Lecture 7

ME756A

Principles of Vibration Control

Instructor : ***Prof. Bishakh Bhattacharya***
Department of Mechanical Engineering
IIT Kanpur
E-mail: *bishakh@iitk.ac.in*

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Laboratory
IIT Kanpur

Field Balancing of Rigid Rotors

Field Balancing of a Rigid Disc Rotor

- Method of balancing a rotor is based on an extension of balancing of a thin disc during rotation

- Following the text book "Principles of Vibration Control", one can show that for balancing of thin disc one needs to measure
  a. amplitude (A_i) and phase (θ_i) of vibration
  b. the effect of adding additional mass (m_i) to the disc for the amplitude (A_t) and phase (Φ_t)

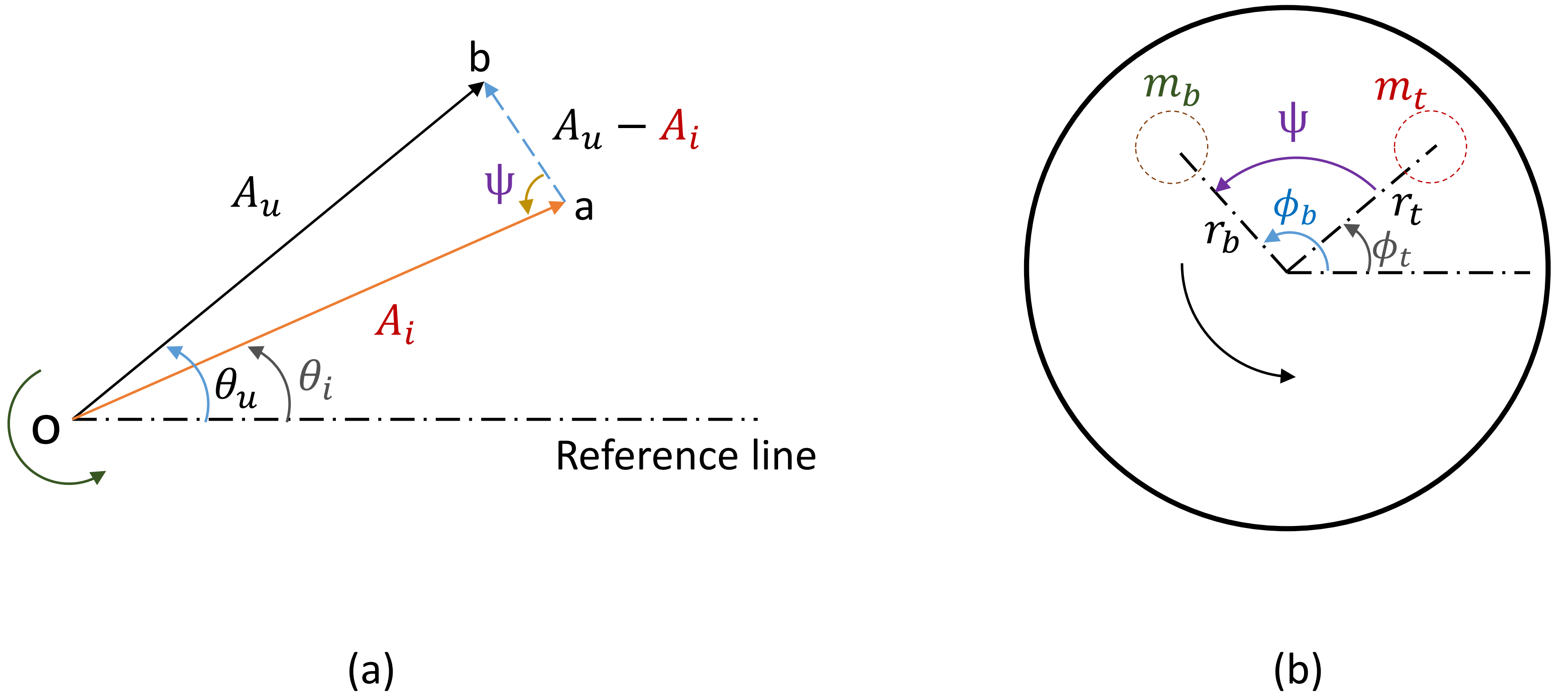
- Accordingly, denoting $\boldsymbol{\delta = -\frac{A_i}{A_t - A_i}}$ one can find the balancing mass

(m_b) and it's location (r_b, Φ_b) in polar coordinates as

$$m_b r_b = \delta\, m_t r_t$$

$$\Phi_b = \Phi_t + \Phi_\delta$$

where, r_t is the radial distances of the trial mass and Φ_t is the angular location.



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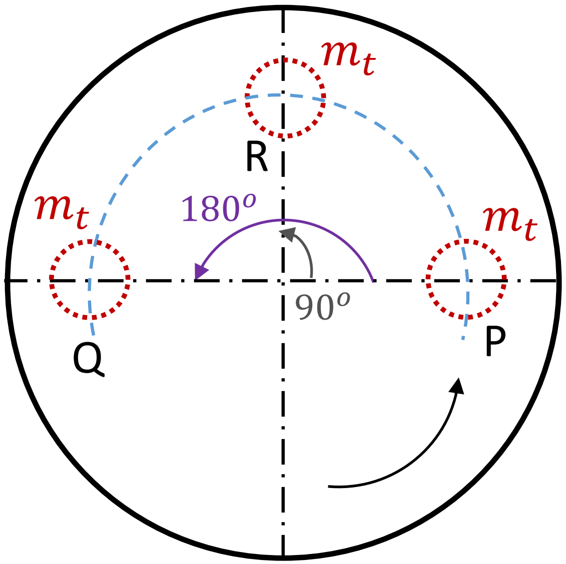
Amplitude Based Measurement



Balancing of Rigid Rotors

- For balancing of a rotor one needs three test runs and measure the amplitude and phase of vibration at two different locations 1 and 2.

- At first, amplitudes and phases corresponding to unbalanced rotor at locations 1 and 2 denoted as $\bar{A}_{1i}$ and $\bar{A}_{2i}$ are measured.

- Next for a trial mass m_{t1} the same are noted. Let these be $\bar{A}_{1t1}$ and $\bar{A}_{2t2}$

- Finally, in the third run, for a trial mass m_{t2} one needs to obtain $\bar{A}_{1t2}$ and $\bar{A}_{2t2}$

Balancing of Rigid Rotors

- One can show that the balancing masses required are $(mr)_{b1} = \delta_1 \, (mr)_{t1}$

  and $(mr)_{b2} = \delta_2 \, (mr)_{t2}$

where

$$\delta_1 = \frac{\bar{A}_{2_i}\left(\bar{A}_{1_{t_2}} - \bar{A}_{1_i}\right) - \bar{A}_{1_i}(\bar{A}_{2_{t_2}} - \bar{A}_{2_i})}{\Delta}$$

$$\delta_2 = \frac{\bar{A}_{1_i}\left(\bar{A}_{2_{t_1}} - \bar{A}_{2_i}\right) - \bar{A}_{2_i}(\bar{A}_{2_{t_1}} - \bar{A}_{2_i})}{\Delta}$$

$$\Delta = \left[\left(\bar{A}_{1_{t_1}} - \bar{A}_{1_i}\right)\left(\bar{A}_{2_{t_2}} - \bar{A}_{2_i}\right) - \left(\bar{A}_{1_{t_2}} - \bar{A}_{1_i}\right)\left(\bar{A}_{2_{t_1}} - \bar{A}_{2_i}\right)\right]$$

Balancing of Flexible Rotors

- A rotor operating at a speed higher than (or close to) its first critical speed is termed as a flexible rotor since it undergoes a significant transverse deflection at this speed.
- Unlike a rigid rotor, a flexible rotor cannot be balanced by adding two masses placed in two arbitrarily chosen planes.
- The principle used for balancing a flexible rotor is entirely different from that applied in the case of a rigid rotor. The objective of attaching the balancing masses to a rigid rotor, as already stated, is to neutralize the unbalanced forces and moments.
- In a flexible rotor, on the other hand, the balancing masses are attached to suitably modify the dynamic deflection characteristics of the rotor. The technique to do this is known as the modal balancing technique .

✓ Vibration Control Strategies

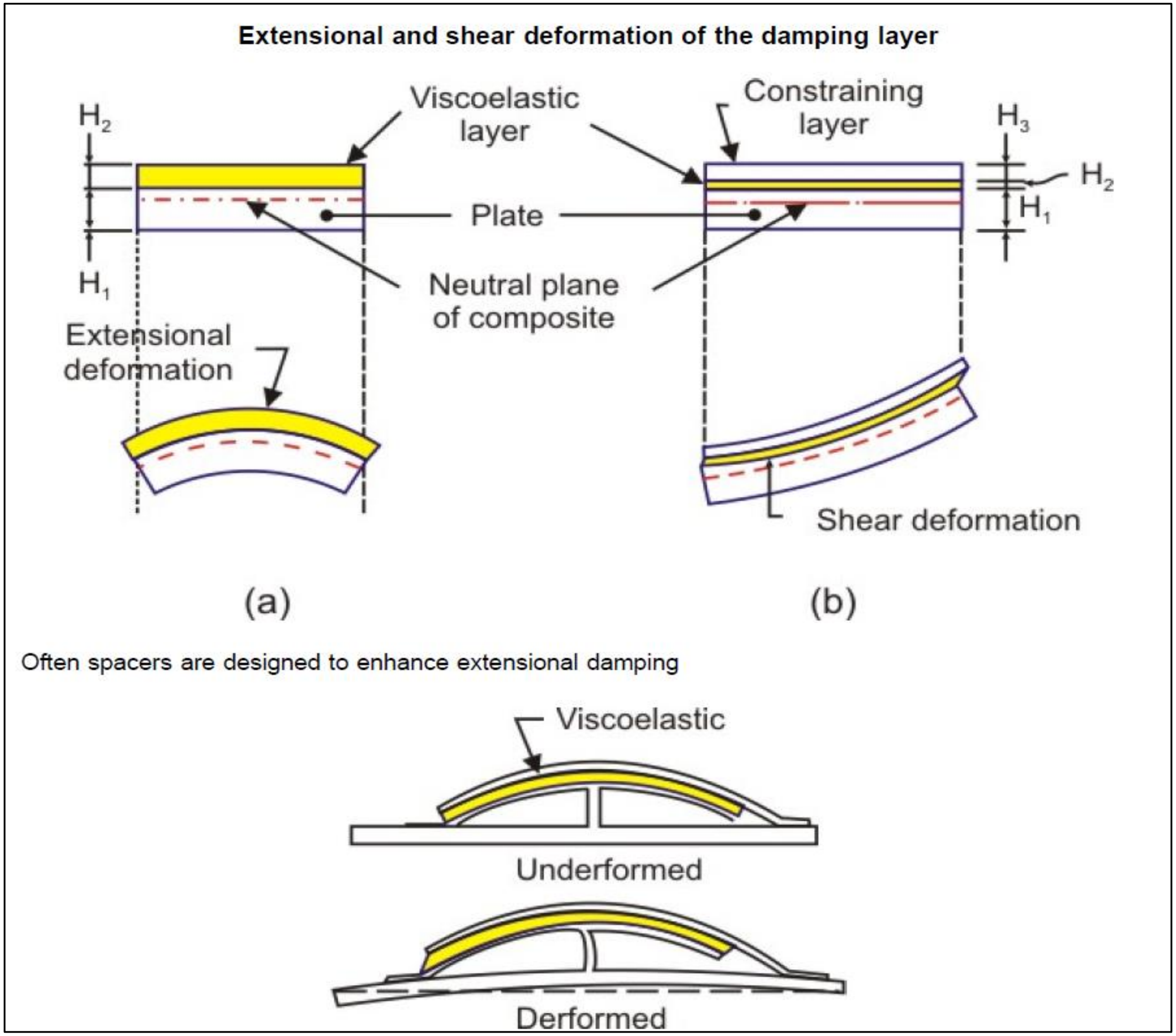
- Reduction of excitation at the source
- Isolation of the Source
- System Redesign
- **Remedial Measures**

(iv) *Remedial measures*

- ✓ **Addition** of a **secondary vibratory system** to the original (primary) vibratory system which is under excitation.
- ✓ Some secondary systems are vibration neutralizer, vibration absorber, tuned , self-tuned, impact absorbers.
- ✓ This strategy has been successfully used for suppressing vibration in very small to very large systems.
- ✓ **Examples**: electric hair clippers, DC-9 aircraft, tractors, foot bridges, pipelines etc.

Viscoelastic materials are used as additive damping treatments:

Constrained and Unconstrained Layers



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Ref.: Principles of Vibration Control, A. K. Mallik

Energy Dissipation
in
Structural Materials

Introduction

Selection of structural materials corresponding to high inherent energy dissipation or damping depends on three factors:-

- ✓ Material properties
- ✓ Geometric property of the structural member and
- ✓ Type of loading (bending, torsion, etc.)

Material properties are connected with the system parameters as follows:

- Inertia – Depends on density and cross-section
- Stiffness – Governed by Young’s modulus (E) and Shear modulus (G)

  Also depends on geometry and mode of loading.

  Depending on loading & geometry, only one elastic constant may be involved in some situations

These values for all structural materials are readily available but not for damping capacity.

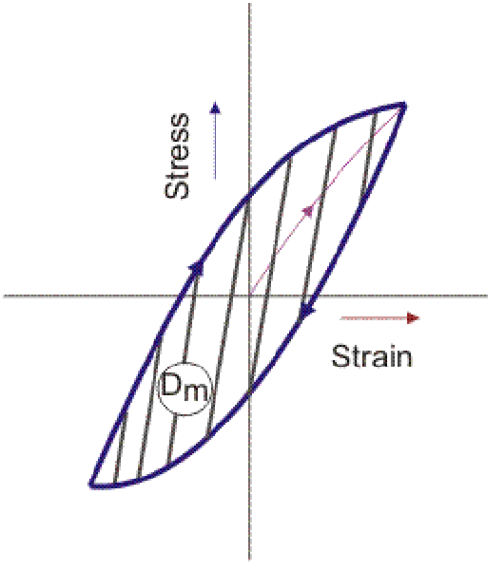
Damping capacity of structural materials may depend on the following mechanisms:-

- ✓ **Dislocation movement**: Occurs due to the presence of slip planes in crystalline materials.
- ✓ **Grain boundary slip**: Movement of one grain over the other causes energy dissipation.
- ✓ **Magneto-elastic effect**: Interaction between magnetization and strain of a magnetic material.
- ✓ **Thermo-elastic effect**: Interaction between thermal and mechanical deformation.
- ✓ **Localized plastic strain** : Presence of defects like shear bands can entangle dislocations preventing the crystal from sliding. This may also create energy dissipation.

Stress Dependence of Energy Dissipation

The **stress-strain plot** of **structural materials** (metals and metallic alloys are considered here) under **harmonic loading** and **low stress level** may be plotted as

- The energy dissipated per unit volume of a structural material per unit cycle, $\mathbf{D_m}$ is given by the area of the hysteresis loop (also known as mechanical hysteresis loop).



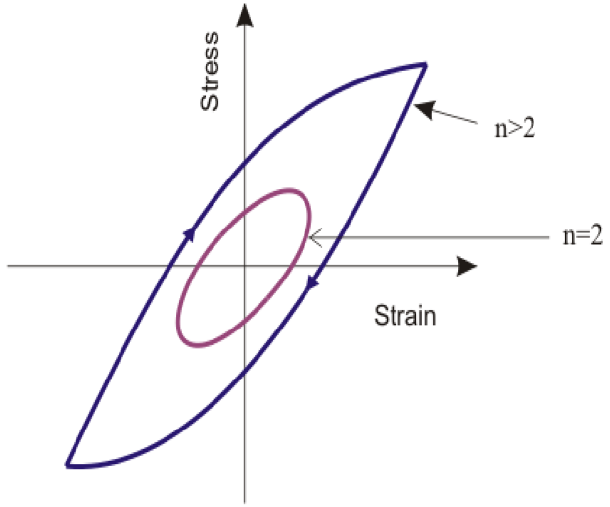
D_m is related to the Stress (based on experiment uniaxial harmonic loading) as-

$$D_m = J\sigma^n$$

J = damping constant

n = damping index, usually 2-3 in the working stress range

- At a **very low stress level**, n = 2 and the stress-strain diagram becomes **elliptic** instead of showing pointed tip.

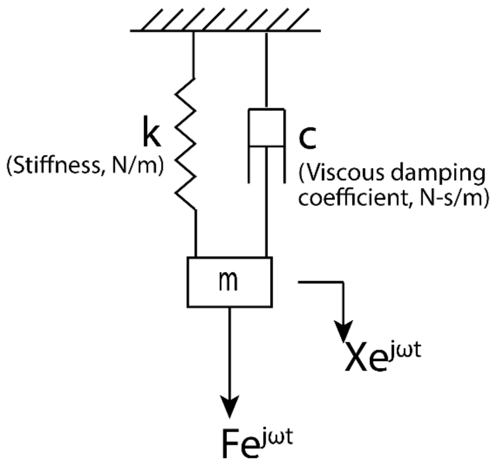


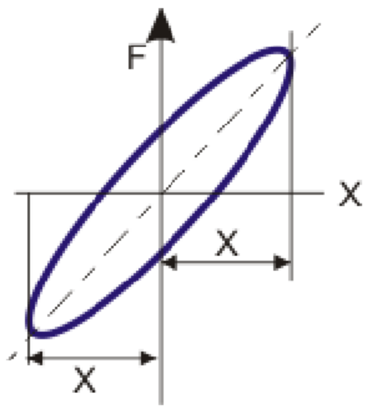
- D_m is independent of frequency. So, hysteresis loop doesn't alter with it.

- Now, **comparing** with **viscous damping**. In both the cases, the **energy dissipated** per cycle is **proportional** to the **square of the amplitude**.

- But in **viscous damping**, energy dissipation **increases linearly** with **frequency** (ω).

$$\boxed{D_m = \pi c \omega X^2}$$





For higher values of "**n**" (n>2), a modified relationship is used as follows

$$\boxed{D_m = J_1\sigma^2 + J_2\sigma^n}$$

J_1, J_2 are the damping constants

For **multi axial loading** of a structural member, the relationship is given as

$$\boxed{D_m = J\left(\sigma_{eq}^2\right)^{n/2}}$$

Uniaxial stress, σ is replaced by equivalent stress, σ_{eq}

$where, \sigma_{eq}^2 = (1-\lambda_1)(I_1^2 - 3I_2) + \lambda_1 I_1^2$ and $(0 < \lambda_1 < 1)$

$with\ stress\ invariants\ as\ \boldsymbol{I_1 = \sigma_1 + \sigma_2 + \sigma_3}, \boldsymbol{I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1}$

and $\boldsymbol{\sigma_1, \sigma_2, \sigma_3}$ $as\ the\ principal\ stress\ amplitudes$

The material loss factor could be expressed as,

$$\boxed{\eta_m = \frac{D_m}{2\pi W_m}}$$

Where, M$aximum\ elastic\ energy\ per\ unit\ volume\ in\ the\ cycle, W_m = \frac{I_1^2}{2E} - \frac{I_2(1+\mu)}{E}$

μ = Poisson's ratio
E =Young's modulus

For small I_2, $W_m \approx \frac{I_1^2}{2E}$

The total loss factor of a composite specimen η_s can be obtained as,

$$\boxed{\eta_s = \frac{D_s}{2\pi W_s}}$$

Where, $$D_s = \sum_{i=1}^{n} \left(\int_0^l D_i dx \right) b_i t_i$$

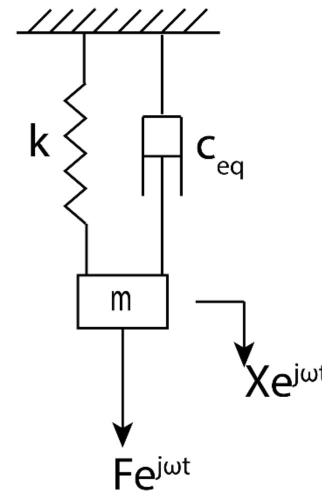
$$W_s = \sum_{i=1}^{n} \left(\int_0^l W_i dx \right) b_i t_i$$

b_i = width of ith layer
t_i = thickness of ith layer

Material	Density (ρ) (kg/ m^3)	Young's modulus (E) (GPa)	Order of loss factor (η_m)
Aluminium	2800	72	10^{-5}
Brass	8500	104	10^{-4}
Steel	7800	210	10^{-3}
Cast Iron	7300	103	10^{-2}
Concrete (dense)	2300	27	10^{-2}

Selection Criteria for Linear Hysteretic Materials

Consider harmonic force excitation of the given system where linear hysteretic damping is replaced by equivalent viscous damping.



For hysteretic damping, the energy dissipated per cycle is proportional to the square of the amplitude.

$$D = \alpha\,|X|^2 \qquad , with\ \boldsymbol{\alpha}, \text{a constant}$$

Comparing it with energy dissipation in a viscous dashpot

$$\alpha\,|X|^2 = \pi c_{eq}\omega\,|X|^2$$

So, the equivalent viscous damping coefficient is

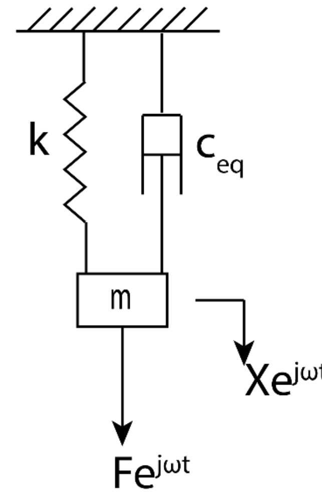
$$\boxed{c_{eq} = \frac{\alpha}{\pi\omega} = \frac{h}{\omega}}$$

where, **h** (= $\frac{\alpha}{\pi}$) is called **hysteretic damping coefficient**

$$\boxed{\text{Loss factor, } \eta = \frac{\alpha\,|X|^2/2\pi}{\left[\frac{1}{2}k|X|^2\right]} = \frac{\alpha}{\pi k} = \frac{h}{k}}$$

Now , the equation of motion for the mass can be written as,

$$\boxed{m\ddot{x} + \frac{h}{\omega}\dot{x} + kx = Fe^{j\omega t}}$$



Substituting, $x = X\, e^{j\omega t}$ in above equation, we have

$$X = \frac{\frac{F}{k}}{[\,1 - \frac{m\omega^2}{k} + \frac{jh}{k}\,]} = \frac{\frac{F}{k}}{[\,1 - \frac{m\omega^2}{k} + j\eta\,]}$$

The amplitude of displacement is

$$|X| = \frac{\frac{|F|}{k}}{[\,(1 - \boldsymbol{\Omega}^2)^2 + \eta^2\,]^{1/2}}$$

and $\boldsymbol{\Omega} = \frac{\omega}{\omega_n}$

Material selection criteria against damping

Example: A beam of specific mass and rectangular cross-section (width "b" and thickness "h") has to be designed. The length and width of the beam are specified but its height is left as a variable.

Determine the figure of merit (FOM) for selecting the beam material so that the **maximum displacement amplitude** is **minimum** under a harmonic loading.

$$|X| = \frac{\frac{|F|}{k}}{[\,(1-\boldsymbol{\Omega}^2)^2 + \eta^2\,]^{1/2}} \qquad \text{and } \boldsymbol{\Omega} = \frac{\omega}{\omega_n}$$

Now, when $\boldsymbol{\Omega} = 1$

$$|X| = \frac{\frac{|F|}{k}}{[\,(1-\mathbf{1}^2)^2 + \eta^2\,]^{1/2}} = \frac{|\boldsymbol{F}|}{\eta\boldsymbol{k}}$$

Ref.: Principles of Vibration Control, A. K. Mallik

- For the maximum displacement amplitude to be minimum, the quantity $\frac{|\boldsymbol{F}|}{\eta \boldsymbol{k}}$ should be minimum.

- In this problem, $\eta \boldsymbol{k}$ should be maximum for the material to be chosen. With a given width, for bending vibration, the stiffness $\boldsymbol{k} \propto \boldsymbol{E}\boldsymbol{h}^{\boldsymbol{3}}$, where ‘ h’ is the height of the beam and ‘E’ is Young's modulus.

- Now, for a specified mass and with **h** as the only variable. (mass = density x volume)

$$h \propto \frac{1}{\rho}$$ where, $\boldsymbol{\rho}$ is the density

So, $$k \propto \frac{E}{\rho^3}$$

Hence, for $\eta \boldsymbol{k}$ to be maximum, the quantity $\frac{\boldsymbol{E}\eta}{\boldsymbol{\rho}^{\boldsymbol{3}}}$ should be maximum

Thus, the **best material** is the one for which **this quantity is maximum**. This quantity (a function of only the material properties) to be **maximized** is called the **FOM** against damping.

Figure of merit to be **maximized** , FOM = $\frac{E\eta}{\rho^3}$

Considering the following structural materials

Material	Youngs Modulus (GPa)	Loss factor order	Density (kg/m^3)	FOM
Steel	200	10^{-3}	7500	4.74×10^{-4}
Brass	105	10^{-4}	8500	1.71×10^{-5}
Aluminium	70	10^{-5}	2700	3.18×10^{-5}

The table suggests **steel** to be the **best structural material** against **minimization of maximum displacement** and **aluminium** to be the **next best material** even though brass has higher loss factor than aluminium.