

TOPIC 2 STOCHASTIC PROGRAMMING

Stochastic Programming

- All the optimization we have done so far has been completely deterministic
 - Once you make a decision, you know exactly what the outcome will be
 - Even in the portfolio optimization problem we thought of it as a deterministic problem
- What if we don't know what the outcome will be?
 - We pick a price for a product, but demand is random
 - If demand is random, profit is unknown

Simulation

- It will frequently be useful to use **simulation** to generate samples from the randomness!
- I like to think that simulation is statistics in reverse
 - We have a model, and we generate data from that model
 - Usually so we can answer complicated questions
- We use a computer to flip a coin a bunch of times
 - Each coin toss is some possible scenario we're interested in
 - All the coin tosses together tell us about the distribution of these scenarios

Simulation

- I have a fair coin that I flip 15 times
 - What is the probability of getting 4 or more heads in a row at least once in these 10 flips?
 - This is a hard question to answer using traditional probability
 - With simulation we can answer this question with relative ease
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1. Use the computer to flip 15 coins
 2. Check if there is a streak of 4 or more
 3. Repeat steps 1-2 a bunch of times and calculate how many times there is a streak

Simulation

- Let's try this in python

Simulation Framework

- The general process for simulation is:
 1. Define a quantity of interest
 2. Articulate a set of assumptions about the problem
 3. Simulate the random process
 4. Compute the quantity of interest
 5. Repeat steps 3-4 many times and examine the long-term distribution of the quantity of interest

Simulation

- You're saving for retirement, and you have \$2M saved
 - Your portfolio has a random (normal) annual return with a mean of 7% and std of 10%
 - You spend \$150k per year
 - Will you have enough money to live for 35 years?
 - What is the probability you go bankrupt before you die?
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1. Simulate returns and withdrawals for 35 years
 2. Repeat this process many times
 3. How often do you go bankrupt?

Class Participation

- You may not actually live 35 more years
- You are equally likely to live 25, 26, ..., 35 years
- Re-simulate probability of going bankrupt before you die if your death occurs at a random time

Simulation

- Simulation really just approximates the quantity we're interested in!
- Modern computers are very fast though
- We can often simulate things MANY times
 - Hundreds of thousands or millions
- Do you remember the law of large numbers from stats?
 - More data leads to more accurate approximations!
 - If we simulate enough, we're guaranteed to get very close to the truth