Classmate

Date
Page

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1)

$\mu_1 = 9.2$ $\mu_2 = 9.6$

$\sigma_1 = 1.6$ $\sigma_2 = 1.2$.

(a) Given,

$$P_1 + P_2 = 1$$

let P_1 be x

then $P_2 = 4x$

where P_1 is Probability that rock is from A

$$5x = 1$$

$$x = \frac{1}{5}$$

The probability the rock is from B is $\frac{4}{5}$

$$P_A = \frac{1}{5} \quad P_B = \frac{4}{5}$$

(b) Weight of rock $r_1 = 9.3$

" " rock $r_2 = 8.8$

" " " $r_3 = 9.8$

Posterior Probability = $P(A/D)$ - probability that the rock is from location A.

$$P(A/D) = \frac{P(A) \cdot P(D/A)}{P(D)}$$

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Date
Page

$P(A)$ = prior probability = $\frac{1}{5}$

$P(D/A)$ = likelihood.

$P(D)$ = Evidence = $P(A)\,P(D/A) + P(B)\,P(D/B)$

P.D.f of Gaussian distribution.

$$= \frac{1}{\sqrt{2\pi} * \sigma_1} e^{-(x_i-\mu_1)^2/2\sigma_1^2}$$

$p(D = x_1 x_2 x_3 / A)$

$$= \frac{1}{\sqrt{2\pi}\,\sigma_1} e^{-(x_1-\mu_1)^2/2\sigma_1^2} * \frac{1}{\sqrt{2\pi}\,\sigma_1} e^{\frac{-(x_2-\mu_1)^2}{2\sigma_1^2}} * \frac{1}{\sqrt{2\pi}\sigma_1} e^{\frac{-(x_3-\mu_1)^2}{2\sigma_1^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma_1}\right)^3 e^{\frac{-1}{2\sigma_1^2}\left[(x_1-\mu_1)^2 + (x_2-\mu_1)^2 + (x_3-\mu_1)^2\right]}$$

$$= \left(\frac{1}{\sqrt{2\times3.14}\times1.6}\right)^3 e^{\frac{-1}{2\times(1.6)^2}\left[(9.3-9.2)^2 + (8.8-9.2)^2 + (9.8-9.2)^2\right]}$$

$$= \left(\frac{1}{4.009}\right)^3 \times e^{\frac{-1}{5.12}\left[(0.1)^2 + (0.4)^2 + (0.6)^2\right]}$$

$$= \left(\frac{1}{4.009}\right)^3 \times e^{\frac{-1}{5.12}\left[0.1 + 0.16 + 0.36\right]}$$

$$= \left(\frac{1}{4.009}\right)^3 \times e^{\frac{-1}{5.12} \times 0.53} = \frac{1}{(4.)^3} \times e^{-0.103516}$$

$= 0.014$

Date
Page 2

$$P(D/B) = \left(\frac{1}{\sqrt{2\pi}\,\sigma_2}\right)^3 \times e^{-\frac{(\mu_2 - x_1)^2}{2\sigma^2}} \times e^{-\frac{1}{2\sigma^2}(x_2-\mu_2)^2} \times e^{-\frac{1}{2\sigma^2}(x_3-\mu_2)^2}$$

$$= \left(\frac{1}{\sqrt{2 \times 3.14} \times 1.2}\right)^3 \times e^{-\frac{1}{2 \times 1.2^2}\left[(9.6-9.3)^2 + (8.8-9.6)^2 + (9.8-9.6)^2\right]}$$

$$= \left(\frac{1}{0.33}\right)^3 \times e^{-\frac{1}{2.88}\left[(0.3)^2 + (0.8)^2 + (0.2)^2\right]}$$

$$= 0.037 \times e^{-\frac{1}{2.88}\left[0.09 + 0.64 + 0.04\right]}$$

$$= 0.037 \times e^{-0.267}$$

$$= ~~0.037 \times~~ \; 0.028$$

$$P(D) = P(A) \times P(D/A) + P(B) \times P(D/B)$$

$$= \frac{1}{5} \times 0.014 + \frac{4}{5} \times 0.028$$

$$= 0.0028 + 0.022$$

$$= 0.025$$

$$P(A/D) = \frac{1}{5} \times 0.014 \times \frac{1}{0.025}$$

$$= 0.112$$

Posterior Probability that rocks are from A

$= 0.112$

(c) $P(D/B) > P(D/A)$

$P(D/B)$ is the maximum likelihood

So the max. likelihood hypothesis is that weights of the rocks are from Location B.

2)

H_1 – has the disease.

H_2 – does not have the disease.

D_1 : +ve (result obtained.

$$P(H_1/D_1) = \frac{P(D/H)\,P(H)}{P(D)}$$

$$P(D) = P(H_1) \times P(D/H_1) + P(H_2) \times P(D/H_2)$$

$P(D)$ is the total Probability.

$$P(H_1) = \frac{0.02}{100}$$ (0.02% has the disease)

$$= 0.0002$$

$$P(H_2) = 0.9998$$

Date ______
Page ______

False Negative $F_{N1} = 10$

False Positive $F_{P1} = 15$

$P(D_1/H_1) = 1 - F_{N1} = 1 - 10\% = 0.9$

$P(D_1/H_2) = F_{P1} = 0.15$

$$P(H_1/D_1) = \frac{0.9 \times 0.0002}{0.0002 \times 0.9 + 0.9998 \times 0.15}$$

$$= \frac{0.9 \times 0.0002}{0.150150}$$

$= 0.001199$

$= 0.1\%$

$$P(H_2/D_1) = \frac{0.15 \times 0.9998}{0.150150}$$

$= 99.9\%$

(a) MAP hypothesis: The person does not have the disease. It shows false positive.

(b) ML hypothesis: The person has the disease.

argmax $P(x/c)$

$0.9 > 0.15$

So Patience has the disease.

(c) Given:
both the results are independent.

D_2 : +ve. D_1 : +ve.

$$P(H_1/D_1 D_2) = \frac{P(D_1 H_1)\, P(D_2 H_1) \times P(H_1)}{P(D_1, D_2)}$$

$P(D_1, D_2)$ is the total evidence (2 positive)

$P(H_1) = 0.0002$ $P(H_2) = 0.9998$

$P(D_1/H_1) = 0.9$ $P(D_1/H_2) = 0.15$

$P(D_2/H_2) = 1 - F_{N} = 1 - 3\% = 0.97$

$P(D_2/H_2) = 5\% = 0.05$

$$P(D_1 D_2) = P(H_1) \times P(D_1/H_1) * P(D_2/H_1) + P(H_2) P(D_1/H_2) P(D_2/H_2)$$

$= 0.0002 \times 0.9 * 0.97 + 0.9998 \times 0.15 \times 0.05$
$= 0.007673.$

$$P(H_1/D_1 D_2) = \frac{0.9 * 0.97 \times 0.0002}{0.007673}$$

$= \boxed{0.02\%}$

Date
Page

3) A) let us take $x = 1$, if head comes up.
$x = 0$, if tails comes up.

if only has two possible values 1 and 0 with probabilities p and 1-p

$$p(x) = \begin{cases} \theta^{x}(1-\theta)^{1-x} & \text{for } x \in (0,1) \\ 0 & \text{otherwise} \end{cases}$$

(a) $$P(TTHTTH|\theta) = \prod_{t=1}^{N} \theta^{x^t}(1-\theta)^{1-x^t}$$

$$= \theta^{0}(1-\theta)^{1-0} * \theta^{0}(1-\theta)^{1-0} * \theta^{1}(1-\theta)^{1-1} * \theta^{0}(1-\theta)^{1-0} * \theta^{0}(1-\theta)^{1-0} \times \theta^{1}(1-\theta)^{1-1}$$

$$= 1(1-\theta) * 1(1-\theta) \times \theta \times 1(1-\theta) * (1-\theta) \times \theta$$

$$= \theta^{2}(1-\theta)^{4}$$

(b) $$\log P(TTHTTH|\theta) = \log \prod_{t=1}^{N} \theta^{(x^t)}(1-\theta)^{(1-x^t)}$$

$$= \sum_{t}^{N} x^t \log\theta + \left(N - \sum_{t} x^t\right)\log(1-\theta)$$

$$= (0\times\log\theta + 0\times\log\theta + 1\times\log\theta + 0*\log\theta + 0*\log\theta + 1\log\theta) + (6 - (0+0+1+0+0+1)) * \log(1-\theta)$$

$$= 2\log\theta + (6-2)\log(1-\theta) = 2\log\theta + 4\log(1-\theta)$$

classmate

Date
Page

$a = 2 \quad b = 4$

(c) To find the ML of θ. we need to find argmax $P(TTHTTH|\theta)$. But this is tough so we find argmax log $P(HTTHH|\theta)$

Since from (b), we know that:

$P(\theta) = 2\log\theta + 4\log(\theta - 1)$

$\frac{\partial P(\theta)}{\partial\theta} \rightarrow$ this is the argmax.

$$\frac{\partial P(\theta)}{\partial\theta} = \frac{\partial(2\log\theta + 4\log(\theta-1))}{\partial\theta}$$

$$\Rightarrow \frac{2}{\theta} - \frac{4}{1-\theta} = 0$$

$$\Rightarrow \frac{2(1-\theta) - 4\theta}{\theta(1-\theta)} = 0$$

$$2 - 2\theta - 4\theta = 0$$

$$2 = 6\theta$$

$$\theta = \frac{1}{3}$$

4)

A)

A (a) We know that estimate θ_i can be calculated as $\frac{t}{N}$ for an unsmoothed

$$X = \{3, 1, 1, 2, 3\}, \quad S = 3$$

ML Estimate for Θ_1 -

$v = 1 \qquad t = 2, \; N = 5$

$$\Rightarrow \frac{t}{N} = \frac{2}{5}$$

(b) If we use add-1 smoothening, with

$$X = \{3, 1, 1, 2, 3\} \qquad S = 3$$

$$\text{estimate} = \frac{t+1}{N+S} = \frac{2+1}{5+3}$$

$$= \frac{3}{8}$$

(c) ~~pdf $p(\theta_1) = 6\theta(1-\theta)$~~

~~$S = 2$~~

Page

4(C)

A) $\delta = 2$

$P(\theta) = 6\theta(1-\theta)$

We are given that N independent occurrences are observed, ~~where~~

$P(x/\theta) = \theta^{t}(1-\theta)^{N-t}$. Since only 2 values are possible,

Now, we know

$$MAP = \underset{\theta}{\operatorname{argmax}}\, P(\theta)\, p(x/\theta)$$

$P(\theta) = 6\theta(1-\theta)$ $\quad P(x/\theta) = \theta^{t}(1-\theta)^{N-t}$

$$= \underset{\theta}{\operatorname{argmax}}\, 6\theta(1-\theta)\left(\theta^{t}(1-\theta)^{N-t}\right)$$

On taking the differentiation of a maxed out value we get 0 $\frac{d(f(x))}{dx} = 0 \Rightarrow$ when x maxes out $f(x)$

$$\cancel{\frac{d}{d\theta}\left(6\theta(1-\theta)\left(\theta^{t}(1-\theta)^{N-t}\right)\right)} = 0$$

Date
Page

Since, its difficult to calculate.

$$\underset{\theta}{\arg\max}\ 6\theta(1-\theta)\left(\theta^t(1-\theta)^{N-t}\right)$$

we can take log.

$$\underset{\theta}{\arg\max}\ \log\left(6\theta(1-\theta)\left(\theta^t(1-\theta)^{N-t}\right)\right)$$

$$\underset{\theta}{\arg\max}\left[\log 6\theta + \log(1-\theta) + \log(\theta^t) + \log(1-\theta)^{N-t}\right]$$

Now on differentiating

$$\frac{d}{d\theta}\left[\log(6\theta) + \log(1-\theta) + \log\theta^t + \log(1-\theta)^{N-t}\right] = 0$$

$$\cancel{\log 6} + \frac{1}{\theta} + \frac{(-1)}{1-\theta} + \frac{(t\theta^{t-1})}{\theta^t} + \frac{(N-t)(1-\theta)^{N-t-1}(-1)}{(1-\theta)^{N-t}} = 0$$

$$\cancel{\log 6}\ \theta^{-1} - (1-\theta)^{-1} + t\theta^{-1} + (N-t)(1-\theta)^{-1}(-1) = 0$$

$$\theta^{-1}(1+t) - (1-\theta)^{-1}\left[1+N-t\right] = 0$$

$$\theta^{-1}(1+t) = (1-\theta)^{-1}\left[1+N-t\right]$$

$$\frac{1+t}{\theta} = \frac{1+N-t}{1-\theta}$$

$$(1+t)(1-\theta) = \theta + N\theta - \theta t$$

$$1-\theta+t-\cancel{\theta t} = \theta + N\theta - \cancel{\theta t}$$

$$1+t = 2\theta + N\theta$$

$$\Rightarrow \quad 1+7 = \theta(N+2)$$

$$\theta = \frac{1+7}{2+N}$$

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5)

Given:

C_1 and C_2 are classes.

$C_1 = +$, $C_2 = -$

$x_1 \in \{High, Medium, low\}$

$x_2 \in \{Yes, no\}$

$x_3 \in \{Red, Green\}$

[redacted]

[redacted]

(a) $P(x_1 = low/+) = \frac{1 + 0.3}{2 + 0.3 \times 3} = \frac{1.3}{2.9} = \frac{13}{29}$.

$P(x_2 = Yes/+) = \frac{0 + 0.3}{2 + 2 \times 0.3} = \frac{0.3}{2.6} = \frac{3}{26}$

$P(x_3 = Green/+) = \frac{1 + 0.3}{2 + 0.3 \times 2} = \frac{1.3}{2.6} = \frac{1}{2}$

$P(x_1 = low/-) = \frac{1 + 0.3}{3 + 0.3 \times 3} = \frac{1.3}{3.9} = \frac{1}{3}$

$P(x_2 = Yes/-) = \frac{2 + 0.3}{3 + 0.3 \times 2} = \frac{2.3}{3.6} = \frac{23}{36}$

$$P(x_3 = \text{Green}/-) = \frac{2 + 0.3}{3 + 0.3 \times 2} = \frac{2.3}{3.6}.$$

(b)

$$P(X/C) = P(x_1 = \text{low}/C) * P(x_2 = \text{Yes}/C) * P(x_3 = \text{Green}/C)$$

$$P(X/+) = P(\text{low}/+) * P(\text{Yes}/+) * P(\text{Green}/+)$$

$$= \frac{\cancel{13}\,1}{\cancel{29}} * \frac{3}{\cancel{26}\,2} * \frac{1}{2}$$

$$= \frac{3}{116} = 0.025$$

$$P(X/-) = P(\text{low}/-) * P(\text{Yes}/-) * P(\text{Green}/-)$$

$$= \frac{1}{3} \times \frac{23}{36} \times \frac{23}{36}.$$

$$= \cancel{0.076}\ \cancel{0.077}\ 0.13$$

(c)

$$P(X/-) > P(X/+)$$

which implies that

ML is that label is '−' ve.

classmate

Date

Page

(d)

$$P(+/x) = \frac{P(+) \times P(x/+)}{P(x)}$$

$$P(-/x) = \frac{P(-) \times (P(x/-))}{P(x)}$$

$$P(+/x) \;?\; P(-/x)$$

$$\frac{P(+) \times P(x/+)}{\cancel{P(x)}} \;?\; \frac{P(-)(P(x/-))}{\cancel{P(x)}}$$

$$\frac{2}{\cancel{5}} P(x/+) \;?\; \frac{3}{\cancel{5}} P(x/-)$$

$$2 \times 0.025 \;?\; 3 \times 0.13$$

$$0.050 \;?\; 0.39$$

$$0.050 < 0.39.$$

The MAP label is '−' ve.