LIVE

# JEE MAIN 2024

ATTEMPT – 02 , 09th April 24, SHIFT - 02

# PAPER DISCUSSION

# Mathematics

$|z-2i|$ $\quad$ $|z-(-6i-8)|$

If $\dfrac{z-2i}{z+2i}$ is purely imaginary then find the maximum value of $|z+8+6i|$.

$(-8,-6)$

CNF

$\leq |z|+10$

$$z=-\bar{z}$$

$$\frac{z-2i}{z+2i}=-\left(\frac{\bar{z}+2i}{\bar{z}-2i}\right)$$

$$(z-2i)(\bar{z}-2i)=-(\bar{z}+2i)(z+2i)$$

$$z\cdot\bar{z}-\cancel{z(2i)}-\cancel{\bar{z}(2i)}+(-4)=-(z\bar{z}+\cancel{2iz}+\cancel{2i\bar{z}}-4)$$

$$2(z\bar{z})=8$$

$$z\bar{z}=4$$

$$|z|^2=4$$

$$x^2+y^2=4 \qquad C(0,0),\ R=2$$

$(0,0)$ $\quad$ $(-8,-6)$

$$\sqrt{6^2+8^2}+2$$

$$10+2=12$$

In the expansion of $\left(x^{2/3}+\frac{1}{2}x^{-2/5}\right)^9$ find the sum of coefficients of $x^{2/3}$ and $x^{-2/5}$

$T_6$

$T_7$

$${}^9C_r\left(x^{2/3}\right)^{9-r}\left(\frac{1}{2}x^{-2/5}\right)^r$$

$x \rightarrow \frac{2}{3}(9-r)-\frac{2r}{5}$

$6-\frac{2r}{3}-\frac{2r}{5}$

$6-\frac{2r(8)}{15}$

$6-\frac{16r}{15}=\frac{2}{3}$

$6-\frac{2}{3}=\frac{16r}{15}$, $r=5$

$6-\frac{16r}{15}=-\frac{2}{5}$

$6+\frac{2}{5}=\frac{16r}{15}$

$2\,\frac{\cancel{32}}{5}=\frac{\cancel{16}r}{15}$, $r=6$

$$ {}^{9}C_{5}\left(\frac{1}{2}\right)^{5}+{}^{9}C_{6}\left(\frac{1}{2}\right)^{6} $$

$$\lim_{x\to 0}\frac{e-(1+2x)^{1/2x}}{x}$$

$$\lim_{x\to 0}(1+x)^{1/x}=e\left[1-\frac{x}{2}+\frac{11}{24}x^2\right]$$

$$=e\left[1-\frac{\cancel{2}x}{\cancel{2}}+\frac{11}{24}(2x)^2\right]$$

$$\frac{\cancel{e}-\cancel{e}\left[1-x+\frac{11}{24}(2x)^2\right]}{x}$$

$e$

Number of integers between 100 to 1000 whose sum of digits is 14.

$x_1 x_2 x_3$

$1 \le x_1 \le 9,\ 0 \le x_2, x_3 \le 9$

$x_1 + x_2 + x_3 = 14$

$x^{14} \Rightarrow (x^1 + x^2 \cdots + x^9)(x^0 + x \cdots + x^9)^2$

$$\left(\frac{1-x^{10}}{1-x}\right)^2 \times \left(\frac{1-x^9}{1-x}\right)$$

$\underline{\underline{\text{Coeff } x^{13}}} \Rightarrow (1-x^{10})^2(1-x^9)(1-x)^{-3}$

$(1-x^9)(1-2x^{10}+x^{20})(1-x)^{-3}$

$\| \quad \Rightarrow (1-2x^{10}-x^9)(1-x)^{-3}$

$^{3+12}C_{13} - 2\left(^{3+3-1}C_3\right) - {}^{4+2}C_4$

$^{15}C_{13} - 2(^5C_3) - {}^6C_4$

$x^{r}$

$(1-x)^{-n}$:

$^{n+r-1}C_{r}$

$$\sum_{n=0}^{\infty} ar^n = 57 \sum_{n=0}^{\infty} a^3 r^{3n} = 9747$$ What is $a + 18r = ?$

$-1 < r < 1$

$a^3(1 + r^3 \cdots \infty)$

$a[1 + r \cdots \infty) = 57$

$\frac{a}{1-\frac{2}{3}} = 57$

$a = 19$

$\frac{a}{1-r} = 57$ — 1 (cube)

$\frac{a^3}{1-r^3} = 9747$ — 2

$19 + 18\left(\frac{2}{3}\right) = 31$

$57)\,9747\,(171$

$57$

$404$

$57$

$$\frac{1-r^3}{(1-r)^3} = \frac{(57)^3}{9747}$$

$$\frac{\cancel{(1-r)}(1+r+r^2)}{\cancel{(1-r)}(r^2-2r+1)} = \frac{57^3}{9747}$$

$$= \frac{57^3}{57^2(3)} = 19$$

$(r^2+r+1) = 19(r^2-2r+1)$

$18r^2 - 39r + 18 = 0,\ r = \frac{2}{3}$

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$$I = \int_{-1}^{2} 1 \cdot \ln\left(x + \sqrt{1 + x^2}\right) dx$$

$$\int_{-1}^{1} + \int_{1}^{2}$$

$$\int_{-a}^{a} \text{odd} = 0 \qquad 2\int_{0}^{a}$$

$$x \ln\left(x+\sqrt{1+x^2}\right) - \int \frac{x}{\left(x+\sqrt{1+x^2}\right)}\left(1+\frac{x}{\sqrt{1+x^2}}\right)dx$$

$$\left. x \ln\left(x+\sqrt{1+x^2}\right)\right]_1^2 - \int \frac{x\,dx}{\sqrt{1+x^2}}$$

$$1+x^2 = t$$

$$2x\,dx = dt$$

$$\int \frac{dt}{2\sqrt{t}} = \frac{t^{1/2}}{\frac{1}{2}(2)}$$

$$\left. x \ln\left(x+\sqrt{1+x^2}\right) - \sqrt{1+x^2}\right]_1^2$$

$$\left[2 \ln(2+\sqrt{5}) - 1 \ln(1+\sqrt{2})\right]$$

$$-\left[\sqrt{5}-\sqrt{2}\right]$$

$f(x) = -f(-x)$ (odd f(n))

$f(x) = \ln\left(x + \sqrt{1+x^2}\right)$

$f(-x) = \ln\left(-x + \sqrt{1+x^2}\right)$

$f(x) + f(-x) = \ln\left(\left(\sqrt{1+x^2}\right)^2 - x^2\right)$

$f(x) + f(-x) = 0$

$2\sin^{-1}(x) + 3\cos^{-1}(x) = \frac{7\pi}{5}$, find number of real solution of equation

$2\pi/5$

easy.

1

$2\left[\underbrace{\sin^{-1}(x)+\cos^{-1}}_{\pi/2}(x)\right]+\cos^{-1}(x)=7\pi/5$

$\cos^{-1}(x)=7\pi/5-\pi$

$=2\pi/5$

$[-1,1]$

$x=\cos(72^\circ)$

$\sin 18^\circ=\frac{\sqrt{5}-1}{4}$

$\cos 36^\circ=\frac{\sqrt{5}+1}{4}$

$\cos^{-1}(x)=2\pi/5-\pi$

$\cos^{-1}(x)=-3\pi/5$ (Not possible]

$[0,\pi]$

$f(x) = \dfrac{1}{2 + \sin 3x + \cos 3x}$ if range of $f(x)$ is $[a, b]$, then ratio of AM of $a, b$ and GM of $a, b$ is

$-\sqrt{a^2+b^2} \leq a\sin x + b\cos x \leq \sqrt{a^2+b^2}$

$a \Rightarrow \dfrac{1}{2-\sqrt{2}}$

$b \Rightarrow \dfrac{1}{2+\sqrt{2}}$

$A \Rightarrow \dfrac{a+b}{2} \Rightarrow \left(\dfrac{\frac{1}{2-\sqrt{2}} + \frac{1}{2+\sqrt{2}}}{2}\right) = 1$

$G \Rightarrow \sqrt{\dfrac{1}{2-\sqrt{2}} \dfrac{1}{2+\sqrt{2}}} = \dfrac{1}{\sqrt{2}}$

$\dfrac{A}{G} = \sqrt{2}$

$9e^{\pi/2}$

If $\ln(y) = \sin^{-1}(x)$ then find the value of $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} = ?$ at $x = \dfrac{1}{2}$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\left(\frac{dy}{dx}\right) = y, \quad \frac{dy}{dx} = \frac{y}{\sqrt{1-x^2}}$$

$$\frac{d^2y}{dx^2}\sqrt{1-x^2} + \left(\frac{dy}{dx}\right)\left(\frac{-x}{\sqrt{1-x^2}}\right) = \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2}(1-x^2) - x\frac{dy}{dx} = \frac{dy}{dx}\sqrt{1-x^2}$$

$$= y$$

$$\ln y = \pi/6$$

$$y = e^{\pi/6}$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{3}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\,\frac{dy}{dx} = 3y$$

$$\frac{d^2y}{dx^2}\sqrt{1-x^2} + \left(\frac{dy}{dx}\right)\left(\frac{-x}{\sqrt{1-x^2}}\right) = 3\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2}(1-x^2) - x\frac{dy}{dx} = 3\sqrt{1-x^2}\,\frac{dy}{dx}$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 9y$$

$$\ln y = 3\sin^{-1}(x)$$

$$\ln y = \cancel{3}\left(\frac{\pi}{\cancel{6}_2}\right)$$

$$y = e^{\pi/2}$$

ANS $\Rightarrow$ $9e^{\pi/2}$

Given $f'(x) = 3f(x) + \alpha$, If $f(0) = 7$ and $\lim_{x \to -\infty} f(x) = 0$

Find $f\left(\frac{1}{3}\right)$

$f(-\ln 3)$

$f(x) = 7e^{3x}$

$7e^{-3\ln 3}$

$\frac{7}{3^3} = \frac{7}{27}$

$\frac{dy}{dx} = 3y + \alpha$

$\frac{dy}{3y+\alpha} = dx$

$\frac{\ln(3y+\alpha)}{3} = x + c$

$\ln(3y+\alpha) = 3x + 3c$

$(3y+\alpha) = e^{3x+3c} = c \cdot e^{3x}$

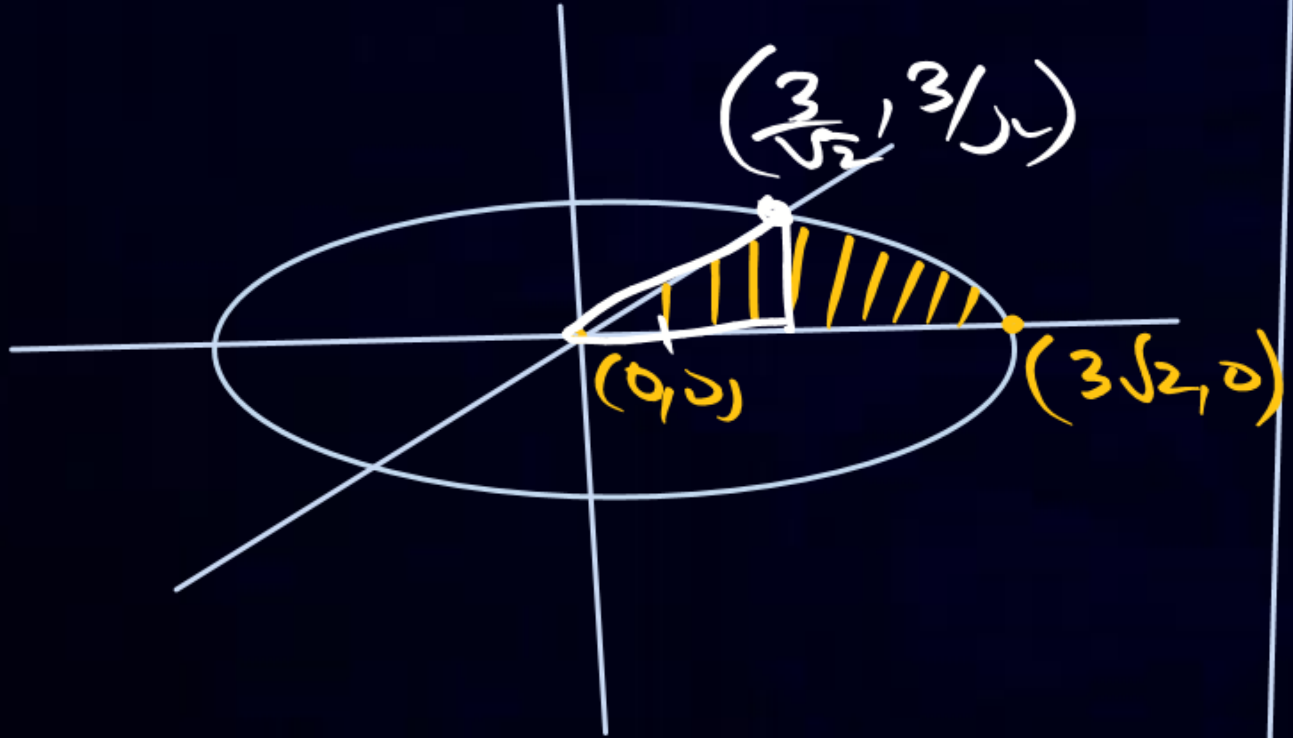
$(3y+\alpha) = c \cdot e^{3x}$

$21 + \alpha = c$ — (1)

$\alpha = 0$, $c = 21$

$y = 7e^{3x}$

Find the area bounded by $y = x, \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and $x$–axis in first quadrant $(a = 3\sqrt{2}, b = \sqrt{6})$.




$$\frac{x^2}{18} + y^2/6 = 1, \quad y = \sqrt{6}\sqrt{1 - \frac{x^2}{18}}$$

$$y = \frac{1}{\sqrt{3}}\sqrt{18 - x^2}$$

$$x^2\left(\frac{1}{18} + \frac{1}{6}\right) = 1$$

$$x^2\left(\frac{4}{18}\right) = 1, \quad x^2 = \frac{\cancel{18}}{4}\,\frac{9}{2}$$

$$x = \frac{3}{\sqrt{2}}$$

$$\Delta = \frac{1}{2}\left(\frac{3}{\sqrt{2}}\right)\left(\frac{3}{\sqrt{2}}\right) + \int_{3/\sqrt{2}}^{3\sqrt{2}} \frac{1}{\sqrt{3}}\sqrt{18 - x^2}$$

$$\Delta = \frac{1}{2}\left(\frac{9}{2}\right) + \frac{1}{\sqrt{3}}\left(\frac{x}{2}\sqrt{18-x^2} + 9\sin^{-1}\left(\frac{x}{\sqrt{18}}\right)\right]_{3/\sqrt{2}}^{3\sqrt{2}}$$

$$\Delta = \frac{9}{4} + \frac{1}{\sqrt{3}}\left[9\left(\frac{\pi}{2}\right) - \left(\frac{3}{\sqrt{2}(2)}\sqrt{18-\frac{9}{2}} + 9\sin^{-1}\left(\frac{1}{2}\right)\right)\right.$$

$$\Delta = 9/4 + \frac{1}{\sqrt{3}}\left(\frac{9\pi}{2} - \left(\frac{3}{2\sqrt{2}}\frac{\sqrt{27}}{\sqrt{2}} + 9\left(\frac{\pi}{6}\right)\right)\right)$$

$$\Delta = 9/4 + \frac{1}{\sqrt{3}}\left(\left(\frac{9\pi}{2} - 9\pi/6\right) - \frac{9\sqrt{3}}{4}\right)$$

A dice is thrown three times such that the outcomes are $x_1, x_2, x_3$ respectively. Find the probability of getting the outcomes such that $x_1 < x_2 < x_3$.

Total $= 6^3$

FAV $\Rightarrow {}^6C_3$

$$\frac{5 \times 4}{6^3}$$

$$\frac{20}{216}$$

$(1, 2, 3)$ ✓

$(2, 1, 3)$ ✗

$(1, 2, 3)$

$(1, 2, 4)$

$(1, 2, 5)$

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$$\int_{\frac{1}{4}}^{\frac{3}{4}} \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$$

$$\cos 2\tan^{-1}\left(\sqrt{\frac{1+x}{1-x}}\right) \quad \theta$$

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\tan\theta = \sqrt{\frac{1+x}{1-x}}$$

$$\cos 2\theta = \frac{1-\left(\frac{1+x}{1-x}\right)}{1+\left(\frac{1+x}{1-x}\right)} = \frac{-2x}{2} = -x$$

$$-\int_{\frac{1}{4}}^{3/4} x\,dx$$

$$-\frac{1}{2}\left[\left(3/4\right)^2-\left(1/4\right)^2\right]$$

$$-1/2\left(\frac{1}{2}\right) = -1/4$$

$x^2$ $(y-1)^2$ $\frac{x^2}{100}+\frac{y^2}{75}=1$

Ellipse $\frac{(x-1)^2}{100}+\frac{y^2}{75}=1$ and A Hyperbola of same focus as ellipse whose major axis is α and minor axis is β & $ee'=1$ (where $e'$ is eccentricity of hyperbola and $e$ is eccentricity of ellipse) find $3\alpha^2+2\beta^2$.

$75=100(1-e^2)$

$1-e^2=3/4$

$e^2=1/4$

$e_{ellipse}=1/2$

$e_H=2$ ✓

$ae=5$

$\frac{x^2}{(\alpha/2)^2}-\frac{y^2}{(\beta/2)^2}=1$

$\alpha=5$

$\beta^2=\alpha^2(e^2-1)$

$\beta^2=25(3)$

$\beta^2=75$

$3(25)+2(75)$

$75+150=225$

If $\int_0^x \sqrt{1-(y')^2}\,dx = \int_0^x y(x)\,dx, y(0) = 0.$

Find $|y'' + y + 1|$ at $x = 1/2$

$\sqrt{1-\left(\frac{dy}{dx}\right)^2} = y$

$1-\left(\frac{dy}{dx}\right)^2 = y^2$

$\therefore y = \pm \sin x$

$\frac{dy}{dx} = \pm \cos x$

$\frac{d^2y}{dx^2} = \mp \sin x$

$1-y^2 = \left(\frac{dy}{dx}\right)^2$

$\frac{dy}{dx} = \pm\sqrt{1-y^2}$

$\int \frac{dy}{\sqrt{1-y^2}} = \int \pm\, dx$

$\sin^{-1}(y) = \pm x + C$

$\boxed{y = \pm \sin x}$

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Let $\frac{1}{\alpha+1}+\frac{1}{\alpha+2}+\cdots+\frac{1}{\alpha+1012}=\frac{1}{1\cdot 2}+\frac{1}{3\cdot 4}+\frac{1}{5\cdot 6}+\cdots+\frac{1}{2023\cdot 2024}$, then find the value of $\alpha$.

$$\left(1-\frac{1}{2}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)\cdots+\left(\frac{1}{2023}-\frac{1}{2024}\right)$$

$$\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\cdots+\frac{1}{2023}+\frac{1}{2024}\right)$$

$$-2\left(\frac{1}{2}+\frac{1}{4}\cdots+\frac{1}{2024}\right)$$

$$\left(1+\frac{1}{2}\cdot+\frac{1}{2024}\right)-\left(1+\frac{1}{2}\cdot+\frac{1}{1012}\right)$$

$$\left(\frac{1}{1013}+\frac{1}{1014}\cdots+\frac{1}{2024}\right)$$

$$\alpha+1=1013$$

$$\alpha=1012$$

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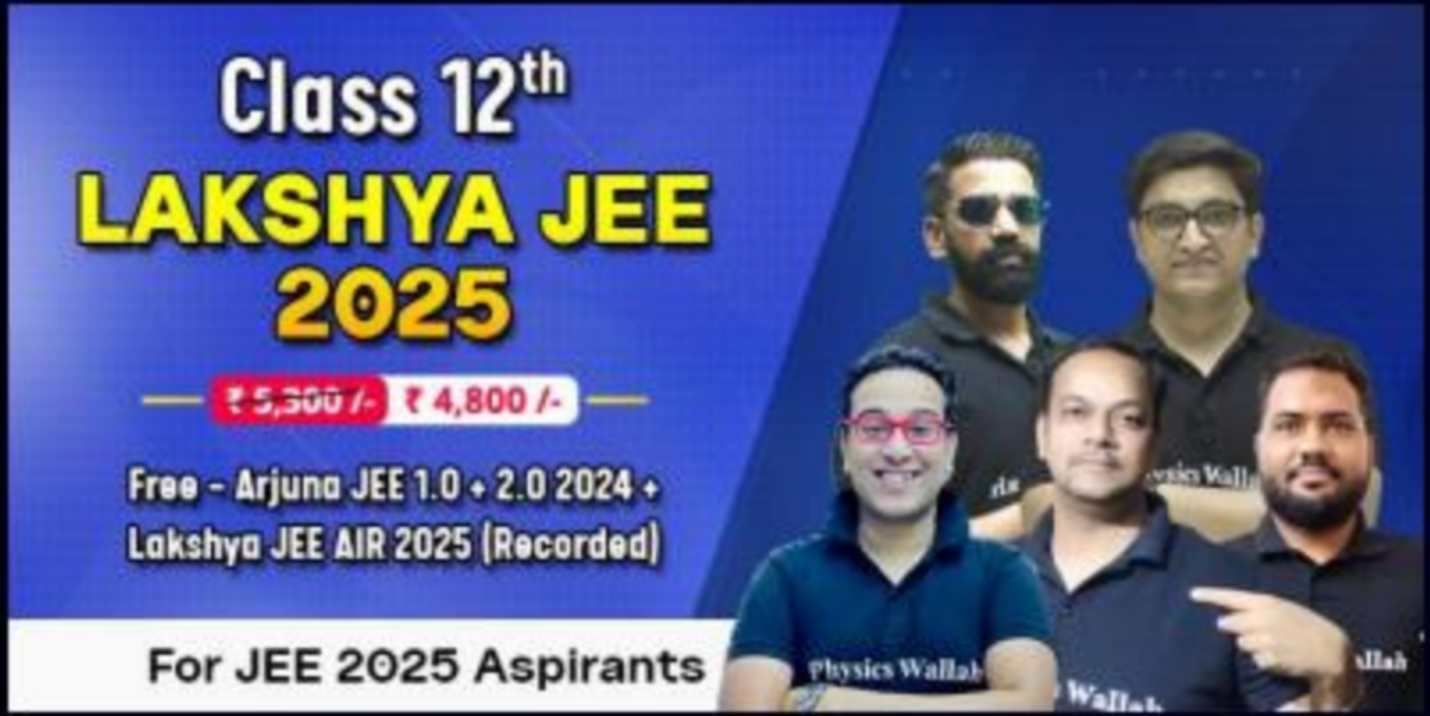


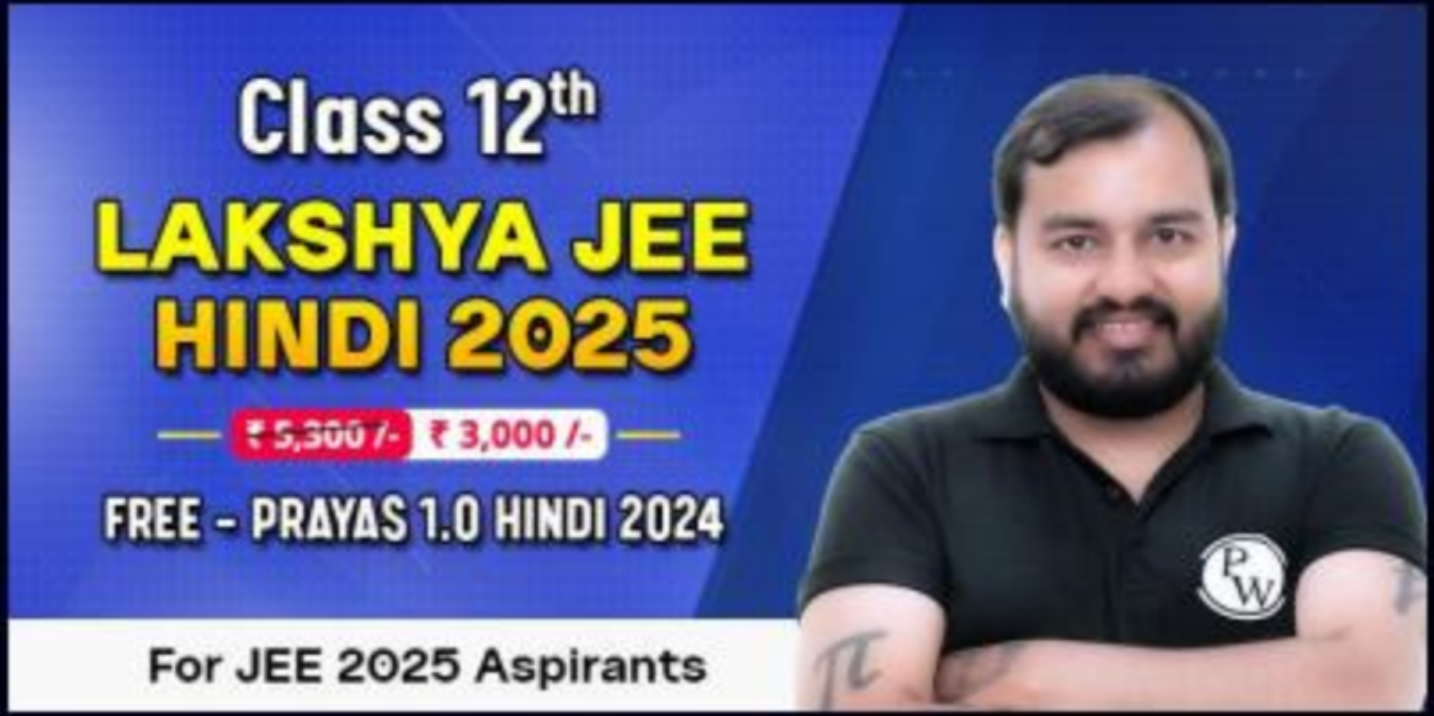


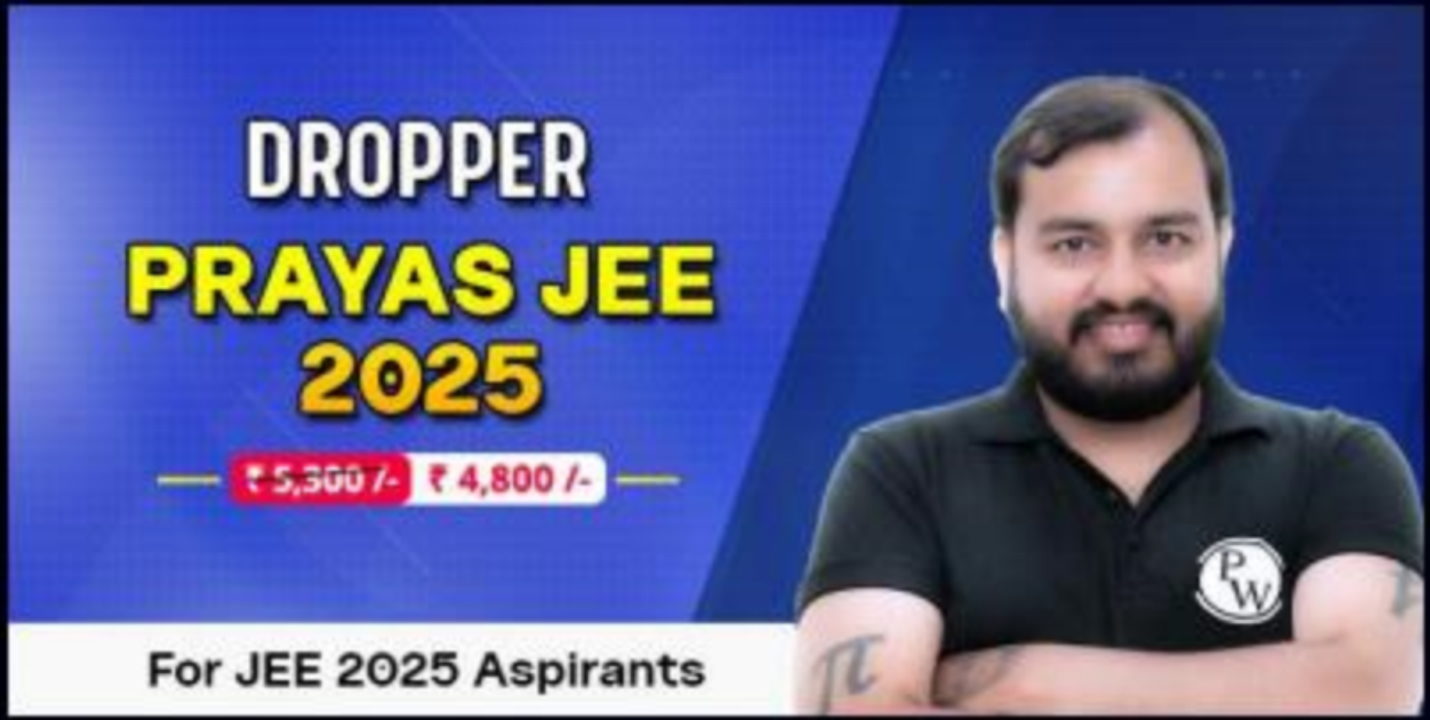


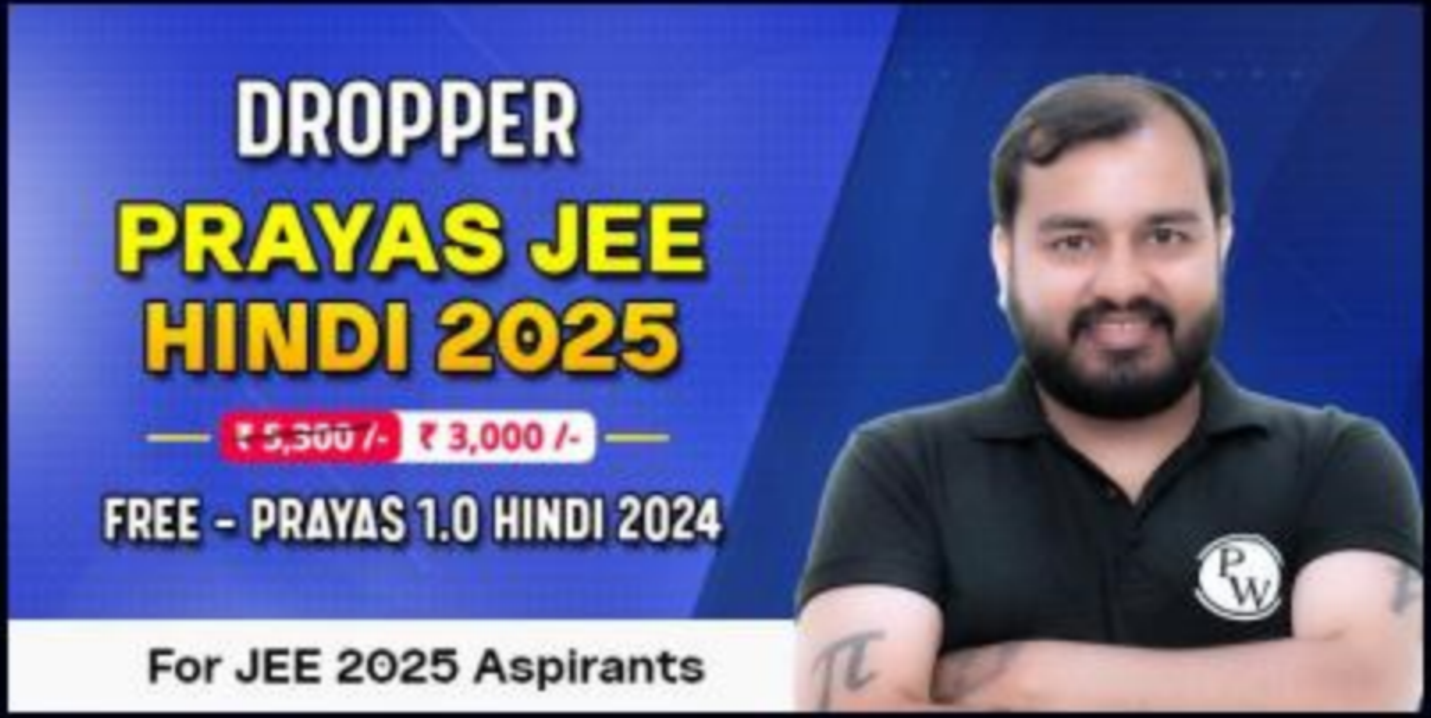

# THANK YOU