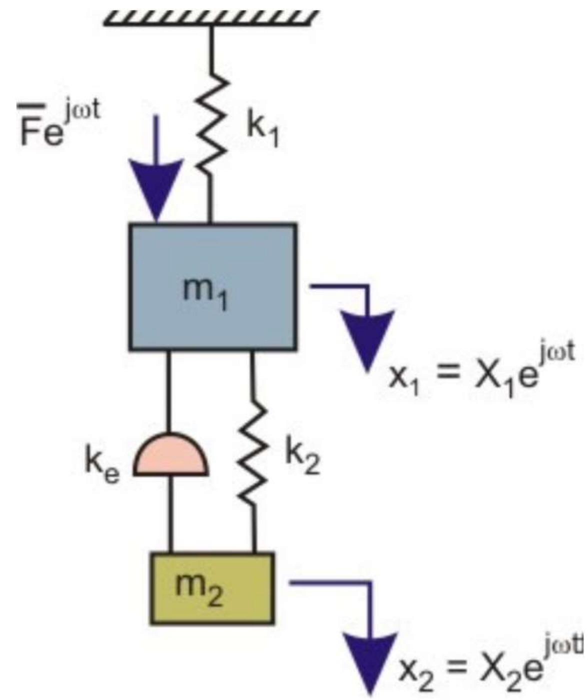
Active Dynamic Vibration Absorber

- It may be noted that Passive Neutralizer **eliminates primary response** only at a **particular frequency**.
- Use of **active element** - for example, a hydraulic actuator would increase the advantage of tuned mass damping for a broad frequency range.

Here, $\mathbf{m_1}$ denotes the **primary mass** and $\mathbf{k_1}$ the **primary stiffness**. The **damping** of the **primary system** is neglected. The system is subjected to a harmonic excitation $\bar{F}e^{j\omega t}$.

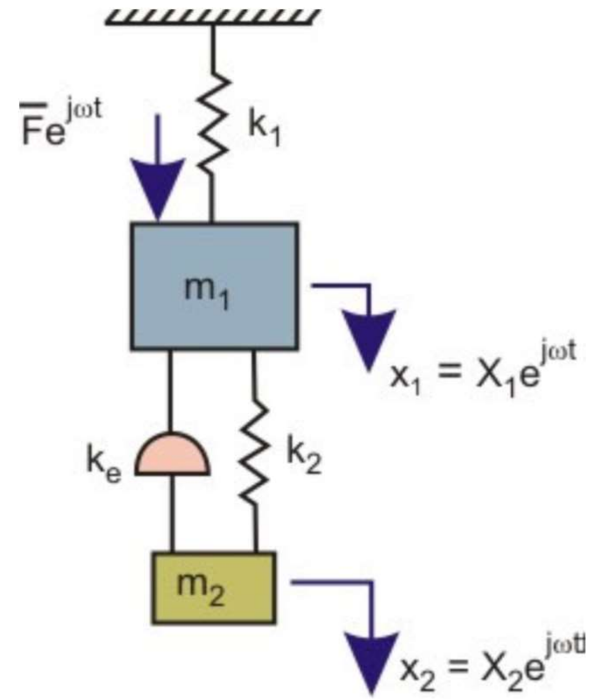
The primary system is **attached** to a secondary system of fixed mass $\mathbf{m_2}$ and stiffness $\mathbf{k_2}$. However, there is an additional spring element with **variable stiffness** '$\mathbf{k_e}$' representative of a hydraulic actuator.



The governing EOM of the two DOF system may be written as

$$m_1\ddot{x}_1 + k_1x_1 + k_2(x_1 - x_2) = \bar{F}e^{j\omega t} + k_e x_2 \qquad (1)$$

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) = k_e x_2 \qquad (2)$$



Using $x_1 = X_1e^{j\omega t}$ *and* $x_2 = X_2e^{j\omega t}$, we get

From equation (2),

$$-\omega^2 m_2X_2 + k_2(X_2 - X_1) = -k_eX_2$$

or

$$(k_2 - m_2\omega^2 + k_e)X_2 = k_2X_1$$

or

$$X_2 = \frac{k_2}{k_2 - m_2\omega^2 + k_e}X_1 \qquad (3)$$

Similarly, from Equation 1, we get $\boxed{m_1\ddot{x}_1 + k_1x_1 + k_2(x_1 - x_2) = \bar{F}e^{j\omega t} + k_e x_2}$

$$(k_1 - \omega^2 m_1)X_1 + k_2(X_1 - X_2) = \bar{F} + k_e X_2$$

or

$$(k_1 + k_2 - \omega^2 m_1)X_1 - (k_2 + k_e)X_2 = \bar{F}$$

or

$$(k_1 + k_2 - \omega^2 m_1)X_1 - (k_2 + k_e)X_2 = \bar{F}$$

or

$$\left[(k_1 + k_2 - \omega^2 m_1) - \frac{k_2^2 + k_e k_2}{k_2 - \omega^2 m_2 + k_e}\right]X_1 = \bar{F}$$

$$\boxed{\left[(k_1+k_2-\omega^2 m_1)-\frac{k_2^2+k_e k_2}{k_2-\omega^2 m_2+k_e}\right]X_1=\bar{F}}$$

Thus, when the hydraulic actuator is switched on the active displacement of the primary mass X_{1a} may be written as:

$$X_{1a}=\bar{F}\,\frac{(k_2-\omega^2 m_2+k_e)}{(k_1+k_2-\omega^2 m_1)(k_2-\omega^2 m_2+k_e)-k_2^2-k_e k_2}$$

When the hydraulic system is switched off, the passive displacement of the primary mass X_{1p} may be written as:

$$X_{1p}=\bar{F}\,\frac{(k_2-\omega^2 m_2)}{(k_1+k_2-\omega^2 m_1)(k_2-\omega^2 m_2)-k_2^2}$$

The ratio of active and passive displacement of the primary mass brings out the efficiency of the new system. Therefore,

$$\frac{X_{1a}}{X_{1p}}=\frac{(k_2-\omega^2 m_2+k_e)}{(k_2-\omega^2 m_2)}\times\frac{(k_1+k_2-\omega^2 m_1)(k_2-\omega^2 m_2)-k_2^2}{(k_1+k_2-\omega^2 m_1)(k_2-\omega^2 m_2+k_e)-k_2^2-k_e k_2}$$

$$\frac{X_{1a}}{X_{1p}} = \frac{(k_2 - \omega^2 m_2 + k_e)}{(k_2 - \omega^2 m_2)} \times \frac{(k_1 + k_2 - \omega^2 m_1)(k_2 - \omega^2 m_2) - k_2^2}{(k_1 + k_2 - \omega^2 m_1)(k_2 - \omega^2 m_2 + k_e) - k_2^2 - k_e k_2}$$

For a simple case, use $k_1 = k_2 = k$, $m_1 = m_2 = m$, $\Omega^2 = \frac{\omega^2}{(k/m)}$

$$\frac{X_{1a}}{X_{1p}} = \frac{\left(1 + \frac{k_e}{k} - \Omega^2\right)}{1 - \Omega^2} \times \frac{(2 - \Omega^2)(1 - \Omega^2) - 1}{(2 - \Omega^2)\left(1 + \frac{k_e}{k} - \Omega^2\right) - 1 - \frac{k_e}{k}}$$

As a test case,

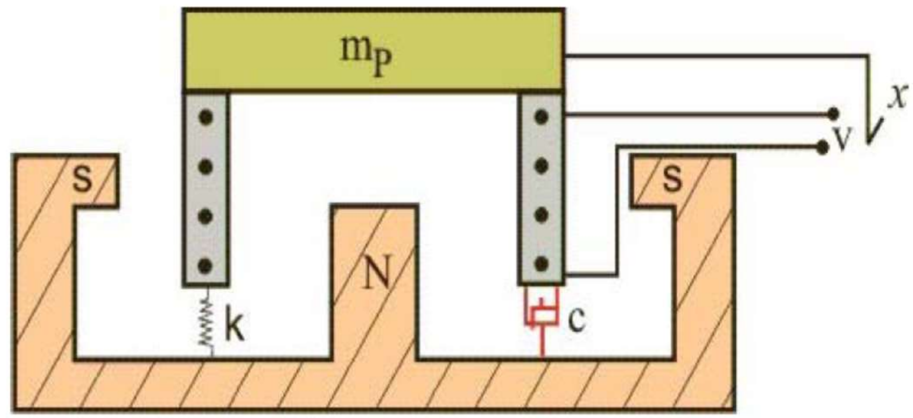
$$for\ \frac{k_e}{k} = -2, \qquad \frac{X_{1_a}}{X_{1_p}} = \left|\frac{\Omega^6 - 2\Omega^4 - 2\Omega^2 + 1}{\Omega^6 - 2\Omega^4 + 1}\right|$$

$$for\ \frac{k_e}{k} = +2, \qquad \frac{X_{1_a}}{X_{1_p}} = \left|\frac{\Omega^6 - 6\Omega^4\_ + 10\Omega^2 - 3}{\Omega^6 - 6\Omega^4 + 8\Omega^2 - 3}\right|$$

From these expressions, one can check that the negative feedback system with $k_e/k = -2$ works better for a wider frequency range.

Active DVA

Another Active DVA – **Proof Mass Actuator**



- A **Proof mass** m_p is connected to a **magnetic base** with **spring k** and **damper c**.
- The proof mass is placed over a solenoid in which magnetic field could be generated by passing current through coils.
- The **resistance** of the coil is **R**, **inductance L**, **current** passing through the coil is **i** and the **proportionality constant** corresponding to **back EMF** is k_b.

What is **back EMF**?

- When device like a refrigerator or an air conditioner (anything with a motor) first turns on in our house, the lights often dim momentarily.

- Because, a motor has coils turning inside magnetic field, and a coil turning inside a magnetic field induces a **back emf**, which acts against the applied voltage and reduces the current flowing through the coils of the motor.

If the applied voltage is ΔV, then the initial current flowing through a motor with coils of resistance R is:

$$I = \frac{\Delta V}{R}, \text{ for example } I = 120/6 = 20A$$

- A device drawing **that much current reduces** the **voltage and current** provided to other electrical equipment in your house, causing **lights to dim**.

- When the motor is spinning and generating a back emf E_b the current is reduced to:

$$I = \frac{\Delta V - E_b}{R}, \text{ for example } I = \frac{120-108}{6} = 2A$$

$$\boxed{E_b = \frac{PZ}{A}\frac{\emptyset N}{60} = k_b \frac{\emptyset N}{60}}$$

Back EMF regulates the flow of armature current i.e., it automatically changes the armature current to meet the load requirement.

P – Number of poles of the machine
φ – Flux per pole in Weber.
Z – Total number of armature conductors.
N – Speed of armature in r.p.m.
A – Number of parallel paths in the armature winding
k_b - Back emf constant

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The EOM are:

$$Ri + L\frac{di}{dt} = V - k_b\dot{x} \qquad (1)$$

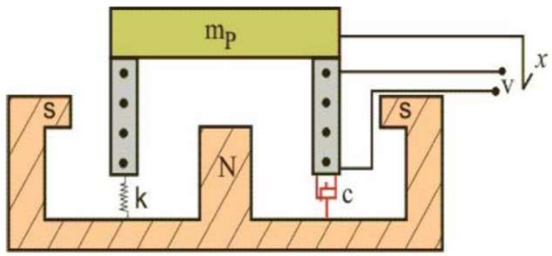
$$m_p\ddot{x} + k_p x + c_p\dot{x} = k_a i \qquad (2)$$

k_a= Current constant
k_p= Proof mass stiffness
c_p= Proof mass damping
m_p= Proof mass
L = inductance
k_b = back emf constant

Converting equation (1) into frequency domain using Laplace transform,

$$RI + LsI = \bar{V} - k_b sX$$

$$I = \frac{1}{R+Ls}(\bar{V} - k_b sX) \qquad (3)$$



Similarly for equation (2),

$$s^2 m_p X + k_p X + c_p sX = k_a I$$

$$(s^2 m_p + k_p + c_p s)X = k_a I \qquad (4)$$

Now, using equation (3) in (4), we get

$$(s^2 m_p + k_p + c_p s)X = k_a \frac{1}{R + Ls}(\bar{V} - k_b s X)$$

Denoting, $\frac{\mathbf{1}}{\mathbf{R+Ls}} = \frac{\mathbf{1}}{\mathbf{Z(s)}} = \tilde{\mathbf{A}}(\mathbf{s})$

$$(s^2 m_p + k_p + c_p s + s k_a k_b \tilde{A}(s))X = k_a \tilde{A}(s)\bar{V}$$

$$X = \frac{k_a \tilde{A}(s)}{(s^2 m_p + k_p + s(c_p + k_a k_b \tilde{A}(s))}\bar{V} \qquad (5)$$

k_a= Current constant
k_p= Proof mass stiffness
c_p= Proof mass damping
m_p= Proof mass
L = inductance
k_b = back emf constant
Z(s) = Impedance
$\tilde{A}(s)$= Admittance

Also, force exerted by the proof mass actuator on the base is $F(t) = -m_p \ddot{x}$

Using Laplace transform, thus, $F(s) = -s^2 m_p X$

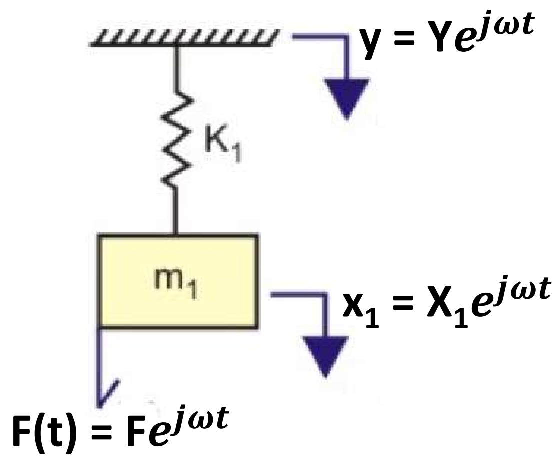
Using equation (5), we get

$$\boxed{F(s) = \frac{-s^2 m_p k_a \tilde{A}(s)}{(s^2 m_p + k_p + s(c_p + k_a k_b \tilde{A}(s))}\bar{V}} \qquad (6)$$

This relationship tells us how **force F** will be **generated** by the **proof mass actuator** upon **application** of **voltage $\bar{V}$** .

Application of active DVA

Consider a SDOF system (undamped, mass m_1) subjected to base excitation.



$$m_1\ddot{x}_1 + k_1(x_1 - y) = F(t)$$

Converting above equation into frequency domain,

$$s^2m_1X_1 + k_1X_1 - k_1Y = F(s)$$

$$(s^2m_1 + k_1)X_1 = F(s) + k_1 Y$$

Therefore,

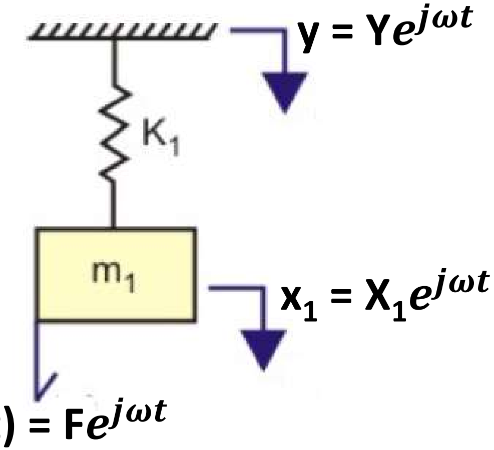
$$X_1 = \frac{F(s) + k_1 Y}{s^2m_1 + k_1}$$

Now, we already derived this equation

$$\boxed{F(s) = \frac{-s^2m_pk_a\tilde{A}(s)}{(s^2m_p+k_p+s(c_p+k_ak_b\tilde{A}(s))}\bar{V}} \quad (6)$$

Plugging the output of the Active DVA into the system and using Equation (6), we get the new displacement of m_1 as

$$\boxed{X_1 = \frac{k_1 Y}{s^2 m_1 + k_1} - \frac{s^2 m_p k_a \tilde{A}(s)}{(s^2 m_p + k_p + s(c_p + k_a k_b \tilde{A}(s))}\bar{V}} \qquad (7)$$



A Special case:

When $\tilde{A}(s)$ *is constant*, $\omega_1^2 = \frac{k_1}{m_1}$ $\omega_p^2 = \frac{k_p}{m_p}, \bar{V} = kX_1$

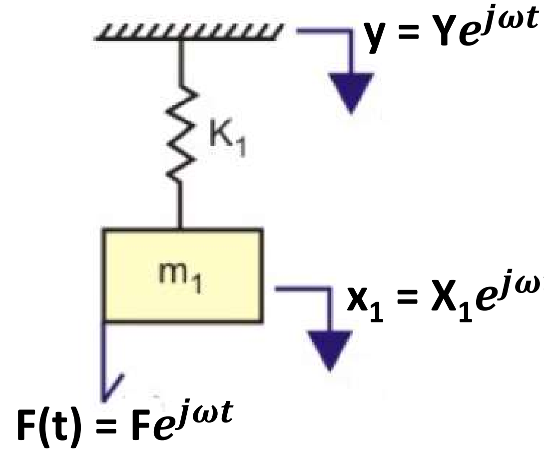
$$X_1 = \frac{\omega_1^2}{s^2 + \omega_1^2} Y - \frac{\bar{\alpha} s^2}{(s^2 + \bar{\beta} s + \omega_p^2)} X_1$$

where,

$$\bar{\alpha} = \frac{\tilde{A} k_a}{k} \qquad \bar{\beta} = \frac{c_p + \tilde{A} k_a k_b}{m_p}$$

Therefore, $(1 + \frac{\bar{\alpha} s^2}{(s^2 + \bar{\beta} s + \omega_p^2)}) X_1 = \frac{\omega_1^2}{s^2 + \omega_1^2} Y$

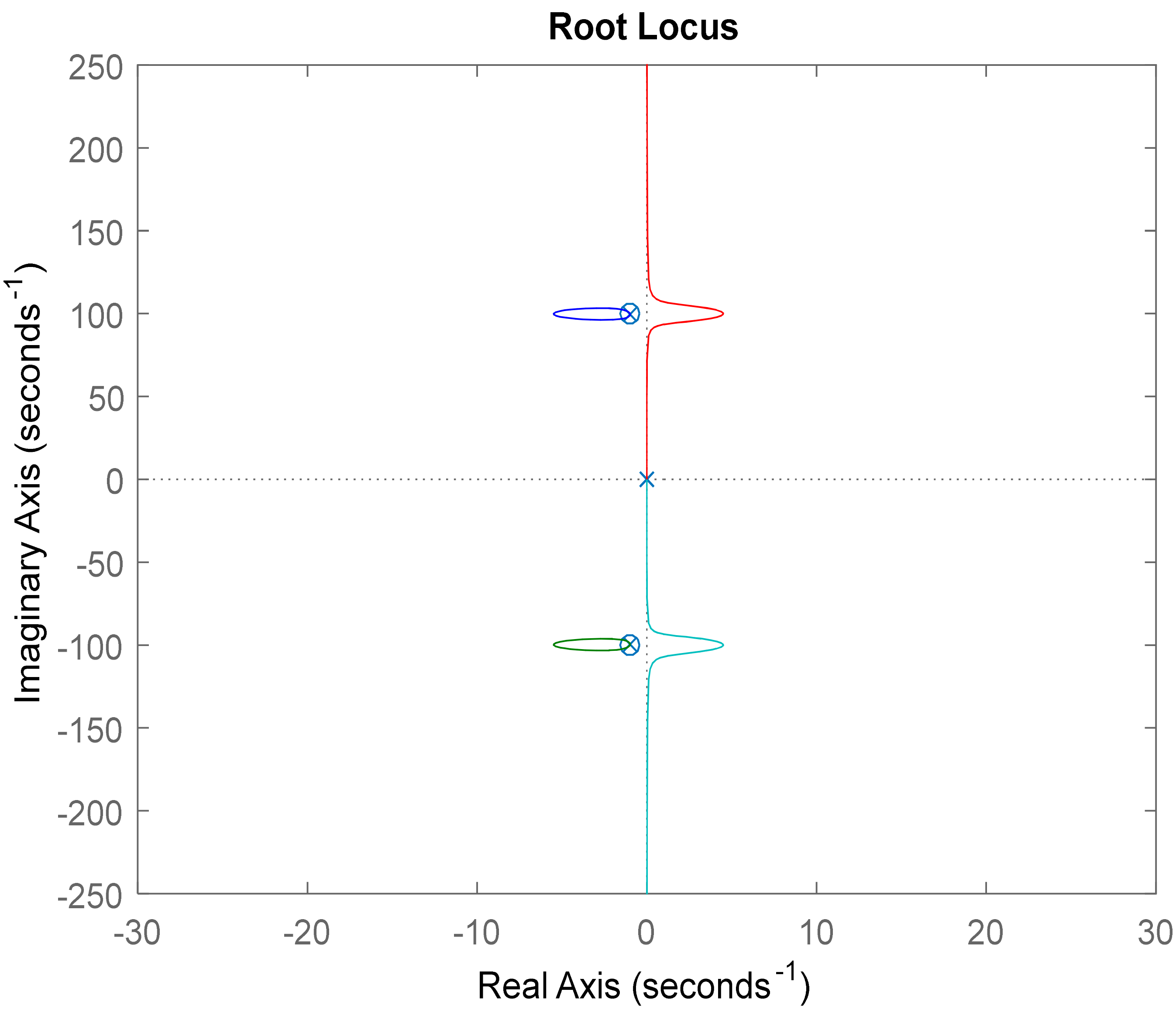
One can show a wide band amplitude reduction of the primary mass (X_1) by suitably choosing k_a and k_b in equation (7)



$$\frac{X_1}{Y} = \frac{\omega_1^2(s^2+\bar{\beta}s + \omega_p^2)}{(s^2 + \omega_1^2)[(s^2(1+\bar{\alpha}) + \bar{\beta}s + \omega_p^2]}$$

$$= \frac{\omega_1^2}{1+\bar{\alpha}} \frac{s^2 + \bar{\beta}s + \omega_p^2}{(s^2 + \omega_1^2)[s^2 + \frac{\bar{\beta}}{1+\bar{\alpha}}s + \frac{\omega_p^2}{1+\bar{\alpha}}]}$$

$$\bar{\alpha} = \frac{\tilde{A}k_a}{k} \qquad \bar{\beta} = \frac{c_p + \tilde{A}k_a k_b}{m_p}$$



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