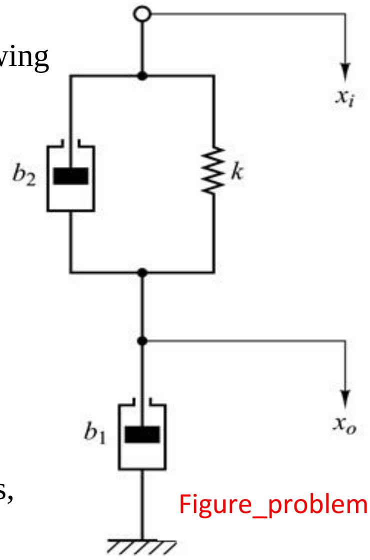
ME756A: Principles of Vibration Control

Total marks: 75 **<u>Assignment-2</u>** **Last date: 15/04/2019**

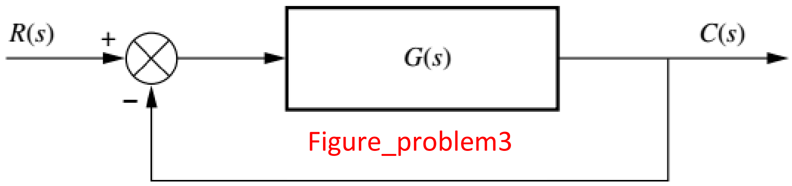
Q1. Show that the root loci for a control system with open loop transfer function $G(s) = \frac{K(s^2+6s+10)}{(s^2+2s+10)}$ and negative feedback H(s) =1 are arc of the circle with centre at the origin and radius equal to $\sqrt{10}$. **[5 marks]**

Q2. Consider the mechanical system shown in the Figure_problem2. It consists of a spring stiffness $\boldsymbol{k}$ and two dashpots with damping coefficients $\boldsymbol{b_1}, \boldsymbol{b_2}$. Obtain the transfer function of the system. The displacement $\boldsymbol{x_i}$ is the input and displacement $\boldsymbol{x_o}$ is the output. Verify and prove whether this system a mechanical lead network or lag network? **[5 marks]**



Figure_problem2

Q3. For the unity feedback system, where $G(s) = \frac{K(s-1)(s-2)}{s(s+1)}$. Obtain the following



Figure_problem3

a) Sketch the root locus. **[5 marks]**
b) The break-away and break-in points with gain values. **[2 marks]**
c) The jω-axis crossing **[1 marks]**
d) The value of **K** to yield a stable system with second-order complex poles, with a damping ratio of 0.5 **[2 marks]**

Q4. Using the knowledge of state space analysis of control system, answer the following.

a) A system is described by the following differential equation

$$\frac{d^3x}{dt^3} + 3\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = u_1(t) + 3u_2(t) + 4u_3(t)$$

And outputs are

$$y_1 = 4\frac{dx}{dt} + 3u_1(t)$$
$$y_2 = \frac{d^2x}{dt^2} + 4u_2(t) + u_3(t)$$

Obtain the state space representation of the system. **[5 marks]**

b) A system is characterized by the given transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2}{s^3 + 6s^2 + 11s + 6}$$

Find the state and output equation in matrix form. **[5 marks]**
Also, test the controllability and observability of the system. **[5 marks]**

c) The system equations are given by

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

Find the transfer function of the system. **[5 marks]**

Q5. A unity feedback control system is characterized by the open loop transfer function

$$G(s) = \frac{K(s+13)}{s(s+3)(s+7)}$$

a) Using Routh criterion, calculate the range of K for the system to be stable. **[5 marks]**

b) Check if for K=1, all these roots of the characteristic equation of the above system have damping ratio greater than 0.5 **[5 marks]**

Q6. Design a lag compensator for a system whose open loop transfer function is

$$G(s) = \frac{K}{s(s+1)(s+4)}$$

To meet the following specifications.

Damping ratio = 0.5, settling time = 10 sec, velocity error constant, $k_v \geq 5$

Compare the root locus of uncompensated and compensated system. **[10 marks]**

Q7. A unity feedback control system is characterized by the open loop transfer function

$$G(s) = \frac{K}{s(s+10)}$$

Determine the gain K such that the system will have a damping ratio of 0.5. For this value of K determine settling time, peak overshoot and peak time for unit step input. **[5+1+2+2 marks]**

Q8. Obtain the response y(t) of the following system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u, \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where u(t) is the unit-step input occurring at t = 0. **[5 marks]**