Q1 Ans

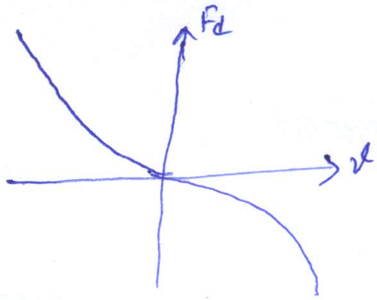
$$F_d = \alpha_q \, sgn(v) \, v^2$$

$$v = \dot{x}$$

Let steady state sol^n be $x = X \sin \omega t$

Energy dissipated in one cycle

$$\Delta E = \oint F_d \, dx$$



$$= \alpha_q \, sgn(\dot{x}) \oint (\dot{x})^2 \frac{dx}{dt} dt$$

$$= \alpha_q \, sgn(\dot{x}) \oint (\dot{x})^3 dt$$

Eliminating the signum function by integrating over a quarter of the period & ~~multiplying~~ quadrupling the result

$$\Delta E = 4 \alpha_q \omega^3 X^3 \int_0^{\frac{2\pi}{\omega} \times \frac{1}{4}} \cos^3 \omega t \, dt$$

$$= \frac{8}{3} \alpha_q \omega^2 X^3$$

$$\pi C_{eq} \omega X^2 = \frac{8}{3} \alpha_q \omega^2 X^3$$

$$C_{eq} = \frac{8}{3} \frac{\alpha_q \omega X}{\pi}$$

$$|X| = \frac{F_0}{\sqrt{(K - m\omega^2)^2 + \frac{64 \alpha_q^2 \omega^4 |X|^2}{9\pi^2}}}$$

or $\frac{64 \alpha_q^2 \omega^4}{9\pi^2} |X|^4 + (K - m\omega^2)^2 |X|^2 - F_0^2 = 0$

$$\therefore |X| = \frac{3\pi}{8\sqrt{2} \alpha_q \omega^2} \sqrt{\sqrt{\frac{256 F_0^2 \alpha_q^2}{9\pi^2} \omega^4 + (K - m\omega^2)^4} - (K - m\omega^2)^2}$$

Q2 Ans

$$\Delta E = \Delta E_{visc} + \Delta E_{cool}$$

$$= \pi c \omega X^2 + 4 \mu m g X$$

Equating to the equivalent viscously damped system

$$\pi C_{eq} \omega X^2 = \pi c \omega X^2 + 4 \mu m g X$$

$$\therefore C_{eq} = c + \frac{4 \mu m g}{\pi \omega X}$$

$$\frac{C_{eq}}{C_c} = \frac{c}{C_c} + \frac{4 \mu m g}{\pi \omega X C_c}$$

$$\xi_{eq} = \xi + \frac{2 \mu g}{\pi \omega \omega_n X} \qquad \because C_c = 2 m \omega_n$$

$$X = \frac{F_0/K}{\sqrt{(1-r^2)^2 + (2 \xi_{eq} r)^2}}$$

$$= \frac{F_0/K}{\sqrt{(1-r^2)^2 + \left(2 \xi r + \frac{4 \mu m g}{\pi K X}\right)^2}}$$

Q3 Ans The strouhal number is related to the frequency at which vortices are shed is given by

$$S = \frac{fD}{V} = \frac{\omega D}{2\pi V}$$

The excitation amplitude is related to drag coefficient C_D by

$$C_D = \frac{F_0}{\frac{1}{2}\rho V^2 D L}$$

for, $S = 0.2$, $C_D = 1$

$$1 = \frac{F_0}{\frac{1}{2}\rho \left(\frac{\omega D}{0.4\pi}\right)^2 D L}$$

$$\therefore F_0 = 0.317 \rho D^3 L \omega^2$$

$$\therefore F_0 \propto \omega^2$$

Proportionity constant $= 0.317$

Q4 Ans: The stiffness of the system is

a)

$$K = \frac{F}{\Delta y} = \frac{25 \times 9.81}{0.05}$$

$$= 4905 \text{ N/m}$$

$\therefore$ Natural frequency of vibration, $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{4905}{25}}$

$= 14.01$ rad/sec

= Resonating angular velocity

b) The Periodic force which causes the fan to vibrate is the centrifugal force due to unbalanced rotor.

$$f_0 = m_o r \omega^2$$

$$= 3.5 (0.1)(10)^2$$

$$= 35 \text{ N}$$

$\therefore$ The steady-state vibration for undamped system

$$X = \left| \frac{F_0/K}{1 - (\omega/\omega_n)^2} \right|$$

$$= \left| \frac{35/4905}{1 - (10/14.01)^2} \right| = 0.0146 \text{ m}$$

$$= 14.6 \text{ mm}$$

Q5 Ans $m = 10\,kg, \; K = 1.5 \times 10^{4}\,N/m, \; F_0 = 90\,N$

$$\omega = 25(2\pi) = 50\pi \text{ rad/s}$$

$$\omega_n = \sqrt{\frac{K}{m}} = 38.73 \text{ rad/s}$$

$$\mu = 0.1$$

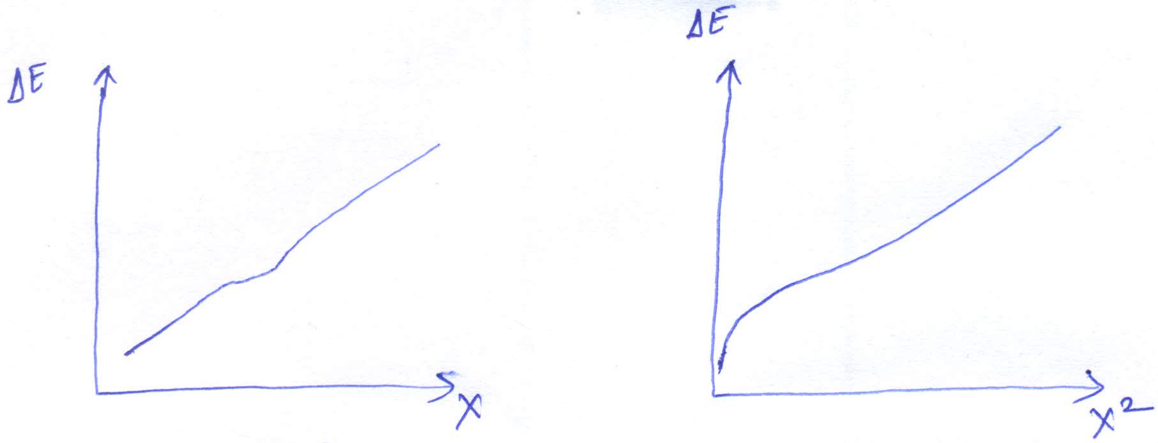
$$X = \frac{\frac{F_0}{K}\sqrt{1 - \left(\frac{4\mu mg}{\pi F_0}\right)^2}}{1 - r^2} = 3.85 \times 10^{-4}\,m$$

$$C_{eq} = \frac{4\mu mg}{\pi \omega X} = \frac{4(0.1)(10)(9.81)}{\pi(50\pi)(3.85 \times 10^{-4})} = 206.7 \text{ Ns/m}$$

Q6 Ans

For viscous damping, $\Delta E = \pi c \omega X^2$

For coulomb damping, $\Delta E = 4 \mu m g X$



straight line

curve

$\therefore$ Damping is likely to be Coulomb.

Q7 Ans

Plot start at $|H(j\omega)| = \frac{1}{K}$

$$0.05 = \frac{1}{K} \Rightarrow K = 20\ N/m$$

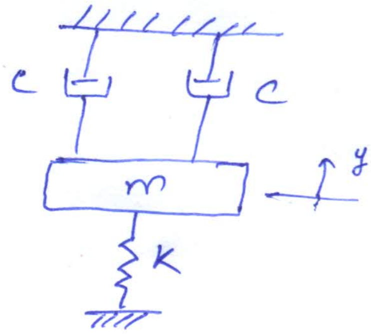
At the peak, $\omega_n = 3$ rad/s

$$\therefore m = \frac{K}{\omega_n^2} = 2.22\,kg$$

$$\frac{1}{c\omega} = 0.11 \Rightarrow c = 3.03$$

$$\xi = \frac{c}{2\sqrt{mK}} = 0.227$$

Q 8 Ans



Two dashpots are in parallel, so have the same velocity.

$\therefore$ Force produced, $F_d = c\dot{y} + c\dot{y}$

$= 2c\dot{y}$

Equivalent damping coefficient, $C_{eq} = \frac{F}{\dot{y}} = \frac{2c\dot{y}}{\dot{y}} = 2c$

For underdamped system $C_{eq} < C_c$

$2c < 2m\sqrt{\frac{K}{m}}$ $\quad \because C_c = 2m\omega_n$

$\therefore \boxed{c < \sqrt{mK}}$

Q9 Ans $m_0 = 10kg$, $m = 100kg$, $K = 2 \times 3.2$ N/mm

$e = 0.1m$

$\omega_r = 1750$ RPM

$= 183.26$ rad/s

$$\omega_n = \sqrt{\frac{2 \times 3200}{100}} = 8 \text{ rad/s}$$

$$r = \frac{\omega_r}{\omega_n} = \frac{183.3}{8} = 22.9$$

$$X = \frac{m_0 e}{m} \frac{r^2}{1-r^2} = 0.01m$$

Q10 Ans $$X = \frac{m_0 e}{m} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

At resonance, $X = 10mm = \frac{m_0 e}{m}\left(\frac{1}{2\xi}\right)$

$$\therefore \frac{10m}{m_0 e} = \frac{1}{2\xi} \quad -1$$

when $r >> 1$

$$X = 1 = \frac{m_0 e}{m} \quad -2$$

from 1 & 2

$$10(1) = \frac{1}{2\xi}$$

$$\boxed{\xi = 0.05}$$