsymbol{\pi}_\infty}_{\text{3D plane: the scene in the projective geometry}} = \underbrace{(0,0,0,1)^T}_{\text{3D plane at infinity}}$$

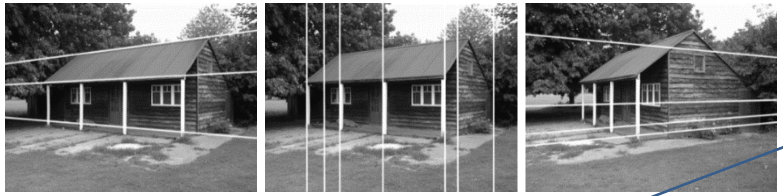
3D plane at infinity

3D plane: the scene in the projective geometry
(need to be determined, ex. use 3 points to find out)

- Translation motion
- Parallel lines
- Distance ratios on a lines (angle)
- The infinite homography
- One of the cameras is affine

3D reconstruction for projective geometry

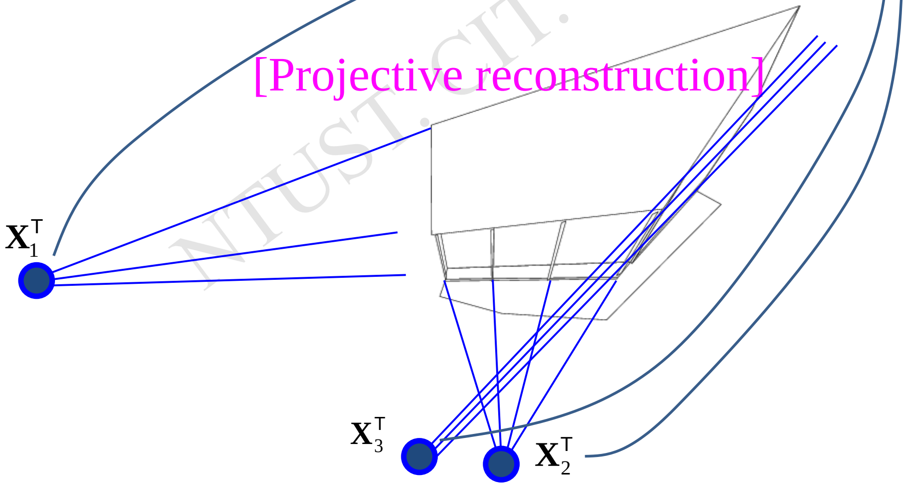
- Affine reconstruction
  - Projective transformation, then affine trans..



$\mathbf{X}_1^{\mathrm{T}}$ $\mathbf{X}_2^{\mathrm{T}}$ $\mathbf{X}_3^{\mathrm{T}}$

Recall slide: "Projective 3D geometry“
Determine a 3D plane

$$\begin{bmatrix}\mathbf{X}_1^{\mathrm{T}}\\ \mathbf{X}_2^{\mathrm{T}}\\ \mathbf{X}_3^{\mathrm{T}}\end{bmatrix}\pi=0 \qquad \begin{bmatrix}(X_1)_1 & (X_1)_2 & (X_1)_3 & (X_1)_4\\ (X_2)_1 & (X_2)_2 & (X_2)_3 & (X_2)_4\\ (X_3)_1 & (X_3)_2 & (X_3)_3 & (X_3)_4\end{bmatrix}\begin{bmatrix}\pi_1\\ \pi_2\\ \pi_3\\ \pi_4\end{bmatrix}=0$$

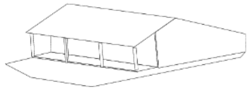


Find $\boldsymbol{\pi}_\infty$

Then, $\mathbf{H}=\begin{bmatrix}\mathbf{I}\,|\,0\\ \boldsymbol{\pi}_\infty^{\mathrm{T}}\end{bmatrix}$

Again, calculate final **X**
(here is affine reconstruction)

$\mathbf{P}_2=\mathbf{P}_1\mathbf{H}^{-1}$
$\mathbf{P}_2'=\mathbf{P}_1'\mathbf{H}^{-1}$
$\mathbf{X}_2=\mathbf{H}\mathbf{X}_1$

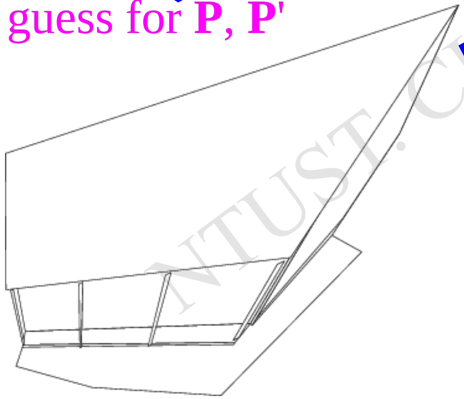


3D reconstruction for projective geometry

- Metric reconstruction

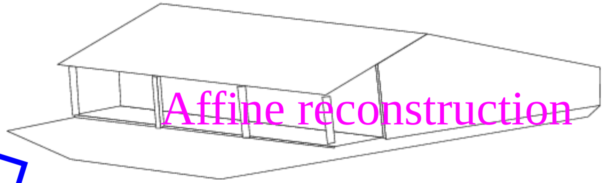
Two views geometry (**F**)

A guess for **P**, **P'**



[Projective reconstruction]

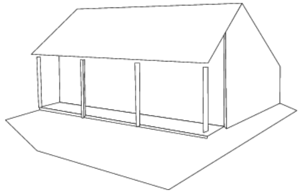
$$\mathbf{H} = \begin{bmatrix} \mathbf{I} \mid 0 \\ \boldsymbol{\pi}_\infty^T \end{bmatrix}$$



Affine reconstruction

$$\mathbf{H} = \begin{bmatrix} \mathbf{A}^{-1} & 0 \\ 0 & 1 \end{bmatrix}$$

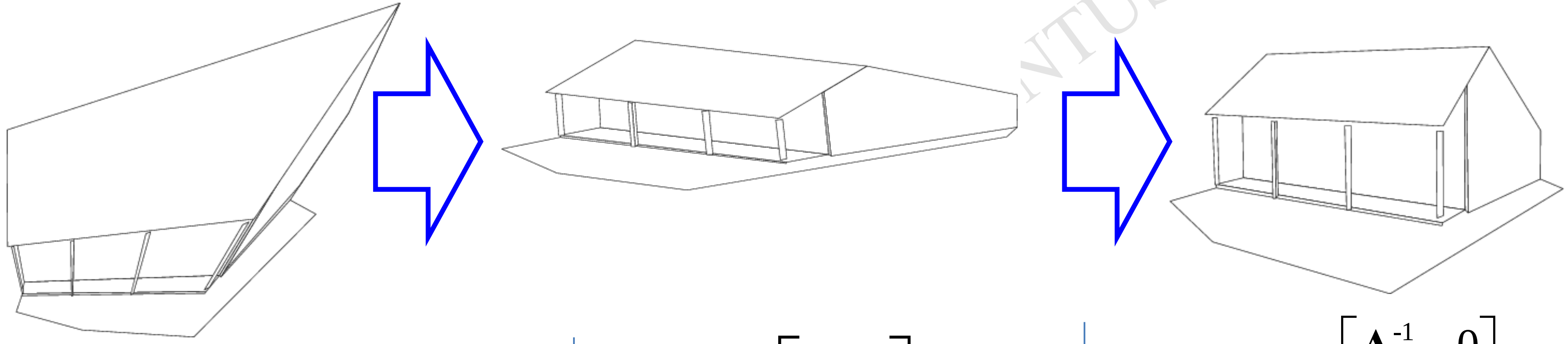
$$\mathbf{A}\mathbf{A}^T = (\mathbf{M}^T\boldsymbol{\omega}\mathbf{M})^{-1}$$



Metric reconstruction

3D reconstruction for projective geometry

- Metric reconstruction—cont.



Either

$$\mathbf{P}_1=[\mathbf{I}\,|\,0]\quad \mathbf{P}_1'=[[\mathbf{e}']_\times\mathbf{F}+\mathbf{e}'\mathbf{v}^{\mathrm{T}}\,|\,\lambda\mathbf{e}']$$

Or

$$\mathbf{P}_1=[\mathbf{I}\,|\,0]\quad \mathbf{P}_1'=[[\mathbf{e}']_\times\mathbf{F}\,|\,\mathbf{e}']$$

Triangulation for all 3D points ($\mathbf{X}_{1i}$), ex

$$\begin{bmatrix} u\mathbf{p}_3^{\mathrm{T}}-\mathbf{p}_1^{\mathrm{T}} \\ v\mathbf{p}_3^{\mathrm{T}}-\mathbf{p}_2^{\mathrm{T}} \\ u'\mathbf{p}_3'^{\mathrm{T}}-\mathbf{p}_1'^{\mathrm{T}} \\ v'\mathbf{p}_3'^{\mathrm{T}}-\mathbf{p}_2'^{\mathrm{T}} \end{bmatrix}\mathbf{X}_{4\times1}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$\mathbf{H}=\begin{bmatrix}\mathbf{I}\,|\,0 \\ \boldsymbol{\pi}_\infty^{\mathrm{T}}\end{bmatrix}$$

$$\mathbf{P}_2=\mathbf{P}_1\mathbf{H}^{-1}$$
$$\mathbf{P}_2'=\mathbf{P}_1'\mathbf{H}^{-1}$$
$$\mathbf{X}_{2i}=\mathbf{H}\mathbf{X}_{1i}$$

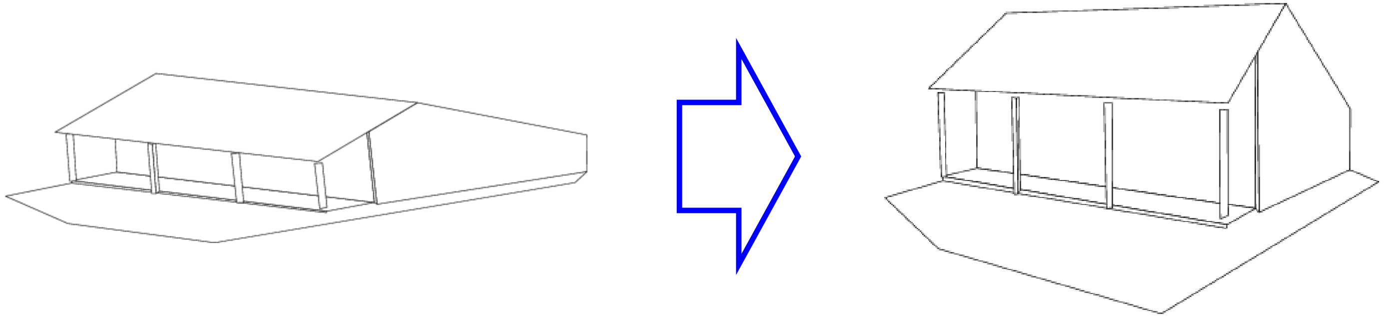
$$\mathbf{H}=\begin{bmatrix}\mathbf{A}^{-1} & 0 \\ 0 & 1\end{bmatrix}$$

$$\mathbf{A}\mathbf{A}^{\mathrm{T}}=(\mathbf{M}^{\mathrm{T}}\boldsymbol{\omega}\mathbf{M})^{-1}$$

$$\mathbf{P}_3=\mathbf{P}_2\mathbf{H}^{-1}$$
$$\mathbf{P}_3'=\mathbf{P}_2'\mathbf{H}^{-1}$$
$$\mathbf{X}_{3i}=\mathbf{H}\mathbf{X}_{2i}$$

3D reconstruction for projective geometry

- Metric reconstruction—cont.



$$\mathbf{H}=\begin{bmatrix}\mathbf{A}^{-1} & 0\\ 0 & 1\end{bmatrix}$$

$$\mathbf{A}\mathbf{A}^{\mathrm{T}}=(\mathbf{M}^{\mathrm{T}}\boldsymbol{\omega}\mathbf{M})^{-1}$$

Cholesky factorization

$\mathbf{P}_3=\mathbf{P}_2\mathbf{H}^{-1}$

$\mathbf{P}_3'=\mathbf{P}_2'\mathbf{H}^{-1}$

$\mathbf{X}_{3i}=\mathbf{H}\mathbf{X}_{2i}$

Here

$$\boldsymbol{\omega}=(\mathbf{K}\mathbf{K}^{\mathrm{T}})^{-1}$$

→you may assume intrinsic parameter **K** is known, or under some constraints.

$$\mathbf{P}_2=[\mathbf{M}\,|\,\mathbf{m}]$$

The projection matrix of affine reconstruction

3D estimation Direct Triangulation method

- If the projected point **PX** is very close to x, then

$$\mathbf{x} \times \mathbf{x} = \mathbf{0} \quad \rightarrow \text{zero vector}$$

So, one constraint for **X** is

$$\mathbf{x} \times (\mathbf{P}\mathbf{X}) = \mathbf{0}$$

For convenience, rewrite **P** as

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{31} & p_{32} & p_{34} \end{bmatrix} = \begin{bmatrix} \mathbf{p}_1^{\mathrm{T}} \\ \mathbf{p}_2^{\mathrm{T}} \\ \mathbf{p}_3^{\mathrm{T}} \end{bmatrix}_{3\times 4}$$

1x4 vector

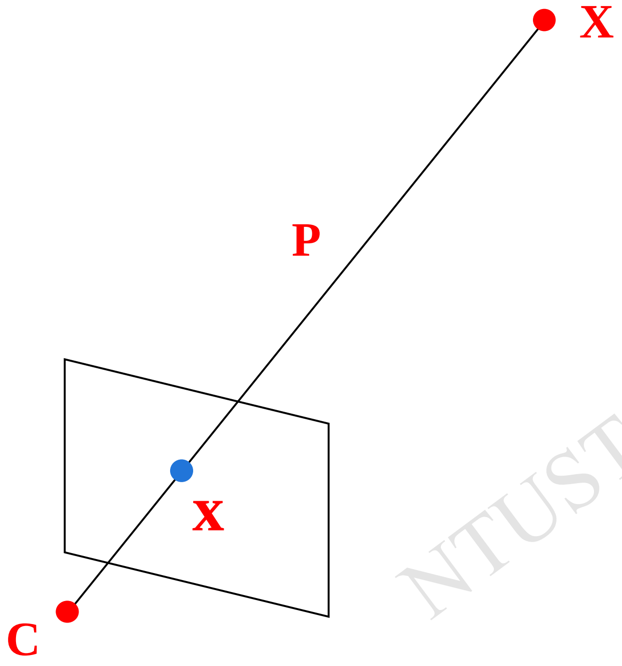
1x4 vector

1x4 vector

$$\mathbf{p}_1^{\mathrm{T}} = [p_{11} \quad p_{12} \quad p_{13} \quad p_{14}]$$

$$\mathbf{p}_2^{\mathrm{T}} = [p_{21} \quad p_{22} \quad p_{23} \quad p_{24}]$$

$$\mathbf{p}_3^{\mathrm{T}} = [p_{31} \quad p_{32} \quad p_{33} \quad p_{34}]$$



3D estimation Direct Triangulation method—cont.

- To solve the equation: $\mathbf{x} \times (\mathbf{P}\mathbf{X}) = \mathbf{0}$

$$\rightarrow \begin{bmatrix} x \\ y \\ w \end{bmatrix} \times \begin{bmatrix} \mathbf{p}_1{}^{\mathrm{T}}\mathbf{X} \\ \mathbf{p}_2{}^{\mathrm{T}}\mathbf{X} \\ \mathbf{p}_3{}^{\mathrm{T}}\mathbf{X} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{cases} y\mathbf{p}_3{}^{\mathrm{T}}\mathbf{X} - w\mathbf{p}_2{}^{\mathrm{T}}\mathbf{X} = 0 \\ -x\mathbf{p}_3{}^{\mathrm{T}}\mathbf{X} + w\mathbf{p}_1{}^{\mathrm{T}}\mathbf{X} = 0 \\ x\mathbf{p}_2{}^{\mathrm{T}}\mathbf{X} - y\mathbf{p}_1{}^{\mathrm{T}}\mathbf{X} = 0 \end{cases}$$

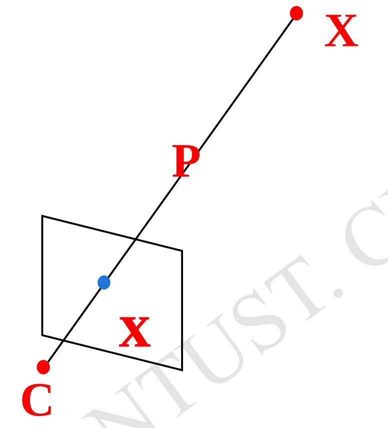
$$\rightarrow \begin{cases} -\mathbf{p}_2{}^{\mathrm{T}}\mathbf{X} + \dfrac{y}{w}\mathbf{p}_3{}^{\mathrm{T}}\mathbf{X} = 0 \\ -\mathbf{p}_1{}^{\mathrm{T}}\mathbf{X} + \dfrac{x}{w}\mathbf{p}_3{}^{\mathrm{T}}\mathbf{X} = 0 \\ y\mathbf{p}_1{}^{\mathrm{T}}\mathbf{X} - x\mathbf{p}_2{}^{\mathrm{T}}\mathbf{X} = 0 \end{cases}$$

Two independent eqs. only. (neglect 3rd eq.)

$\rightarrow$

Let $(u,v) = (x/w, y/w)$ as the image point

$$\begin{cases} (v\mathbf{p}_3{}^{\mathrm{T}} - \mathbf{p}_2{}^{\mathrm{T}})\mathbf{X} = 0 \\ (u\mathbf{p}_3{}^{\mathrm{T}} - \mathbf{p}_1{}^{\mathrm{T}})\mathbf{X} = 0 \end{cases}$$



The unknown **X** must statisfy the above eqs. for this image (says **C**)

3D estimation Direct Triangulation method—cont.

- Of course, in the second image (says C'), the point X will have the same property as:

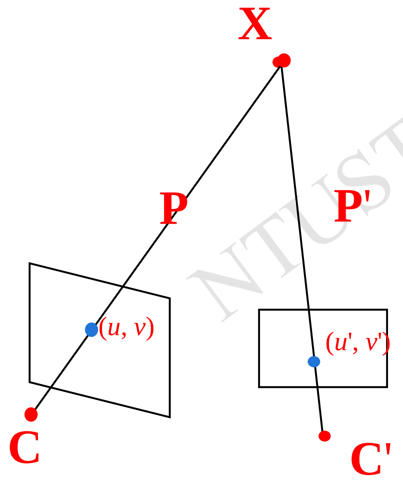
$$\begin{cases} (v'\mathbf{p'}_3{}^{\mathrm{T}} - \mathbf{p'}_2{}^{\mathrm{T}})\mathbf{X} = 0 \\ (u'\mathbf{p'}_3{}^{\mathrm{T}} - \mathbf{p'}_1{}^{\mathrm{T}})\mathbf{X} = 0 \end{cases}$$

- Finally, from one correspondence (u, v) and (u', v'), we will have

$$\begin{cases} (v\mathbf{p}_3{}^{\mathrm{T}} - \mathbf{p}_2{}^{\mathrm{T}})\mathbf{X} = 0 \\ (u\mathbf{p}_3{}^{\mathrm{T}} - \mathbf{p}_1{}^{\mathrm{T}})\mathbf{X} = 0 \end{cases} \quad \begin{cases} (v'\mathbf{p'}_3{}^{\mathrm{T}} - \mathbf{p'}_2{}^{\mathrm{T}})\mathbf{X} = 0 \\ (u'\mathbf{p'}_3{}^{\mathrm{T}} - \mathbf{p'}_1{}^{\mathrm{T}})\mathbf{X} = 0 \end{cases} \rightarrow \boxed{\begin{bmatrix} u\mathbf{p}_3{}^{\mathrm{T}} - \mathbf{p}_1{}^{\mathrm{T}} \\ v\mathbf{p}_3{}^{\mathrm{T}} - \mathbf{p}_2{}^{\mathrm{T}} \\ u'\mathbf{p'}_3{}^{\mathrm{T}} - \mathbf{p'}_1{}^{\mathrm{T}} \\ v'\mathbf{p'}_3{}^{\mathrm{T}} - \mathbf{p'}_2{}^{\mathrm{T}} \end{bmatrix} \mathbf{X}_{4\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}$$

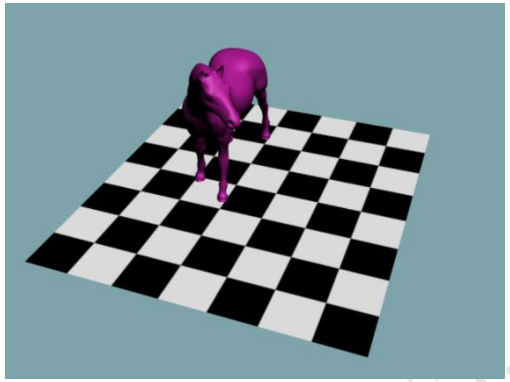
Solve it by SVD

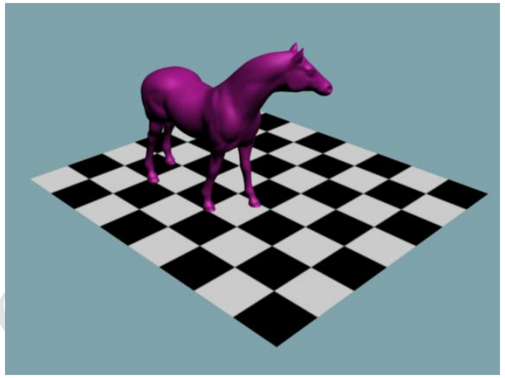
remeber: $\mathbf{p}_1{}^{\mathrm{T}} = [p_{11} \quad p_{12} \quad p_{13} \quad p_{14}]$ ...

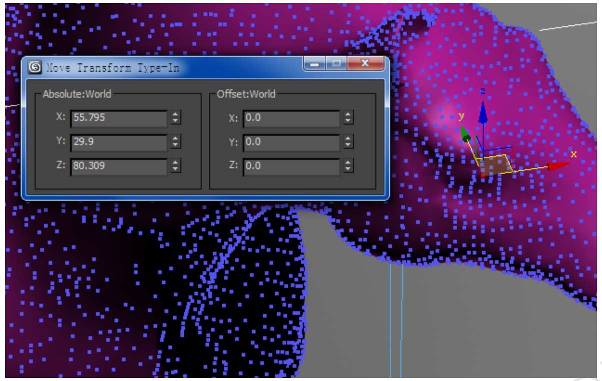


3D estimation Direct Triangulation method—cont.

- Example1 for Condition-1
  - **P**, **P'**, $\mathbf{x}_i$, $\mathbf{x}'_i$: known
  - **X** : unknown (to be solved)







P =

$$\begin{matrix} 2.0179 & 1.5967 & -0.5695 & 113.8802 \\ 0.2820 & -0.7636 & -2.4258 & 305.7125 \\ -0.0009 & 0.0023 & -0.0018 & 1.0000 \end{matrix}$$

Pp=

$$\begin{matrix} 2.8143 & -1.3450 & -0.5673 & 347.4957 \\ -0.4439 & -0.4444 & -3.0134 & 371.1864 \\ 0.0023 & 0.0023 & -0.0018 & 1.0000 \end{matrix}$$

Ground truth :
$[55.795, 29.9, 80.309, 1]^T$

To determine the 3D coordinate of the horse’s right eye, we pick up this feature on these two images, then we have
$x=[259, 120, 1]^T$
$x'=[395, 89, 1]^T$

→X?

3D estimation Direct Triangulation method—cont.

- Example1 for Condition-1—cont.

$\mathbf{P}, \mathbf{P}', \mathbf{x}_i, \mathbf{x}'_i$: known

$\mathbf{X}$: unknown (to be solved)

$$\begin{bmatrix} u\mathbf{p}_3^{\mathrm{T}} - \mathbf{p}_1^{\mathrm{T}} \\ v\mathbf{p}_3^{\mathrm{T}} - \mathbf{p}_2^{\mathrm{T}} \\ u'\mathbf{p}'^{\mathrm{T}}_3 - \mathbf{p}'^{\mathrm{T}}_1 \\ v'\mathbf{p}'^{\mathrm{T}}_3 - \mathbf{p}'^{\mathrm{T}}_2 \end{bmatrix} \mathbf{X}_{4\times1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\mathbf{p}_1$	$\mathbf{p}_2$	$\mathbf{p}_3$	$[u \quad v]$
p1 =	p2 =	p3 =	u =
2.0179	0.2820	-0.0009	259
1.5967	-0.7636	0.0023	v =
-0.5695	-2.4258	-0.0018	120
113.8802	305.7125	1.0000	

$\mathbf{p}'_1$	$\mathbf{p}'_2$	$\mathbf{p}'_3$	$[u' \quad v']$
pp1 =	pp2 =	pp3 =	up =
2.8143	-0.4439	0.0023	395
-1.3450	-0.4444	0.0023	vp =
-0.5673	-3.0134	-0.0018	89
347.4957	371.1864	1.0000	

In Matlab (note the notation ' in Matlab means transpose operation)

```
A=[u*p3'-p1';
v*p3'-p2';
up*pp3'-pp1';
vp*pp3'-pp2'];
[U,S,V]=svd(A)
…
V=
   0.0038    0.7662   -0.3417   -0.5441
   0.0030   -0.5582   -0.7735   -0.3002
   0.0088   -0.3183    0.5337   -0.7834
  -0.9999   -0.0015    0.0010   -0.0099
```

normalize →

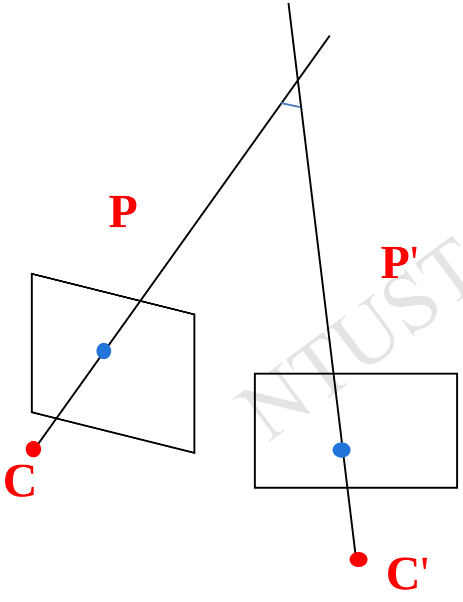
```
X =
   55.1137
   30.4065
   79.3529
    1.0000
```

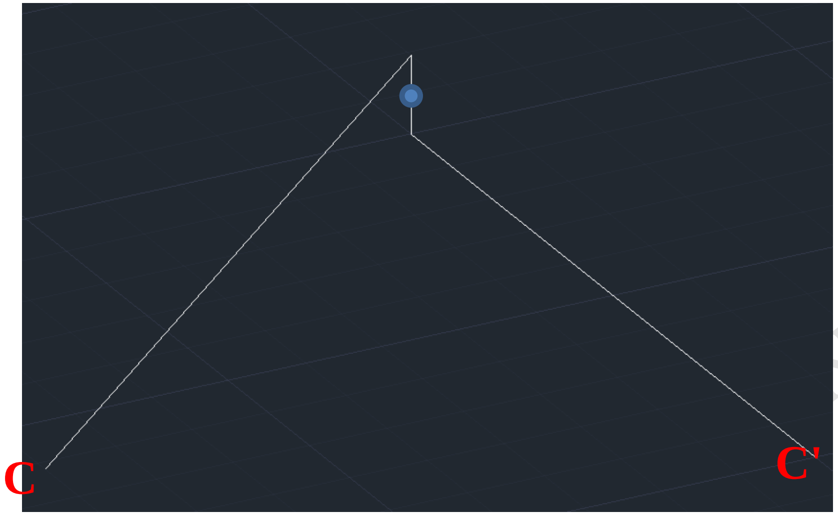
Ground truth : $[55.795, 29.9, 80.309, 1]^{\mathrm{T}}$

3D estimation Direct Triangulation method—cont.

- Example1 for Condition-1—cont.

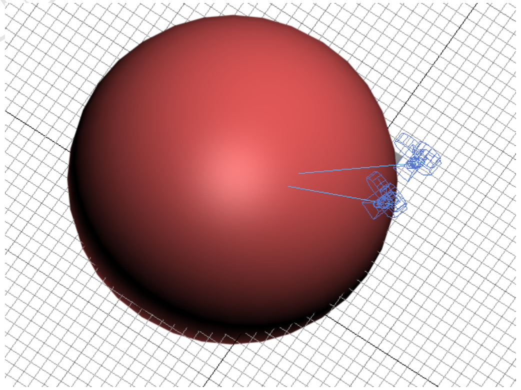
Note!
These two spatial lines are not necessary to intersect. The solution is the point whose residual error is smallest.





3D estimation Direct Triangulation method—cont.

- Example2 for Condtion-1

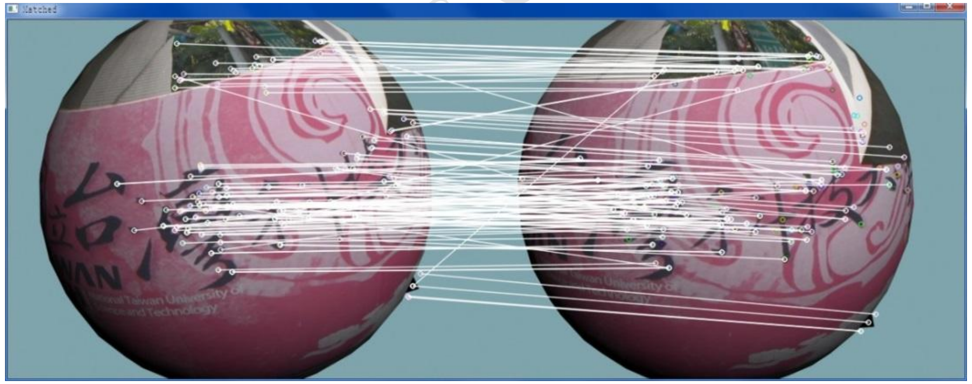


P=

$$\begin{matrix} 2.5873 & -0.3302 & -0.9174 & 246.8139 \\ -0.5844 & -1.8389 & -1.8654 & 401.7092 \\ 0.0004 & 0.0014 & -0.0029 & 1.0000 \end{matrix}$$

Pp =

$$\begin{matrix} 2.5737 & 0.2628 & -0.9172 & 207.1313 \\ -0.1437 & -1.9363 & -1.8245 & 382.3570 \\ 0.0001 & 0.0014 & -0.0029 & 1.0000 \end{matrix}$$



The question is:

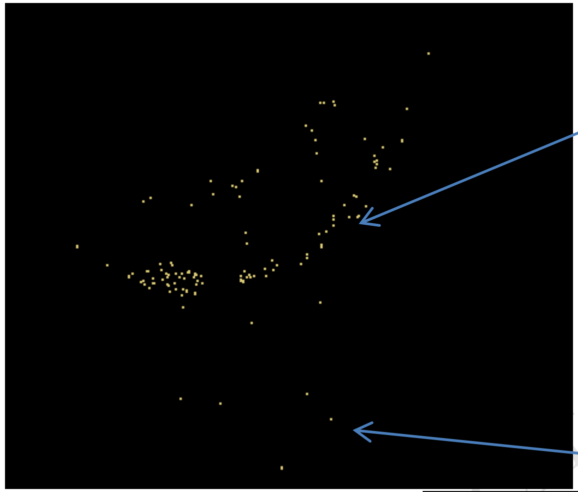
P, P' : known

$\mathbf{x}_i$, $\mathbf{x}'_i$: Generated by Feature Matching algorithm (including outlier)

X?

3D estimation Direct Triangulation method—cont.

- Example2 for Condtion-1—cont.

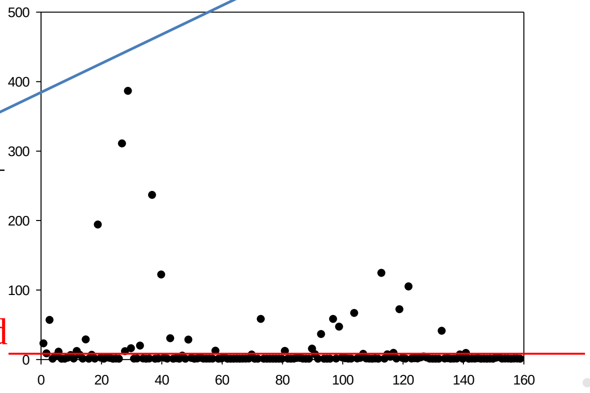


inlier

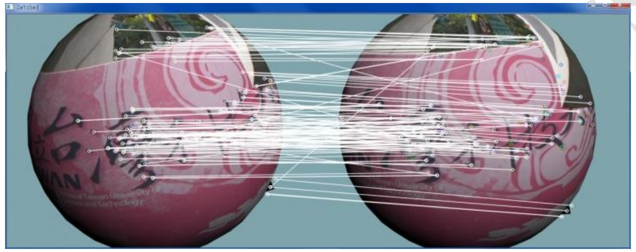
outlier

$$\begin{bmatrix} u\mathbf{p}_3^{\mathrm{T}} - \mathbf{p}_1^{\mathrm{T}} \\ v\mathbf{p}_3^{\mathrm{T}} - \mathbf{p}_2^{\mathrm{T}} \\ u'\mathbf{p}'^{\mathrm{T}}_3 - \mathbf{p}'^{\mathrm{T}}_1 \\ v'\mathbf{p}'^{\mathrm{T}}_3 - \mathbf{p}'^{\mathrm{T}}_2 \end{bmatrix} \mathbf{X}_{4\times 1} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix}$$

$$\sqrt{\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2}$$

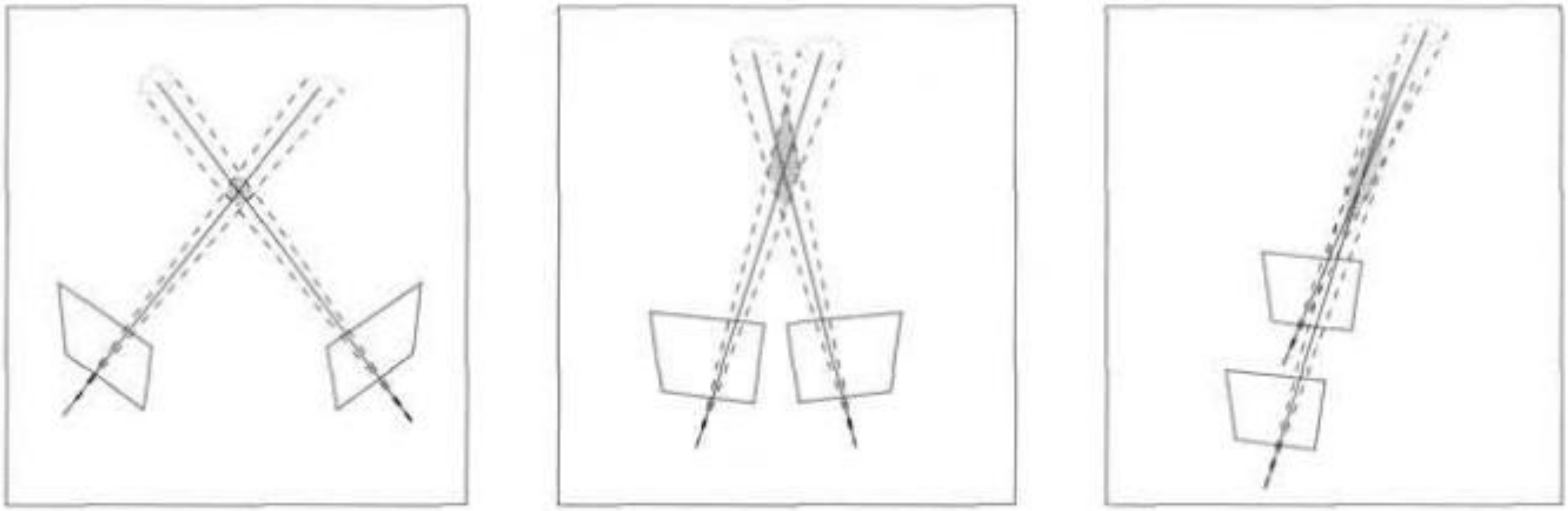


threshold



In practice, the outliers are usually removed by Fundamental Matrix constraints.

3D estimation Direct Triangulation method—cont.



More Difficult to find correspondences (feature matching)

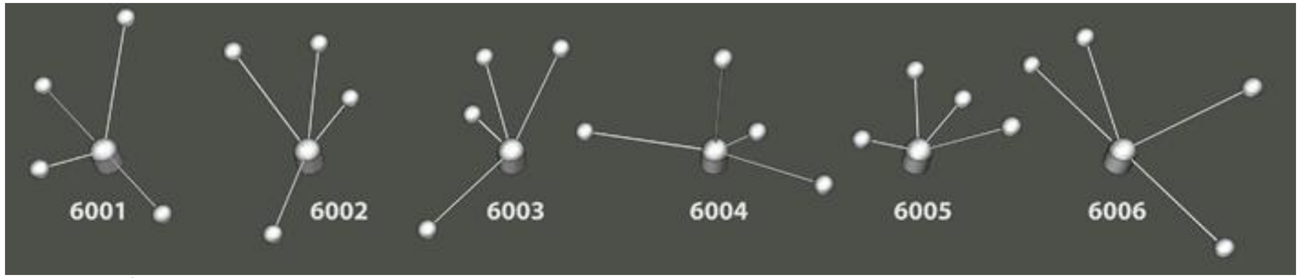
Smaller uncertainty (Better 3D reconstruction)

Easier to find correspondences (feature matching)

Larger uncertainty (Poor 3D reconstruction)

3D reconstruction–Applications

- Motion Tracking (other tracking issue)
- How to? 3D→2D (handling features)



http://www.iotracker.com/

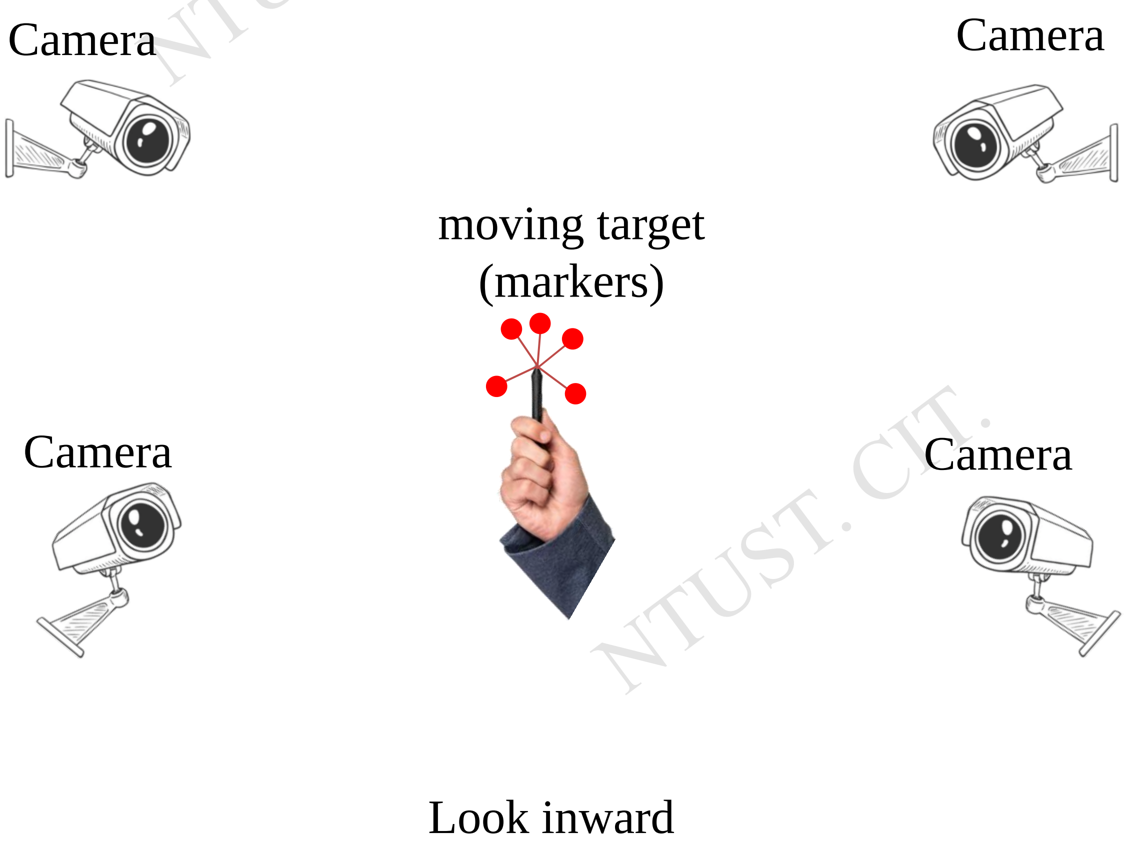
3D reconstruction–Applications

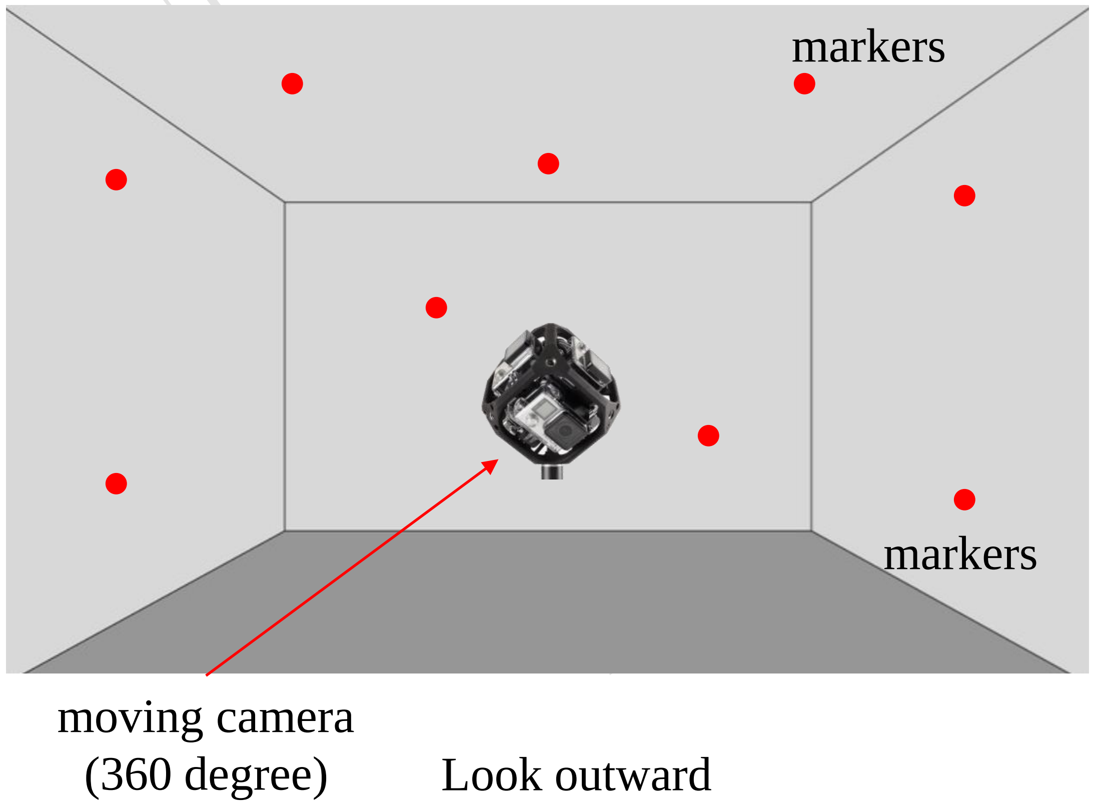
- Motion Tracking (other tracking issue)

HTC VIVE, Meta Quest 2

3D reconstruction–Applications

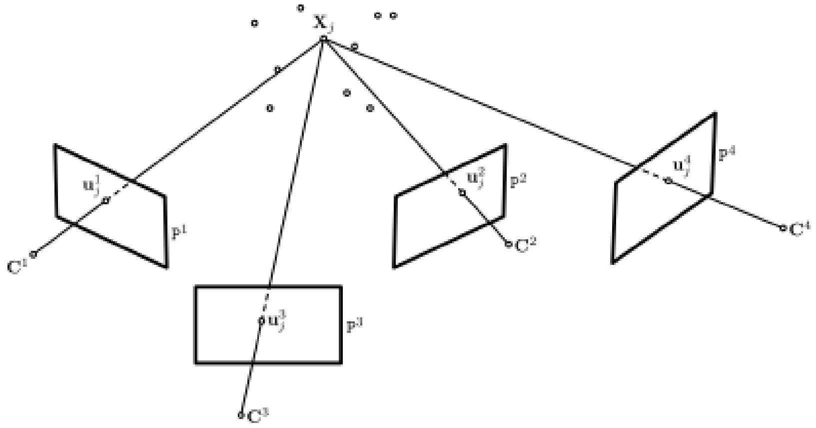
- Motion Tracking

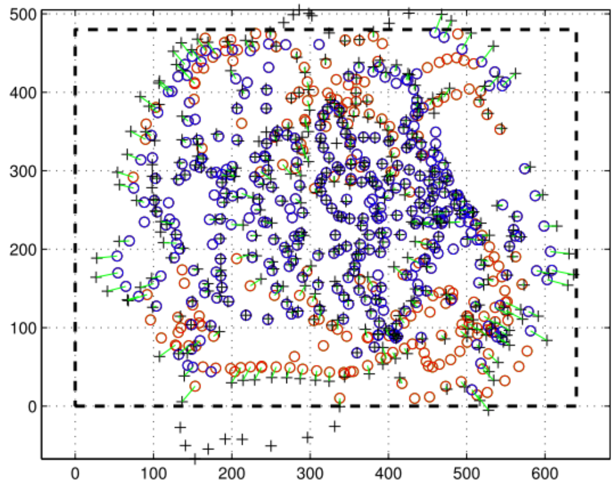




3D reconstruction–Applications

- Multi-camera calibration





T. Svoboda, D. Martinec, and T. Pajdla, "A convenient multicamera self-calibration for virtual environments," *Teleoperators Virtual Environments*, vol. 14, no. 4, pp. 407-422, 2005.

3D reconstruction–Applications

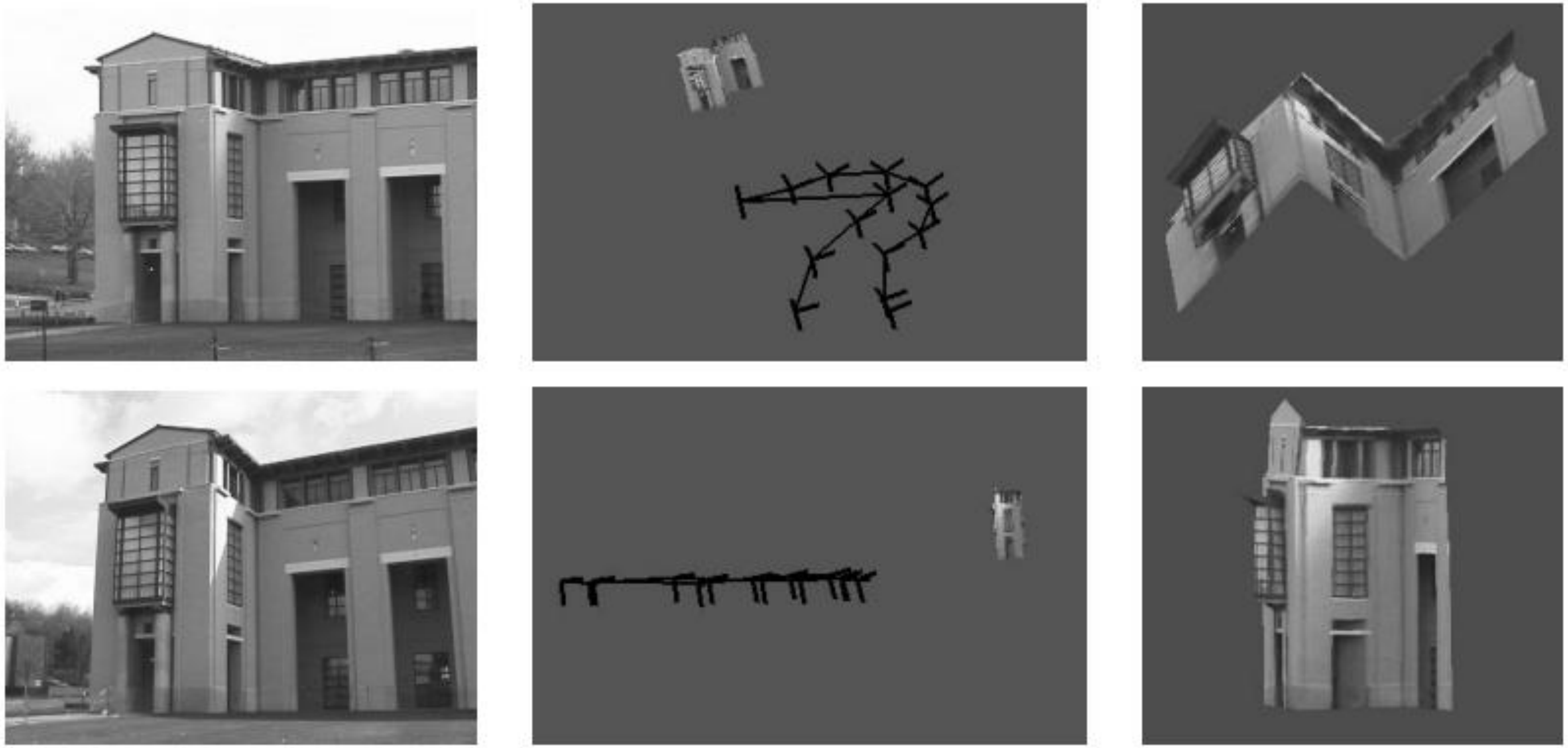
- Other issues
  - 1. Bundle adjustment?
  - 2. Structure From motion?
  - 3. Factorization?

- Select paper:

M. Han and T. Kanade, "Creating 3D models with uncalibrated cameras," in IEEE Workshop on Applications of Computer Vision, 2000, pp. 178-185.

3D reconstruction–Applications

- Factorization

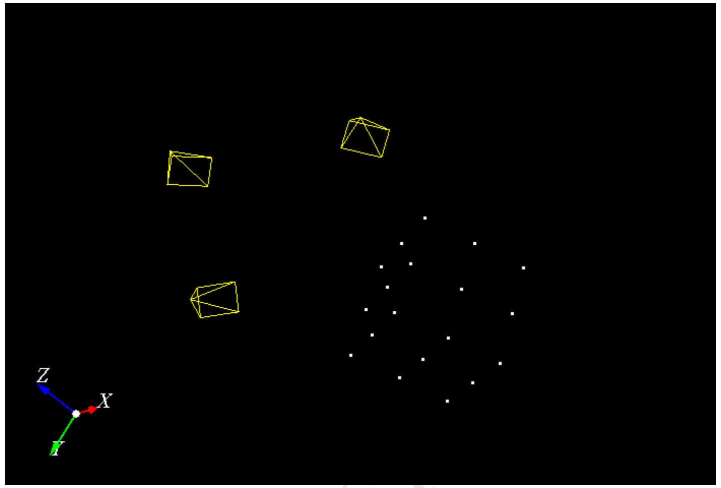


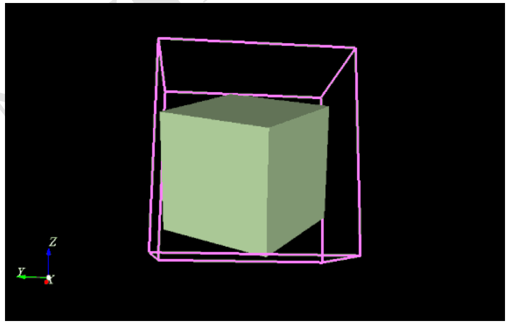
(a) (b) (c)

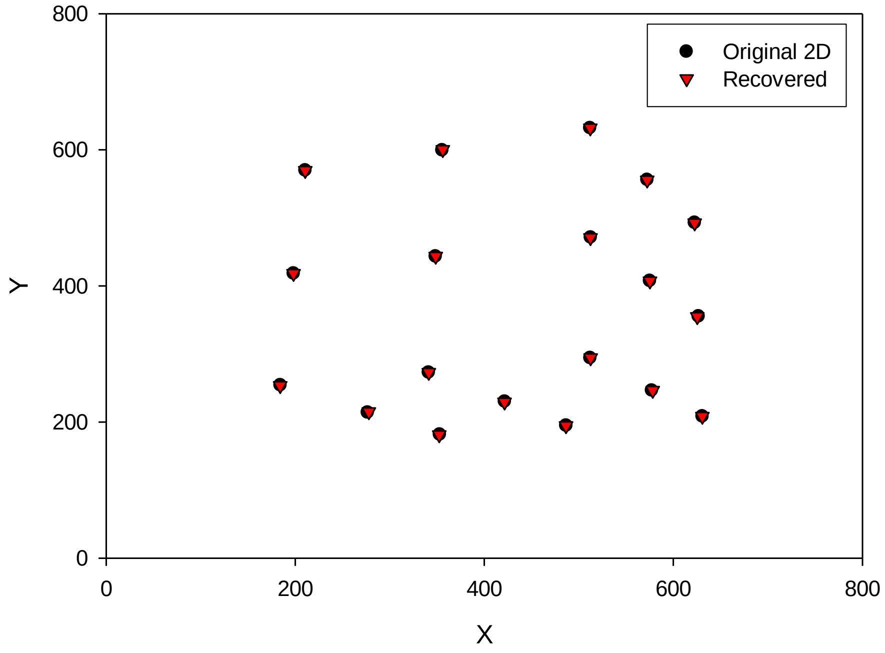
M. Han and T. Kanade, "Creating 3D models with uncalibrated cameras," in *IEEE Workshop on Applications of Computer Vision*, 2000, pp. 178-185.

3D reconstruction–Applications

- Factorization—Implementation result

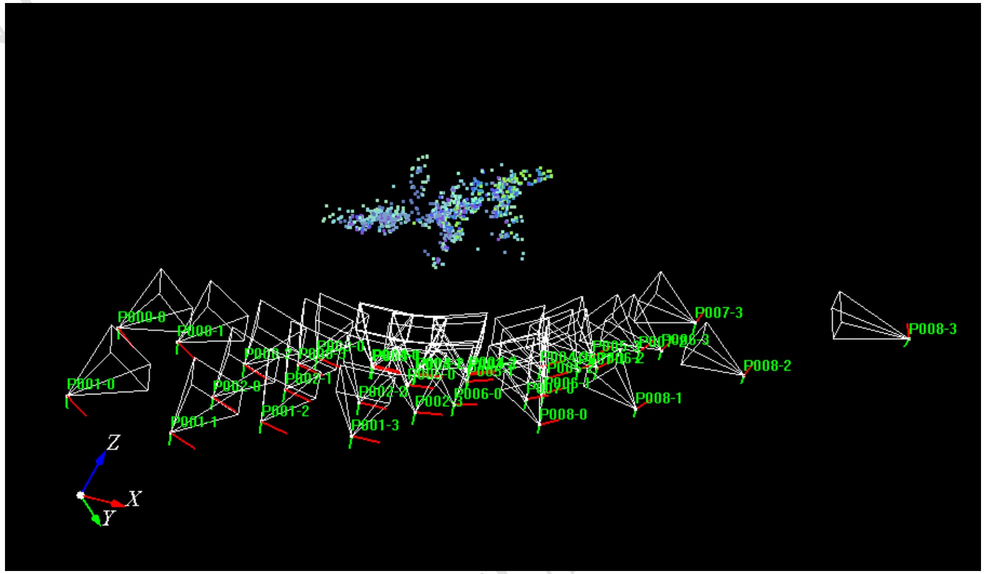


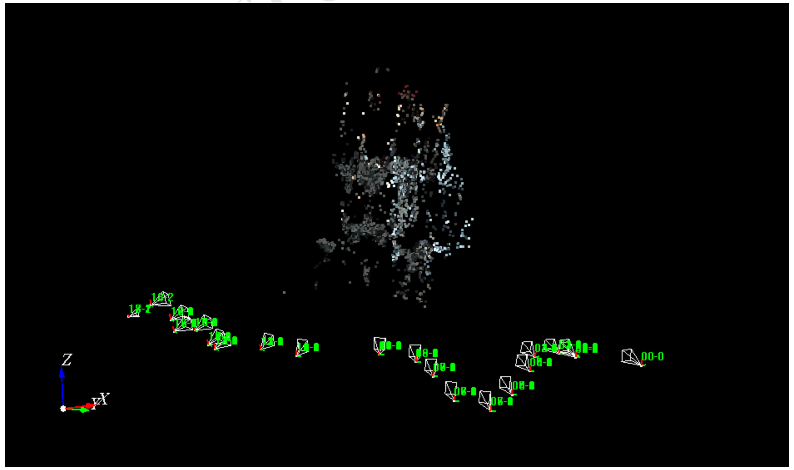


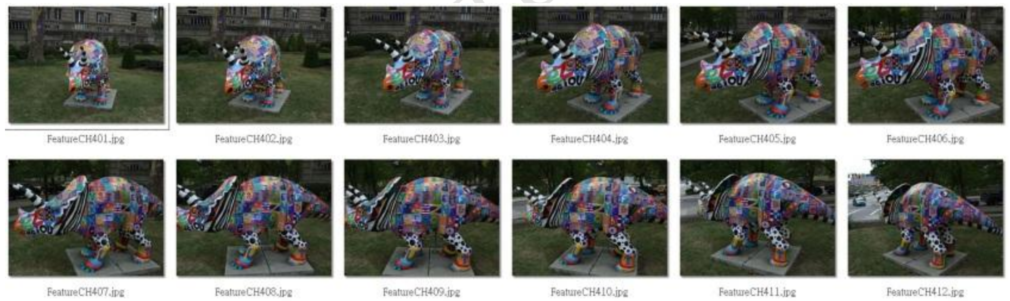


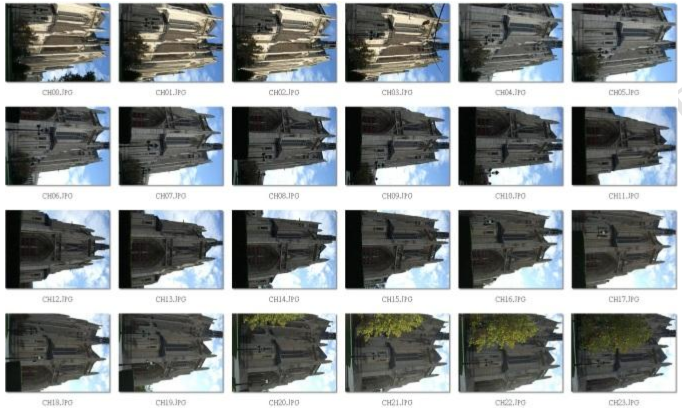
3D reconstruction–Applications

- Factorization—Implementation result—cont.



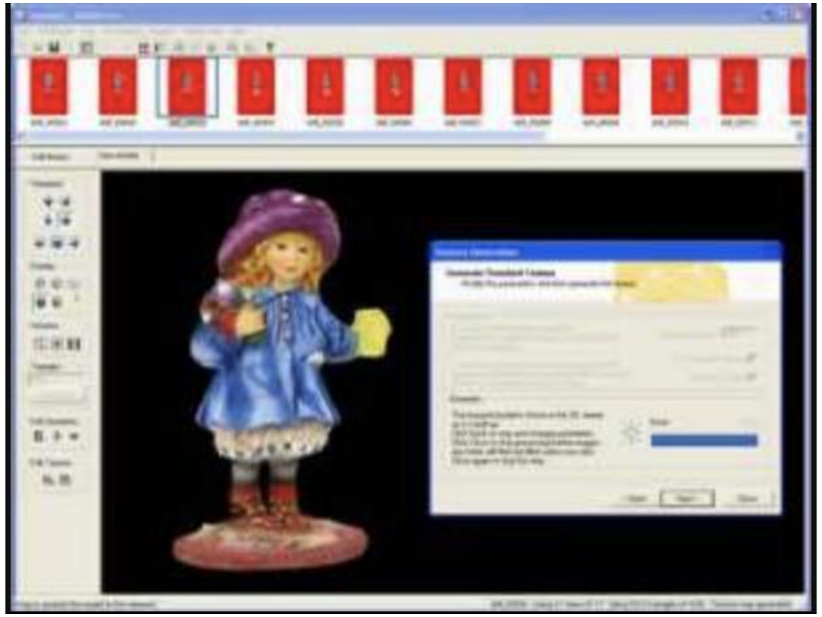






3D reconstruction–Applications

- Maker based solution (calibration on-time)



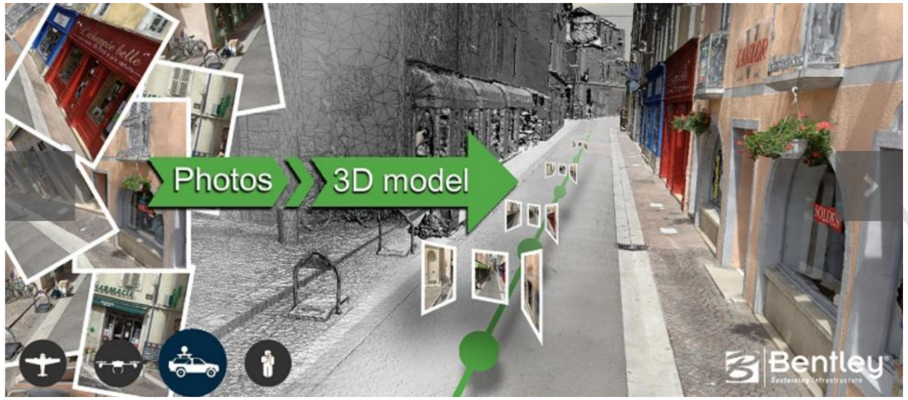
3DSOM (Strata Foto)

Qlone (App)

http://www.3dsom.com/
https://www.qlone.pro/

Commercial / Noncommercial Tools

- Agisoft PhotoScan
- RealityCapture
- PhotoModeler
- Autodesk Remake
- Strata Foto 3D CX2



http://www.agisoft.com/
https://www.capturingreality.com/
http://www.photomodeler.com/index.html
https://remake.autodesk.com/about
https://www.strata.com/foto-3d-cx-create-textured-3d-models-from-your-digital-camera/

Commercial / Noncommercial Tools

- 3DF Zephyr Pro
- PIX4D
- DroneDeploy
- senseFly
- VisualSFM
- Regard3D
- Meshroom
- openMVG
- insight3d

…

http://www.3dflow.net/3df-zephyr-pro-3d-models-from-photos/
https://pix4d.com/
https://www.dronedeploy.com/
https://www.sensefly.com/drones/ebee.html
…

Structure from motion

CMPMVS

- Image-based 3D reconstruction (freeware)

CMPMVS - Multi-View Reconstruction Software

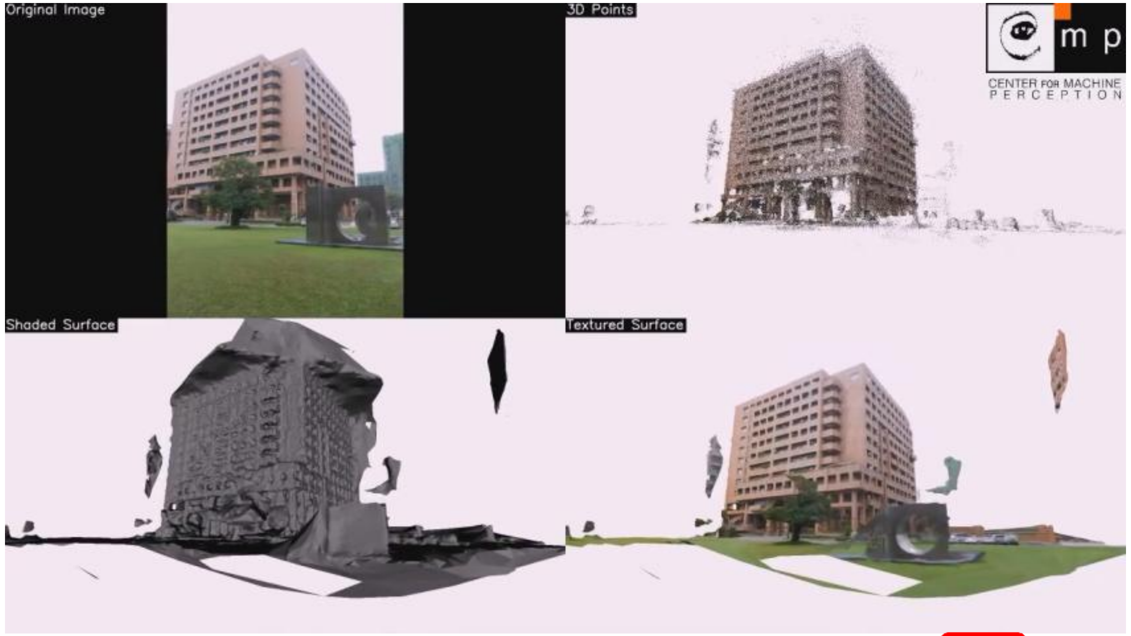
Authors: Michal Jancosek & Tomas Pajdla
Software written by: Michal Jancosek
Latest version: 0.6.0
Release date: September 28, 2012
Reference to cite:
[1] *M. Jancosek, T. Pajdla. Multi-View Reconstruction Preserving Weakly-Supported Surfaces*, CVPR 2011 - IEEE Conference on Computer Vision and Pattern Recognition 2011 (pdf).

Introduction

CMPMVS is a multi-view reconstruction software. The input to our software is a set of perspective images and camera parameters (internal and external camera calibrations). The output is a textured mesh of the rigid scene visible in the images. Non-rigid objects are implicitly ignored.

For discussion on the software please visit our Google group at

http://groups.google.com/group/cmpmvs

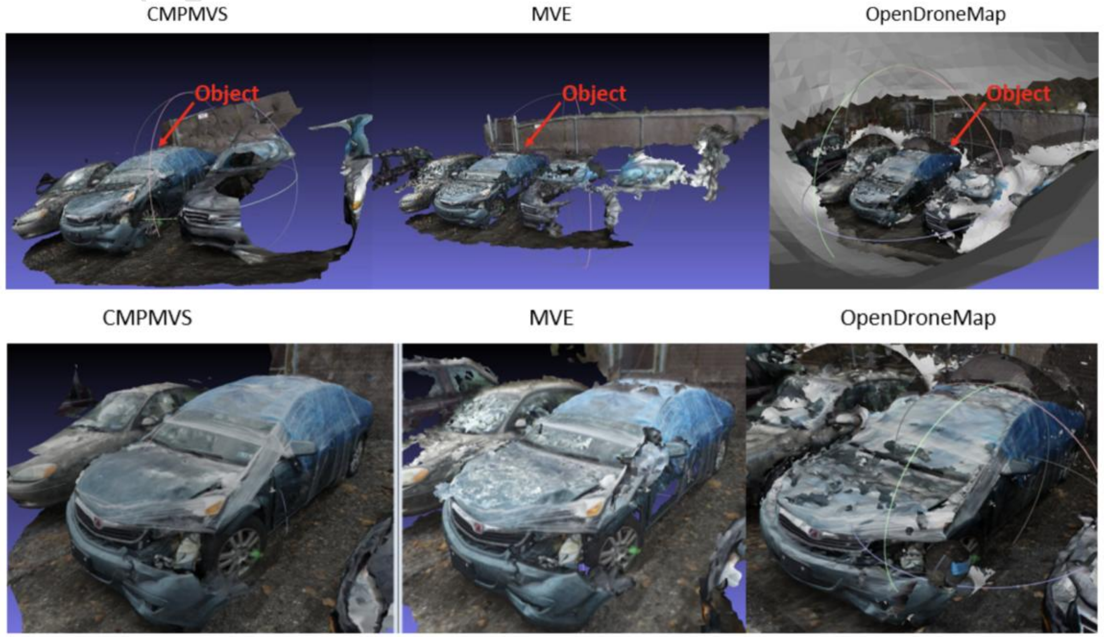


http://ptak.felk.cvut.cz/sfmservice/websfm.pl?menu=cmpmvs

Structure from motion

- openSfM/OpenDroneMap (from Carnegie Mellon University)

Reconstructed scenes: From top to bottom are original image, 3D scene and colorized 3D scene. Left to right: Penny on table (very small object), crashed motorcycle, crashed vehicle, indoor crime scene, full house (very large object), indoor multi-room.



https://www.cs.cmu.edu/~reconstruction/index.html

Structure from motion–summary from a peer review paper

Table 1. Summary of artefacts in three-dimensional reconstructions.

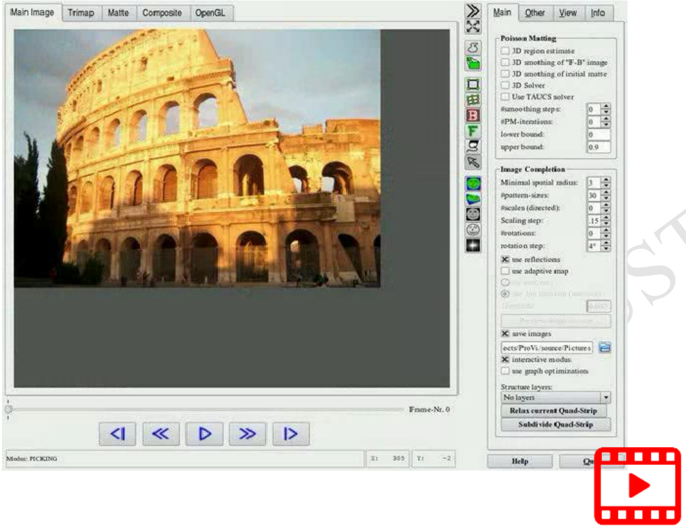
software	floating artefacts virtual versus real scene	attached artefacts	partial reconstructions	background issues	ghosts
CMVS/ PMVS	150 / 411	few, thin layer of grass on synthetic tree branches	incomplete real tree, missing most of its upper half	tree and synthetic scene targets partially reconstructed	small pieces of real tree foliage reconstructed elsewhere
CMPMVS	39 / 58	few in synthetic scene. Large number of artefacts attached to the top of real trees	a few missing virtual tree branches	ground discontinuities	large sections of the synthetic tree reconstructed elsewhere in scene
MVE	1 / 7	sky attached on trees	large missing section in the upper middle of the real tree	object shape in real scene background deteriorates with distance	no ghosts
SURE	127 / 18	sky artefacts on upper parts of crowns, more pronounced in the real scene	complete tree reconstructions	practically no background in real scene	no ghosts
PhotoScan (lowest quality)	93 / 54	sky artefacts at the top of tree crown, larger in the real scene	complete tree reconstruction but hazy shape with hollow appearance	ground discontinuities, distorted background trees	no ghosts
PhotoScan (low quality)	35 / 70	small grass and sky artefacts on synthetic tree. Large sky artefacts in real scene	complete reconstruction of trees but somewhat hazy shape	ground discontinuities, distorted background trees	no ghosts
PhotoScan (medium quality)	6 / 24	thin layers of grass mixed in synthetic tree crown. Large upper crown artefact in real scene	complete reconstruction of trees	ground discontinuities	no ghosts
PhotoScan (high quality)	3 / 27	misplaced thin layers of grass and sky in synthetic scene. Small sky artefacts attached to upper parts of the real tree	complete reconstruction of trees except for selected branches	discontinuities in real scene ground. Missing parts of synthetic scene targets	no ghosts
PhotoScan (highest quality)	0 / 29	no synthetic scene artefacts, small layer of sky to the real scene tree	almost half of the synthetic tree is missing	discontinuities in real scene ground. Partially reconstructed ground.	no ghosts

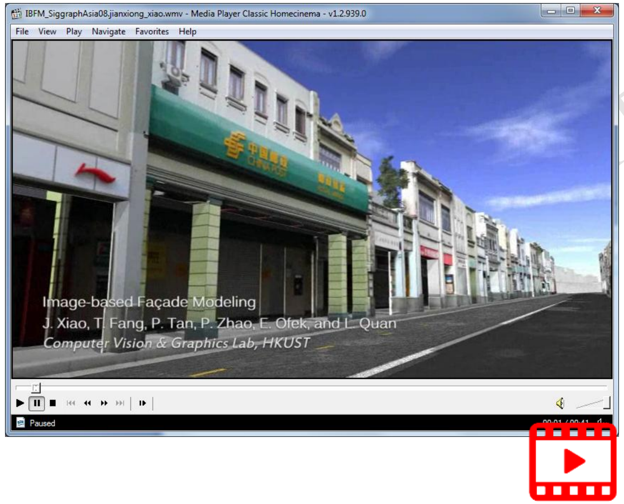
A. Probst, D. Gatziolis, and N. Strigul, "Intercomparison of photogrammetry software for three-dimensional vegetation modelling," *R. Soc. Open Sci.*, vol. 5, no. 7, 2018

Structure from motion (software pre-product)

Applications

- Video trace: image synthesis
- Authoring tools to 3D (static object)
- Reconstruct urban 3D:Deal with frontal textures



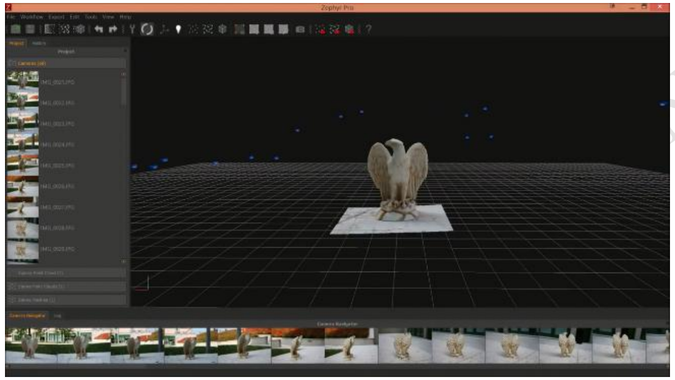


2006 - Pavić, Schönefeld, Kobbelt - Interactive image completion with perspective correction
Siggraph 2007 Video trace
Image based façade modeling, SIGGRAPH ASIA 2008

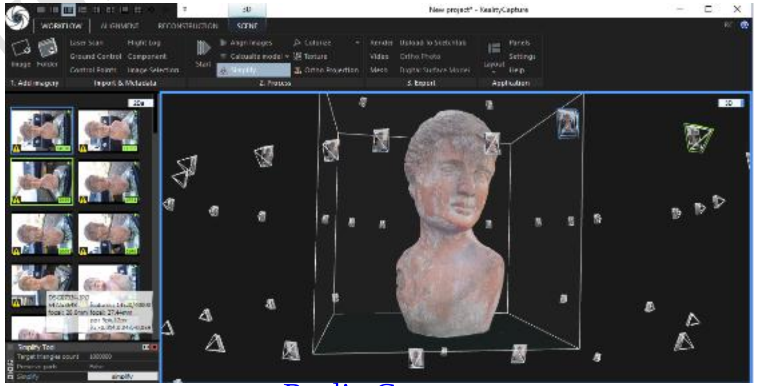
Multi-view in 3D reconstruction (photometric)

- 3D software or services

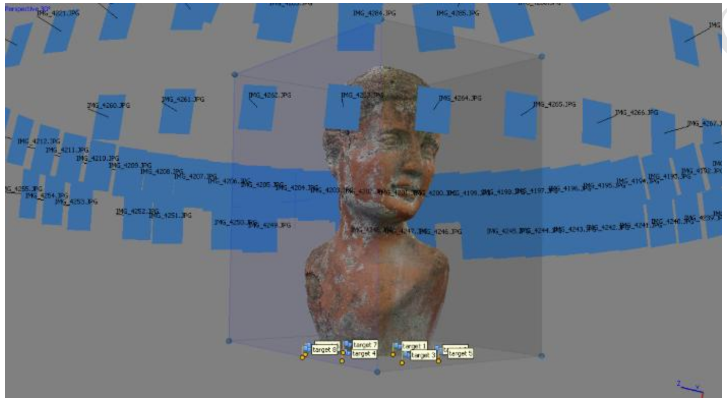
Autodesk Remake



3DF Zephyr Pro

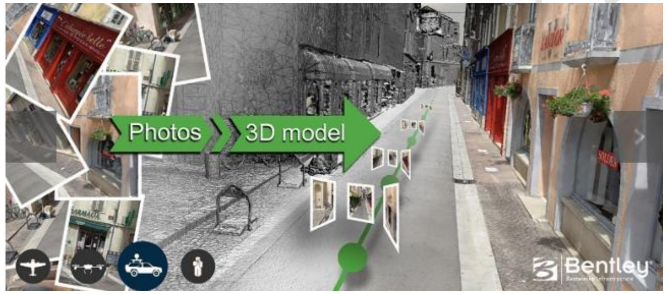


RealityCapture



Agisoft PhotoScan

Photo Modeler



http://www.acute3d.com/

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