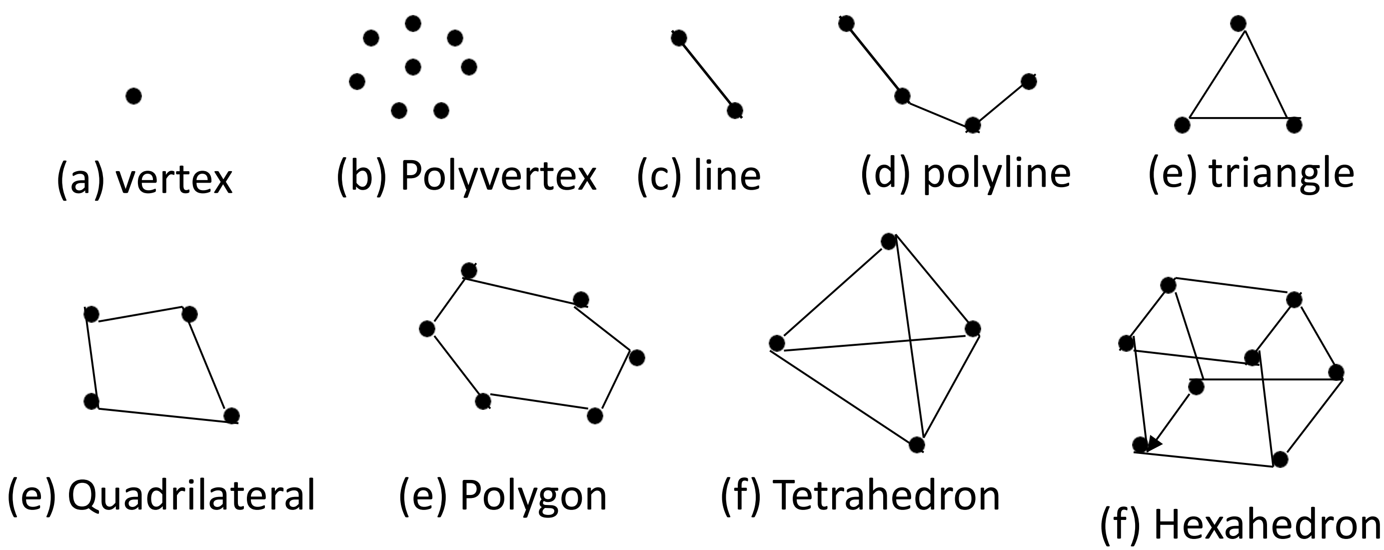
Linear Interpolation from Cells

Courtesy of Prof. Han-Wei Shen
(CSE 5544, The Ohio State University)

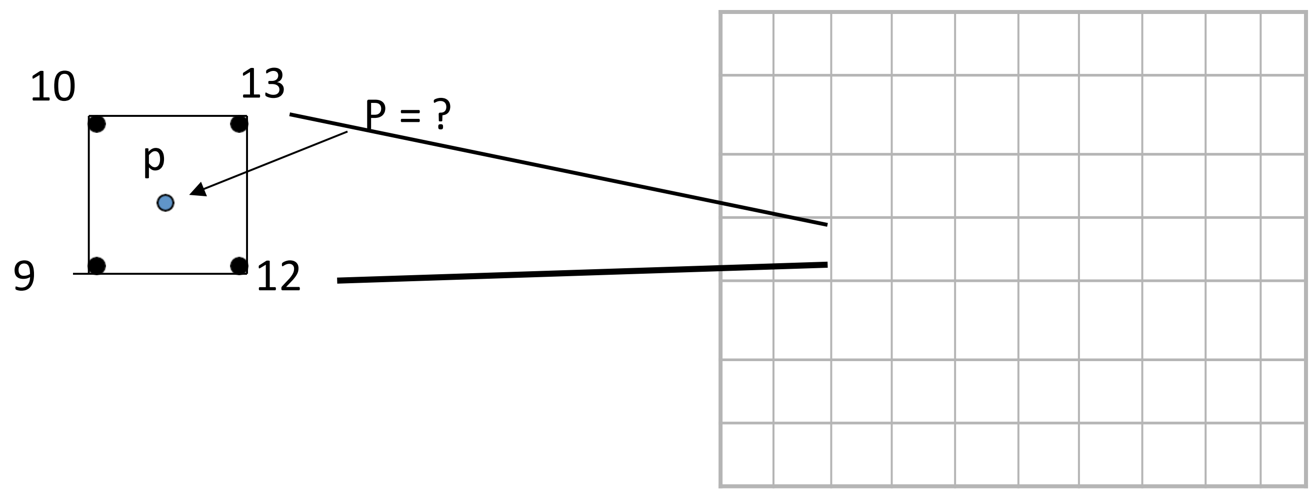
Why Interpolation?

- Most visualization algorithms have to deal with discrete data
  - Data attributes that define at the cell vertices



Why Interpolation?

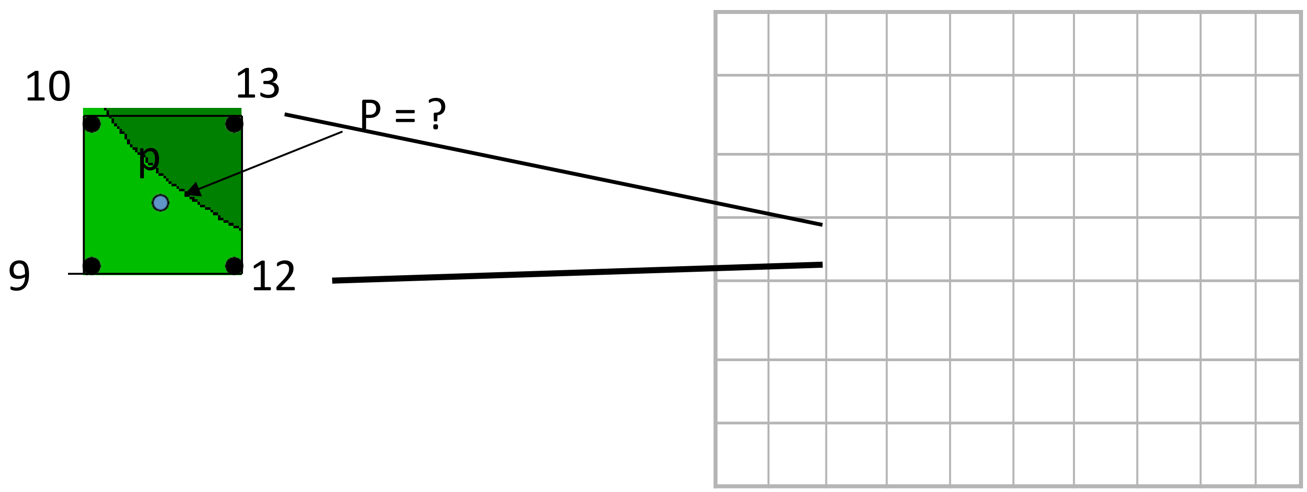
- Example: Produce a color map from a 2D regular grid



1. Interpolate the values from the cell corners to get the value of P
2. Apply a color to P

Why Interpolation?

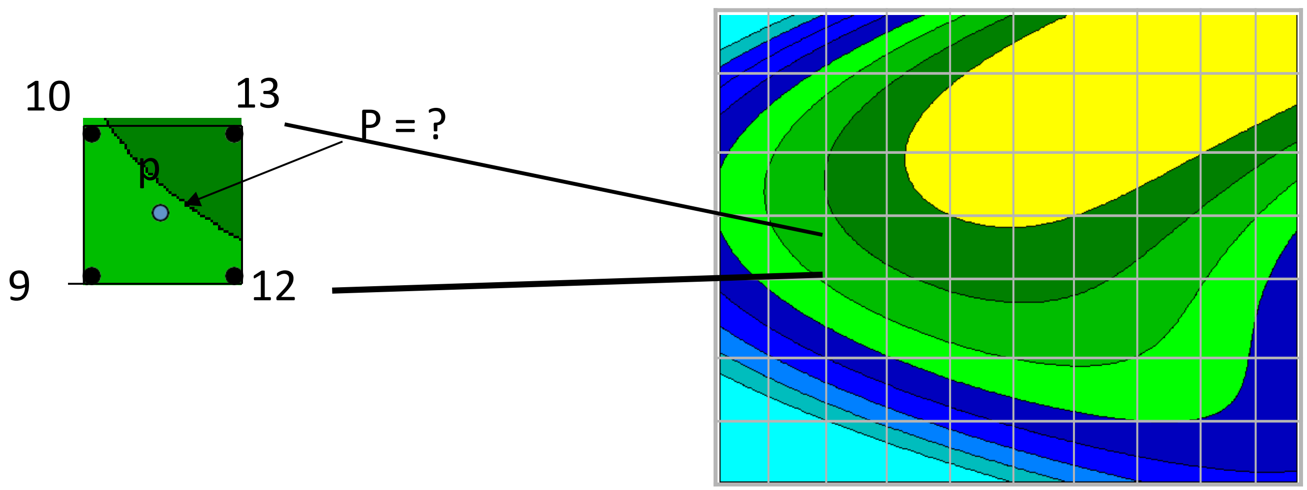
- Example: Produce a color map from a 2D regular grid



1. Interpolate the values from the cell corners to get the value of P
2. Apply a color to P

Why Interpolation?

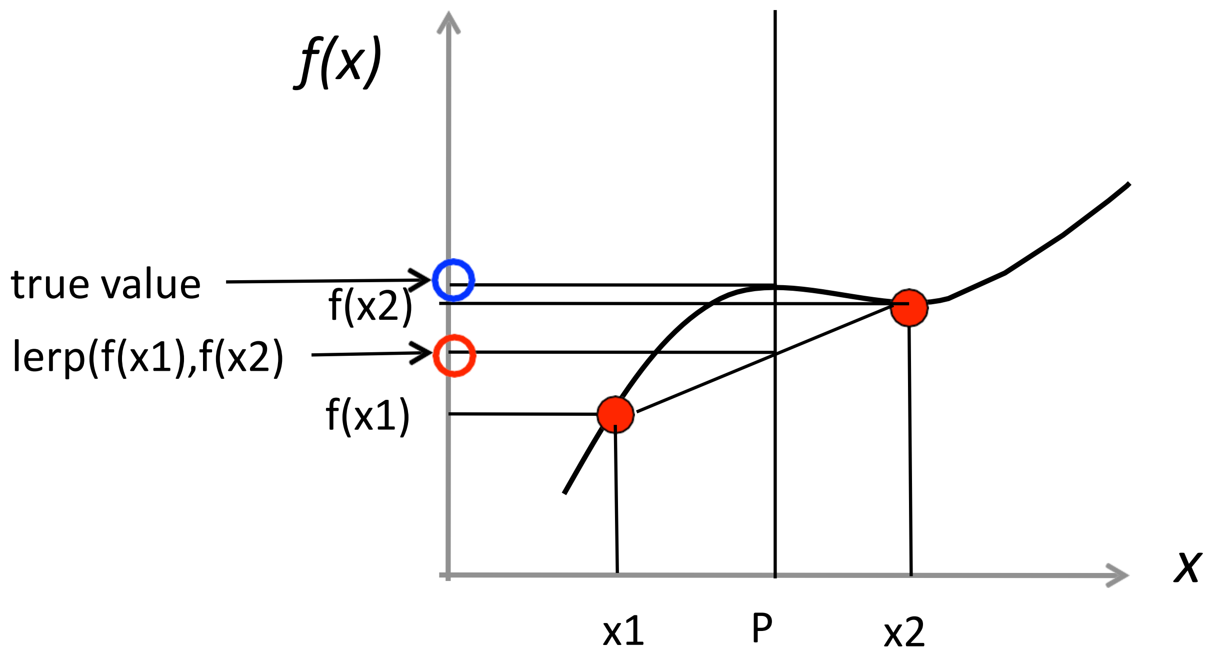
- Example: Produce a color map from a 2D regular grid



1. Interpolate the values from the cell corners to get the value of P
2. Apply a color to P

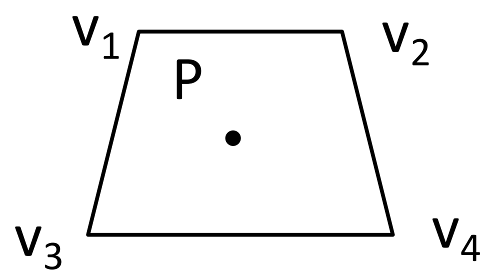
Linear Interpolation (LERP)

- Linear interpolation (lerp): connecting two points with a straight line in the function plot



Linear Interpolation (LERP)

- General form: $V_p = \Sigma \ \ w_i * v_i$ (weighted sum)

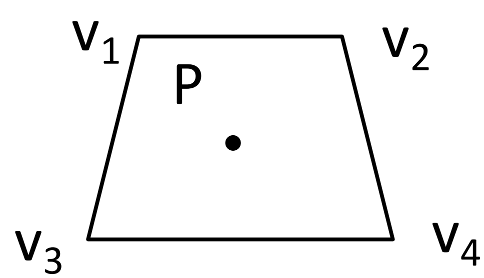


v_i : value at vertex i

$_{wi:}$ weight for v_i

Linear Interpolation (LERP)

- General form: $V_p = \Sigma \; w_i * v_i$ (weighted sum)

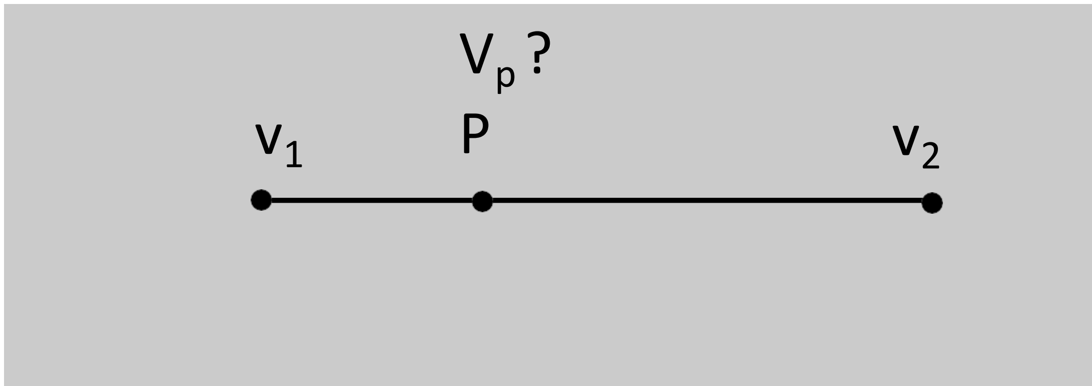


v_i : value at vertex i

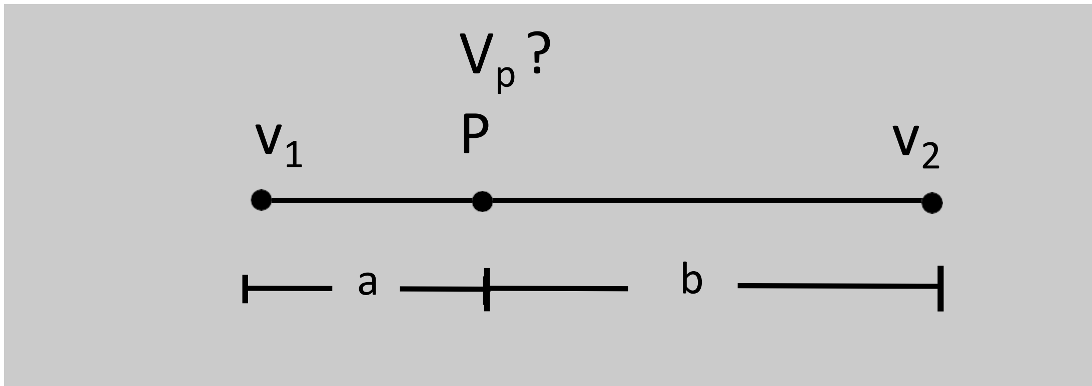
$_{wi:}$ weight for v_i

- Essential information needed:
  - Cell type
  - Data at cell corners
  - Parametric coordinates of the point in question (P)
    - Related to the position of point P in the cell

LERP in Line

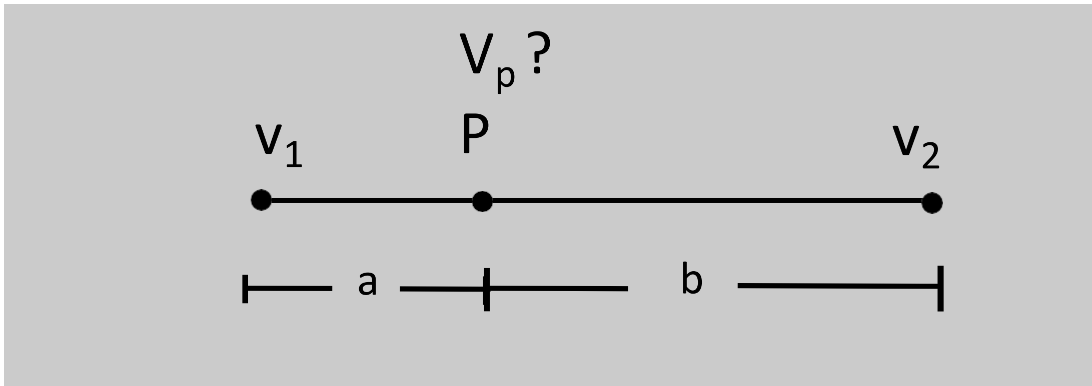


LERP in Line



- Parametric coordinate of P: α = a / (a+b)

LERP in Line

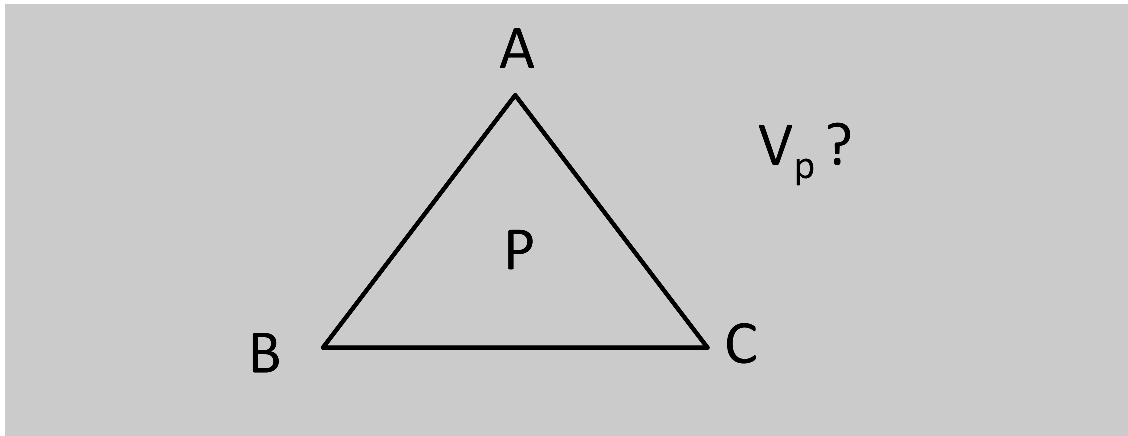


- Parametric coordinate of P: $\alpha = a / (a+b)$
- Linearly interpolated value of P:

$$\boxed{V_p = (1-\alpha) * V_1 + \alpha * V_2}$$

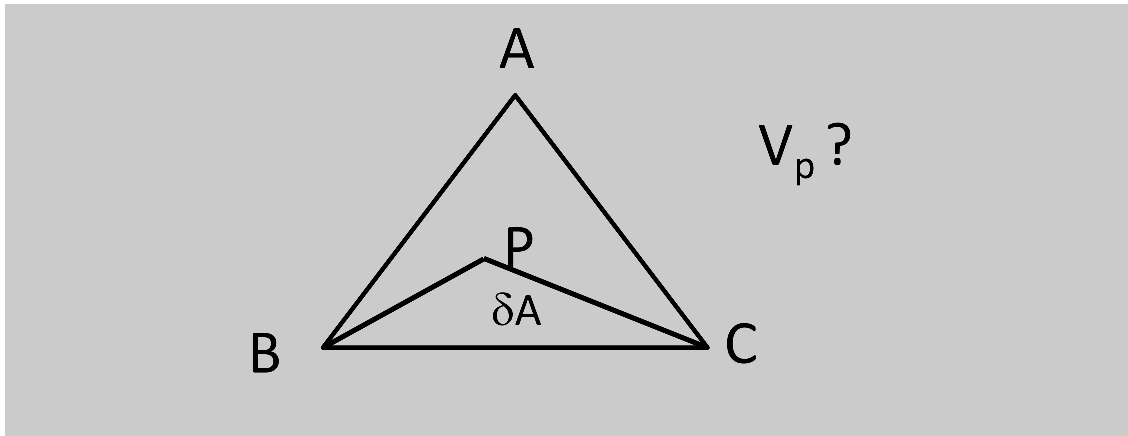
lerp(v1,v2, α)

Lerp in Triangle

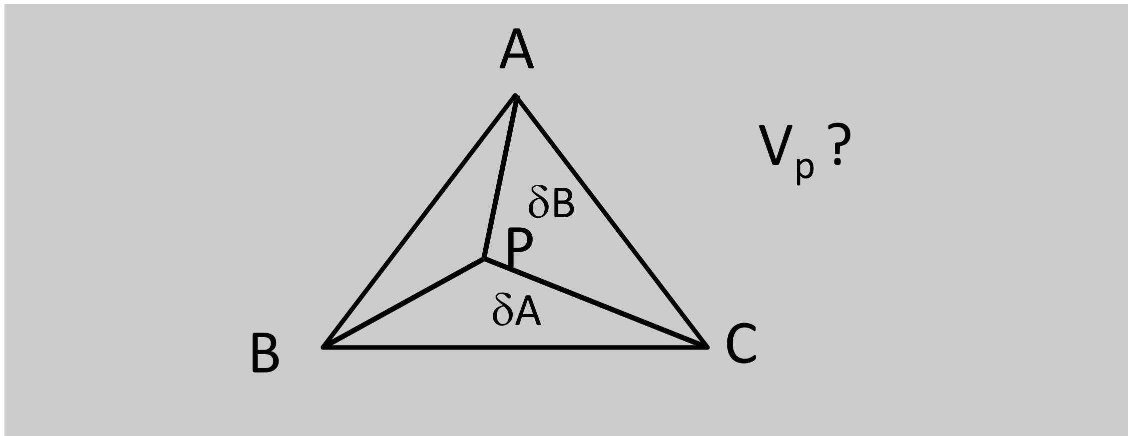


S04-01

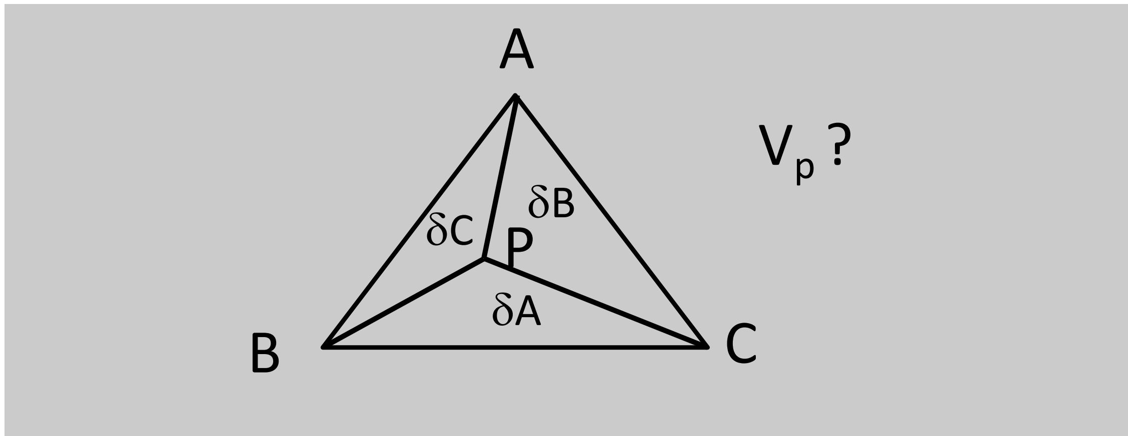
Lerp in Triangle



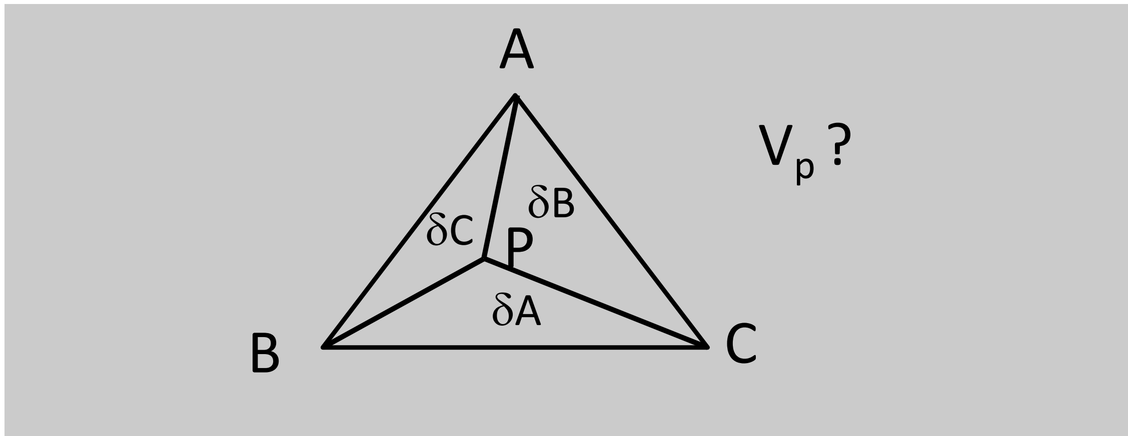
Lerp in Triangle



Lerp in Triangle



Lerp in Triangle



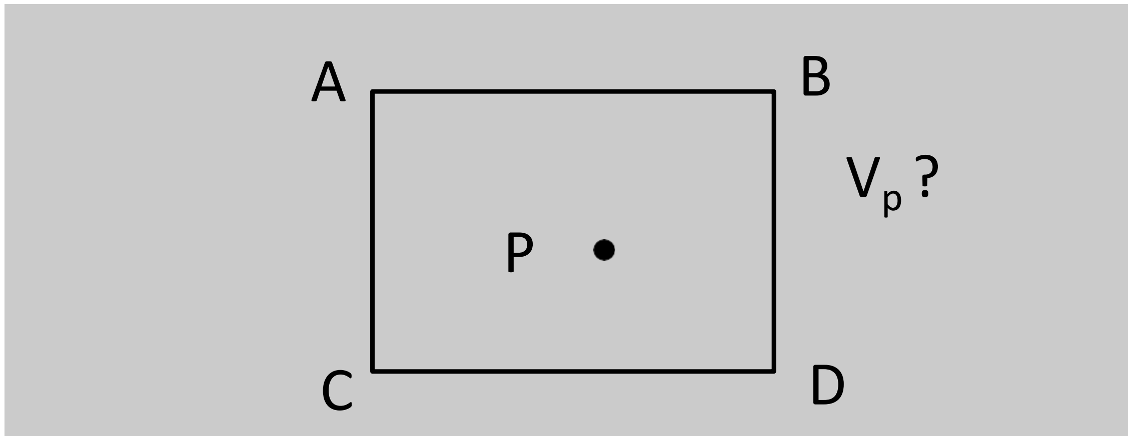
- Parametric coordinates of P: (α,β,γ)

$$\alpha = \delta A / (\delta A + \delta B + \delta C)$$
$$\beta = \delta B / (\delta A + \delta B + \delta C)$$
$$\gamma = \delta C / (\delta A + \delta B + \delta C)$$

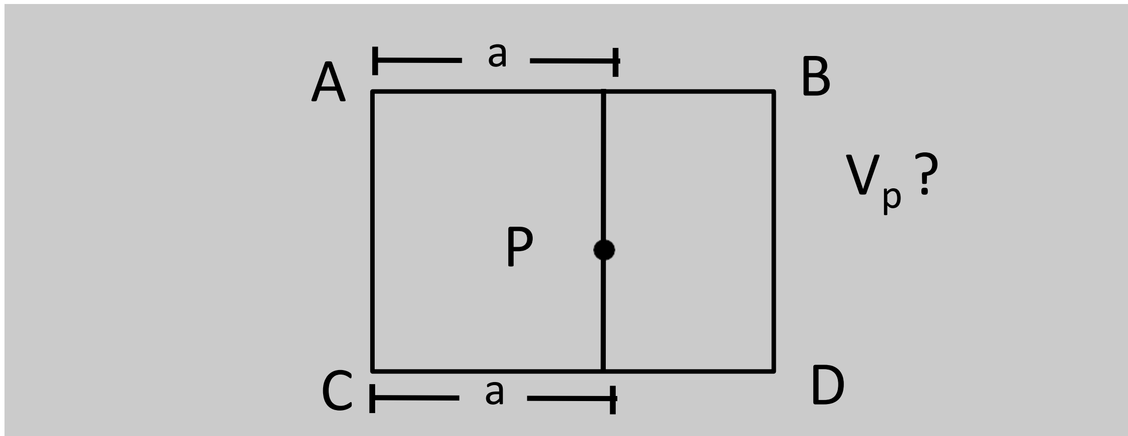
Baricentric Coordinates

- Linearly interpolated value of P: $\boxed{V_A * \alpha + V_B * \beta + V_C * \gamma}$

Lerp in Rectangle



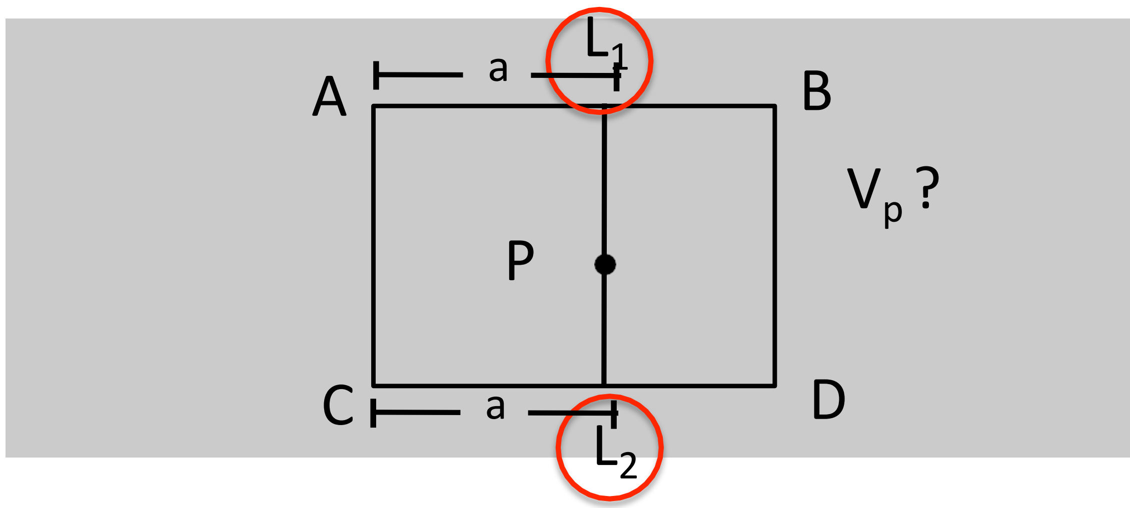
Lerp in Rectangle



- Parametric coordinates of P: (α,β)

$$\alpha = \text{a / width;}$$

Lerp in Rectangle

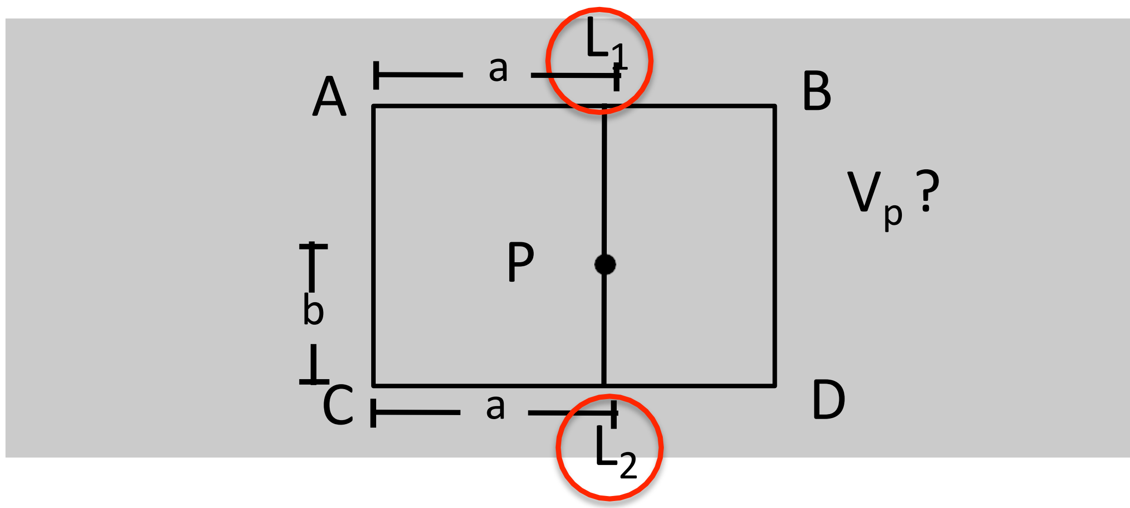


- Parametric coordinates of P: (α,β)

$$\alpha = a / width;$$

- Value at $L_1 = Lerp(V_A, V_B, \alpha)$;
- Value at $L_2 = Lerp(V_C, V_D, \alpha)$;

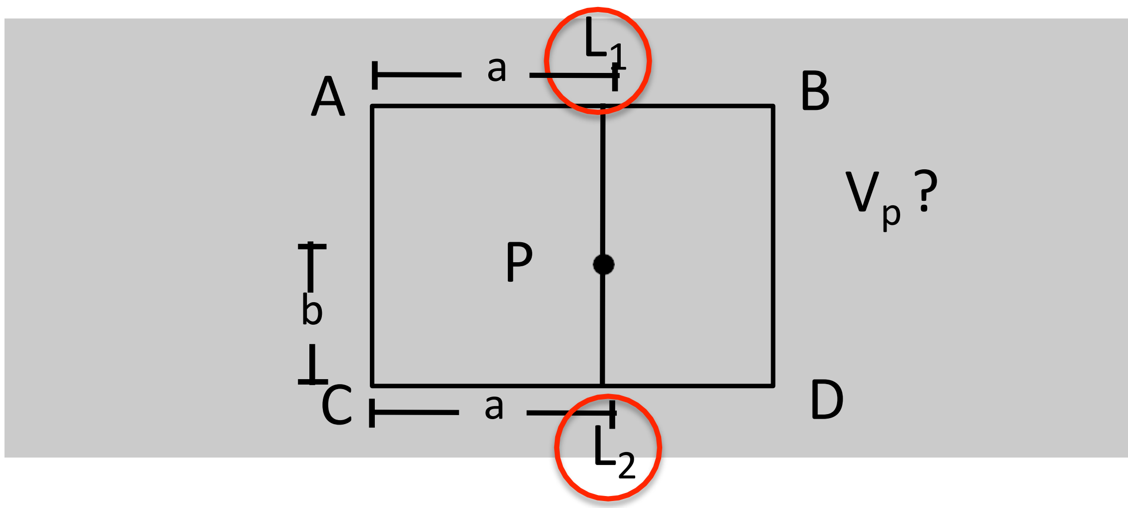
Lerp in Rectangle



- Parametric coordinates of P: (α,β)

$$\alpha = a / \text{width};$$

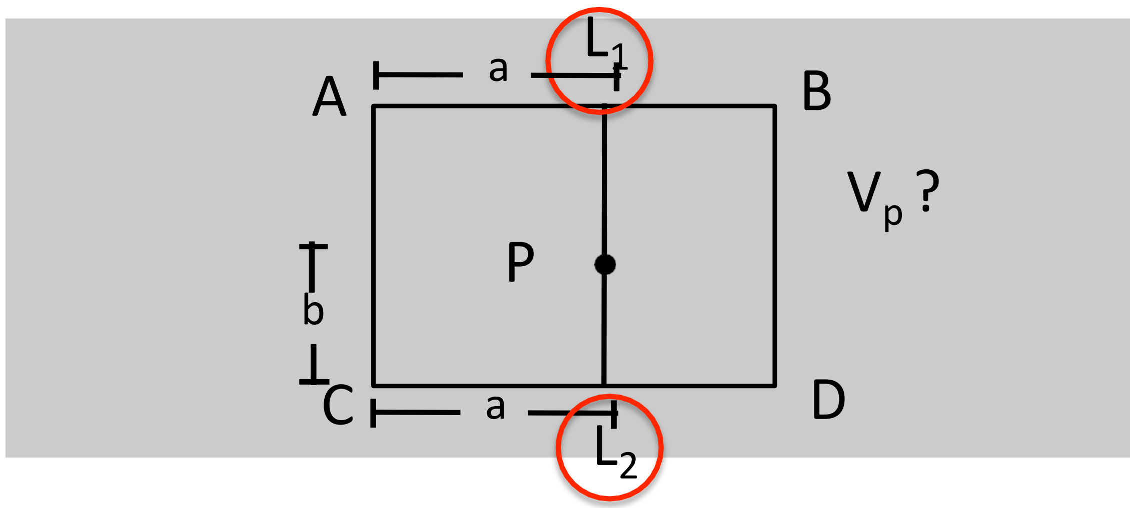
Lerp in Rectangle



- Parametric coordinates of P: (α,β)

$$\alpha = a / \text{width}; \ \beta = b / \text{height}$$

Lerp in Rectangle

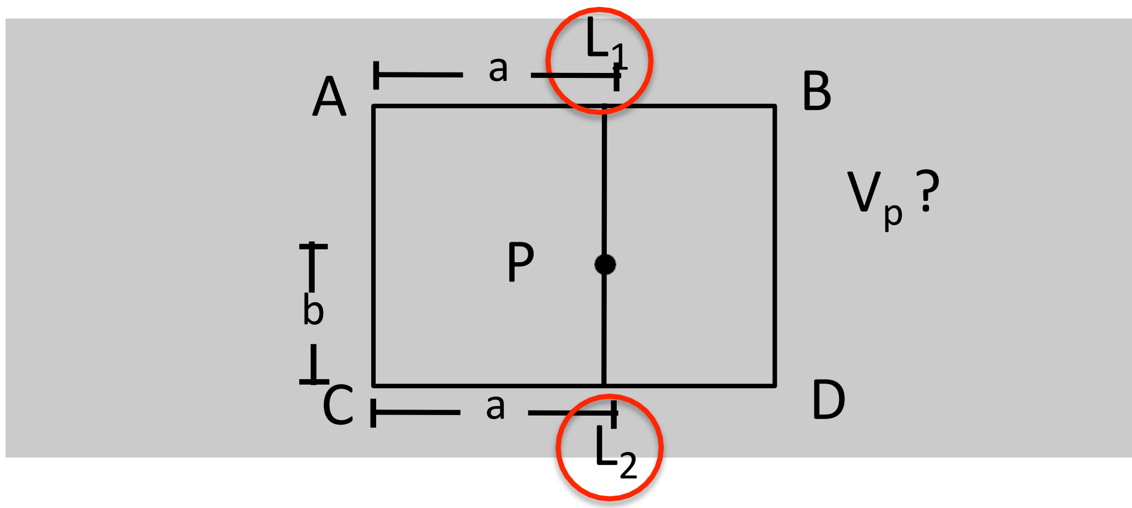


- Parametric coordinates of P: (α,β)

  $\alpha =$ a / width; $\beta =$ b / height

- Linearly interpolated value of P: $\boxed{\text{Lerp}(V_{L1}, V_{L2}, \beta)}$

Lerp in Rectangle



- Parametric coordinates of P: (α,β)

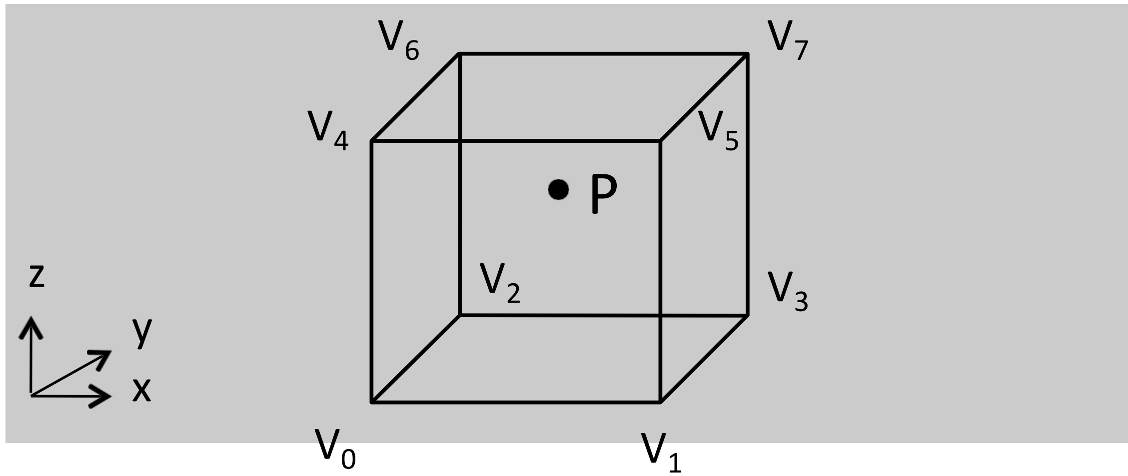
  $\alpha =$ a / width; $\beta =$ b / height

Bi-linear interpolation
Bi-Lerp(V_A,V_B, V_C,V_D)

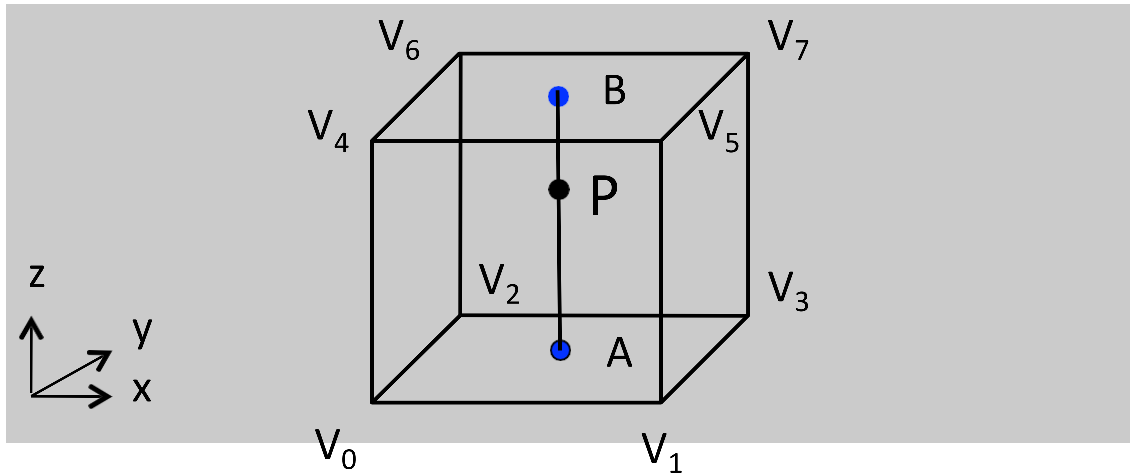
- Linearly interpolated value of P: $\boxed{\text{Lerp}(V_{L1}, V_{L2}, \beta)}$

S04-02

Lerp in Cube

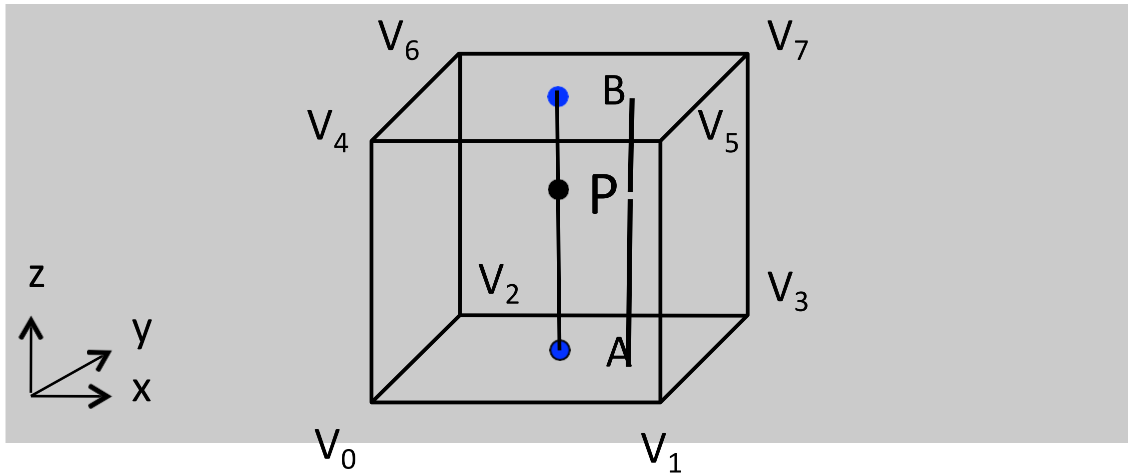


Lerp in Cube



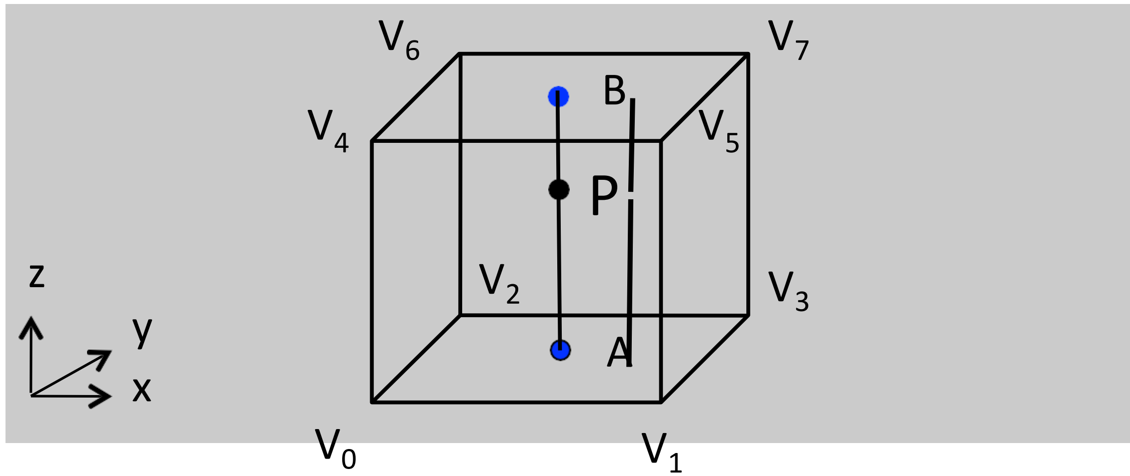
- Value at A = Bi-Lerp(V_0,V_1,V_2,V_3) ;
- Value at B = Bi-Lerp(V_4,V_5,V_6,V_7) ;

Lerp in Cube



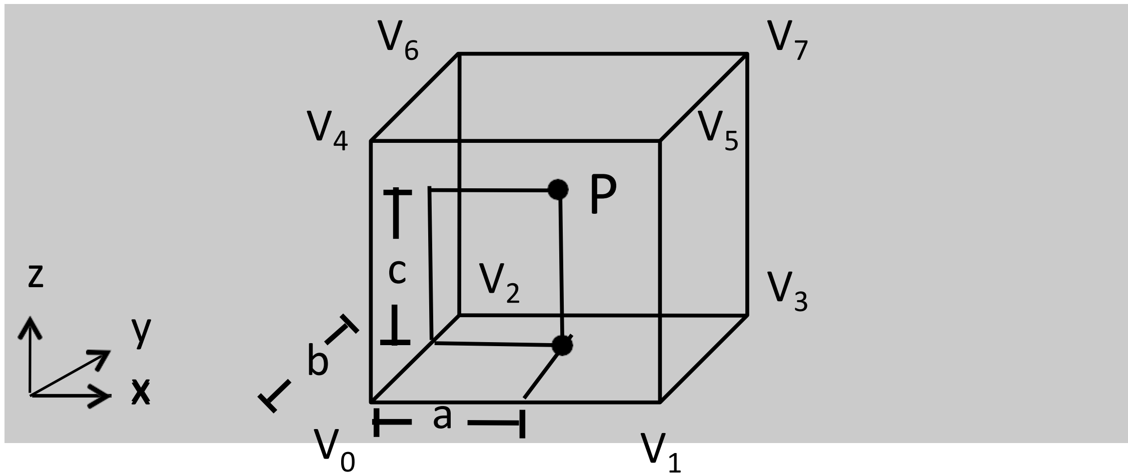
- Value at A = Bi-Lerp(V_0,V_1,V_2,V_3) ;
- Value at B = Bi-Lerp(V_4,V_5,V_6,V_7) ;

Lerp in Cube



- Value at A = Bi-Lerp(V_0,V_1,V_2,V_3) ;
- Value at B = Bi-Lerp(V_4,V_5,V_6,V_7) ;
- Value at P = Lerp(A,B, PA/AB); ⟵ Tri-linear interpolation

Lerp in Cube



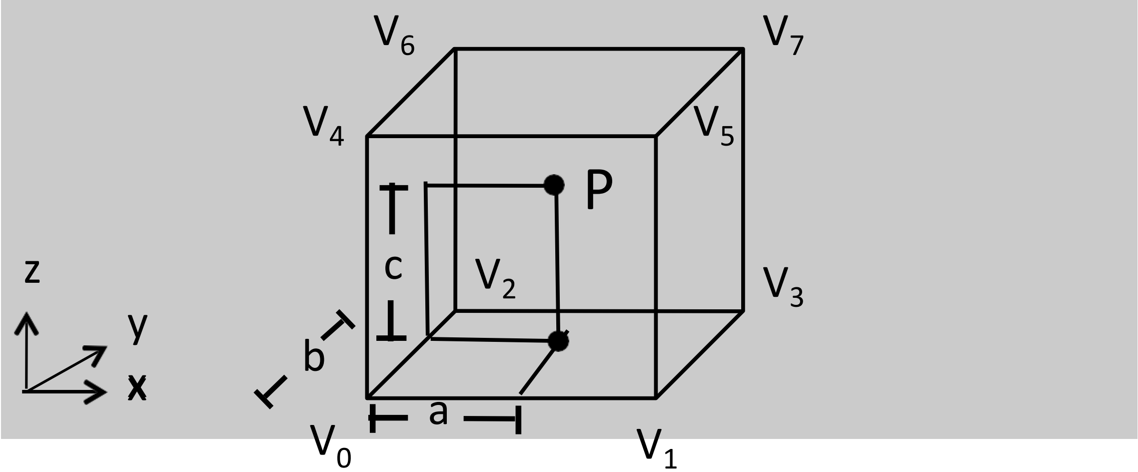
Another way to perform calculate the value at P:

- Parametric coordinates of P: (α,β)

  $\alpha =$ a / width; β = b / depth (along y);

  γ = c / height

Lerp in Cube



Another way to perform calculate the value at P:

- Parametric coordinates of P: (α,β,γ)

  $\alpha =$ a / width; β = b / depth (along y);

  γ = c / height
- Value at P =

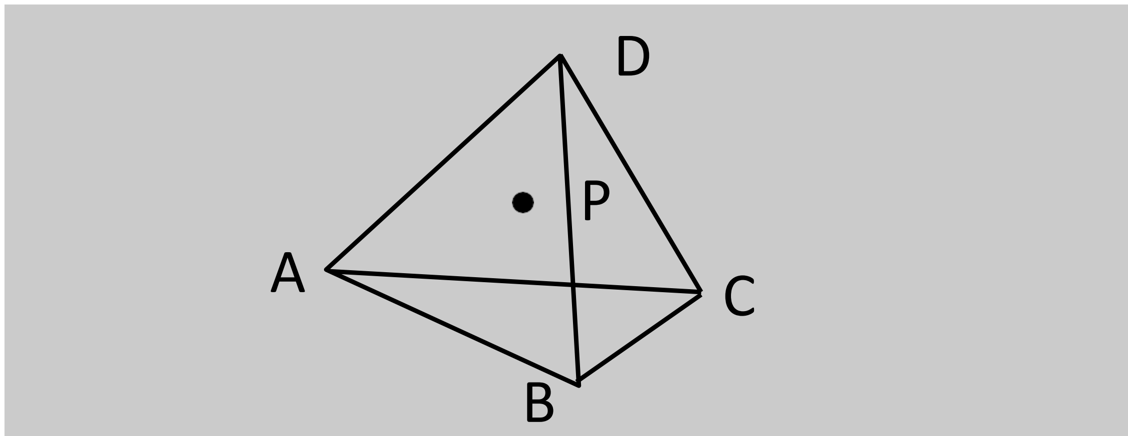
  $(1-\alpha)(1-\beta)(1-\gamma)V_0 + \alpha(1-\beta)(1-\gamma)V_1 +$

  $(1-\alpha)\beta(1-\gamma)V_2 + \alpha\beta(1-\gamma)V_3 +$

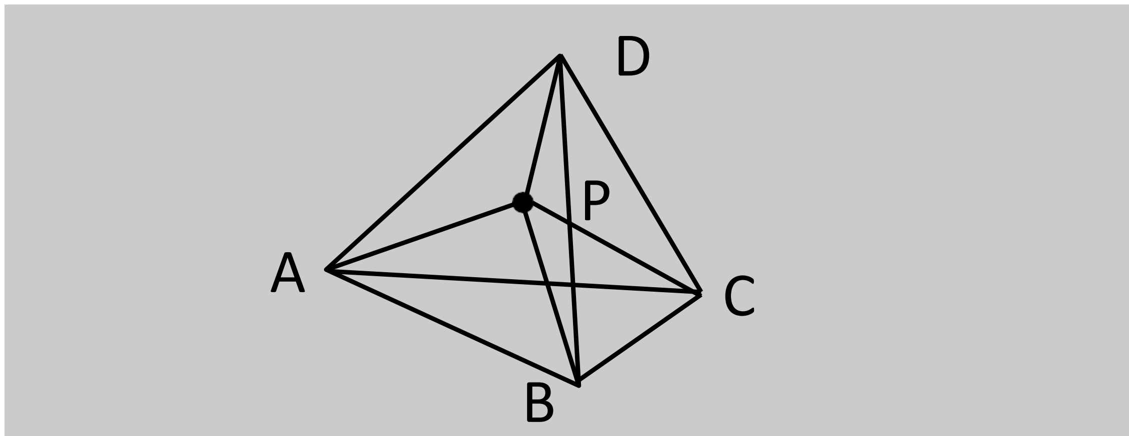
  $(1-\alpha)(1-\beta)\gamma V_4 + \alpha(1-\beta)\gamma V_5 +$

  $(1-\alpha)\beta\gamma V_6 + \alpha\beta\gamma V_7$

Lerp in Tetrahedron

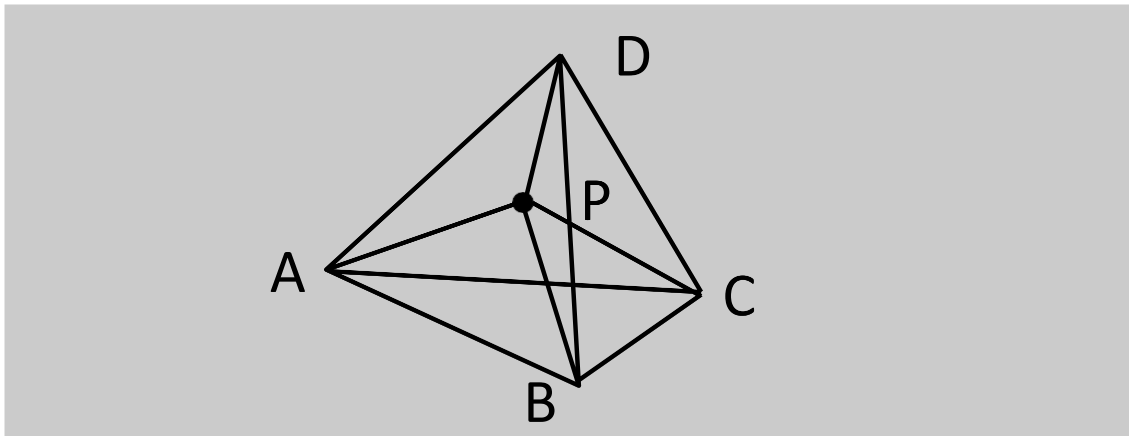


Lerp in Tetrahedron



- Break the tetrahedron ABCD into four sub tetrahedra: ABCP, BDCP, ACDP, ADBP
- Calculate the volume of each small tetrahedra
- Calculate P's parametric (tetrahedral) coordinates based on the ratios of the volumes

Lerp in Tetrahedron

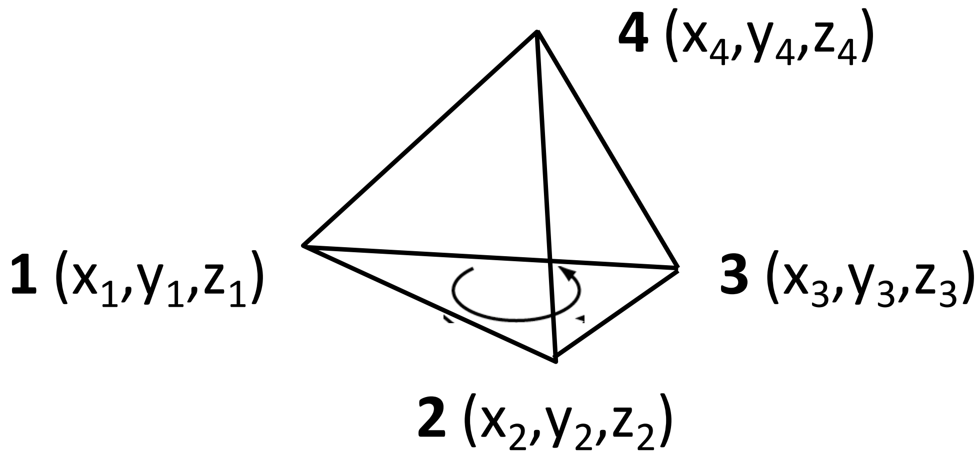


- Tetrahedral coordinates of P: $(\alpha,\beta,\gamma,\delta)$

$$\alpha = V_{BDCP} \,/\, V_{ABCD}$$
$$\beta = V_{ACDP} \,/\, V_{ABCD}$$
$$\gamma = V_{ADBP} \,/\, V_{ABCD}$$
$$\delta = V_{ABCP} \,/\, V_{ABCD}$$

- Linearly interpolated value of P: $\boxed{V_A * \alpha + V_B * \beta + V_C * \gamma + V_D * \delta}$

Volume of Tetrahedron



$$V = \frac{1}{6}\det\begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{bmatrix} = \frac{1}{6}\det(\mathbf{J}) = \frac{1}{6}J.$$

V will be positive if when you look at the triangle $_{123}$ from vertex 4, vertex 1 2 3 are In a counter clockwise order