



Answers & Solutions

Time : 3 hrs.

for

M.M. : 300

JEE (Main)-2023 (Online) Phase-2

(Mathematics, Physics and Chemistry)

15/04/2023

Morning

IMPORTANT INSTRUCTIONS:

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper consisting of **Mathematics, Physics and Chemistry** having 30 questions in each part of equal weightage. Each part (subject) has two sections.
 - (i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
 - (ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

Sol. $x|x| - 5|x + 2| + 6 = 0$

Case-I :

$$x < -2$$

$$-x^2 + 5(x + 2) + 6 = 0$$

$$\Rightarrow x^2 - 5x - 16 = 0$$

$$\Rightarrow x = \frac{5 \pm \sqrt{25 + 64}}{2}$$

$$\therefore x = \frac{5 - \sqrt{89}}{2} \text{ is accepted}$$

Case-II :

$$-2 \leq x < 0$$

$$-x^2 - 5(x + 2) + 6 = 0$$

$$\Rightarrow x^2 + 5x + 4 = 0$$

$$\Rightarrow (x + 1)(x + 4) = 0$$

$$x = -1 \text{ is accepted}$$

Case-III :

$$x \geq 0$$

$$x^2 - 5(x + 2) + 6 = 0$$

$$\Rightarrow x^2 - 5x - 4 = 0$$

$$x = \frac{5 \pm \sqrt{25 + 16}}{2}$$

$$= \frac{5 \pm \sqrt{41}}{2}$$

$$x = \frac{5 \pm \sqrt{41}}{2} \text{ is accepted}$$

\therefore 3 real roots are possible.

Option (4) is correct.

11. Let A_1 and A_2 be two arithmetic means and G_1 , G_2 and G_3 be three geometric means of two distinct positive numbers. Then $G_1^4 + G_2^4 + G_3^4 + G_1^2 G_3^2$ is equal to

$$(1) (A_1 + A_2)^2 G_1 G_3$$

$$(2) 2(A_1 + A_2) G_1 G_3$$

$$(3) (A_1 + A_2) G_1^2 G_3^2$$

$$(4) 2(A_1 + A_2) G_1^2 G_3^2$$

Answer (1)

Sol. Let the two numbers are a , b .

$$A_1 = a + \frac{b-a}{3} = \frac{2a+b}{3}$$

$$A_2 = a + \frac{b-a}{3} \cdot 2 = \frac{a+2b}{3}$$

$$G_1 = a \left(\frac{b}{a} \right)^{\frac{1}{4}}$$

$$G_2 = a \left(\frac{b}{a} \right)^{\frac{2}{4}}$$

$$G_3 = a \left(\frac{b}{a} \right)^{\frac{3}{4}}$$

$$(G_1)^4 + (G_2)^4 + (G_3)^4 + (G_1)^2 \cdot (G_3)^2$$

$$\Rightarrow a^4 \cdot \frac{b}{a} + a^4 \cdot \frac{b^2}{a^2} + a^4 \cdot \frac{b^3}{a^3} + a^4 \cdot \frac{b^2}{a^2} \\ = ba^3 + b^2 a^2 + b^3 a + a^2 b^2 \\ = ab(a^2 + b^2 + 2ab) = ab(a + b)^2$$

$$(A_1 + A_2)^2 \cdot G_1 G_3 = (a + b)^2 \cdot ab$$

\therefore Option (1) is correct.

12. Let $[x]$ denote the greatest integer function and $f(x) = \max\{1+x+[x], 2+x, x+2[x]\}$, $0 \leq x \leq 2$.

Let m be the number of points in $[0, 2]$, where f is not continuous and n be the number of points in $(0, 2)$, where f is not differentiable. Then $(m+n)^2 + 2$ is equal to

$$(1) 2$$

$$(2) 11$$

$$(3) 6$$

$$(4) 3$$

Answer (4)

$$\text{Sol. } f(x) = \begin{cases} \max\{x+1, x+2, x\} & 0 \leq x < 1 \\ \max\{x+2, x+2, x+2\} & 1 \leq x < 2 \\ \max\{5, 4, 6\} & x = 2 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} x+2 & 0 \leq x < 1 \\ x+2 & 1 \leq x < 2 \\ 6 & x = 2 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} x+2 & 0 \leq x < 2 \\ 6 & x = 2 \end{cases}$$

f is not continuous at $x = 2$

f is differentiable in $(0, 2)$

$$\therefore m = 1, n = 0$$

$$(m+n)^2 + 2 = 1 + 2 = 3.$$

Option (4) is correct

Answer (4)

Sol. $I = \frac{1}{2} \int_0^1 \frac{1}{\left(\frac{5}{2} - (x^2 - x)\right) \left(1 + e^{-4(x-\frac{1}{2})}\right)} dx$

 $= \frac{1}{2} \int_0^1 \frac{1}{\left(\left(\frac{11}{4}\right) - \left(x - \frac{1}{2}\right)^2\right) \left(1 + e^{-4(x-\frac{1}{2})}\right)} dx$

Let $x - \frac{1}{2} = t, dx = dt$

 $= \frac{1}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\left(\left(\frac{\sqrt{11}}{2}\right)^2 - t^2\right) \left(1 + e^{-4t}\right)} dt$
 $= \frac{1}{2} \int_0^{\frac{1}{2}} \frac{1}{\left(\left(\frac{\sqrt{11}}{2}\right)^2 - t^2\right) \left(1 + e^{-4t}\right)} + \frac{1}{\left(\left(\frac{\sqrt{11}}{2}\right)^2 - t^2\right) \left(1 + e^{4t}\right)} dt$
 $= \frac{1}{2} \int_0^{\frac{1}{2}} \frac{1}{\left(\left(\frac{\sqrt{11}}{2}\right)^2 - t^2\right)} dt = \frac{1}{2} \times \frac{1}{2\left(\frac{\sqrt{11}}{2}\right)} \ln \left| \frac{\frac{\sqrt{11}}{2} + t}{\frac{\sqrt{11}}{2} - t} \right|_0^{\frac{1}{2}}$
 $= \frac{1}{2\sqrt{11}} \ln \left(\frac{\sqrt{11} + 1}{\sqrt{11} - 1} \right) = \frac{1}{2\sqrt{11}} \ln \left(\frac{(\sqrt{11} + 1)^2}{10} \right)$

$$\Rightarrow \alpha = \sqrt{11}, \beta = \sqrt{10} \Rightarrow \alpha^4 - \beta^4 = 21$$

Answer (4)

$$\text{Sol. } 4x^2 + 11x + 6 > 0 \Rightarrow (4x+3)(x+2) > 0$$

$$\Rightarrow x \in (-\infty, -2) \cup \left(\frac{-3}{4}, \infty \right) \quad \dots(i)$$

$$-1 \leq 4x + 3 \leq 1 \Rightarrow -4 \leq 4x \leq -2$$

$$-1 \leq x \leq -\frac{1}{2} \quad \dots \text{(ii)}$$

$$-1 \leq \frac{10x+6}{3} \leq 1 \Rightarrow -3 \leq 10x+6 \leq 3$$

$$\Rightarrow -9 \leq 10x \leq -3$$

$$\frac{-9}{10} \leq x \leq \frac{-3}{10} \quad \dots \text{(iii)}$$

$$(i), (ii), (iii) \Rightarrow \frac{-3}{4} < x \leq \frac{-1}{2}$$

$$\alpha = \frac{-3}{4}, \beta = \frac{-1}{2}$$

$$36|\alpha + \beta| = 36 \times \frac{5}{4} = 45$$

19. Let $x = x(y)$ be the solution of the differential equation

$$2(y+2)\log_e(y+2)dx + (x+4 - 2\log_e(y+2))dy = 0,$$

$y > -1$ with $x(e^4 - 2) = 1$. Then $x(e^9 - 2)$ is equal

to

(1) 3

(2) $\frac{4}{9}$

$$(3) \frac{32}{9}$$

(4) $\frac{10}{3}$

Answer (3)

Sol. Let $x + 4 = u$, $y + 2 = v$

$$dx = du, dy = dv$$

$$(2v \ln v)du = -(u - 2 \ln v)dv$$

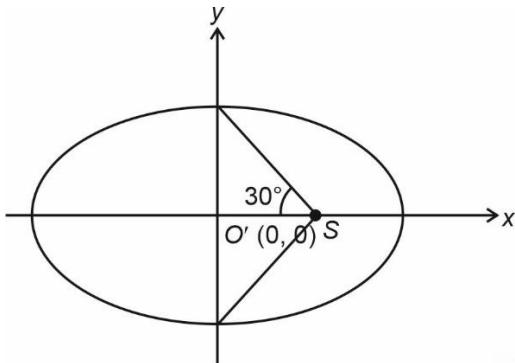
$$2v \ln v \frac{du}{dv} + u = 2 \ln v$$

$$\frac{du}{dy} + \frac{1}{2y \ln y} \cdot u = \frac{1}{y}$$

22. Let an ellipse with center $(1, 0)$ and latus rectum of length $\frac{1}{2}$ have its major axis along x -axis. If its minor axis subtends an angle 60° at the foci, then the square of the sum of the lengths of its minor and major axes is equal to _____.

Answer (9)

Sol.



$$\frac{2b^2}{a} = \frac{1}{2}, \quad \tan 30^\circ = \frac{b}{ae}$$

$$b^2 = \frac{a}{4}, \frac{1}{3} = \frac{b^2}{a^2 - b^2} \Rightarrow a^2 - b^2 = 3b^2 \Rightarrow b^2 = \frac{a^2}{4}$$

$$\Rightarrow a = 1, b^2 = \frac{1}{4} \Rightarrow b = \frac{1}{2}$$

$$\Rightarrow (2a + 2b)^2 = 9$$

23. A person forgets his 4-digit ATM pin code. But he remembers that in the code all the digits are different, the greatest digit is 7 and the sum of the first two digits is equal to the sum of the last two digits. Then the maximum number of trials necessary to obtain the correct code is _____.

Answer (72)

Sol. $abcd$

$$a + b = c + d$$

0, 1, 2, 3, 4, 5, 6, 7

Let $a + b = c + d = \lambda$

There must be 7

$$\Rightarrow a + b = c + d = \lambda \geq 7$$

$$\lambda = 7, (a, b)/(c, d) \rightarrow (0, 7)/(1, 6)$$

or $(2, 5)(3, 4) \rightarrow 24$ numbers

$$\lambda = 8, (a, b)/(c, d) \rightarrow (1, 7)/(2, 6)$$

or $(3, 5) \rightarrow 16$ numbers

$$\lambda = 9, (a, b)/(c, d) \rightarrow (2, 7)/(3, 6)$$

or $(4, 5) \rightarrow 16$ numbers

$$\lambda = 10, (a, b)/(c, d) \rightarrow (3, 7)/(4, 6) \rightarrow 8$$
 numbers

$$\lambda = 11, (a, b)/(c, d) \rightarrow (4, 7)/(5, 6) \rightarrow 8$$
 numbers

$$\lambda = 12, (a, b)/(c, d) \rightarrow (5, 7) \text{ Not possible}$$

$$\lambda = 13, (a, b)/(c, d) \rightarrow (6, 7) \text{ Not possible}$$

$$\lambda = 14, (a, b)/(c, d) \rightarrow (7, 7) \text{ Not possible}$$

$$\Rightarrow \text{Total numbers} = 72$$

24. The number of elements in the set $\{n \in \mathbb{N} : 10 \leq n \leq 100 \text{ and } 3^n - 3 \text{ is a multiple of 7}\}$ is _____.

Answer (15.00)

$$\text{Sol. } 3^n - 3 = 7k$$

$$3 \equiv 3 \pmod{7}$$

$$3^2 \equiv 2 \pmod{7}$$

$$3^3 \equiv -1 \pmod{7}$$

$$3^6 \equiv 1 \pmod{7}$$

$$3^7 \equiv 3 \pmod{7}$$

:

$$3^{13} \equiv 3 \pmod{7}$$

n can be 13, 19, 25, ... 97

Total 15 such n exist

25. Let $A = \{1, 2, 3, 4\}$ and R be a relation on the set $A \times A$ defined by $R = \{(a, b), (c, d) : 2a + 3b = 4c + 5d\}$. Then the number of elements in R is _____.

Answer (06.00)

$$\text{Sol. } 2a + 3b = 4c + 5d$$

Maximum value of $2a + 3b = 20$ at $(4, 4)$

Minimum value of $4c + 5d = 9$ at $(1, 1)$

So, $4c + 5d$ can be equal to 9, 13, 14, 17, 18, 19

$$2a + 3b \text{ can be 9} \Rightarrow (a, b) = (3, 1)$$

$$(c, d) = (1, 1)$$

$$2a + 3b \text{ can be 13} \Rightarrow (a, b) = (2, 3)$$

$$(c, d) = (2, 1)$$

$$2a + 3b \text{ can be 14} \Rightarrow (a, b) = (4, 2) \text{ OR } (1, 4)$$

$$(c, d) = (1, 2)$$

$$2a + 3b \text{ can be 17} \Rightarrow (a, b) = (4, 3)$$

$$(c, d) = (3, 1)$$

$$2a + 3b \text{ can be 18} \Rightarrow (a, b) = (3, 4)$$

$$(c, d) = (2, 2)$$

26. Consider the triangles with vertices $A(2, 1)$, $B(0, 0)$ and $C(t, 4)$, $t \in [0, 4]$. If the maximum and the minimum perimeters of such triangles are obtained at $t = \alpha$ and $t = \beta$ respectively, then $6\alpha + 21\beta$ is equal to _____.

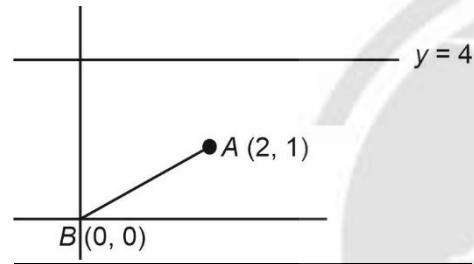
Answer (48.00)

Sol. To minimize $CA + CB$, take image of B in

$$y = 4$$

$$B' = (0, 8)$$

AB'



$$y - 8 = \frac{-7}{2}(x - 0)$$

when $y = 4$

$$-4 = \frac{-7}{2}(x)$$

$$x = \frac{8}{7} \Rightarrow \beta = \frac{8}{7}$$

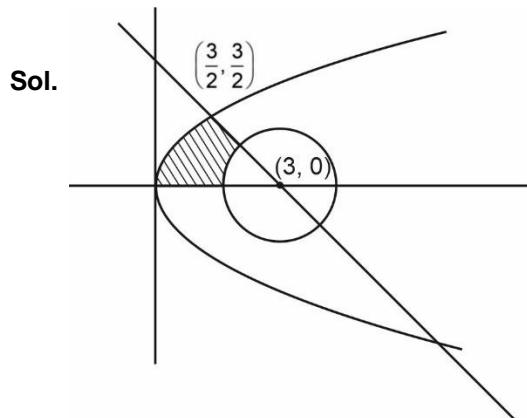
Maximization will be possible if $\alpha = 0$ or 4

When compared $\alpha = 4$

$$6\alpha + 21\beta = 48$$

27. If the area bounded by the curve $2y^2 = 3x$, lines $x + y = 3$, $y = 0$ and outside the circle $(x - 3)^2 + y^2 = 2$ is A , then $4(\pi + 4A)$ is equal to _____.

Answer (42.00)



$$A = \int_0^{3/2} \left((3 - y) - \frac{2y^2}{3} \right) dy - \pi(\sqrt{2})^2 \cdot \frac{1}{8}$$

$$= \frac{36 - 9 - 6}{8} - \frac{\pi}{4} = \frac{21}{8} - \frac{\pi}{4}$$

28. If the line $x = y = z$ intersects the line $x\sin A + y\sin B + z\sin C - 18 = 0 = x\sin 2A + y\sin 2B + z\sin 2C - 9$, where A, B, C are the angles of a triangle ABC , then $80 \left(\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \right)$ is equal to _____.

Answer (5)

Sol. $x = y = z = k$ (let)

$$\therefore k(\sin A + \sin B + \sin C) = 18$$

$$\Rightarrow k \left(4 \cos \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \cos \frac{C}{2} \right) = 18 \dots (i)$$

$$k(\sin 2A + \sin 2B + \sin 2C) = 9$$

$$\Rightarrow k(4 \sin A \cdot \sin B \cdot \sin C) = 9 \dots (ii)$$

(ii)/(i)

$$8 \sin \frac{A}{2} \cdot \sin \frac{B}{2} \cdot \sin \frac{C}{2} = \frac{9}{18}$$

$$\Rightarrow 80 \sin \frac{A}{2} \cdot \sin \frac{B}{2} \cdot \sin \frac{C}{2} = 5$$

29. If the sum of the series

$$\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{2^2} - \frac{1}{2 \cdot 3} + \frac{1}{3^2}\right) + \left(\frac{1}{2^3} - \frac{1}{2^2 \cdot 3} + \frac{1}{2 \cdot 3^2} - \frac{1}{3^3}\right) +$$

$$\left(\frac{1}{2^4} - \frac{1}{2^3 \cdot 3} + \frac{1}{2^2 \cdot 3^2} - \frac{1}{2 \cdot 3^3} + \frac{1}{3^4}\right) + \dots$$

is $\frac{\alpha}{\beta}$, where α and β are co-prime, then $\alpha + 3\beta$ is equal to _____.

Answer (7)

Sol. Let $a = \frac{1}{2}$, $b = \frac{1}{3}$

Given: $(a - b) + (a^2 - ab + b^2) + (a^3 - a^2b + ab^2 - b^3) + \dots$

$$\Rightarrow \frac{1}{a+b} \left((a^2 - b^2) + (a^3 + b^3) + (a^4 - b^4) + \dots \right)$$

$$\Rightarrow \left(\frac{1}{a+b} \right) \left(a^2 + a^3 + a^4 - (b^2 + b^3 + b^4) \dots \right)$$

$$= \left(\frac{1}{a+b} \right) \left(\frac{a^2}{1-a} - \frac{b^2}{1+b} \right)$$

$$= \frac{1}{\frac{1}{2} + \frac{1}{3}} \left(\frac{\frac{1}{4}}{1 - \frac{1}{2}} - \frac{\frac{1}{9}}{1 + \frac{1}{3}} \right)$$

$$= \frac{6}{5} \left(\frac{1}{2} - \frac{1}{12} \right)$$

$$= \frac{6}{5} \left(\frac{5}{12} \right) = \frac{1}{2} = \frac{\alpha}{\beta}$$

$$\alpha + 3\beta = 1 + 6 = 7$$

30. Let the plane P contain the line $2x + y - z - 3 = 0 = 5x - 3y + 4z + 9$ and be parallel to the line $\frac{x+2}{2} = \frac{3-y}{-4} = \frac{z-7}{5}$. Then the distance of the point $A(8, -1, -19)$ from the plane P measured parallel to the line $\frac{x}{-3} = \frac{y-5}{4} = \frac{2-z}{-12}$ is equal to _____.

Answer (26)

Sol. Plane containing the line $2x + y - z - 3 = 0 = 5x - 3y + 4z + 9$ is $2x + y - z - 3 + \lambda(5x - 3y + 4z + 9) = 0$

$$\Rightarrow x(2 + 5\lambda) + y(1 - 3\lambda) + z(4\lambda - 1) + 9\lambda - 3 = 0$$

This plane is parallel to the line

$$\frac{x+2}{2} = \frac{3-y}{-4} = \frac{z-7}{5}$$

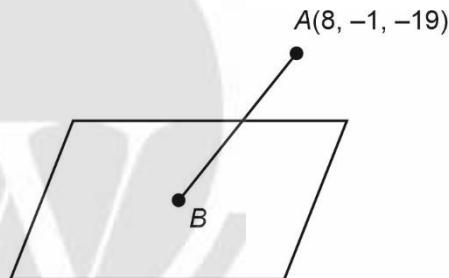
$$\therefore (2 + 5\lambda)(2) + (1 - 3\lambda)(4) + (4\lambda - 1)5 = 0$$

$$\Rightarrow 4 + 10\lambda + 4 - 12\lambda + 20\lambda - 5 = 0$$

$$\Rightarrow 18\lambda = -3 \Rightarrow \lambda = \frac{-1}{6}$$

$$\therefore \text{Plane } P: \frac{7}{6}x + \frac{3}{2}y - \frac{5}{3}z - \frac{9}{2} = 0$$

$$\Rightarrow 7x + 9y - 10z - 27 = 0$$



$$AB: \frac{x-8}{-3} = \frac{y+1}{4} = \frac{z+19}{12} = k$$

$$B = (-3k + 8, 4k - 1, 12k - 19)$$

B lies on plane P .

$$7(-3k + 8) + 9(4k - 1) - 10(12k - 19) = 27$$

$$\Rightarrow -21k + 56 + 36k - 9 - 120k + 190 = 27$$

$$\Rightarrow -105k = -210$$

$$\Rightarrow k = 2$$

$$\therefore B = (2, 7, 5)$$

$$AB = \sqrt{36 + 64 + 576}$$

$$= \sqrt{676} = 26$$

43. A vector in $x - y$ plane makes an angle of 30° with y -axis. The magnitude of y -component of vector is $2\sqrt{3}$. The magnitude of x -component of the vector will be :

- (1) $\frac{1}{\sqrt{3}}$ (2) 6
 (3) 2 (4) $\sqrt{3}$

Answer (3)

Sol. $a_y = 2\sqrt{3}$

$$\therefore a_x = a_y \tan 30^\circ$$

$$= 2\sqrt{3} \times \frac{1}{\sqrt{3}}$$

44. The position of a particle related to time is given by $x = (5t^2 - 4t + 5)$ m. The magnitude of velocity of the particle at $t = 2$ s will be :

- (1) 06 ms^{-1} (2) 14 ms^{-1}
 (3) 10 ms^{-1} (4) 16 ms^{-1}

Answer (4)

Sol. $x = 5t^2 - 4t + 5$

$$\Rightarrow v = 10t - 4$$

$$\Rightarrow v = 16 \text{ m/s}$$

45. The electric field due to a short electric dipole at a large distance (r) from center of dipole on the equatorial plane varies with distance as :

- (1) r
 (2) $\frac{1}{r^2}$
 (3) $\frac{1}{r^3}$
 (4) $\frac{1}{r}$

Answer (3)

Sol. $E = \frac{2kp}{r^3}$

46. In a linear Simple Harmonic Motion (SHM)
- (A) Restoring force is directly proportional to the displacement.
 - (B) The acceleration and displacement are opposite in direction.
 - (C) The velocity is maximum at mean position.
 - (D) The acceleration is minimum at extreme points.

Choose the correct answer from the options given below:

- (1) (C) and (D) only
 (2) (A), (C) and (D) only
 (3) (A), (B) and (C) only
 (4) (A), (B) and (D) only

Answer (3)

Sol. $F = -kx$

$$a = -\omega^2 x$$

Velocity is maximum at mean position.

47. The speed of a wave produced in water is given by $v = \lambda^a g^b \rho^c$. Where λ , g and ρ are wavelength of wave, acceleration due to gravity and density of water respectively. The values of a , b and c respectively, are

- (1) 1, -1, 0
 (2) $\frac{1}{2}, 0, \frac{1}{2}$
 (3) 1, 1, 0
 (4) $\frac{1}{2}, \frac{1}{2}, 0$

Answer (4)

Sol. $[v] = [\lambda^a g^b \rho^c]$

$$\Rightarrow [LT^{-1}] = [L]^a [LT^{-2}]^b [ML^{-3}]^c$$

48. A body is released from a height equal to the radius (R) of the earth. The velocity of the body when it strikes the surface of the earth will be:

(Given g = acceleration due to gravity on the earth.)

- (1) $\sqrt{2gR}$
 (2) \sqrt{gR}
 (3) $\sqrt{4gR}$
 (4) $\sqrt{\frac{gR}{2}}$

Answer (2)

Sol. $-\frac{GMm}{2R} = -\frac{GMm}{R} + \frac{1}{2}mv^2$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{2R}$$

$$v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

49. Given below are two statements:

Statement I : The equivalent resistance of resistors in a series combination is smaller than least resistance used in the combination.

Statement II: The resistivity of the material is independent of temperature.

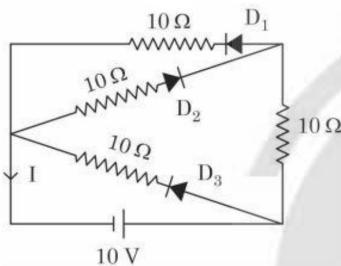
In the light of the above statements, choose the correct answer from the options given below:

- (1) Both Statement I and Statement II are false
- (2) Both Statement I and Statement II are true
- (3) Statement I is true but Statement II is false
- (4) Statement I is false but Statement II is true

Answer (1)

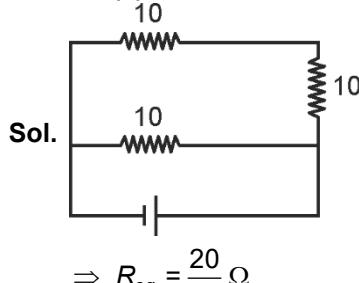
Sol. $R_{\text{series}} > R_1 \text{ or } R_2$
as $R_{\text{series}} = R_1 + R_2$
 $\rho = \rho_0 (1 + \alpha \Delta T)$

50. In the given circuit, the current (I) through the battery will be



- (1) 2.5 A
- (2) 1 A
- (3) 2 A
- (4) 1.5 A

Answer (4)



$$I = \frac{30}{20} \text{ A} = 1.5 \text{ A}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

51. The fundamental frequency of vibration of a string between two rigid support is 50 Hz. The mass of the string is 18 g and its linear mass density is 20 g/m. The speed of the transverse waves so produced in the string is _____ ms⁻¹.

Answer (90)

Sol. $\frac{V}{2l} = 50$

$$v = 100 \times l = \frac{100 \times 18}{20} = 90 \text{ m/s}$$

52. A block of mass 10 kg is moving along x-axis under the action of force $F = 5x$ N. The work done by the force in moving the block from $x = 2$ m to 4 m will be _____ J.

Answer (30)

Sol. $F = 5x$

$$W = \frac{5}{2} (x_f^2 - x_i^2) = \frac{5}{2} \times 12 = 30 \text{ J}$$

53. A solid sphere and a solid cylinder of same mass and radius are rolling on a horizontal surface without slipping. The ratio of their radius of gyration respectively ($k_{\text{sph}} : k_{\text{cyl}}$) is $2 : \sqrt{x}$. The value of x is _____.

Answer (5)

Sol. Considering rotational axis as the diametrical axis for sphere and axis of cylinder. Then

$$K_1^2 = \frac{2}{5} R^2 \text{ and } K_2^2 = \frac{1}{2} R^2$$

$$\therefore \frac{K_1}{K_2} = \sqrt{\frac{2/5}{1/2}} = \sqrt{\frac{4}{5}}$$

$$\Rightarrow \frac{K_1}{K_2} = \frac{2}{\sqrt{5}}$$

$$\therefore x = 5$$

54. An electron in a hydrogen atom revolves around its nucleus with a speed of $6.76 \times 10^6 \text{ ms}^{-1}$ in an orbit of radius 0.52 Å. The magnetic field produced at the nucleus of the hydrogen atom is _____ T.

Answer (40)

Sol. $B = \frac{\mu_0 I}{2r} = \frac{\mu_0 e \times \omega}{2\pi \times 2 \times r}$

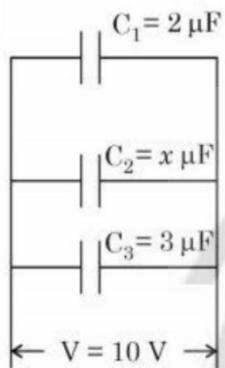
$$= \frac{10^{-7} \times 1.6 \times 6.76 \times 10^6 \times 10^{-19}}{0.52 \times 0.52 \times 10^{-20}} = 40$$

55. There is an air bubble of radius 1.0 mm in a liquid of surface tension 0.075 Nm^{-1} and density 1000 kg m^{-3} at a depth of 10 cm below the free surface. The amount by which the pressure inside the bubble is greater than the atmospheric pressure is _____ Pa ($g = 10 \text{ ms}^{-2}$)

Answer (1150)

$$\text{Sol. } \Delta P = \rho gh + \frac{2T}{r} = 1000 + \frac{2 \times 0.075}{10^{-3}} = 1000 + 150 = 1150$$

56. In the given figure the total charge stored in the combination of capacitors is $100 \mu\text{C}$. The value of 'x' is _____.



Answer (5)

$$\text{Sol. } 10(2 + x + 3) = 100$$

$$\Rightarrow x = 5$$

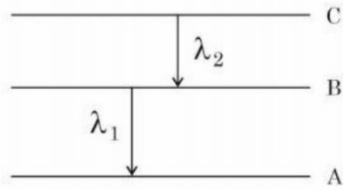
57. A 20 cm long metallic rod is rotated with 210 rpm about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field 0.2 T parallel to the axis exists everywhere. The emf developed between the centre and the ring is _____ mV.

$$\text{Take } \pi = \frac{22}{7}$$

Answer (88)

$$\begin{aligned} \text{Sol. } \varepsilon &= \frac{B_0 I^2}{2} \\ &= 0.2 \times \frac{210 \times 2\pi}{60} \times \frac{(0.2)^2}{2} \\ &= \frac{0.2 \times 210 \times 22 \times 0.04}{7 \times 60} \\ &= 88 \text{ mV} \end{aligned}$$

58. As per given figure A, B and C are the first, second and third excited energy levels of hydrogen atom respectively. If the ratio of the two wavelengths $\left(i.e. \frac{\lambda_1}{\lambda_2} \right)$ is $\frac{7}{4n}$, then the value of n will be



Answer (5)

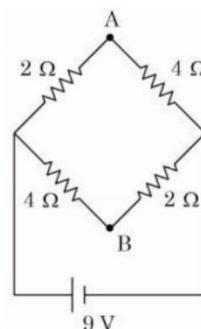
$$\begin{aligned} \text{Sol. } \frac{\lambda_1}{\lambda_2} &= \frac{\left(\frac{1}{9} - \frac{1}{16}\right)}{\left(\frac{1}{4} - \frac{1}{9}\right)} = \frac{\frac{7}{16}}{\frac{5}{36}} \\ &\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{7}{20} \end{aligned}$$

59. The refractive index of a transparent liquid filled in an equilateral hollow prism is $\sqrt{2}$. The angle of minimum deviation for the liquid will be _____ °.

Answer (30)

$$\begin{aligned} \text{Sol. } \sqrt{2} &= \frac{\sin\left(30 + \frac{\delta}{2}\right)}{\frac{1}{2}} \\ &\Rightarrow 45^\circ = 30^\circ + \frac{\delta}{2} \\ &\delta = 30^\circ \end{aligned}$$

60. A network of four resistances is connected to 9 V battery, as shown in figure. The magnitude of voltage difference between the points A and B is _____ V.



Answer (3)

$$\text{Sol. } I_A = I_B = 1.5 \text{ A}$$

$$V_A - V_B = 4 \times 1.5 - 2 \times 1.5 = 3$$

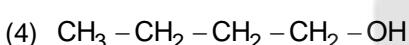
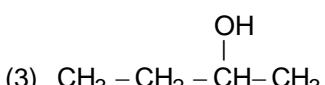
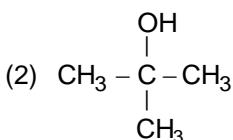
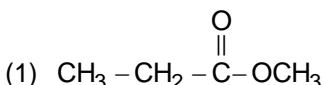
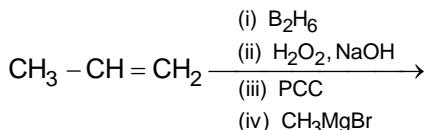
CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

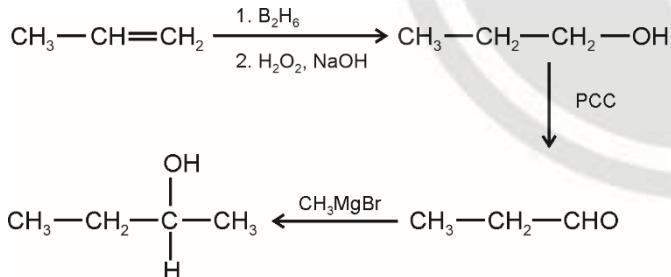
Choose the correct answer:

61. The product formed in the following multistep reaction is:

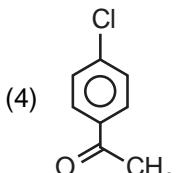
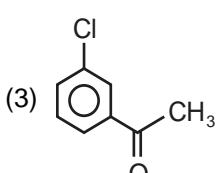
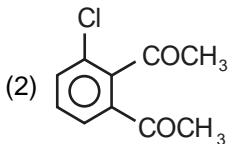
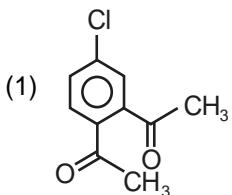


Answer (3)

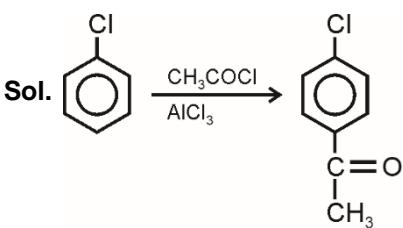
Sol.



62. The major product formed in the Friedel-Crafts acylation of chlorobenzene is



Answer (4)

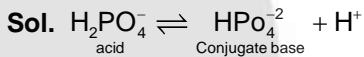


63. Which of the following statement(s) is/are correct?
 (A) The pH of 1×10^{-8} M HCl solution is 8.
 (B) The conjugate base of H_2PO_4^- is HPO_4^{2-} .
 (C) K_w increases with increase in temperature.
 (D) When a solution of a weak monoprotic acid is titrated against a strong base at half neutralisation point, $\text{pH} = \frac{1}{2}\text{pK}_a$.

Choose the correct answer from the options given below:

- | | |
|-------------------|-------------------|
| (1) (B), (C) | (2) (A), (D) |
| (3) (A), (B), (C) | (4) (B), (C), (D) |

Answer (1)



K_w increase with increase in temperature as dissociation of water increases with increase in temperature.

When weak monoprotic acid is titrated against strong base, at half neutralization point, the solution becomes acidic buffer and its pH is given by

$$\text{pH} = \text{pK}_a + \log \frac{[\text{Salt}]}{[\text{Acid}]}$$

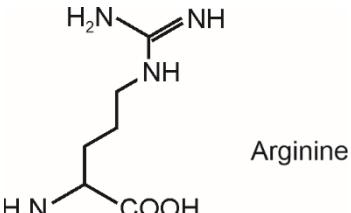
$$\Rightarrow \boxed{\text{pH} = \text{pK}_a + 0} \text{ as } [\text{salt}] = [\text{acid}]$$

64. Which is not true for arginine?

- (1) It has a fairly high melting point.
- (2) It is associated with more than one pK_a values.
- (3) It has high solubility in benzene.
- (4) It is a crystalline solid.

Answer (3)

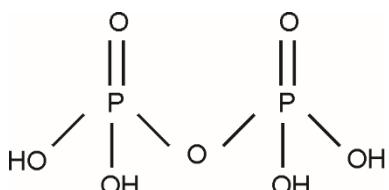
Sol.



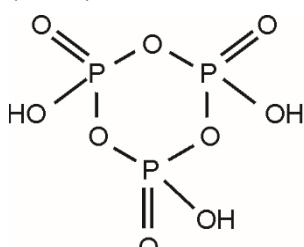
Since arginine is polar, it is not soluble in benzene.

Answer (4)

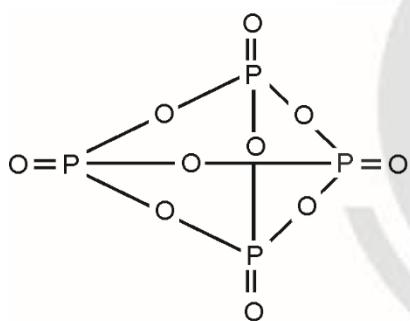
Sol. H₄P₂O₇:



$$(\text{HPO}_3)_3$$



P₄O₁₀



66. The possibility of photochemical smog formation will be minimum at

 - Srinagar, Jammu and Kashmir in January
 - Kolkata in October
 - Mumbai in May
 - New-Delhi in August (Summer)

Answer (1)

Sol. Photochemical smog occurs in warm, dry and sunny climate

Hence the correct answer is option (1).

67. The complex with highest magnitude of crystal field splitting energy (Δ_0) is

(1) $[\text{Ti}(\text{OH}_2)_6]^{3+}$ (2) $[\text{Cr}(\text{OH}_2)_6]^{3+}$
(3) $[\text{Mn}(\text{OH}_2)_6]^{3+}$ (4) $[\text{Fe}(\text{OH}_2)_6]^{3+}$

Answer (2)

Sol.	Complex	CFSE
[Ti(OH ₂) ₆] ³⁺	-0.4 Δ ₀	
[Cr(H ₂ O) ₆] ³⁺	-1.2 Δ ₀	
[Mn(H ₂ O) ₆] ³⁺	-0.6 Δ ₀	
[Fe(H ₂ O) ₆] ³⁺	0	

68. During water-gas shift reaction

 - (1) Carbon monoxide is oxidized to carbon dioxide.
 - (2) Water is evaporated in presence of catalyst.
 - (3) Carbon is oxidized to carbon monoxide.
 - (4) Carbon dioxide is reduced to carbon monoxide.

Answer (1)

Sol. $\text{C} + \text{H}_2\text{O} \longrightarrow \text{CO} + \underbrace{\text{H}_2}_{\text{syn gas}}$



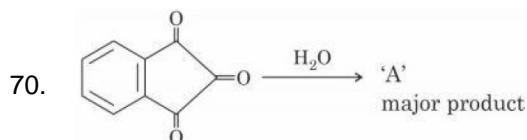
Water gas shift reaction

69. Which of the following statement is correct for paper chromatography?

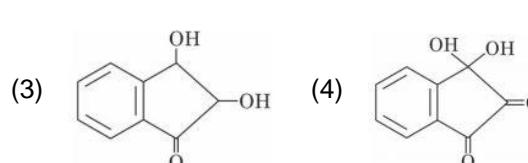
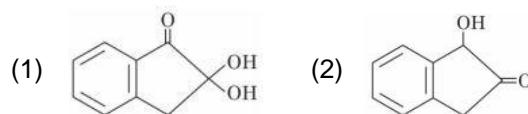
 - (1) Water present in the pores of the paper forms the stationary phase.
 - (2) Paper sheet forms the stationary phase.
 - (3) Water present in the mobile phase gets absorbed by the paper which then forms the stationary phase.
 - (4) Paper and water present in its pores together form the stationary phase.

Answer (1)

Sol. In paper chromatography water present in the pores of the paper forms the stationary phase.

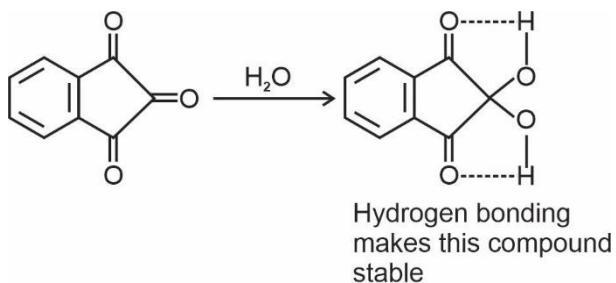


'A' formed in the above reaction is

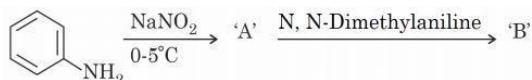


Answer (1)

Sol.



71. Consider the following sequence of reactions

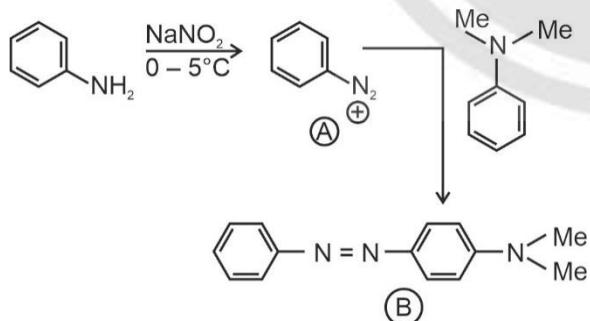


The product 'B' is

- (1)
- (2)
- (3)
- (4)

Answer (3)

Sol.



72. Given below are two statements

Statement I : According to Bohr's model of hydrogen atom, the angular momentum of an electron in a given stationary state is quantised.

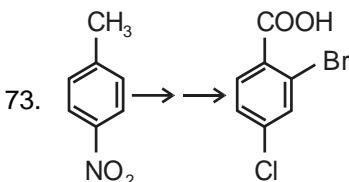
Statement II : The concept of electron in Bohr's orbit, violates the Heisenberg uncertainty principle.

In the light of the above statements, choose the *most appropriate* answer from the options given below

- (1) Statement I is incorrect but Statement II is correct
- (2) Both Statement I and Statement II are correct
- (3) Both Statement I and Statement II are incorrect
- (4) Statement I is correct but Statement II is incorrect

Answer (2)

Sol. Both statements (I) and (II) are correct.

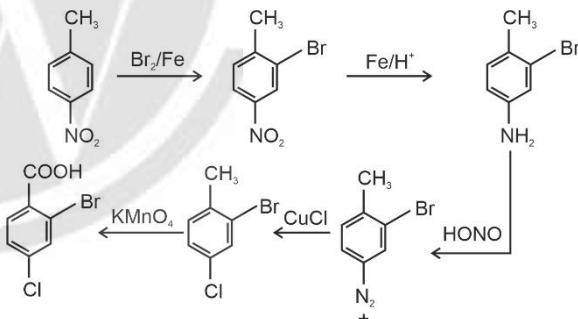


In the above conversion the correct sequence of reagents to be added is

- (1) (i) KMnO4, (ii) Br2/Fe, (iii) Fe/H+, (iv) Cl2
- (2) (i) Br2/Fe, (ii) Fe/H+, (iii) KMnO4, (iv) Cl2
- (3) (i) Fe/H+, (ii) HONO, (iii) CuCl, (iv) KMnO4, (v) Br2
- (4) (i) Br2/Fe, (ii) Fe/H+, (iii) HONO, (iv) CuCl (v) KMnO4

Answer (4)

Sol.



74. Which one of the following is not an example of calcination?

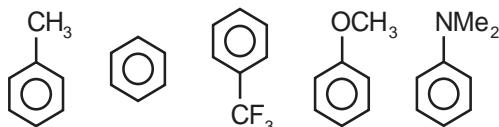
- (1) $\text{CaCO}_3 \xrightarrow{\Delta} \text{CaO} + \text{CO}_2$
- (2) $\text{Fe}_2\text{O}_3 \cdot x\text{H}_2\text{O} \xrightarrow{\Delta} \text{Fe}_2\text{O}_3 + x\text{H}_2\text{O}$
- (3) $2\text{PbS} + 3\text{O}_2 \xrightarrow{\Delta} 2\text{PbO} + 2\text{SO}_2$
- (4) $\text{CaCO}_3 \cdot \text{MgCO}_3 \xrightarrow{\Delta} \text{CaO} + \text{MgO} + 2\text{CO}_2$

Answer (3)

Sol. $2\text{PbS} + 3\text{O}_2 \xrightarrow{\Delta} 2\text{PbO} + 2\text{SO}_2$

The above reaction is an example of roasting

80. Decreasing order of reactivity towards electrophilic substitution for the following compounds is:

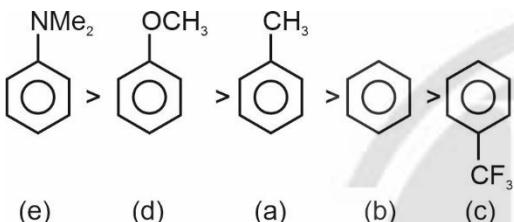


- (1) d > a > e > c > b (2) e > d > a > b > c
 (3) a > d > e > b > c (4) c > b > a > d > e

Answer (2)

Sol. Reactivity of aromatic compounds towards electrophilic substitution depends on the electron density on benzene ring. Higher the electron density on benzene ring, more be the reactivity towards electrophilic substitution reaction.

Hence, the correct decreasing order is



SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. In Chromyl chloride, the oxidation state of chromium is (+) _____.

Answer (6)

Sol. Chromyl chloride is $\text{Cl}-\overset{\text{O}}{\underset{\text{O}}{\text{||}}} \text{Cr}-\text{Cl}$

Hence the O.S. of Cr is +6

82. The vapour pressure of 30% (w/v) aqueous solution of glucose is _____ mm Hg at 25°C.

[Given: The density of 30% (w/v), aqueous solution of glucose is 1.2 g cm⁻³ and vapour pressure of pure water is 24 mm Hg.]

(Molar mass of glucose is 180 g mol⁻¹)

Answer (23)

Sol. Wt. of solution = $100 \times 1.2 = 120$ gm

Wt. of water = $120 - 30 = 90$ gm

Now using formula

$$\frac{P_0 - P}{P} = \frac{\text{moles of glucose}}{\text{moles of water}}$$

$$\frac{24 - P}{P} = \frac{\frac{30}{180}}{\frac{90}{90}} = \frac{3}{90}$$

$$24 \times 90 - P \times 90 = 3 P$$

$$\Rightarrow P = 23.22$$

83. The volume (in mL) of 0.1 M AgNO₃ required for complete precipitation of chloride ions present in 20 mL of 0.01 M solution of [Cr(H₂O)₅Cl]Cl₂ as silver chloride is _____

Answer (4)

Sol. $(\text{Cr}(\text{H}_2\text{O})_5\text{Cl})\text{Cl}_2 \longrightarrow (\text{Cr}(\text{H}_2\text{O})_5\text{Cl}) + 2\text{Cl}^-$

Molarity of Cl⁻ ions = $0.01 \times 2 = 0.02$ M

∴ Millimoles of Cl⁻ ions in the given solution
 $= 20 \times 0.02$

∴ Millimoles of AgNO₃ required = 20×0.2

$$V \times 0.1 = 20 \times 0.02$$

$$V = 4 \text{ mL}$$

84. For a reversible reaction A ⇌ B, the $\Delta H_{\text{forward}}$ reaction = 20 kJ mol⁻¹. The activation energy of the uncatalyzed forward reaction is 300 kJ mol⁻¹. When the reaction is catalysed keeping the reactant concentration same, the rate of the catalysed forward reaction at 27°C is found to be same as that of the uncatalyzed reaction at 327°C. The activation energy of the catalysed backward reaction is _____ kJ mol⁻¹.

Answer (130)

Sol. As per the question

$$Ae^{-\frac{300 \times 10^3}{600 \times R}} = Ae^{-\frac{-Ea}{300 \times R}}$$

$$\Rightarrow \frac{10^3}{2} = \frac{Ea}{300}$$

$$\Rightarrow Ea = 150 \times 10^3 \text{ J mol}^{-1}$$

$$\Rightarrow Ea = 150 \text{ kJ mol}^{-1}$$

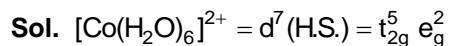
∴ Activation energy of catalysed backward reaction

$$= Ea - \Delta H$$

$$= 150 - 20 = 130 \text{ kJ mol}^{-1}$$

85. The homoleptic and octahedral complex of Co^{2+} and H_2O has _____ unpaired electron(s) in the t_{2g} set of orbitals.

Answer (1)



86. The number of correct statements from the following is _____

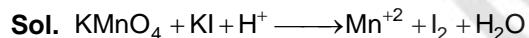
- (A) Conductivity always decreases with decrease in concentration for both strong and weak electrolytes.
- (B) The number of ions per unit volume that carry current in a solution increases on dilution.
- (C) Molar conductivity increases with decrease in concentration.
- (D) The variation in molar conductivity is different for strong and weak electrolytes.
- (E) For weak electrolytes, the change in molar conductivity with dilution is due to decrease in degree of dissociation.

Answer (3)

Sol. Except (B) and (E), all statements are correct.

87. The total change in the oxidation state of manganese involved in the reaction of KMnO_4 and potassium iodide in the acidic medium is _____.

Answer (5)



Hence the change in O.S. of Mn is (5).

88. The total number of isoelectronic species from the given set is _____.

$\text{O}^{2-}, \text{F}^-, \text{Al}, \text{Mg}^{2+}, \text{Na}^+, \text{O}^+, \text{Mg}, \text{Al}^{3+}, \text{F}$

Answer (5)

Sol.

Species	No. of electrons
O^{2-}	10
F^-	10
Al	13
Mg^{2+}	10
Na^+	10
O^+	7
Mg	12
Al^{3+}	10
F	9

Hence five species among the given are isoelectronic species.

89. 20 mL of 0.5 M NaCl is required to coagulate 200 mL of As_2S_3 solution in 2 hours. The coagulating value of NaCl is _____.

Answer (50)

Sol. Coagulating value = $\frac{20 \times 0.5}{200} \times 1000 = 50$

90. 30.4 kJ of heat is required to melt one mole of sodium chloride and the entropy change at the melting point is $28.4 \text{ J K}^{-1} \text{ mol}^{-1}$ at 1 atm. The melting point of sodium chloride is _____ K (Nearest Integer)

Answer (1070)

Sol. Melting point = $\frac{\Delta H_{\text{fus}}}{\Delta S}$
 $= \frac{30.4 \times 10^3}{28.4} = 1070 \text{ K}$

