Dark matter vs. visible matter in the SPARC Galaxy sample

Last week we saw how the Doppler shift effect could be used to measure the rotational speeds of visible matter in our neighboring Andromeda galaxy (a.k.a. M31). You calculated the total mass of M31 and then compared it to total mass of the visible matter (stars, dust, and gas) in the galaxy. You also made a very simple model of how the visible matter was distributed in the galaxy, and then used that to estimate what the rotation curve would look like if the visible matter constituted the entire mass of the galaxy.

While the predicted rotation curve from visible matter was very different from the actual observed rotational motion of gas in M31, one might argue that the model we used for the spatial distribution of visible matter in the galaxy was too simple—perhaps the data could be explained without the need for dark matter if we had more realistic models of the visible mass in the galaxy. Today you will use more realistic models of the gas and stellar content to make more robust measurements of the dark matter content and its spatial distribution across a wide variety of galaxies. You will also investigate how much the *mass-to-light* ratio (one of the main sources of uncertainties in these type of dark matter studies) might affect your determination of the dark matter in the galaxies, and whether adjusting this mass-to-light ratio might allow you to explain the data without the need for dark matter.

Materials

The accompanying python data analysis notebook, the galaxy information, and the data files are all found at the URL:

https://www.dropbox.com/sh/yg6ovlzxcokam47/AADqujCpa8IIOA1MDEjbld8ba?dl=0

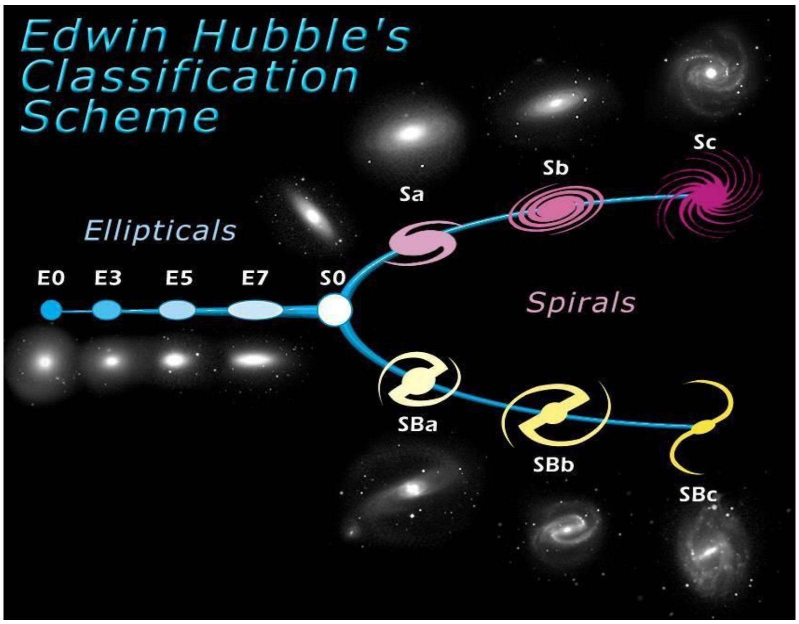
The dataset

The data you will use is from the Spitzer Photometry and Accurate Rotation Curves (SPARC) database. For more information about the survey, visit http://astroweb.cwru.edu/SPARC/ .
To quote their own website,

> The new Spitzer Photometry and Accurate Rotation Curves (SPARC) database is publicly available online. Created by team leaders Federico Lelli and Stacy McGaugh (CWRU Astronomy) and Jim Schombert (UOregon Physics), SPARC is a sample of 175 disk galaxies
>
> covering a broad range of morphologies (S0 to Irr), luminosities (10^7 to 10^{12} Lsun), and sizes (0.3 to 15 kpc). For each galaxy, the team collected both HI data and Spitzer images at 3.6 um. The HI data provide the gas distribution and the galaxy rotation curve, tracing the total gravitational potential: these data are the fruits of 30 years of observations from an entire community of radio astronomers. The Spitzer images probe the stellar mass distribution: these data were homogeneously analysed by our group and used to calculate the gravitational contribution of stars. The SPARC dataset is used to study the interplay between visible and dark matter in galaxies, test the LCDM cosmological model, and investigate possible alternatives.

There are many different types of galaxies in this sample. Since you don't have time to look at all 150+ galaxies in one day, pick one first that you think would be interesting. You can go back and pick out other galaxies to analyze after you complete the first one.

Download the file 'galaxy_sample_summary.txt' in your text editor. (Not MS Word or whatever its Mac equivalent is, use the basic text editor.) This file lists some properties for each galaxy in the sample. Column 1 is the name of each galaxy. You might find column 2 the most interesting—it classifies each galaxy by its Hubble Type, which is a category based on what the galaxy looks like. The sample includes S0 types and any type that is to the right of S0 on the diagram below (spiral types). There are also “irregular” galaxies, with types abbreviated “Im” or “Irr”. Choose your favorite type and try to find a galaxy of that type in the sample. Take note of its name.



Modeling the visible mass of a galaxy

Last week you were simply given the mass of the stars in M31 without much explanation as to how that number was determined. Now you will see how that figure was calculated, and how you can use the data you have to make a more accurate model for how the stars (the bulk of the visible matter) in other galaxies are distributed.

The *observable* property of the stars in galaxies that can be measured directly is their luminosity, or intrinsic brightness. We can't see individual stars in these distant galaxies, but we can measure the *surface brightness* at different positions in the galaxy.

Download the file 'SPARC_data.zip' and unzip it. You should end up with a bunch of files '<galaxy>_rotmod.dat' in a folder called 'SPARC_data'. Download and open the python notebook 'Visible and dark matter distribution of mass in galaxies.ipynb' and execute the code up to and including the section 'Plot the surface brightness at different radii'.

After plotting this for your galaxy (save the plot with the button in the interactive plotting area) try looking at this for different galaxies in the dataset to get an idea of how different the brightness distributions of galaxies can be.

1. Can you find a galaxy in the sample that has a dense central bulge component? What would this look like in the surface brightness plot?

NGC 3741 is an irregular galaxy with a bright central bulge due to its distinctly steep slope within a very short kpc radius (2kpc), meaning its brightest stars are concentrated greatly within the center, and leaving the central bulge means significantly dimmer material resulting in a sharp declining slope.

Determining the distribution of visible and dark matter in a galaxy

The SPARC public dataset does not provide their mass estimates the visible mass and dark matter mass in each galaxy. Instead they provide the *total* observed rotation speed (already corrected for the overall motion of each galaxy and its inclination) and the component of the total rotational velocity that is due to the stellar mass and gas mass. We need to use the total rotational velocities to work backwards and figure out the total mass enclosed within each radius. We can then do the same thing for the visible mass, using the rotational velocity attributed to the influence of the visible mass. Then we can subtract the visible mass from the total mass to find the difference, which we ascribe to the dark matter.

Remember the expression for rotational velocity: $\boxed{v_c = \sqrt{\frac{GM}{r}}}$

where M is the mass enclosed within the orbit radius r, and $G=4.43\times10^{-6}$ kpc(km/s)^2/M_{sun} is the gravitational constant.

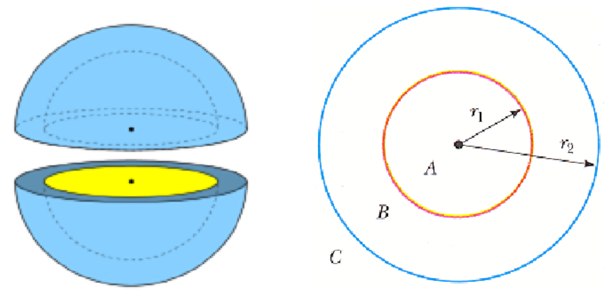
2. Rewrite the equation for rotational velocity so that it is expressing the mass enclosed M in terms of G, v_{rot}, and r.

$$M = \frac{v^2 r}{G}$$

However, it is more useful to describe the *density* of matter at each radius, and not the total mass enclosed. For now, just consider the first two radii measured in your galaxy (the closest two points from the center). Imagine drawing two spheres: one has radius r_1 (the smallest radius) and the other has radius r_2 (next smallest radius). The observed velocities from the data at these radii are v_1 and v_2. **Write your answers below in terms of r_1, r_2 , v_1, v_2, M, and G.**

3. What is the volume of the spherical shell S_{12} that is defined by radii r_1 and r_2? The volume of a sphere is given by $V_{sphere} = (4/3)\ \pi\ r^3$

$$V=(4/3)\ \pi\ (r2^3 - r1^3)$$



4. What is the mass enclosed within the spherical shell S_{12} ?

$$M = \frac{(v_2{}^2 r_2 - v_1{}^2 r_1)}{G}$$

5. What is the average density of matter in the shell S_{12}? Remember: density = mass/volume.

$$\rho = \frac{3}{4\pi G} \frac{(v_2{}^2 r_2 - v_1{}^2 r_1)}{(r_2{}^3 - r_1{}^3)}$$

The expression you arrived at above can be generalized to find the average density of any shell between two measured radii. Go back to the python notebook and read/execute the sections 'Plot the rotational velocity data provided in the SPARC data' and 'Plot the density distribution of visible and dark matter'. The first section will discuss how you infer the contribution of the enclosed dark matter mass to the rotational velocity at some radius. The second will implement the function you just wrote above for the density of visible/dark material as a function of radius.

6. Look at the plot you produced for the density distribution of the total/dark/visible mass. Depending on the galaxy you are looking at, the density of visible matter might be calculated to be

higher than the total density in the central region of the galaxy (this effect will also be evident in your rotation curve plot). How is this possible? (Ask your TA if you're not sure.)

For a galaxy whose density of visible is higher than the total observed, this discrepancy could be due to it being far enough away such that the luminosity measured from the bulge could have been distorted and decreased by the interstellar medium by a significant enough amount as to cause a difference between the visible matter calculated and total observed.

7. Around what radius (in kpc) does the dark matter density dominate over the visible matter density? If you go back to the file 'galaxy_sample_summary.txt' in your text editor, you can look up the effective radius R_{eff} (defined as the radius within which half of the total luminosity is contained) and the stellar disk radius R_{disk}. These are columns 7 and 9, respectively. Write their values below. How do these radii compare to the radius at which dark matter becomes the dominant component of the mass density?

For NGC 3741, the radius at which dark matter becomes more dominant is at around .46 kpc. The effective Radius and Stellar Disk Radius are 4 kpc, and 0.001 kpc respectively.

Can the rotation curves be explained without the need for dark matter?

How do we use the measured brightness of a galaxy to determine its stellar mass? Astronomers use what is known as the stellar mass-to-light ratio **Υ.** The mass to light ratio is defined such that our Sun has a mass-to-light ratio of 1:

$\mathbf{\Upsilon_{sun} = 1\ M_{sun}/\ L_{sun}}$

These units are useful because we can measure the luminosity of other star in other galaxies in units of solar luminosity L_{sun} (e.g. a star whose intrinsic brightness is twice that of the sun has $L = 2\ L_{sun}$).

8. If you assumed that all stars were like the sun, i.e. they had mass-to-light ratios of Υ=1, then what would the mass be for a galaxy that was observed to have a luminosity $L=10^{11}\ L_{sun}$?

For a galaxy with mass-to-light ratio of 1, if the luminosity were $10^{11}\ L_{sun}$, the mass should also be $10^{11}\ M_{sun}$.

In reality, not all stars are like the Sun. Stars of different masses have different temperatures and different brightness. The most common types of stars are red dwarfs, which have masses 7.5-60% that of the Sun and luminosities 0.015-7% that of the Sun.

9. Do red dwarfs have mass to light ratios greater than or less than 1?

Red Dwarfs have an MtL ratio greater than 1.

10. If the stellar content of a galaxy with $L=10^{11}$ M_{sun} was predominantly composed of red dwarfs, would you estimate it to have a higher or lower mass in stars than you did in (2)?

The galaxy should have a mass higher than ones found in 2 as that mass is based off solar mass.

In order to convert a galaxy's luminosity into a mass, you would multiply the luminosity by the mass-to-light ratio. As questions 8 and 10 just illustrated, the value that you assume for the mass-to-light ratio can have a big effect on the final mass in stars that you calculate. In what follows, you will investigate whether or not changing the mass-to-light ratio could explain the observed galactic rotation curves without the need for dark matter in the galaxies. The SPARC dataset assumes a fixed mass-to-light ratio of Y=1 M_{sun}/L_{sun} when calculating the predicted rotational speeds from stellar matter . Your goal is to try and adjust the mass-to-light ratio such that the predicted velocity curve from the visible matter in the galaxy matches up with the observed data points.

11. Does the value of Y=1M_{sun}/L_{sun} seem like a reasonable value to use? Do you think it should be higher or lower?

A value higher than 1 seems more reasonable since there is a very large density of stars within the central bulge of a galaxy, meaning there will most likely be a large collection of red dwarfs contained within it, averaging the Y to be greater than 1.

12. If you wanted to change the mass-to-light ratio and try to fit the observed rotation curve using only the visible matter, could you simply scale the entire visible rotation curve up or down by multiplying it by your new value of Y? If not, by what factor should you scale the stellar velocity curve?

Scaling the entire rotation curve would not be enough to fit to the observed curve as the visible matter curve has a downward rate of change as you move further from the center. On the other hand, the observed shows a clear incline as we move outward from the center, and no amount of scaling will account for that change in rate.

In just a bit, you will try to fit the data using only the visible matter in the galaxy. To answer the question "Can the data be fit without dark matter?" you can use a Chi-square goodness of fit test. The Chi-Square test require you to find the value of the quantity

$$\chi^2 = \sum \frac{(O_i - C_i)^2}{\sigma_i^2}$$

O_i is the *observed* rotational speed for at the radius of each data point, and C_i is the *calculated* speed that same radius, given your hypothesis that the data can be fit with the rotation curve of the visible matter. σ_i is the size of the error bar on that particular measurement of the total rotational velocity. The Greek letter sigma out front of the right hand side (looks sort of like a captial E) means that you are going to calculate this quantity $(O\text{-}C)^2/\sigma$ for each data point and then add all these terms together. As you might be able to infer from the way this is calculated, the better your prediction fits the data, the smaller the value of χ^2 will be.

Finally, we will then divide χ^2 by the (number of data points+1) to come up with a value for the "χ^2 per degree of freedom". The number of degrees of freedom = number of data points - number of free model parameters. Since the only parameter you are varying when you are trying to fit the data with the visible matter is the mass-to-light ratio, we subtract 1 to the number of data points. Typically, if your χ^2 per degree of freedom is 1 or lower, your model is a good fit to the data.

Go to the section "Varying the mass-to-light ratio" in your python notebook. You can change the mass-to-light ratio for the stars and then see how well the resulting visible matter rotation curve fits the observations. The python code will take care of the χ^2 per degree of freedom calculation for you, but we want you to know what this test for goodness of fit means.

13. What is the value of Y that gives the lowest χ^2 per degree of freedom for your galaxy when you try to fit the data with the visible matter only? What is the χ^2 per degree of freedom in this fit?

The Value of Y that gives the lowest χ^2 per degree of freedom is 9, with a χ^2 of 7.5. However, the visible matter curve does not fit the observed curve whatsoever.

14. The python notebook is set up to make it pretty efficient for you to examine different galaxies in the dataset. Look at as many different galaxies as you can in the remaining time, and try to fit the observed rotational speeds in each of them with visible matter only by playing with the value of the mass-to-light ratio. Are there any galaxies that can be fit without the need for dark matter (χ^2 per degree of freedom <1)? If you find any, write their names down and take note of their galaxy type.

Galaxy Name	Type	Mass to Light Ratio	Chi Squared per degree of Freedom
D512-2	Irregular - Im	2.21	0.9507651650058855
F561-1	Irregular “Magellanic” Spiral -Sm	0.9	0.6352477401718724
F563-V1	Irregular - Im	1.0	0.17410504733432194
F563-V2	Irregular - Im	6.1	0.49263563458701864
F567-2	Irregular - Sm	1.9	0.29975905493872046
F574-2	Irregular - Sm	0.6	0.10001591843163175

15. What value(s) of Y seem to fit the data the best when you try to match the total rotational speeds by changing the mass-to-light ratio? Do these seem to be consistent with the range of mass-to-light ratios that we think the typical star might have?

For NGC 3741, the best ***visible*** fit was with a Y of ~1.35, but this only matches up the first half parsec or so. The same goes for most other galaxies, save for a select few that seemed to line up with an adjusted mass-to-light ratio.