1 Algebra

1. Show that the relation R on $\mathbb{R}$ defined as $R = \{(a, b) : a \leq b\}$, is reflexive, and transitive but not symmetric.

2 Differentiation

2. If $\log(x^2 + y^2) = 2\tan^{-1}(\frac{y}{x})$, show that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

3. If $x^y - y^x = a^b$, find $\frac{dy}{dx}$.

4. If y = $(\sin^{-1} x)^2$, prove that $(1 - x^2)\ \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$.

5. Find the order and the degree of the differential equation

$$x^2 \frac{d^2y}{dx^2} = \{1 + (\frac{dy}{dx})^2\}^4$$

6. $If x = \cos t + \log\tan(\frac{t}{2}), y = \sin t$, then find the values of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$

7. Write the order and the degree of the following differential equation: $x^3(\frac{d^2y}{dx^2})^2 + x(\frac{dy}{dx})^4 = 0$

8. If $y = (\sin^{-1} x)^2$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$.

9. Solve the differential equation : $\frac{dy}{dx} - \frac{2x}{1+x^2}y = x^2 + 2$

10. Solve the differential equation : $(x + 1)\frac{dy}{dx} = 2e^{-y} - 1; y(0) = 0$.

3 Functions

11. Examine Whether the operation $*$ defined on R by a $* \, b = ab + 1$ is
    (a) a binary or not.
    (b) if a binary operation is it associative or not?

12. Prove that the function f : N $\rightarrow N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto. Find inverse of f : N $\rightarrow S$, where S is range of f.

4 Geometry

13. Find the equation of tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$ Also, write the equation of normal to the curve at the point of contact.

14. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs ₹70 per square metre for the base and ₹45 per square metre for the sides, what is the cost of least expensive tank?

15. Find the area of the region lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$.

16. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by sides $x = 0, x = 4, y = 4$ and $y = 0$ into three equal parts.

5 Integration

17. Find: $\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}} dx$.

18. Find: $\int \sqrt{1 - \sin 2x}\, dx$, $\frac{\pi}{4} < x < \frac{\pi}{2}$.

19. Find: $\sin^{-1}(2x)dx$.

20. Form the differential equation representing the family of curves $y = e^{2x}(a+bx)$, where a and b are arbitrary constants.

21. Find $\int \frac{3x+5}{x^2+3x-18} dx$

22. Prove that $\int\limits_0^a f(x)\, dx = \int\limits_0^a f(a-x)\, dx$, hence evaluate $\int\limits_0^\pi \frac{x \sin x}{1+\cos^2 x}\, dx$.

23. Solve the differential equation : $xdy - ydx = \sqrt{x^2+y^2}$ dx given that $y = 0$ when $x = 1$.

24. Solve the differential equation:$(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$, subject to the initial condition $y(0) = 0$.

25. If $f(x) = x + 7$ and $g(x) = x–7$, $(x \in R)$ then find $\frac{d}{dx}(fog)(x)$.

26. Find: $\int \frac{\tan^2 x \sec^2 x}{1-\tan^6 x} dx$.

27. Find :$\int \sin x \cdot \log \cos x dx$.

28. Evaluate : $\int\limits_{-\pi}^{\pi} (1-x^2) \sin x \cos^2 x dx$.

29. Find:$\int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx$

6 Matrices

30. Find a matrix A such that $2A - 3B + 5C = 0$, Where $B = \begin{pmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{pmatrix}$

31. Using properties of determinants, prove that
$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$ =$(a-1)^3$

32. If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{pmatrix}$, find A^{-1} Hence, solve the system of equations

$$x + y + z = 6, x + 2z = 7, 3x + y + z = 12.$$

33. Find the inverse of the following matrix using elementary operations A=$\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$

34. Find the value of x–y, if

$$2\begin{pmatrix} 1 & 3 \\ 0 & x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$$

35. If $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$, then find $(A^2 - 5A)$.

36. Using properties of determinants, prove the following
$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a).$

37. If $A = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$ then find A^{-1}. Hence solve the following system of equations :

$$2x–3y + 5z = 11, 3x + 2y–4z = –5, x + y–2z = –3.$$

38. Obtain the inverse of the following matrix using elementary operations :

$$A = \begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$$

39. If $3A - B = \begin{pmatrix} 5 & 0 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 3 \\ 2 & 5 \end{pmatrix}$ then find the matrix A.

40. Using properties of determinants, prove the following : $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$
$= a^3 + b^3 + c^3 - 3abc.$

7 Optimization

41. A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model A is ₹15 and on an item of model B is ₹10. How many of items of each model should be made per day in order to maximize daily profit ? Formulate the above LPP and solve it graphically and find the maximum profit.

8 Probability

42. A die marked $1, 2, 3$ in red and $4, 5, 6$ in green is tossed. Let A be the event "number is even" and B be the event "number is marked red". Find whether the events A and B are independent or not.

43. A die is thrown 6 times. If "getting an odd number" is a "success", what is the probability of

    (a) 5 successes ?

    (b) at most 5 successes ?

44. The random variable X has a probability distribution $P(X)$ of the following form, where k is some number:

$$P(X = x) = \begin{cases} k, & \text{if x=0} \\ 2k, & \text{if x=1} \\ 3k, & \text{if x=2} \\ 0, & \text{Otherwise} \end{cases}$$

Determine value of $'k'$.

45. A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B

and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A ?

46. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean and variance of the number of kings.

9 Trigonometry

47. Solve $\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{4}$

48. Solve $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$.

10 Vectors

49. If a line makes angles $90°$, $135°$, $45°$ with the x, y and z axes respectively, find its direction cosines.

50. Find the vector equation of the line which passes through the point (3,4,5) and is parallel to the vector

$$2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$$

51. If $\mathbf{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}, \mathbf{b} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}, \mathbf{c} = -3\hat{\imath} + \hat{\jmath} + 2\hat{k}$, Find $\left[\mathrm{a} \quad \mathbf{b} \quad \mathbf{c}\right]$

52. If $\hat{\imath} + \hat{\jmath} + \hat{k}, 2\hat{\imath} + 5\hat{\jmath}$ and $3\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\hat{\imath} - 6\hat{\jmath} - \hat{k}$ k respectively are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether **AB** and **CD** are collinear or not.

53. Find the value of λ, So that the lines $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

54. Using integration, find the area of triangle ABC, whose vertices are $A(2, 5), B(4, 7)$ and $C(6, 2)$.

55. Find the vector and Cartesian equations of the plane passing through the points $(2, 2–1), (3, 4, 2)$ and $(7, 0, 6)$. Also find the vector equation of a plane passing through $(4, 3, 1)$ and parallel to the plane obtained above.

56. Find the vector equation of the plane that contains the lines $\mathbf{r} = (\hat{\imath} + \hat{\jmath}) + \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k})$ and the point $(–1, 3, –4)$. Also, find the length of the perpendicular drawn from the point $(2, 1, 4)$ to the plane, thus obtained.

57. If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

58. Using integration, find the area of the triangle whose vertices are $(2, 3)$, $(3, 5)$ and $(4, 4)$.