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Volatility Models and their Performance in Indian Capital Markets

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Executive Summary

Estimation and forecasting of volatility of asset returns is important in various applications related to financial markets such as valuation of derivatives, risk management, etc. Till early eighties, it was commonly assumed that the volatility of an asset is constant and estimation procedures were based on this assumption even though some of the pioneering studies on property of stock market returns did not support this assumption. Following the pioneering work of Engle and Bollerslev in eighties on developing models (ARCH/GARCH type models) to capture time-varying characteristics of volatility and other stock return properties, extensive research has been done world over in modeling volatility for estimation and forecasting.

There are broadly four possible approaches for estimating and forecasting volatility. These are:

- ***Traditional Volatility Estimators*** — These estimators assume that 'true' volatility is unconditional and constant. The estimation is based on either squared returns or standard deviation of returns over a period.
- ***Extreme Value Volatility Estimators*** — These estimators are similar to traditional estimators except that these also incorporate high and low prices observed unlike traditional estimators which are based on closing prices of the asset.
- ***Conditional Volatility Models*** — These models (ARCH/GARCH type models) take into account the time-varying nature of volatility. There have been quite a few extensions of the basic conditional volatility models to incorporate 'observed' characteristics of asset/ stock returns.
- ***Implied Volatility*** — In case of options, most of the parameters relevant for their valuation can be directly observed or estimated, except volatility. Volatility is, therefore, backed out from the observed option values and is used as volatility forecast.

The empirical research across countries and markets has not been equivocal about the effectiveness of using these approaches. This study compares the result of the first three approaches in estimating and forecasting Nifty returns.

Based on four different criteria related to bias and efficiency of the various estimators and models, this study analysed the estimation and forecasting ability of three different traditional estimators, four extreme value estimators, and two conditional volatility models. As a benchmark, it used 'realized' volatility estimates. The findings of this study are as follows:

- For estimating the volatility, the extreme value estimators perform better on efficiency criteria that the conditional volatility models.
- In terms of bias, conditional volatility models perform better than the extreme value estimators.
- As far as predictive power is concerned, extreme value estimators estimated from sample of length equal to forecast period perform better than the conditional volatility estimators in providing five-day and month ahead volatility forecasts.

KEY WORDS

Volatility Estimation and Forecasting

Conditional Volatility Models

Extreme Value Volatility Estimators

Modeling and forecasting stock market volatility is of considerable interest to the practitioners[1] and researchers alike. This has led to considerable research in this area in the past decade or so. The autoregressive conditional heteroskedasticity (ARCH) model, introduced by Engle (1982) and later generalized by Bollerslev (1986), spawned numerous empirical studies modeling volatility in developed markets.[2] Later, there have been quite a few studies focusing on emerging stock markets[3] as well. Researchers have increasingly used conditional volatility models such as ARCH, generalized autoregressive conditional heteroskedasticity (GARCH), and their extensions as these models have helped them to model some of the empirical regularities.

Starting with the pioneering work of Mandelbrot (1963) and Fama (1965), various features of stock returns have been extensively documented[4] in the literature which are important in modeling stock market volatility. It has been found that stock market volatility changes with time (i.e., it is 'time varying') and also exhibits positive serial correlation or 'volatility clustering' (Mandelbrot, 1963). This implies that the changes in volatility are non-random. Moreover, the volatility of returns can be characterized as a long-memory process as it tends to persist (Bollerslev, Chou and Kroner, 1992). Volatility shocks at daily or lower frequency tend to decay very slowly (Andersen and Bollerslev, 1996). It was Black (1976) who first noted that even the changes in volatility have been found to be negatively correlated with changes in the stock prices. This has been termed as 'leverage effect in volatility.' He argued that the changes in stock volatility are too large in response to changes in return direction to be explained by the leverage effect alone. The works of Christie (1982) and Schwert (1989) later supported this conclusion. The volatility has also been found to be low during non-trading periods (Fama, 1965; French and Roll, 1986). It is also predictable in the sense that it is typically much higher at the beginning and close of this trading period than the rest of the trading period (Harris, 1986; Baillie and Bollerslev, 1991). It is also high at the time of earning announcements for individual stocks (Cornell, 1978; Patell and Wolfson, 1981). At the individual stock level, the volatility seems to change across stocks (Black, 1976). In other words, there seems to be co-movement in volatility across assets. Co-movement in volatility has also been found in the context of exchange rates (Diebold and Nerlove, 1989; Harvey, Ruiz and Sentana, 1992), yields of different maturities in bond market (Engle, Ng and Rothschild, 1990), and stock markets across countries (Engle and Susmel, 1993; Hamao, Masulis and Ng, 1990).

The ARCH model proposed by Engle (1982) and its various extensions[5] have been developed to model some of the above-mentioned characteristics of financial time series. In particular, different models have tried to capture time varying second moments of return distributions, time varying and mean reverting second moments, leverage effect, varying first moment, time varying 'baseline' second moment, undefined second moment, etc. Despite the ability of the ARCH/GARCH type models to capture some of the stylized facts about volatility and return distribution characteristics, their usefulness ultimately depends on their ability to forecast volatility as pointed out by Engle and Patton (2001).

As far as volatility forecasting is concerned, Akgiray (1989) in an early work found that GARCH (1,1) outperformed models based on historical prices. Later works[6] across different countries using different data sets have reported different results. From the literature on forecasting volatility, it is clear that it is difficult to forecast volatility and that the results on the forecasting ability of various models are mixed. In the Indian context, we have not come across any study comparing forecasting ability of various volatility models even though ARCH/GARCH type models have been used in various empirical works to model time-varying second moment and serial correlation in volatility.

In addition to testing comparative performance of various conditional volatility models, another strand has evolved in the literature in the form of research on extreme value volatility estimators following the work of Parkinson (1980). These estimators are historical unconditional estimators and are based on the range of prices observed during trading unlike the classical or traditional volatility estimator which uses closing price. These estimators, though not popular, have been shown to be theoretically much more efficient compared to the traditional estimator. One reason for the lack of interest in these estimators is that they could be downward-biased compared to the traditional estimator due to discreteness of prices and trading in the stock markets. However, recently, Li and Weinbaum (2000) argued that the assumed unbiasedness of the traditional estimator is contingent on the validity of assumption of return generating process. They contend that both the bias and efficiency of extreme value estimators and the traditional estimator is more of an empirical issue. In their empirical work, they use the realized volatility measure using high frequency data, developed by Andersen *et al*. (2001a) as the benchmark to evaluate the empirical performance of extreme value estimators *vis-à-vis* traditional estimators. Besides the issue of bias, extreme-value estimators, unlike conditional volatility models, do not explicitly incorporate the empirical features of returns distribution discussed above, and are, therefore, not as attractive as conditional volatility models.

Though a plausible reason for relatively less research on and application of extreme value estimators could be the time-varying characteristic of volatility, yet, the use of extreme value estimators may still be preferred if they are as efficient empirically as implied by the theory. In that case, conditional volatility models, efficient extreme value volatility estimators, and high frequency data-based realized volatility model could possibly compete for modeling and forecasting* volatility for various applications. An attractive feature of extreme value volatility estimators is that they are, by construction, closer to the realized volatility than the models using only daily closing or opening prices. On the other hand, while conditional volatility models based on low-frequency daily data do not take into account intra-day price variations, they explicitly recognize time-varying characteristics of volatility of asset returns. The estimates and forecasts given by conditional volatility models, unlike traditional or extreme value estimators, are based on the model fitted on a larger data set or a longer time-period as the estimates and forecasts are based as much on the parameters of the model as much they are on the returns characteristics during the relevant period. The relevant period, of course, is the period over which volatility is being estimated for estimation and period just prior to the period for which forecast is sought. The trade-off in the use of extreme-value estimators *vis-à-vis* conditional volatility models can be set forth as a result of efficiency gained by use of intra-day data *vs*. explicit modeling of time-varying characteristics of volatility. The former class of models use some additional data (intra-day prices) and, therefore, might provide efficient estimates and forecasts based on a short time-series without recognizing time-varying characteristics of volatility. However, the latter, despite not using intra-day data, could still be more accurate for estimation and forecasting based on longer time-series data having incorporated time-varying characteristics of volatility. This trade-off can be evaluated over different horizons only empirically and by using a measure of 'true' volatility independent of either of the two classes of models. Realized volatility measure provides such a measure as a benchmark for the evaluation of this trade-off.

The primary motivation of this work is to evaluate and compare different volatility models in the Indian capital markets for their estimation and forecasting ability empirically. In this paper, we report the empirical performance of both historical, unconditional volatility estimators and conditional volatility models using realized volatility measure as the benchmark. In some ways, the work is similar to the study by Li and Weinbaum (2000) and its replication and extension in the Indian capital markets by Pandey (2002). However, these studies did not include conditional volatility models for comparison. Besides including estimates from two conditional volatility models (GARCH and Exponential GARCH (EGARCH)), we also extend the scope of previous studies by investigating the predictive power of the estimators and models. The latter part is similar to studies by Day and Lewis (1992) and Pagan and Schwert (1990). Unlike these, however, we use realized volatility measure constructed from high frequency data as the 'true' volatility to be forecasted by the estimators and models. The realized volatility measure has been shown to be model free by Andersen *et al*. (2001a, 2001b). The use of 'realized volatility' based on high frequency data could be a key factor in determining the forecasting ability of the conditional volatility models as shown by Andersen and Bollerslev (1998).

REVIEW OF VOLATILITY MODELS*

There are various classes of models and estimators which have been proposed in the literature for measuring volatility of asset returns. Models and estimators, assuming volatility to be constant, are the oldest ones among the models which have been used to estimate and forecast volatility. These models and estimators measure 'unconditional volatility.' With the recognition of empirical regularity that the volatility in financial markets

* Forecasting models would also include volatility forecasts based on implied volatility of options.

* Some parts (excluding the section on conditional volatility models) of this section of the paper draw extensively from Li and Weinbaum (2000) and Pandey (2002).

is clustered in time and is time-varying, these models gave way to models measuring 'conditional volatility.' In addition, volatility estimated from the value of options in which typically volatility is the only unobservable parameter for valuation allowed researchers and practitioners to use 'implied volatility,' i.e., the market forecast of volatility in valuing the traded options. Finally, as shown by Andersen *et al*. (2001a, 2001b), volatility becomes observable and does not remain latent if high frequency data are available. The 'realized volatility' estimated using high frequency data is model-free under very weak assumptions.

'Unconditional Volatility' Estimators and Models

Traditional Estimators

Traditionally, the unconditional volatility of asset returns has been estimated using close-to-close returns. The traditional close-to-close volatility (or variance) estimator (scc) for a driftless security is estimated using squared returns and is given by:

$$\sigma^2_{cc} = 1/n\ \Sigma\ (c)^2 \tag{1}$$

where,

n = number of days (or, periods) used to estimate the volatility

$c = \ln C_t - \ln C_{t-1}$

C_t = closing price of day t

The mean-adjusted version of the close-to-close estimator (σ_{acc}) is estimated using sample standard deviation and is given by:

$$\sigma^2_{acc} = 1/(n-1)^*\ [\Sigma\ (c)^2 - n\underline{c}^2] \tag{2}$$

where,

$\underline{c} = (\ln C_n - \ln C_0)/n$

While equation (2) provides an unbiased estimate of variance, the square root of the estimator is a biased estimator of volatility due to Jensen inequality (Fleming, 1998). The statistical properties of the sample mean make it a very inaccurate estimator particularly for small samples (Figlewski, 1997). He suggests that taking deviations around zero, i.e., using equation (1), improves the volatility forecast accuracy.

Extreme Value Estimators

Parkinson (1980), following the work of Feller (1951) on the distribution of the trading range of a security following geometric Brownian motion (GBM), was the first to propose an extreme value volatility estimator for a security following driftless[7] GBM which is theoretically five times more efficient compared to traditional close-to-close estimator. His estimator (σ_p) is given by:

$$\sigma^2_p = 1/(4n\ \ln 2)^*\ \Sigma\ (\ln H_t/L_t)^2 \tag{3}$$

where,

H_t = highest price observed on day t

L_t = lowest price observed on day t

Extending his work, Garman and Klass (1980) constructed an extreme value estimator incorporating the opening and closing prices in addition to the trading range which is theoretically 7.4 times more efficient than its traditional counterpart. Their estimator (σ_{gk}) is given by:

$$\sigma^2_{gk} = 1/\ n\ *\ \Sigma\ [0.511(\ln H_t/L_t)^2 - 0.019\ (\ln (C_t/O_t)^*\ln (H_t\ L_t/O_t^2)\ - 2\ \ln (H_t/O_t)^*\ \ln (Lt/O_t))\ -0.383(\ln C_t/O_t)^2] \tag{4}$$

where,

O_t = opening price of day t

Both the Parkinson and Garman-Klass estimators, despite being theoretically more efficient, are based on assumption of driftless GBM process. Rogers and Satchell (1991) relaxed this assumption and proposed an estimator (σ_{rs}), which is given by:

$$\sigma^2_{rs} = 1/n^*\ \Sigma\ [\ln (H_t/C_t)\ \ln (H_t/O_t) + \ln (L_t/C_t)\ \ln (L_t/O_t)] \tag{5}$$

Kunitomo (1992) also proposed an extreme value estimator based on the range of a Brownian Bridge process constructed from price process which is two times more efficient than the Parkinson estimator. His estimator, however, cannot be computed directly from the daily data. Later, Spurgin and Schneeweis (1999) proposed an estimator based on the distribution of the range of binomial random walk. Their estimator (σ_{ss}) is given by:

$$\sigma^2_{ss} = 1/n^2\ *\ 0.3927^*\ \Sigma\ (\ln H_t/L_t)^2 - 0.4986\ S \tag{6}$$

where,

S = the tick-size of the trades

Recently, Yang and Zhang (2000) proposed an estimator independent of drift which also takes into account an estimate of closed market variance. The estimators proposed earlier, including the Rogers-Satchell estimator, do not take into account the closed market variance. This means that the prices at the opening of the market are implicitly considered the same as that of closing price on the previous day. The Yang-Zhang estimator is based on the sum of estimated overnight variance and estimated open market variance. The estimated open market variance, in turn, is based on the weighted average sum of the open market returns sample variance and the Rogers-Satchell estimator with the weights chosen to minimize the variance of estimator. The Yang-Zhang estimator (σ_{yz}) is given by:

$$\sigma^2_{yz} = 1/(n-1) *\Sigma (\ln O_t/C_{t-1} - \underline{o})^2 + k/(n-1)*\Sigma (\ln C_t/O_t - \underline{c})^2 + (1-k)* \sigma^2_{rs} \quad (7)$$

where,

$\underline{o} = 1/n* \Sigma (\ln O_t/ C_{t-1})$

$\underline{c} = (\ln C_n - \ln C_0)/n$ or, $1/n*\Sigma (\ln C_t/O_t)$

$k = 0.34/ [1.34 + (n+1)/ (n-1)]$

The extreme value estimators proposed in the literature have been usually derived under strong assumptions. As pointed out earlier, attempts have been made to relax the assumption of driftless price process and closed market variance by Rogers and Satchell (1991) and Yang and Zhang (2000) respectively. Besides these, it is argued that the observed extreme values may reflect certain liquidity-motivated trades (Li and Weinbaum, 2000). This could make them less representative of 'true' prices as compared to the closing prices.

Besides extreme values being potentially less representative of the true prices, extreme values observed are in markets where the trading is discrete whereas extreme value estimators are derived under the assumption of continuous trading. This can induce downward 'discrete trading' bias in extreme value estimators as the observed highest prices are lower than the 'true' highest price and the observed lowest price is higher than the 'true' lowest price (Rogers and Satchell, 1991; Li and Weinbaum, 2000). Rogers and Satchell (1991) addressed this issue by proposing adjustment in their extreme value estimator by taking into account the number of steps (trades) explicitly. The adjusted Rogers-Satchell estimator (σ_{ars}) is positive root of the following equation:

$$\sigma^2_{ars} = (0.5594/N_{obs})* \sigma^2_{ars} + (0.9072/ N^{1/2}_{obs})* \ln (H_t/L_t)* \sigma_{ars} + \sigma^2_{rs} \quad (8)$$

where,

N_{obs} = number of observations/ transactions

σ_{rs} = unadjusted Rogers-Satchell estimator

Rogers and Satchell also proposed similar correction to the Garman and Klass (1980) estimator. The adjusted Garman-Klass estimator (σ_{agk}) is positive root of the following equation:

$$\sigma^2_{agk} = 0.511*[(\ln H_t/L_t)^2 + (0.9079/N_{obs})*\sigma^2_{agk} + (1.8144/ N^{1/2}_{obs})*\ln H_t/L_t*\sigma_{agk}] + 0.038*[\ln H_t/O_t* \ln L_t/O_t - (0.2058/ N_{obs})* \sigma^2_{agk} - (0.4536/ N^{1/2}_{obs})* \ln (H_t/L_t)*\sigma_{agk}] - 0.019* \ln (C_t/O_t)*\ln (H_t L_t/ O_t^2) - 0.383*(\ln C_t/O_t)^2 \quad (9)$$

While theoretically extreme value estimators are shown to be more efficient (5 to 14 times), yet they have not been very popular. This is mainly because these estimators are derived under strong assumptions about underlying returns generating process in the asset markets. It is assumed that the asset prices follow GBM and are observable in a market trading continuously. While extreme value estimators of volatility could be biased if the returns generating process is mis-specified, Li and Weinbaum (2000) point out that the assumed 'unbiasedness' of the traditional estimator itself is contingent on the validity of assumed return generating process. In particular, they show that the traditional estimator based on the sample standard deviation/ variance of returns is not an unbiased estimator of the true instantaneous volatility/ variance for the trending Ornstein-Uhlenbeck process having predictable returns and constant volatility. They argue that the bias in the traditional or extreme value estimators is more of an empirical issue, more so, when it is possible to assess the efficiency and/or bias of the traditional and extreme value estimators of volatility using realized volatility measured from high frequency data.

The extreme value estimators proposed in the literature have been tested using simulated stock prices, actual stock prices, and recently, using realized volatility measures. Using simulated data with discrete price changes, Garman and Klass (1980) showed that extreme value estimators are downward-biased. Beckers (1983) using actual data also found downward bias in extreme value estimators. Studies by Wiggins (1991, 1992) also reached similar conclusions. However, Spurgin and Schneeweis (1999) found that the binomial estimator developed by them outperformed traditional and other extreme value estimators on daily and intra-day data of two futures — CME SP500 and CBT Treasury Bonds contracts. Li and Weinbaum (2000), using intra-day high frequency data to measure realized volatility, found overwhelming support for extreme value estimators for stock indices (S&P 500 and S&P 100) data set, but confirmed the bias of extreme value estimators for currencies and S&P 500 futures data set despite efficiency gains. Li and Weinbaum investigated the performance of extreme value estimators for two stock indices (S&P 500 and S&P 100), a stock index futures (on S&P 500), and three exchange rates (Deutsche Mark: US$, Yen: US$, and UK Pound: US$).

Conditional Volatility Models

Conditional volatility models, unlike the traditional or extreme value estimators, incorporate time varying characteristics of second moment/volatility explicitly. Following the pioneering work of Engle (1982), various models have been proposed in the literature. The specification of an ARCH (q) model (Engle, 1982) is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \; \varepsilon^2_{t-i} \qquad (10)$$

where,

ω, $\alpha_1,...,\ \alpha_q$ = parameters to be estimated

σ_t^2 = conditional variance at period t

q = number of lags included in the model

e_t = innovation in return at time t

In the ARCH (q) model, the volatility at time t is a function of q past squared returns. For the ARCH model to be well-defined, the parameters should satisfy $\omega>0$ and $\alpha_1,..,...,\alpha_q>0$. Equation (10) gives the conditional variance equation. In the ARCH/GARCH type models, standard conditional mean equation is usually[8] modeled as r_t = constant + ε_t. Since empirical application of ARCH (q) model required long lag length and a large number of parameters to be estimated, Bollerslev (1986) proposed GARCH (p,q) model in which volatility at time t is also affected by p lags of past estimated volatility. The specification of a GARCH (p,q) is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \; \varepsilon^2_{t-i} + \sum_{j=1}^{p} \beta_j \; \sigma^2_{t-j} \qquad (11)$$

where,

ω, $\alpha_1,...,\ \alpha_q, \beta_1,..,\beta_p$ = parameters to be estimated

q = number of return innovation lags included in the model

p = number of past volatility lags included in the model

The coefficients of the model should satisfy certain conditions for the conditional variance in the GARCH (p,q) model to be well-defined. β_j's in the model capture GARCH coefficients, whereas α_i's capture ARCH coefficients. For the GARCH (1,1) model, the conditions to ensure positive conditional variance are: $\omega>0$, $0\leq\alpha$, $0\leq\beta$, and $\alpha+\beta<1$. As pointed out elsewhere in the paper, the basic ARCH/GARCH models have been extended and new models proposed to model returns distribution better. EGARCH is one such model. In this work, we have used only GARCH and EGARCH models to model volatility in the Indian stock market. In the EGARCH model proposed by Nelson (1991), the conditional variance depends upon both the size and the sign of lagged residuals. EGARCH as well as other asymmetric volatility models have been developed to incorporate the 'leverage effect' and observed 'asymmetric volatility changes with the change in return sign.' The specifications of EGARCH (1,1) model are given by:

$$\ln \sigma_t^2 = \omega + \beta . \ln \sigma^2_{t-1} + \gamma \; \varepsilon_{t-1}/\sigma_{t-1} + \alpha \left[|\varepsilon_{t-1}|/\sigma_{t-1} - (2/\Pi)^{1/2} \right] \qquad (12)$$

In equation (12), ω, α, β, γ are the parameters to be estimated while other symbols are the same as in equation (10) and (11). Unlike GARCH, EGARCH is a model of logarithm of conditional variance and hence no non-negativity constraints are required for conditional variance to be positive. As shown by Nelson (1991), $|\beta|<1$ ensures ergodicity and stationarity of EGARCH (1,1). Besides EGARCH, there have been quite a few extensions of basic ARCH/GARCH model proposed in the literature for modeling volatility. In addition, there is a separate class of conditional volatility model called stochastic volatility models in which the conditional variance specification contains two error terms. In case of ARCH/GARCH models, the conditional variance equation is determined by the information available at that time with only one error term associated with the past return. In this work, we have used relatively basic GARCH and EGARCH conditional volatility models. These models have been quite popular[9] amongst the researchers and practitioners despite emergence of extensions aimed at modeling the empirical regularities of return time-series.

Realized Volatility

If high-frequency data are available, the volatility becomes observable and does not remain latent. The realized volatility measure developed by Andersen *et al.* (2001a), therefore, can be used to directly compare performance various volatility models and estimators[10]. The realized volatility measure for day t is given by:

$$\sigma_t = (\Sigma \, r^2_{j,t})^{1/2} \qquad (13)$$

where,

$r^2_{j,t}$ = squared return series of intra-day data

j = intra-day interval over which returns are being measured

It is possible to annualize the realized volatility so measured by scaling it up with an annualizing factor. The annualizing factor is simply square root of number of trading days in a year. Measuring realized volatility requires choosing appropriate interval over which the squared returns are used to measure the realized volatility. While shorter time intervals reduce the measurement error, they are also likely to be biased by the microstructure effects (Andersen and Bollerslev, 1998;

Andersen *et al*., 1999). Andersen *et al*. (2001a, 2001b) and Li and Weinbaum (2000) found that sampling the returns over 5-minute interval is optimal. Without investigating the desirability of using 5-minute returns series on our data set, we have used 5-minute returns to compute the realized volatility.

CHARACTERISTICS OF NIFTY DAILY RETURNS TIME-SERIES: THE METHODOLOGICAL ISSUES

In this study, our objective is to empirically investigate the performance of some of the popular volatility models and estimators proposed in the literature. With the availability of high frequency data being compiled by the National Stock Exchange, a direct comparison of estimates with the model-free realized volatility estimates is possible and hence the realized volatility estimates have been used in the study to assess the bias and efficiency of various volatility models estimators. Traditional close-to-close estimators, various extreme value estimators, and two popular conditional volatility models are estimated and compared with the realized volatility estimates. We also test the ability of these estimators and models to forecast one-day, five-day (approximately weekly), and monthly volatility. In this section, we describe the data set used, the characteristics of daily returns of the index chosen to represent the Indian stock market, and discuss the performance criteria used to assess the performance of various models and estimators.

Nifty Daily Returns Characteristics

In this paper, we use high frequency data on S&P CNX Nifty, a value-weighted stock index of National Stock Exchange (NSE), Mumbai. S&P CNX Nifty is constructed from the prices of 50 large capitalization stocks. The NSE started compiling high frequency data for research purposes since 1999. Our data set covers the period of January 1999-December 2001, i.e., the first three years for which NSE has high frequency data. In order to make full use of high frequency data to estimate 'realized volatility' for testing the forecasting ability of conditional volatility based on out-of-the-sample volatility forecasts, we also use daily (low frequency) data on S&P CNX Nifty for the period 1st January 1996 to 31st December 1998. In addition, we also use low-frequency daily data for the period 1st January 1996 to 31st December 2001 for forecasting conditional volatility on the basis of model fitted using data on a rolling basis. The forecast of conditional volatility for the first day for which realized volatility estimate is available, i.e., 1st January 1999, is based on the models estimated for the period 1st January 1996 to 31st December 1998 and for forecasts of later periods and days, additional data are used. Similarly, for estimation of conditional volatility, we estimate conditional volatility models based on daily data for the period 1st January 1999 to 31st December 2001, the period which coincides with the period for which the realized volatility estimates are computed. We also do estimation using daily data on S&P CNX Nifty for the period 1st January 1996 to 31st December 2002, some part of which falls outside the period for which realized volatility has been measured and compared. The latter is simply to check the robustness of results. As pointed out earlier, the use of different period data sets for the conditional volatility models results in changes in the parameters (ω, α, β and γ for GARCH(1,1) and EGARCH(1,1)) of the model fitted over the data used but actual estimates and forecasts are contingent on the return characteristics (σ^2_{t-1}, ε_{t-1}, and $|\varepsilon_{t-1}|$) as well. The descriptive statistics of the entire returns series used, i.e., 1st January 1996 to 31st December 2002, and its parts, 1st January 1996 to 31st December 1998, 1st January 1999 to 31st December 2001, are given in Table 1a.

As Table 1a shows, the index had a small negative average return in the first sub-period and a small positive average return during the second period. The standard deviation of daily return is of the order of 1.7 per cent in both the sub-periods, implying average annualized volatility of around 27 per cent. The kurtosis of daily returns in each of the period is higher than three, the kurtosis of Gaussian distribution. It is, however, closer to normal in the second sub-period. The Jarque-Bera test for normality of returns distribution yield statistics much greater than any critical value at conventional confidence levels in both the sub-periods.

Time Dependence in Daily Index Returns and Volatility

In order to ascertain time-dependence in daily returns, we compute autocorrelation coefficients for up to 36 lags for each of the three series and associated Ljung-Box Q statistic. The first-order correlation coefficient at 0.050 for the entire data set is significant at 5 per cent level. The partial and autocorrelation coefficients for lag up to five are given in Table 1a.

For the entire data set, the autocorrelation coefficients* (simple and partial) associated with 6th, 10th, 18th, and 26th lags are also significant with the maximum absolute value at 0.069 (pac at 18th lag). As pointed out earlier, we use the entire data set only for one set of estimates of conditional volatility models; the other sets are based on the period for which the high frequency data were available, i.e., between January 1999 and December 2001. In the time-series for the period January 1996-December 1998, the autocorrelation coefficients associated with 18th lag are significant with values of –0.100 and –0.108 respectively. In the other series of returns between January 1999-December 2001, significant auto-correlation coefficients are associated with 23rd lag with values of 0.086 and 0.091 respectively. Using Schwarz's Information Criterion (SIC) for deciding the order of ARMA (p,q) models, we find that SIC is minimized for p=0, q=0 for the entire return series for $p\in$ [1, 6] and $q\in$ [1, 6]. Lags of up to order six were explored in case there are any settlement cycle[11] induced autocorrelations in time-series. As we have not explicitly modeled any calendar or settlement cycle effects in the return series, henceforth we model return series without incorporating autoregressive and moving average processes.

A more general test for time-based dependence in the returns series, due to Brock *et al.* (1996), viz., BDS test statistics, was also conducted for epsilon ranging from 0.5 to 2 times of standard deviation and embedding dimensions up to three (for the sub-sets of the entire time-series) and five (for the entire time-series). This test can detect a variety of departures from randomness including non-linear dependence and deterministic chaos. The BDS test strongly rejects the null hypothesis of randomness in the Nifty return series in each of the data set. One possible reason for the non-randomness in the returns series is attributed to predictability of volatility or autocorrelation in volatility.

We provide descriptive statistics of the squared return series of each of the periods in Table 1b. In order to check the presence of volatility clustering, we compute auto and partial autocorrelation functions of the squared return series for the entire data set (part of the same is reported in Table 1b). The Ljung-Box Q statistic is significant at 1 per cent level for up to 36 lags in the entire data set. This confirms volatility clustering in the Indian markets, just as it has been found and reported in case of other markets. In the first sub-period of January 1996 to December 1998, the first order autocorrelation of squared returns, though significant at 5 per cent, is not as high as in the remaining data set.

The use of ARCH/GARCH-type conditional volatility models is motivated by the presence of volatility clustering and time-varying characteristics of volatility. In order to test the presence of 'ARCH effect,' we compute and report the F-statistics and the LM-statistics associated with ARCH-LM test on each data set of Nifty

Table 1a: Descriptive Statistics–S&P CNX Nifty Daily Returns

	1st Jan 1996-31st Dec 1998	1st Jan 1999-31st Dec 2001	1st Jan 1996-31st Dec 2002
Observations	743	753	1747
Mean	-0.000036	0.000240	0.000106
Median	-0.000625	0.000625	0.000207
Maximum	0.099339	0.075394	0.099339
Minimum	-0.088405	-0.077099	-0.088405
Std. Dev.	0.017043	0.018416	0.016901
Skewness	0.154152	-0.134935	0.003877
Kurtosis	6.902943	4.928598	6.114683
Jarque-Bera	474.5297	118.9840	706.1745
Probability	0.000000	0.000000	0.000000
Autocorr.-(1)	0.050	0.059	0.050*
Ljung-Box Statistic	(1.8996)	(2.6429)	(4.3963*)
Autocorr.-(2)	-0.003	-0.039	-0.023
Ljung-Box Statistic	(1.9083)	(3.7762)	(5.3323)
Autocorr. (3)	0.033	-0.014	0.010
Ljung-Box Statistic	(2.7446)	(3.9284)	(5.5226)
Autocorr. (4)	0.015	0.035	0.033
Ljung-Box Statistic	(2.9056)	(4.8800)	(7.3954)
Autocorr. (5)	0.023	0.015	0.020
Ljung-Box Statistic	(3.3157)	(5.0419)	(8.0639)

Table 1b: Squared S&P Nifty Returns

	1st Jan 1996-31st Dec 1998	1st Jan 1999-31st Dec 2001	1st Jan 1996-31st Dec 2002
Mean	0.000290	0.000339	0.000285
Median	0.000086	0.000106	0.000084
Maximum	0.009868	0.005944	0.009868
Minimum	0.000000	0.000000	0.000000
Std. Dev.	0.000705	0.000671	0.000646
Skewness	7.802061	4.398913	6.572535
Kurtosis	83.05484	27.85253	64.98853
Autocorr.-(1)	0.088	0.247	0.174
Ljung-Box Statistic	(5.7139*)	(46.247**)	(52.718**)
Autocorr. (2)	0.085	0.084	0.097
Ljung-Box Statistic	(11.169**)	(51.623**)	(69.218**)
Autocorr. (3)	0.018	0.090	0.064
Ljung-Box Statistic	(11.418**)	(57.744**)	(76.471**)
Autocorr. (4)	-0.010	0.046	0.030
Ljung-Box Statistic	(11.488*)	(59.330**)	(78.063**)
Autocorr. (5)	0.124	0.085	0.115
Ljung-Box Statistic	(23.050**)	(64.869**)	(101.32**)

* Significant at 5% level

** Significant at 1% level.

* Not reported in this paper for brevity.

Table 1c: ARCH-LM Test Statistics on Nifty Daily Returns

	1st Jan 1996-31st Dec 1998	1st Jan 1999-31st Dec 2001	1st Jan 1996-31st Dec 2002
F-Statistics	4.491289**	11.34598**	17.10076**
LM-Statistics	21.96662**	53.12680**	81.77179**

** Significant at 1% level.

daily returns in Table 1c. While computing these, we use the residuals of OLS residuals of the daily returns regressed on a constant. The number of lags included is five in the results reported in Table 1c. The results in Table 1c indicate the presence of 'ARCH effect' in the Nifty daily return series in each of the data set. Increasing the number of lags for the test (not reported here) did not affect the results reported here.

To sum up, our analysis indicates that the daily return series of the index is non-normal and exhibits 'ARCH effect.'

Conditional Volatility Models

Symmetric Conditional Volatility Model: GARCH

As pointed out, the return series of the index exhibits ARCH effect in all the periods studied. We, therefore, use GARCH (p, q) model, the most popular member of the ARCH class of models, to model volatility of Nifty returns. We use EViews[12] software for model estimation. Using the Schwarz Information Criterion, we find that the best model in the GARCH (p, q) class for $p \in [1, 5]$ and $q \in [1, 5]$ is GARCH (1, 1). The results from the model estimated for different periods are reported in Table 2a. The sum of ARCH and GARCH coefficients (α and β respectively) estimated by the model is close to 0.9 in both the sub-periods as well for the entire data set. However, during the 1999-2001 period, the volatility in Indian capital markets was spikier (higher α) and less persistent (lower β) than the 1996-1998 period and the entire data set. The sum of coefficients being significantly less than one indicates that volatility is mean reverting. The coefficients of the estimated GARCH (1, 1) models are significant as can be seen from the z-statistics reported in Table 2a. This inference from the z-statistics as reported in the table is valid only if errors are conditionally normally distributed. Table 2a also reports the descriptive statistics of residuals from the estimated models. Standardized residuals from estimated GARCH models in each of the period are not normally distributed as indicated by the Jarque-Bera statistic. The standard errors (and, therefore, associated z-statistics) computed under the assumption of conditionally normally distributed error terms are not consistent if the errors are not normally distributed. As can be seen from the descriptive statistics related to standardized residuals given in Table 2a, the standardized residuals are non-normal. However, Bollerslev and Woolridge (1992) provided a method for obtaining consistent and robust estimates of the standard errors. The z-statistics computed following the Bollerslev and Woolridge procedure (not reported here) for each of these models is generally lower and is also insignificant for a few parameters (GARCH term β is significant in all the three data sets) as compared to the ones reported in Table 2a. In particular, the z-statistics falls below two for the constant term in variance equation for all the three data sets and is between two and three for the ARCH term (α) for the periods 1999-2001 and 1996-2002. The ARCH term in the data set of 1996-1998 period is below two and hence insignificant.

In order to test whether the GARCH (1, 1) model has adequately captured the persistence in volatility and there is no ARCH effect left in the residuals from the models, ARCH-LM test was conducted for lags up to five. The tests (not reported here) indicate that the standardized residuals do not exhibit any ARCH effects.

Asymmetric Conditional Volatility Model: EGARCH

Conditional volatility of returns may not only be dependent on the magnitude of error terms or innovations but also on its sign. In order to test for asymmetries in volatility, we compute cross-correlation between the squared residuals of the GARCH (1, 1) model and the lagged residuals. In the presence of the asymmetry of conditional volatility, these correlations should be negative. As shown in Table 2b, the cross-correlation for the entire data set as well as for the period 1999-2001 is significant for up to lags of three.

Since there is asymmetry in volatility in the period used for comparing performance of various estimators and models with the realized volatility, i.e., January 1999 to December 2001, we estimate EGARCH (1,1) models for each of the three periods. The results for estimated EGARCH (1,1) model are reported in Table 2c. The results are consistent with the test for asymmetry in conditional volatility as reported in Table 2b. The asymmetry term, γ, is insignificant in the period January 1996 to December 1998, but is significant in the other sub-period as well as for the entire data set. The other coefficients are significant at conventional significance levels. The Bollerslev-Woolridge robust standard errors (not reported here) are higher and z-statistic lower than under the assumption of conditionally normally distributed error terms. Like in the case of GARCH model, the standardized residuals from the estimated models are not normally distributed. The ARCH-LM test on residual (not reported here) indicates that there is no ARCH effect left after estimating the model. While the insignificance of asymmetry term γ for the period 1996-1998 does not affect the evaluation of EGARCH model for estimation, it does affect its evaluation for its forecasting ability as the period between 1999-2001 over which the forecasts are made, this term is large and significant. For evaluation of models for their forecasting ability, we have used the period of 1996-1998 for estimation of the model which then has been used to predict out-of-sample (1999-2001) volatility. This constraint was due to availability of high frequency data only after 1999.

In this paper, we use only these two commonly used conditional volatility models from the class of ARCH/GARCH type models to test their performance *vis-à-vis* traditional and extreme value unconditional volatility estimators.

Close-to-close Market Variance Estimates and Extreme Value Estimators

In conditional volatility models and traditional volatility estimators, using closing daily prices does not pose any problems. However, as pointed out elsewhere in the paper, extreme value estimators prior to Yang and Zhang (2000) did not take the closed-market variance (between the closing prices of the previous day and opening prices) into account. Similarly, the realized volatility measure, in the absence of continuous trading markets, is essentially a measure of open-market variance of volatility. In order to compare, therefore, some of the extreme value estimators and realized volatility measures need to be modified for estimating close-to-close market variance. In the absence of any observation for the period during which market is closed, treatment of the closed-market variance, however, has to be alike for all the estimators. For incorporating the closed-market variance in such estimators, we use traditional unadjusted estimator, as given in equation (1), for one-day period and traditional mean-adjusted estimator, as given in equation (2), for longer periods. The closed-market variance is computed using close-to-open returns.

Realized Volatility: Descriptive Statistics

In computing volatility measures for the chosen index (S&P CNX Nifty), we faced measurement problems due to trading breaks. On quite a few days during the period, trading was stopped (and later resumed) at NSE because of communication and operational reasons. Since the

Table 2a: Results from GARCH (1,1) Model

	1st Jan 1996-31st Dec 1998	1st Jan 1999-31st Dec 2001	1st Jan 1996-31st Dec 2002
Constant (Mean eq.)	-0.000092 (-0.139151)	0.001097 (1.783237)	0.000386 (1.134965)
ω	0.000028 (4.124338**)	0.000033 (3.857869**)	0.000016 (7.086951**)
α	0.056814 (4.117929**)	0.153048 (5.602553**)	0.102514 (9.857610**)
β	0.846745 (33.39316**)	0.756544 (19.11235**)	0.847406 (84.94265**)
Annualized long-run volatility implied by ω, α, and β	27.13%	29.98%	27.82%
Standardized Residuals: Descriptive Statistics			
Mean	9.43E-05	-0.049790	-0.023647
Std. dev.	0.999903	0.999320	0.999791
Skewness	0.011157	-0.044657	-0.043485
Kurtosis	7.262294	4.692043	6.204649
Jarque-Bera	562.4400	90.07721	748.1046
Probability	0.000000	0.000000	0.000000

Table 2b: Cross-correlation of Squared Residuals from GARCH (1,1) Model with the Lagged Residuals

	1st Jan 1996-31st Dec 1998	1st Jan 1999-31st Dec 2001	1st Jan 1996-31st Dec 2002
Lag=1	-0.04226	-0.10034**	-0.07240**
Lag=2	-0.06436	-0.07691*	-0.06780**
Lag=3	-0.01743	-0.08322*	-0.05107*

Table 2c: Results from EGARCH (1,1) Model

	1st Jan 1996-31st Dec 1998	1st Jan 1999-31st Dec 2001	1st Jan 1996-31st Dec 2002
Constant (Mean eq.)	-0.000339 (-0.499085)	0.000424 (0.686293)	4.56E-05 (0.117468)
ω	-0.943289 (-2.982314**)	-1.158396 (-4.752127**)	-0.756000 (-7.896799**)
α	0.133435 (5.769960**)	0.272820 (6.661729**)	0.212420 (13.76127**)
β	0.896355 (23.06698**)	0.882940 (31.20140**)	0.927577 (80.11228**)
γ	-0.030392 (-1.699621)	-0.118988 (-4.542789**)	-0.064712 (-5.346620**)
Standardized Residuals: Descriptive Statistics			
Mean	0.015696	-0.008306	0.003416
Std. dev.	0.999373	1.001506	0.999910
Skewness	0.005987	0.015417	-0.024480
Kurtosis	7.412521	4.560810	6.420721
Jarque-Bera	602.7737	76.46337	851.9340
Probability	0.000000	0.000000	0.000000

Note: Figures in parentheses are z-statistics associated with coefficients.
* Significant at 5% level.
** Significant at 1% level.

extreme value estimators and the traditional estimator are based on extreme values and closing prices are reported for the entire day, we use the squared return series even if there are trading breaks. In other words, the returns between the trading breaks are treated as if they are 5-minute returns. This is likely to introduce measurement errors in the realized volatility measure and make them slightly downward-biased. The extreme-value estimators also suffer from a similar bias in case of trading breaks, as pointed out earlier.

The descriptive statistics of daily realized volatility during the period January 1999-December 2001 is given in Table 3. For making comparisons with Table 1b easy, the descriptive statistics in Table 3 is reported for the variance rather than volatility. As is clear from the comparison, the mean of realized daily variance is slightly higher than the squared returns. On the other hand, the standard deviation of realized daily variance is lower. As opposed to squared return series, the autocorrelation in realized variance decays slowly. For the period 1999-2001, the auto correlation at first lag is somewhat similar in magnitude in both the cases, but for lags between 2 and 5, the autocorrelation coefficients drop considerably in case of squared returns, whereas, they drop extremely gradually in case of realized daily variance series indicating greater 'volatility clustering or persistence.' On closer examination, we find that partial autocorrelation for lags 2 to 5 are considerably lower in case of squared return series. The mean daily realized variance implies annualized volatility of around 31 per cent. The volatility during this period was slightly higher than the average long run volatility as the capital markets in India were volatile during this period (driven by boom in technology, telecom, and media stocks) as is evident from Table 1c.

Table 3: Descriptive Statistics: Daily Realized Variance of S&P CNX Nifty

	Jan 1999-Dec 2001
Observations	737
Mean	0.000383
Median	0.000208
Maximum	0.007979
Minimum	0.000019
Std. dev.	0.000589
Skewness	5.899638
Kurtosis	57.08921
Autocorr.-(1) Ljung-Box Statistic	0.261 (50.248**)
Autocorr. (2) Ljung-Box Statistic	0.245 (94.743**)
Autocorr. (3) Ljung-Box Statistic	0.237 (136.25**)
Autocorr. (4) Ljung-Box Statistic	0.255 (184.67**)
Autocorr. (5) Ljung-Box Statistic	0.192 (211.98**)

** Significant at 1% level.

Performance Criteria for Evaluation of Estimators and Models

In order to compare bias and efficiency of various estimators and models for estimation, we use the following finite sample criteria:

- bias of the estimator
- mean square error of the estimator
- relative bias of the estimator
- mean absolute error of the estimator.

The first and the second criteria measure bias and efficiency respectively and are standard measures. The third criterion is to assess the magnitude of bias with respect to the true parameter (realized volatility measure in this case) as the first criterion gives only absolute amount. The fourth criterion is another measure of efficiency like the second one but is less likely to be affected by the presence of outliers in the data set.

If true volatility (realized volatility) on day t is σ_t and the estimated volatility given by an estimator or model is σ_{est}, then the four performance criteria are computed as under:

$$\text{Bias} = E\,(\sigma_{est} - \sigma_t)$$

$$\text{Mean square error (MSE)} = E\,[(\sigma_{est} - \sigma_t)^2\,]$$

$$\text{Relative bias} = E\,[(\sigma_{est} - \sigma_t)/\,\sigma_t]$$

$$\text{Mean absolute error (MAE)} = E\,[Abs\,(\sigma_{est} - \sigma_t)]$$

For forecasting, we use h-period volatility estimates of 'unconditional volatility' estimators for forecasting volatility h-period ahead. In case of conditional models, we forecast based on model parameters estimated from the period outside the period of study, i.e., 1st January 1996 to 31st December 1998. In case of GARCH model, we also report results based on estimation of model on a rolling basis. For example, for forecasting volatility on 1st January 1999, we use the model estimated on daily data from 1st January 1996 to 31st December 1998. In case of forecast for 2nd January 1999, we re-estimate the model parameters from data of period 1st January 1996 to 1st January 1999 and so on. For evaluating the forecasts given by the models, we use the same criteria as we do for estimation. We use the term 'forecast error' in place of 'bias' in the context of evaluating predictive power of estimators and models. In addition to forecast error, mean square forecast error, relative forecast error, and mean absolute forecast error, we also report the results of OLS regressions of the realized volatility on a constant and forecast value given by the various models and estimators following one of the approaches used by Day and Lewis (1992) to test out-of-sample predictive power of volatility models. We also report the results of the other approach followed by them, i.e., forecast encompassing regressions based on a procedure due to Fair and Shiller (1990). Following this approach, we regress realized volatility on a constant and the forecast values obtained from different models and estimators.

EMPIRICAL RESULTS

In this paper, we analysed the empirical performance of the volatility models and estimators *vis-à-vis* the realized volatility measure for the S&P CNX Nifty stock index in terms of (a) estimation, and (b) predictive power. Accordingly, we report the results separately on bias and efficiency of these models and estimators for estimation and for their out-of-sample predictive power.

Volatility Models and Estimation

Estimation of volatility from data over a given horizon is important for researchers and practitioners alike. In order to test empirical performance for estimation, we compute volatility estimates from unconditional estimators and conditional volatility models and compare them with the realized volatility in Table 4. For comparison, we use three time periods of one-day, non-overlapping five-days, and calendar months. We chose these horizons partly because we had only three years' high frequency data and partly because shorter horizons are more likely to be used in the case of time-varying volatility, particularly for unconditional estimators. While estimating conditional volatility estimates using GARCH and EGARCH models, we report the results based on the estimates from the sub-period (in sample) of January 1999-December 2001 as well as from the estimates of the entire data set (complete data set from January 1996-December 2002 including the sample period). In case of traditional estimators, two estimates are reported. The first one is based on separate adjustment for the closed market variance while the second one is more commonly used and is based on close-to-close daily returns.

In panel A of Table 4, we report the results for one-day period. While volatility estimates given by the GARCH/EGARCH models exhibit lower bias than traditional and extreme value estimators, the extreme value estimators given by Garman-Klass and Rogers-Satchell exhibit lower relative bias and higher efficiency in terms of both MSE and MAE criteria. In case of five-day period as reported in panel B, the results are similar. Garman-Klass, Rogers-Satchell, and Yang-Zhang extreme value estimators perform well compared to the GARCH/EGARCH estimates on relative bias, MSE, and MAE criteria despite exhibiting higher bias. For results on one-month (calendar month) period, as reported in panel C, these three estimators perform well on both efficiency criteria but exhibit higher absolute and relative bias than conditional volatility models. The extent of bias in case of these three extreme value estimators as well as conditional volatility models increases with the increase in horizon. Even then, the bias in terms of annualized volatility is less than 1 per cent for one-day period, around 2 per cent for five-day periods, and less than 3 per cent for one-month period. In case of conditional volatility models, it is about 1 per cent less than extreme-value estimators for all horizons. The bias exhibited by Parkinson estimator is exceptionally high.

Higher efficiency of extreme value estimators compared to traditional estimators and conditional volatility models in the Indian capital markets is in line with the earlier findings of Pandey (2002) wherein extreme value estimators were compared with traditional estimators. Higher efficiency of extreme value estimators in comparison with conditional volatility models is somewhat surprising though. This could be because while conditional volatility models have been estimated using daily closing prices, the extreme value estimators take into account intra-day information on prices, similar to realized volatility measure, the benchmark used for making comparisons. These results are also similar to some of the recent works discussed earlier (Li and Weinbaum, 2000; Spurgin and Schneeweis, 1999).

Predictive Power of Volatility Models

Besides estimation, the other important application and use of unconditional and conditional volatility models is forecasting volatility. In case of unconditional estimators, generally h-period volatility estimates are used to forecast h-period volatility ahead. For evaluating the predictive power, we use estimates over a given horizon as the forecast for next horizon of equal length and compare it with the realized volatility next period. In case of volatility models, the forecasts are based on the model estimated on out-of-sample data and forecasts are

Table 4: Performance of Volatility Models (and Estimators) for Estimation

Panel A: One-day Period (Number of Observations = 737)

Model/Estimator	Bias	Relative Bias	Mean Square Error	Mean Absolute Error
Traditional Cl-O-Cl✦	-0.003258	-0.193098	0.000103	0.007680
Traditional Cl-Cl✦	-0.003505	-0.193992	0.000126	0.008435
Parkinson	-0.005915	-0.340108	0.000062	0.006031
Garman-Klass	-0.001502	-0.065399	<u>0.000028</u>	<u>0.003543</u>
Rogers-Satchell	-0.001475	<u>-0.064936</u>	0.000038	0.003962
GARCH (1,1)- In sample	0.000681	0.228127	0.000071	0.005854
GARCH (1,1)- Complete	0.001092	0.257060	0.000070	0.005964
EGARCH-In sample	<u>0.000450</u>	0.213318	0.000069	0.005718
EGARCH-Complete	0.000903	0.242548	0.000070	0.005840

Panel B: Five-day Period* (Number of Observations = 147)

Model/Estimator	Bias	Relative Bias	Mean Square Error	Mean Absolute Error
Traditional Cl-Cl✦	-0.002295	-0.132712	0.000026	0.003847
Traditional Cl-O-Cl✦	-0.001673	-0.099923	0.000021	0.003443
Traditional adj. Cl-Cl✦	-0.002190	-0.132206	0.000032	0.004329
Traditional adj.Cl-op-Cl✦	-0.002009	-0.122123	0.000031	0.004104
Parkinson	-0.006173	-0.340889	0.000052	0.006173
Garman-Klass	-0.001582	-0.076001	0.000011	0.002247
Rogers-Satchell	-0.001311	<u>-0.060231</u>	0.000012	0.002349
Yang-Zhang	-0.001399	-0.067837	<u>0.000010</u>	<u>0.002118</u>
GARCH (1,1)- In sample	-0.000162	0.090241	0.000032	0.004098
GARCH (1,1)- Complete	0.000280	0.117163	0.000029	0.004028
EGARCH-In sample	-0.000397	0.078539	0.000032	0.004087
EGARCH-Complete	<u>0.000097</u>	0.106139	0.000028	0.004057

Panel C: Calendar Month* (Number of Observations = 36)

Model/Estimator	Bias	Relative Bias	Mean Square Error	Mean Absolute Error
Traditional Cl-Cl✦	-0.001371	-0.067712	0.000010	0.002362
Traditional Cl-O-Cl✦	-0.001328	-0.067828	0.000009	0.002244
Traditional adj. Cl-Cl✦	-0.001294	-0.063739	0.000010	0.002282
Traditional adj.Cl-op-Cl✦	-0.001205	-0.061844	0.000009	0.002144
Parkinson	-0.006420	-0.343027	0.000049	0.006420
Garman-Klass	-0.001717	-0.085808	0.000005	0.001843
Rogers-Satchell	-0.001440	-0.071387	0.000005	0.001716
Yang-Zhang	-0.001413	-0.070288	<u>0.000004</u>	<u>0.001553</u>
GARCH (1,1)- In sample	-0.000620	0.023725	0.000015	0.002762
GARCH (1,1)- Complete	-0.000131	0.054742	0.000014	0.002718
EGARCH-In sample	-0.000858	<u>0.014670</u>	0.000016	0.002893
EGARCH-Complete	<u>-0.000300</u>	0.046885	0.000014	0.002933

✦ Traditional Cl-op-cl estimator is based on the sum of closed market and open market squared returns whereas traditional Cl-Cl estimator is based on close-to-close squared returns. Similarly, traditional adjusted Cl-Cl estimator is estimated using close-to-close returns whereas in case of traditional adjusted Cl-op-Cl estimator, open and closed variances are separately measured and added to arrive at daily variance/ volatility.

* The volatility estimates for these comparisons are based on average daily volatility estimated over the relevant period and have not been annualized. For converting them in percentage annualized volatility, the volatility needs to be multiplied with $(N)^{1/2} * 100$ where N is approximately 250. The same factor will also scale up the reported bias and mean absolute error while relative bias will remain unaffected. The mean square error needs to be scaled up by multiplying with N instead of its square root.

obtained for different length of periods. We use data from the period 1st January 1996 to 31st December 1998 to estimate GARCH (1, 1) and EGARCH model for forecasting. As pointed out earlier, we also forecast on the basis of rolling estimation of GARCH model by successively estimating the model to include the data just prior to the forecast period.

The results for one-day ahead forecast performance are reported in panel A of Table 5. Among the various estimators, conditional volatility models (GARCH and EGARCH) perform well for one-day period on all parameters except relative forecast errors. However, as can be seen from results for five-day and one-month period in panel B and C, extreme value estimators perform as well for these horizons on both efficiency criteria (mean square forecast error and mean absolute forecast error). The relative forecast error for these estimators is also lower than the conditional volatility models even though mean forecast errors are higher.

In order to test the ability of the estimators and models to forecast volatility, we also regress realized volatility on the forecasted value given by each model and a constant following Day and Lewis (1992). The specification of the OLS regression is given by:

$$\sigma^2_{t+1} = \beta_0 + \beta_1 \sigma^2_{ft} + \varepsilon_{t+1} \quad (14)$$

where,

σ^2_{t+1} = actual value of 'realized variance' at time $t+1$

σ^2_{ft} = value forecasted for the realized variance of time $t+1$ at time t

In case the forecasts are accurate, we would expect value of β_0 to be 0 and that of β_1 to be equal to 1. The sign and magnitude of coefficients and R-squared values of these regressions, therefore, can be interpreted to assess the predictive power of various models and

Table 5: Predictive Power of Volatility Models and Estimators

Panel A: One-day Period (Number of Observations = 736)

Model/Estimator	Mean Forecast Error	Relative Forecast Error	Mean Square Forecast Error	Mean Absolute Forecast Error
Traditional Cl-O-Cl✦	-0.003495	-0.158813	0.000157	0.009154
Traditional Cl-Cl✦	-0.003743	-0.156675	0.000173	0.009755
Parkinson	-0.006151	-0.296215	0.000120	0.007392
Garman-Klass	-0.001739	-0.005943	0.000102	0.006353
Rogers-Satchell	-0.001713	0.004317	0.000121	0.007058
GARCH (1,1)	0.000274	0.243228	0.000083	0.006269
GARCH (1,1)- Rolling	0.000634	0.259618	0.000081	0.006297
EGARCH	0.000253	0.239605	0.000083	0.006196

Panel B: Five-day Period✦ (Number of Observations = 146)

Model/Estimator	Mean Forecast Error	Relative Forecast Error	Mean Square Forecast Error	Mean Absolute Forecast Error
Traditional Cl-Cl*	-0.002295	-0.077617	0.000076	0.006309
Traditional Cl-O-Cl*	-0.001669	-0.038811	0.000074	0.006304
Traditional adj. Cl-Cl*	-0.002166	-0.060767	0.000088	0.007045
Traditional adj.Cl-op-Cl*	-0.001985	-0.051693	0.000087	0.006814
Parkinson	-0.006181	-0.302628	0.000084	0.006947
Garman-Klass	-0.001589	-0.028514	0.000050	0.005045
Rogers-Satchell	-0.001320	-0.012849	0.000050	0.005071
Yang-Zhang	-0.001404	-0.017729	0.000050	0.005076
GARCH (1,1)	-0.000465	0.122342	0.000052	0.005242
GARCH (1,1)- Rolling	-0.000034	0.144828	0.000051	0.005295
EGARCH	-0.000407	0.125388	0.000052	0.005256

Panel C: Calendar Month (Number of Observations = 35)

Model/Estimator	Mean Forecast Error	Relative Forecast Error	Mean Square Forecast Error	Mean Absolute Forecast Error
Traditional Cl-Cl✦	-0.001225	0.008519	0.000062	0.005938
Traditional Cl-O-Cl✦	-0.001178	0.004903	0.000057	0.005687
Traditional adj. Cl-Cl✦	-0.001156	0.011293	0.000061	0.005915
Traditional adj. Cl-op-Cl✦	-0.001062	0.010622	0.000057	0.005711
Parkinson	-0.006333	-0.299390	0.000075	0.006540
Garman-Klass	-0.001608	-0.028542	0.000040	0.004855
Rogers-Satchell	-0.001345	-0.014075	0.000040	0.004831
Yang-Zhang	-0.001311	-0.010940	0.000041	0.004914
GARCH (1,1)	-0.001126	0.050896	0.000044	0.004854
GARCH (1,1)- Rolling	-0.000534	0.086979	0.000044	0.004940
EGARCH	-0.000949	0.063854	0.000044	0.005043

✦ As cited in Table 4.

* As cited in Table 4.

Table 6: Results of OLS Regression on Predictive Power of Volatility Models and Estimators

Panel A: One-day Period (Number of Observations = 736)

Model/Estimator	β_0	β_1	R-Squared
Traditional Cl-O-Cl✦	0.000284	0.340427	0.138275
	(11.76185)	(10.85265)	
Traditional Cl-Cl✦	0.000284	0.346103	0.125434
	(11.56929)	(10.26028)	
Parkinson	0.000255	0.864431	0.10449
	(9.414865)	(9.254452)	
Garman-Klass	0.000257	0.448345	0.085098
	(9.099892)	(8.262659)	
Rogers-Satchell	0.000297	0.308335	0.049506
	(10.5897)	(6.183036)	
GARCH (1,1)	-0.00037	2.45939	0.158784
	(-5.39779)	(11.77057)	
GARCH (1,1)-Rolling	-0.00021	1.863083	0.145622
	(-3.64259)	(11.18503)	
EGARCH	-0.00042	2.631491	0.149532
	(-5.59641)	(11.36021)	

Panel B: Five-day Period (Number of Observations = 146)

Model/Estimator	β_0	β_1	R-Squared
Traditional Cl-O-Cl✦	0.000262	0.366746	0.141589
	(6.727447)	(4.873579)	
Traditional Cl-Cl✦	0.000266	0.375357	0.144801
	(7.011435)	(4.937798)	
Traditional adj. Cl-Cl✦	0.000277	0.332819	0.112006
	(7.053345)	(4.261833)	
Traditional adj. Cl-op-Cl✦	0.000293	0.277347	0.099136
	(7.678961)	(3.980774)	
Parkinson	0.000184	1.219383	0.233093
	(4.449891)	(6.615672)	
Garman-Klass	0.000158	0.720093	0.289836
	(3.943864)	(7.666167)	
Rogers-Satchell	0.00016	0.687434	0.293897
	(4.04632)	(7.741847)	
Yang-Zhang	0.000153	0.718399	0.296900
	(3.818194)	(7.797907)	
GARCH (1,1)	-0.00024	2.024221	0.173425
	(-2.05022)	(5.496624)	
GARCH (1,1)-Rolling	-0.00009	1.469621	0.149816
	(-0.9534)	(5.037380)	
EGARCH	-0.00031	2.226531	0.180424
	(-2.43191)	(5.630326)	

Panel C: Calendar Month (Number of Observations = 35)

Model/Estimator	β_0	β_1	R-Squared
Traditional Cl-O-Cl✦	0.000270	0.348629	0.093451
	(3.299997)	(1.844389)	
Traditional Cl-Cl✦	0.000289	0.294417	0.063213
	(3.440199)	(1.492245)	
Traditional adj. Cl-Cl✦	0.000288	0.293773	0.065862
	(3.464967)	(1.525348)	
Traditional adj. Cl-op-Cl✦	0.000271	0.338724	0.093691
	(3.349638)	(1.847000)	
Parkinson	0.000171	1.308826	0.255559
	(2.16143)	(3.365789)	
Garman-Klass	0.000140	0.778776	0.334506
	(1.87633)	(4.072741)	
Rogers-Satchell	0.000139	0.756023	0.359427
	(1.928762)	(4.303069)	
Yang-Zhang	0.000147	0.730613	0.323700
	(1.961677)	(3.974285)	
GARCH (1,1)	-0.00010	1.60115	0.046000
	(-0.25220)	(1.26143)	
GARCH (1,1)-Rolling	0.00007	0.972845	0.026216
	(0.211446)	(0.942551)	
EGARCH	-0.00030	2.234786	
	(-0.60958)	(1.397152)	0.055849

Note: Figures in parentheses are t-statistic associated with the coefficient.

✦As cited in Table 4.

estimators. We report the values of coefficients, associated *t*-statistics, and *R*-squared values of these regressions in Table 6. The forecasted values in regressions based on equation (14) are as given by different models and estimators. In panel A, B, and C of the table, we report regressions for one-day period, five-day period, and one-month period forecasts respectively. As can be seen from panel A, the values of β_0 in all the regressions are significantly different from zero. In terms of *R*-squared values, conditional volatility models perform the best though only slightly better than the traditional estimator. In case of five-day and monthly forecasts however, the extreme value estimators (Garman-Klass, Rogers-Satchell, and Yang-Zhang) perform well. In contrast, the conditional volatility models perform extremely poorly on monthly forecasts. This is expected to some extent as the forecasts by conditional volatility models are extremely sensitive to recent volatility and error term by construction. Volatility forecasts by conditional volatility models far out into the future are likely to result in considerable error if the volatility is not as persistent as estimated by them. Despite significant volatility persistence observed in the squared Nifty returns series and in realized volatility series, the latter being more persistent than the former, the forecasting power of conditional volatility models is not vastly greater than the traditional estimator. Another plausible reason for the poor performance of conditional volatility models over longer horizons could be due to improper modeling of long-run dependencies in volatility (Andersen and Bollerslev, 1998) and may require using FIGARCH model (Baillie, Bollerslev and Mikkelsen, 1996).

We also performed forecast encompassing regression, similar to Day and Lewis (1992), wherein different set of forecasts given by different models and estimators are used as independent variables to test whether they contain different sets of information from each other. The results for forecasts of one-day, five-day, and one-month periods are given in panel A, B, and C of Table 7. For forecast encompassing regression, we chose forecasts of the best traditional estimator from among the traditional estimators, two best performing extreme value estimators for each horizon from the class of extreme value estimators, and the GARCH and EGARCH forecasts. The specification used in the regression is given by:

$$\sigma^2_{t+1} = \beta_0 + \beta_1\,\sigma^2_{Tt} + \beta_2\,\sigma^2_{E1t} + \beta_3\sigma^2_{E2t} + \beta_4\,\sigma^2_{Et} + \beta_5\,\sigma^2_{EGt} + \varepsilon_{t+1} \qquad (15)$$

where,

σ^2_{t+1} = actual value of 'realized variance' at time $t+1$

σ^2_{Tt} = value forecasted for the realized variance of time $t+1$ at time t by the best traditional estimator

σ^2_{E1t} = value forecasted for the realized variance of time $t+1$ at time t by the best extreme value estimator

σ^2_{E2t} = value forecasted for the realized variance of time $t+1$ at time t by the second-best extreme value estimator

σ^2_{Gt} = value forecasted for the realized variance of time $t+1$ at time t by GARCH (1, 1) model

σ^2_{EGt} = value forecasted for the realized variance of time $t+1$ at time t by EGARCH model

As is evident from panel A of Table 7, the results from forecast encompassing regressions are in line with the results discussed earlier. In case of one-day period, the GARCH forecasts and traditional estimators have significant coefficients and, put together, they forecast realized volatility better. Both the extreme value estimators (Parkinson and Garman-Klass) for one-day period perform poorly. In case of five-day period, however, only extreme value estimators (Yang-Zhang and Rogers-Satchell) have significant coefficients and predictive power. In the results of regression using both the extreme value estimators and other forecasts, the coefficients for both are insignificant as the forecasts from both the extreme value estimators are highly correlated (0.99+). For one-month period also, extreme value estimators (Rogers-Satchell and Garman-Klass) forecast volatility much better than others and are the only ones to have significant coefficients. In the regression involving all the forecasts, the coefficients are once again insignificant due to high correlation (0.99+) of the forecasts given by two extreme value estimators. Another interesting aspect of one-month period results is that, unlike the five-day period, traditional and conditional volatility model forecasts add somewhat to the predictive power. The poor performance of EGARCH model in terms of incremental predictive ability compared to GARCH is understandable as the model estimated for forecasting over 1996-1998 data did not have significant asymmetric term whereas during the 1999-2001 period, the asymmetric term was found to be significant and large. On examining the forecasts given by conditional volatility models closely, it was clear that the variance in their forecasts was much lower than other estimators' forecasts as well as that of realized volatility for each of the three horizons. This implies that the realized volatility is not as persistent as forecasted by conditional volatility models.

Besides relatively superior performance of extreme value estimators in forecasting volatility for five-day and one-month period ahead, the other striking aspect of our result is that the R-square values are of much higher order than the results of Day and Lewis (1992). Most of the explanatory power in volatility prediction comes from extreme value estimators which have mostly not been used in other studies. To that extent, it would be interesting to replicate the study on different samples and contexts.

SUMMARY AND CONCLUSIONS

Modeling and forecasting volatility of capital markets has been an important area of inquiry and research in financial economics with the recognition of time-varying volatility, volatility clustering, and asymmetric response of volatility to market movements. This stream of research has been aided by various conditional volatility (ARCH/GARCH type) models proposed to handle these empirical regularities. Nonetheless, researchers have found that forecasting volatility is difficult.

Estimating and forecasting of volatility is not only of interest to academia but is required and used by practitioners in financial and other sectors of the economy. Volatility estimates or forecasts are one of the inputs required in valuing OTC derivatives written on asset prices. They are required in estimating corporate and financial risk exposures and along with the return correlations across assets are the main ingredients of popular risk management frameworks such as value at risk.

In this paper, we model the volatility of S&P CNX Nifty, an index of 50 stocks of the NSE, Mumbai, using different class of estimators and models. Our results indicate that while conditional volatility models perform well in estimating volatility for the past in terms of bias, extreme value estimators based on observed trading range perform well on efficiency criteria. As far as forecasting is concerned, the extreme value estimators are able to forecast five-day (approximately a week) and one-month volatility ahead — much better than conditional volatility models. The implication of this for the practitioners is that if volatility is being estimated or forecasted for a week or a month, the extreme value estimators such as Garman-Klass, Rogers-Satchell, and Yang-Zhang estimators could be easily used instead of

Table 7: Results of Forecast Encompassing Regression

Panel A: One-day Period (Number of Observations = 736)

Forecast Comparisons	β_0	β_1	β_2	β_3	β_4	β_5	*R-squared*
Garch vs. EGARCH	-0.0004	-	-	-	1.935	0.610	0.1596
	(-5.27)				(2.96)	(0.85)	
Garch, EGARCH, Trad.	0.000	0.198	-	-	1.566	0.176	0.1917
	(-2.62)	(5.40)			(2.43)	(0.25)	
GARCH vs. Trad.	0.000	0.200	-	-	1.714	-	0.1917
	(-2.77)	(5.46)			(6.96)		
EGARCH vs. Trad.	0.000	0.208	-	-	-	1.773	0.1852
	(-2.74)	(5.67)				(6.50)	
Garch vs. EV(P)	0.000	-	0.374	-	1.961	-	0.1718
	(-3.77)		(3.39)		(7.72)		
EGARCH vs. EV(GK)	0.000	-	-	0.151	-	2.239	0.1558
	(-4.25)			(2.33)		(7.84)	
Garch, EGARCH, EV(P)	0.000	-	0.371	-	1.906	0.069	0.1718
	(-3.40)		(3.28)		(2.94)	(0.09)	
All	0.000	0.341	-1.041	0.504	1.916	-0.487	0.2044
	(-1.77)	(5.46)	(-3.15)	(3.39)	(2.94)	(-0.65)	

Panel B: Five-day Period (Number of Observations = 146)

Forecast Comparisons	β_0	β_1	β_2	β_3	β_4	β_5	*R-squared*
Garch vs. EGARCH	0.000	-	-	-	0.710	1.506	0.1829
	(-2.39)				(0.65)	(1.29)	
Garch, EGARCH, Trad.	0.000	0.130	-	-	0.104	1.596	0.1885
	(-1.06)	(0.99)			(0.08)	(1.36)	
GARCH vs. Trad.	0.000	0.117	-	-	1.552	-	0.1780
	(-0.77)	(0.89)			(2.40)		
EGARCH vs. Trad.	0.000	0.136	-	-	-	1.680	0.1885
	(-1.09)	(1.19)				(2.77)	
Garch vs. EV(YZ)	0.000	-	0.735	-	-0.078	-	0.2970
	(1.26)		(5.01)		(-0.14)		
EGARCH vs. EV(RS)	0.000	-	-	0.647	-	0.223	0.2947
	(0.72)			(4.81)		(0.40)	
Garch, EGARCH, EV(YZ)	0.000	-	0.736	-	-0.046	-0.042	0.2970
	(1.13)		(4.80)		(-0.04)	(-0.04)	
All	0.000	0.087	0.046	0.597	-0.037	-0.080	0.2982
	(1.00)	(0.47)	(0.03)	(0.43)	(-0.03)	(-0.07)	

Panel C: Calendar Month (Number of Observations = 35)

Forecast Comparisons	β_0	β_1	β_2	β_3	β_4	β_5	*R-squared*
Garch vs. EGARCH	0.000	-	-	-	0.610	1.645	0.0586
	(-0.61)				(0.31)	(0.66)	
Garch, EGARCH, Trad.	0.000	0.337	-	-	0.048	-0.037	0.0937
	(0.37)	(1.10)			(0.02)	(-0.01)	
GARCH vs. Trad.	0.000	0.335	-	-	0.035	-	0.0937
	(0.55)	(1.30)			(0.02)		
EGARCH vs. Trad.	0.000	0.339	-	-	-	-0.002	0.0937
	(0.39)	(1.16)				(0.00)	
Garch vs. EV(RS)	0.000	-	0.829	-	-0.861	-	0.3694
	(1.10)		(4.05)		(-0.71)		
EGARCH vs. EV(GK)	0.001	-	-	1.104		-3.237	0.3933
	(2.02)			(4.22)		(-1.76)	
Garch, EGARCH, EV(RS)	0.001	-	0.984	-	0.596	-3.181	0.4053
	(1.76)		(4.25)		(0.37)	(-1.37)	
All	0.001	-1.780	-5.723	8.828	1.216	-2.793	0.5441
	(1.07)	(-2.97)	(-2.10)	(2.55)	(0.80)	(-1.26)	

Notes:

1. Figures in parentheses are t-statistic associated with the coefficient.
2. For one-day period, the traditional estimator used for forecasts is traditional close-open-close estimator and extreme-value estimators used for forecasts are Parkinson and Garman-Klass Estimators. The choice was based on *R-squared* as reported in Table 6.
3. For five-day period, the traditional estimator used for forecasts is traditional close-close estimator and extreme-value estimators used for forecasts are Yang-Zhang and Rogers-Satchell Estimators. The choice was based on *R-squared* as reported in Table 6.
4. For one-month period, the traditional estimator used for forecasts is traditional adj. close-open-close estimator and extreme-value estimators used for forecasts are Rogers-Satchell and Garman-Klass Estimators. The choice was based on *R-squared* as reported in Table 6.

conditional volatility models. Computationally, they may find extreme value estimators easier to work with and relate to.

In this paper, we have not used 'implied volatility' forecasts, used extensively elsewhere, for two reasons. Firstly, the options in the Indian capital markets have been introduced only recently and, therefore, long enough time-series is not available. Secondly, as pointed out by Varma (2002), Indian markets seem to underprice (by implication, underestimate) volatility of index options in its short history of pricing options. Nonetheless, comparisons incorporating 'implied volatility' forecasts remain potentially an area worth investigating at a future date. Similarly, even though we have used five-minute returns for computing realized volatility, the optimality of use of lower or higher frequency returns needs to be verified empirically. Another area requiring further work in the Indian context is to model volatility explicitly for non-trading days and for any plausible 'day-of-the-week' or calendar effects in returns and volatility. As far as ARCH models are concerned, we have used relatively basic conditional volatility models. It would be interesting to use various extensions proposed in the literature to evaluate their performance in the Indian context. In particular, for testing the forecast performance over a horizon greater than one-day, long-run dependencies in volatility may be critical and the use of FIGARCH model may improve forecast performance. Similarly, intra-day data characteristics and high frequency return volatility dynamics are other areas requiring further work.

ENDNOTES

1. Practitioners require volatility estimates and forecasts of returns on asset prices (not always that of stock market or stock prices) for valuation of derivatives, real options, and in estimating and managing risk exposures. Though this work is focused on the Indian stock market, much of the issues involved in estimating and forecasting volatility are relevant to other markets such as foreign exchange markets, commodity markets, etc.
2. For an early review of empirical applications of ARCH models on low frequency data, see Bollerslev, Chou and Kroner (1992) and Bollerslev, Engle and Nelson (1994).
3. For example, Varma (1999) evaluates the performance of GARCH-GED and EWMA models in terms of goodness of fit in the context of Indian capital markets.
4. These features are essentially same as discussed in Bollerslev, Engle and Nelson (1994). Later, empirical studies have found long-run dependencies in financial market volatility despite intra-day volatility shocks being dissipated quickly. Andersen and Bollerslev (1996) explain this anomaly theoretically. They show that the aggregate volatility can be characterized as a long-memory or fractionally integrated process which might be an aggregation of numerous independent components having their own dependence structure.
5. We do not discuss in detail various ARCH-type models which have been proposed in the literature as we use only two basic ARCH-type models in this paper — GARCH EGARCH.
6. See Poon and Granger (2003) for a detailed review of 93 papers on volatility forecasting. The papers reviewed by them do not provide overwhelming support for use of any particular model for forecasting volatility.
7. Driftless means that log price process is driftless, i.e., $\mu = \sigma^2/2$. The process is specified as $dS_t = \mu S_t dt + \sigma S_t dW_t$, where W_t is a standard Brownian motion and S_t is the price of asset at time t.
8. In case the time-series of returns being used for modeling exhibits serial dependence or day-of-the week (or any other seasonality/calendar effects), the conditional mean equation needs to be modified based on the characteristics of the return series. The specification of the conditional mean equation also depends upon the data frequency as serial dependence in returns is more likely in high frequency data.
9. "Many studies find that the simple GARCH (1,1) model provides a good first approximation to the observed temporal dependencies" (Andersen and Bollerslev, 1998).
10. Li and Weinbaum (2000) use the realized volatility measure to evaluate empirical performance of extreme value estimators. In a similar vein, the study by Day and Lewis (1992) use variance of daily returns multiplied by the number of trading days to compute weekly variance for evaluating out-of-sample predictive power of various volatility models.
11. In the early part of the time-series (approximately half), NSE and other major Indian markets used to operate with weekly settlement cycle, i.e., trades during a settlement cycle were aggregated (and netted) and settled at one go. While settlement cycle induced autocorrelations required examining lags of up to five, six lagged terms were explored for ARMA modeling due to the presence of significant autocorelation coefficients (simple and partial) associated with the sixth lag.
12. EViews uses maximum likelihood procedure to estimate the model under the assumption that the errors are conditionally normally distributed. For initialization of variance, by default, EViews first uses the coefficient values to compute the residual of mean equation and then computes an exponential smoothing estimator of the initial values with smoothing parameter, λ=0.7. Even though the software provides for options for initialization, we have used the default initialization procedure in this work throughout. Different initial variances for maximum likelihood procedure in conditional volatility models could lead to different estimates affecting model performance.

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Each time a man stands up for an ideal, or acts to improve the lot of others, or strikes out against injustice, he sends forth a tiny ripple of hope ... and crossing each other from a million different centers of energy and daring those ripples build a current that can sweep down the mightiest walls of oppression and resistance.

Robert F. Kennedy