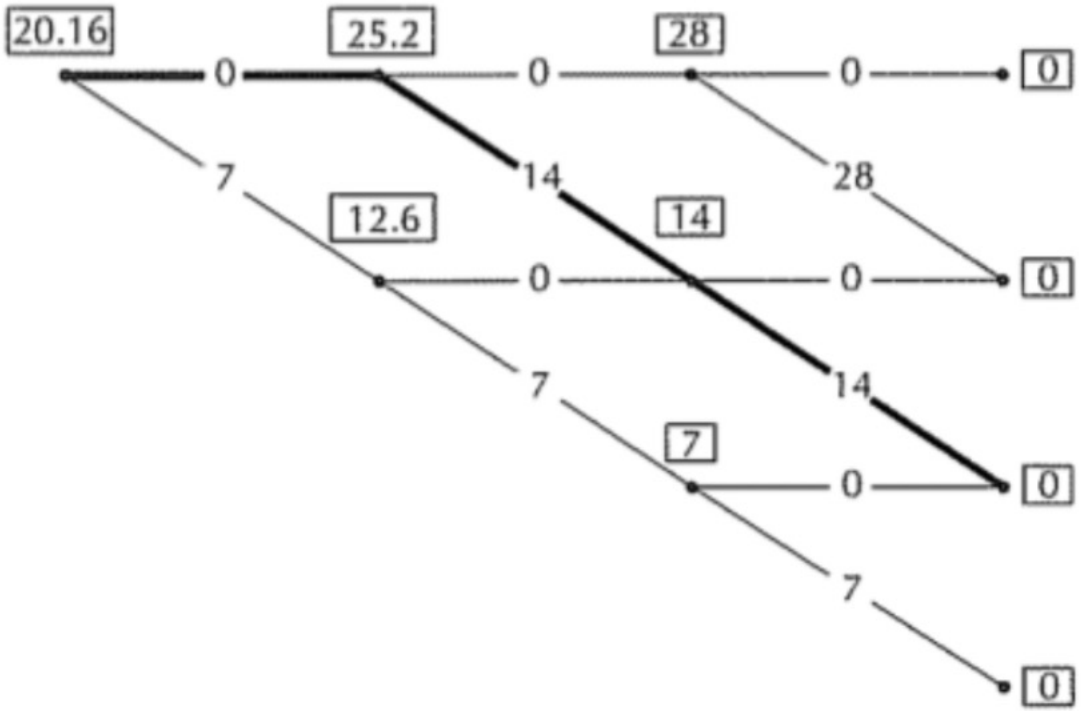

# TOPIC 6 DYNAMIC PROGRAMMING

# Dynamic Programming

- ALL dynamic programming problems need a few common elements
- State variables
  - What information do you need to describe where you are
- Choice/Decision variables
  - What can you choose to do
- Dynamics
  - How do choice variables combine with state variables to evolve through time
- Value Function
  - Discounted value of all future payoffs
- Bellman Equation
  - Value today is immediate payoff plus discounted payoff tomorrow
- Terminal/Boundary Condition
  - Value function after the last time step

# Fishing Example

What are the state variables, choice variables, dynamics, value function, Bellman equation and terminal condition?


20.16
25.2
28
0
0
0
0
7
14
28
12.6
14
0
0
0
7
14
7
0
0
7
0

# Fishing Example

- How would we put this in python to find the optimal policy?

# Fishing Example

- Let’s rearrange the triangle upside down into a box!

# Dynamic Programming

- Value Function
  - $v(S_t, t) = \max_x \sum_{i=t}^{T} \delta^{i-t} r_i$
  - $v(S_t, t) = \max_x r_t + \delta \sum_{i=t+1}^{T} \delta^{i-t-1} r_i$
  - $v(S_t, t) = \max_x r_t + \delta v(S_{t+1}, t+1)$
- If I know the value function for all possible values of s tomorrow, then I can calculate it for all possible values of s today!
- In general, tomorrow's state is dependent on today's state and our choice today

# Mining Example

- You must decide how much ore to extract from a mine that will be shut down and abandoned after T years of operation.
- The sales price of extracted ore is p dollars per ton, and the total cost of extracting x tons of ore in any year, given that the mine contains s tons at the beginning of the year, is $x^2/(1+s)$ dollars.
- The mine currently contains M tons of ore
- This discount factor is $\delta$
- Assuming the amount of ore extracted in any year must be an integer number of tons, what extraction schedule maximizes profits?

# Mining Example

- Can we pose this as a traditional optimization problem (not a DP)?

# Mining Example - DP

What are the state variables, choice variables, dynamics, value function, Bellman equation and terminal condition?

# Dynamic Programming

- The general Bellman Eq is
  - $v(S_t, t) = \max_x r_t + \delta v(S_{t+1}, t+1)$
  - $S_t$ is a dynamic variable that changes through time
- For the mining example this is
  - $v(s, t) = \max_{0 \le x \le s} px - \frac{x^2}{1+s} + \delta v(s - x, t+1)$
  - $s$ is one particular value that variable could take on
  - If $S_t$ takes on the value $s$, then $S_{t+1}$ takes on the value $s - x$

# Mining Example

- How would we code it in python?