Design for Enhanced Material Damping

ME 756

Lecture 8

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Contents

- Overall loss factor in a beam
- Dependence of loss factor on Material, Geometry and Stress distribution
- Design for Enhanced Material Damping

In this lecture, we will discuss about various issues related to the design of structural systems from damping perspective specially for materials with damping index, n>2

Consider a beam subjected to pure bending and having a symmetric cross-section with respect to X and Y axes as shown in the figure below.

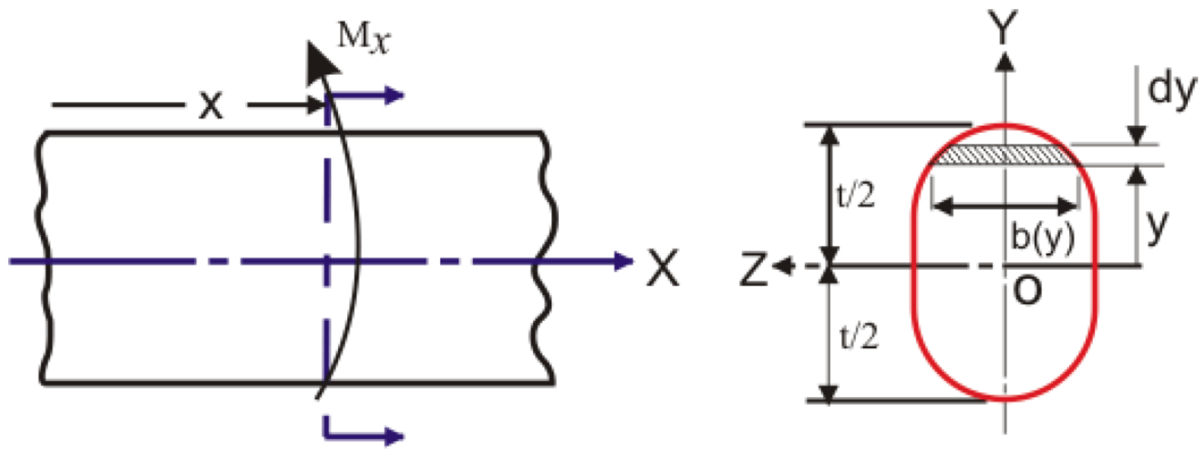

Figure 11.1: Beam subjected to pure bending

Following simple theory of bending one can write (11.1)

$$\frac{M_x}{I_{zz}} = \frac{(\sigma_x)_{max}}{t/2} = \frac{\sigma_x}{y}$$

where M_x = bending moment amplitude at the section, t= depth of the beam, $(\sigma_x)_{max}$= maximum bending stress amplitude at the section x(i.e., in the fibre at a distance t/2 from the neutral axis), and σ_x = bending stress amplitude in the fibre at a distance y from the neutral axis where the width of the beam is b(y).

The maximum elastic energy stored in the beam for a complete cycle of vibration may be expressed as

$$W_s = [1/(2EI_{zz})]\int_0^l M_x^2 dx \tag{11.2}$$

where *l* = length of the beam and E= Young's modulus. Substituting eqn. (11.1) in eqn. (11.2), we obtain

$$\begin{aligned} W_s &= 2[1/(EI_{zz})](I_{zz}^2/t^2)\int_0^l (\sigma_x)_{max}^2 dx \\ &= \left[\frac{2I_{zz}}{Et^2}\right]\int_0^l (\sigma_x)_{max}^2 dx \\ &= \frac{2I_{zz}}{Et^2}\int_0^l (\sigma_x)_{max}^2\, dx \end{aligned} \tag{11.3}$$

Using the damping-stress amplitude relationship, we get the energy dissipated per cycle from the entire beam as

$$D_s = 2J\int_0^l \int_{-t/2}^{t/2} \sigma_x^n b(y)dx\,dy$$

$$= 4J\int_0^t b\left({}^{y}/_{t/2}\right)^n dy\int_0^l (\sigma_x)_{max}^n dx$$

$$= 4J\int_0^{t/2} b(y)\left(\frac{y}{t/2}\right)^n dy\int_0^l (\sigma_x)_{max}^n dx \qquad (11.4)$$

Hence, the overall loss factor $\eta_s = \dfrac{D_s}{2\pi W_s}$ may be written as

$$\eta_s = \frac{\frac{2^n \times 4j}{t^n}\int_0^l (\sigma_x)_{max}^n dx \times \int_0^{t/2} b(y)y^n dy}{2\pi \times 2I_{zz}\int_0^l \left(\sigma_{x_{max}}\right)^2 dx} \times Et^2$$

$$= \frac{2^{n+2}JE\int_0^l \left(\sigma_{x_{max}}\right)^n dx \times \int_0^{t/2} b(y)y^n\,dy}{2^2\pi \times I_{zz} \times t^n\int_0^l \left(\sigma_{x_{max}}\right)^2 dx} t^2$$

$$= \frac{2^n JE}{\pi I_{zz} t^{n-2}}\int_0^l \left(\sigma_{x_{max}}\right)^{n-2} dx \times \int_0^{t/2} b(y)y^n\,dy$$

$$= \frac{2^n JE\,\sigma_{en}^{n-2}}{\pi} \times \int_0^l \left[\left(\sigma_{x_{max}}\right)/\sigma_{en}\right]^{n-2} dx \times \frac{\int_0^{t/2} b(y)y^n dy}{t^{n-2} I_{zz}} \qquad (11.5)$$

Note σ_{en} , the endurance strength against fatigue is introduced for nondimensionalization of the maximum stress.

- the first factor on the right-hand side, namely, $\beta_m = 2^n J E \sigma_{en}^{n-2}/\pi$ depends only on the material properties, and is therefore called the **Material Factor**.

- The second factor, i.e. $\beta_s = \int_0^l \left[\frac{\sigma_{x_{max}}}{\sigma_{en}}\right]^{n-2} dx$ is governed by the bending stress distribution along the beam length. This is called the **Longitudinal Stress Distribution Factor**.

- The third factor, namely, $\beta_c = \int_0^{t/2} b(y) y^n dy / t^{n-2} I_{zz}$ depends only on the cross-section shape and is referred to as the **Cross-sectional Shape Factor**.

For the same material and loading condition, different cross sections result in varying values of Loss Factor. The following table shows the value of for three different cross-sections, each having the same depth.

Cross Section		ß$_c$ for n=2	ß$_c$ for n=3
Rectangular		1	3/4
Circular		1	$16/\pi \int_0^{\pi/2} \frac{Sin^3\theta \ d\theta}{5}$
Diamond		1	3/5

It should be noted that, for n=2, β_c =2, for all the cross-sections since, for uniaxial loading of a hysteretic material. $\eta_s = \eta_m = JE/\pi$.

Any section having more material away from the neutral axis has a better damping capacity than a section in which most of the material is near the neutral axis.

Based on the expression of overall loss factor, it can be inferred that another way to increase the same is by increasing the maximum stress at the outer layers. Indeed, it has been shown that by using a series of cylindrical inserts one can enhance damping. If the inserts are made of high damping material the effect is further enhanced.

The method of enhancing damping capacity of a structure by high-damping inserts can be extended to design composite materials with high-damping spherical inclusions, which has a good balance of stiffness and damping. Composites of viscoelastic materials with suitable choices for relaxation times of the constituents can also be designed, which can maintain high damping over a wide frequency range.