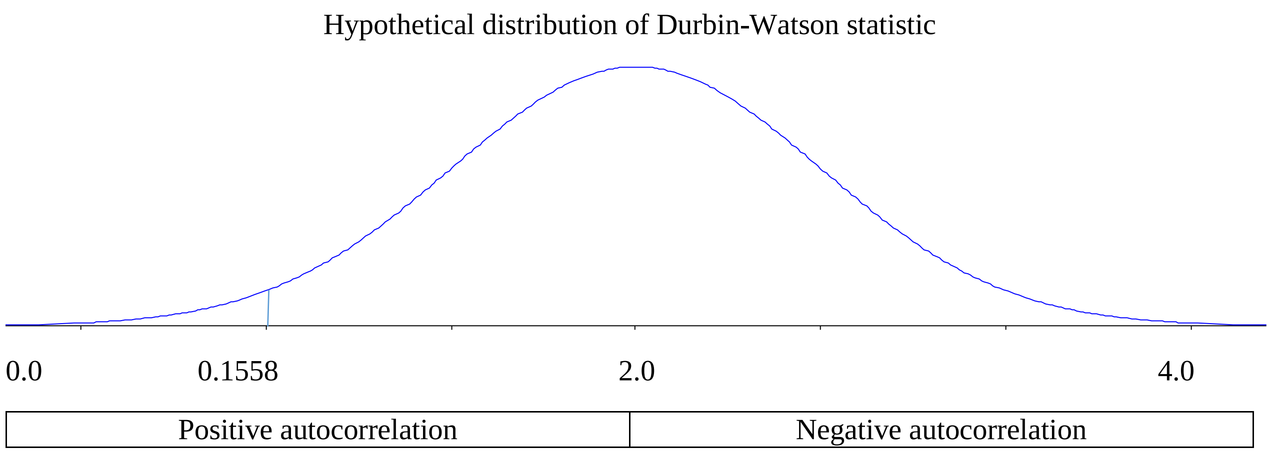
Interpreting the Durbin-Watson Tests for Positive and Negative Autocorrelation

Recall that $DW \cong 2(1 - autocorr)$. This implies that $0 \leq DW \leq 4$ and that $DW < 2$ corresponds to positive autocorrelation, and $DW > 2$ corresponds to negative autocorrelation. The reversal of the usual directions for positive and negative in the DW creates some confusion in interpreting the results of DW tests. This document addresses this confusion.



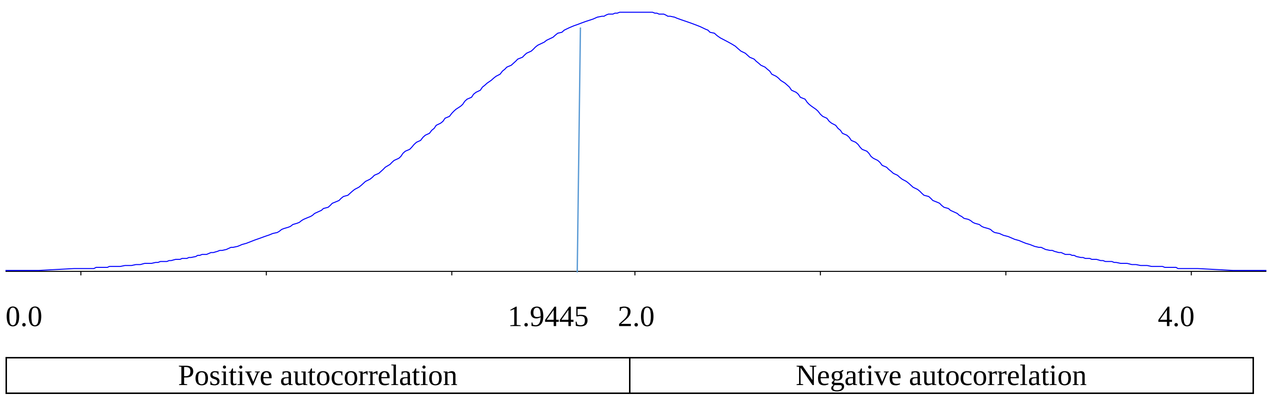
When you test *Ho: autocorr* $\leq$ *0*, you want to know if you can reject the autocorr being on the **right-hand** side. To reject the right side, you need a DW value sufficiently far on the left. Like in the illustration, with DW = 0.1558 (not shown to scale). You know that the DW value is sufficiently far on the left if the probability below the DW value is small. That is, Pr < DW is small. In the illustration, Pr (DW < 0.1558) is about 0.0001. This is the p-value. That is small enough for nearly all analysts. So Ho is rejected (negative and zero autocorr is rejected). Conclude that the autocorr is positive. This is a test for the presence of positive autocorrelation. (Can you conclude that positive autocorr is present? – Yes.)

When you test *Ho: autocorr* $\geq$ *0*, you want to know if you can reject the autocorr being on the **left-hand** side. To reject the left side, you need a DW value sufficiently far on the right. In the illustration, with DW = 0.1558 (not shown to scale), the DW value is not even on the right – it is on the left. You would know that the DW value is sufficiently far on the right if the probability above the DW value were small. That is, Pr > DW is small. In the illustration, Pr (DW > 0.1558) is about 1.0. This is the p-value. That is too big for all analysts. So Ho is not rejected (positive and zero autocorr is not rejected). Conclude that the data are consistent with autocorr being positive or zero. This is a test for the presence of negative autocorrelation. (Can you conclude that negative autocorr is present? – No.)

Putting the preceding two tests together, **you conclude that the autocorr is positive**.

[See next page for another example.]

Hypothetical distribution of Durbin-Watson statistic



When you test *Ho: autocorr ≤ 0*, you want to know if you can reject the autocorr being on the **right-hand** side. To reject the right side, you need a DW value sufficiently far on the left. In the illustration, with DW = 1.9445 (not shown to scale), the DW value is just barely on the left. You know that the DW value is sufficiently far on the left if the probability below the DW value is small. That is, Pr < DW is small. In the illustration, Pr (DW < 1.9445) is about 0.3896. This is the p-value. That is too big for nearly all analysts. So Ho is not rejected (negative and zero autocorr is not rejected). Conclude that the data are consistent with autocorr being negative or zero. (Can you conclude that positive autocorr is present? – No.)

When you test Ho: autocorr ≥ 0, you want to know if you can reject the autocorr being on the **left-hand** side. To reject the left side, you need a DW value sufficiently far on the right. In the illustration, with DW = 1.9445 (not shown to scale), the DW value is not even on the right – it is on the left. You would know that the DW value is sufficiently far on the right if the probability above the DW value were small. That is, Pr > DW is small. In the illustration, Pr (DW > 1.9445) is about 0.6104. This is the p-value. That is too big for nearly all analysts. So Ho is not rejected (positive and zero autocorr is not rejected). Conclude that the data are consistent with autocorr being positive or zero. (Can you conclude that negative autocorr is present? – No.)

Putting the preceding two tests together, **you conclude that the data are consistent with the autocorr being zero** (cannot reject autocorr being zero).

[See next page for an example of reading SAS output]

Reading Durbin-Watson Output in SAS

The following SAS program fits an autoregressive order 1 (AR1) model to daily Houston ozone measurements, and tests the "I" specification for both the Random Sample and the AR1 model:

```
proc autoreg data=Houston;
  model ozone = / nlags = 1 dwprob;
run;
```

The following part of the output applies the Durbin-Watson test to the residuals of the AR1 model:

Durbin-Watson Statistics				
Order	**DW**	**Pr < DW**	**Pr > DW**	
1	2.0058	0.5128	0.4872	
NOTE: Pr<DW is the p-value for testing positive autocorrelation, and Pr>DW is the p-value for testing negative autocorrelation				

The value of DW is 2.0058, which is very close the value 2 that indicates zero autocorrelation. The output shows two DW tests:

- The test of Ho: autocorr ≤ 0 has p-value = Pr < DW = 0.5128, so do not reject negative autocorrelation.
- The test of Ho: autocorr ≥ 0 has p-value = Pr > DW = 0.4872, so do not reject positive autocorrelation.

We conclude that the DW test is consistent with the AR1 model for Houston ozone having zero autocorrelation in its residuals. This supports the AR1 model for Houston ozone.

On the other hand, the following is an earlier part of the same output. This output applies the Durbin-Watson test to the residuals of the Random Sample model:

Durbin-Watson Statistics				
Order	**DW**	**Pr < DW**	**Pr > DW**	
1	1.1836	<.0001	1.0000	
NOTE: Pr<DW is the p-value for testing positive autocorrelation, and Pr>DW is the p-value for testing negative autocorrelation				

The value of DW is 1.1836, which is substantially less than 2 and well into the region of positive autocorrelation. DW = 1.1836 corresponds roughly to an autocorrelation of (2 – DW)/2 = (2 – 1.1836)/2 = 0.4082.

The output shows two DW tests:

- The test of Ho: autocorr ≤ 0 has p-value = Pr < DW = <.0001, so reject negative autocorrelation.
- The test of Ho: autocorr ≥ 0 has p-value = Pr > DW = 1.0000, so do not reject positive autocorrelation.

We conclude that the Houston ozone data are not consistent with the Random Sample model because the empirical residuals for that model have positive autocorrelation. This undermines the Random Sample model for the Houston ozone data.