Active Vibration Control: Design of Controller using Transform Method

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Jargons!

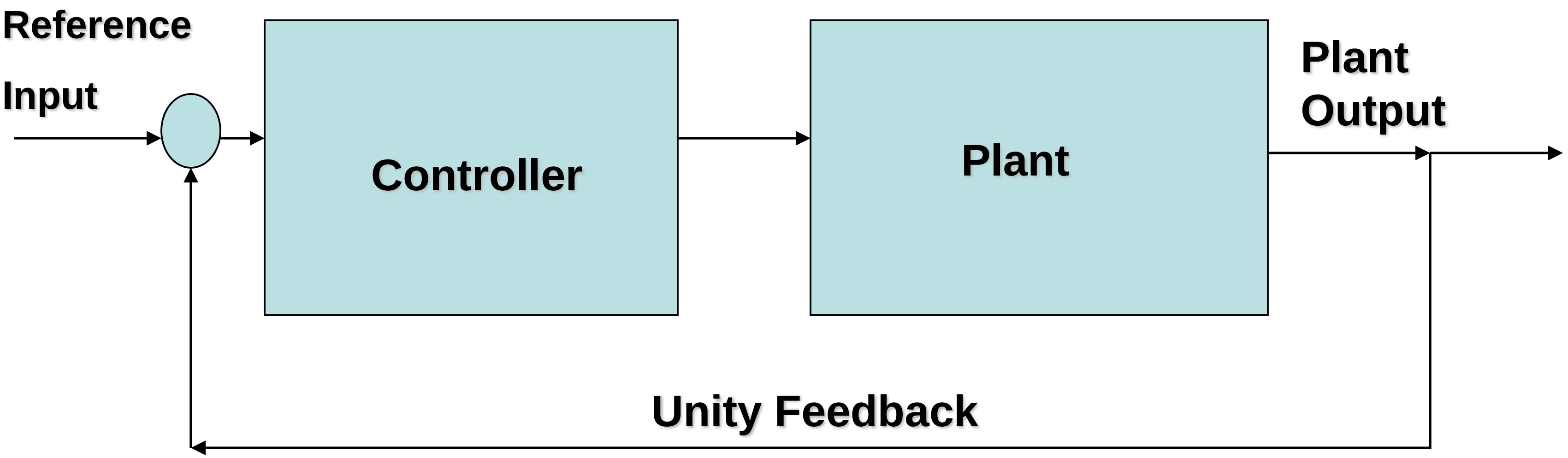
- Cascade – If object A is cascaded over object B $\longrightarrow$
A is falling or hanging over B. We imply: A and B are in the forward path, output of A is input of B

- Compensator – A controller that is used to compensate the unwanted dynamics of a Plant/Process or System

- Lead Controller - The transfer function of this type of controller introduces a dominant zero – thereby creating phase lead in the transfer function – eqv. To PD control
- Lag Controller - Reverse – here it introduces a dominant pole – and hence phase lag in the transfer function – eqv. To PI
- Lead-Lag – eqv. To PID

- Notch The frequency response shows as if a notch has been cut in an otherwise flat response.

Cascaded Controller



Lag Compensator, Lead Compensator, Lead-Lag Compensator, Notch Filter

Transient response of a second order system

A Second order system may be represented as follows:

$$Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} R(s)$$

Where, R(s) is the excitation signal and the second order plant is defined by it's natural frequency ω_n and damping ratio ζ. The response of the system in time domain may be written as:

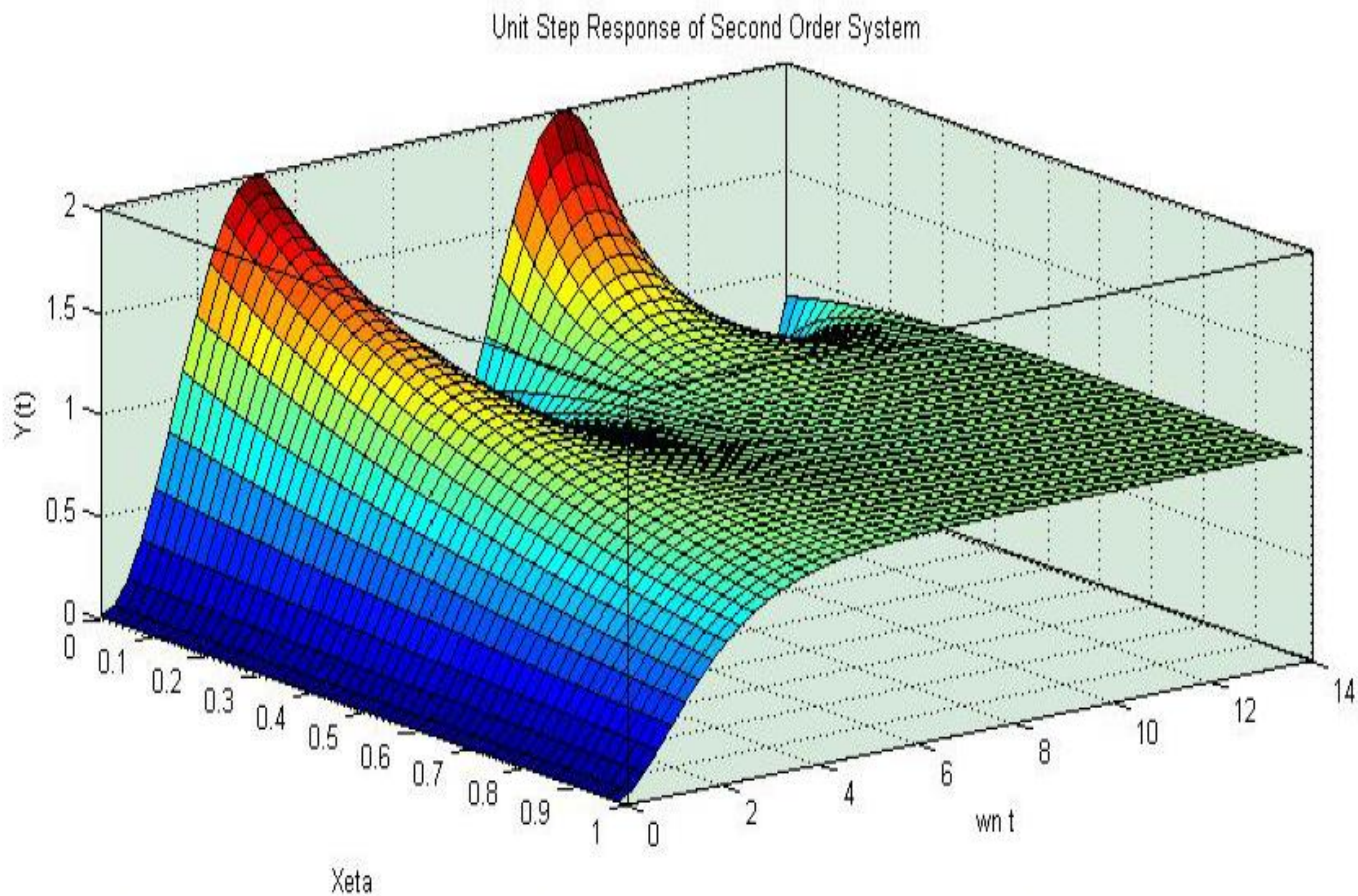
$$y(t) = 1 - \frac{1}{\beta} e^{\zeta\omega_n t} \sin(\omega_n \beta t + \theta)$$

$$\beta = \sqrt{(1 - \zeta^2)}$$

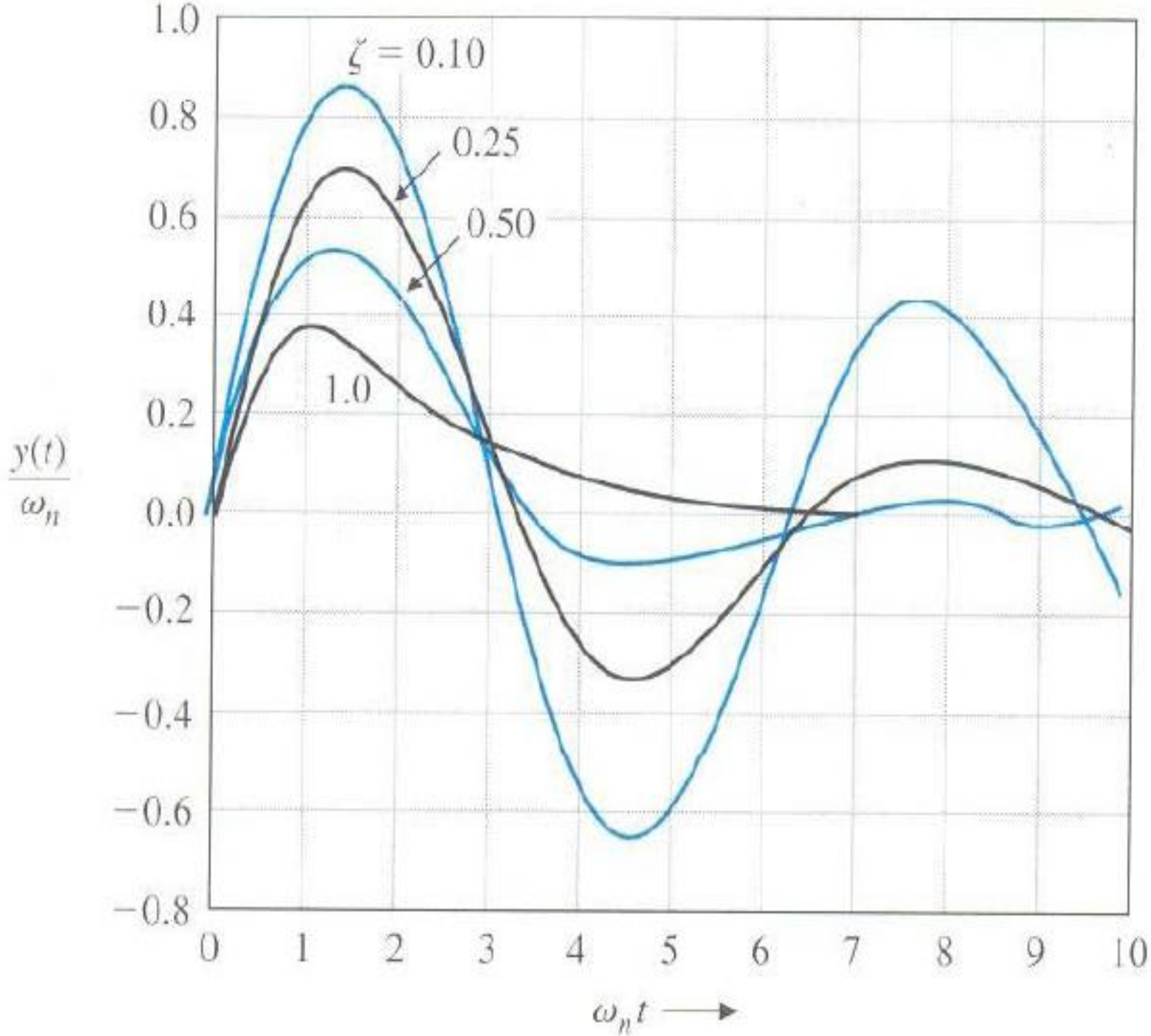
$$\theta = \cos^{-1}\zeta$$

$$\zeta - 0 \text{to} 1$$

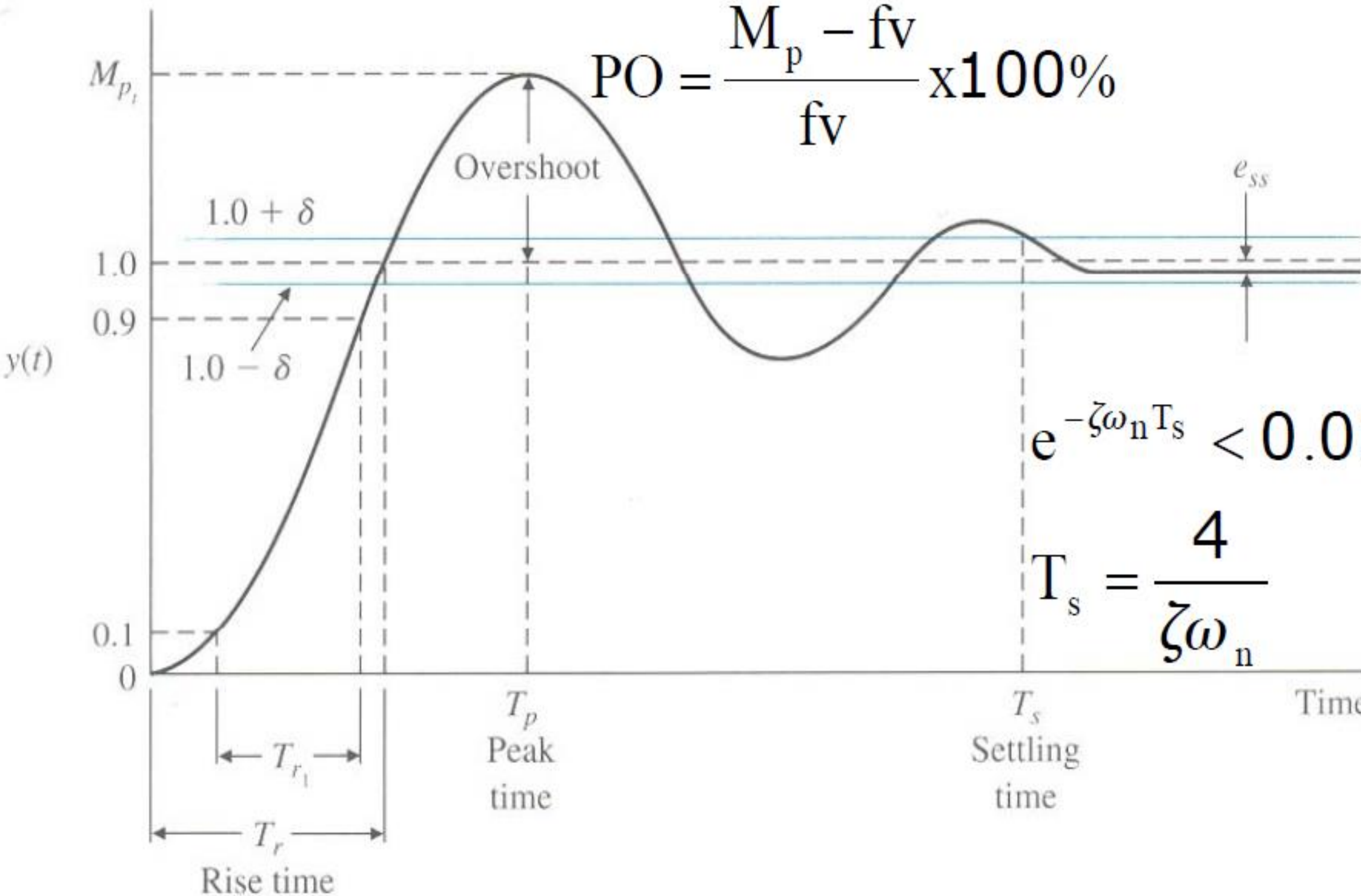
A 3-D representation of unit step response



Unit Impulse Response



Unit step response



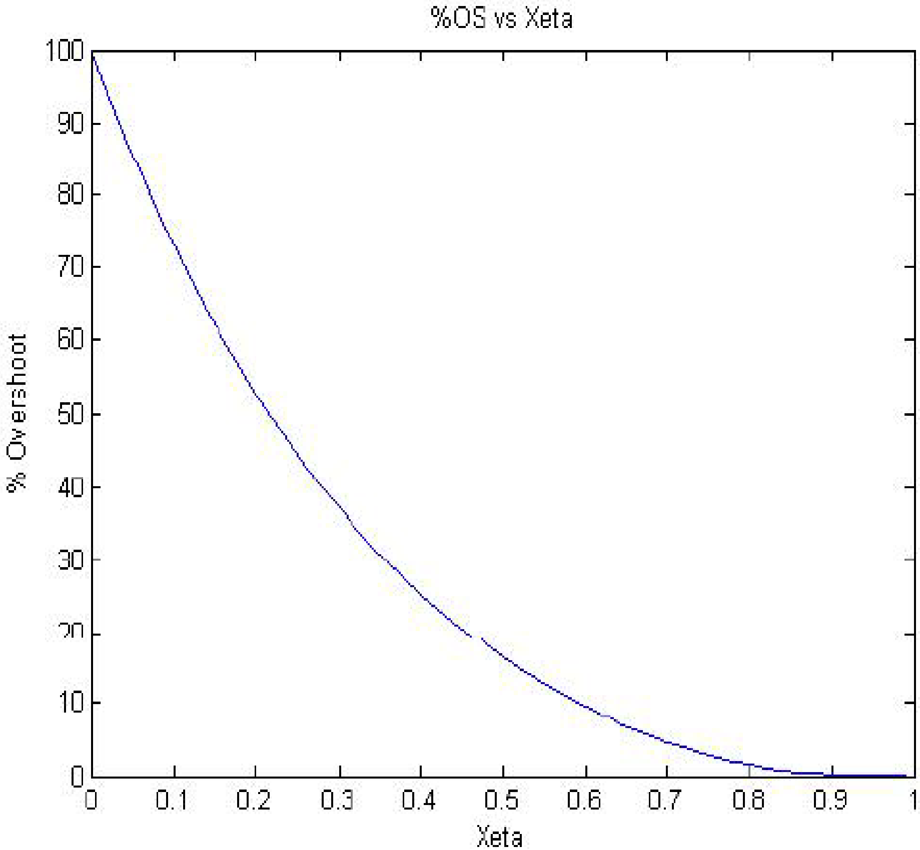
$$PO = \frac{M_p - fv}{fv} \times 100\%$$

$$e^{-\zeta\omega_n T_s} < 0.02$$

$$T_s = \frac{4}{\zeta\omega_n}$$

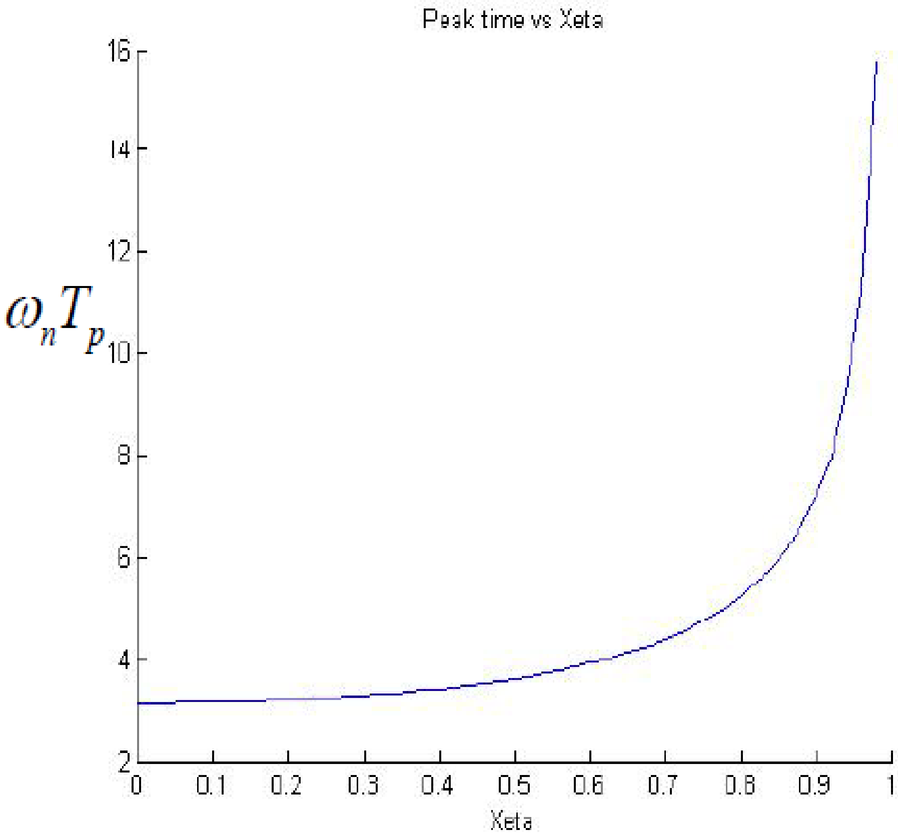
PO: A Measure of Closeness of response

$$PO = 100e^{-\zeta\pi/(\sqrt{(1-\zeta^2)}}$$

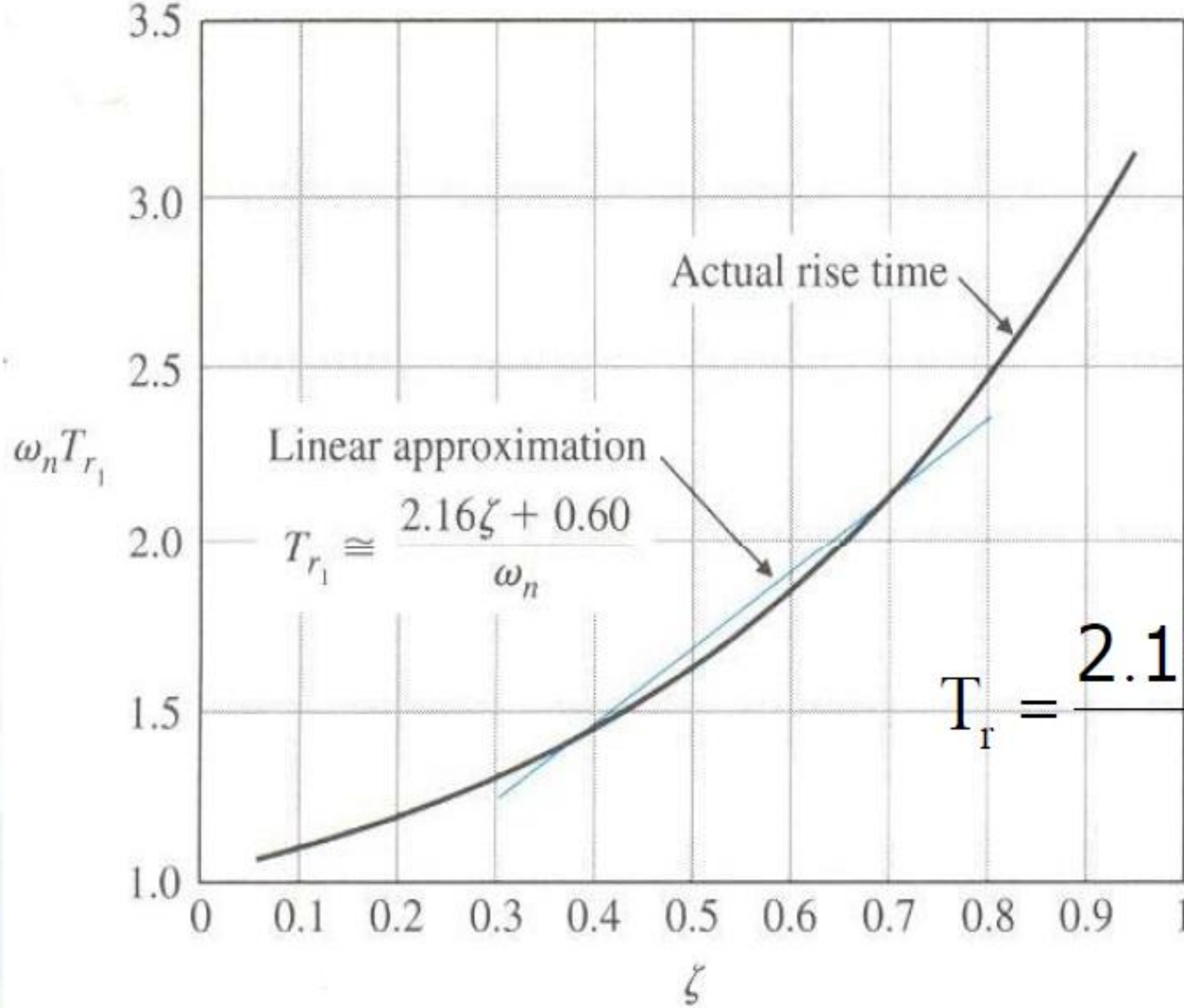


Peak Time: A Measure of swiftness

$$T_p = \frac{\pi}{\omega_n\sqrt{(1-\zeta^2)}}$$



Normalized Rise-time



$$T_r = \frac{2.16\zeta + 0.60}{\omega_n}$$

Steady state error - Introduction

- Steady state error refers to the long-term behavior of a dynamic system.
- The Type of a system is significant to predict the nature of this error.
- A system having no pole at the origin is referred as Type-0 system.
- Thus, Type-1, refers to one pole at the origin and so on.
- It will be shown in this lecture that, it is the type of a system which can directly determine whether a particular command will be followed by a system or not.
- We will consider three common commands: namely, step, ramp and parabolic ramp and find out the steady state response/error of a system to follow these commands.
- A closed loop control system shows remarkable performance in reducing the steady state error of a system.

Steady state error of a system

- Error in a system: E(s) = U(s) /(1+G(s))

$$\boxed{\lim_{t\to\infty} e(t) = e_{ss} = \lim_{s\to 0} \frac{sU(s)}{1+G(s)}}$$

- For a step input

$$\boxed{e_{ss} = \frac{A}{1+G(0)}}$$

- Plant Transfer function G(s) is defined as

$$\boxed{G(s) = \frac{K\prod_{i=1}^{M}(s+z_i)}{s^k \prod_{j=1}^{N}(s+p_j)}}$$

Error Constants

- **Position error constant** $$K_p = \lim_{s \to 0} G(s)$$

- **Steady state error of a step input of magnitude A is** $e_{ss} = A/(1+K_p)$

- **Steady state error will be zero for system with type greater than or equal to 1**

- **For ramp input** $$e_{ss} = \lim_{s\to 0} \frac{A}{sG(s)}$$

- **Define velocity constant as** $$K_v = \lim_{s\to 0} sG(s)$$

- **Hence steady state error is** A/K_v

- **Error will be zero for k greater than or equal to 2**

Summary of Steady State Errors

Type	Step (A/s)	Ramp (A/s^2)	Parabolic Ramp (A/s^3)
0	$E_{ss} = A/(1+K_p)$	Inf	Inf
1	0	A/K_v	Inf
2	0	0	A/K_a

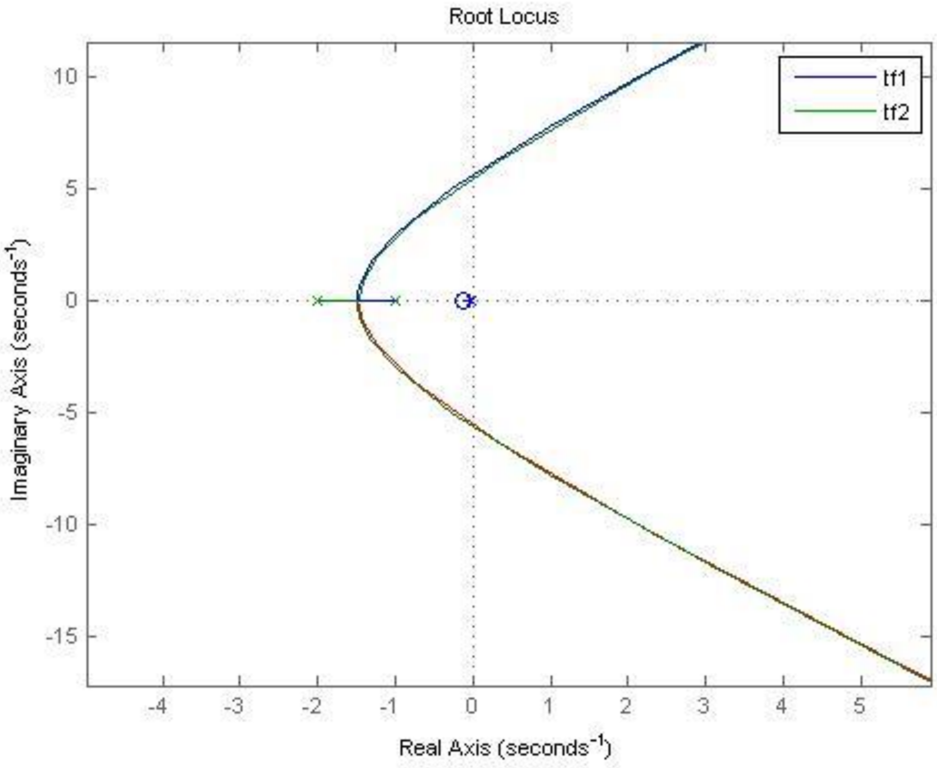
Lag Filter/Compensation

- Introduces a set of Pole and Zero
- Improves Steady State Performance if the Pole is very near to the origin and the zero is little away in the left side from the pole.
- K_0 (uncompensated) = Kz1z2…/p_1p_2….
- K (compensated) = $K_o\, z_c/p_c$

Design of a Lag Compensator

- Consider a Plant with open loop poles at -1,-2 and -10, Find K_p, Improve steady state error by a factor of 10 and get the closed loop poles. Damping ratio should be around 0.174.
- K_p = 8.23 (Dominant Pair Gain/$p_1p_2p_3$)
- e_α= 1/1+K_p=.108
- Desired e_α =(1/10) e_α =.0108, K_p= 91.59
- z_c/p_c = 91.59/8.23 = 11.13, Use p_c=0.01, z_c = 0.111

Root Loci for the two systems

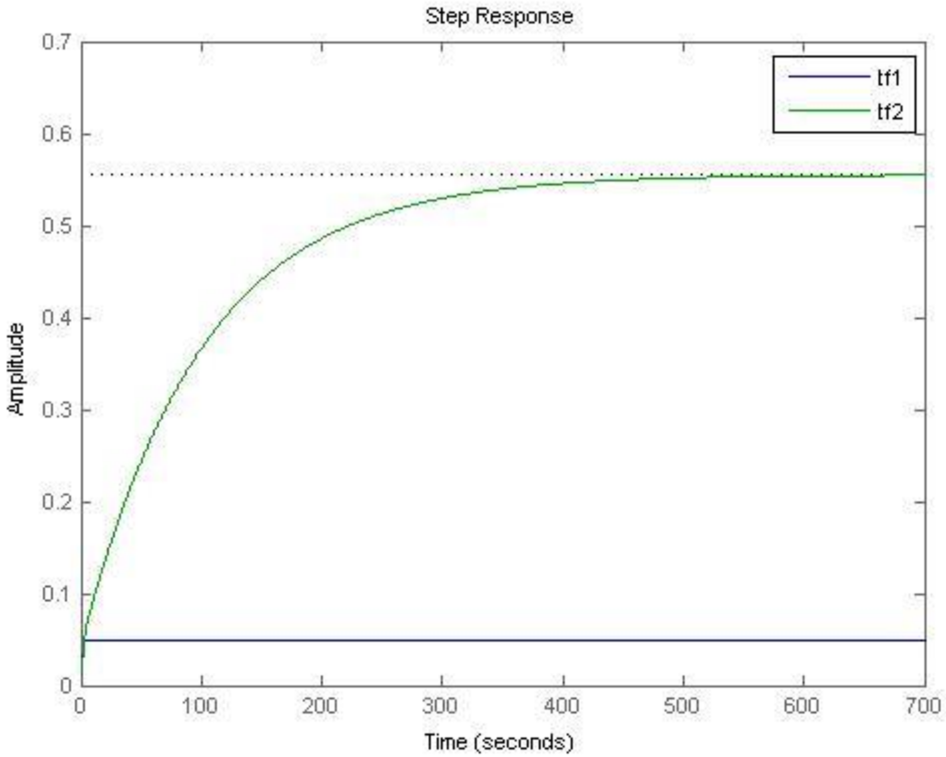


$$tf_1 = \frac{1}{(s+1)(s+2)(s+10)}$$

$$tf_2 = \frac{(s+0.11)}{(s+0.01)(s+1)(s+2)(s+10)}$$

There's almost no effect of compensator on the transient behavior and stability of the system

Steady State Response of the two systems



$$tf_1 = \frac{1}{(s+1)(s+2)(s+10)}$$

$$tf_2 = \frac{(s+0.11)}{(s+0.01)(s+1)(s+2)(s+10)}$$

You may have observed that the steady state response has increased more than five times.

Application of Lag Controller

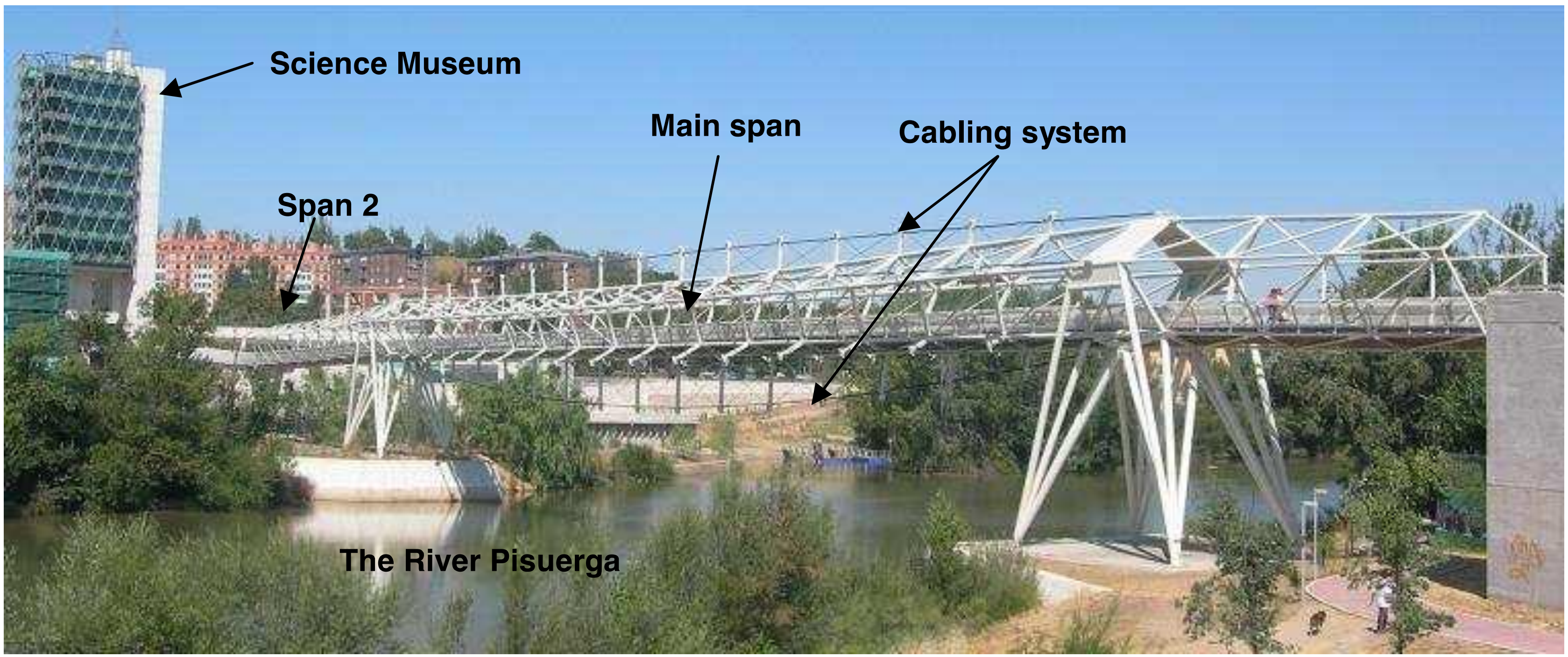


Figure 1. General view of the structure

Vibration Control of the Central Span

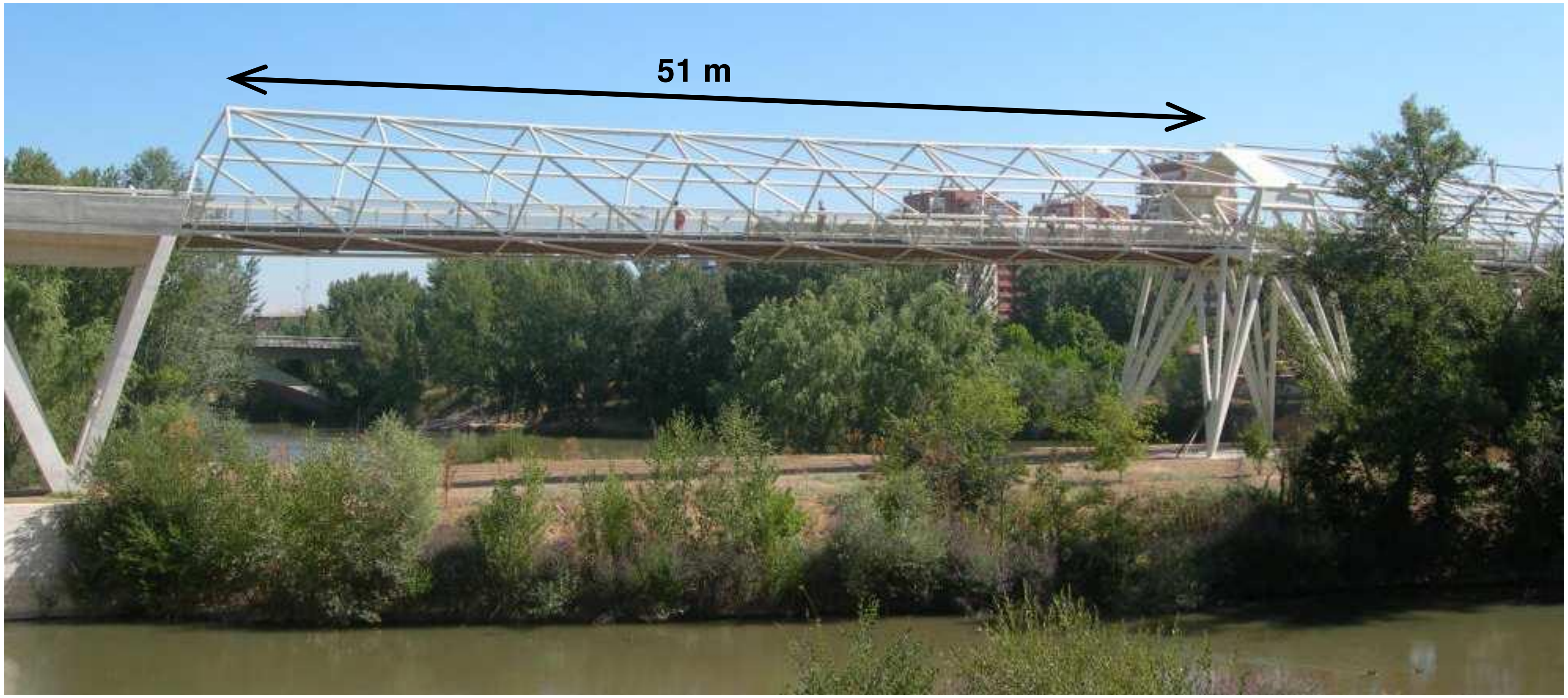
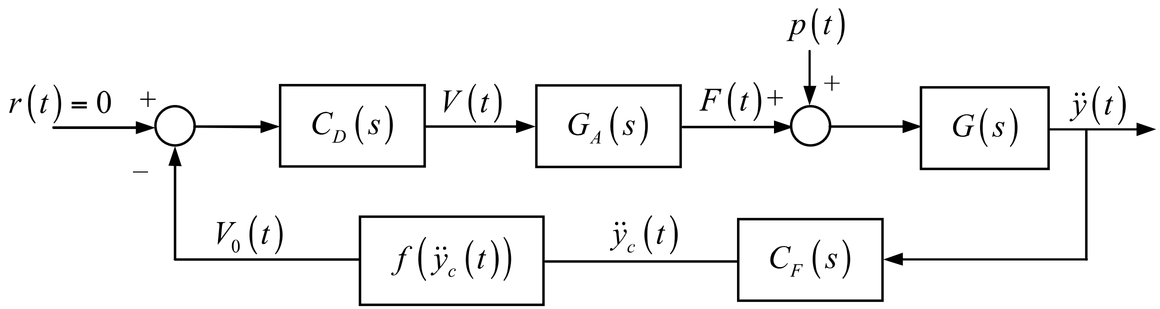


Figure 2. View of Span 2

Three vibration modes were identified in the frequency range 1–15 Hz

$$G(s)=\sum_{i=1}^{3}\frac{\alpha_i s^2}{s^2+2\zeta_i\omega_i s+\omega_i^2}=\frac{7.13\cdot 10^{-5}s^2}{s^2+0.264s+483.6}+\frac{4.54\cdot 10^{-6}s^2}{s^2+0.279s+2162}+\frac{5.85\cdot 10^{-5}s^2}{s^2+0.591s+3488},$$

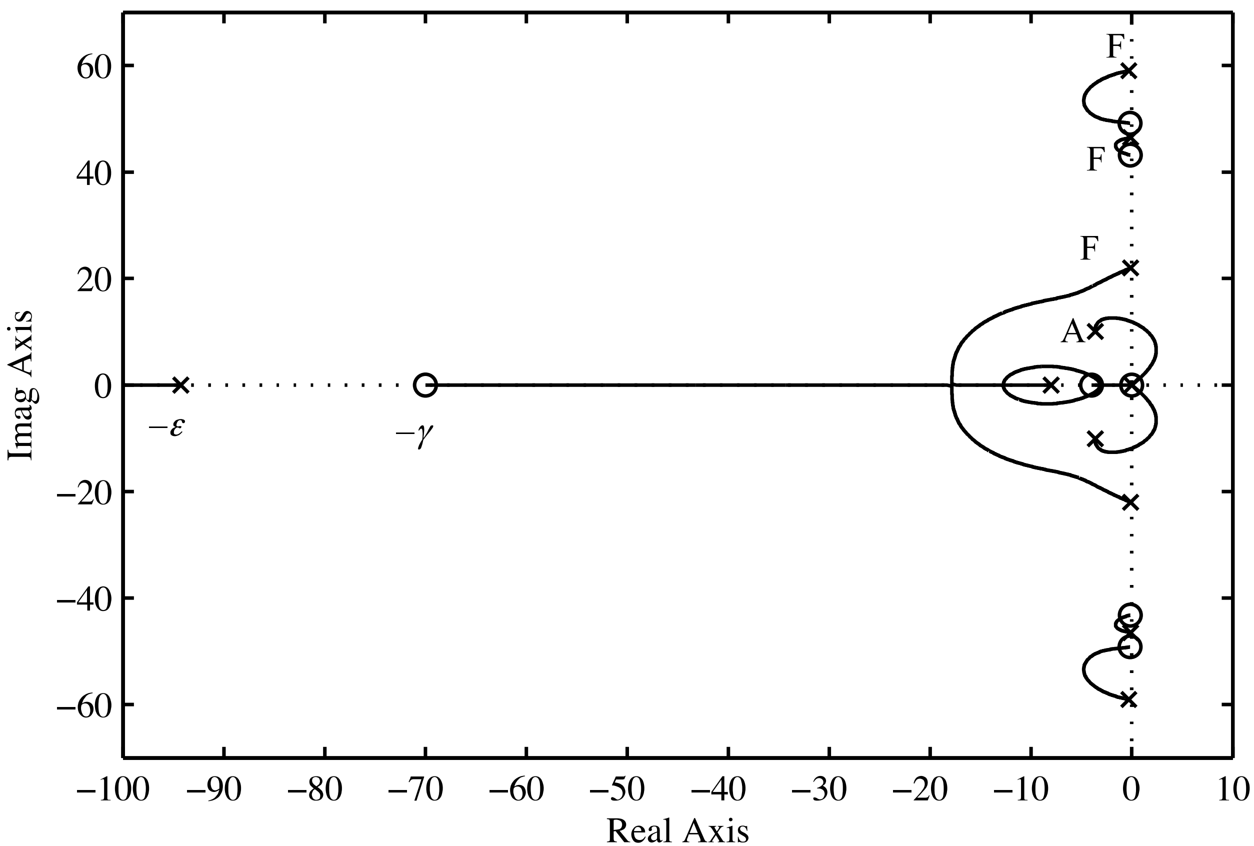
Lag Compensator in the feedback loop



$r(t)$	Reference command	$\ddot{y}(t)$:	Acceleration response
$V(t)$:	Control voltage	$\ddot{y}_c(t)$:	Compensated acceleration
$F(t)$:	Actuator force	$V_0(t)$:	Initial control voltage
$p(t)$:	Plant disturbance	$f(\ddot{y}_c)$:	Nonlinear element
$C_D(s)$:	Transfer function of the direct compensator		
$G_A(s)$:	Transfer function of the proof-mass actuator		
$G(s)$:	Transfer function of the floor structure		
$C_F(s)$:	Transfer function of the feedback compensator		

Figure 4. General control scheme

Root Locus Diagram



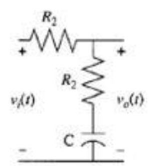
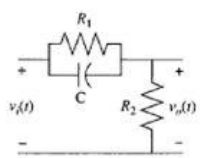
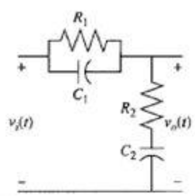
Root locus of the total transfer function G_T. (×) pole; (o) zero; (F) footbridge; (A) actuator

Practical Realization of a Lag Compensator

The figure below shows the practical realization of a Lag Compensator with the help of resistors and capacitor. In terms of mechanical elements we can realize the same by using dashpot and springs. Just replace t the resistors by dashpots and capacitor by spring.

Passive-Circuit Realization

- Lag, lead, and lag-lead compensators can also be implemented with passive networks (Table 2) .

Function	Network	Transfer function, $\frac{V_o(s)}{V_i(s)}$
Lag compensation		$\frac{R_2}{R_1+R_2}\frac{s+\frac{1}{R_2C}}{s+\frac{1}{(R_1+R_2)C}}$
Lead compensation		$\frac{s+\frac{1}{R_1C}}{s+\frac{1}{R_1C}+\frac{1}{R_2C}}$
Lag-lead compensation		$\frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)s+\frac{1}{R_1R_2C_1C_2}}$

What is a Lead Compensator?

- Lead Compensator is similar in structure as a Lag Compensator, the transfer function could be written as:

$$K_{lead}(s) = \frac{K_c\,(s+a)}{s+b}$$

- However, in this case the zero is dominating over the pole and hence *b>a*.

- Lead Compensator improves the stability of a system by shifting the root locus towards the left of the origin. Thus while a lag compensator improves the steady state response sacrificing the stability, lead improves the stability of the system.

Design of a Lead filter

- Consider a Plant with open loop poles at 0,-4 and

-6, even if this is a 3rd order system, the third pole is quite away.

Objective: Get 16% overshoot with three-fold reduction in settling time.

- ζ=.504 (for 16% OS) (see the chart in the next page)
- Poles at -1.20 +- j2.06, K=43.20
- The uncompensated system is shown in the next slide.