

TOPIC 3 DYNAMIC PROGRAMMING

Dynamic Programming

- Most of the optimization we have done so far is for a decision at a single time
 - Linear, integer, non-linear, stochastic optimization
- What if we need to make multiple successive decisions?
 - Playing chess or GO
 - Finding the shortest route through traffic
 - Making quarterly investments in your 401k
 - Dynamically managing inventory in a store

Dynamic Programming

- **Dynamic programming** is the process of finding optimal sequential decisions
- If you have taken any computer science classes, you may have heard of **reinforcement learning**
- Reinforcement learning is a specific type of dynamic programming
 - Approximate Dynamic Programming (ADP)
 - We will do RL in a few weeks

Let's play a game!

- There are 30 matches on a table
- You and I alternate picking up matches
- At each turn we can pick up 1, 2, or 3 matches
- The person who picks up the last match wins
- Let's play!

The Match Game

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

The Match Game

- I would like to pick up the 30th match
- I can be guaranteed to pick up the 30th match if I pick up the 26th match
- I can be guaranteed to pick up the 26th match if I pick up the 22th match
- ...
- I can be guaranteed to pick up the 6th match if I pick up the 2nd match
- Therefore, if I go first and pick up 2 matches, I can be guaranteed to win!

Dynamic Programming

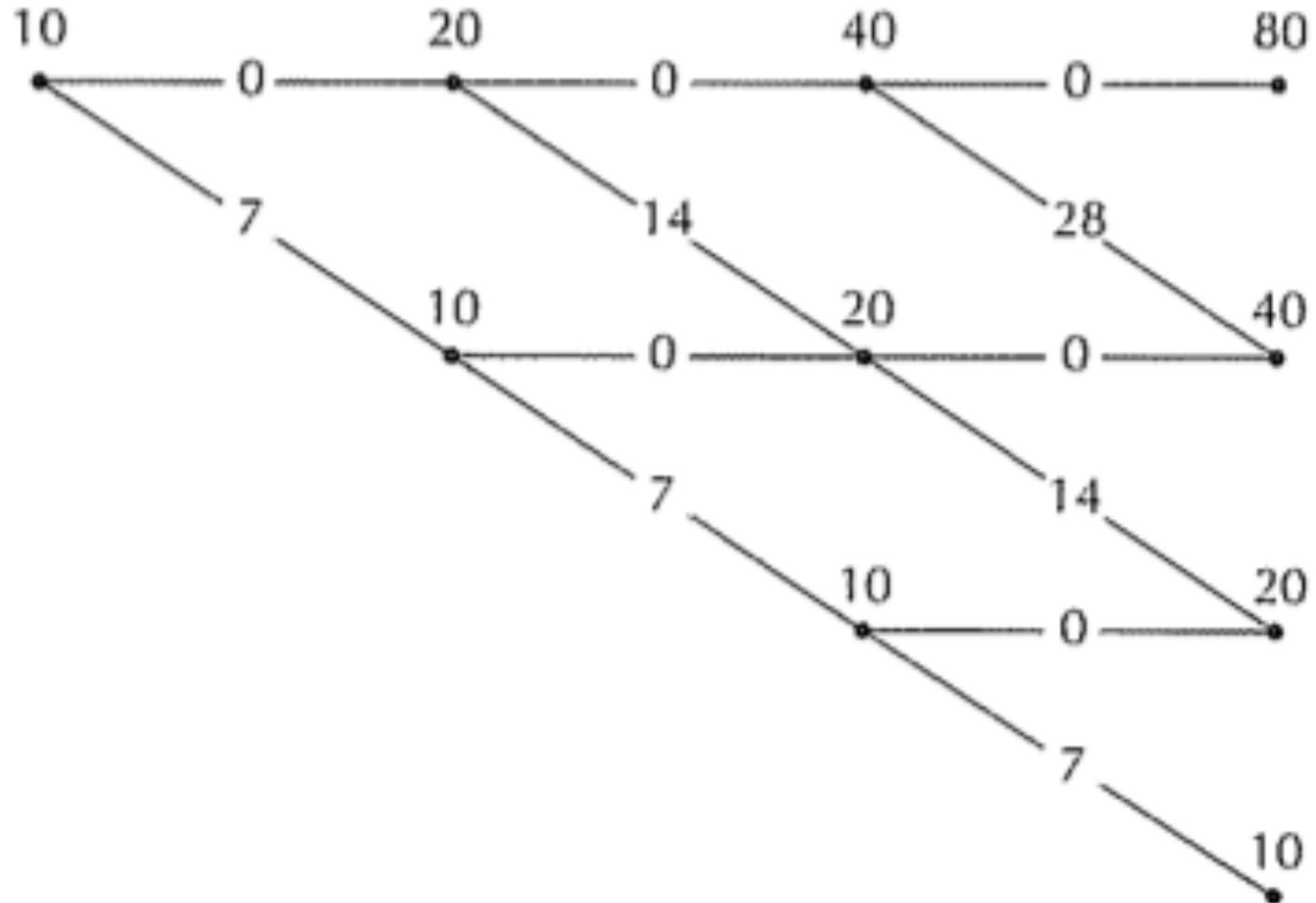
- In **ALL** dynamic programming problems, we will need to solve the problem by working backwards in time!
- Once we solve the problem backwards, we then execute that solution forwards
 - I figured out which matches to pick up by working backwards in time
 - I actually pick them up forwards in time

Fishing Example

- You have a 3-year lease for fishing on a big lake
- Each year you must decide if you want to fish or not
- If you don't fish, the fish in the lake will flourish, and the population of fish will double next year
- If you do fish, you will take 70% of all fish in the lake
 - Afterwards the fish will rebound to the population at the beginning of the year
- You make \$1 per ton of fish you take from the lake
- Your discount factor is 0.8
- The initial fish population is 10 tons
- Which years should you fish?

Fishing Example

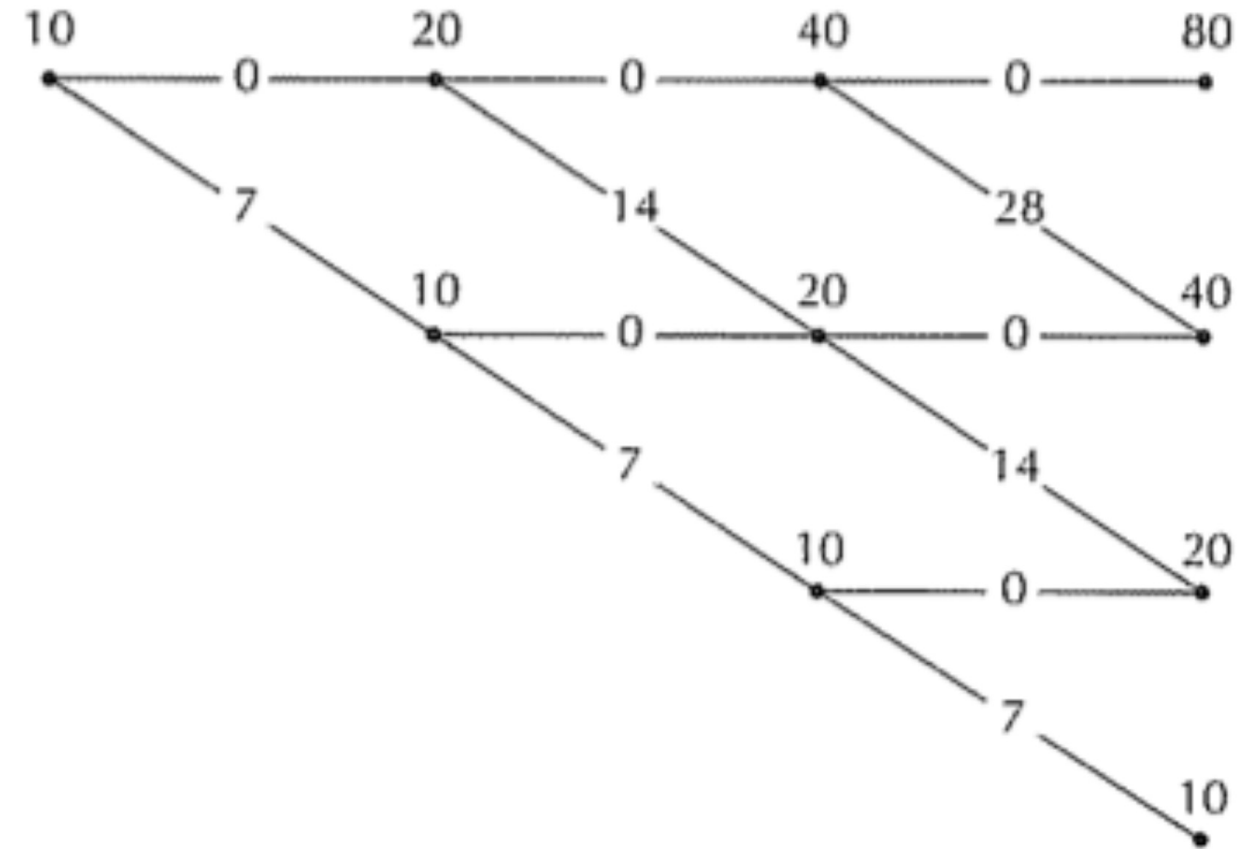
Fishing Example



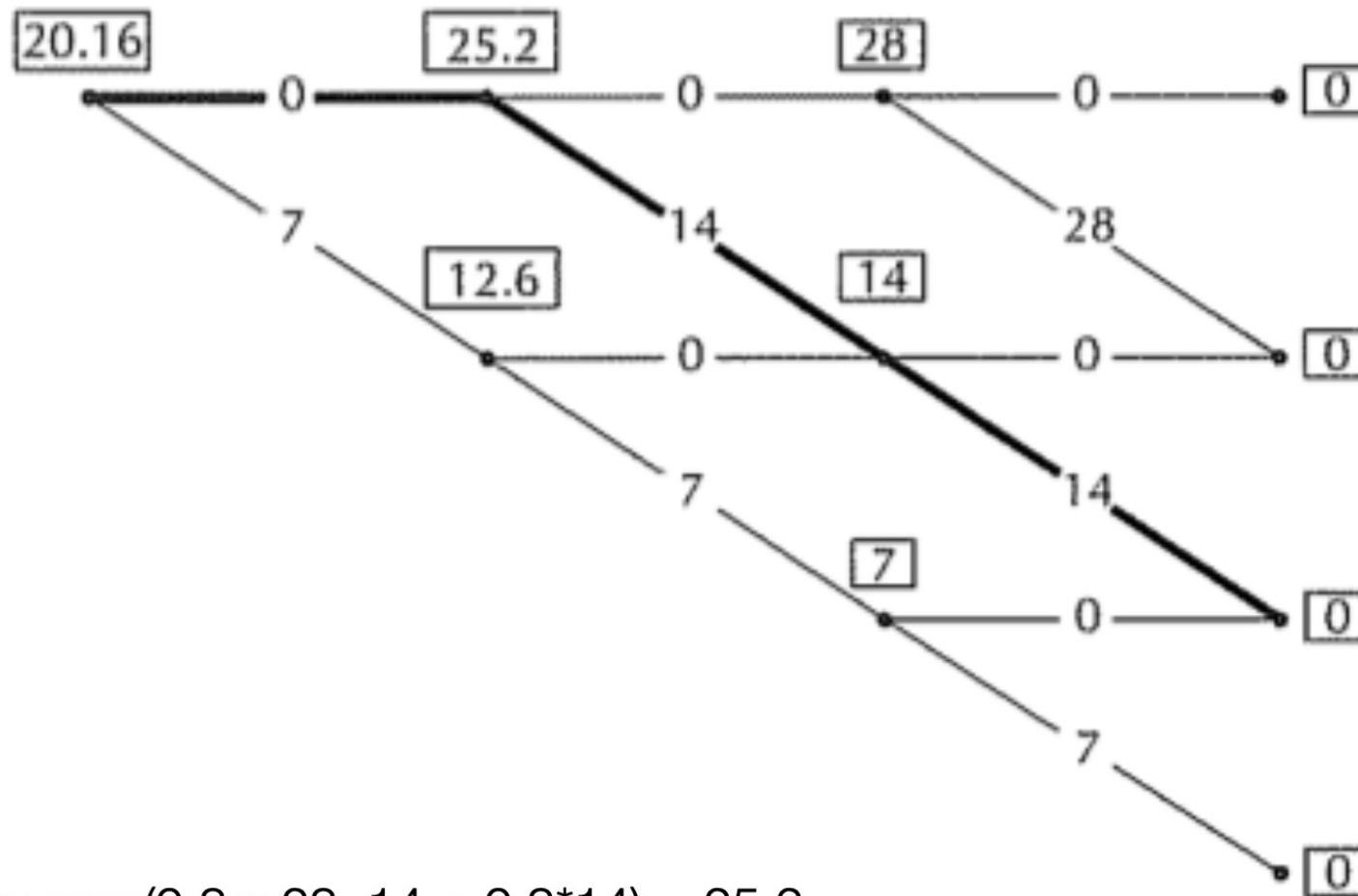
Value at a Node

- At the node where there are 40 tons of fish
- If you fish, you'll make \$28 and the population of fish will be 40 tons next year
- If you don't fish, you'll make \$0 and the population of fish will be 80 tons next year
- YOU CAN'T FISH NEXT YEAR
- You should pick the strategy to
 - $\max(28+0.8*0, 0+0.8*0)$
- So you should fish if at the beginning of year 3 there are 40 tons of fish in the lake!

Fishing Example



Value at a Node



Eg: $\max(0.8 \times 28, 14 + 0.8 \times 14) = 25.2$