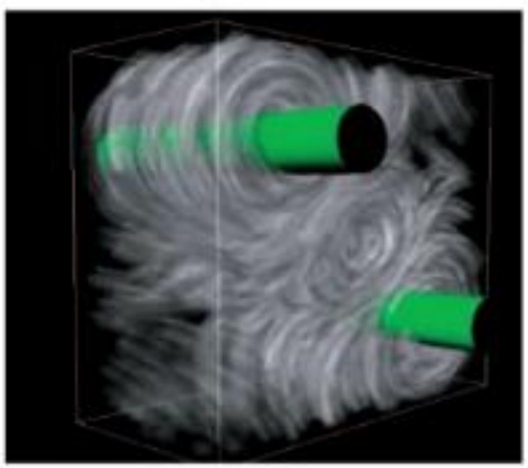
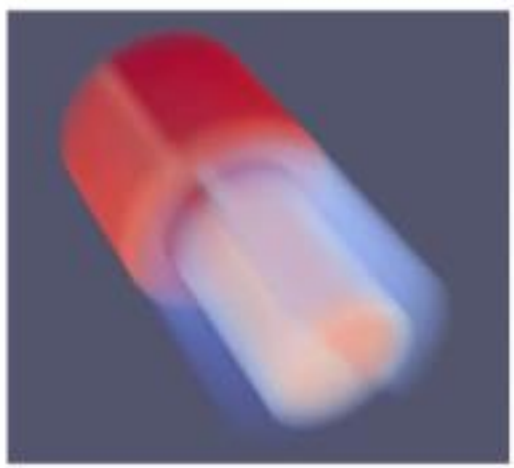
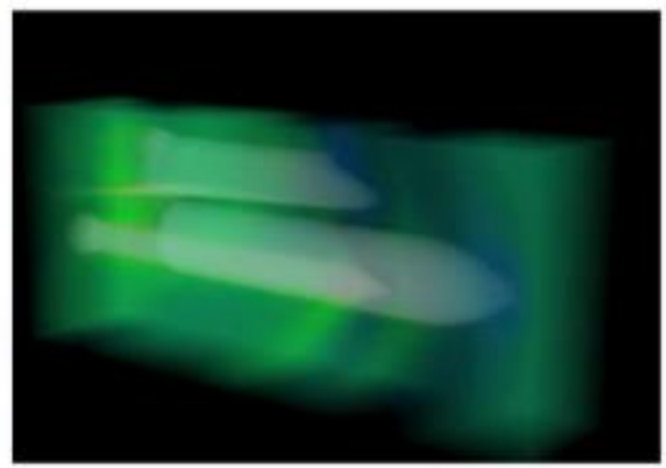
Direct Volume Rendering
Optical Model

Courtesy of Prof. Han-Wei Shen
(CSE 5544, The Ohio State University)

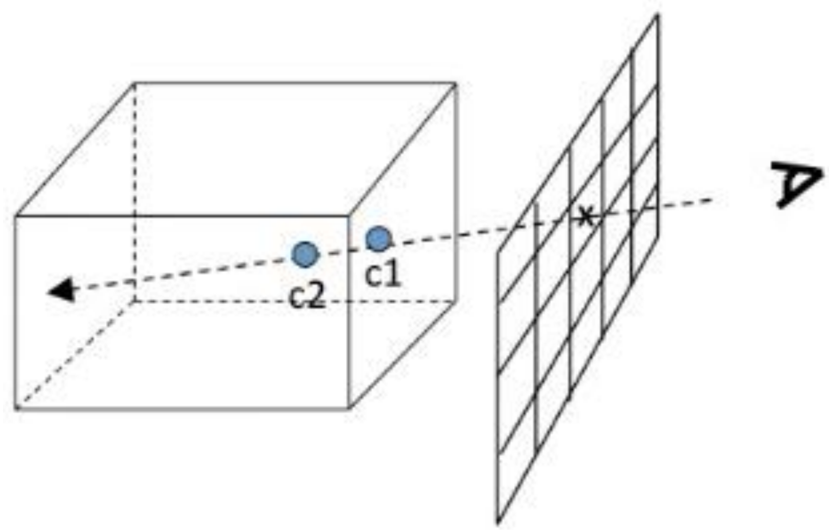
Volume Rendering

- A method to visualize the entire 3D data set by simulating light transport across the volume
- A 2D projection of 3D discrete samples

Direct Volume Rendering

- Simulate light transport through a continuous volume
- Data are interpolated from the samples at the grid points
- Optical properties such as colors and opacities are assigned to the interpolated data
- Optical properties must be integrated along each viewing ray

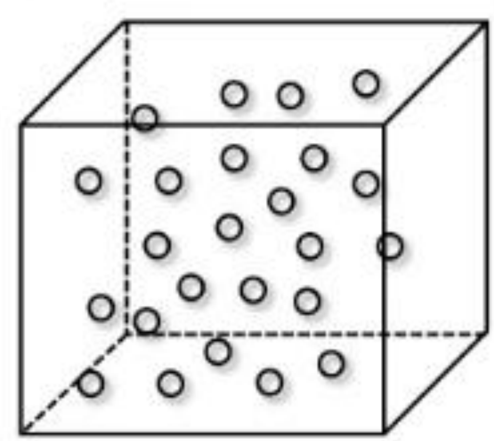


Optical Model

Participating medium (the voxels) can absorb, emit, or both absorb and emit light

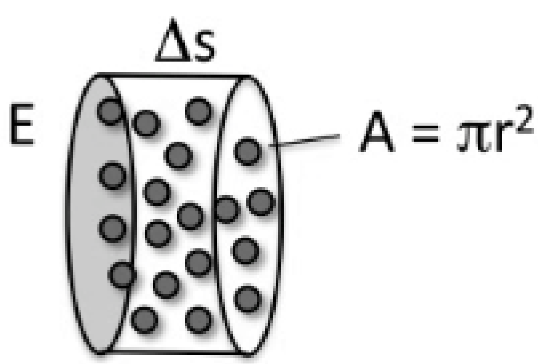
How much light will reach the eye?

- Absorption Model
- Emission Model
- Absorption + Emission



Absorption Only

- The simplest participating medium
- Consists of perfectly black particles that absorb all the light that they intercept
- Assume
  - Each particle has an area of $A = \pi r^2$
  - Number of particles per unit volume = ρ
  - A small cylindrical slab with a base area E and thickness Δs



- Total number of particles = $E \, \Delta s \, \rho$
- Total area occluded by particles = $A \, E \, \Delta s \, \rho$
- The fraction of occluded area = $AE\Delta s\rho \, / \, E = A \, \Delta s \, \rho$

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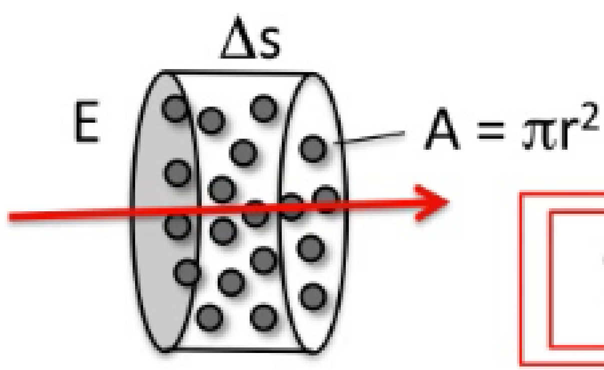

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$$\boxed{\frac{d\, I(s)}{ds}} = - I(s) \times \frac{A \, \Delta s \, \rho(s)}{\Delta s} = - A \, \rho(s) \, I(s)$$

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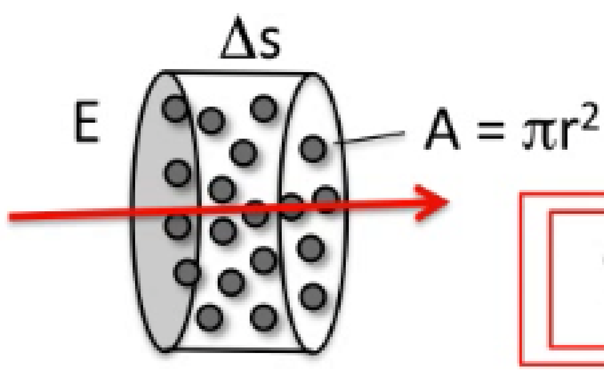

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$$\boxed{\boxed{\frac{d\,\mathbf{I(s)}}{ds}} = -\,I(s) \times \frac{A\ \Delta s\ \rho(s)}{\Delta s} = -\,A\ \rho\ (s)\ I(s)}$$

Solve this ODE

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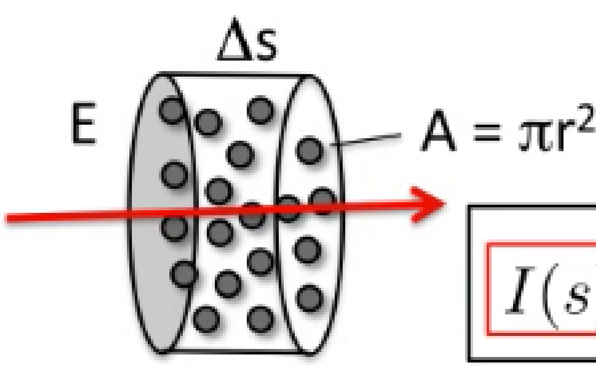
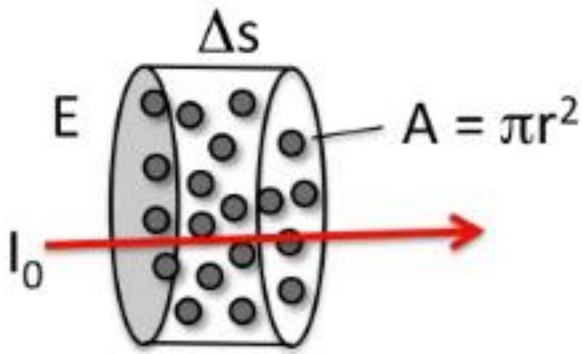


$$\frac{dI}{ds} = -A\,\rho\,(s)\,I(s)$$

Solve this ODE

$$\boxed{I(s)} = \boxed{I_0} \times e^{-\int_0^s \rho(t) A dt} = I_0 \times e^{-\int_0^s \tau(t) dt}$$

Absorption Only



$$I(s) = I_0 \times e^{-\int_0^s \boxed{\rho(t)A} dt} = I_0 \times e^{-\int_0^s \boxed{\tau(t)} dt}$$

$\boxed{\rho(t)A = \tau(t)}$: extinction coefficient

I_0 = Initial light intensity

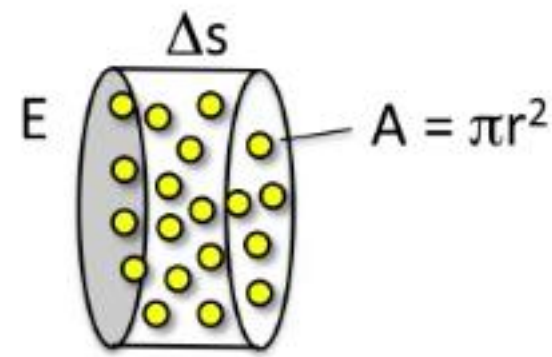
$$\boxed{e^{-\int_0^s \tau(t)dt}}$$

Can be seen as the transparency, or
1 – opacity of the medium from 0 to s

Emission Only

- Each particle will glow diffusively with an intensity C
- In a small cylindrical slab, the total area occupied by the particle is $AE\Delta s\rho$
- So the glow flux will be $\mathrm{C}AE\Delta s\rho$
- Then the glow per unit area is $\mathrm{C}AE\Delta s\rho/\mathrm{E} = \mathrm{C}A\Delta s\rho$

$$\boxed{\frac{dI}{ds} = C(s)\, A\, \rho(s) = C(s)\tau(s) = g(s)} \longrightarrow \boxed{\text{Solve this ODE}}$$



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- Then the glow per unit area is $AE\Delta s\rho$/E = C$A\Delta s\rho$

$$\frac{dI}{ds} = C(s)\, A\, \rho(s) = C(s)\tau(s) = g(s)$$

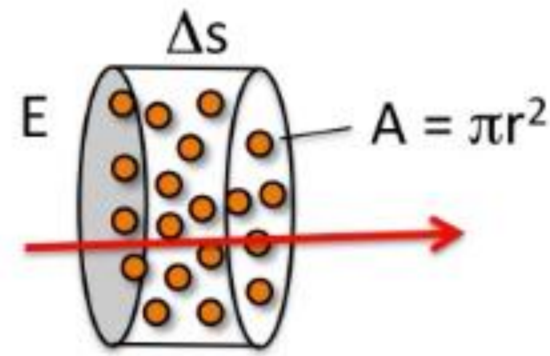
Solve this ODE

$$I(s) = I_0 + \int_0^s g(t)dt = I_0 + \int_0^s C(t)\tau(t)dt$$

Emission plus Absorption

- Simply add emission and absorption together

$$\boxed{\frac{dI}{ds} = C(s)\tau(s) - A\,\rho\,(s)\,I(s) = \underbrace{g(s)}_{\text{emission}} - \underbrace{\tau(s)I(s)}_{\text{absorption}}}$$

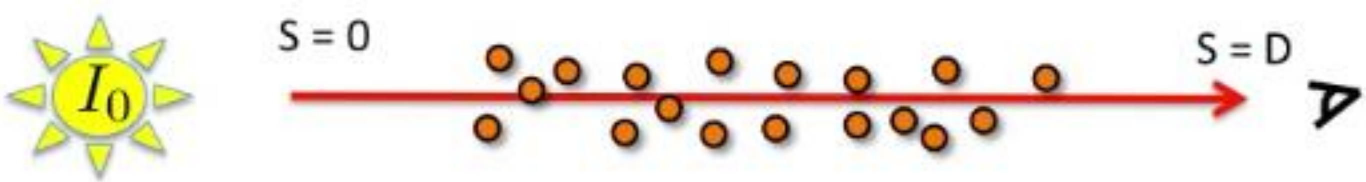


$\downarrow$

Solve this ODE

$\downarrow$

$$I(D) = I_0 \times e^{-\int_0^D \tau(t)dt} + \int_0^D g(s) e^{-\int_s^D \tau(t)dt} ds$$



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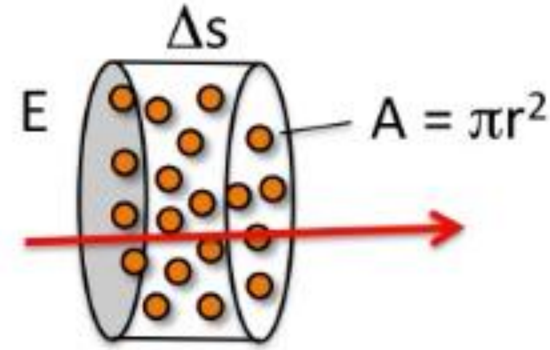

↓

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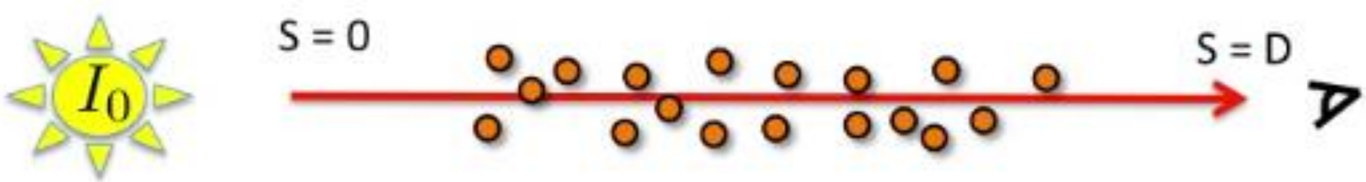

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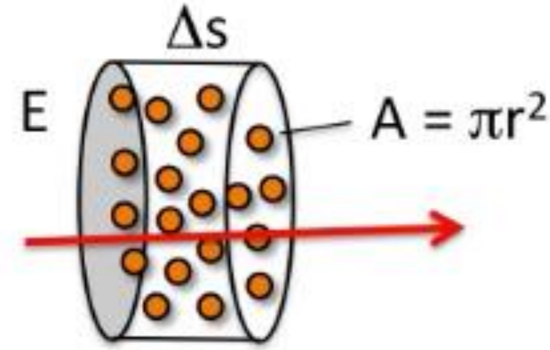

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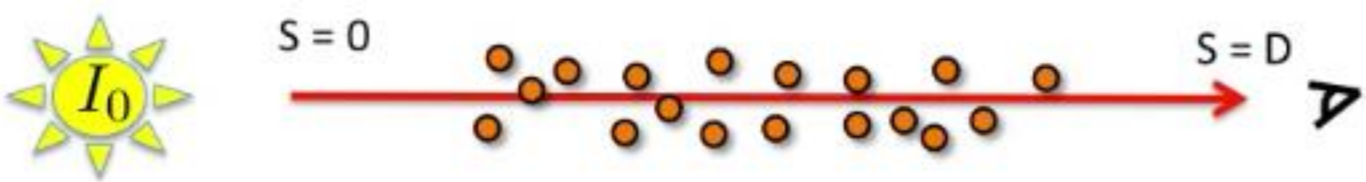

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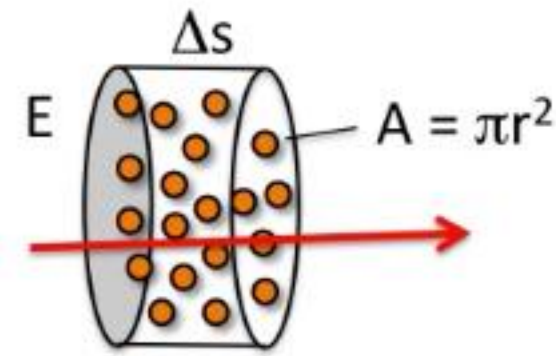

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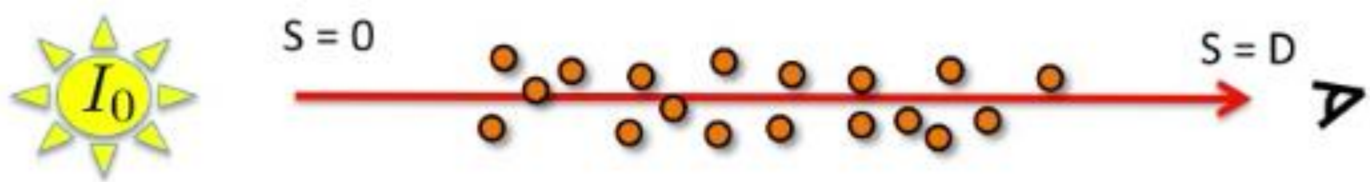

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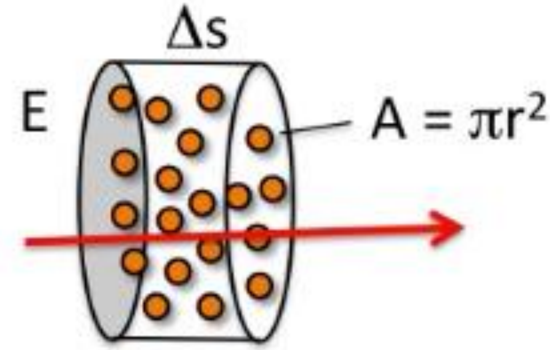

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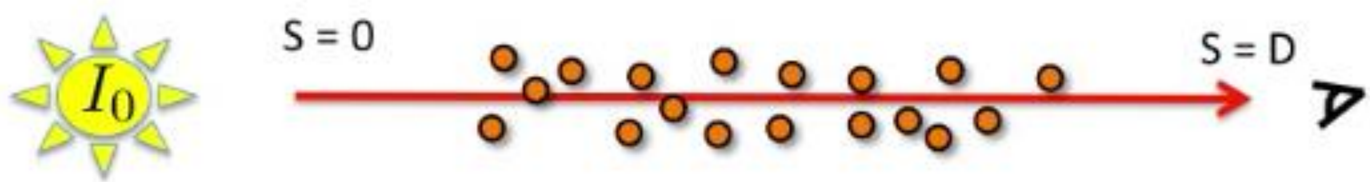
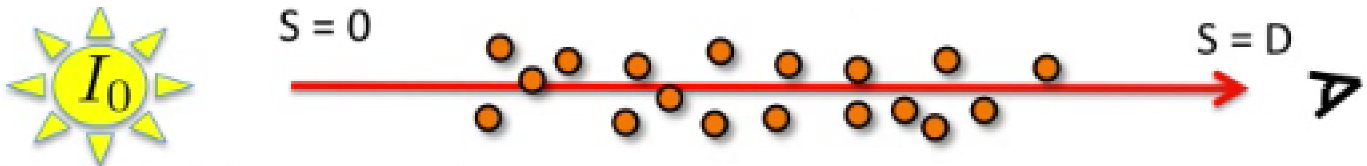


Put It All Together



- Absorption only:

$$\boxed{I(s) = I_0 \times e^{-\int_0^s \rho(t) A dt} = I_0 \times e^{-\int_0^s \tau(t)dt}}$$

- Emission only:

$$\boxed{I(s) = I_0 + \int_0^s g(t)dt = I_0 + \int_0^s C(t)\tau(t)dt}$$

- Emission plus absorption:

$$\boxed{I(D) = I_0 \times e^{-\int_0^D \tau(t)dt} + \int_0^D g(s) e^{-\int_s^D \tau(t)dt} ds}$$

Look More Closely

$$\boxed{I(D) = I_0 \times e^{-\int_0^D \tau(t)dt} + \int_0^D g(s)e^{-\int_s^D \tau(t)dt}ds}$$

I_0 : background light

$\tau(t)$: extinction coefficient at t , related to the rate that light is occluded

D : total distance light will travel

$e^{-\int_0^D \tau(t)dt}$: transparency of medium between 0 and D

$1 - e^{-\int_0^D \tau(t)dt} = \alpha$: opacity of medium between 0 and D

$g(s)$: source term at point s, typically derived from the data value