Producer Theory (Cost and Cost Minimization Analysis-1)

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Introduction

- **cost information** cannot make optimal production decisions without cost information.

- The key question: **how are costs related to output**?

- Virtually any business decision requires comparison between costs and benefits.

Introduction

- Information about cost is necessary for a variety of basic managerial decisions:
- Pricing output
- Strategic behavior against actual and potential competitors: Implies actions **today** that are chosen *because* they will change rivals' behavior **now** (actual competitors) or **in the future** (potential entrants)—shaping prices, choices, and even whether new firms enter at all.
- Cost control and planning for future production needs.

Cost Minimization Problem

- In making cost minimizing choices, firms must look at the cost of using capital and labor in production decisions.
- The cost-minimization problem is an example of constrained optimization.
- We want to minimize the firm's total costs, subject to the requirement that the firm produce a given amount of output.

- Let's study the long-run cost-minimization problem (why long-run?) for a firm that uses two inputs; labour and capital (both inputs are variable inputs)

- Each input has a price. The price of a unit of labour services also called the wage rate is *w*. This price per unit of capital services is *r*.

- The price of labour could be either an explicit cost or an implicit cost.
- It would be an explicit cost if the firm (as most firms do) hires workers in the open market.

- It would be an implicit cost if the firm's owner provides her own labour to run the firm and, in so doing, sacrifices outside employment opportunities.

- Similarly, the price of capital could either be an explicit cost or an implicit cost.

- It would be an explicit cost if the firm leased capital services from another firm (e.g., a firm that leases computer time on a server to host its website).

- It would be an implicit cost if the firm owned the physical capital and, by using it in its own business, sacrificed the opportunity to sell capital services to other firms.

- The firm has decided to produce Q_0 units of output during the next year.
- For now, the quantity Q_0 is exogenous (e.g., as if the manufacturing manager of the firm has been told how much to produce).
- The long-run cost-minimization problem facing the manufacturing manager is to figure out how to produce that amount in the cost-minimizing way.

- The manager must choose a quantity of capital K and a quantity of labour L that minimize the total cost $TC = wL + rK$ of producing Q_0 units of output.

- This total cost is the sum of all the economic costs the firm incurs when it uses labour and capital services to produce output.

Economic Vs Accounting costs

- **Economic cost** is the total value of all resources used to produce a good or service, measured in terms of the **best alternative use** of those resources.
- It includes **both**:
- **Explicit costs** – actual monetary payments made to acquire resources (e.g., wages, rent, materials).
- **Implicit costs** – the *opportunity costs* of using resources you already own (e.g., foregone salary if you use your time to run your own business instead of working elsewhere, or the interest you could have earned if you invested your capital in another project).

- Thus,
- Economic Cost=Explicit Cost + Implicit Cost
- **Example:**
- Suppose you run a bakery:
- **Explicit costs:**
  - Flour, sugar, butter: ₹50,000/month
  - Wages to workers: ₹40,000/month
  - Rent: ₹30,000/month

    Total explicit costs = ₹1,20,000/month

- **Implicit costs:**
  - You gave up a ₹60,000/month job to run the bakery.
  - You invested ₹5,00,000 of your own savings, which could have earned ₹5,000/month in interest.
    Total implicit costs = ₹65,000/month
- **Economic cost:**
- ₹1,20,000+₹65,000=₹1,85,000 per month
- **Key difference from accounting cost:**
- **Accounting cost** only considers **explicit costs** (₹1,20,000 in this case).
- **Economic cost** also includes **opportunity costs**, making it a better measure for decision-making.

Cost minimizing input choices

- The Isocost Line
  - A line showing all combinations of L & K that can be purchased for the same cost
  - Total cost of production is sum of firm's labor cost, wL and its capital cost rK

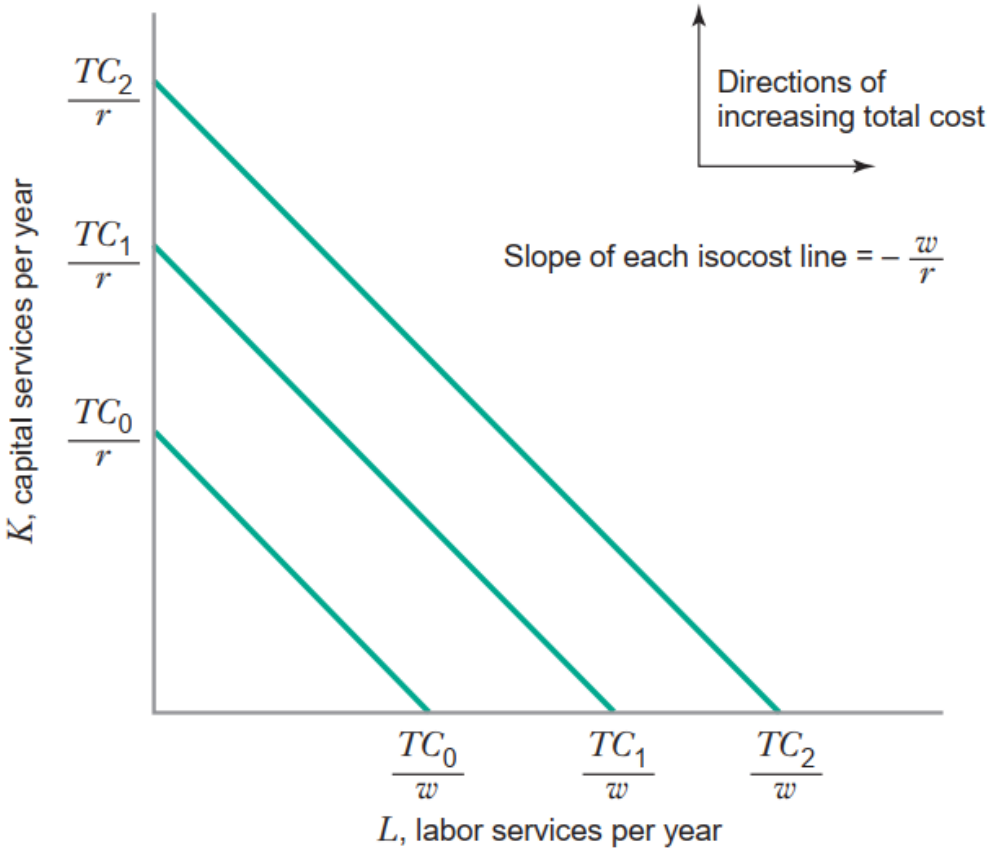
$$TC = wL + rK$$

  - For each different level of cost, the equation shows another isocost line

Cost in the Long Run

- Rewriting TC as an equation for a straight line:
  - K = TC/r - (w/r)L
  - Slope of the isocost: $\Delta K / \Delta L = -(w/r)$
    - -(w/r) – is the ratio of the wage rate to rental cost of capital.
    - This shows the rate at which capital can be substituted for labor with no change in cost.

Figure A


Directions of
increasing total cost
Slope of each isocost line = $-\frac{w}{r}$
$\frac{TC_2}{r}$
$\frac{TC_1}{r}$
$\frac{TC_0}{r}$
K, capital services per year
$\frac{TC_0}{w}$
$\frac{TC_1}{w}$
$\frac{TC_2}{w}$
L, labor services per year

- There are an infinite number of isocost lines, one corresponding to every possible level of total cost.

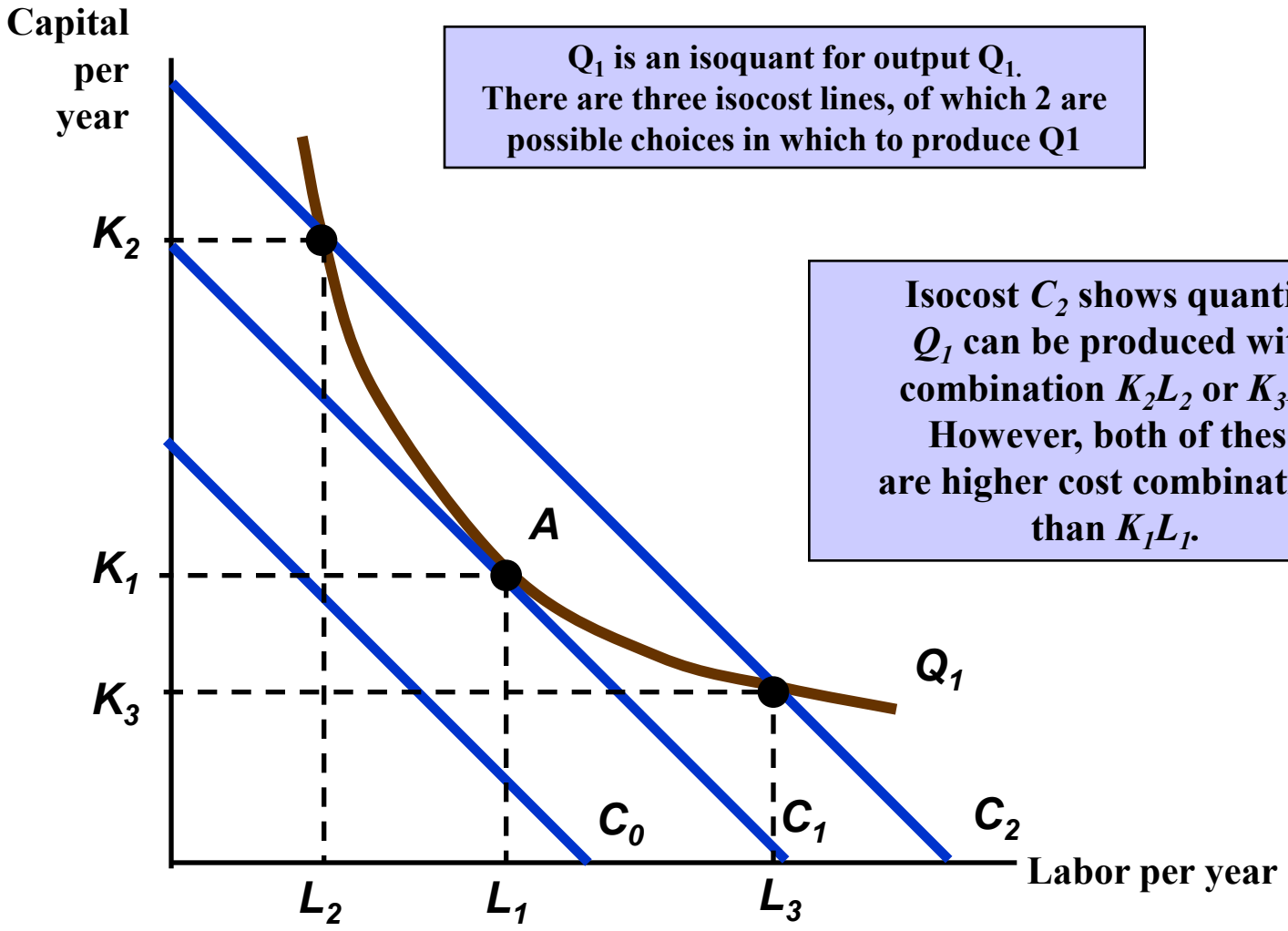
- Figure-A illustrates that the slope of every isocost line is the same:

- With K on the vertical axis and L on the horizontal axis, that slope is $-w/r$ (the negative of the ratio of the price of labour to the price of capital).

Choosing Inputs

- We will address how to minimize cost for a given level of output by combining isocosts with isoquants
- We choose the output we wish to produce and then determine how to do that at minimum cost
  - Isoquant is the quantity we wish to produce
  - Isocost is the combination of K and L that gives a set cost

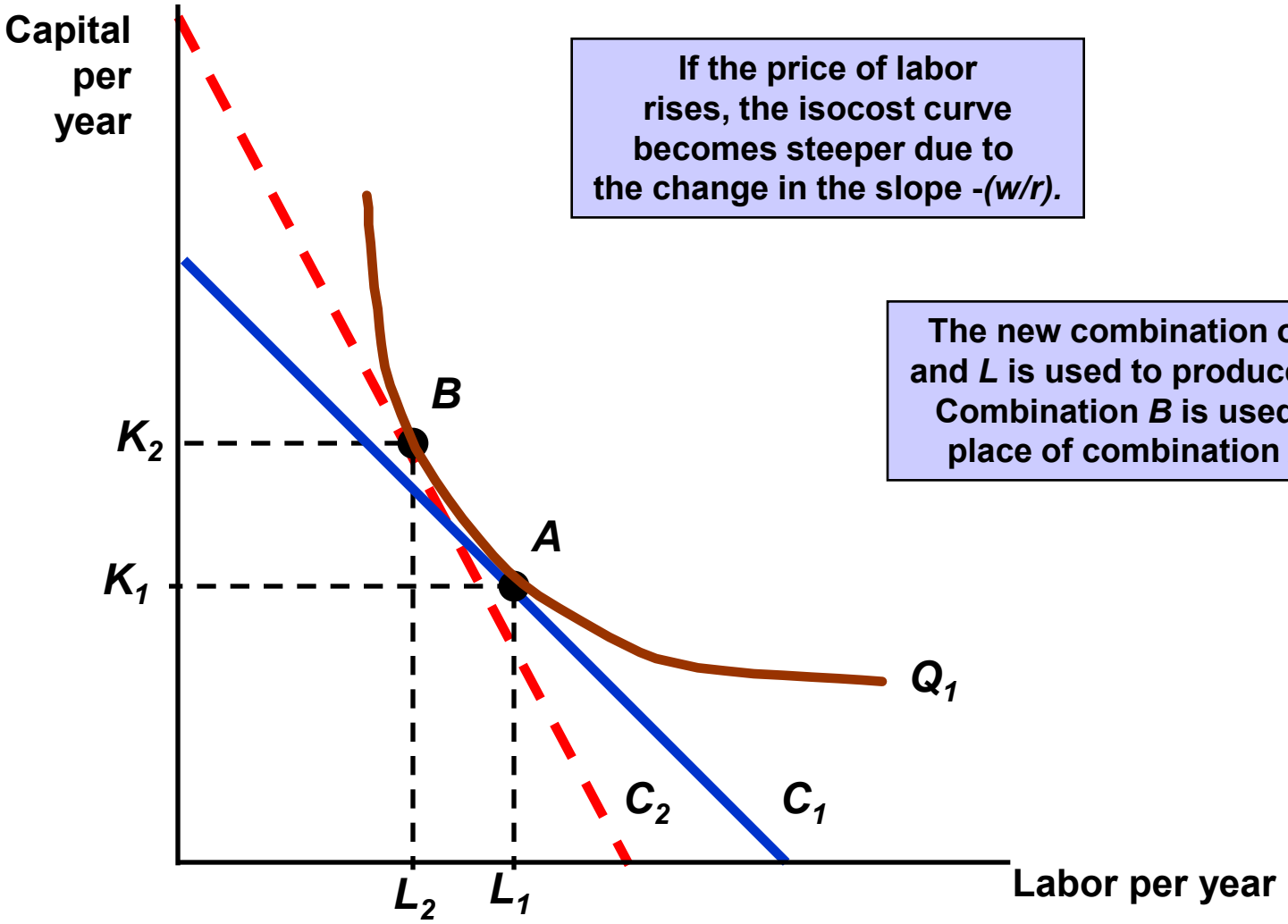
Producing a Given Output at Minimum Cost



Input Substitution When an Input Price Change

- If the price of labor changes, then the slope of the isocost line change.
- It now takes a new quantity of labor and capital to produce the output.
- If price of labor increases relative to price of capital, then capital is substituted for labor.

Input Substitution When an Input Price Change (Figure B)



Cost minimization

- How does the isocost line relate to the firm's production process?

$$\text{MRTS} = -\Delta K/\Delta L = \text{MP}_L/\text{MP}_K$$

$$\text{Slope of isocost line} = \Delta K/\Delta L = -w/r$$

$$MP_L/MP_K = w/r \text{ when firm minimizes cost}$$

Cost minimization

- The optimal input combination A is an interior optimum. An interior optimum involves positive amounts of both inputs ($L>0$ and $K>0$), and the optimum occurs at a tangency between the isoquant and an isocost line.
- The minimum cost combination can then be written as:

$$MP_L/w = MP_K/r$$

- Minimum cost for a given output will occur when each rupee/dollar of input added to the production process will add an equivalent amount of output.

- To see why equation (previous slide) must hold, consider a non–cost-minimizing point in Figure C, such as *E*. At point *E,* the slope of the isoquant is more negative than the slope of the isocost line. Therefore, $-\left(\frac{MP_L}{MP_K}\right) < -(\frac{w}{r})$ or $(\frac{MP_L}{MP_K})$>(w/r)

- Or $\frac{MP_L}{w} > \frac{MP_K}{r}$

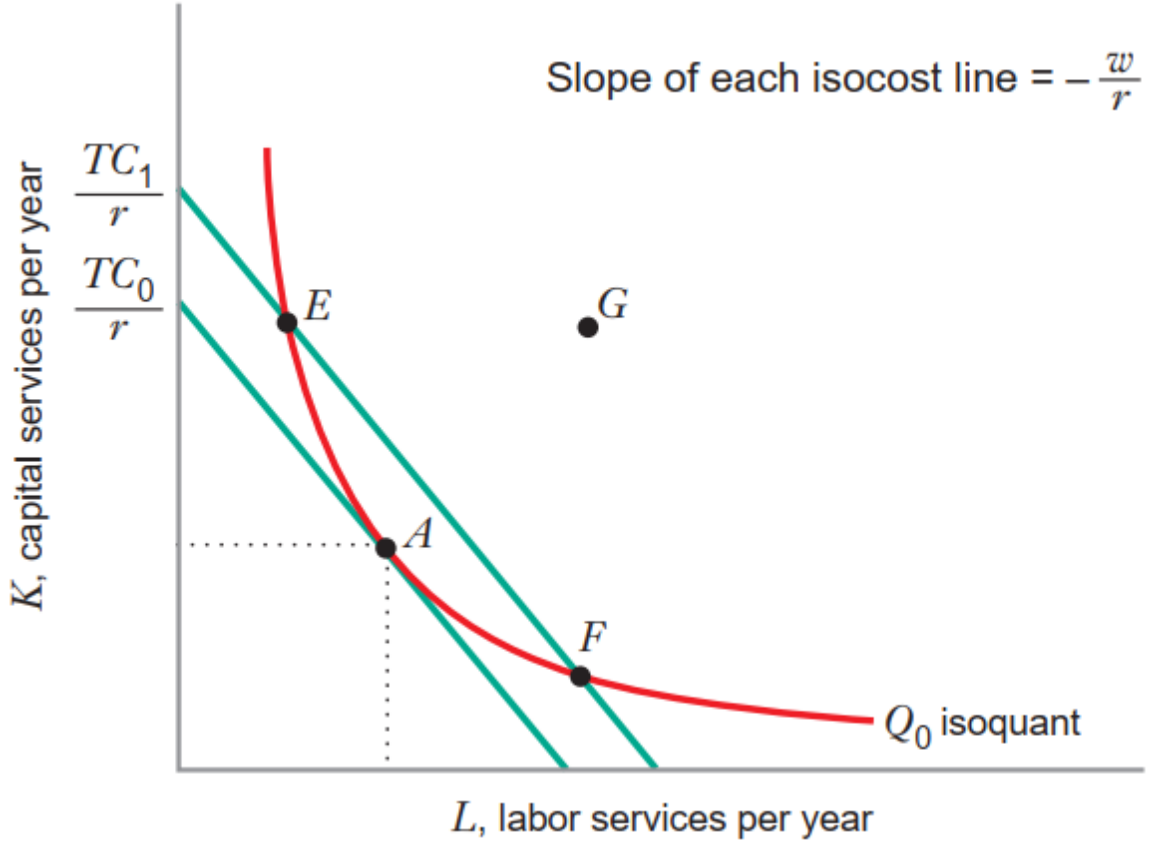
- This condition implies that a firm operating at *E* could spend an additional dollar on labour and save *more than one dollar* by reducing its employment of capital services in a manner that keeps output constant.

- Since this would reduce total costs, it follows that an interior input combination, such as *E,* at which equation

$$MP_L/w = MP_K/r$$

does not hold and hence cannot be cost-minimizing.

Figure C


Slope of each isocost line = $-\frac{w}{r}$
$\frac{TC_1}{r}$
$\frac{TC_0}{r}$
E
G
A
F
Q_0 isoquant
K, capital services per year
L, labor services per year

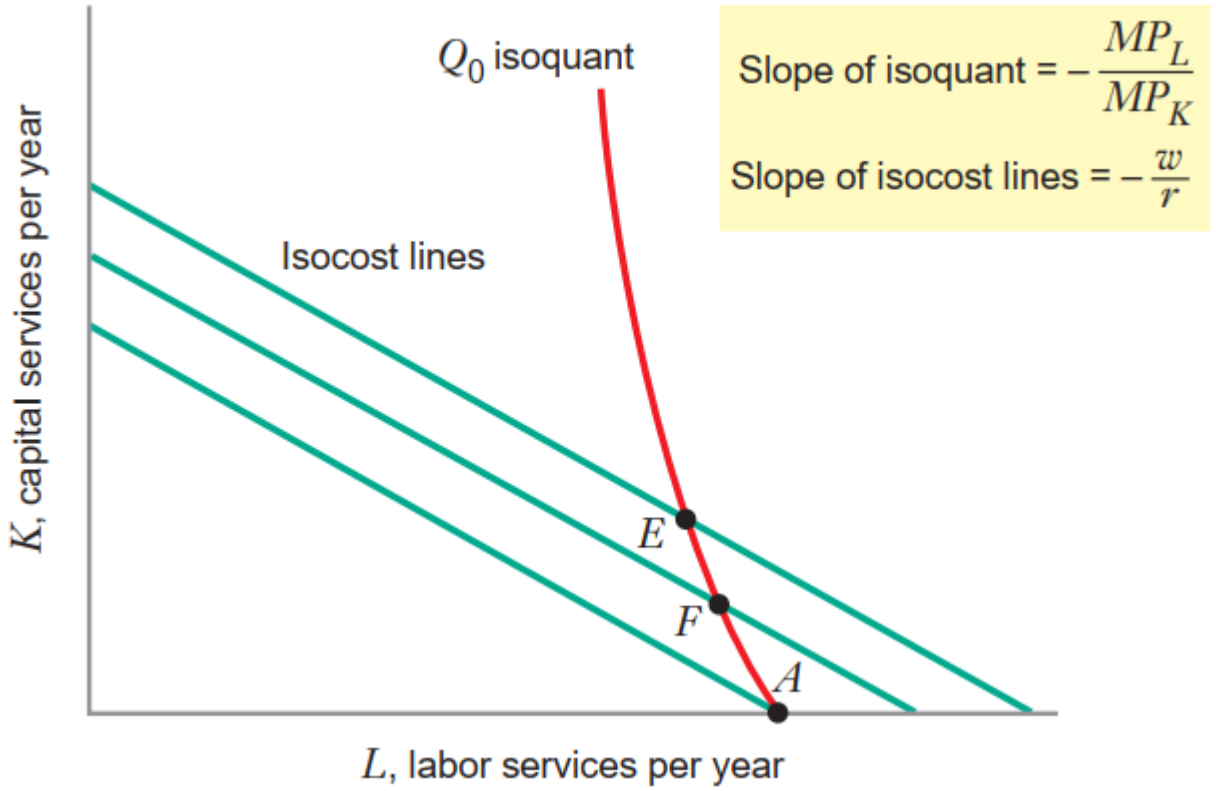
Cost minimization

- If w = \$10, r = \$2, and suppose adding a unit of labor and capital will increase output by 20 units which input would the producer use more of?
- The additional output per dollar of labor and capital inputs would be 20/10=2 and 20/2=10 respectively.
- Because a dollar spent for capital is five times more productive than a dollar spent for labor the firm will want to use more
  - Capital and less labor
  - Increasing capital lowers MPK
  - Decreasing labor raises MPL
  - Substitute capital for labor until $\frac{MP_L}{w} = \frac{MP_K}{r}$
  - And firm is minimizing its cost of production

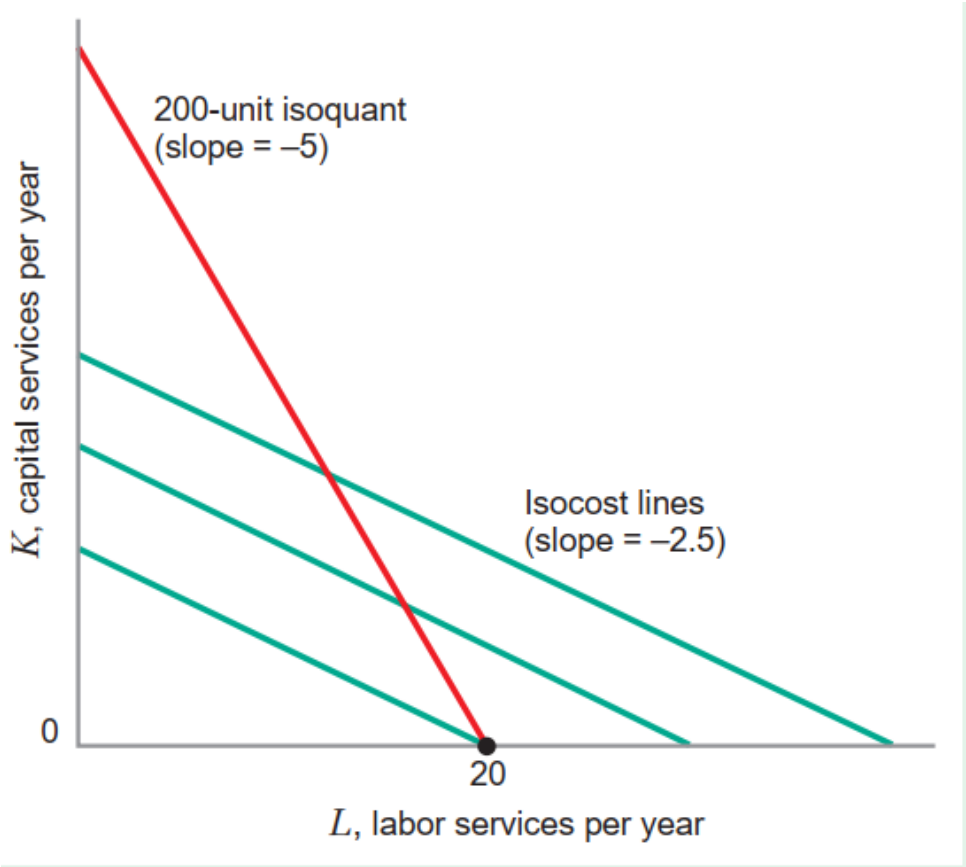
Finding an interior cost minimization optimum

- Problem to be discussed in the class.

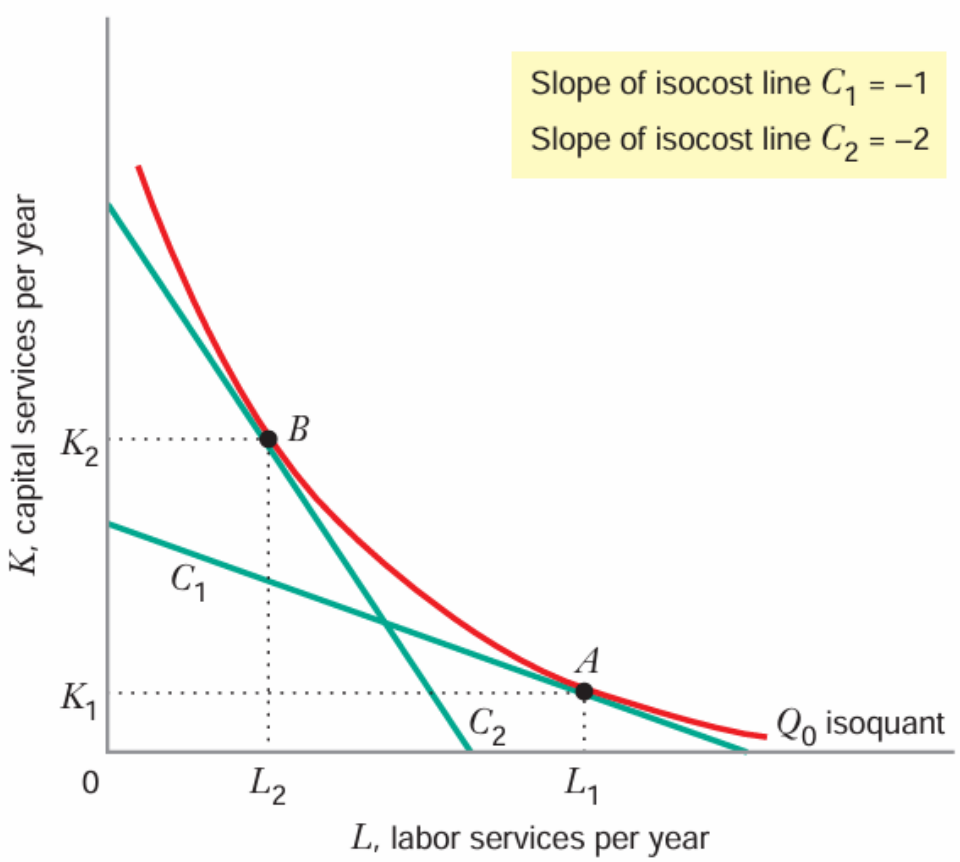
Corner Point Solution



Finding Corner Point with Perfect substitutes



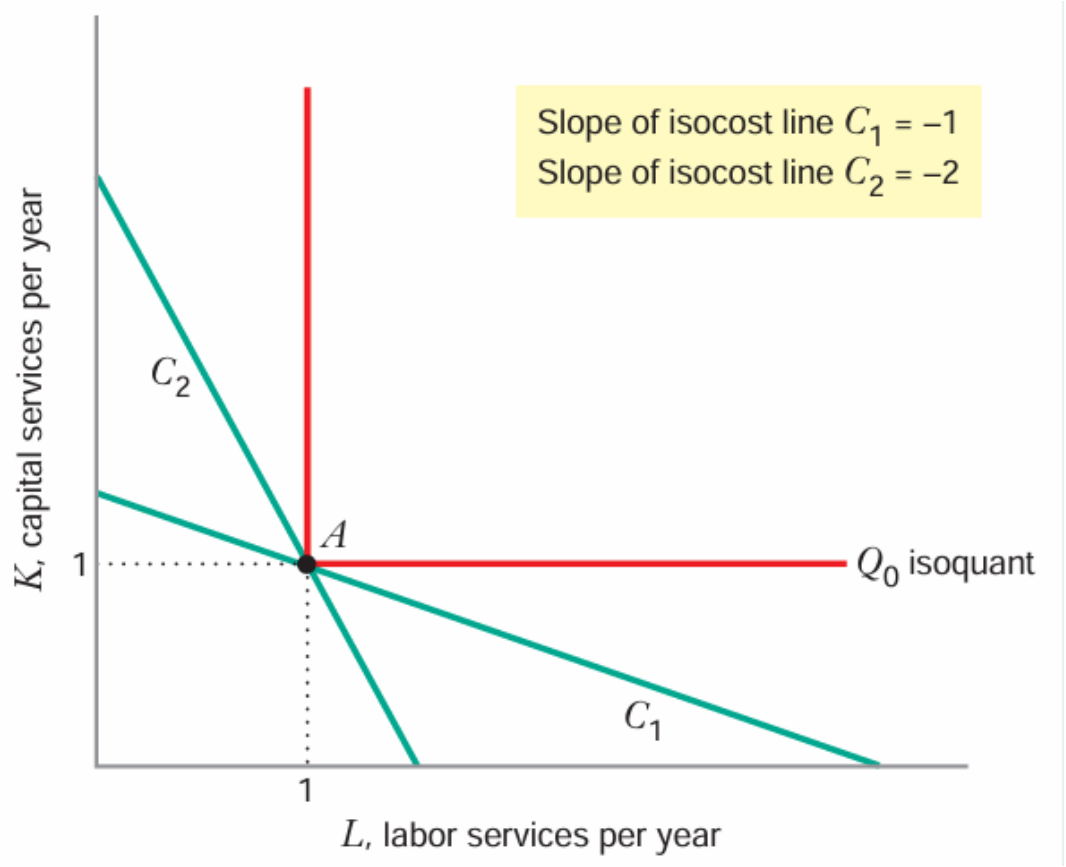
Comparative Static Analysis of Changes in Input Prices with respect to price of Labour



Explanation

- The price of capital $r = 1$ and the quantity of output Q0 are held constant.
- When the price of labor is $w = 1$, the isocost line is C1 and the ideal input combination is at point A (L1, K1).
- When the price of labor is $w = 2$, the isocost line is C2 and the ideal input combination is at point B (L2, K2).
- Increasing the price of labor causes the firm to substitute capital for labor.

For Fixed Proportion Production Function



Explanation

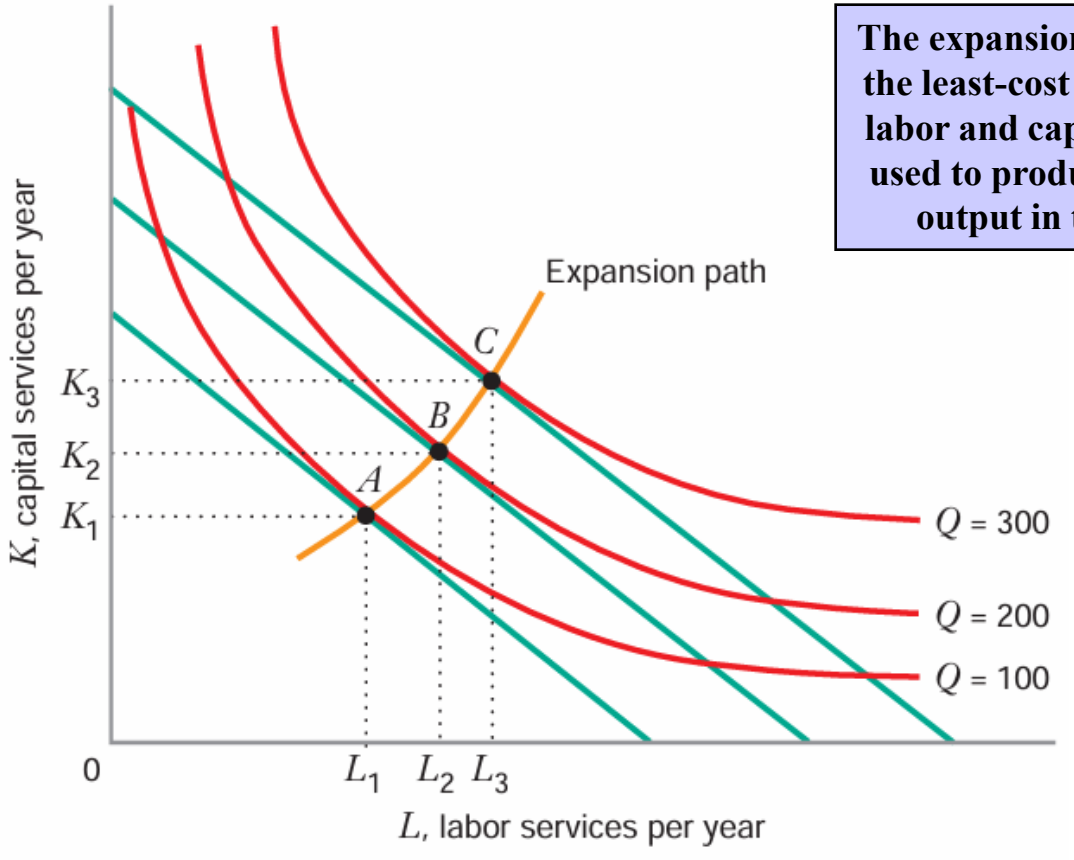
- The price of capital r =1, and the quantity of output Q0are held constant.

- When the price of labor w =1, the isocost line is C1and the ideal input combination is at point A (L 1, K 1).
- When the price of labor w =2, the isocost line is C2and the ideal input combination is still at point A.
- Increasing the price of labor does not cause the firm to substitute capital for labor.

Comparative Static Analysis of Changes in Output: Both inputs are normal

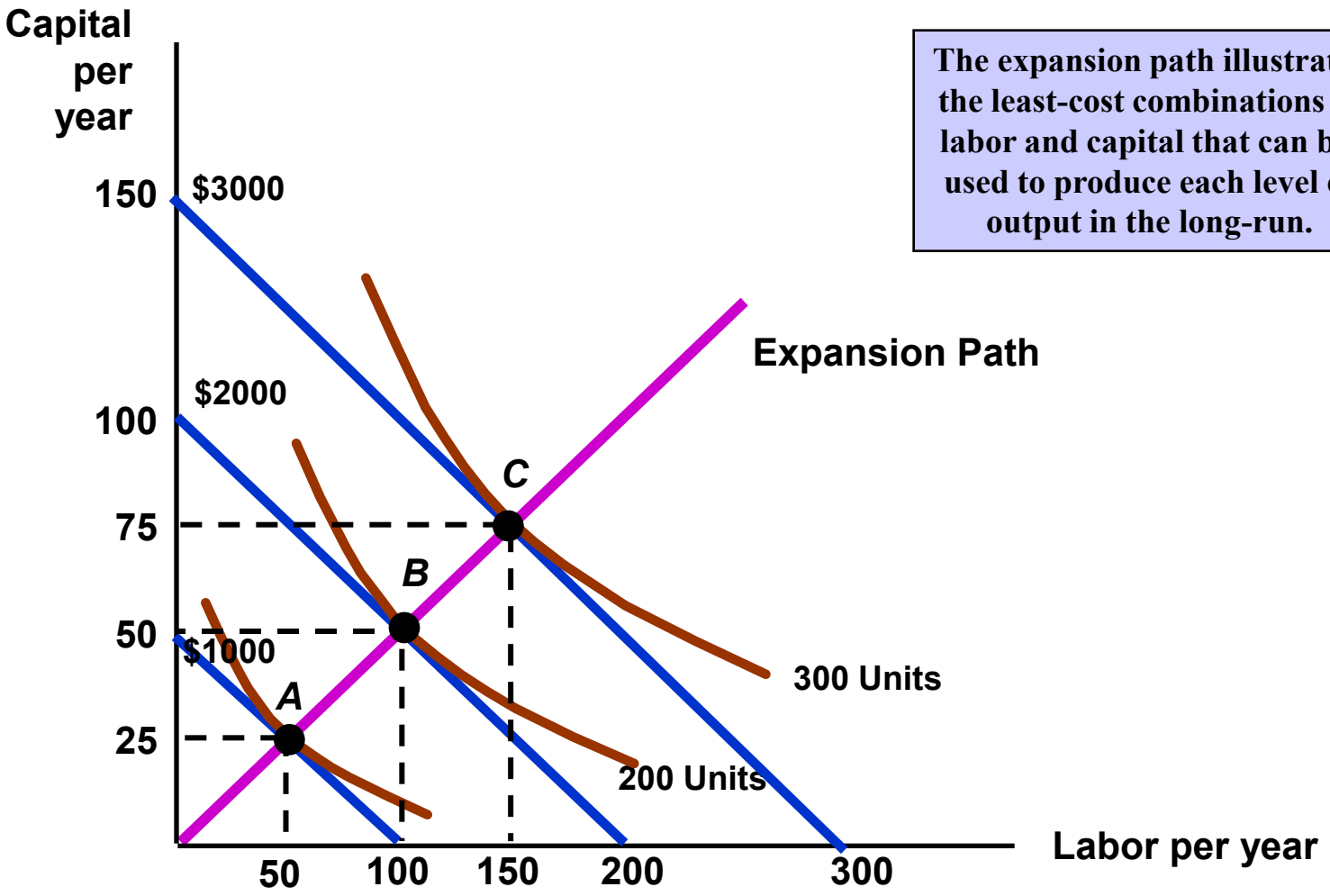
- Cost minimization with Varying Output Levels
  - For each level of output, there is an isocost curve showing minimum cost for that output level
  - A firm's expansion path shows the minimum cost combinations of labor and capital at each level of output.
  - Slope equals $\Delta K/\Delta L$
  - Both inputs are normal
  - An input whose cost-minimizing quantity increases as the firm produces more output.

Figure A



The expansion path illustrates the least-cost combinations of labor and capital that can be used to produce each level of output in the long-run.

A Firm's Expansion Path

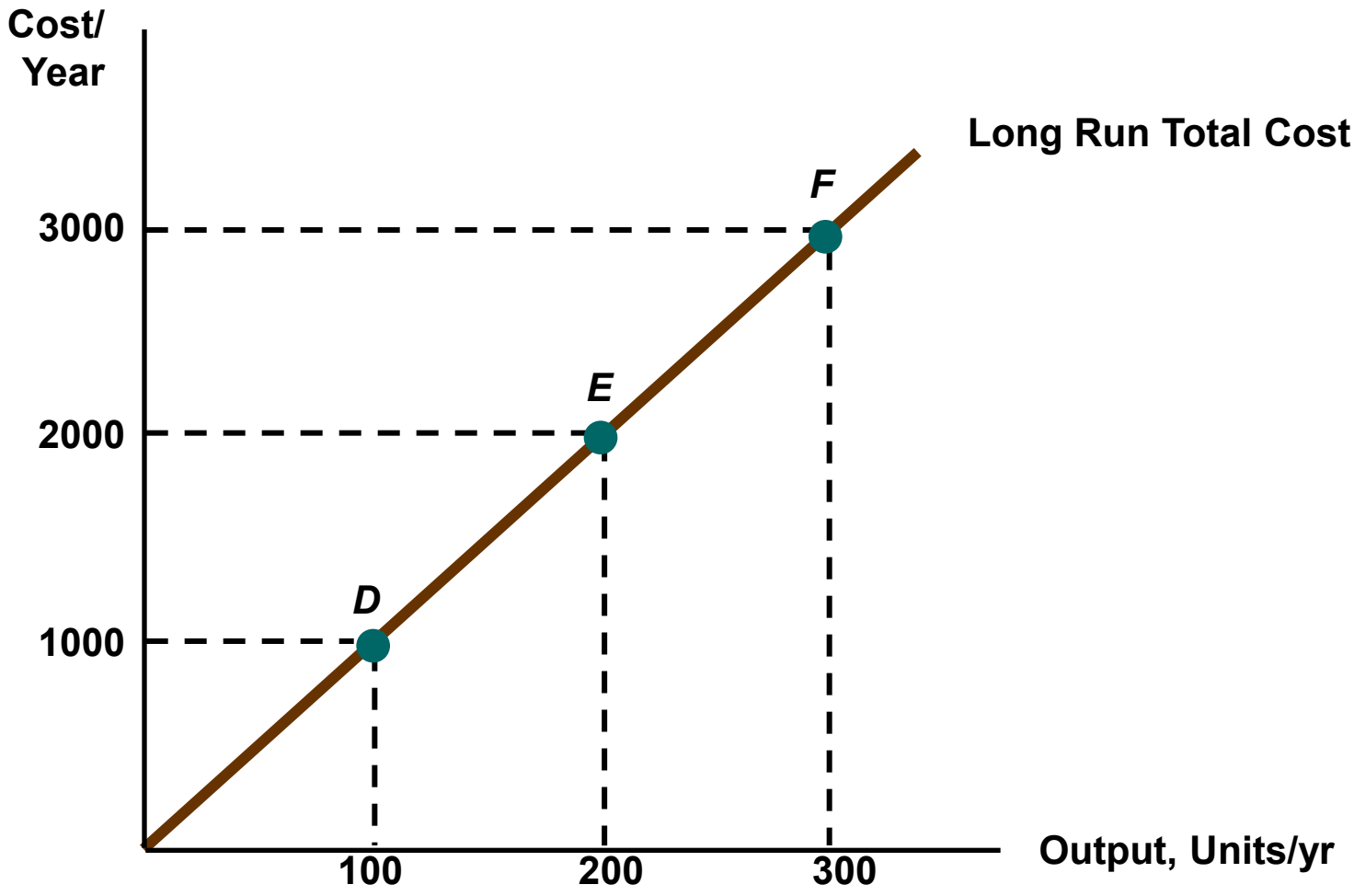


The expansion path illustrates the least-cost combinations of labor and capital that can be used to produce each level of output in the long-run.

Expansion Path & Long-run Costs

- Firms' expansion path has same information as long-run total cost curve
- To move from expansion path to LR cost curve
  - Find tangency with isoquant and isocost
  - Determine min cost of producing the output level selected
  - Graph output-cost combination

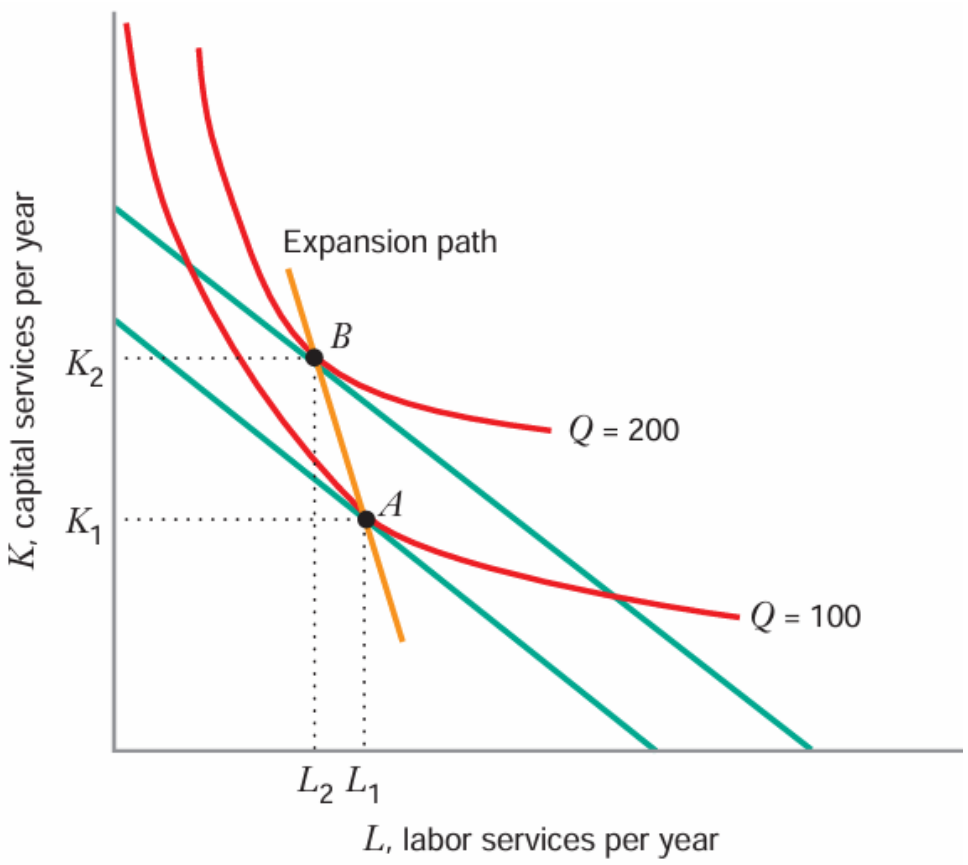
A Firm's Long-Run Total Cost Curve



Comparative Static Analysis of Changes in Output: One input is inferior

- What if one of the inputs is not normal, but is an inferior input—that is, the firm uses less of it as output increases?
- This situation can arise if the firm drastically auto mates its production process to increase output, using more capital but less labor, as shown in Figure-B
- (in this case, labor is an inferior input).
- When one of the inputs is inferior, the expansion path is downward sloping, as the figure-B shows.

(Fig.-B)


Expansion path
B
A
K_2
K_1
Q = 200
Q = 100
K, capital services per year
L_2 L_1
L, labor services per year

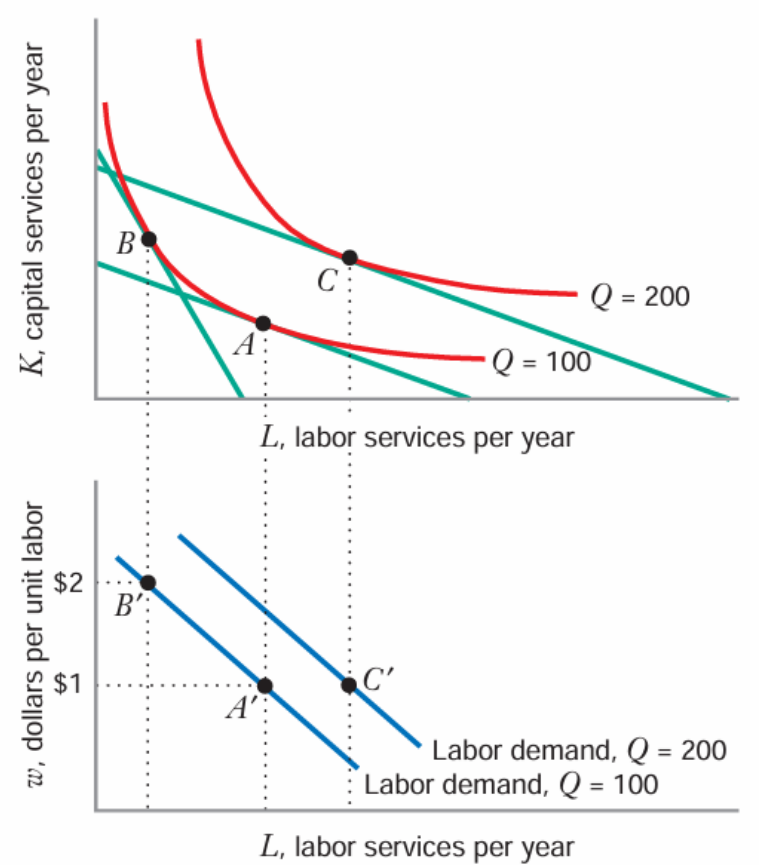
Input Demand Curves

- The solution to the cost-minimization problem is an optimal input combination:
  - a quantity of capital and a quantity of labor.

- We've also seen that this input combination depends on how much output the firm wants to produce and the prices of labor and capital.

- Figure-D shows how the cost minimizing quantity of labor varies with the price of labor.

Comparative Static Analysis: Derivation of Input Demand: (Figure-D)



Explanation

- The labor demand curve shows how the firm's cost minimizing amount of labor varies as the price of labor varies.

- For a fixed output of 100 units, an increase in the price of labor from \$1 to \$2 per unit moves the firm along its labor demand curve from point $\acute{A}$to point $\acute{B}$.

- Holding the price of labor fixed at \$1 per unit, an increase in output from 100 to 200 units per year shifts the labor demand curve rightward and moves the firm from $\acute{A}$ point to point $\acute{C}$.

Deriving the Input Demand Curves from a Production Function (Numerical Problem)

- Suppose that a firm faces the production function $Q = 50\sqrt{LK}$.

- What are the demand curves for labor and capital?

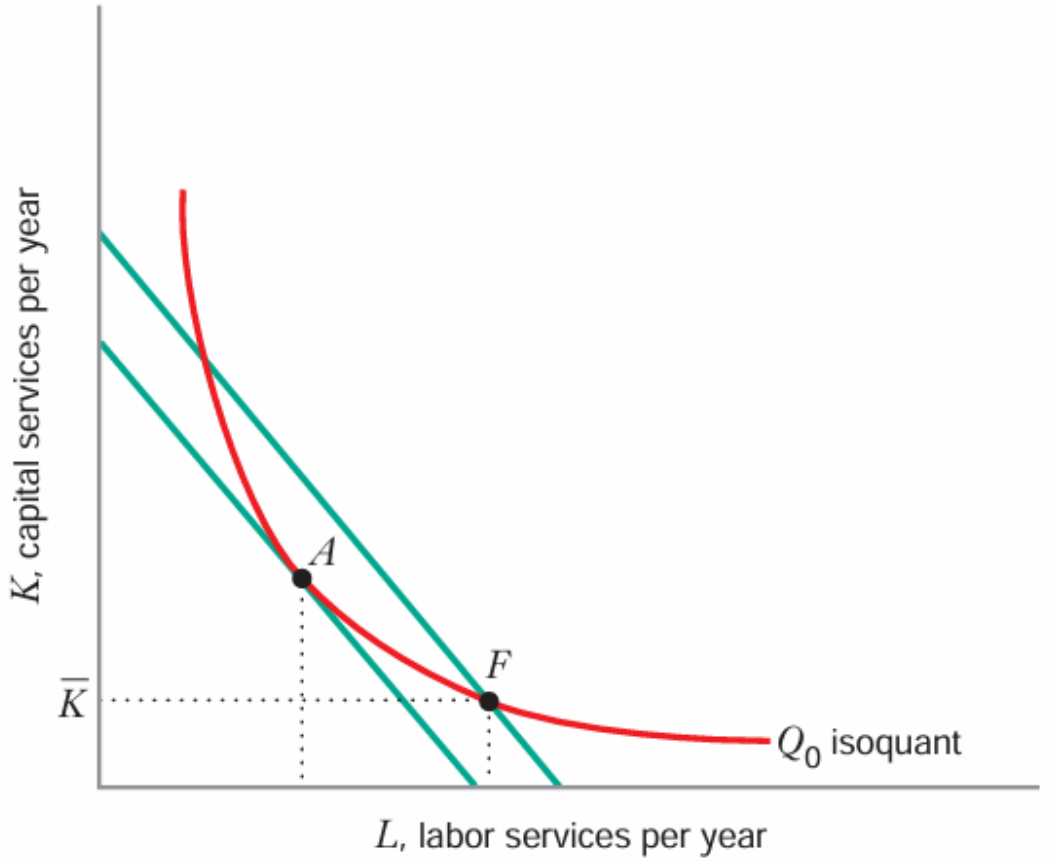
Short run cost minimization problem

- The firm's cost-minimization problem in the short run occurs, when the firm faces the constraint that one or more of the firm's inputs cannot be changed.

- We assume here that in the short- run, capital is constant and defined as $\overline{K}$.

- The firm's only technically efficient combination of inputs occurs at point F, where the firm uses the minimum quantity of labor that, in conjunction with the fixed quantity of capital $\overline{K}$ and allows the firm to produce exactly the desired output Q_0 (Figure-E)

Figure-E


K, capital services per year
A
F
$\bar{K}$
Q_0 isoquant
L, labor services per year

- This short-run cost-minimizing problem has only one variable factor (labor).

- Because the firm cannot substitute between capital and labor, the determination of the optimal amount of labor does not involve a tangency condition (i.e., no isocost line is tangent to the Q_0 isoquant at point F).

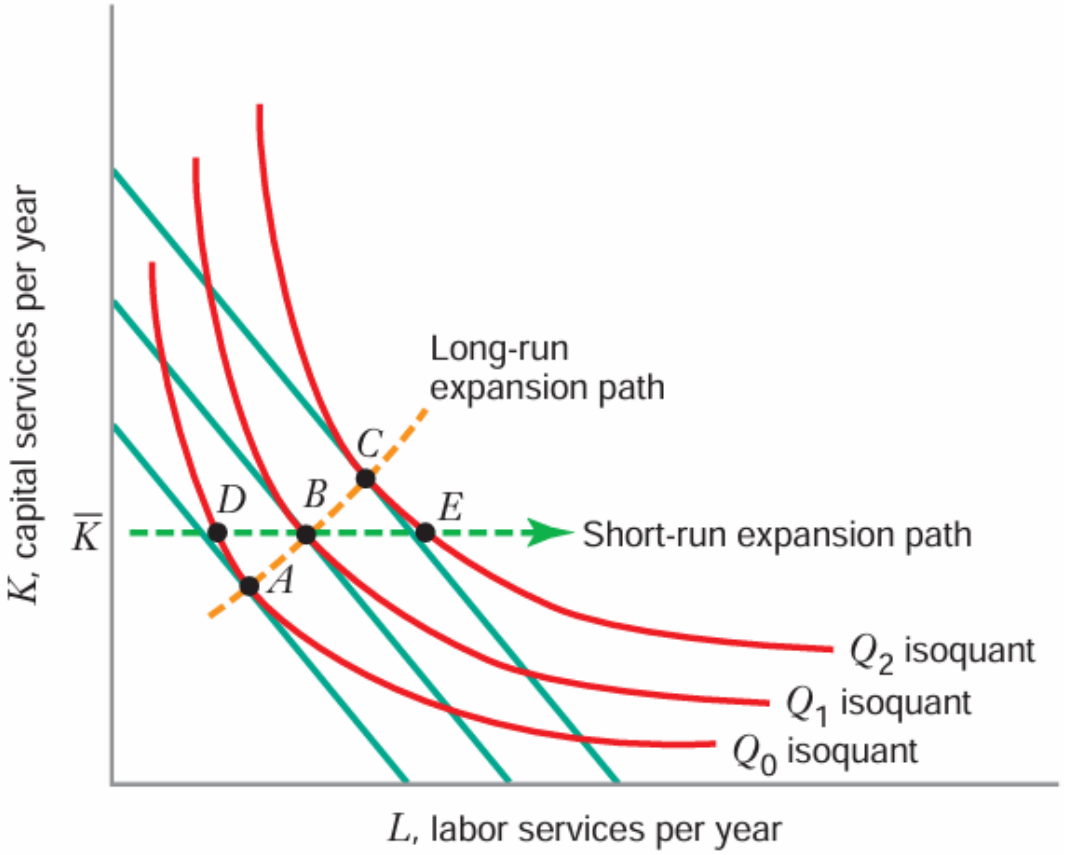
- By contrast, in the long run, when the firm can adjust the quantities of both inputs, it will operate at point A, where an isocost line is tangent to the isoquant.

- Thus, the cost minimization in the short run will not, in general, involve the same combination of inputs as cost minimization in the long run.

- In the short run, the firm will typically operate with higher total costs than it would if it could adjust all its inputs freely.

- There is, however, one exception, illustrated in Figure-F.

Figure-F


K, capital services per year
$\overline{K}$
Long-run
expansion path
C
B
D
E
A
Short-run expansion path
Q_2 isoquant
Q_1 isoquant
Q_0 isoquant
L, labor services per year

- Suppose the firm is required to produce Q_1. In the long run, it will operate at point B, freely choosing $\bar{K}$ units of capital.

- However, if the firm is told that in the short run it must produce with the amount of capital fixed at $\bar{K}$, it will also operate at point B.

- In this case the amount of capital the firm would choose in the long run just happens to be the same as the amount of capital fixed in the short run.

- Therefore, the total cost the firm in curs in the short run is the same as the total cost in the long run.

- Short-Run Cost Minimization with One Fixed Input.
- Problem to be discussed.

SHORT-RUN INPUT DEMAND VERSUS LONG-RUN INPUT DEMAND

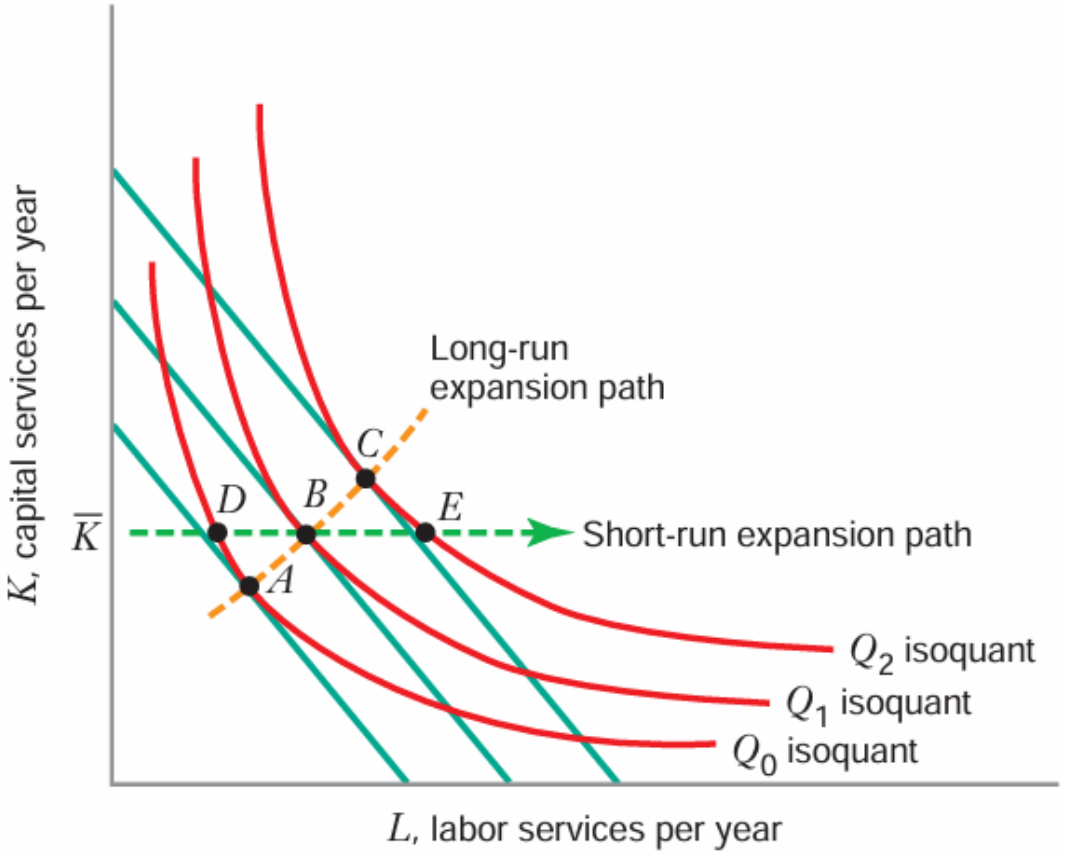
- In the case of a firm that uses just two inputs, labor and capital, the long-run cost-minimizing demand for labor will vary with the price of both inputs (as discussed earlier).

- By contrast, in the short run, if the firm cannot vary its quantity of capital, its demand for labor will be independent of input prices.

- The firm's demand for labor in the short run will, however, vary with the quantity of output.

- Figure-F shows this relationship using the concept of an expansion path.

Figure -F


K, capital services per year
$\overline{K}$
Long-run
expansion path
C
B
D
E
A
Short-run expansion path
Q_2 isoquant
Q_1 isoquant
Q_0 isoquant
L, labor services per year

- As the firm varies its output from Q_0 to Q_1 to Q_2, the long-run cost-minimizing input combination moves from point A to point B to C, along the long-run expansion path.

- But in the short run, when the quantity of capital is fixed at $\overline{K}$, the cost-minimizing input combination moves from point D to point B to point E, along the short-run expansion path.

- (As noted above, point B illustrates a cost-minimizing input combination that is the same both in the long run and in the short run, if the quantity of output is Q1.)
- Short-Run Cost Minimization with One Fixed Input.
- Numerical Problem to be discussed.