Responses & Applications of Viscoelastic Materials

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The frequency-dependence of the complex modulus we have just discussed can be explained through a linear viscoelastic model. For example, consider the simple, three-element model shown in the figure below:

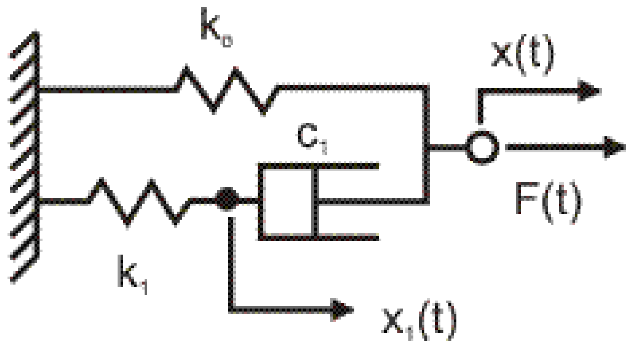


Figure 14.2: 3 Element model

The stress-strain relation for this model is given by the following equation

$$[1 + (c_1/k_1)(\partial/\partial t)]\sigma = \Lambda[k_0 + \{c_1 + (c_1/k_1)k_0\}(\partial/\partial t)]\epsilon$$

where Λ is a geometric parameter.

Assuming a harmonic loading of frequency ω, we substitute $(j\omega)$ for the operator $\partial/\partial t$ in this equation. Then, we get the complex modulus as

$$E^*_\omega = \Lambda[\,\{k_0 k_1 + c_1(k_0 + k_1)j\omega\}/(k_1 + jc_1\omega)]$$

Taking the real and imaginary parts of this equation, we obtain

$$E_{r,\omega} = \Lambda[\{k_0 k_1^2 + c_1^2\omega^2(k_0 + k_1)\}/(k_1^2 + \omega^2 c_1^2)]$$

$$E_{i,\omega} = \Lambda[c_1\omega k_1^2/(k_1^2 + \omega^2 c_1^2)]$$

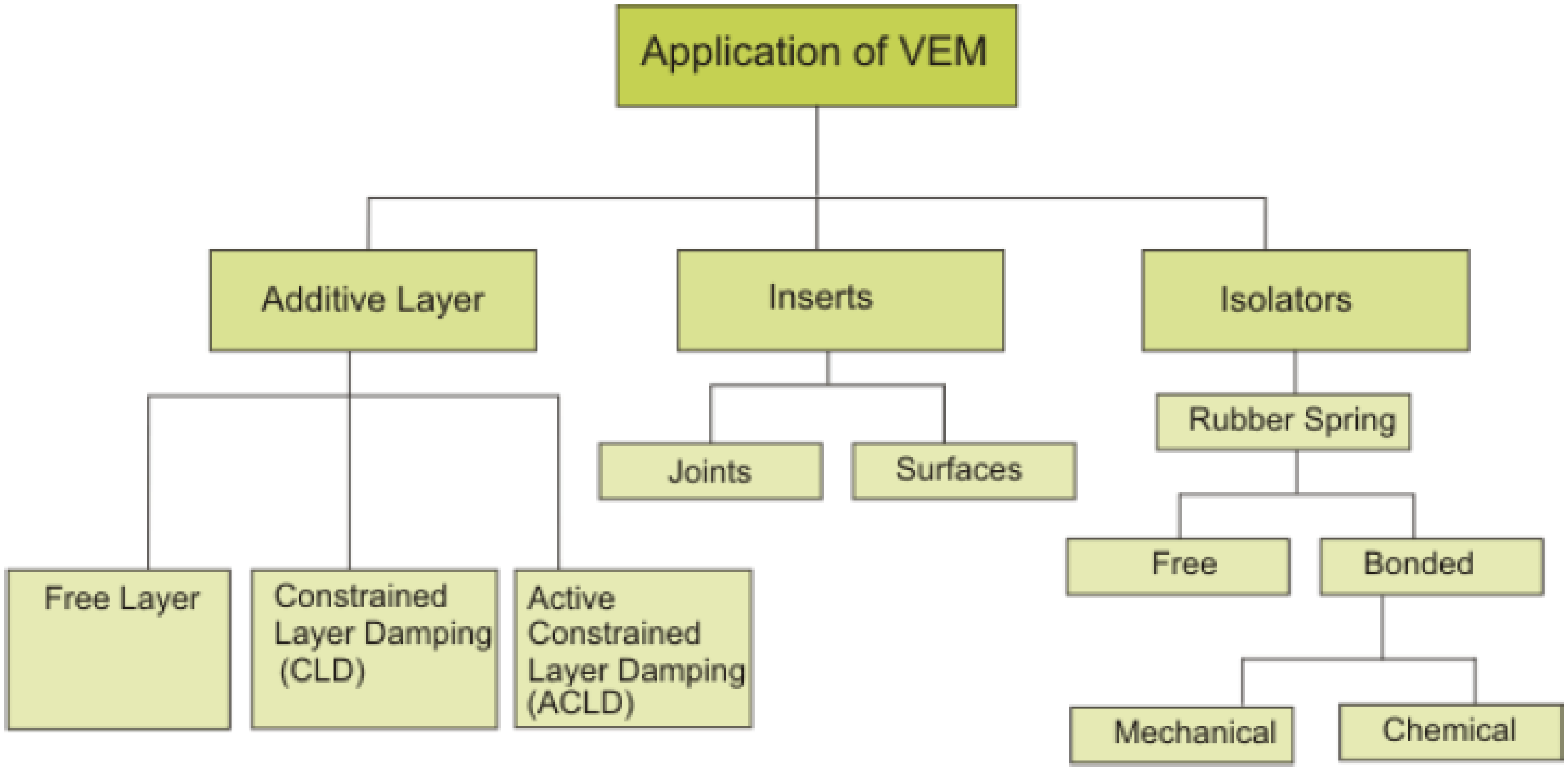
It can be seen from eqns. that the loss modulus $E_{i,\omega}$ has a maxima at $\omega = (k_1 \,/\, c_1) = \lambda_1$, where λ_1 is the relaxation parameter of the viscous branch.

Deborah Number (D_e) = Time of Relaxation/Time of Observation.
Originally defined by Reiner while working with Bingham
When Deborah Number is Low – Material behaves lie a Fluid
For Large Deborah Number – Non Newtonian Fluid
Very Large Deborah Number - Solid

Applications
of
Viscoelastic Materials (VEM)

Classification

The application of the VEM for vibration control could be classified in the following groups:

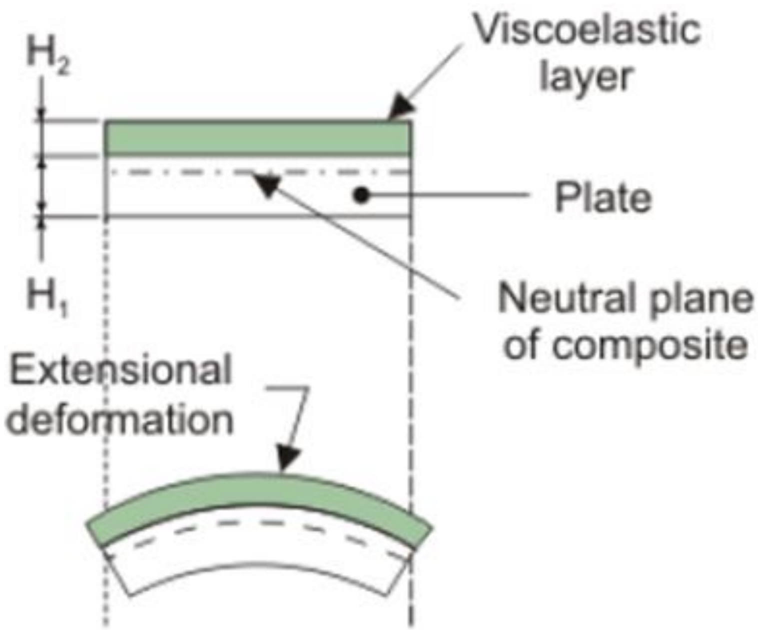


Use of Viscoelastic Laminae: Additive Layer Damping using VEM

Layers of Viscoelastic Materials are used often for vibration control. These are of two types:

- ✓ Unconstrained
- ✓ Constrained

For **unconstrained damping**, the V.E. layer is placed over one of the surfaces.



- The **vibrational energy** is dissipated due to the **extensional deformation** of the high damping viscoelastic layer
- assuming the **base plate** to be **non-dissipative** and the **extensional stiffness** of **the viscoelastic layer** is **much less** than that of the **base plate**.

Overall loss factor, $\eta \approx \dfrac{(\eta_{E_2})eh(3+6h+4h^2)}{[1+eh(3+6h+4h^2)]}$

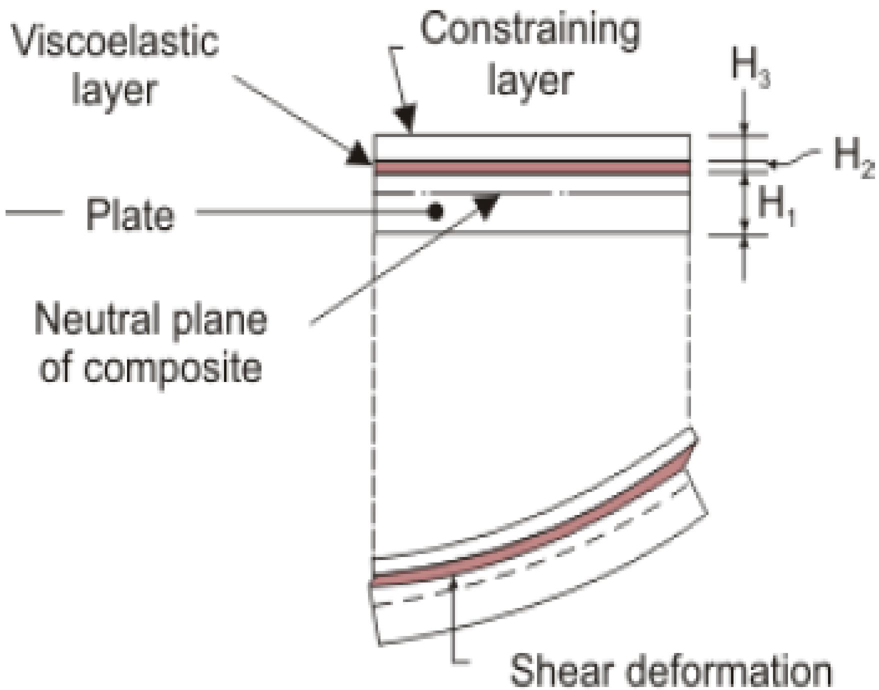
η_{E_2} = loss factor of the viscoelastic layer in longitudinal deformation

E_1 = Young's modulus of the base layer

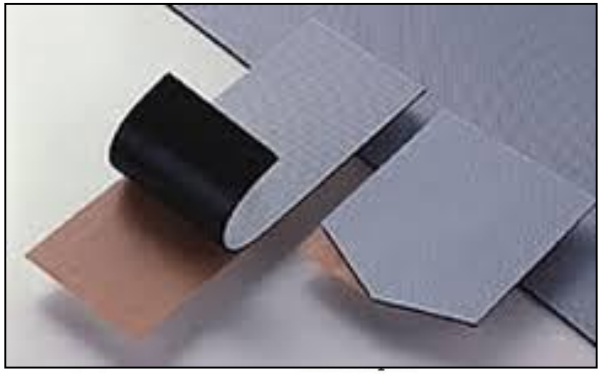
E_2 = Young's modulus of the viscoelastic layer

$$e = \frac{E_2}{E_1}, h = \frac{H_2}{H_1}$$

- **For constrained layer damping (CLD)**, the damping layer is sandwiched between the vibrating surface and a stiff constraining layer.
- In this treatment, most of the energy is dissipated due to the **shear deformation** of the viscoelastic layer.



- **CLD** is normally **more effective** than an **unconstrained treatment**.
- The **base layer** and the **constraining layer** are assumed to be non-dissipative.
- During flexural vibration of the base plate, the viscoelastic layer is subjected to large shear deformation and the **shear damping** is likely to exceed the **extensional damping**.



If the **extensional stiffness** of the viscoelastic layer is **negligible** as compared to the **stiffnesses of the bottom and top layers** (as is usually the case in **real life**), then the **overall loss factor**, neglecting the **extensional damping**, is given by

$$\text{Overall loss factor, } \eta \approx \frac{(\eta_{G_2} Y g)}{[1+(2+Y)g+(1+Y)(1+\{\eta_{G_2}\}^2)g^2]}$$

η_{G_2} = loss factor of the viscoelastic material in shear

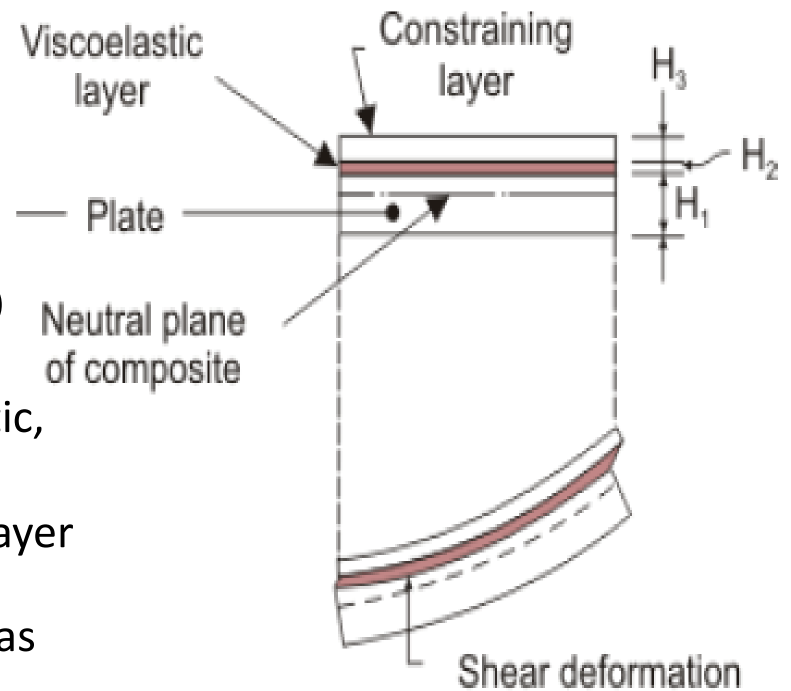
- The parameter Y, called the **stiffness parameter** is given by

$$Y = \frac{12ehH^2}{[(1+eh)(1+eh^3]}$$

$$e = \frac{E_3}{E_1}, \boldsymbol{h} = \boldsymbol{H_3} = \boldsymbol{H_1}, H = \frac{1}{2} + \frac{H_2}{H_1} + H_3 = (2H_1)$$

With **H_1, H_2, H_3** as the **thickness** of the base, viscoelastic, constraining layers,

E_1, E_3 as Young's moduli of the base and constraining layer



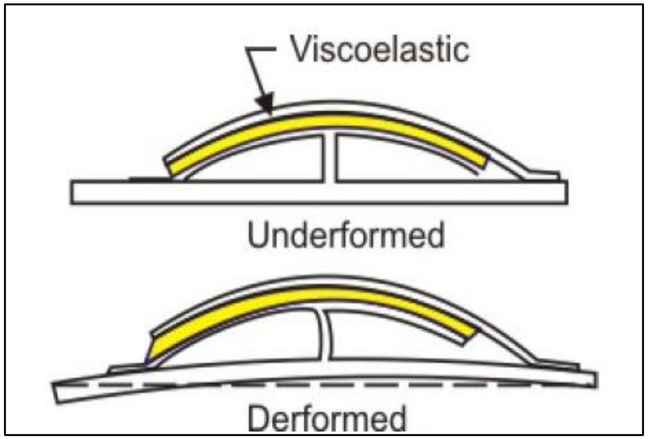
- The parameter g, called the **shear factor** is expressed as

$$g = \left[\frac{G_2}{(\frac{4\pi^2}{\lambda^2})H_2}\right]\left[\frac{1}{E_1H_1} + \frac{1}{E_3H_3}\right]$$

G_2= Storage shear modulus of VEM

λ = wavelength of flexural vibration

- The **shear damping** can be **enhanced** if the **shear strain in the viscoelastic layer** is amplified.
- One approach, known as **corrugated damping** configuration, whose undeformed and deformed states are shown below.



Corrugated damping

- Another approach is the **sandwich construction** where the **original member** is **divided** in two equal halves with a **viscoelastic layer inserted** between them.