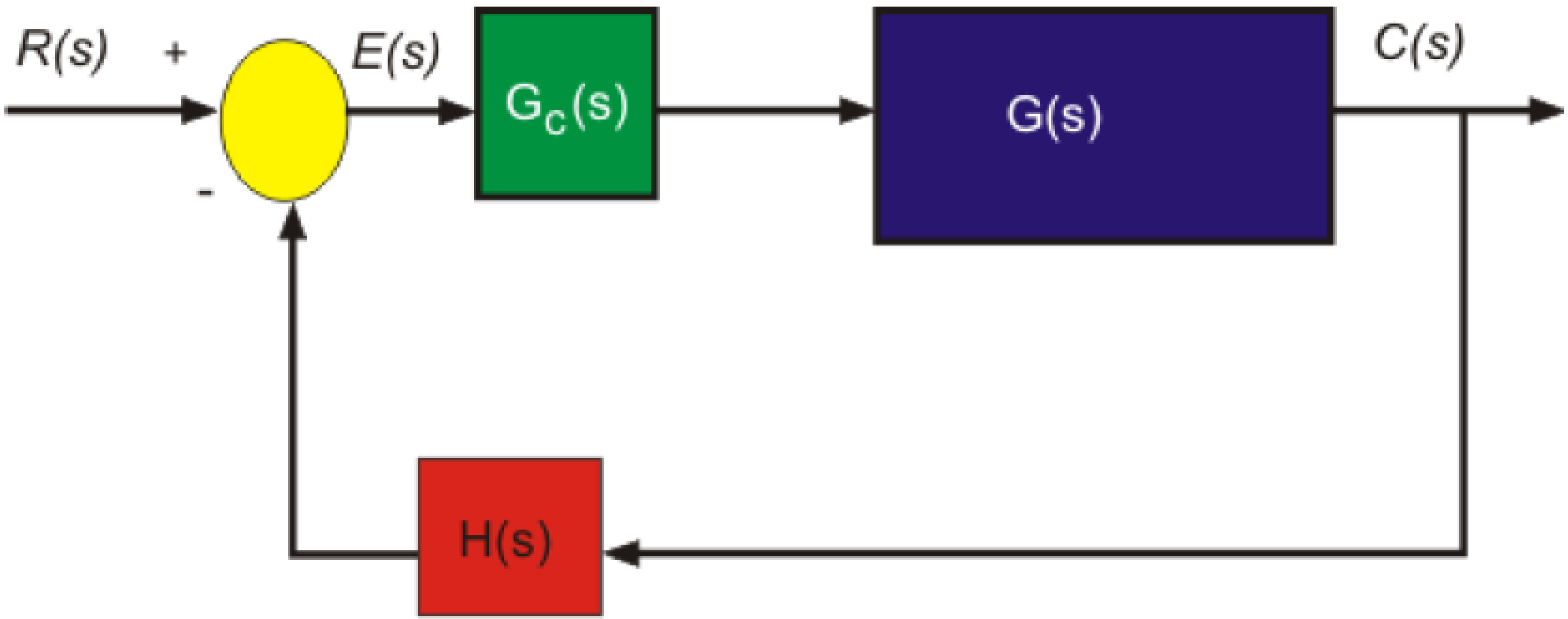
# Introduction to
# Active Vibration Control

## Three Basic Control Structures for AVC

AVC is implemented broadly using either of the three control systems–

- Single input-single output (SISO) vibration controller.
- State-space based multiple input and multiple output (MIMO) controller
- distributed vibration control.

# Classical SISO Control



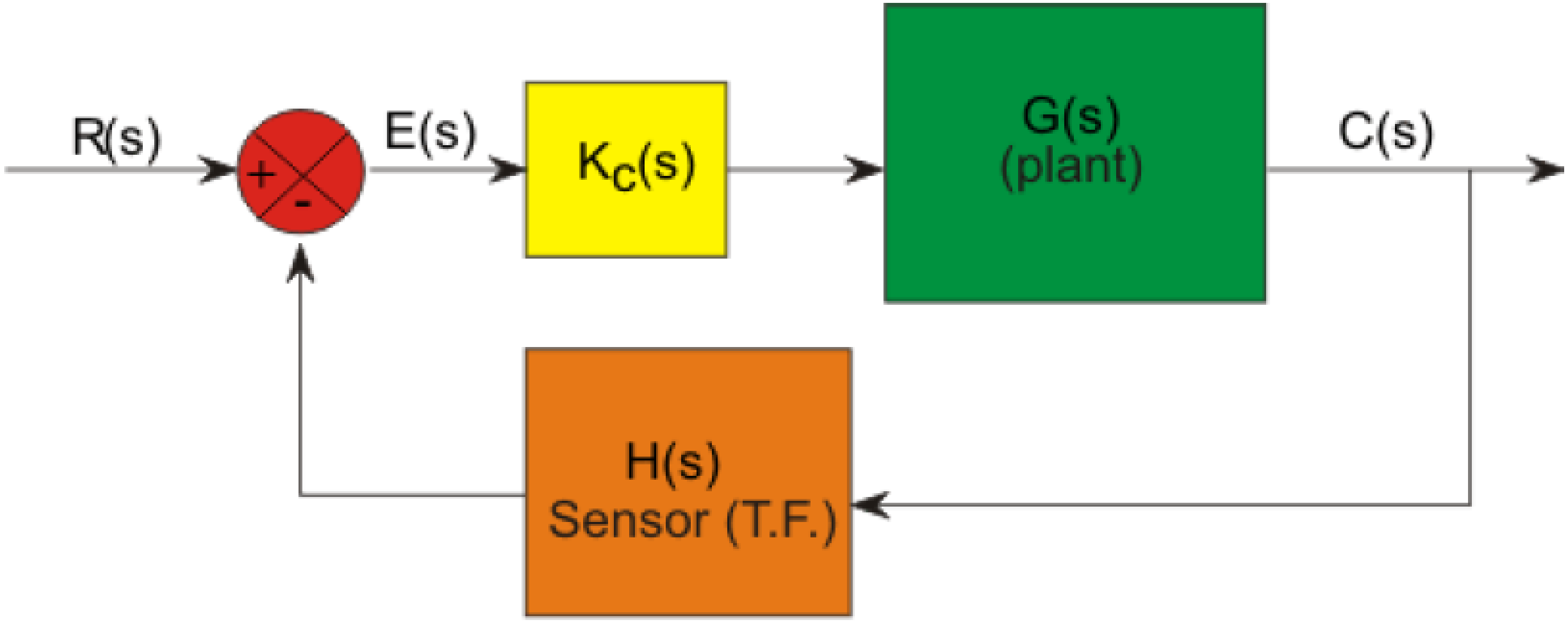

A typical Classical Control System where G(s) represents the transfer function of a single degree of freedom vibrating system, H(s) the transfer function of a sensor and $G_C(s)$ the transfer function of the controller + actuator. This type of system works well when the vibrating system could be represented as several decoupled SDOF vibrating system.

## MIMO Control

- For a multi DOF coupled system, the equation of motion is represented as a Matrix Equation of motion in the time domain.
- The entire analysis is carried out directly in the time domain for reducing real-time computational effort. This is also known as 'State-Space Analysis'.
- The controller is also represented in matrix form and the system performance is predicted by analysing the open and closed-loop plant matrices.

## Distributed Control

- The MIMO model could trigger problems of spillover as discussed earlier. This happens more in the case of control of flexible body system.
- One-way to avert such problem is to represent the governing equation of motion in the form of partial differential equation (without spatial and temporal decomposition). Thus, error due to model truncation is avoided.
- Again, for stability of such system, one would need continuous control force distributed over the entire system. The use of smart material based distributed actuator could resolve this problem.
- However, the difficulty arises while the controller is implemented in the real system. A continuously distributed controller is very difficult to achieve in real-time. An approximate discretely distributed controller is used in practice. Such systems indeed have limited applicability.




**The Closed loop Transfer Function (TF) of the system:**

$$\frac{C(s)}{R(s)} = \frac{K_c(s)G(s)}{1 + K_c(s)G(s)H(s)}$$

Laplace Transformation is a very useful tool for Designing Controller in the Frequency-Domain. This transformation helps to transform differential equation into the form of algebraic equations which is easier to manipulate.

A time-domain signal *f(t)* which may represent a forcing function or the response of a system may be transformed into frequency domain by using the following transformation:

$$L|f(t)| = F(s) = \int_{0}^{\infty} f(t)\, e^{-st}\, dt \qquad (23.1))$$

$f(t) = A\, e^{-pt}\, u(t)$

$$F(s) = \int_0^{\infty} A e^{-pt} e^{-st} u(t)\, dt = A \int_0^{\infty} e^{-(s+p)t}\, dt$$

$$= -\frac{A}{s+p} e^{-(s+p)t} \Big|_{t=0}^{\infty} = \frac{A}{s+p}$$

Applying the same principle on a differential equation one can obtained an algebraic equation. Consider a second order system as

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=F\sin(\omega t) \tag{23.2}$$

Applying Laplace transformation and assuming zero initial condition the above equation could be transformed as

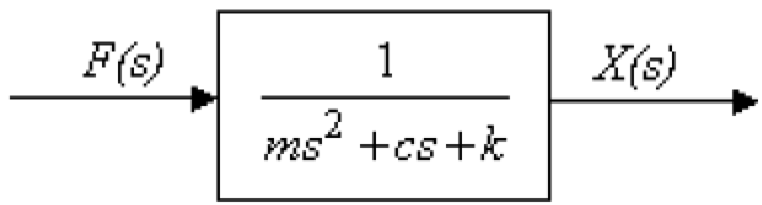
$$ms^2X(s)+csX(s)+kX(s)=\frac{F\omega}{s^2+\omega^2}$$

Denoting the right hand side of the above equation as $\overline{F}(s)$, one can express the ratio of frequency-domain response *X(s)* and $\overline{F}(s)$ as

$$T(s)=\frac{X(s)}{\overline{F}(s)}=\frac{1}{ms^2+cs+k}$$

*T(s)* is also known as **transfer function** of the system.

In a block diagram form, this can be represented as



## What will be the structure of $K_C(s)$?

For a generalized PID Controller

$$K_c(s) = \frac{K}{s}[(K_D s + 1)(s + K_I)]$$

for a simplified PD Controller

$$K_I = 0, \quad K_c(s) = K(K_D s + 1)$$

For a simplified PI Controller

$$\begin{aligned} K_D = 0, \quad K_c(s) &= \frac{K}{s}(s + K_I) \\ &= K\left(\frac{K_I}{s} + 1\right) \end{aligned}$$

The response of a system in time domain could be obtained by carrying out **Inverse Laplace Transformation** of the transfer function. The inverse Laplace Transform is written as

$$L^{-1}[F(s)] = \frac{1}{2\pi j} \int_{\sigma - j\omega}^{\sigma + j\omega} F(s) e^{st}\, ds \qquad (23.3)$$

However, this relationship is seldom used. If F(s) is rational, one commonly uses the method of partial fraction expansion. Consider a rational function F(s) expressed as:

$$F(s) = \frac{b_1 s^m + b_2 s^{m-1} + \ldots + b_{m+1}}{s^n + a_1 s^{n-1} + \ldots + a_n} \qquad (23.4)$$

Factoring the numerator and denominator polynomials one can also write

$$F(s) = K \frac{\prod_{i=1}^{m}(s - z_i)}{\prod_{i=1}^{n}(s - p_i)} \tag{23.5}$$

Corresponding to the numerator polynomial, $z_i$s are referred as the **zeros** of the transfer function while the roots of the numerator polynomial $p_i$'s are known as the **poles** of the transfer function.

Now, the transfer function *F(s)* may be expressed as

$$F(s) = \frac{C_1}{s - p_1} + \frac{C_2}{s - p_2} + \ldots + \frac{C_n}{s - p_n} \tag{23.6}$$

$$C_i = (s - p_i)\,F(s)\;\Big|_{s=p_i}$$

Finally, the response of the system may be expressed as

$$x(t) = \sum_{i=1}^{n} C_i e^{p_i t}$$

## A Simple Numerical Simulation

- Consider a SDOF system subjected to unit step input. Due to flexibility in the system, some part of the energy induces vibration in the system. A PID controller with $K$ =1.5, $K_D$ = 0.1 and $K_I$ = 0.5 is used to control the vibration.
- Find out the response of the system with and without closed-loop control. The following are the system parameters; m=0.2, c=0.001, k=0.5, H=1 (assume unity feedback).

## Sample Solution

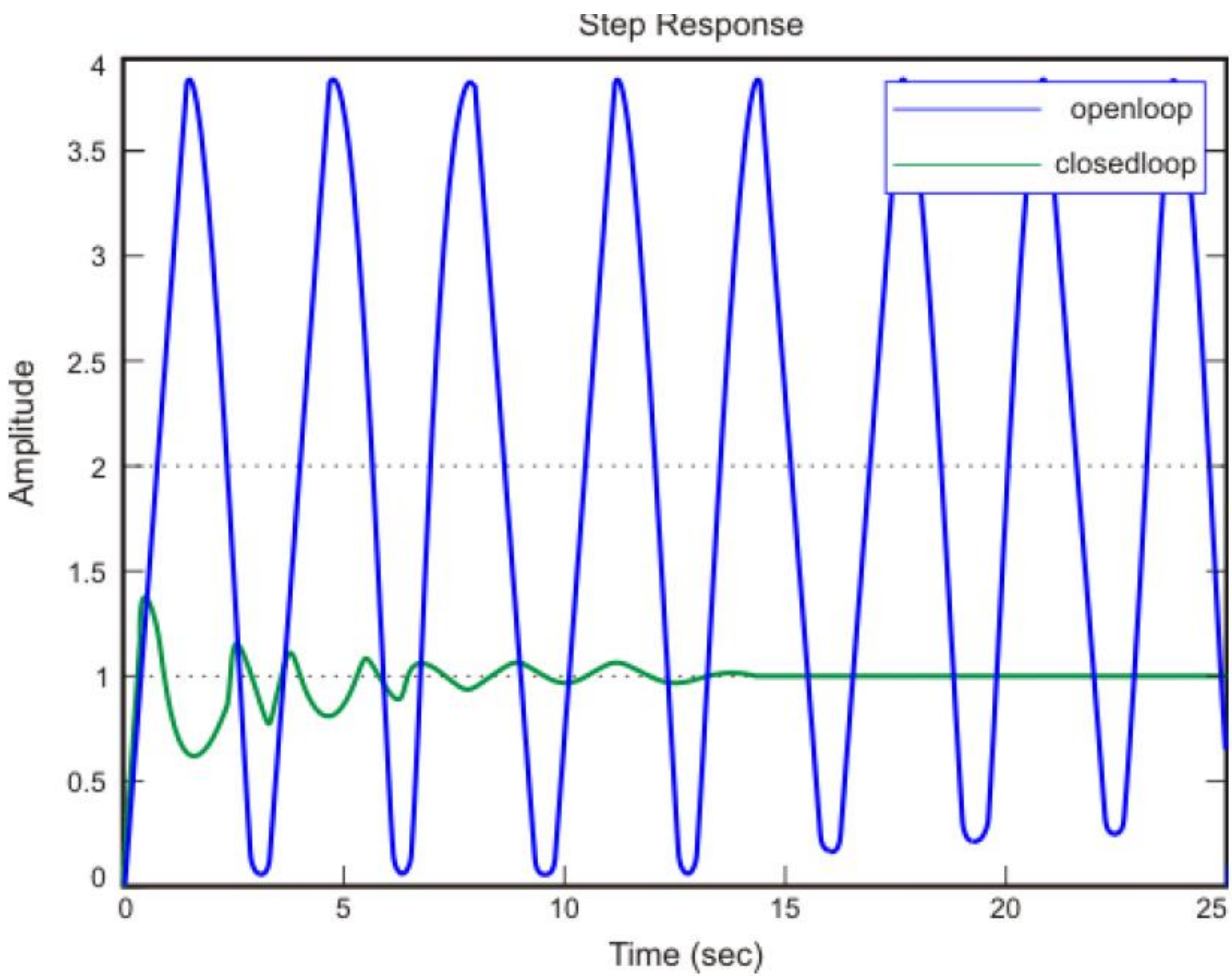
- The open loop transfer function of the system is given by:

$$T_o(s) = 1/(0.2s^2 + 0.001\,s + 0.5)$$

- The closed loop transfer function of the system is given by:

$$K_c(s) = 1.5 \times \frac{(0.1s + 1)(s + 0.5)}{s} = \frac{(s + 10)(2s + 1)}{13.3\,s}$$

$$T_{cl}\,(s) = \frac{2s^2 + 21s + 10}{2s^3 + 2.02s + 31s + 10}$$


Step Response
openloop
closedloop
Amplitude
Time (sec)
0
0.5
1
1.5
2
2.5
3
3.5
4
0
5
10
15
20
25