Real-Coded Evolutionary Algorithms with Parent-Centric Recombination

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Abstract- Due to an increasing interest in solving real-world optimization problems using evolutionary algorithms (EAs), researchers have developed a number of real-parameter genetic algorithms (GAs) in the recent past. In such studies, the main research effort is spent on developing an efficient recombination operator. Such recombination operators use probability distributions around the parent solutions to create offspring. Some operators emphasize solutions at the center of mass of parents and some emphasize solutions around the parents. In this paper, we propose a generic parent-centric recombination operator (PCX) and compare its performance with a couple of commonly-used mean-centric recombination operators (UNDX and SPX). With the help of a steady-state, elite-preserving, and computationally fast EA model, the simulation results show the superiority of PCX over mean-centric operators on three test problems.

1 Introduction

Over the past few years, there have been a surge of studies related to real-parameter genetic algorithms (GAs), despite the existence of specific real-parameter evolutionary algorithms, such as evolution strategy and evolutionary programming. Although in principle such real-parameter GA studies have been shown to have a similar theoretical behavior on certain fitness landscapes with proper parameter tuning in an earlier study (Beyer and Deb, 2001), in this paper we investigate the performance of a couple of popular real-parameter genetic algorithms.

The unimodal normal distribution crossover (UNDX) operator (Ono and Kobayashi 1997) uses multiple parents and create offspring solutions around the center of mass of these parents. A small probability is assigned to solutions away from the center of mass. On the other hand, the simplex crossover (SPX) (Higuchi et al., 2000) assigns a uniform probability distribution for creating offspring in a restricted search space around the region marked by the parents. These mean-centric operators have been applied to a number of test problems and superiority of the SPX operators has been established (Higuchi et al., 2000). These studies have used a specific GA model (they called the minimum generation gap or MGG model (Satoh et al., 1996)). In their MGG model, 200 offspring solutions are created from a few parent solutions and two better solutions are selected. Since the SPX operator uses a uniform probability distribution for creating offspring, such a large offspring pool was necessary to find a useful offspring. On the other hand, the UNDX operator uses a normal probability distribution to create offspring, giving more emphasis to solutions close to the center of mass of the parents. Although more than two good offspring could have been created using the UNDX operator in 200 offspring solutions, only two were kept in the population and the rest were rejected. Such a waste of good solutions resulted in a poor performance of the UNDX operator under the MGG model. In this paper, we perform a parametric study by varying the offspring pool size and observe an interesting scenario. The UNDX operator performs much better than the SPX operator for small offspring pool size on a number of test problems. Moreover, in order to make the MGG model computationally faster, we suggest a generalized generation gap model (G3), which replaces the roulette-wheel selection with the tournament selection. The proposed G3 model is a steady-state, elite-preserving, and computationally fast algorithm for real-parameter optimization.

In addition, we propose a parent-centric recombination operator, which assigns more probability for creating offspring near parents than anywhere in the search space. This recombination operator is an extension of the simulated binary crossover (SBX) operator, a two-parent, parent-centric recombination operator suggested elsewhere (Deb and Agarwal, 1995). The efficacy of the G3 model with the proposed parent-centric recombination operator on three test problems suggests their application to more complex problems.

2 Real-coded Genetic Algorithms

Over the past few years, many researchers have been paying attention to real-coded evolutionary algorithms, particularly for solving real-world optimization problems. Among numerous studies on development of different recombination operators, blend crossover (BLX), simulated binary crossover (SBX), unimodal normal distribution crossover (UNDX), simplex crossover (SPX) are commonly used. A number of other recombination operators, such as arithmetic crossover, intermediate crossover, extended crossover are similar to BLX operator. A detailed study of many such operators can be found elsewhere (Deb, 2001; Herrara et al, 1998). In the recent past, GAs with some of these recombination operators have been demonstrated to exhibit self-adaptive behavior similar to that in evolution strategy (ES) and evolutionary programming approaches.

Beyer and Deb (2001) argued that a recombination operator may have the following two properties:

1. Population mean decision vector should remain the

same before and after the recombination operator.

2. Variance of the intra-member distances must increase due to the application of the recombination operator.

Since the recombination operator does not use any fitness function information explicitly, the first argument makes sense. The second argument comes from the realization that selection operator has a tendency to reduce the population variance. Thus, population variance must be increased by the recombination operator to preserve adequate diversity in the population.

The population mean can be preserved by several ways. One method would be to have individual recombination events preserving the mean between the participating parents and resulting offspring. We call this approach as the *mean-centric* recombination. The other approach would be to have individual recombination event biasing offspring to be created near the parents, but assigning each parent an equal probability of creating offsprings in its neighborhood. This will also ensure that the population mean of the entire offspring population is identical to that of the parent population. We call this latter approach the *parent-centric* recombination.

Recombination operators such as unimodal normal distribution crossover (UNDX), simplex crossover (SPX), and blend crossover (BLX) are mean-centric approaches, whereas the simulated binary crossover (SBX) and fuzzy recombination (Voigt et al., 1995) are parent-centric approaches. Beyer and Deb (2001) have also shown that these operators may exhibit similar performances if the variance growth under recombination operator can be matched by fixing their associated parameters. In this paper, we use UNDX and SPX as representative mean-centric recombination operators and a multi-parent version of SBX as a parent-centric recombination operator.

2.1 Mean-Centric Recombination

In the UNDX operator (Kita et al., 1999), $(\mu-1)$ parents are randomly chosen and their mean $\vec{g}$ is computed. From this mean, the $(\mu-1)$ direction vectors $\vec{d}^{(i)}=\vec{x}^{(i)}-\vec{g}$ is formed. Let the direction cosines be $\vec{e}^{(i)} = \vec{d}^{(i)}/|\vec{d}^{(i)}|$. Thereafter, from another randomly chosen parent $\vec{x}^{(\mu)}$, the length D of the vector $(\vec{x}^{(\mu)}-\vec{g})$ orthogonal to all $\vec{e}^{(i)}$ is computed. Let $\vec{e}^{(j)}$ (for $j=\mu,\ldots,n$, where n is the size of the variable vector $\vec{x}$) be the orthonormal basis of the subspace orthogonal to the subspace spanned by all $\vec{e}^{(i)}$ for $i=1,\ldots,(\mu-1)$. Then, the offspring is created as follows:

$$\vec{y}=\vec{g}+\sum_{i=1}^{\mu-1} w_i|\vec{d}^{(i)}|\vec{e}^{(i)}+\sum_{i=\mu}^{n} v_i D\vec{e}^{(i)}, \qquad (1)$$

where w_i and v_i are zero-mean normally distributed variables with variances σ_ζ^2 and σ_η^2, respectively. Kita and Yamamura (1999) suggested $\sigma_\zeta = 1/\sqrt{\mu-2}$ and $\sigma_\eta = 0.35/\sqrt{n-\mu-2}$, respectively and observed that $\mu=3$ to 7 performed well. It is interesting to note that each offspring is created around the mean vector $\vec{g}$. The probability of creating an offspring away from the mean vector reduces and the maximum probability is assigned at the mean vector. Figure 1 shows three parents and a few offspring created by the UNDX operator. The complexity of the above procedure in creating one offspring is $O(\mu^2)$, governed by the Gram-Schmidt orthonormalization needed in the process.

The SPX operator also creates offspring around the mean, but restricts them within a predefined region (in a simplex similar but $\gamma=\sqrt{\mu+1}$ times bigger than the parent simplex). A distinguishing aspect of SPX from UNDX operator is that the SPX assigns a uniform probability distribution for creating any solution in a restricted region. Figure 2 shows the density of solutions with three parents for the SPX operator. The computational complexity for creating one offspring here is $O(\mu)$.

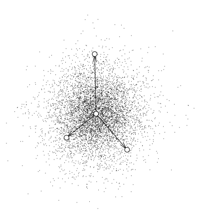

Figure 1: UNDX operator.

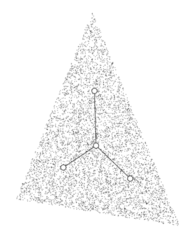

Figure 2: SPX operator.

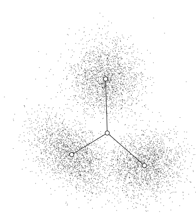

Figure 3: PCX operator.

2.2 Parent-Centric Recombination (PCX)

The SBX operator assigns more probability for an offspring to remain closer to the parents than away from parents. We use this parent-centric concept and modify the UNDX operator as follows. The mean vector $\vec{g}$ of the chosen μ parents is computed. For each offspring, one parent $\vec{x}^{(p)}$ is chosen with equal probability. The direction vector $\vec{d}^{(p)}=\vec{x}^{(p)}-\vec{g}$ is calculated. Thereafter, from each of the other $(\mu-1)$ parents perpendicular distances D_i to the line $\vec{d}^{(p)}$ are computed and their average $\bar{D}$ is found. The offspring is created as follows:

$$\vec{y}=\vec{x}_p+w_\zeta|\vec{d}^{(p)}|+\sum_{\substack{i=1\\ i\neq p}}^{\mu} w_\eta \bar{D}\vec{e}^{(i)}, \qquad (2)$$

where $\vec{e}^{(i)}$ are the $(\mu-1)$ orthonormal bases that span the subspace perpendicular to $\vec{d}^{(p)}$. Thus, the complexity of the PCX operator to create one offspring is $O(\mu)$, instead of $O(\mu^2)$ required for the UNDX operator. The parameters w_ζ and w_η are zero-mean normally distributed variables with variance σ_ζ^2 and σ_η^2, respectively. The important distinction with UNDX operator is that offspring solutions are centered around each parent. The probability of creating an offspring closer to the parent is more. Figure 3 shows the distribution of offspring solutions with three parents.

3 Evolutionary Algorithm Model

Besides the recombination operator, researchers have also realized the importance of a different genetic algorithm model than a vanilla genetic algorithm for real-parameter optimization. In the following, we describe a commonly-used model originally suggested by Satoh et al. (1996) and later used in a number of studies (Kita et al., 1999; Tsutsui et al. (1999)).

3.1 Minimal Generation Gap (MGG) Model

This is a steady-state model, where the recombination and selection operators are intertwined in the following manner:

1. From the population $P(t)$, select μ parents randomly.
2. Generate λ offspring from μ parents using a recombination scheme.
3. Choose two parents at random from μ chosen parents.
4. Of the two parents, one is replaced with the best of λ offspring and the other is replaced with a solution chosen by a roulette-wheel selection procedure from a combined population of λ offspring and two chosen parents.

The above procedure completes one iteration of the MGG model. For the SPX study (Higuchi et al., 2000), authors have used $\mu = n+1$ and $\lambda = 200$ and for the UNDX study, $\mu = 3$ and $\lambda = 200$ are used. No mutation operator was used. With the above parameters, that study showed that MGG model with the SPX operator and a population size of 300 was able to solve a number of test problems better than that using the UNDX operator.

However, that study did not show any justification for using $\lambda = 200$ and for using a population size of $N = 300$. Here, we use the MGG model with both recombination operators and perform a parametric study with λ on three standard test problems:

$$F_{\text{elp}} = \sum_{i=1}^{n} i x_i^2 \quad \text{(Ellipsoidal function)} \tag{3}$$

$$F_{\text{sch}} = \sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_j \right)^2 \quad \text{(Schwefel's function)} \tag{4}$$

$$F_{\text{ros}} = \sum_{i=1}^{n-1} \left(100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2 \right) \tag{5}$$

(Generalized Rosenbrock's function)

In all problems, we have used $n = 20$. The first two problems have their minimum at $x_i^* = 0$ with $F^* = 0$ and the third function has its minimum at $x_i^* = 1$ with $F^* = 0$. In order not to bias the search, we have initialized the population in $x_i \in [-10, -5]$ for all i, in all problems.

First, we fix $N = 300$ and vary λ from 2 to 300. All other parameters are kept as they were used in the original study (Higuchi et al. 2000), except that in UNDX $\mu = 6$ is used, as this value was found to produce better results. In all experiments, we have run the MGG model till a pre-defined number of function evaluations F^t are elapsed. We have used the following values of F^t for different functions: $F^t_{\text{elp}} = 0.5(10^6)$, $F^t_{\text{sch}} = 1(10^6)$ and $F^t_{\text{ros}} = 1(10^6)$. In all experiments, 50 runs with different initial populations were taken and the median, smallest, and highest best fitness values are recorded. Figure 4 shows the best fitness values obtained by the SPX and the UNDX operators on F_{elp} with different values of λ. The figure shows that $\lambda = 50$ produced the best reliable per-

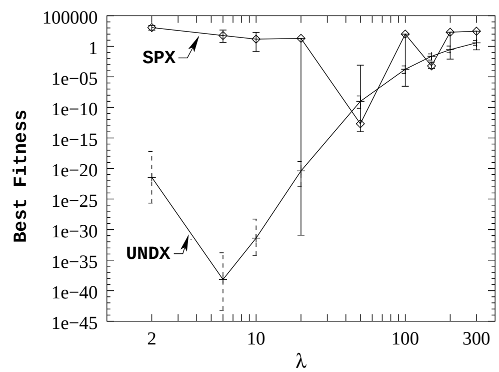

Figure 4: Best fitness for different λ on the ellipsoidal function using the MGG model with SPX and UNDX operators.

formance for the SPX operator. Importantly, the MGG model with $\lambda = 200$ (which was suggested in the original study) did not perform well. Similarly, for the UNDX operator, the best performance is observed at $\lambda = 6$, which is much smaller than the suggested value of 200.

Figure 5 shows the population best fitness for the MGG model with SPX and UNDX operators applied to the F_{sch} function. Once again, the best performance is observed at

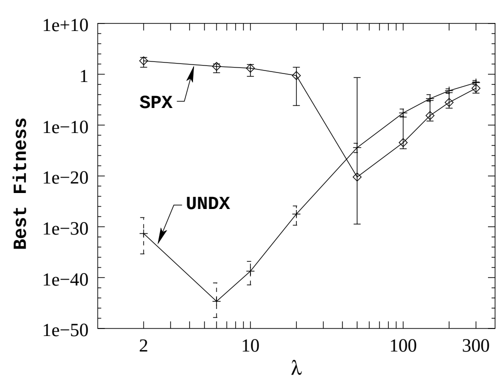

Figure 5: Best fitness for different λ on the Schwefel's function using the MGG model with SPX and UNDX.

$\lambda = 50$ for SPX and $\lambda = 6$ for UNDX. Figure 6 shows the population best fitness for the MGG model with SPX and

UNDX operators applied to the $F_{\rm ros}$ function. Here, the best

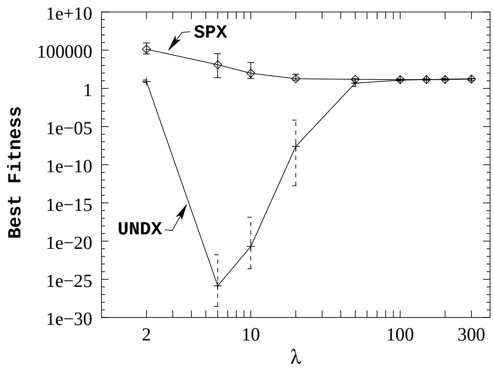

Figure 6: Best fitness for different λ on the generalized Rosenbrock's function using the MGG model with SPX and UNDX.

performance is observed at $\lambda = 100$ to 300 for the SPX operator and $\lambda = 6$ for the UNDX operator.

Thus, it is clear from the above experiments that the suggested value of $\lambda = 200$ is not optimal for either recombination operator. Instead, a smaller value of λ has exhibited better performance. It is also clear from the figures that the SPX operator works better with a large offspring pool size, whereas the UNDX works well with a small offspring pool size. Since a uniform probability distribution is used in the SPX operator, a large pool size is intuitive. With a biased probability distribution, the UNDX operator does not rely on the sample size, rather it relies on large number of iterations, each providing a careful choice of an offspring close the center of mass of the chosen parents.

3.2 Generalized Generation Gap (G3) Model

Here, we modify the MGG model to make it computationally faster by replacing the roulette-wheel selection with a tournament selection operator. This model also preserves elite solutions from the previous iteration.

1. From the population $P(t)$, select the best parent and $\mu - 1$ other parents randomly.
2. Generate λ offspring from the chosen μ parents using a recombination scheme.
3. Choose two parents at random from μ chosen parents.
4. Form a combined subpopulation of chosen two parents and λ offspring, choose the best two solutions and replace the chosen two parents with these solutions.

Only in the first iteration, μ parents are chosen at random in step 1.

3.2.1 Simulation Results

In all experiments with the G3 model, we record the number of function evaluations needed to achieve the best fitness value equal to 10^{-20}. Figure 7 shows the performance of the G3 model with all three operators (PCX, UNDX, and SPX) on the ellipsoidal problem. For the PCX and UNDX operators $N = 100$ is used and for the SPX operator $N = 300$ is used. In all PCX runs, we have used $\sigma_\eta = \sigma_\zeta = 0.1$. In PCX and UNDX runs, we have used $\mu = 3$ and in SPX runs we have used $\mu = n + 1$ or 21. The minimum, median, and maximum number of required function evaluations, as shown in the figure, suggest the robustness of the G3 model with the PCX operator. The G3 model with the PCX operator has performed better (minimum function evaluations is 5,818) than that with the UNDX operator (minimum function evaluations is 16,602). For the SPX operator, not all 50 runs have found a

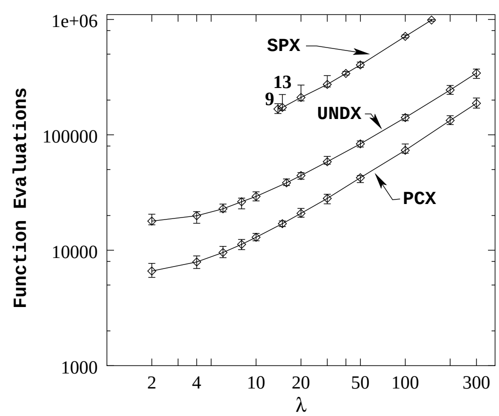

Figure 7: Function evaluations needed to find a solution of fitness 10^{-20} for different λ on $F_{\rm elp}$ using the G3 model with PCX, UNDX, and SPX.

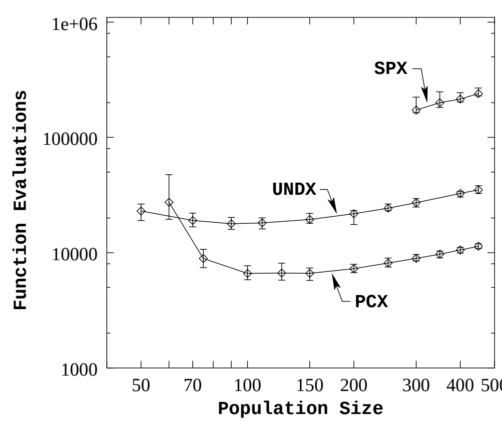

Figure 8: Function evaluations versus population sizes on $F_{\rm elp}$ using the G3 model with PCX, UNDX, and SPX.

solution having a fitness value as small as the required value of 10^{-20} for $\lambda = 12$ and 15. The number of runs (out of 50) where such a solution was found is marked on the plot. The best run of the SPX operator requires 1,63,380 function evaluations.

The figure shows that smaller the offspring pool size (λ), the better is the performance for PCX and UNDX operators. Thus, we choose $\lambda = 2$ for these operators and perform a parametric study for the population size. For the SPX operator, we use $\lambda = 15$, below which satisfactory results were not found. Figure 8 shows that there exists an optimum population size, at which the performance is the best for the PCX (with 5,744 function evaluations) and the UNDX (with 15,914 function evaluations) operators. For the 20-variable ellipsoidal problem, $N \sim 100$ seems to be optimum for these two operators. An interesting aspect is that for the SPX operator with $\lambda = 15$ all runs with $N < 300$ did not find the desired solution. Since SPX creates solutions within a fixed range proportional to the location of the parents, its search power is limited. Moreover, since random samples are taken from a wide region in the search space, the success of the algorithm depends on a large population size.

Next, we apply the G3 model with all three recombination operators to $F_{\rm sch}$. Figures 9 and 10 show the parametric studies with λ and the population size for the PCX and the UNDX operators, respectively. Once again, $N = 100$ and $\lambda = 2$ are found to perform the best for both operators. However, the PCX operator is able to find the desired solution with a smaller number of function evaluations (14,643 for PCX versus 27,556 for UNDX). However, the SPX operator does not

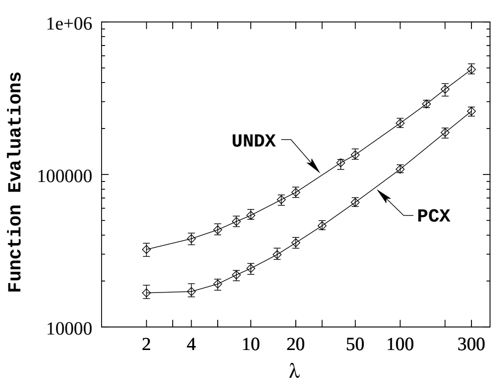

Figure 9: Function evaluations needed to find a solution of fitness 10^{-20} for different λ on $F_{\rm sch}$ using the G3 model with PCX and UNDX. $N = 100$ is used.

perform well on Schwefel's function. The minimum function evaluations needed with any parameter setting to find the desired solution is 4,14,350, which is an order of magnitude more than the best results obtained using the PCX and the UNDX operators. Thus, we do not present any results with the SPX operator.

Figure 11 shows the population best fitness values (of 50 runs) of $F_{\rm sch}$ with number of function evaluations in the case of G3 model with best-performing parameters for PCX

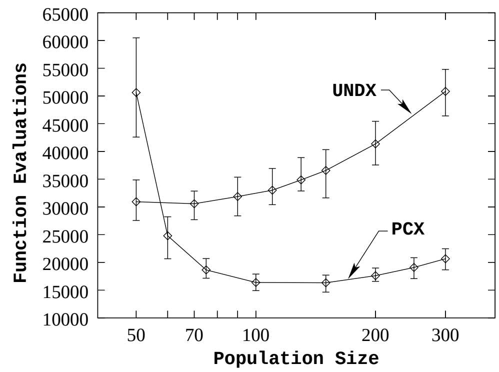

Figure 10: Function evaluations versus population sizes on $F_{\rm sch}$ using the G3 model with PCX and UNDX. $\lambda = 2$ is used.

($\lambda = 2$, and $N = 150$), UNDX ($\lambda = 2$ and $N = 70$), and SPX ($\lambda = 70$ and $N = 300$) operators. The figure shows the superiority of the PCX operator in achieving a desired accuracy with the smallest number of function evaluations.

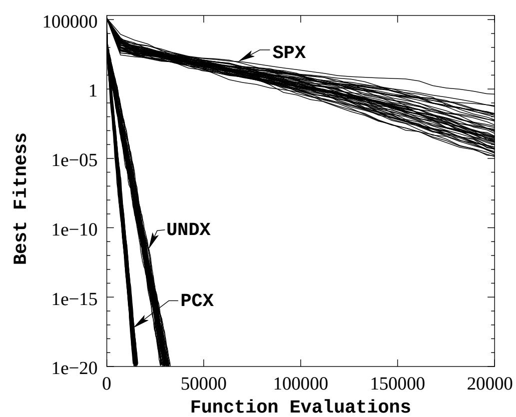

Figure 11: Best fitness versus function evaluations for $F_{\rm sch}$.

Next, we attempt to solve the $F_{\rm ros}$ function. This function is more difficult to solve than the previous two functions. Here, no algorithm is able to find a solution very close to the global optimum (with a fitness value 10^{-20}) in all 50 runs each within one million function evaluations. Runs with PCX and UNDX operators sometimes get stuck to the local optimum solution. Interestingly, the SPX operator failed to find the required solution in *any* of the 50 runs. However, Figures 12 and 13 show that PCX operator (with a minimum of 14,847 function evaluations) has performed much better than the UNDX operator (with a minimum of 58,205 function evaluations). The number of times where a run has converged near the global optimum is shown in the figures. Once again, a small λ (~ 4) and a small population size (100 to 150) were found to produce optimum behavior for PCX and UNDX operators.

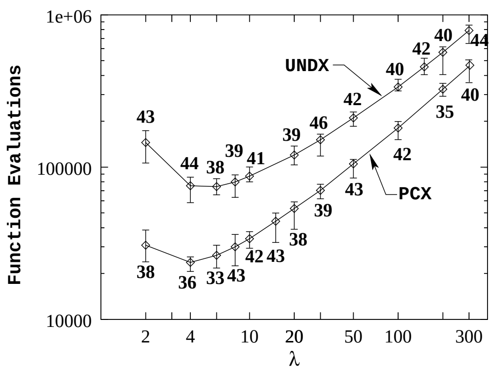

Figure 12: Function evaluations needed to find a solution of fitness 10^{-20} for different λ values on the $F_{\rm ros}$ using the G3 model with PCX and UNDX operators.

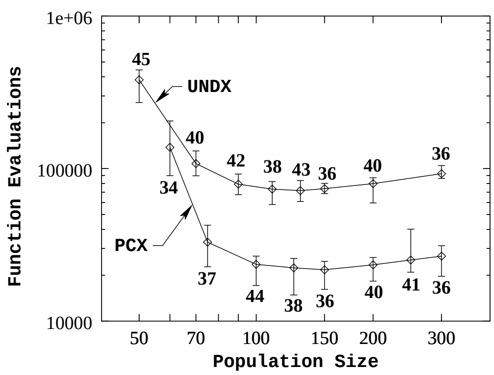

Figure 13: Function evaluations versus population sizes on $F_{\rm ros}$ using the G3 model with PCX and UNDX.

Finally, we compare the number of function evaluations needed to achieve a solution with fitness 10^{-20} using MGG and G3 models with the UNDX operator. The following table shows that the G3 model is an order of magnitude better than the MGG model in all three test problems.

Function	MGG	G3
$F_{\rm elp}$	2,97,546	18,120
$F_{\rm sch}$	5,03,838	30,568
$F_{\rm ros}$	9,38,544	73,380

4 Conclusions

The ad-hoc use of a uniformly distributed probability for creating an offspring used in many real-parameter evolutionary algorithms (such as SPX and BLX) and the use of mean-centric probability distribution (such as that used in UNDX) have been found to be not efficient in this paper. Systematic studies on three 20-variable test problems have shown that a parent-centric recombination is a meaningful and efficient way of solving real-parameter optimization problems. In any case, the use of a uniform probability distribution for creating offspring has not been found to be efficient compared to a biased probability distribution near the search region represented by the parent solutions.

Moreover, the use of an elite-preserving, steady-state, and computationally fast evolutionary model (named as generalized generation gap (G3) model) has been found to be effective with both PCX and UNDX recombination operators. In all simulation runs, the G3 model with the PCX operator has outperformed both the UNDX and SPX operators in terms of number of function evaluations in achieving a desired accuracy. Moreover, the proposed PCX operator is also computationally faster. The efficacy of the proposed G3 model and the PCX operator must now be investigated by applying them to more complex problems.

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