QUANTITATIVE TECHNIQUES
A REPORT ON TATA MOTORS

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Linear Programming – I

TATA motors use a combination of parts such as seats and wiring harnesses from companies such as Johnson controls limited and Yazaki AutoComp Limited. Based on demand of Sedan cars in the company, the company gives order to Yazaki and Johnson controls. The total distribution limit on Seats is 5000. Wiring harness distribution limit is up to 3000. Based on the demand, the total requirement of the total sets by TATA motors for sedans cannot exceed 7500. On the basis of discount structure provided by the companies, TATA motors gets a discount of 1000 per seat set and 750 per wire harness. What is the optimal number of inventories that TATA should order to maximize the total amount in discount?

Maximizing the profit helps the company reduce the costs which in return helps the company to increase their reserves.

Let Number of Seat sets=x

Number of Wire harnesses=y

Maximization function-

Max g=1000x+750y

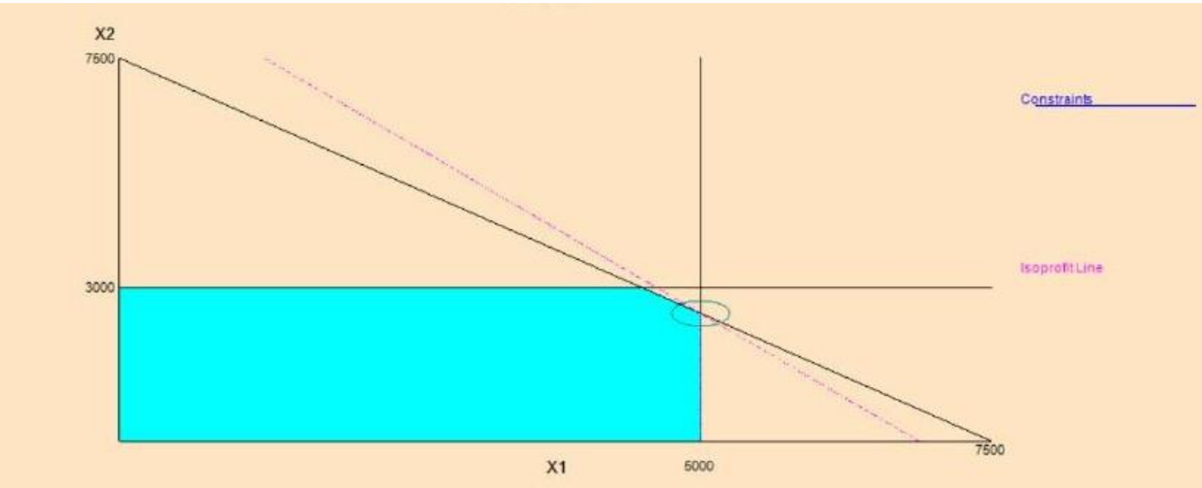
subject to-

x<=5000

y<=3000

x+y<=7500

where x>=0, y>=0



Where the shaded region denotes the feasible region.

Linear Programming – II

Considering Commercial Vehicles segment, Tata Motors produces 3 models, Sedan, SUV and Hatchbacks. Capacity of production is no more than 7500 Sedan cars and 6000 SUV cars and 1500 Hatchbacks by the Pune Plant. To capacity limit, a total of 13700 cars can be developed monthly. If each Sedan car sold results in a 20000 profit, but each SUV car produces a 10000 profit, each Hatchback car produces a 12000 profit, how many of each type should be made monthly to maximize net profits?

Here TATA wants to find the optimum quantity of each type of vehicles that they must manufacture for maximizing their profits based on the capacity present.

Developing the objective function along with the constraints in order to find the maximum profit-

Let Number of Sedans=x

Number of SUVs=y

Number of Hatchbacks=z

Maximization function-

Max g=20000x+10000y+12000z

subject to

x+y+z<=13700

x<=7500

y<=6000

z<=1500

x>=0,y>=0,z>=0

Using SIMPLEX method-

The initial simplex tableau is as follows:

x	y	z	s1	s2	s3	s4	b
1	1	1	1	0	0	0	13700
1	0	0	0	1	0	0	7500
0	1	0	0	0	1	0	6000
0	0	1	0	0	0	1	1500
-20000	-10000	-12000	0	0	0	0	0

As there are negative elements in z, we proceed onto further iterations to get the optimal solution.

The departing variable is S2 and entering variable is x based on SIMPLEX rules-

- The column having the most negative element of the bottom row denotes the entering element
- The departing variable corresponds to the smallest nonnegative ratio of b/to elements of entering variable belonging to the same row

x	y	z	s1	s2	s3	s4	b
0	1	1	1	-1	0	0	6200
1	0	0	0	1	0	0	7500
0	1	0	0	0	1	0	6000
0	0	1	0	0	0	1	1500
0	-10000	-12000	0	20000	0	0	15M

Using the same rules applied before, the entering variable is z and the departing variable is s4

x	y	z	s1	s2	s3	s4	b
0	1	0	1	-1	0	-1	4700
1	0	0	0	1	0	0	7500
0	1	0	0	0	1	0	6000
0	0	1	0	0	0	1	1500
0	-10000	0	0	20000	0	12000	16.8M

In the next iteration, the entering variable is y and departing variable is s1.

x	y	z	s1	s2	s3	s4	b
0	1	0	1	-1	0	-1	4700
1	0	0	0	1	0	0	7500
0	0	0	-1	1	1	1	1300
0	0	1	0	0	0	1	1500
0	0	0	10000	10000	0	2000	21.5M

Solving the entire problem, a total of 7500 sedan cars, 4700 SUV cars and 1500 Hatchbacks must be produced within the capacity of production to maximize the monthly profit of TATA motors to 21.5 Million

Solution through QM software

Linear Programming Results

simplex Solution

	X1	X2	X3		RHS	Dual
Maximize	20000	10000	12000			
Constraint 1	1	1	1	<=	13700	10000
Constraint 2	1	0	0	<=	7500	10000
Constraint 3	0	1	0	<=	6000	0
Constraint 4	0	0	1	<=	1500	2000
Solution	7500	4700	1500		215000000	

Iteration 1									
0	slack 1	7,500	1	0	0	1	0	0	0
0	slack 2	6,000	0	1	0	0	1	0	0
0	slack 3	1,500	0	0	1	0	0	1	0
0	slack 4	13,700	1	1	1	0	0	0	1
	zj	0	0	0	0	0	0	0	0
	cj-zj		20,000	10,000	12,000	0	0	0	0
Iteration 2									
20000	X1	7,500	1	0	0	1	0	0	0
0	slack 2	6,000	0	1	0	0	1	0	0
0	slack 3	1,500	0	0	1	0	0	1	0
0	slack 4	6,200	0	1	1	-1	0	0	1
	zj	15,0...	20000	0	0	20000	0	0	0
	cj-zj		0	10,000	12,000	-20,0...	0	0	0
Iteration 3									
20000	X1	7,500	1	0	0	1	0	0	0
0	slack 2	6,000	0	1	0	0	1	0	0
12000	X3	1,500	0	0	1	0	0	1	0
0	slack 4	4,700	0	1	0	-1	0	-1	1
	zj	16,8...	20000	0	12000	20000	0	12000	0
Iteration 4									
20000	X1	7,500	1	0	0	1	0	0	0
0	slack 2	1,300	0	0	0	1	1	1	-1
12000	X3	1,500	0	0	1	0	0	1	0
10000	X2	4,700	0	1	0	-1	0	-1	1
	zj	21,5...	20000	10000	12000	10000	0	2000	10000

Transportation problem

TATA motors have production units spread all over India. 4 of them are located Bengaluru, Pune, Lucknow and Jamshedpur. Demand for vehicles is heavy in 4 cities - Mumbai, Delhi, Hyderabad and Chennai. The entire cost of transportation is provided in the matrix below. At what minimum cost can TATA motors deliver these vehicles to their destinations?

TATA motors want to minimize the cost of transportation of the manufactured vehicles to their destination. The transportation matrix is based on the distance between the source and destinations.

		Destination				
		Mumbai	Delhi	Hyderabad	Chennai	**Supply**
Source	Bengaluru	3	6	2	1	410
	Pune	1	3	4	3	1730
	Lucknow	5	1	5	5	300
	Jamshedpur	4	2	2	4	120
	Demand	670	1500	210	180	**2560**

For finding the initial basic solution of this problem, we have used the Least cost method of transportation where the least distances will be allotted the lowest of the supply or demand inputs available.

The sequence of steps that have been followed to obtain the minimum cost is as follows:

3	6	2	1	410	
1 (670)	3	4	3	~~1730~~	1060
5	1	5	5	300	
4	2	2	4	120	
~~670~~	1500	210	180	**2560**	
0					

3	6	2	1	410	
1 (670)	3	4	3	~~1730~~	1060
5	1 (300)	5	5	~~300~~	0
4	2	2	4	120	
~~670~~	~~1500~~	210	180	**2560**	
0	1200				

3	6	2	1 (180)	~~410~~	230
1 (670)	3	4	3	~~1730~~	1060
5	1 (300)	5	5	~~300~~	0
4	2	2	4	120	
~~670~~	~~1500~~	210	~~180~~	**2560**	
0	1200		0		

3	6	2	1 (180)	~~410~~	230
1 (670)	3	4	3	~~1730~~	1060
5	1 (300)	5	5	~~300~~	0
4	2 (120)	2	4	~~120~~	0
~~670~~	~~1500~~	210	~~180~~	**2560**	
0	~~1200~~		0		
	1080				

3	6	2 (210)	1 (180)	~~410~~	~~230~~	20
1 (670)	3	4	3	~~1730~~	1060	
5	1 (300)	5	5	~~300~~	0	
4	2 (120)	2	4	~~120~~	0	
~~670~~	~~1500~~	~~210~~	~~180~~	**2560**		
0	~~1200~~	0	0			
	1080					

3	6	2 (210)	1 (180)	~~410~~	~~230~~	20
1 (670)	3 (1060)	4	3	~~1730~~	~~1060~~	0
5	1 (300)	5	5	~~300~~	0	
4	2 (120)	2	4	~~120~~	0	
~~670~~	~~1500~~	~~210~~	~~180~~	**2560**		
0	~~1200~~	0	0			
	~~1080~~					
	20					

3	6 (20)	2 (210)	1 (180)	~~410~~	~~230~~	~~20~~	0
1 (670)	3 (1060)	4	3	~~1730~~	~~1060~~	0	
5	1 (300)	5	5	~~300~~	0		
4	2 (120)	2	4	~~120~~	0		
~~670~~	~~1500~~	~~210~~	~~180~~	**2560**			
0	~~1200~~	0	0				
	~~1080~~						
	~~20~~						
	0						

Solving the question, the total cost of transportation from 4 factories to 4 major cities on the basis of supply and demand is 5110

Similarly solving using the northwest corner method

3 (410)	6	2	1	~~410~~	0
1	3	4	3	1730	
5	1	5	5	300	
4	2	2	4	120	
~~670~~	1500	210	180	**2560**	
260					

3 (410)	6	2	1	~~410~~	0
1 (260)	3	4	3	~~1730~~	1470
5	1	5	5	300	
4	2	2	4	120	
~~670~~	1500	210	180	**2560**	
~~260~~					
0					

3 (410)	6	2	1	~~410~~	0		
1 (260)	3 (1470)	4	3	~~1730~~	~~1470~~	0	
5	1	5	5	300			
4	2	2	4	120			
~~670~~	~~1500~~	210	180	**2560**			
~~260~~	30						
0							

3 (410)	6	2	1	~~410~~	0		
1 (260)	3 (1470)	4	3	~~1730~~	~~1470~~	0	
5	1 (30)	5	5	~~300~~	270		
4	2	2	4	120			
~~670~~	~~1500~~	210	180	**2560**			
~~260~~	~~30~~						
0	0						

3 (410)	6	2	1	~~410~~	0		
1 (260)	3 (1470)	4	3	~~1730~~	~~1470~~	0	
5	1 (30)	5 (210)	5	~~300~~	~~270~~	60	
4	2	2	4	120			
~~670~~	~~1500~~	~~210~~	180	**2560**			
~~260~~	~~30~~	0					
0	0						

3 (410)	6	2	1	~~410~~	0		
1 (260)	3 (1470)	4	3	~~1730~~	~~1470~~	0	
5	1 (30)	5 (210)	5 (60)	~~300~~	~~270~~	~~60~~	0
4	2	2	4	120			
~~670~~	~~1500~~	~~210~~	~~180~~	**2560**			
~~260~~	~~30~~	0	120				
0	0						

3 (410)	6	2	1	~~410~~	0		
1 (260)	3 (1470)	4	3	~~1730~~	~~1470~~	0	
5	1 (30)	5 (210)	5 (60)	~~300~~	~~270~~	~~60~~	0
4	2	2	4 (120)	~~120~~	0		
~~670~~	~~1500~~	~~210~~	~~180~~	**2560**			
~~260~~	~~30~~	0	~~120~~				
0	0		0				

Therefore, the total cost = (3*410) + (1*260) + (3*1470) + (1*30) + (5*210) + (5*60) + (4*120)
= 6920

Moving to the optimum solution, where the cost needs to be reduced to the least, we perform Stepping Stone method.

3	6 (20)	2 (210)	1 (180)	u1
1 (670)	3 (1060)	4	3	u2
5	1 (300)	5	5	u3
4	2 (120)	2	4	u4
v1	v2	v3	v4	**2560**

For the first iteration, taking u1=0 and using the occupied cells to calculate rest of the u's and v's using the formula: $C_{ij} = u_i + v_j$

The calculations take place as follows:

$C_{12} = u_1 + v_2$ $6 = 0 + v_2$ $v_2 = 6$	$C_{13} = u_1 + v_3$ $2 = 0 + v_3$ $v_3 = 2$	$C_{14} = u_1 + v_4$ $1 = 0 + v_4$ $v_4 = 1$
$C_{22} = u_2 + v_2$ $3 = u_2 + 6$ $u_2 = -3$	$C_{21} = u_2 + v_1$ $1 = -3 + v_1$ $v_1 = -4$	
$C_{32} = u_3 + v_2$ $1 = u_3 + 6$ $u_3 = -5$		
$C_{42} = u_4 + v_2$ $2 = u_4 + 6$ $u_4 = -4$		

And for the unoccupied cells, using these same u_i and v_j values, we calculate the costs associated to unoccupied cells as follows: $C_{ij} - u_i - v_j$

The calculations are as follows:

(1,1) = 3 – 0 – 4 = -1
(2,3) = 4 – (-3) – 2 = 5
(2,4) = 3 – (-3) – 1 = 5
(3,1) = 5 – (-5) – 4 = 6
(3,3) = 5 – (-5) – 2 = 8
(3,4) = 5 – (-5) – 1 = 9
(4,1) = 4 – (-4) – 4 = 4
(4,3) = 2 – (-4) – 2 = 4
(4,4) = 4 – (-4) – 1 = 7

Therefore, on calculation of reduced costs, following is the output

3 (-1)	6 (20)	2 (210)	1 (180)	u1=0
1 (670)	3 (1060)	4 (5)	3 (5)	u2=-3
5 (6)	1 (300)	5 (8)	5 (9)	u3=-5
4 (4)	2 (120)	2 (4)	4 (7)	u4=-4
v1=4	v2=6	v3=2	v4=1	**2560**

Looping and reallocating weights in order to find a closed loop around the negative elements present (in this case -1 in cell(1,1))

The loop is formed between subtraction elements (2,1) and (1,2) and addition elements (1,1) and (2,2) with the cost of (1,2) taken for arithmetic operations

(1,1) = 0 + 20 = 20
(1,2) = 20 − 20 = 0
(2,1) = 670 − 20 = 650
(2,2) = 1060 + 20 = 1080

The next tableau is as follows:

3 (20)	6	2 (210)	1 (180)	u1
1 (650)	3 (1080)	4	3	u2
5	1 (300)	5	5	u3
4	2 (120)	2	4	u4
v1	v2	v3	v4	**2560**

For the next iteration, we take u1=0 again and use the same formulas for calculating the u's and v's from the occupied cells and using the same values calculating the reduced costs.

$C_{11} = u_1 + v_1$ $3 = 0 + v_1$ $v_1 = 3$	$C_{13} = u_1 + v_3$ $2 = 0 + v_3$ $v_3 = 2$	$C_{14} = u_1 + v_4$ $1 = 0 + v_4$ $v_4 = 1$
$C_{21} = u_2 + v_1$ $1 = u_2 + 3$ $u_2 = -2$	$C_{22} = u_2 + v_2$ $3 = -2 + v_2$ $v_2 = 5$	
$C_{32} = u_3 + v_2$ $1 = u_3 + 5$ $u_3 = -4$		

$C_{42} = u_4 + v_2$ $2 = u_4 + 5$ $u_4 = -3$		

(1,2) = 6 – 0 – 5 = 1
(2,3) = 4 – (-2) – 2 = 4
(2,4) = 3 – (-2) – 1 = 4
(3,1) = 5 – (-4) – 3 = 6
(3,3) = 5 – (-4) – 2 = 7
(3,4) = 5 – (-4) – 1 = 8
(4,1) = 4 – (-3) – 3 = 4
(4,3) = 2 – (-3) – 2 = 3
(4,4) = 4 – (-3) – 1 = 6

Therefore, on calculation of reduced costs, following is the output

The output of the MODI method is as follows:

3 (20)	6 (1)	2 (210)	1 (180)	u1=0
1 (650)	3 (1080)	4 (4)	3 (4)	u2=-2
5 (6)	1 (300)	5 (7)	5 (8)	u3=-4
4 (4)	2 (120)	2 (3)	4 (6)	u4=-3
v1=3	v2=5	v3=2	v4=1	**2560**

And as there are no negative weights assigned to unallocated cells, the optimum solution has been reached with a value of:

(3*20) + (2*210) + (1*180) + (1*650) + (3*1080) + (1*300) + (2*120) = 5090.

After calculating the values from the allocated cells, the total optimal cost that TATA can obtain in transportation is 5090.

Solution of the entire problem through QM software

Transportation Results

transportation solution

solution value = $5090	Mumbai	Delhi	Hyderabad	Chennai
Bangluru	20		210	180
Pune	650	1080		
Lucknow		300		
Jamshedpur		120		

Iterations

transportation solution

	Mumbai	Delhi	Hyderabad	Chennai
Iteration 1				
Bangluru	(-1)	20	210	180
Pune	670	1060	(5)	(5)
Lucknow	(6)	300	(8)	(9)
Jamshedpur	(4)	120	(4)	(7)
Iteration 2				
Bangluru	20	(1)	210	180
Pune	650	1080	(4)	(4)
Lucknow	(6)	300	(7)	(8)
Jamshedpur	(4)	120	(3)	(6)

Assignment Problem

Pune Plant of TATA motors have a number of assembly lines and large testing centers. Each testing center receives a certain number of cars for testing. What can be the ideal assignment of cars on the basis of their time for travel to the testing station so that the overall time could be reduced?

Taking two scenarios in this case, TATA wants to optimize the allocation in such a way that the total time is minimized.

Assignment problem having 4 assembly lines (considering only consumer vehicles) and 4 assembly lines-

		Testing Areas			
		I	II	III	IV
Assembly Lines	1	10	8	5	5
	2	12	7	15	4
	3	12	7	20	2
	4	10	3	12	6

Balanced system operations are followed (Hungarian Method) in this case-

First the row operations are performed to identify the least element in each row and subtracting it from all the elements in that row-

	I	II	III	IV
1	5	3	0	0
2	8	3	11	0
3	10	5	18	0
4	7	0	9	3

Performing column operations on those columns where zeros are not present –

	I	II	III	IV
1	0	3	0	0
2	3	3	11	0
3	5	5	18	0
4	2	0	9	3

Crossing out zeros (highlighting) in order to perform the optimality test-

	I	II	III	IV
1	0	3	0	0
2	3	3	11	0
3	5	5	18	0
4	2	0	9	3

Performing the optimality test (in this case, since the number of lines for crossing out zeros is not equal to the number of rows, we need to take in the smallest uncrossed element and subtract it from the rest of the uncrossed elements while adding it to the intersection line elements)-

	I	II	III	IV
1	0	3	0	3
2	0	0	8	0
3	2	2	15	0
4	2	0	9	6

Checking optimality again-

	I	II	III	IV
1	0	3	0	3
2	0	0	8	0
3	2	2	15	0
4	2	0	9	6

Final feasible elements-

	I	II	III	IV
1	0	3	0	3
2	0	0	8	0
3	2	2	15	0
4	2	0	9	6

The optimal matrix provides the minimized cost of transportation of vehicles to the testing areas which is 22 minutes.

Assignment Problem for the unbalanced system

This is a more realistic situation where all the assembly lines in Pune Plant (including assembly lines for consumer vehicles and light transportation vehicles which is 6). Here TATA motors want to find the ideal assignment of vehicles from all the assembly lines 6 to the testing centers which are 4.

		Testing Areas			
		I	II	III	IV
Assembly Lines	1	10	8	5	5
	2	12	7	15	4
	3	12	7	20	2
	4	10	3	12	6
	5	15	4	13	8
	6	5	10	6	9

This system being unbalanced (number of rows not equal to columns), we need to create dummy columns (testing areas) for balancing the matrix with their weights (time) as 0.

	I	II	III	IV	V	VI
1	10	8	5	5	0	0
2	12	7	15	4	0	0
3	12	7	20	2	0	0
4	10	3	12	6	0	0
5	15	4	13	8	0	0
6	5	10	6	9	0	0

Performing column operations as this system doesn't require row reductions (due to dummy columns)-

	I	II	III	IV	V	VI
1	5	5	0	3	0	0
2	7	4	10	2	0	0
3	7	4	15	0	0	0
4	5	0	7	4	0	0
5	10	1	8	6	0	0
6	0	7	1	7	0	0

Performing the optimality test-

	I	II	III	IV	V	VI
1	5	5	0	3	0	0
2	7	4	10	2	0	0
3	7	4	15	0	0	0
4	5	0	7	4	0	0
5	10	1	8	6	0	0
6	0	7	1	7	0	0

Since the optimality is proved in this case (number of lines being equal to number of rows), we move to the feasible assignments-

	I	II	III	IV	V	VI
1	5	5	0	3	0	0
2	7	4	10	2	0	0
3	7	4	15	0	0	0
4	5	0	7	4	0	0
5	10	1	8	6	0	0
6	0	7	1	7	0	0

Minimized time for transportation of cars to the testing areas- 5+2+3+5 = 15 minutes.

Game theory

Tata has a number of suppliers for Automotive supplier parts wherein the two major competitors are Bosch and Continental. Control electronics are the major components that are manufactured by the companies. Every new model and innovation require new control electronic model which is devised and introduced to Tata motors.

Payoff table- Deal struck for the new control electronic. (Won by Bosch, Lost by Continental)

		Continental	
Bosch	10% Discount -b1	Extended services-b2	Extended Warranty-b3
10% Discount-a1	3	2	3
Transportation services-a2	-1	1	4
Extended Warranty	4	-3	-2

Game theory is used here as to prove a competition for getting the new deals from the company on the basis of additional services provided with the products (in this case control electronics)

Performing Row operations, identifying the lowest element in each row-

		Continental	
Bosch	10% Discount -b1	Extended services-b2	Extended Warranty-b3
10% Discount-a1	3	2	3
Transportation services-a2	-1	1	4
Extended Warranty	4	-3	-2

Moving to the column operations-identifying the highest value element in each column-

		Continental		
Bosch	10% Discount -b1	Extended services-b2	Extended Warranty-b3	
10% Discount-a1	3	2	3	2
Transportation services-a2	-1	1	4	-1
Extended Warranty	4	-3	-2	3
	4	2	4	

Finding the Maximin of the rows and Minmax of the columns, a saddle point is obtained where the optimal strategies can be devised for both companies.

Best strategy for Bosch is a1 where providing a discount on the control electronics would help them grab the new component deal.

Best Strategy for Continental is b2 where providing extended services with the control electronics would make them lose at most 1 deal with TATA.

Value of the game is 2.

Game theory by applying the mixed strategy where saddle point is not obtained is for the following problem:

TATA Motors has 2 consumer segments in vehicles- Sedan, SUV. Based on the additional features provided TATA motors wants to devise a plan on a Sedan winning the order and the SUV losing as such and vice versa.

Sedan	SUV: ABS - b1	SUV: GPS - b2	SUV: Extended Insurance-b3
ABS - a1	1	2	4
GPS - a2	9	1	7
Extended Insurance - a3	5	-3	11

Applying the concept of dominant strategies where one strategy for a category outperforms the other, we then reduce the matrix into a 2X2 form as follows:

Sedan	SUV: ABS - b1	SUV: Extended Insurance-b3
ABS - a1	1	4
GPS - a2	9	7
Extended Insurance - a3	5	11

Sedan	SUV: ABS - b1	SUV: Extended Insurance-b3
GPS - a2	9	7
Extended Insurance - a3	5	11

And on checking for the saddle point it can be observed that no such point exists and hence we apply the probabilistic method of mixed strategy

Sedan	SUV: ABS - b1	SUV: Extended Insurance-b3
GPS - a2	9	7
Extended Insurance - a3	5	11

Let p = probability of selection of GPS of sedan
1-p = probability of selection of extended insurance of sedan

If ABS (b1) is selected
EV = 9p+ 5(1-p)
= 4p+5

If Extended Insurance (b3) is selected
EV = 7p+11(1-p)
= -4p+11

For optimal probabilities for Sedan,

$4p+5 = -4p+11$

$8p = 6$

$p = 6/8 = 3/4 = 0.75$

$1-p = 2/8 = 0.25$

Let q = probability of selection of ABS of SUV

$1-q$ = probability of selection of extended insurance of SUV

If GPS (a2) is selected

$EV = 9q+7(1-q)$

$= 2q+7$

If Extended Insurance (a3) is selected

$EV = 5q+11(1-q)$

$= -6q+11$

For optimal probabilities for SUV,

$2q+7 = -6q+11$

$8q = 4$

$q = 1/2 = 0.5$

$1-q = 1/2 = 0.5$

Value of game

For SUV

$EV= 9p+ 5(1-p) = 9(0.75) + 5(0.25) = 8.5$

$EV= 7p+11(1-p) = 7(0.75) + 11(0.25) = 8.5$

Value of game

For Sedan

$EV = 9q+ 7(1-q) =9 (0.5) + 7(0.5) =8$

$EV = 5q+11(1-q) = 8$

Solution of game theory sums via the QM software

Pure strategy

Game Theory Results

game theory solution

	1	2	3	Row Mix
1	3	2	3	1
2	-1	1	4	0
3	4	-3	-2	0
Column Mix--->	0	1	0	
Value of game (to row)	2			

Maximin/Minimax

(untitled) Solution

	1	2	3	Row Minimum	Maximin
1	3	2	3	2	2
2	-1	1	4	-1	
3	4	-3	-2	-3	
Column Maximum	4	2	4		
Minimax		2			
Value=2					

Mixed strategy

Game Theory Results

(untitled) Solution

	1	2	Row Mix
1	9	7	.75
2	5	11	.25
Column Mix--->	.5	.5	
Value of game (to row)	8		

Maximin/Minimax

(untitled) Solution

	1	2	Row Minimum	Maximin
1	9	7	7	7
2	5	11	5	
Column Maximum	9	11		
Minimax	9			
7 <= value <=9				

Queuing Theory

Considering the Karnataka factory of TATA motors manufacturing a total of 410 cars in a 10-hour operating window from 9am - 6pm. Deepak an operator, requires 0.8 minutes to conduct automated fitness tests of these vehicles using machines. With the data provided, make calculations on the entire queuing system for the single operator with orders being received in a 20-minute window.

The queuing model is represented as an M/M/1 : N/FCFS

Where,

- The distributions for inter-arrival and inter-service times follow Poisson and Exponential distributions respectively.
- Single operator defines the number of servers which conducts the tests.
- The testing unit at TATA motors taking only a finite number of vehicles for testing in a day.
- The testing of cars occurring on a First Come First Serve basis due to it being an assembly line.

The arrival time is calculated as: 410 cars being manufactured / 10-hour window

λ = 41/hour

= An order in every 1.46 minutes

Therefore, the arrival rate within the 10-minute interval can be calculated as = 10/1.46

= 6.84

≈ 7 approx

Mean service rate for this M/M/1 queuing system will be calculated as:

μ = 1 / (Mean service time)

where, the mean service time is given as 0.8 / 60 hour

μ = 1 / (0.8/60)

= 75 / hour

To acquire the distribution of arrivals, with the expected number of events being discrete valued, Poisson distribution can be used to calculate the probabilities of the events occurring in the time frame given.

Using the same example,

Calculating the Probability of receiving 4 cars to be tested within the 10-minute window

Given, λ = 7

x = 4

$P(x=4) = (e^{-7} * 7^4) / 4!$

$= 0.0912$

$= 9.12\ \%$

Therefore, there is a 9.12 % chance of receiving 4 cars within the 10-minute window

Calculating the Probability of receiving more than 7 cars to be tested within the 10-minute window

Given, $\lambda = 7$

$x = 0$ to 6

$P(x>6) = 1 - \Sigma P(x=0 \text{ to } 6) = 1 - (0.0009 + 0.0063 + 0.022 + 0.052 + 0.091 + 0.127 + 0.149)$

$= 1 - (0.4482)$

$= 0.5518$

$= 55.18\ \%$

Therefore, there is a 55.18 % chance of receiving more than 7 cars within the 10-minute window

To calculate the percentage of cars requiring more than 2 minutes to process (test), we take the cumulative distribution function which focusses on a continuous range of time specified rather than a singular instance. It is formulated as:

$P(T<=t) = 1 - e^{-\mu t}$

And for values greater than the time specified the formula changes to:

$P(T>t) = 1 - (1 - e^{-\mu t})$

$= e^{-\mu t}$

where, μ=mean service rate 75/hour

t=processing time (2/60)

$P(T>2/60) = e^{-75(2/60)}$

$= e^{-2.5}$

$= 0.082$

$= 8.2\ \%$ of cars require more than 2 minutes to process

The turnaround time of the system denotes the amount of time between the process of collection of the cars for testing and the testing taking place. It ideally should be lesser and is represented as:

$W = 1 / (\mu - \lambda)$

$= 1 / (75-41)$

$= 1 / 34$ hour

or 1.76 minutes

The average number of cars waiting in the queue to be tested is expressed as follows

$Lq = \lambda^2 / [\mu(\mu - \lambda)]$

$= 41^2 / (75*34)$

$= 0.659$

This denotes that the average number of cars in the queue of testing are very low, indicating that it is a quick assembly line and quicker it is to automate the process of fitness testing

To calculate the processing of the system, the utilization factor can be calculated which is denoted as $= \lambda / \mu$

$= 41 / 75$

$= 0.5466$

$= 54.66\ \%$

Modelling and Simulation

Monte Carlo Simulation-

Tata Nexon of TATA motors has a greater than every sales in the last 6 months. The total info regarding the sales of Tata Nexon in these six months (180 days) are as follows-

Demand (No of cars)	350	400	820	900	1000	1050	1100
No of days	70	45	32	12	8	7	6
Probability	0.389	0.25	0.178	0.067	0.045	0.038	0.033

What will be the sales for the next 10 days?
15 random numbers- 300, 320, 680,290,560,700,480,920, 990,300

Demand (No of cars)	350	400	820	900	1000	1050	1100
No of days	70	45	32	12	8	7	6
Probability	0.389	0.25	0.178	0.067	0.045	0.038	0.033
Cumulative Probability	0.389	0.639	0.817	0.884	0.929	0.967	1
Random number interval	0-388	389-638	639-816	817-883	884-928	929-966	967-999

Simulation for the next 10 days-

Day	1	2	3	4	5	6	7	8	9	10
Random No	300	320	680	290	560	700	480	920	990	300
Demand	350	350	820	350	400	820	350	1000	1100	350