Q1 Ans

$$\frac{1}{4}\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + y = u(t)$$

Taking Laplace transform both sides

$$\frac{1}{4}\left[s^2Y(s) - s\,y(0) - \dot{y}(0)\right] + 3\left[sY(s) - y(0)\right] + Y(s) = \frac{1}{s} \quad - 3$$

Putting I.C's to be zero, $y(0) = \dot{y}(0) = 0$

$$Y(s) = \frac{4}{s(s^2+12s+4)} \quad — 2$$

Using Partial fraction

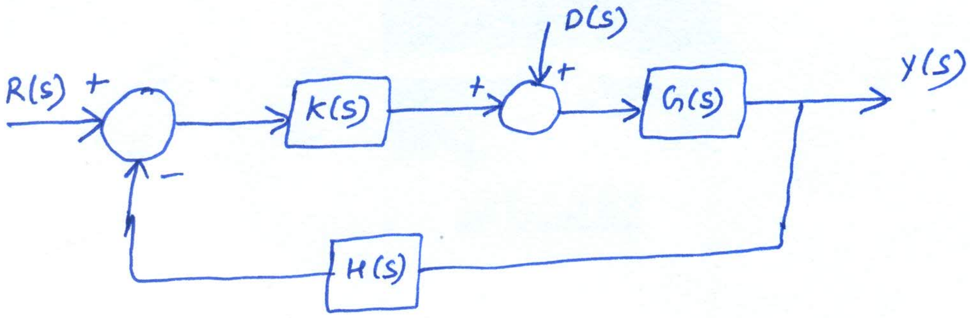
$$Y(s) = \frac{1}{s} + \frac{0.0303}{s+11.656} - \frac{1.0308}{s+0.343}$$

Taking Inverse transform

$$y(t) = 1 + 0.0303\,e^{-11.656t} - 1.0308\,e^{-0.343t} \quad — 5$$

Q2 Ans

$$K(s) = 10 \; ; \; G(s) = \frac{2}{(3s+1)} \; , \; H(s) = \frac{1}{(0.5s+1)}$$



a) forward transfer function from R(s) to Y(s)

$$Y(s) = K(s) \cdot G(s) \cdot R(s)$$

$$\frac{Y(s)}{R(s)} = \frac{20}{(3s+1)}$$ ✓ 2

b) forward transfer function from D(s) to Y(s)

$$Y(s) = D(s) \cdot G(s)$$

$$\frac{Y(s)}{D(s)} = \frac{2}{(3s+1)}$$ ✓ 2

c) CLTF for both R(s) & D(s)

$$E = R - YH \quad —① $$

$$Y = (KE + D)G \quad —②$$

Substitute ① in ②

$$Y = [K(R - YH) + D]G$$

$$\therefore Y = \frac{RKG}{1+KGH} + \frac{DG}{1+KGH}$$

$$\frac{Y(s)}{R(s)} = \boxed{\frac{20(0.5s+1)}{(3s+1)(0.5s+1)+20}}$$ 3

$$\frac{Y(s)}{D(s)} = \boxed{\frac{2(0.5s+1)}{(3s+1)(0.5s+1)+20}}$$ 3

Q3 Ans $G(s) = \dfrac{(s-5)}{(s+1)(s+2)(s+4)}$

Poles $= 3$ $[-1, -2, -4]$

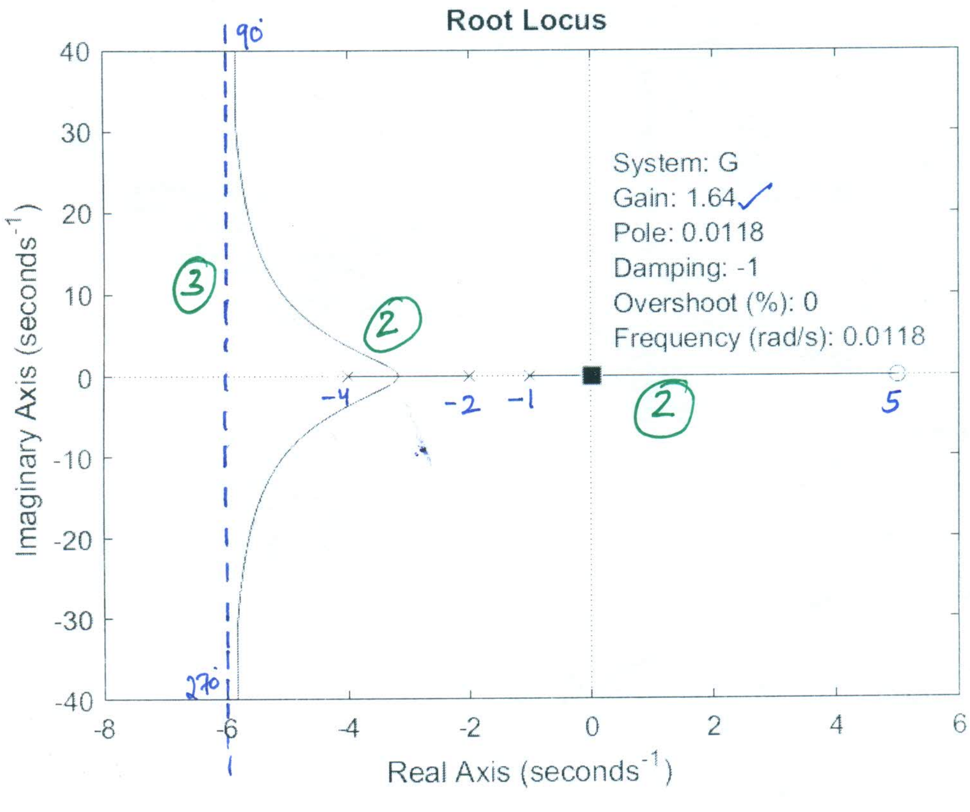
Zeros $= 1$ $[5]$

Branches $= 3>1 = 3$

No. of asymptotes $= 3-1 = 2$

Angle of asymptotes $= \dfrac{(2q+1)}{P-Z} \times 180° = 90°, 270°$

Centroid $= \dfrac{(-1-2-4)-5}{2} = -6$



$1 + \dfrac{K(s-5)}{(s+1)(s+2)(s+4)} = 0$

$(s+1)(s+2)(s+4) + K(s-5) = 0$

At $s = 0$

$8 + K(-5) = 0$

$\boxed{K = 1.6}$ — 3

$K = -\dfrac{(s^3+7s^2+14s+8)}{(s-5)}$

$\dfrac{dK}{ds} = 0$

$-2s^3 + 8s^2 + 70s + 78 = 0$

$s = 8.598, \boxed{-3.165}, -1.433$

Break-away pt. ($\because$ part of root locus)

Q4 Ans $G(s) = \frac{1}{s(s+2)}$ Type-I system

$K_u = 10$

Appropriate steady state error is K_v

$$K_v = \lim_{s\to 0} s K_u G(s)$$

$$= \lim_{s\to 0} s \times \frac{10}{s(s+2)} = 5$$ — 2

Uncompensated steady state error $= \frac{1}{K_v} = 0.2$

Compensated error $= \frac{0.2}{10} = 0.02$ — 2

$$K_v' = 1/0.02 = 50$$

$$\frac{Z_c}{P_c} = \frac{K_v'}{K_v} = \frac{50}{5} = 10$$

Pole is nearer to the origin than zero.

$$G_c(s) = \frac{K(s+Z_c)}{(s+P_c)}$$ — 2

Take $p_c = -0.01$, $Z_c = -0.1$

$$G_c(s) = \frac{(s+0.1)}{s(s+0.01)(s+2)}$$

Q5 Ans

$$1 + KG(s) = 0$$

$$1 + \frac{K(s+1)}{s(s-1)(s+6)} = 0$$

$$s^3 + 5s^2 + s(K-6) + K = 0$$

s^3	1	$K-6$	0
s^2	5	K	0
s^1	$\frac{5(K-6)-K}{5}$	0	0
s^0	K	0	

To have stability → No sign changes required in 1st column

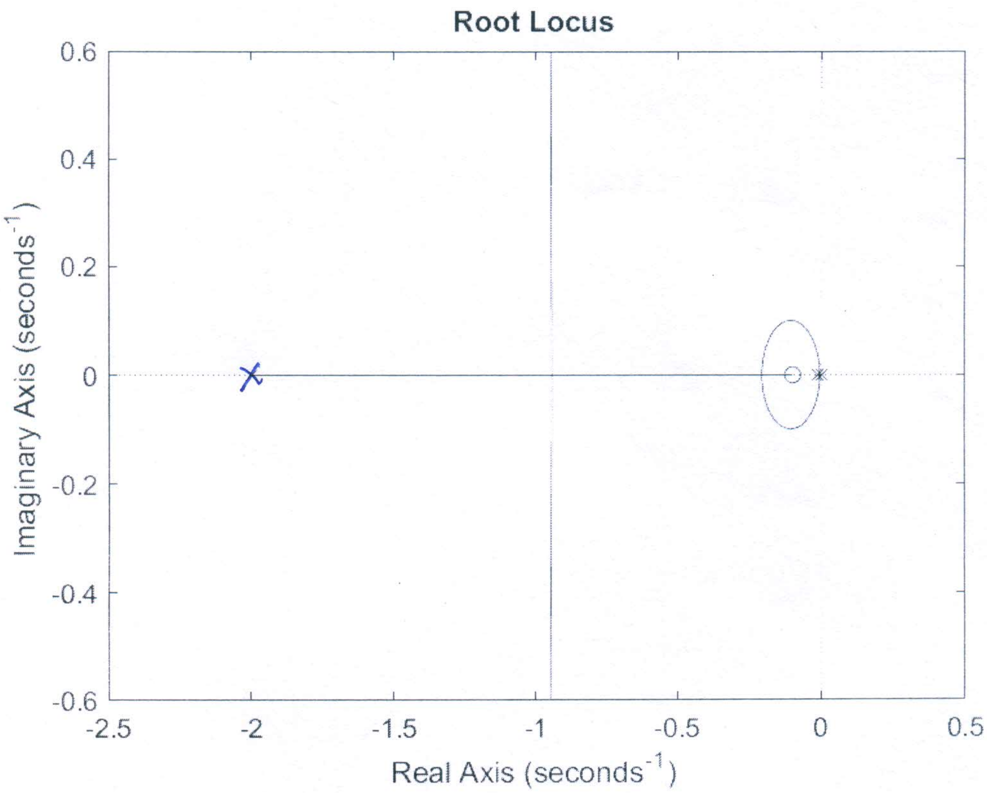
$$\frac{5(K-6)-K}{5} > 0$$

$$\therefore \boxed{K > 7.5}$$

$K > 0$

Range $0 < K > 7.5$ or

Q4 Root locy



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