

Travelling Salesman Problem

The "Travelling Salesman Problem" is very similar to the task problem except that previously, there are additional restrictions that a seller starts from his city, visits each city once and returns to his city, so the total distance (cost or time) is the minimum.

If there are n cities, there are $(n - 1)!$ possible ways for his tour. For example, if the number of cities to be visited is 5, then there are $4!$ different combinations. Such type of problems can be solved by the assignment method.

Let c_{ij} be the distance (or cost or time) between City i to City j and

$$x_{ij} = \begin{cases} 1 & \text{if a tour includes travelling from city } i \text{ to city } j \text{ (for } i \neq j) \\ 0 & \text{otherwise} \end{cases}$$

The following example will help you in understanding the travelling salesman problem of operation research.

Steps in solving Travelling Salesman Problem

The 'Travelling salesman problem' is very similar to the assignment problem except that in the former, there are additional restrictions that a salesman starts from his city, visits each city once and returns to his home city, so that the total distance (cost or time) is minimum.

Procedure:

Step 1: Solve the problem as an assignment problem.

Step 2: Check for a complete cycle or alternative cycles. If the cycle is complete, Go to Step 4. If not, go to the Step 3.

Step 3: To start with, assign the next least element other than zero, (only for first allocation) and complete the assignment. Go to Step 2.

Step 4: Write the optimum assignment schedule and calculate the cost/time. (Note: If there are two non-zero values in the matrix, it means that there are two optimal solutions. Calculate the cost for the two allocations and find the optimal solution.)

Example:

A Travelling salesman has to visit five cities. He wishes to start from a particular city, visit each city once and then return to his starting point. The travelling cost (in Rs.) of each city from a particular city is given below.

Travelling Salesman Problem

		To city				
		A	B	C	D	E
From city	A	α	2	5	7	1
	B	6	α	3	8	2
	C	8	7	α	4	7
	D	12	4	6	α	5
	E	1	3	2	8	α

What should be the sequence of the salesman's visit, so that the cost is minimum?

Solution: The problem is solved as an assignment problem using Hungarian method; an optimal solution is reached as shown in Table.

Optimal Solution Reached Using Hungarian Method

		To city				
		A	B	C	D	E
From city	A	α	1	3	6	0
	B	4	α	0	6	∞
	C	4	3	α	0	3
	D	8	0	1	α	1
	E	0	2	∞	7	α

In this assignment, it means that the travelling salesman will start from city A, then go to city E and return to city A without visiting the other cities. The cycle is not complete. To overcome this situation, the next highest element can be assigned to start with. In this case it is 1, and there are three 1's. Therefore, consider all these 1's one by one and find the route which completes the cycle.

Case 1: Make the assignment for the cell (A, B) which has the value 1. Now, make the assignments for zeros in the usual manner. The resulting assignments are shown in table.

Resulting Assignment

		To city				
		A	B	C	D	E
From city	A	α	1	3	6	∞
	B	4	α	0	6	∞
	C	4	3	α	0	3
	D	8	∞	1	α	1
	E	0	2	∞	7	α

The assignment shown in Table 7.42 gives the route sequence $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow E$ and $E \rightarrow A$.

The travelling cost to this solution is

$$= 2000 + 3000 + 4000 + 5000 + 1000$$

$$= \text{Rs.}15,000.00$$

Case 2: If the assignment is made for cell (D, C) instead of (D, E), the feasible solution cannot be obtained. The route for the assignment will be $A \rightarrow B \rightarrow C \rightarrow D \rightarrow C$. In this case, the salesman visits city C twice and cycle is not complete.

Therefore the sequence feasible for this assignment is

$$A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow A.$$

with the travelling cost of Rs.15,000.00