

Approach

First we calculate σ_X and σ_Y as explained in the problem. ρ is calculated using `pearsonr()` function in `scipy` library.

ϵ_1 and ϵ_2 are drawn from a bivariate normal distribution using `multivariate_normal` function in `numpy` library.

We run 10000 simulation for X and Y values. Year is discretised in 252 days. For all days X and Y are updated as $X=X+dX$ and $Y=Y+dY$. If $T_i = 0.25*i*\text{years}$, payoff is calculated and all payoffs are summed at the end of the year. Expected value of payoff is calculated over all the simulation as mean of all the payoffs. Then price of the product is calculated using the formula given.

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In [2]: r = 0.01
N = 4
X_SPOT = [2443.25,2447.83,2459.27,2459.14,2460.61,2473.83,2473.45,2472.54,24
69.91,2477.13,2477.83,2475.42,2472.1,2470.3,2476.35,2477.57,2472.16,2476.83,
2480.91,2474.92,2474.02,2438.21,2441.32,2465.84,2464.61,2468.11,2430.01,2425
.55,2428.37,2452.51,2444.04,2438.97,2443.05,2444.24,2446.3,2457.59,2471.65,2
476.55,2457.85,2457.85,2465.54,2465.1,2461.43,2488.11,2496.48,2498.37,2495.6
2,2500.23,2503.87,2506.65,2508.24,2500.6,2502.22,2496.66,2496.84,2507.04,251
0.06,2519.36,2529.12,2534.58,2537.74,2552.07,2549.33,2544.73,2550.64,2555.24
,2550.93,2553.17,2557.64,2559.36,2561.26,2562.1,2575.21,2564.98,2569.13,2557
.15,2560.4,2581.07,2572.83,2575.26,2579.36,2579.85,2587.84,2591.13,2590.64,2
594.38,2584.62,2582.3,2584.84,2578.87,2564.62,2585.64,2578.85,2582.14,2599.0
3,2597.08,2602.42,2602.42,2601.42,2627.04,2626.07,2647.58,2642.22,2639.44,26
29.57,2629.27,2636.98,2651.5,2659.99,2664.11,2662.85,2652.01,2675.81,2690.16
,2681.47,2679.25,2684.57,2683.34,2680.5,2680.5,2682.62,2687.54,2673.61,2695.
81,2695.81,2713.06,2723.99,2743.15,2747.71,2751.29,2748.23,2767.56,2786.24,2
776.42,2776.42,2802.56,2798.03,2810.3,2832.97,2839.13,2837.54,2839.25,2872.8
7,2853.53,2822.43,2823.81,2821.98,2762.13,2648.94,2695.14,2681.66,2581,2619.
55,2656,2662.94,2698.63,2731.2,2732.22,2716.26,2716.26,2701.33,2703.96,2747.
3,2779.6,2744.28,2713.83,2677.67,2691.25,2720.94,2728.12,2726.8,2738.97,2786
.57,2783.02,2765.31,2749.48,2747.33,2752.01,2712.92,2716.94,2711.93,2643.69,
2588.26,2658.55,2612.62,2605,2640.87,2581.88,2581.88,2614.45,2644.69,2662.84
,2604.47,2613.16,2656.87,2642.19,2663.99,2656.3,2677.84,2706.39,2708.64,2693
.13,2670.14,2670.29,2634.56,2639.4,2666.94,2669.91,2648.05,2654.8,2635.67,26
29.73,2663.42,2672.63,2671.92,2697.79,2723.07,2727.72,2730.13,2711.45,2722.4
6,2720.13,2712.97,2733.01,2724.44,2733.29,2727.76,2721.33,2689.86,2689.86,27
24.01,2705.27,2734.62,2746.87,2748.8,2772.35,2770.37,2779.03,2782,2786.85,27
75.63,2782.49,2779.66,2773.75,2762.59,2767.32,2749.76,2754.88,2717.07,2723.0
6,2699.63,2716.31,2718.37,2726.71,2713.22,2736.61,2736.61,2759.82,2784.17,27
93.84,2774.02,2798.29]
Y_SPOT = [35.5,35.86,36.35,36.38,36.43,36.47,36.41,36.07,35.82,35.57,35.62,3
5.94,35.77,35.98,34.76,34.82,34.77,35.27,35.3,35.39,35.28,34.88,34.93,35.47,
35.54,35.75,35.34,34.83,34.91,35.3,35.49,35.52,35.6,35.51,35.52,35.82,36.54,37.
36,37.23,37.23,37.67,36.91,37.37,37.35,37.89,38.21,38.79,38.88,38.59,38.7,38.88
,39.1,39.42,40.3,40.26,40.58,40.58,40.38,42.15,43.45,43.78,43.85,44.93,45.33
,45.21,45.47,44.89,45.88,45.76,45.02,45.12,45.35,45.61,45.15,46.48,45.12,45.
25,44.64,43.37,42.98,43.13,42.6,42.34,42.14,41.7,42.11,42.11,42.66,43.57,43,
42.86,43.6,43.88,44.88,44.97,44.29,44.46,44.46,44.17,44.92,43.81,43.09,42.79
,43.05,42.8,42.15,42.02,42.02,41.67,41.53,41.4,40.81,40.95,42.15,42.49,42.52
,42.16,42.02,41.8,41.8,41.31,41.38,40.99,41.8,41.8,42.82,44.14,44.01,44.22,4
4.05,43,44.19,44.07,44.19,44.19,44.03,43.86,43.15,43.29,43.38,44.16,43.16,43
.49,43.02,42.7,42.41,42.43,41.39,54,41.86,42.39,40.75,41.46,42,41.4,41.81,41
.85,41.09,40.77,40.77,40.56,40.91,40.91,41.54,40.17,39.35,37.79,37.43,37.74,
37.93,37.74,37.84,37.84,37.83,38.01,37.69,37.85,37.94,37.01,36.89,37.58,36.3
5,35.17,35.99,34.87,35.47,36.34,35.76,35.76,36.94,38.03,38,37.68,37.83,39.07
,39,38.83,38.73,39.17,39.22,38.93,37.77,37.61,37.69,37.93,38.11,38.25,37.65,
36.74,36.42,36.2,36.15,36.71,36.34,36.33,36.27,37.16,36.89,36.63,36.94,38.03
,38.3,37.79,38.09,38.28,37.85,38.39,38.3,37.38,37.38,37.83,42.7,43.2,43.78,4
3.41,43.93,44.01,44.25,44.85,44.18,44.45,43.57,43.91,43.95,42.26,41.95,41.12
,41.25,40.61,41.01,40.37,40.52,39.4,39.5,38.97,39.47,39.47,39.16,39.75,40.09
,39.3,39.27]

import math
import numpy as np
from scipy.stats.stats import pearsonr

x = [round(math.log(X_SPOT[i+1])-math.log(X_SPOT[i]),4) for i in range(len(X
_SPOT)-1)]
y = [round(math.log(Y_SPOT[i+1])-math.log(Y_SPOT[i]),4) for i in range(len(Y
_SPOT)-1)]

sigmaX = np.std(x)
sigmaY = np.std(y)
#Pearson correlation coefficient
rho = pearsonr(x,y)

#No. of simulations
max iter = 10000

```

0.0