Dynamic Vibration Absorber

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Dynamic Vibration Absorber (DVA)

Basic Concept

Attaching a secondary mass to a primary vibrating system such that:-

- Secondary mass dissipates the energy
- Thus reduce the amplitude of vibration of the primary system

There are many application of DVA, A few are noted below:

✓ Vibration control of transmission cables

✓ Control of torsional oscillation of crankshaft

✓ Control of rolling motion of ships

✓ Chatter control of cutting tools

✓ Control of noise in aircraft cabin

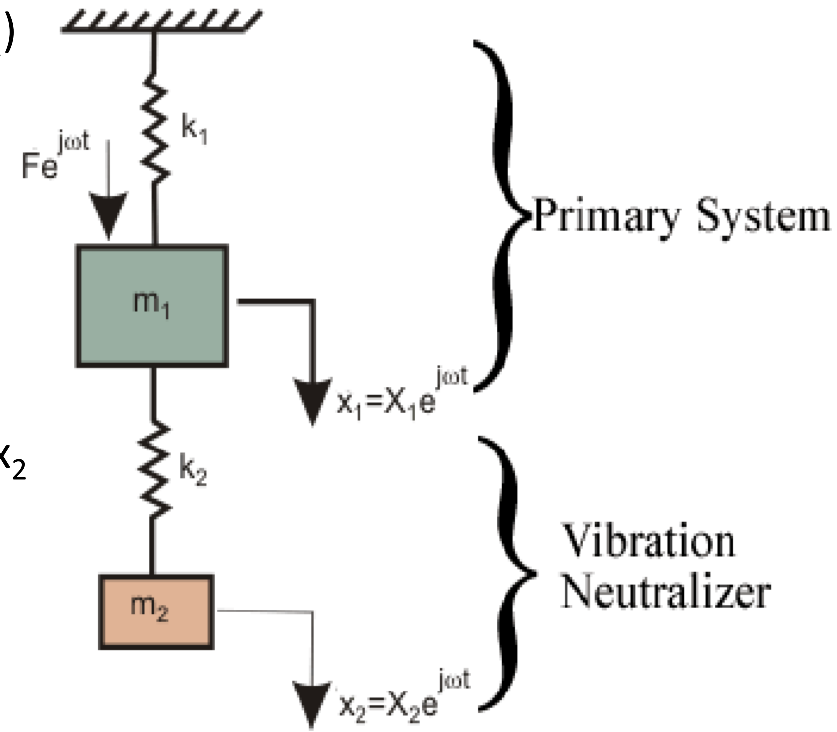
✓ Vibration control of hand held devices

DVAs are generally of three types:-

- **Vibration Neutralizer**: Here the secondary mass is connected to the primary system using a **spring element**.
- **Auxiliary Mass Damper**: Here the secondary mass is connected to the primary system by a **damper/dashpot**.
- **Dynamic Vibration Absorber**: **A general case** where **both spring and damper** are used to connect the secondary mass with the primary system.

Model of a Simple Vibration Neutralizer

Primary System: Assuming undamped SDOF (m_1, k_1)
Secondary System: Neutralizer (m_2, k_2)



The primary system is subjected to a harmonic excitation as $Fe^{j\omega t}$.

The responses of the masses from their respective equilibrium positions are denoted as x_1 for m_1 and x_2 for m_2.

The equations of motion for the masses can be written as

$$m_1\ddot{x}_1 + k_1x_1 + k_2(x_1 - x_2) = Fe^{j\omega t} \qquad (1)$$

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) = 0 \qquad (2)$$

In the steady state, the solutions of the governing equations are assumed to be $x_1 = X_1 e^{j\omega t}$ *and* $x_2 = X_2 e^{j\omega t}$.

Substituting these in equations of motion, we get

$$(k_1 + k_2 - m_1\omega^2)X_1 - k_2 X_2 = F \qquad (3)$$

$$-k_2 X_1 + (k_2 - m_2\omega^2)X_2 = 0 \qquad (4)$$

Solving eqns. (3) and (4), we obtain

$$X_1 = \frac{F(k_2 - m_2\omega^2)}{(k_1+k_2-m_1\omega^2)(k_2-m_2\omega^2)-k_2^2} \qquad (5)$$

$$X_2 = \frac{Fk_2}{(k_1+k_2-m_1\omega^2)(k_2-m_2\omega^2)-k_2^2} \qquad (6)$$

Now, if the secondary system is tuned to the excitation frequency, i.e., its natural frequency $\omega_2 = \sqrt{\frac{k_2}{m_2}}$ is made equal to ω, then X_1=0 and $X_2 = \frac{-F}{k_2}$

This implies that the primary system comes to rest, after tuning.

Design Considerations

- It should be noted that a tuned neutralizer makes the response of the primary system zero only at one frequency, namely, ω_2. So, the **application** of such a neutralizer is **very much limited**.

- Even though the tuned mass damping system could successfully neutralize the vibration response of the primary system when the excitation frequency is ω_2 ; it also **introduces two new resonating frequencies** to the original system. Hence, care should be taken such that the two new frequencies are **kept sufficiently away** from the **expected excitation frequency**.

- From equation (6), one may note that at $\omega = \omega_2$, displacement of the secondary mass, $x_2 \neq 0$. In fact in many system, there is a constraint on maximum permissible value of x_2. This is known as **rattle space**.

$$X_2 = \frac{Fk_2}{(k_1+k_2-m_1\omega^2)(k_2-m_2\omega^2)-k_2^2}$$

$$\omega_2 = \sqrt{\frac{k_2}{m_2}}$$

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Application
of
Dynamic Vibration
Absorption

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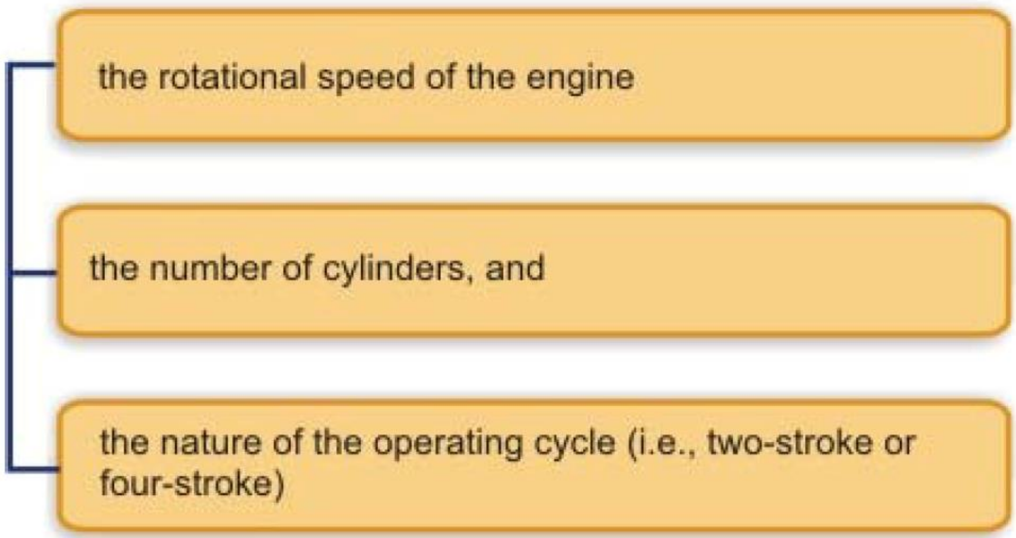
Self-Tuned Pendulum Neutralizer

Self tunable neutralizers are attractive for applications where there is a **possibility of change of excitation frequency.**

It is known that in a multi-cylinder engine, the time period of the variation of the turning moment depends on the following parameters:



This dynamic turning moment gives rise to torsional oscillations of the crank shaft

Self Tuning

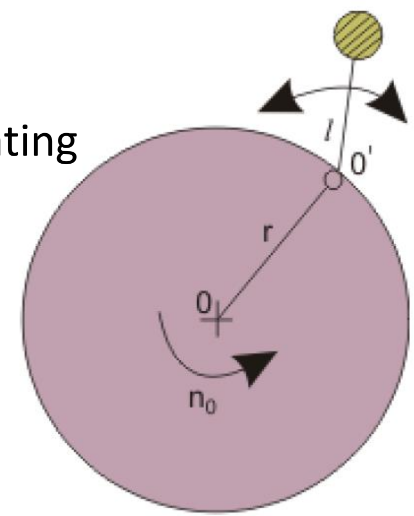
The most dominant or the primary exciting frequency of a multi-cylinder engine is given by,

$$\omega = \alpha N n_0 \qquad (1)$$

Where, α=1 for two stroke cycle and ½ for a four stroke cycle
N= number of cylinders
n_0= rotational speed of the engine, neglecting all the higher harmonics of the system.

Now, the natural frequency of a gravity pendulum of length "l" is $\sqrt{\frac{g}{l}}$, where g is the acceleration due to gravity.

If the pendulum moves in a horizontal plane with its hinge point O' rotating at a speed n_0 with radius r, then the gravity field (g) is replaced by the centrifugal field ($n_0^2 r$).



The natural frequency (ω_n) of this centrifugal pendulum becomes

$$\omega_n = n_0\sqrt{\frac{r}{l}} \qquad (2)$$

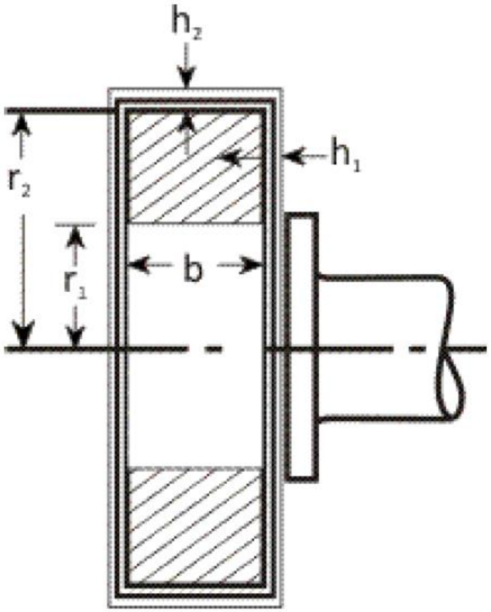
Comparing Eqns. (1) and (2), we see that the necessary condition for self-tuning is

$$\boxed{\frac{r}{l} = (\alpha N)^2}$$

Tuned mass dampers are largely used in vibration control of crankshafts, hand-held devices and transmission cables.

Houdaille Damper - an auxiliary mass damper

In this type of dynamic vibration absorber, a flywheel (as secondary mass) is coupled to the primary crankshaft with fluids.



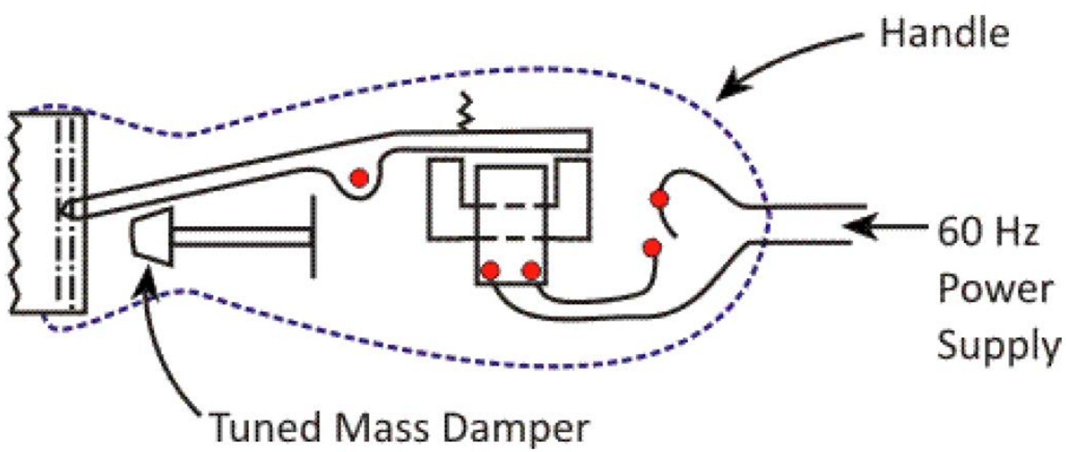
The damping constant is given by:

$$c = 2\pi\mu\left[\frac{r_2^3 b}{h_2} + \frac{1}{2}\frac{r_2^4 - r_1^4}{h_1}\right]$$

where, μ is the viscosity of the fluid

Damping of hand-held devices

Electromagnetic Motors are used extensively to power hand held devices such as Hair clipper, Dry Shaver and similar instruments. Usually, the motors operate at a fixed frequency such as 60 Hz.

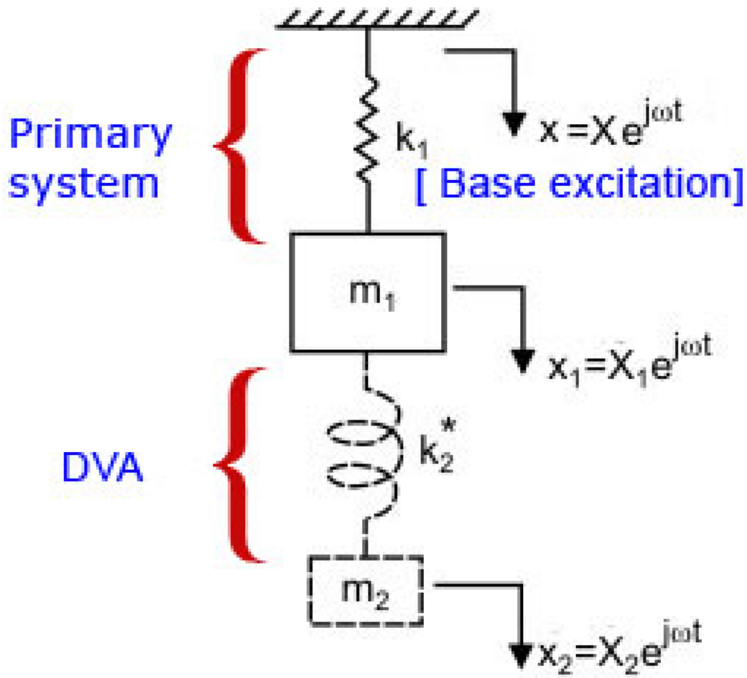


Electric Hair clipper

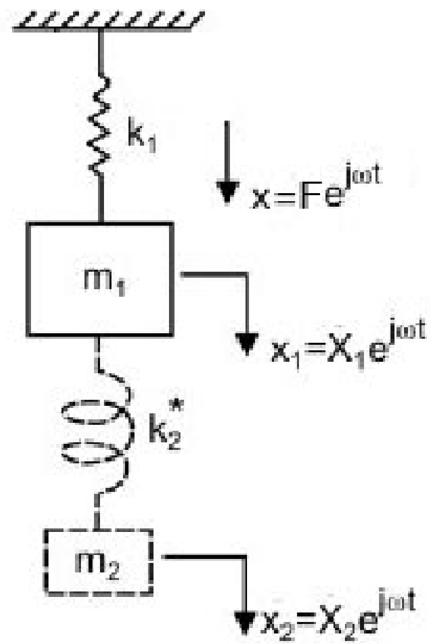
In such hair-clippers, an electro-magnet is used to develop vibrating force for cutting. However, this also generates an unpleasant vibration of the housing. This vibration is neutralized by the application of a pair of mass dampers fixed to the housing at two different points.

Design of Damped DVA

Consider the motion of a SDOF system with which a dynamic vibration absorber (in the form of a lumped mass and a spring with complex stiffness) is attached. The complete system is with two variations is shown below.



Case-I: Primary system subjected to base excitation



Case-II: The primary system is directly excited

In both the cases, the excitations are purely harmonic in nature.

It is clear that the design of DVA involves suitable choice of the following parameters: -

- Inertia of the secondary mass $\mathbf{m_2}$
- Real and Imaginary parts of the complex stiffness

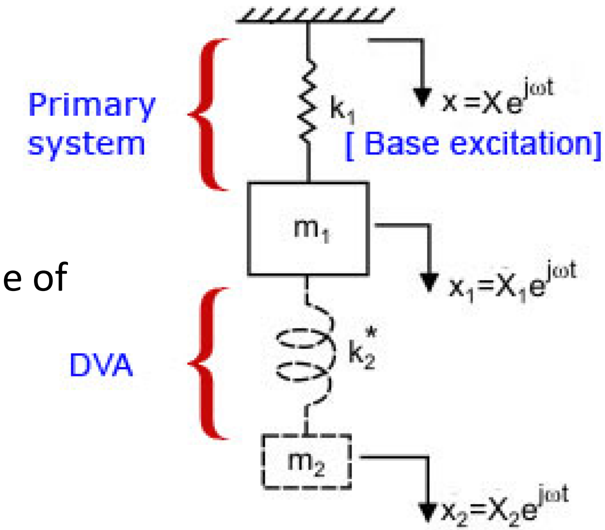
The design objective is to **minimize** the **maximum transmissibility, T_{max}.**

The Transmissibility, T may be defined as

$$\boxed{T = \left|\frac{X_1}{X}\right|}$$

where X_1 is the amplitude of vibration of m_1 and X is the amplitude of vibration of the base.

Often, we also use a different performance index known as **Relative Transmissibility** such that



$$\boxed{T_R = \left|\frac{X_2 - X_1}{X}\right|}$$

$X_2 - X_1$ indicates the difference in amplitude of vibration of m_2 and m_1

The equations of motion of the complete system may be written as

$$m_1\ddot{x}_1 + k_1(x_1 - x) + k_2^*(x_1 - x_2) = 0 \qquad (1)$$

$$m_2\ddot{x}_2 + k_2^*(x_2 - x_1) = 0 \qquad (2)$$

Let us consider the case of harmonic base excitation. Substituting, $x_1 = X_1e^{j\omega t}$, $x_2 = X_2e^{j\omega t}$ and $x = Xe^{j\omega t}$ in the above equations of motion, we get

$$(k_1 + k_2^* - m_1\omega^2)X_1 - k_2^*X_2 = k_1X \qquad (3)$$

$$(k_2^* - m_2\omega^2)X_2 = k_2^*X_1 \qquad (4)$$

Using (3) and (4), one can easily find out the transmissibility as

$$T = \left|\frac{X_1}{X}\right| = \left|\frac{k_1(k_2^* - m_2\omega^2)}{(k_1 - m_1\omega^2)(k_2^* - m_2\omega^2) - k_2^*m_2\omega^2}\right| \qquad (5)$$

Relative Transmissibility, $T_R = \left|\frac{X_2 - X_1}{X}\right| = \left|\frac{k_1m_2\omega^2}{(k_1 - m_1\omega^2)(k_2^* - m_2\omega^2) - k_2^*m_2\omega^2}\right| \qquad (6)$

Since T & T_R are basically two ratios, the RHS could be expressed in terms of the following non-dimensional parameters:

$$\boxed{Inertia\ parameter, \mu = \frac{m_2}{m_1}}$$

$$\boxed{Tuning\ ratio, \vartheta = \frac{\omega_a}{\omega_0}}$$

Where, $\omega_a = \sqrt{\frac{k_{2a}}{m_2}}$, k_{2a} = stiffness of absorber at frequency ω_a

$Locked\ frequency, \omega_o = \sqrt{\frac{k_1}{m_1+m_2}}$, which is the natural frequency of the combined System when the absorber mass is rigidly connected to m_1.

$$\boxed{Excitation\ frequency\ ratio, \boldsymbol{\Omega} = \frac{\omega}{\omega_0}}$$

$$\boxed{Loss\ factor, \eta_2 = \frac{k^*_{2i}(\omega)}{k^*_{2r}(\omega)}}$$

Subscript **r, i** denote real and imaginary parts

$$\text{Thus, } T_{non-dim} = \left| \frac{(\vartheta^2 - \boldsymbol{\Omega}^2\alpha) + j\vartheta^2\eta_{2\omega}}{\frac{\boldsymbol{\Omega}^4\alpha}{1+\mu} - \boldsymbol{\Omega}^2(\vartheta^2+\alpha) + \vartheta^2 + j\vartheta^2\eta_{2\omega}(1-\boldsymbol{\Omega}^2)} \right|$$

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