Unbalanced Assignment Problems

When the cost matrix for the assignment problem is not a square matrix, that is, whenever the number of sources does not equal the number of destinations, the assignment problem is called an unbalanced allocation problem. In such problems, imaginary rows (or columns) are added to the array to complete to form a square array. Phantom rows or columns will contain all cost elements such as zeros. The Hungarian method can be used to solve the problem.

If number of jobs is not equal to the number of operators then the assignment problem is known as unbalanced assignment problem. In other words in a pay off matrix of assignment problem if number of rows and number of columns are not equal then it is unbalanced assignment problem.

To solve unbalanced assignment problem it is necessary to insert dummy row or dummy column such that the given pay off matrix become a square matrix. All values in dummy row or column are assume to be zero.

Example:

In a plant layout, four different machines M1, M2, M3 and M4 are to be erected in a machine shop. There are five vacant areas A, B, C, D and E. Because of limited space, Machine M2 cannot be erected at area C and Machine M4 cannot be erected at area A. The cost of erection of machines is given in the Table.

Assignment Problem

$$
\begin{array}{ccc}
 & & \textbf{Area} \\
 & & \begin{array}{ccccc} \textbf{A} & \textbf{B} & \textbf{C} & \textbf{D} & \textbf{E} \end{array} \\
\textbf{Machine} & \begin{array}{c} M_1 \\ M_2 \\ M_3 \\ M_4 \end{array} & \begin{pmatrix} 4 & 5 & 9 & 4 & 5 \\ 6 & 4 & -- & 4 & 3 \\ 4 & 5 & 8 & 5 & 1 \\ -- & 2 & 6 & 1 & 2 \end{pmatrix}
\end{array}
$$

Find the optimal assignment plan.

Solution:

As the given matrix is not balanced, add a dummy row D5 with zero cost values. Assign a high cost H for (M2, C) and (M4, A). While selecting the lowest cost element neglect the high cost assigned H, as shown in Table below.

Dummy Row D5 Added

$$\begin{array}{ll} & \begin{array}{ccccc} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} & \mathbf{E} \end{array} \\ \begin{array}{c} \mathbf{M_1} \\ \mathbf{M_2} \\ \mathbf{M_3} \\ \mathbf{M_4} \\ \mathbf{D_5} \end{array} & \begin{pmatrix} 4 & 5 & 9 & 4 & 5 \\ 6 & 4 & H & 4 & 3 \\ 4 & 5 & 8 & 5 & 1 \\ H & 2 & 6 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Machine

- Row-wise reduction of the matrix is shown in Table.

Matrix Reduced Row-wise

$$\begin{array}{ll} & \begin{array}{ccccc} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} & \mathbf{E} \end{array} \\ \begin{array}{c} \mathbf{M_1} \\ \mathbf{M_2} \\ \mathbf{M_3} \\ \mathbf{M_4} \\ \mathbf{D_5} \end{array} & \begin{pmatrix} 0 & 1 & 5 & 0 & 1 \\ 3 & 1 & H & 1 & 0 \\ 3 & 4 & 7 & 4 & 0 \\ H & 1 & 5 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Machine

Note: Column-wise reduction is not necessary, as each column has at least one single zero. Now, draw minimum number of lines to cover all the zeros, see Table.

Lines Drawn to Cover all Zeros

$$
\textbf{Machine}\quad
\begin{array}{c}
 \\ M_1 \\ M_2 \\ M_3 \\ M_4 \\ D_5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} & \mathbf{E} \end{array} \\
\begin{pmatrix}
0 & 1 & 5 & 0 & 1 \\
3 & 1 & H & 1 & 0 \\
3 & 4 & 7 & 4 & 0 \\
H & 1 & 5 & 0 & 1 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

Number of lines drawn ≠ Order of matrix. Hence not Optimal. Select the smallest uncovered element, in this case 1. Subtract 1 from all other uncovered element and add 1 with the elements at the intersection. The element covered by single line remains unchanged. These changes are shown in Table. Now try to draw minimum number of lines to cover all the zeros.

Added or Subtracted 1 from Elements

$$
\textbf{Machine}\quad
\begin{array}{c}
 \\ M_1 \\ M_2 \\ M_3 \\ M_4 \\ D_5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} & \mathbf{E} \end{array} \\
\begin{pmatrix}
0 & 1 & 5 & 1 & 2 \\
2 & 0 & H & 1 & 0 \\
2 & 3 & 6 & 4 & 0 \\
H & 0 & 4 & 0 & 1 \\
0 & 0 & 0 & 1 & 1
\end{pmatrix}
\end{array}
$$

Now number of lines drawn = Order of matrix, hence optimality is reached. Optimal assignment of machines to areas is shown in Table.

Optimal Assignment

$$
\text{Machine}\qquad
\begin{array}{cc}
 & \begin{matrix} A & \;B & \;C & \;D & \;E \end{matrix} \\
\begin{matrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ D_5 \end{matrix} &
\begin{pmatrix}
\boxed{0} & 1 & 5 & 1 & 2 \\
2 & \boxed{0} & H & 1 & \cancel{0} \\
2 & 3 & 6 & 4 & \boxed{0} \\
H & 0 & 4 & \boxed{0} & 1 \\
\cancel{0} & \cancel{0} & \boxed{0} & 1 & 1
\end{pmatrix}
\end{array}
$$

Hence, the optimal solution is:

Machines	Area	Erection Cost
M_1	A	4
M_2	B	4
M_3	C	1
M_4	D	1
D_5	E	0
Total Erection Cost = Rs.10.00		