Lecture 11

ME756A

Principles of Vibration Control

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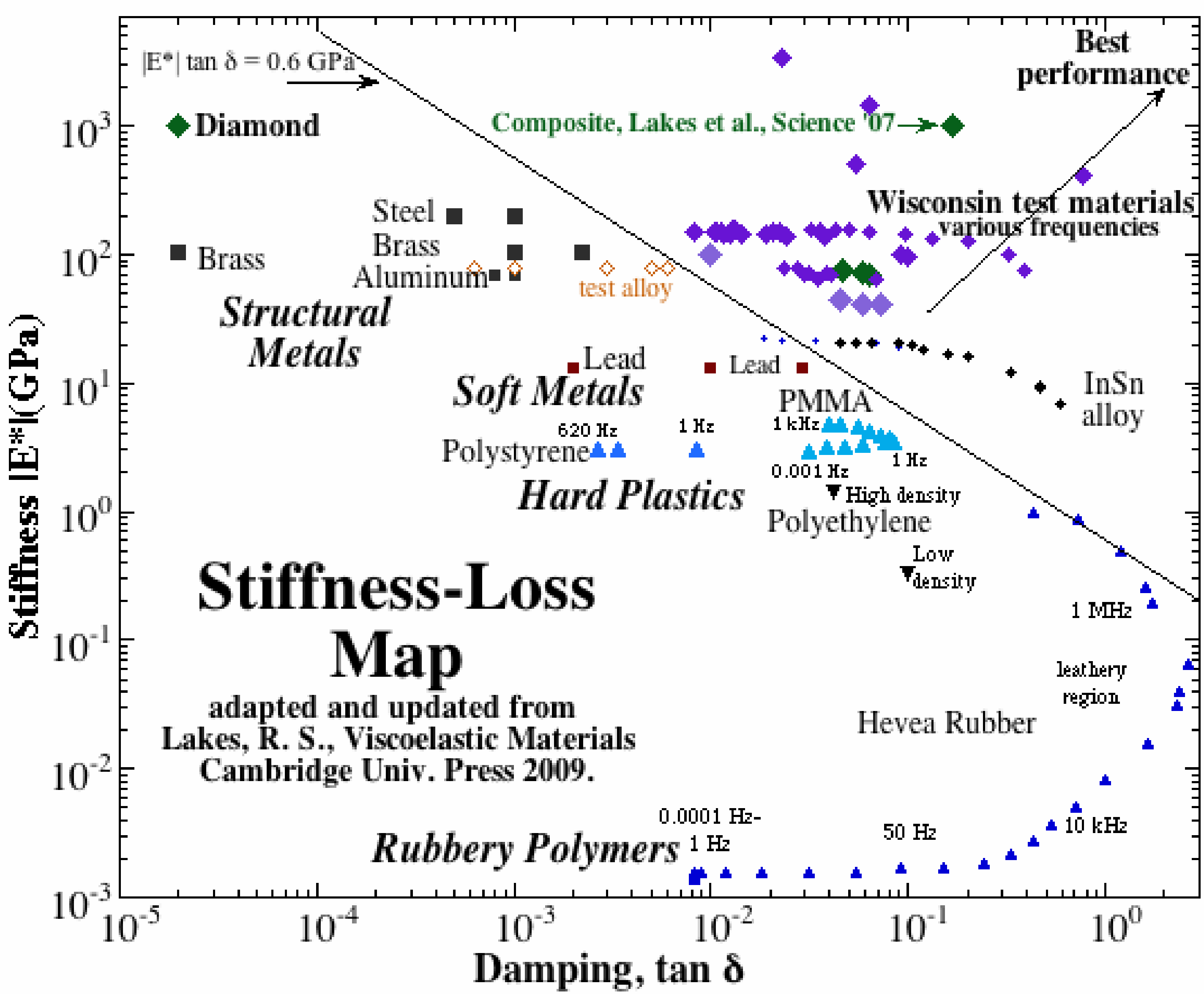

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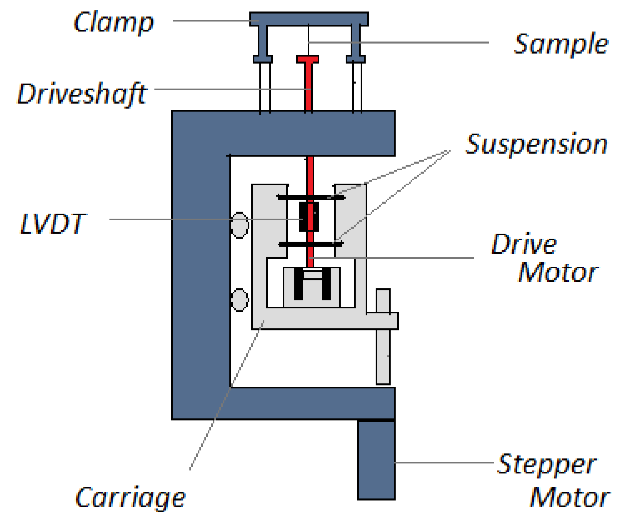
Dynamic Mechanical Analysis (DMA)

- Most useful for studying the viscoelastic behavior of polymers.
- A **sinusoidal stress** is applied and the **strain** in the material is measured to determine the **modulus**.
- The **temperature** of the sample or the **frequency of the stress** are often varied, leading to variations in the modulus.
- This approach can be used to locate the glass transition temperature of the material
- Because **sinusoidal stress** is applied, complex elastic modulus can be expresesed as E* = E' + jE'';
  - ✓ **Storage modulus (E')** - Measure of **elastic response** & the stored energy.
  - ✓ **Loss modulus (E")** - Measure of **viscous response** & the energy dissipated as heat.

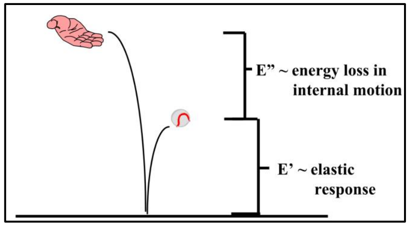


Experiment set-up

Image: Wikipedia

Stress, $\sigma(t) = \sigma_o \sin(\omega t + \delta)$

Strain, $\epsilon(t) = \epsilon_o \sin\omega t$



- **Pure elastic case**, stress is proportional to strain, we have

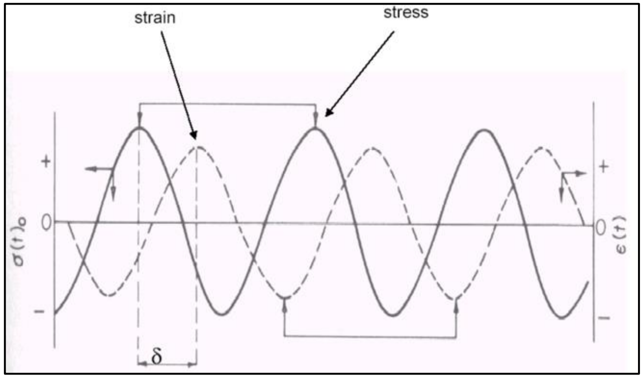
$$\sigma(t) = E\,\epsilon(t)$$

Therefore, $\sigma_o \sin(\omega t + \delta) = E\epsilon_o \sin\omega t$

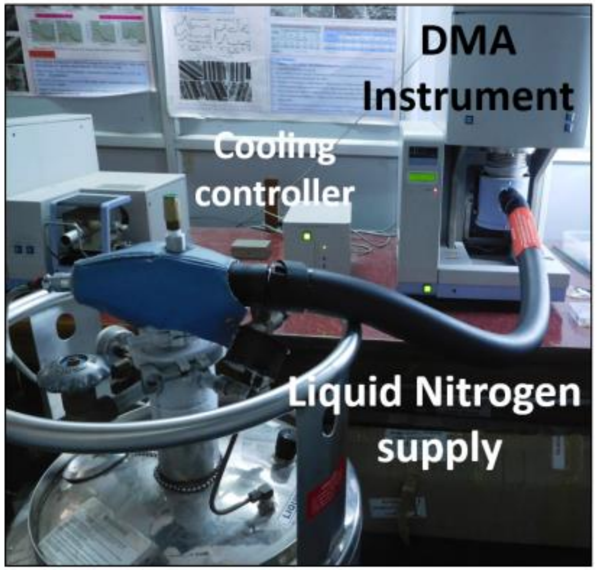
Hence , $\boldsymbol{\delta = 0}$

- **Pure Viscous case,** stress is proportional to strain rate, we have

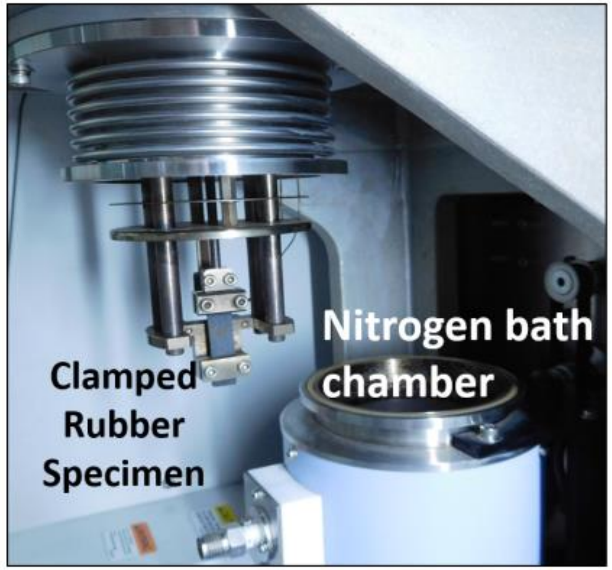
$$\sigma(t) = E\frac{d\epsilon}{dt} \Longrightarrow \sigma_o \sin(\omega t + \delta) = E\epsilon_o \cos\omega t \Longrightarrow \boldsymbol{\delta = \pi/2}$$



Dynamic Mechanical Analysis Test Set-up



(a) DMA instrument along with accessories



(b) Clamped rubber specimen before placing inside nitrogen bath chamber

(c) Specimen under testing in a controlled atmosphere

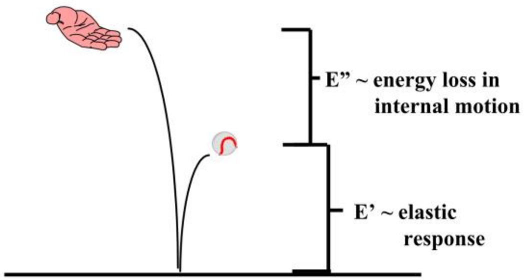
$$\tan\delta = \frac{E''}{E'}$$

where, E’’ = Loss modulus (measure of heat dissipated, viscous behavior)
E’ = Storage modulus (measure of stored energy, elastic behavior)

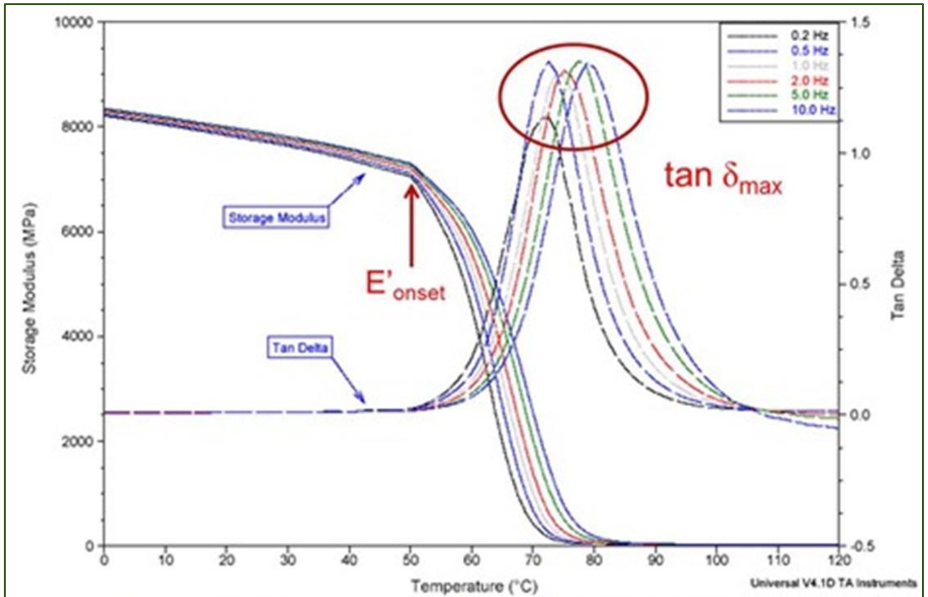
$\tan\delta$ signifies how good a material will be at absorbing energy.

$$Loss\ modulus, E'' = \frac{\sigma_o}{\epsilon_o} sin\delta$$

$$Storage\ modulus, E' = \frac{\sigma_o}{\epsilon_o} cos\delta$$



- δ ranges between 0° - 90°.
- As δ approaches 0° (Purely elastic behavior).
- As δ approaches 90° (Purely viscous behavior).
- At the glass transition, the storage modulus decreases dramatically and the loss modulus reaches a maximum.



E'_{onset} and tan δ_{max} define glass transition range

Reference: www.web.deu.edu.tr/metalurjimalzeme

The complex Young's modulus E*

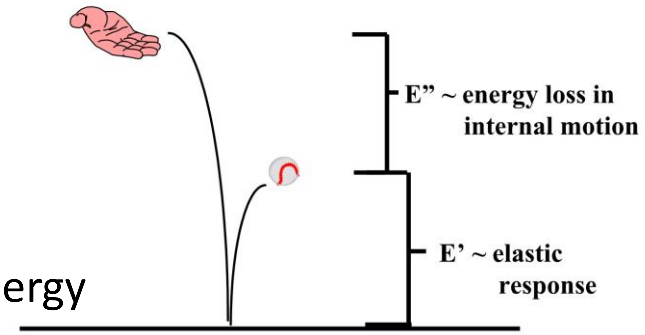
$$E^* = E' + jE''$$

where, E' = Storage modulus (measure of stored energy, elastic behavior)
E'' = Loss modulus (measure of heat dissipated, viscous behavior)

The loss factor η is expressed as

$$\eta = \tan\delta = \frac{E''}{E'}$$

This signifies how good a material will be at absorbing energy



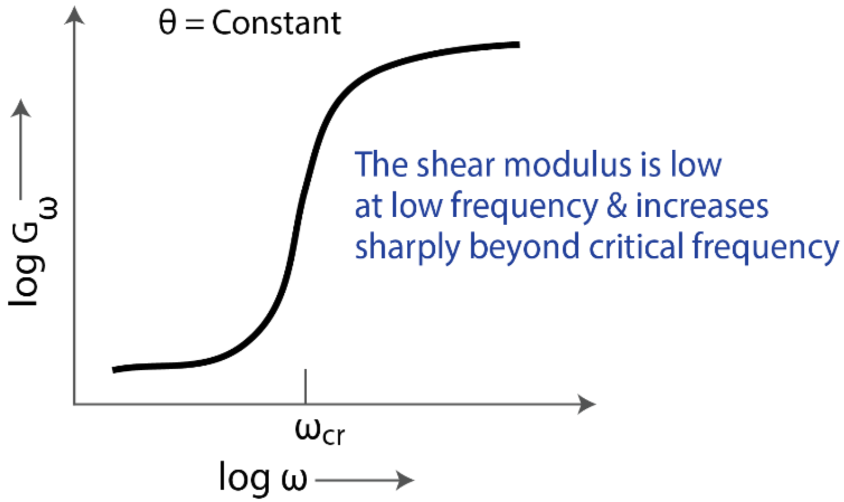
Hence, $\boxed{E^* = E'(1 + j\eta)}$

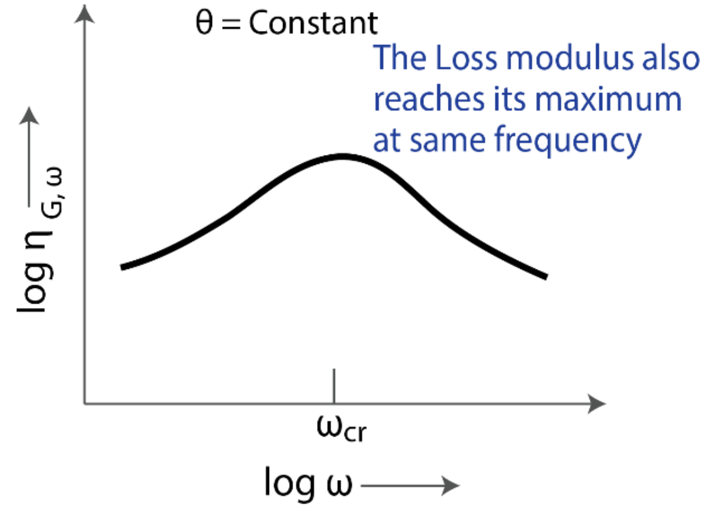
Similarly, the **shear modulus** of VEM $\boxed{G^* = G'(1 + j\eta)}$

and the **bulk modulus** of VEM $\boxed{B^* = B'(1 + j\eta)}$

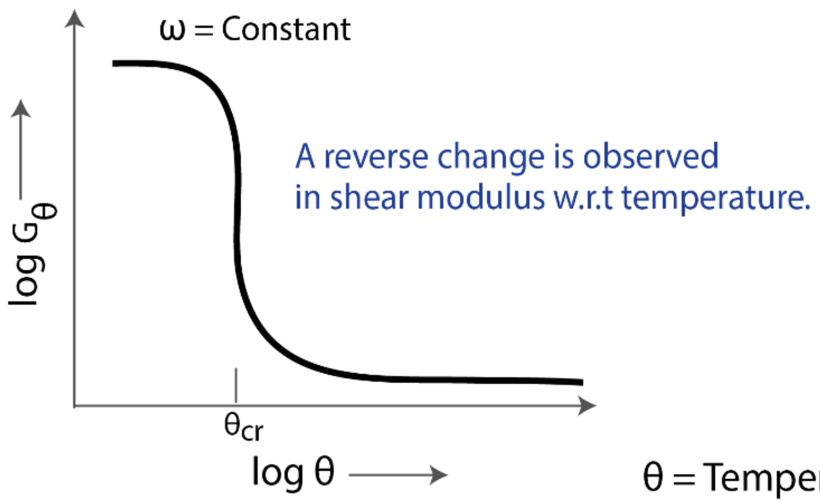
The various moduli are interrelated as $\boxed{E^* = \dfrac{9B^*G^*}{3B^* + G^*}}$

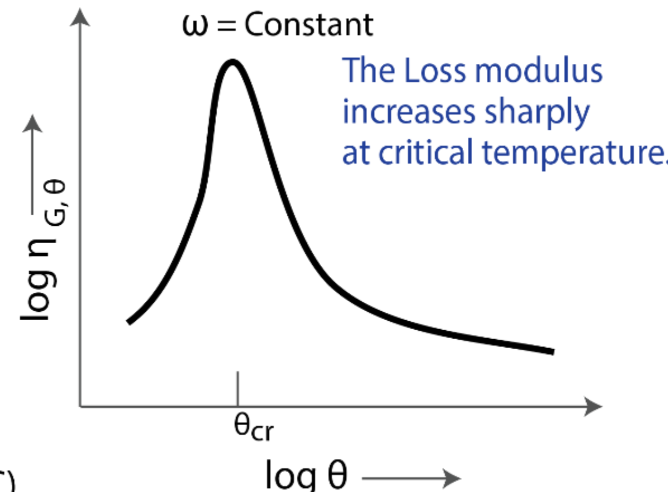
Viscoelastic material: Frequency & temperature dependence of Shear modulus and Loss factor





$$\textit{Loss factor}, \eta = \frac{\textit{Energy dissipated in a cycle}}{\textit{Max. potential energy of the system during that cycle}}$$





θ = Temperature (^{0}C)
ω = Frequency (rad/s)

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Ref.: Principles of Vibration Control, A. K. Mallik