Mechanical Engineering
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Are plant stems structurally optimal?

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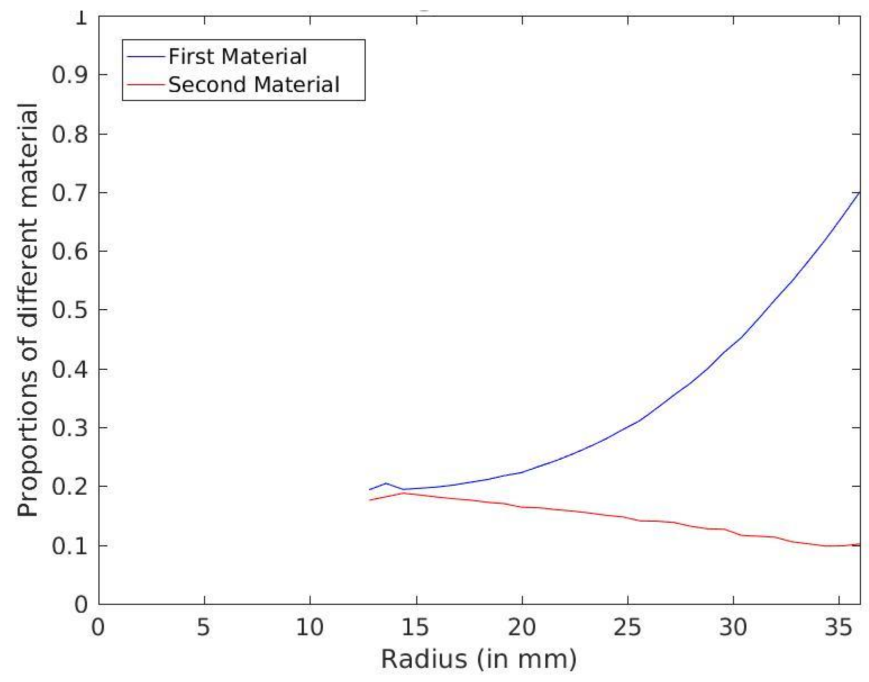
Abstract

Plants have evolved over millions of years into what they are today. Some plants are evolved to have really good structural properties. These include plants like bamboos. In this work, we take the constituent materials of bamboo to simulate material distribution and organisation of a optimal structure that can be formed from given material. Bamboos have desired structural properties like light weight, high yield strength etc. We find that the distribution of material in optimal structure is similar the distribution of raw material found in bamboos.



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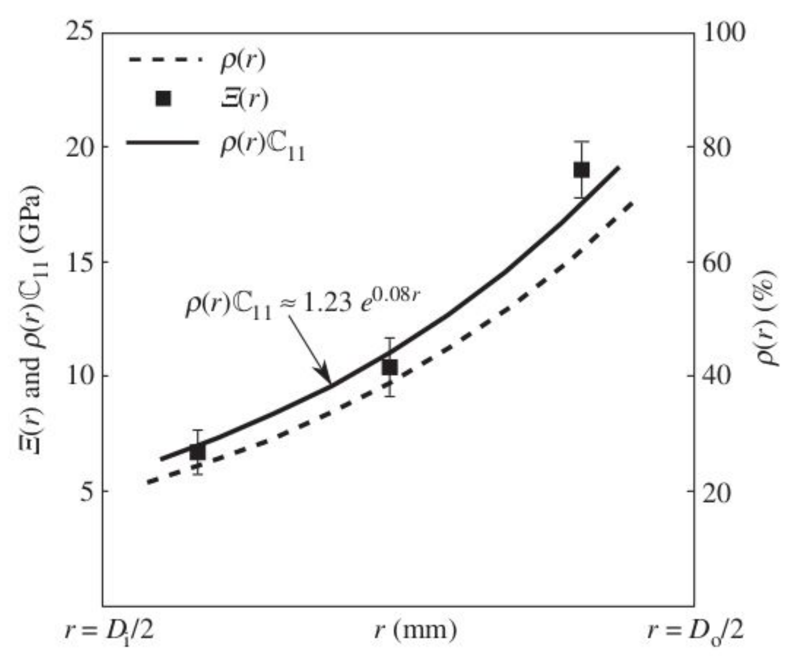


(b)

Figure 1: (a) Distribution of two material in optimal structure subject to constraints. The inner and outer radius taken as taken in [2]. (b) Actual distribution of fibres along radial direction as determined in [2].

Introduction

Plants are versatile and vital for human survival. We depend on plant not only for food and clothes but also for shelter. Due to its structural properties, wood is frequently used in construction works. At this point it is natural to ask this question, are plant stems structurally optimal? Or given the constituent material of plant stem, are plant stems the best in structural strength. We have taken bamboo as a representative of a plant stem because it is mostly used in its raw form for construction. Bending stiffness per unit density is used as measure of structural strength.

Material distribution in radial direction

Distribution of material in plant is axisymmetric. The distribution of hard and soft material (ref to in our simulations as First material and Second material resp.) is known for bamboo. The simulated and actual distributions are compared in Figure 1.

Objective function

$$max \quad \frac{\sum_{r_i}^{r_o} E(r)I(r)}{\sum_{r_i}^{r_o} \rho(r)}$$

subject to:

$$\frac{rE(r)}{R} \leq \sigma_{max} \;\; \forall\, r \in [r_i,\ r_o]$$

$$\frac{EI}{R} > M_{min}$$

Here, we are optimizing Bending stiffness per unit density subjected to maximum stress at every point is less than σ_max and minimum bending moment is greater than M_min. R is the radius of curvature which is constant taken from [1], r_i and r_o are the inner and outer radius of the bamboo.

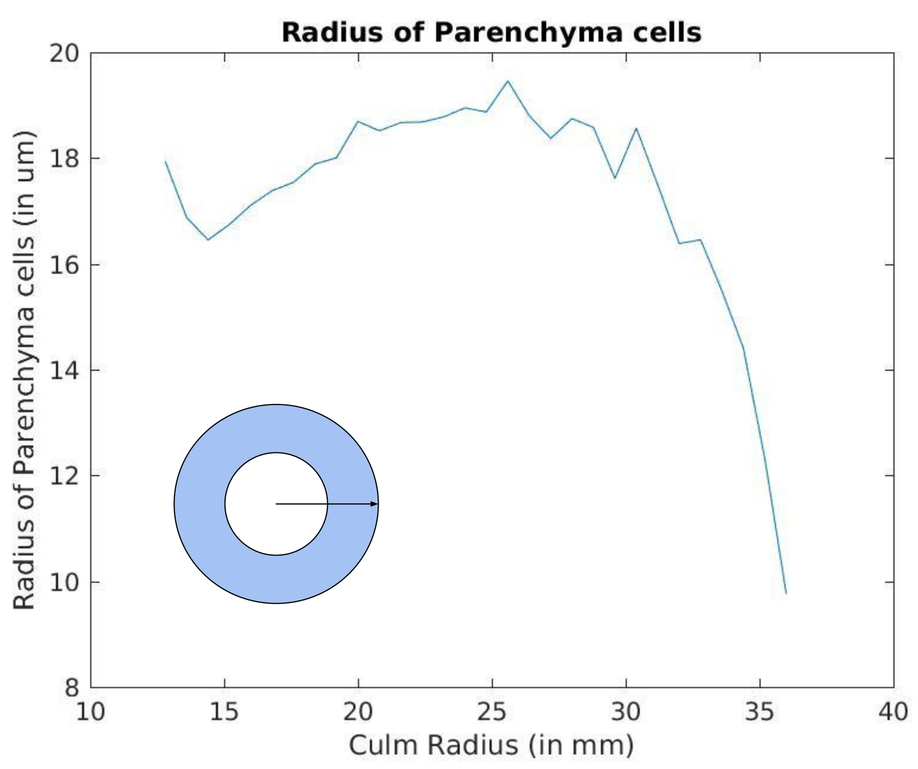
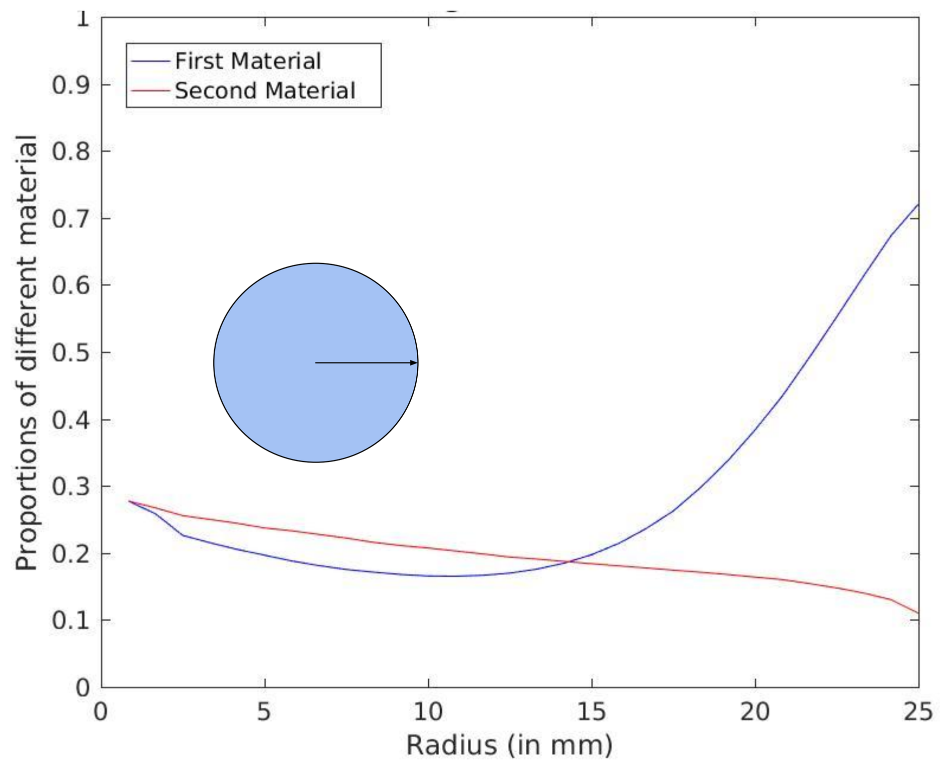


Figure 2: Variation of Parenchyma cells size [simulated]

The parenchyma cell size is calculated from proportion of fibre and parenchyma and the fact that cellular content in parenchyma cell is constant only the air content is changing in radial direction. The actual variation of cell size can be seen in Figure 3 in [2].



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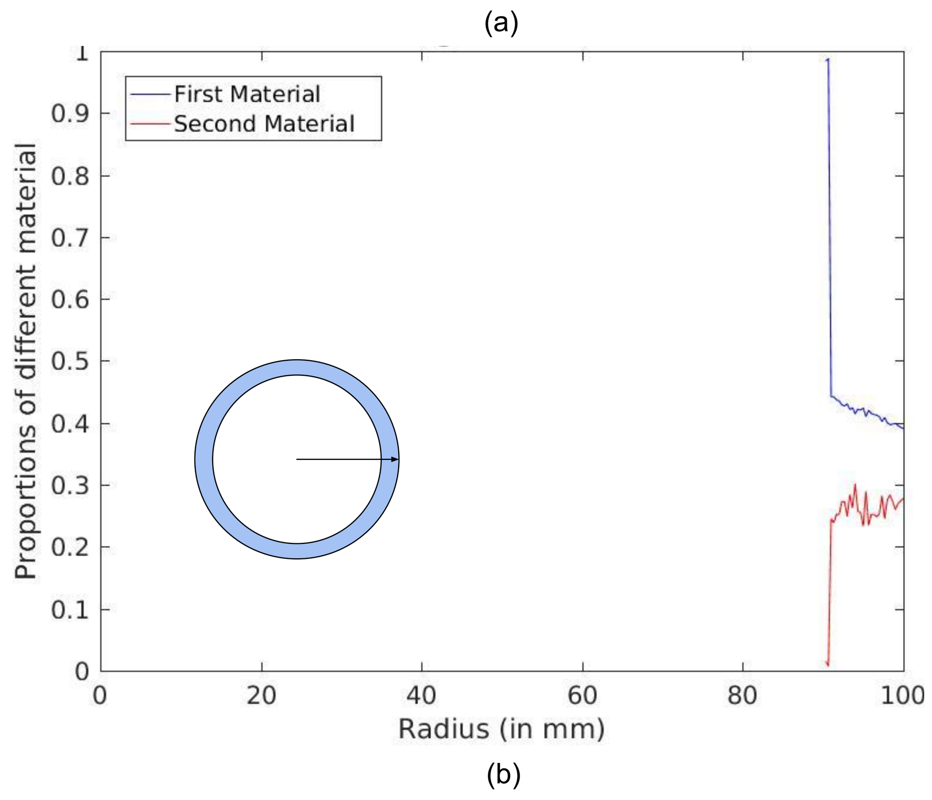

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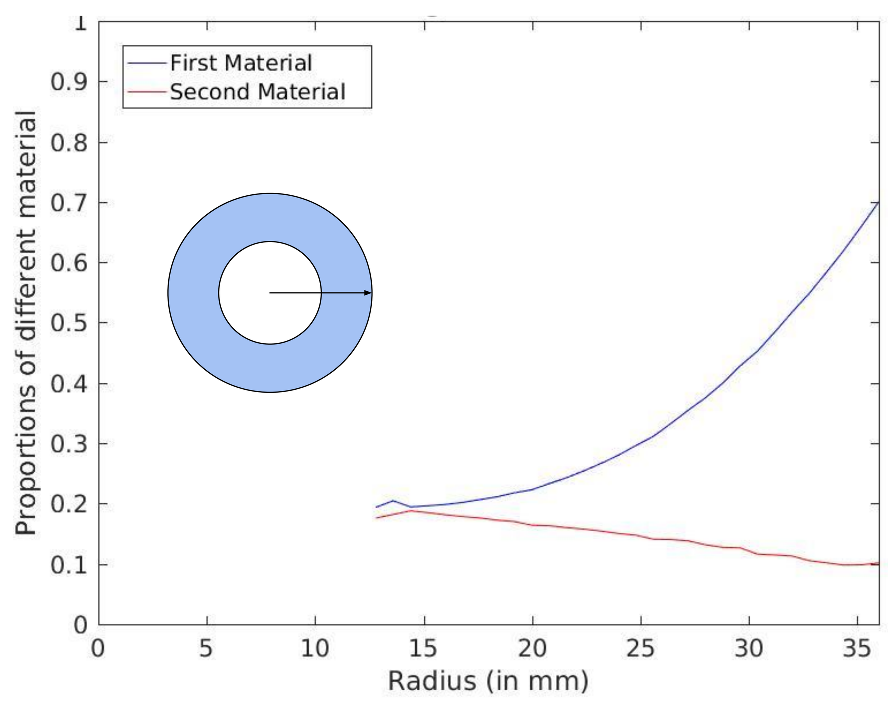

(c)

Figure 3: (a) Show the distribution of two material in completely solid bamboos. (b) Shows the distribution of two material in thin walled large diameter bamboo. (c) Show the distribution for inner and outer radius as used in [1].

Topology Optimization

The organisation of material into hierarchical is obtained through topology optimization. The objective to minimize the ratio of density to Young's modulus.

$$min: \quad \frac{\sum_i \sum_j (x_{1ij}\rho_1 + x_{2ij}\rho_2)}{\sum_i \sum_j (x_{1ij}E_1 + x_{2ij}E_2)}$$

$$subject\ to:$$

$$KU = F$$

$$max(\delta_{ij}) < \delta_{max}$$

$$max(\sigma_{ij}) < \sigma_{max}$$

$$0 \leq x_{min} \leq x_{1ij} \leq 1 \forall i,j$$

$$0 \leq x_{min} \leq x_{2ij} \leq 1 \forall i,j$$

Here we are modelling a very small portion of bamboo such that the force is acting in only one direction. Also, since the dimensions modelled domain are small, Second moment of Inertia I does not very much and bending stiffness can be replaced with Young's modulus. Finite element domain is diagrammatically represented in Figure 4.

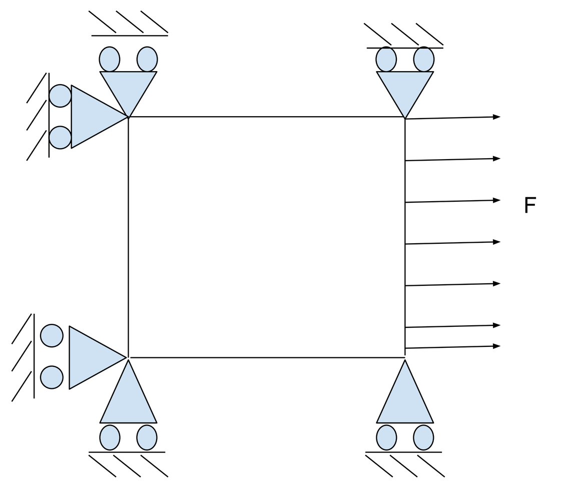

Figure 4: Finite element domain

Conclusion

The radial distribution of two material for a optimum structure is very similar to that of actual bamboo. Also, the parenchyma cell size variation indicate resemblance. Therefore, the material is optimally distributed in bamboo from strength's perspective. But only after the results of topology optimization we can say whether bamboo has optimal structure or not.

References

[1] Mannan, Sayyad, Venkitanarayanan Parameswaran, and Sumit Basu. "Stiffness and toughness gradation of bamboo from a damage tolerance perspective." *International Journal of Solids and Structures* 143 (2018): 274-286.
[2] Mannan, Sayyad, J. Paul Knox, and Sumit Basu. "Correlations between axial stiffness and microstructure of a species of bamboo." *Royal Society open science* 4.1 (2017): 160412.