Hungarian Assignment Methods

Assignment problem is a special type of linear programming problem which deals with the allocation of the various resources to the various activities on one to one basis. It does it in such a way that the cost or time involved in the process is minimum and profit or sale is maximum. Though there problems can be solved by simplex method or by transportation method but assignment model gives a simpler approach for these problems.

In a factory, a supervisor may have six workers available and six jobs to fire. He will have to take decision regarding which job should be given to which worker. Problem forms one to one basis. This is an assignment problem.

Let there be n agents and n tasks. Any agent can be assigned to perform any task, incurring some cost that may vary depending on the agent-task assignment. It is required to perform all tasks by assigning exactly one agent to each task and exactly one task to each agent in such a way that the total cost of the assignment is minimized.

Steps in solving Assignment problems using Hungarian Method

An assignment problem can be easily solved by applying Hungarian method which consists of two phases. In the first phase, row reductions and column reductions are carried out. In the second phase, the solution is optimized on iterative basis.

Phase 1

Step 0: Consider the given matrix.

Step 1: In a given problem, if the number of rows is not equal to the number of columns and vice versa, then add a dummy row or a dummy column. The assignment costs for dummy cells are always assigned as zero.

Step 2: Reduce the matrix by selecting the smallest element in each row and subtract with other elements in that row.

Phase 2:

Step 3: Reduce the new matrix column-wise using the same method as given in step 2.

Step 4: Draw minimum number of lines to cover all zeros.

Step 5: If Number of lines drawn = order of matrix, then optimally is reached, so proceed to step 7. If optimally is not reached, then go to step 6.

Step 6: Select the smallest element of the whole matrix, which is **NOT COVERED** by lines. Subtract this smallest element with all other remaining elements that are **NOT COVERED** by lines and add the element at the intersection of lines. Leave the elements covered by single line as it is. Now go to step 4.

Step 7: Take any row or column which has a single zero and assign by squaring it. Strike off the remaining zeros, if any, in that row and column (X). Repeat the process until all the assignments have been made.

Step 8: Write down the assignment results and find the minimum cost/time.

Note: While assigning, if there is no single zero exists in the row or column, choose any one zero and assign it. Strike off the remaining zeros in that column or row, and repeat the same for other assignments also. If there is no single zero allocation, it means multiple numbers of solutions exist. But the cost will remain the same for different sets of allocations.

Example:

Solve the following assignment problem shown in Table using Hungarian method. The matrix entries are processing time of each man in hours.

Assignment Problem

$$\begin{array}{cc|c}
 & & \text{Men} \\
 & & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\text{Job} & \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \text{V} \end{array} & \begin{pmatrix} 20 & 15 & 18 & 20 & 25 \\ 18 & 20 & 12 & 14 & 15 \\ 21 & 23 & 25 & 27 & 25 \\ 17 & 18 & 21 & 23 & 20 \\ 18 & 18 & 16 & 19 & 20 \end{pmatrix}
\end{array}$$

Solution: The row-wise reductions are shown in Table

Row-wise Reduction Matrix

$$\begin{array}{ccc}
 & & \text{Men} \\
 & & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\text{Job} & \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \text{V} \end{array} & \begin{pmatrix} 5 & 0 & 3 & 5 & 10 \\ 6 & 8 & 0 & 2 & 3 \\ 0 & 2 & 4 & 6 & 4 \\ 0 & 1 & 4 & 6 & 3 \\ 2 & 2 & 0 & 3 & 4 \end{pmatrix}
\end{array}$$

The column wise reductions are shown in Table.

Column-wise Reduction Matrix

$$\begin{array}{ccc} & & \textbf{Men} \\ & & \begin{array}{ccccc} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} \end{array} \\ \textbf{Job} & \begin{array}{c} \mathbf{I} \\ \mathbf{II} \\ \mathbf{III} \\ \mathbf{IV} \\ \mathbf{V} \end{array} & \begin{pmatrix} 5 & 0 & 3 & 3 & 7 \\ 6 & 8 & 0 & 0 & 0 \\ 0 & 2 & 4 & 4 & 1 \\ 0 & 1 & 4 & 4 & 0 \\ 2 & 2 & 0 & 1 & 1 \end{pmatrix} \end{array}$$

Matrix with minimum number of lines drawn to cover all zeros is shown in Table.

Matrix will all Zeros Covered

$$\begin{array}{ccc} & & \textbf{Men} \\ & & \begin{array}{ccccc} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} \end{array} \\ \textbf{Job} & \begin{array}{c} \mathbf{I} \\ \mathbf{II} \\ \mathbf{III} \\ \mathbf{IV} \\ \mathbf{V} \end{array} & \begin{pmatrix} 5 & 0 & 3 & 3 & 7 \\ 6 & 8 & 0 & 0 & 0 \\ 0 & 2 & 4 & 4 & 1 \\ 0 & 1 & 4 & 4 & 0 \\ 2 & 2 & 0 & 1 & 1 \end{pmatrix} \end{array}$$

The number of lines drawn is 5, which is equal to the order of matrix. Hence optimality is reached. The optimal assignments are shown in Table.

Optimal Assignment

$$\begin{array}{cc} & \textbf{Men} \\ & \begin{array}{ccccc} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} \end{array} \\ \textbf{Job} \quad \begin{array}{c} \textbf{I} \\ \textbf{II} \\ \textbf{III} \\ \textbf{IV} \\ \textbf{V} \end{array} & \begin{pmatrix} 5 & \boxed{0} & 3 & 3 & 7 \\ 6 & 8 & \cancel{0} & \boxed{0} & \cancel{0} \\ \boxed{0} & 2 & 4 & 4 & 1 \\ \cancel{0} & 1 & 4 & 4 & \boxed{0} \\ 2 & 2 & \boxed{0} & 1 & 1 \end{pmatrix} \end{array}$$

Therefore, the optimal solution is:

Job	Men	Time
I	2	15
II	4	14
III	1	21
IV	5	20
V	3	16
	Total time =	86 hours