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Working Paper

Components of intraday volatility and their prediction at different sampling frequencies with application to DAX and BUND futures

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No. 15/2014

Components of Intraday Volatility and Their Prediction at Different Sampling Frequencies with Application to DAX and BUND Futures

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SUMMARY

The adjusted measure of realized volatility suggested in [20] is applied to high-frequency orderbook and transaction data of DAX and BUND futures from EU-REX in order to identify the drivers of intraday volatility. Four components are identified to have predictive power: an auto-regressive pattern, a seasonal pattern, long-term memory and scheduled data releases. These components are analyzed in detail. Some evidence for two additional components, market microstrucuture events and unscheduled news, is given. Depending on the sampling frequency we estimate that between one and two thirds of the variation in realized volatility can be predicted by a simple linear model based on the components identified. It is shown how the predictive power of the different components depends on sampling frequencies.

Keywords and phrases: Volatility; realized variance; intraday seasonality; volatility prediction, high-frequency data; tick data; fractional integration, sampling frequency

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1 Introduction

Intraday volatility has been subject to several studies in recent years. Following the distinction made in [2], there is two types of approaches: Firstly, in the parametric approach, volatility is treated as latent variable determining the fluctuation of prices. In this approach GARCH-type models (compare the initial approaches in [11] and [6], also its extensions to intraday volatility in [5] and recent approaches with intraday components, e.g. [12]) are typically used to describe volatility. Secondly, in the non-parametric approach volatility is treated as observable, thus rendered visible by use of some measure of volatility, typically realized volatility (compare e.g. [15] or [7]). The identified drivers of volatility are very similar for both approaches:

• Long-term trend is inherent already in parametric daily volatility models in [11] and [6] but also in intraday models in [5] or [12]

- Pattern of fractional integration in parametric approaches have been introduced in [5] and a become a standard approach since; a nice and recent overview on various derived specifications is provided in [17], for the non-parametric approaches it has been applied rather implicitly by use of higher lag orders, e.g. in [3]
- Seasonal patterns have been studied in both approaches e. g. in [12], [16] or [1]
- Impact of macro-economic news has been studied in non-parametric approaches in [1], [19] and [15] and in the parametric approach suggested in [9].

Both approaches have clear advantages and disadvantages. From a theoretical perspective GARCH-type models are easier integrated into price diffusion models, also they are better set-up to cope with tick size effects, see below. However, when it comes to provide an intuitive explanation of the observed components in volatility it seems to be advantageous to employ direct volatility measures. For example it is not possible to isolate e.g. the distinct spikes in the intraday seasonality patterns within the framework of GARCH-type models, whereas these patterns can easily be isolated by realized volatility. Consequently to the best of our knowledge - the analysis of intraday seasonality patterns associated with opening hours of trading and the publication of scheduled news has been mostly confined to approaches based on direct volatility measurement. In this perspective the rigorous mathematical approaches to capture different seasonal frequencies e.g. in [17] or [16] seem to rather hinder intuitive explanations of the observed intraday patterns. Therefore it is believed that the presented approach based on measured volatility will be of more use at least for practitioners. In the domain of direct volatility measurement, most approaches rely on the calculation of realized volatility as the squared difference of price increments per period, thus considering only values at the grid points. For this analysis we use in contrast an averaged measure of realized volatility following [20] - which allows us to consider more of the information available on price changes, especially the information on changes in prices between grid points. The data sets applied are presented in section 2, the measure of volatility with its detailed specification is discussed in section 3. Based on measured volatility the drivers of intraday volatility are derived and presented in sections 4 and 5. Here we distinguish between drivers which are assumed to have predictive power and those without predictive power. The subsections to each section give the detailed results on each of the identified components. The components deemed to be predictable are combined in a simple linear model. This model is applied for prediction of volatility based on different sampling frequencies. The dependency of forecast power on sampling frequencies has already been studied in [7]. But contrary to this work the present approach emphasises the forecast power of the different components. The results are presented in section 6. Conclusions are given in section 7.

2 Data sets

The analyses performed are based on trade and quote data for two of the most liquid futures traded on EUREX, namely the futures on DAX, and BUND future. The time series employed cover the first half of 2014. The time series is built based on front month future for both instruments at expiry the future with the second shortest available maturity is

Statistic	DAX	BUND
Price level, avg.	9,588	143.9
Price level, max.	10,035	147.1
Price level, min.	8,910	138.7
Tick size, absolute	0.5	0.01
Trades per day, avg.	48,985	35,733
Updates in best bid/best ask per day, avg.	477,054	494,294

Table 1: Some descriptive statistics on the data sets used.

used. The chosen sample contains two expiry dates for each future series. DAX future: March 21 and June 20; BUND future: March 6 and June 6. Table 1 summarizes relevant descriptive statistics. 1

In summary, the data covers continuous trading for every of the 125 German business days in the first half of 2014, starting immediately after the opening auction at 8:00 and ending with the closing auction at 22:00 CET. On February 6, a trading exception occurred for the DAX future when shortly after 13:45 CET a volatility interruption had been triggered by a sudden price drop.² The consecutive volatility interruption and intraday auction lasted three minutes until 13:48 CET. The data during that period has been removed from the sample.

3 Measuring Intraday Volatility

The measurement of intraday volatility is crucial to the presented approach. The choice of measure is known to impact the predictive power, compare e.g. [7]. Let for the presented approach volatility be defined as volatility of the true price of the instrument. It is clear that neither volatility nor true prices are necessarily observable. Consequently, measurement necessarily relies on assumptions about both. The suggested measurement approach is justified by observations and based on three assumptions:

The first assumption is on persistence of volatility. GARCH-type models typically assume volatility to be a moving average of the squared log-increments for a given time intervals. This assumption implies that volatility is persistent. But the strong spikes in the patterns of intraday seasonality (compare e.g. [15], [18] or section 4) or the strong impact of news events (compare section 4.4) imply that this assumption does not hold for intraday volatility³. Consequently, each day should be divided into time bins. Volatility should be measured for each time bin separately - as done in non-parametric approaches. In the pre-

 $^{^{1}\}mathrm{Data}$ has been provided by Deutsche Börse AG.

 $^{^2}$ Refer to http://www.eurexchange.com/exchange-en/about-us/news/822524/.

³The incoherence of the two measurement approaches in their implications on persistence has been discussed in [8].

sented approach six different frequencies corresponding to bin lengths of 10 seconds, 1, 2, 5, 10 and 30 minutes are applied.

Most non-parametric approaches measure volatility with the squared log-increment of the price change between start and the end of the time bin (compare e.g. [15], [3] or [18]). This approach might understate volatility in the case prices changed substantially within a time bin and revert back to the initial level at the end. To avoid this, and given the today's availability of tick data, there is some temptation to use realized volatility as the sample analogue of quadratic variation as

$$\widehat{rv_j} = \sqrt{\sum_{i=1}^{n} (\log(p_i) - \log(p_{i-1}))^2}$$
(3.1)

where p denotes the observe price and the index i loops over every of the n time-ordered trade prices or quote ticks in bin j. But realized volatility is known to exhibit an upward bias, compare e.g. [14], most probably being caused by market microstructure noise. Similar findings are also evident from the data used for this study: an upward bias in realized volatility based on ticks is visible in the realized volatility estimates on the left hand side of figure 1, a pronounced example for the impact of market microstructure is given in figure 2.

The mechanics of market microstructure noise remain unknown, but it is assumed - the second assumption required - that such noise exists, but does not persist for periods longer than several microseconds. In this case, it seems wise to apply the adjusted measure suggested in [20] to a grid with distances big against such noise. The suggested procedure may be described as follows:

- For each time bin, realized volatility is measured on a time grid of 100 ms.
- The calculation is repeated for every time shift of that grid by 10 ms.⁴
- Average realized volatility is calculated as the average of the result for each time shift.
- The adjustment factor suggested in [20] is applied.

Due to the very efficient quoting process with roughly 10 to 20 times more mid-quote updates than trade ticks, compare table 1, the estimator chosen is based on mid-quote rather than trade prices. This approach is further reducing - for the DAX future even eliminating - the upward bias, compare figure 1.

The suggested measure may result in bins with zero volatility in cases where neither bid nor ask quote change. This may frequently happen in future markets for a couple of seconds, but only in rare cases even for minutes. Table 2 gives an overview for all available data sets in the first half of 2014. The frequency of such occurrance increases with decreasing bin length and with increasing tick size. This latter observation indicates that a zero volatility measurement does not necessarily imply zero volatility. It rather stems from volatility falling below tick size threshold. The true volatility may thus be somewhere between zero and the tick size threshold - assumed to equal tick size/price level. Thus the third assumption is to set volatility to half the tick size threshold for all 0 bins.

⁴In order to be able to perform that shift the first 90 ms from the next bin are taken into account. Thus that there is some overlap between two consecutive bins. This effect is considered negligible given that the smallest bin size applied is 10 seconds.

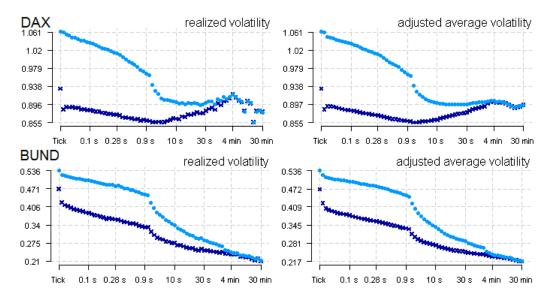


Figure 1: Average estimate of daily volatility for all business days in January 2014, selected frequencies, based on traded prices (light blue, cross) and mid-quote from best bid, best ask (dark blue, circle).

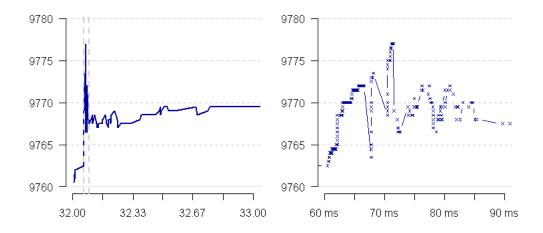


Figure 2: Traded prices of DAX future (front month) on January 17th 2014 at 11:00:32 CET (left hand graph) with zoom into millisecond 60 to 90 (right hand graph).

Bins		DAX		BUND	
Length	Obs.	Zero bins	Perc.	Zero bins	Perc.
1 sec.	$6.30\cdot 10^6$	$3.57 \cdot 10^6$	56.6%	$5.88 \cdot 10^{6}$	93.4%
10 sec.	$6.30\cdot 10^5$	54,655	8.6%	417, 152	66.2%
1 min.	104,875	485	0.5%	25,278	24.1%
2 min.	52,375	74	0.1%	6,237	11.9%
5 min.	20,875	6	0.0%	795	3.8%
10 min.	10,375	0	0.0%	139	1.3%
30 min.	3,375	0	0.0%	7	0.2%
1 hrs.	1,625	0	0.0%	1	0.1%

Table 2: Frequency of zero bins in first half of 2014.

4 Predictable Components of Intraday Volatility

Volatility prediction has long been subject to scientific research, where the first patterns discovered to have high explanatory power are probably the autoregressive structures in daily returns as modelled by ARCH- and GARCH-models in [11] and [6]. Since the publication of these initial approaches, a vast literature has been developed, where several components to be used for prediction have been discussed. Contrary to the existing studies the present work is dedicated to the explicit analysis of each of these components and their interaction. The criterion for judgement on these components is their predictive power in linear prediction rather than pure in-sample fit. Therefore, some initial results with significance in-sample but without improvement of out-of-sample prediction power are omitted.

4.1 Fractional Integration and Autoregressive Pattern

Fractional integration in volatility has been used for GARCH-type models since the mid 1990s, where a rigorous modelling approach has been developed in [5]. Evidence for fractional integration is found in log-volatility for all data sets considered. There is strong indications for non-stationarity in the original price time series as well as significantly negative first order lags with positive and decaying higher order lags in the series of first-order differences.

In order to estimate the degree of fractional integration the procedure suggested in [13] is applied. For this procedure to work well the residuals of the regression on spectral densities should not be auto-correlated, compare e.g. [5]. Applying this procedure a degree of fractional integration (\hat{d}) between 0.15 and 0.5 is found. However, the reliability of the estimates is questioned by weak, but significant autocorrelation indicated by the Durbin-

Bin length	DAX	BUND		
	\hat{d}	DW	\hat{d}	DW
10 seconds	0.229	1.946	0.163	1.977
1 minute	0.326	1.915	0.202	1.928
2 minutes	0.375	1.886	0.244	1.912
5 minutes	0.427	1.857	0.304	1.917
10 minutes	0.468	1.698	0.347	1.893
30 minutes	0.457	1.729	0.370	1.780

Table 3: Degree of fractional integration for different instruments and frequencies.

Watson statistic (DW), see table 3.⁵

There is some evidence that the degree of fractional integration is related to the tick size. For all instruments considered, the degree of fractional integration and the volatility per time bin divided by the respective tick size exhibit a log-linear relation, see figure 3. The relation indicates that the smaller the ratio of volatility per bin versus tick size, the smaller the degree of fractional integration.

Fractional integration implies that the time series is non-stationary. In order to represent this in the framework of a linear model one would be required to include an infinite series of autoregressive terms of the form

$$y_t = \sum_{k=1}^{\infty} \beta_k \cdot y_{t-k} + \epsilon_t, \tag{4.1}$$

compare [5], where y_t is the log-volatility of the time ordered bin t, β a vector of parameters and ϵ_t some error term. As the estimation of infinite lags is impossible and the inclusion of a very high number of lags into the model reduces predictive power, only a finite sum will be included in the model. The question is thus how to determine the lag order to be included into the model.

It is suggested to choose the number of lags such that the predictive power is maximized. Figure 4 gives the predictive power for a simulated $\operatorname{ARFIMA}(0,d,0)$ processes based on different lags. It becomes clear that the optimal lag number depends on the sample size as well as on the degree of fractional integration. It can also be derived from figure 4 that the maximum predictive power that can be achieved depends strongly on the degree of fractional integration.

For the model, the optimal lag order is estimated to maximize the predictive power of using the first 90% of the calibration sample and predict its last 10%. The drawback coming with this approach is that it will slightly underestimate the true optimal lag. The advantage

 $^{^5}$ A Durbin-Watson statistic close to 2 indicates absence of autocorrelation, values of 0 (4) indicate perfect positive (negative) correlation.

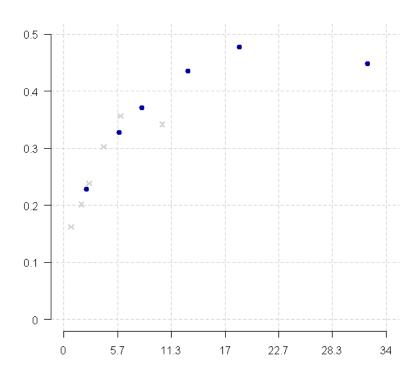


Figure 3: Degree of fractional integration versus the ratio of volatility per bin divided by tick size, for the time bins given in table 3, with DAX: circle, BUND future: cross.

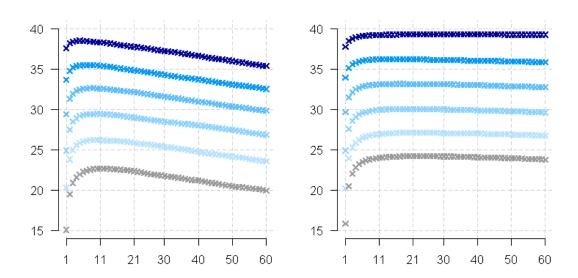


Figure 4: Predictive power in terms of R^2 in percent based on different lag orders for simulated ARFIMA(0, d, 0) processes for (calibration) sample size of 1,000 (left) and 50,000 (right) and different degrees of fractional integration from 0.2 (upper dark blue line) to 0.45 (lower grey line).

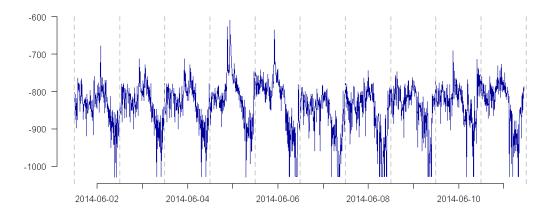


Figure 5: Volatility per 5 minute bin for BUND future on June 2nd to June 11th, log-scale.

is that it is computationally more efficient and that it is less dependent on a reliable estimate for d.

4.2 Intraday Seasonality and calendar effects

Spotting the existence of intraday seasonality patterns does not require sophisticated statistical methods. The repeating patterns of mid-day spikes and end-of-day decay are directly observable in the time series of measured volatility, see e.g. figure 5. The observed pattern is similar for all three instruments investigated: all exhibit clock time dependence with strong spikes at 9:00 CET and - for the months other than March - also at 15:30 CET. This observation motivates an approach where the impact of intraday seasonality is modelled separately for each clock time of the day.

For reason of simplification, the prediction for the seasonal component is based on a linear model where the explained variable is volatility. Due to the strong heteroscedasticity introduced by the strong upward spikes in volatility, logarithm of volatility is used rather than volatility itself. The model is thus setup as

$$y_{ct_j} = \theta_0 + \sum_{i=1}^k \theta_i I_{ct_j,i} + \epsilon_j, \tag{4.2}$$

with y_j as log-volatility of bin ct_j , where ct_j denotes the clock time ct on day j, parameters $\theta_0, ..., \theta_k$ and i = 1 to k numbering the explanatory variables in form of dummy variables. For each time bin a dummy $I_{ct_j,i}$, for macro-economic releases are taken into account, see section 4.4, in order to prevent spurious seasonality at publication times, e.g. 13:45 or 14:30 CET. Following an explorative approach different additional explanatory variables are tried including several derived from conventional hypotheses:

• A week day effect turns out to be significant only for Mondays - but even this sig-

Clock time	Event	Instrument
08:00 CET	Opening auction EUREX	both instruments
09:00 CET	Opening auction XETRA	both instruments
15:30 CET	Opening auction US	both instruments
16:00 CET	unknown	both instruments
17:30 CET	Closing auction XETRA	only DAX

Table 4: Strongest regular spikes in intraday seasonality.

nificance disappears once UK and US bank holidays are taken into account.⁶ Thus a weekday dummy is not taken into the final model.

- Expiry days ("small" expires typically the third Friday a month, or "big" expiries typically the third Friday each quarter) introduce a slightly but significantly higher volatility only at around 13:00 CET the clock time of expiry. Despite this significance the corresponding dummy variable is omitted for reasons of simplicity.
- US, UK and Swiss holidays turn out significant for all 3 instruments, introducing an overall lower volatility with an unique pattern for US holidays, see figure 6. US holidays have no significant impact during morning hours but volatility strongly decreases in the afternoon.
- Different time shifts due to the different starts of daylight saving times in North America and Europe turn out to have significant impact, especially during 14:30 CET and 15:30 CET. This is most probably caused by the US trading start being shifted to 14:30 CET during the two weeks in March concerned. Compare figure 6. Therefore, we introduce a corresponding dummy.

Figure 7 shows the extracted pattern of intraday seasonality for a trading day which is no holiday in US, UK or Switzerland and has the same daylight saving time phase as the US. Even though the linear approach applied may be too sensitive towards outliers, clear patterns can be extracted for the two equity based indices: Volatility seems to spike upwards punctually at auction times only to slowly "calm down" in the following minutes. The most relevant spikes are the given in table 4. Only the very pronounced spike at 16:00 CET can not be linked to an auction or similar event.

The resulting intraday volatility of the BUND future is overall lower and shows a less pronounced pattern. Rather than suddenly spiking after auctions, its peaks are smaller and are observed at auctions as well as at several points of time in the afternoon. The only clear clock time related peaks seem to be at 8:00, 9:00 and 16:00 CET.

⁶Out of the 21 Mondays in the sample, four fall on a US or UK holiday. This high number comes as holidays in both countries are shifted to Mondays in case they fall on the weekend.

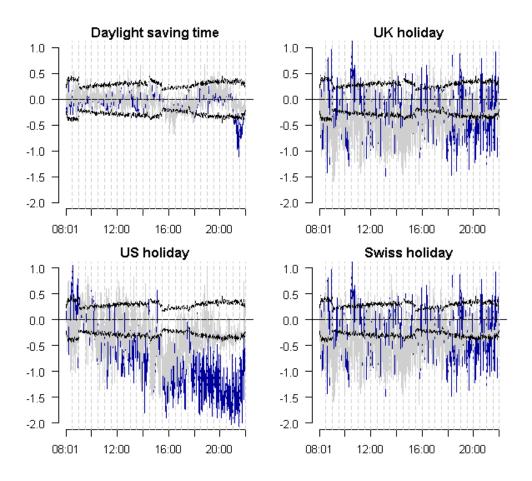


Figure 6: Impact of calendar effects on log-volatility for different clock times, DAX in dark blue and BUND future in grey as estimated by 1 minute based model. The black line gives the standard deviation for DAX.

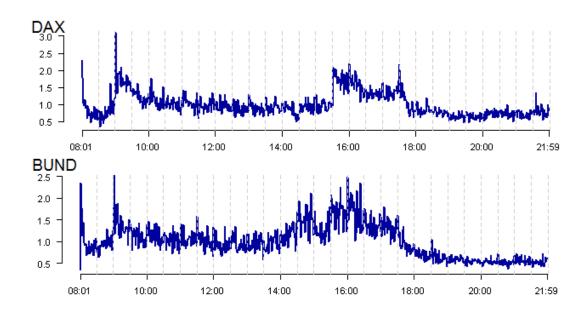


Figure 7: Daily volatility pattern per 1 minute bin for DAX and BUND as predicted for a trading day for a trading day which is no holiday in US, UK or Switzerland and has the same daylight saving time phase as the US. Volatility is divided by average daily volatility.

4.3 Long-term Memory

Long-term memory in intraday volatility has been considered relevant for modelling intraday volatility in recent literature, compare e.g. [12] or [17]. Given the high granularity of the data set employed and the primary goal of the study, namely to derive a short term prediction model form volatility, long-term memory is defined as a memory in volatility over a period of several days rather than over weeks or months. Empirical evidence for such memory is given in the plot of measured volatility aggregated for each business day, compare upper graph in figure 8. Especially for DAX future daily volatility is highly persistent with highest volatility end of Januar, early February and lowest volatility during June.

Our empirical approach for including model component for long term memory is based the findings depicted in figure 8. Comparing the observed long term daily volatility pattern with the patterns emerging for different hours we conclude that the hourly patterns can differ to a large extent from the daily pattern: E.g. the pattern in the hours before 12:00 CET looks roughly independent from the overall long-term pattern, whereas the hours in the early evening seem to strongly reflect the overall pattern.

In some existing approaches, e.g. in [12], long-term trend affects every time bin of a day in the same way. Given above observation, the long-term pattern is rather the included to the intraday seasonality model which models every clock time bin separately. Thus for every time bin an autoregressive component with five lags (from 24 hours up to five days) is included to the intraday seasonality model. This additional component increases the insample fit of the intraday seasonality, compare figure 9. The effect is stronger for DAX than for BUND future and also stronger after 12:00 and before 19:00 CET. This corresponds to the findings on persistence. Long-term memory is treated as part of intraday seasonality in the sequel.

4.4 Scheduled Data Releases

Scheduled data releases are known to have major impact on intraday volatility, compare the results e.g. in [1], [19] or [15]. The relevance of scheduled news for the data set underlying this study has already been established by determining the time series cf. section 2: Immediately after the ECB interest rate decision on February the 6th volatility in DAX future trading increased strongly within a very short time frame leading to a interruption in trading. Another pronounced example for the sensitivity of volatility to news releases is June 5th, where ECB released an interest rate decision at 13:45 CET followed by two data released in the US at 14:30 CET. As figure 10 shows both news events led to pronounced volatility peaks, showing clear deviations from the otherwise (the case of no news releases) expected volatility pattern.

As basis for inclusion of these news events into the model serve Deutsche Börse AlphaFlash data. These data contains time stamps for 857 scheduled events grouped to 107 categories (ranging from Belgium business sentiment to US refinery utilization) referring to 7 different regions (Belgium, Germany, EU, France, Switzerland, UK, and US).

As the main purpose of the model is the prediction of future volatility it has to be assured that the effect of a news event in the future can be estimated. Therefore, only those categories are considered where at least three events occurred during EUREX continuous

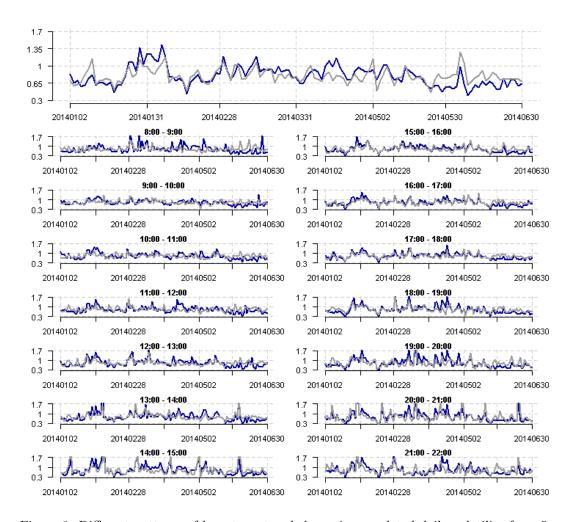


Figure 8: Different patterns of long-term trend shown in cumulated daily volatility from 5 minute bin for first half 2014 divided by half year average, for full day (upper graph) and different hours, DAX in dark blue and BUND future in grey. Clock times given are CET.

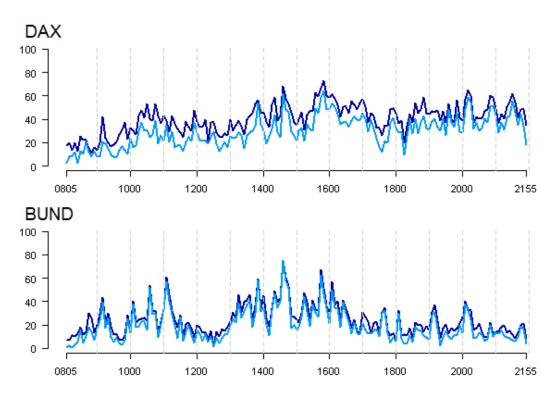


Figure 9: In-sample fit (R^2) of seasonality models calibrated based on Mach 9th to June 9th data, 5 minute bins. Seasonal model without long-term trend in light blue, with long-term trend in dark blue.

trading hours in the first half of 2014. Additionally, all categories whose publication times coincide are combined to one new category. This is the case e.g. for the different ECB rates or UK consumer and producer prices. The CPIs of the different German regions are combined to one category "CPI German region". This approach leaves 78 categories plus an artificially created category as "Other data release" with a total of 810 events.

For the prediction of the impact of future scheduled news events it is assumed that the historical average impact on log-volatility over all events per category is the best predictor for the impact of future events of that category. As most releases are published always at the same clock time (e.g. US initial jobless claims at 14:30 CET) the estimation of the impact of an event category should account for prevailing intraday seasonality. The model should also account for consecutive events spuriously interacting via the autoregressive pattern. Therefore, the estimation of the influence of news events has to consider all model components. The model can be written as

$$\Delta y_t = \alpha + \sum_{i=0}^k \beta_i \cdot y_{t-i} + \sum_{i=-2}^2 \gamma_i \cdot \widetilde{y}_{t-i} + \sum_{i=1}^n \delta_i E_{i,t} + \epsilon_t$$

$$\tag{4.3}$$

where y_t is the log-volatility estimated for time bin t, Δy_t the increase in log-volatility from time bin t to t+1, α a parameter for the trend, β a vector of parameters of the autoregressive component, γ a vector of parameters for the impact of seasonal prediction \tilde{y}_j from the seasonality model in equation 4.2, δ_i a parameter measuring the impact of a news in category i, where $E_{i,t}$ is an event dummy for one of the n categories, equal to 1 if the respective news event occurs during the time represented by bin t and 0 otherwise. ϵ_t denotes the error term for bin t with finite variance, $E(\epsilon_t) = 0$ $\forall t$ and $E(\epsilon_{t_1} \cdot \epsilon_{t_2}) = 0$ $\forall t_1 \neq t_2$.

The model's parameters may be consistently estimated by a linear regression, even though there is strong heteroscedasticity. Heteroscedasticity introduces strong bias to the estimated standard deviations. Assuming that this heteroscedasticity is well explained by intraday seasonality, the residuals of the seasonality regression in equation 4.2 are used for the calculation of standard deviations of impact estimates per category. Based on these adjusted estimates there are only 20 categories for DAX (29 for BUND) with significant impact on volatility for the two minute bins based model (similar results hold for 1, 5 and 10 minute bins). The results for the significant news event categories are presented in table 5 for 2 minute bins based on the calibration period from March 9th to June 9th.

5 Residual Components of Intraday Volatility

Figure 10 not only reveals how well the two spikes in intraday volatility can be explained by scheduled events, but it also shows that the prediction model cannot explain the full pattern of volatility inherent in the data. When comparing the forecasted to the observed volatility, one observes clear differences, e. g. upward spikes in DAX future volatility at 9:20 CET or 13:14 CET. These deviations are clear indications for additional structures in the volatility not covered by the suggested behaviour.

⁷White test indicates strong heteroscedasticity for any model specification and instrument.

An investigation of these differences would, amongst others, require a comprehensive analysis of scheduled news events not covered by the data set employed as well as unscheduled new events. In order to shed some light on this topic we analyze the 30 most important volatility spikes observed in the DAX future (based on 1 minute bins). Table 6 gives an overview

14 out of these 30 spikes coincide with scheduled data release or morning auctions and are thus covered by the model. But another 3 spikes occur in the one minute bin prior to a scheduled event. These events might by explained by an anticipation effect of the news release. Even though the mechanics of the impact of news releases have been studied by impact curves in [1], anticipation effects of scheduled news have not been mentioned and are neither their mechanics nor determinants are known. Scheduled news exhibit an additional residual component: Whereas timing and directional impact of these events is known, their exact impact may substantially vary within one and the same category, compare e.g. impact ECB rate direction in table 6. Both types of events are considered a misspecification of the impact from scheduled news rather than a residual component on their own.

More interestingly, another 4 events coincide with relevant but not scheduled market news: Two of the spikes are observed at times when Mario Draghi publicly clarified the ECB's views on future growth, thus having relevant implications on expected interest rates. The other two or three (one incident falls into both categories) are related to political events with strong links to energy prices, compare table 6.

As unscheduled news releases by their very nature cannot be predicted, they are considered as an unexplainable, residual component of the model.

For the remainder of 9 spikes no such coincidence with news or other events has been identified by the authors. However, when comparing the prevailing trading volumes during these events with the trading volume of at average days, see figure 11 one observes that these events are accompanied by comparably high volumes.

A possible explanation of these high volatilities without the presence of respective news event might be market's microstructure:

Prices may erracticly vary when a single trader executes an unlimited order with high volume into a low volume order book. A stylized example of such a possible event for the DAX future is given in figure 12. Here 4 buy orders with a volume of 10 contracts were executed within several seconds, with the second order causing the market price to drop by 6 points only to recover shortly. Similar patterns of can be found in 7 out of the 9 remaining spikes, compare figure 13.

Such events might be linked to the 2010 'flash crash' event in S&P E-mini and Dow Jones. Methods of prediction of such events have been discussed in the literature on VPIN suggested in [10]. But the discussion is still ongoing and there is no final result on the prediction of such events, compare e.g. [4].

For the purpose of this study the existence of volatility components caused by micro structure events is acknowledged. However as these events are triggered at much shorter time scales than the volatility drivers considered in the model, events caused by market's microstructure are assumed to be unpredictable within the framework of our model.

6 Prediction Based on Four Components

Prediction is based on linear combination of the presented components. The assumption on linearity is justified as being the first best approximation to the true, potentially non-linear, relation. This simplification appears useful as the same functional form is applied to all time horizons (where only the number lags in the autoregressive component is determined dynamically, as mentioned). Equation 4.3 summarizes the final model applied for linear prediction of log-volatility. Calibration is performed in two steps: First the seasonality model, compare equation 4.2, is applied to data from January first to June 9th⁸. Second, the prediction model in equation 4.3 is calibrated - based on the estimated seasonal components - to the observations from March 5th to June 9th for the prediction of June 10th. The interval for this second step is shifted by one business day for 12 times for the prediction of every business day up to June 25th.

The results for modelling the changes in volatility from bin to bin are given in table 7 with both, in-sample fit and prediction, reported in terms of R^2 . Table 8 translates these results to in-sample fit and prediction of levels of volatility. For the instruments considered the results are shown for the different time bin sizes from 10 seconds to 30 minutes. In order to be able to judge the impact of the different model components we show the results for different model set ups: pure auto-regression model (cf. equation 4.1), auto-regression plus seasonal/long-term component (corresponds to equation 4.3 with $\omega_i = 0 \,\forall i$) and full model (cf. equation 4.3).

The results may be summarized as follows:

- Prediction of volatility levels is very poor (below 30%) for BUND below 1 minute bins. This coincides with the fact that for this series more than 10% of the observations are time bins with zero volatility, compare table 2.
- Prediction power for differences based on auto-regressive patterns increases with decreasing bin size. This is in line with the result that lower degree of fractional integration implies higher memory in the time series, compare figure 4 and table 3. This result however does not hold for the 30 minute bin.
- The predictive power of the seasonal component for 10 or 30 minutes bin is as high as 8 to 12 %. For higher frequencies predictive power shrinks and disappears below 2 minute bins.
- The predictive power of scheduled news events for 30 minute bins is as high as 3 (DAX) to 9 % (BUND). For higher frequencies the predictive power shrinks and disappears below 1 minute bins.

The residuals of all estimated models are highly heteroscedastic (not reported), but not auto-correlated, compare table 9. The assumption of normal distributed residuals might be accepted for DAX in 10 second bins, but must be rejected for all other models. Skewness and kurtosis seem to increases with bin size, compare table 9.

 $^{^8\}mathrm{June}$ 9th is important to grant enough observations of Swiss holidays.

7 Conclusion

The suggested approach based on the adjusted measure of realized volatility in [20] applied to mid-quotes allows the prediction of roughly one third to two thirds of changes of log-volatility for most bin lengths. The components of long term trend, seasonality and scheduled news contribute sizably to this prediction if the bin length is two minutes or longer.

The approach fails if quotes remain unchanged over a longer time period and where thus volatility can not be measured. In the data sets applied this is the case for BUND future for bin sizes of 1 minute or lower. For the DAX future the approach is still able to predict 38% of the changes in levels even if 10 second bins are chosen but fails for one second bins, where more than 56% of the bins are zero bins.

The presented approach based on non-parametric volatility measurement is usefull for intraday time bins longer than one or two minutes. For smaller time bins the suggested measures is a bad volatility estimate due to a tick size effect. For such time bins, alternative measures of volatility have to be investigated.

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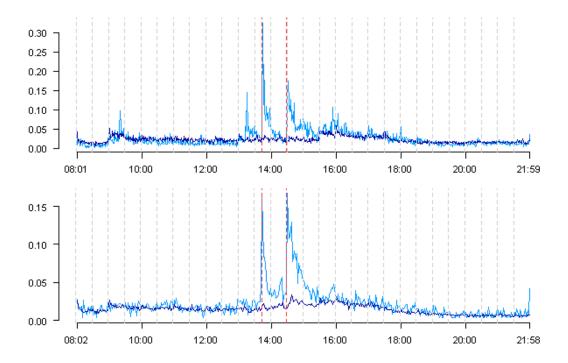


Figure 10: The impact of scheduled data releases - an example on the 5th of June. Dark blue is the out-of-sample prediction from a seasonality and long-term trend model calibrated from March to June 9th, for DAX (1 minute bins, upper graph) and BUND (2 minute bins, lower graph). At 13:45 CET there was an ECB interest rate decision, at 14:30 CET two data releases in the US (building permits and initial jobless claims), both times are marked with a red line.

Region & News	DA	X		BU	ND	
US Nonfarm Payrolls	1	1.998	(0.18)	1	2.058	(0.23)
EU ECB Rate Direction	2	1.688	(0.15)	4	1.548	(0.21)
US FOMC	3	1.375	(0.27)	3	1.761	(0.36)
US GDP	4	1.148	(0.20)	15	0.846	(0.27)
US FOMC Minutes	5	1.021	(0.29)	2	1.875	(0.37)
GE ZEW	6	0.918	(0.19)	8	1.010	(0.20)
US Retail Sales	7	0.911	(0.20)	11	0.976	(0.27)
US Housing Starts	8	0.846	(0.20)	21	0.741	(0.27)
US ADP Unemployment	9	0.767	(0.19)	5	1.248	(0.20)
US Philadelphia Fed	10	0.702	(0.20)	18	0.779	(0.24)
US Construction Spending	11	0.689	(0.20)	7	1.072	(0.24)
US Durable Goods Orders	13	0.487	(0.20)	22	0.705	(0.27)
EU Euro Area Inflation	12	0.487	(0.19)	14	0.864	(0.20)
US New Home Sales	14	0.467	(0.20)	19	0.761	(0.24)
US Producer Price Index	15	0.453	(0.20)	10	0.981	(0.27)
CH KOF Barometer	16	0.446	(0.18)	-	0.369	(0.29)
UK UK Retail Sales	17	0.379	(0.16)	6	1.114	(0.24)
US Initial Jobless Claims	18	0.377	(0.10)	23	0.625	(0.13)
UK Industrial Production	19	0.371	(0.16)	-	0.221	(0.24)
GE IFO Direction	20	0.370	(0.20)	9	1.004	(0.24)
US MNI Chicago Report	-	0.326	(0.15)	12	0.88	(0.21)
US Industrial Production	-	0.319	(0.24)	17	0.800	(0.26)
GE Prelimiry CPI	-	0.246	(0.22)	20	0.743	(0.25)
GE CPI - German Region	-	0.223	(0.14)	16	0.803	(0.19)
UK CPI & PPI	-	0.172	(0.30)	13	0.878	(0.24)
UK Labour Report	-	0.168	(0.16)	24	0.562	(0.24)
US Treasury Announcement	-	0.069	(0.05)	29	0.153	(0.07)
US Treasury Auction	-	0.066	(0.04)	28	0.155	(0.06)
EU HICP Inflation	-	0.054	(0.19)	26	0.463	(0.20)
UK Treasury Auction	-	-0.063	(0.06)	27	0.260	(0.10)
UK Public Sector Finances	-	-0.153	(0.16)	25	0.487	(0.24)

Table 5: List of significant events for DAX and BUND future on 2 minute bin granularity with rank and average impact on log-volatility with standard deviation in brackets, sorted by impact on DAX future. Result of calibration from March 9th to June 9th. Rank has been omitted where not significant.

Day	Time	Impact	Volume	Type	News
06-02	13:45	0.881%	409	sched.	EU ECB Rate Direction
07-02	14:30	0.334%	1845	sched.	US Nonfarm Payrolls
10-03	08:03	0.332%	831	unkn.	none
02-05	14:30	0.296%	1682	sched.	US Nonfarm Payrolls
25-06	08:00	0.286%	721	sched.	Opening auction
05-06	13:45	0.269%	1105	sched.	EU ECB Rate Direction
23-06	09:16	0.262%	2655	unkn.	none
07-02	14:29	0.262%	551	antic.	US Nonfarm Payrolls (at 14:30)
10-01	14:30	0.240%	1251	sched.	US Nonfarm Payrolls
06-03	13:45	0.234%	1268	sched.	EU ECB Rate Direction
07-03	14:30	0.227%	2201	sched.	US Nonfarm Payrolls
02-01	15:54	0.227%	796	news	Bomb series in Bagdag
23-01	14:52	0.226%	1936	unkn.	none
30-04	14:30	0.209%	903	sched.	US Employment Cost Index
17-01	13:04	0.204%	2853	unkn.	none
06-03	14:36	0.203%	1626	news	Draghi voices expectations of recovery
19-03	12:44	0.200%	1435	unkn.	none
07-03	14:29	0.195%	73	antic.	US Nonfarm Payrolls (at 14:30)
29-01	20:00	0.174%	181	sched.	US Federal Open Market Committee
04-04	14:30	0.170%	1174	sched.	US Nonfarm Payrolls
07-05	08:00	0.165%	406	sched.	Opening auction
02-01	10:18	0.163%	1139	news	Announcement of expected Kurdish oil
03-03	15:58	0.162%	738	news	Draghi speech/Russian threats on Krim
20-03	11:36	0.161%	1355	unkn.	none
05-02	14:15	0.155%	906	sched.	US ADP Unemployment
27-02	10:20	0.155%	1938	unkn.	none
27-01	16:28	0.153%	1053	unkn.	none
28-02	15:40	0.142%	1063	${\it unkn.}$	none
31-01	11:00	0.141%	463	sched.	EU Euro Area 'Flash' Inflation Estimate
13-02	14:31	0.138%	673	antic.	US Retail Sales (at 14:32)

Table 6: Top 30 one minute bins with strongest increase in volatility in DAX front month future during first half of 2014. Impact is the deviation of volatility observed to the volatility predicted by the suggested model. Volume is the number of contracts traded within the time bin. Abbreviations: antic. anticipated, sched. scheduled, unkn. unknown. Times in CET.

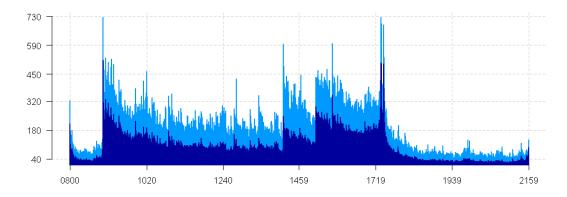


Figure 11: The daily schedule of average traded volumes per minute (dark blue) with one standard deviation added (light blue) for DAX front month future throughout H1 2014. Note that these are unconditional averages, where time shifts to US and other explanatory variables have not been taken into account.

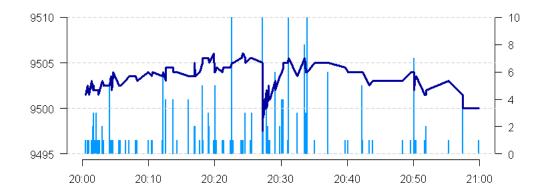


Figure 12: A liquidity event in DAX future as traded on January 2nd 2014 in the minute after 10:20 CET.

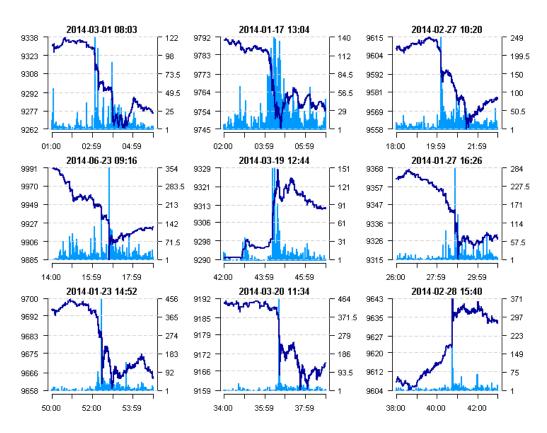


Figure 13: Prices and traded volumes (per second) for the five minutes around the volatility events not explained by news event from table 6.

Asset	Bin size	In-sa	In-sample			iction	
		Full	Seas.	AR	Full	Seas.	AR
DAX	$10 \sec$	42.1	42.1	37.4	35.4	35.4	37.3
	1 min	36.8	36.0	28.3	29.1	28.4	28.9
	$2 \min$	38.2	36.3	23.8	28.1	26.8	24.2
	$5 \min$	43.5	39.1	17.9	33.6	30.2	21.7
	$10 \min$	49.0	43.4	19.1	34.4	30.3	20.1
	$30 \min$	66.2	58.2	50.6	48.8	45.1	37.8
BUND	$10 \sec$	45.4	45.3	40.6	37.9	37.9	40.1
	1 min	44.5	43.9	38.2	37.8	37.2	37.5
	$2 \min$	44.8	43.8	35.0	37.3	36.4	34.9
	$5 \min$	44.9	42.0	28.5	34.5	32.1	28.1
	$10 \min$	48.8	43.6	25.0	40.8	36.1	27.8
	$30 \min$	66.3	55.8	44.4	51.5	42.5	37.6

Table 7: \mathbb{R}^2 in percent for in-sample and for prediction for differences in volatility for full model, model based on seasonality and auto-regressive patterns (Seas.) and a model only based on auto-regressive patterns (AR). Results are averages over 12 model estimates with rolling 3 month calibration periods for predictions from June 10th to June 25th.

Asset	Bin size	In-sa	In-sample			iction	
		Full	Seas.	AR	Full	Seas.	AR
DAX	$10 \sec$	53.5	53.5	49.7	36.1	36.1	37.8
	1 min	70.9	70.6	67.0	48.8	48.3	48.7
	$2 \min$	74.8	74.0	68.9	54.3	53.4	51.8
	$5 \min$	79.1	77.5	69.6	58.9	56.7	51.1
	$10 \min$	81.5	79.4	70.6	59.2	56.5	50.1
	$30 \min$	79.8	75.0	70.5	53.1	49.3	43.3
BUND	$10 \sec$	27.7	27.5	21.4	13.4	13.3	15.3
	1 min	46.8	46.2	40.8	34.5	33.9	33.9
	$2 \min$	56.7	55.9	49.0	45.6	44.8	43.2
	$5 \min$	68.0	66.4	58.5	55.3	53.3	50.1
	$10 \min$	74.2	71.5	62.1	62.5	58.9	52.9
	$30 \min$	80.6	74.6	68.0	69.3	62.0	60.1

Table 8: \mathbb{R}^2 in percent for in-sample and for prediction for levels of volatility for full model, model based on seasonality and auto-regressive patterns (Seas.) and a model only based on auto-regressive patterns (AR). Results are averages over 12 model estimates with rolling 3 month calibration periods for predictions from June 10th to June 25th.

Asset	Bin size	Durbin	-Watson	Skew	Skewness		Kurtosis	
		min	max	min	max	min	max	
DAX	$10 \sec$	2.01	2.01	-0.02	0.04	3.3	3.32	
	1 min	2.02	2.02	-0.16	-0.13	4.76	4.93	
	$2 \min$	2.03	2.04	0.07	0.15	4.71	4.99	
	$5 \min$	2.03	2.04	0.26	0.44	4.45	5.25	
	10 min	2.02	2.04	0.47	0.57	4.52	5.29	
	30 min	1.99	2.03	0.40	0.67	4.09	5.09	
BUND	$10 \sec$	2.01	2.01	0.83	0.9	3.6	3.66	
	1 min	2.02	2.02	-0.36	-0.32	2.88	3.01	
	$2 \min$	2.02	2.03	-0.57	-0.55	3.75	3.80	
	$5 \min$	2.04	2.04	-0.73	-0.65	4.95	5.16	
	10 min	2.04	2.05	-1.03	-0.87	7.48	7.70	
	30 min	1.98	2.01	-0.84	-0.77	9.77	10.15	

Table 9: Error term statistics for the full models. Results over 12 model estimates with rolling 3 month calibration periods for predictions from June 10th to June 25th.

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