ME352 MINI-PROJECT

Free vibration of Beam as SDOF System

Boundary Condition (BC) – Fixed-Fixed

CALCULATOR

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Welcome the world of observing and learning vibrations!

This work of ours is a small contribution to let you learn a basic concept of the Free Vibration of Fixed - Fixed beam.

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THEORY:

Introduction

A system is said to be a Fixed-Fixed beam system if it has a fixed connection at both its ends.

Vibration analysis of a fixed-fixed beam system is important as it can explain and help us analyze a number of real-life systems. The following few real systems can be simplified to a fixed-fixed beam, thereby helping us make design changes accordingly for the most efficient systems.

Few examples are shown below,

Fixed Support – Beam Fixed in Wall

Suspension System

Natural Frequency of Fixed-Fixed Beam

When given an excitation and left to vibrate on its own, the frequency at which a Fixed-Fixed beam will oscillate is its natural frequency. This condition is called Free vibration. The value of natural frequency depends only on system parameters of mass and stiffness. When a real system is approximated to a Fixed-Fixed beam, some assumptions are made for modelling and analysis (Important assumptions for undamped system are given below):

- The mass (mm) of the whole system is considered to be lumped at the middle of the beam
- No energy consuming element (damping) is present in the system i.e. undamped vibration
- The complex cross section and type of material of the real system has been simplified to equate to a Fixed-Fixed beam system

The ***governing equation*** for such a system (spring mass system without damping under free vibration) is as below:

$$m\ddot{x} + kx = 0$$

$$\ddot{x} + \omega_n^2 x = 0$$

k , the stiffness of the system is a property which depends on the length (L), moment of inertia (I) and Young's Modulus (E) of the material of the beam and for a Fixed-Fixed beam is given by:

$$k = \frac{192EI}{l^3}$$

Damping in a Fixed-Fixed Beam

Although there is no visible damper (dashpot) the real system has some amount of damping present in it. When a system with damping undergoes free vibration the damping property must also be considered for the modelling and analysis.

Single degree of freedom mass spring damper system under free vibration is governed by the following differential equation:

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

c is the damping present in the system and ζ is the damping factor of the system which is nothing but ratio of damping and critical damping. Critical damping can be seen as the damping just sufficient to avoid oscillations. At critical condition ζ=1. For real systems the value of ζ is less than 1. For system where ζ<1 the differential equation solution is a pair of complex conjugates. The displacement solution is given by

$$x(t) = e^{-\zeta\omega_n t}\left[x_0 cos(\omega_d t) + \frac{v_0 + \zeta\omega_n x_0}{\omega_d} sin(\omega_d t)\right]$$

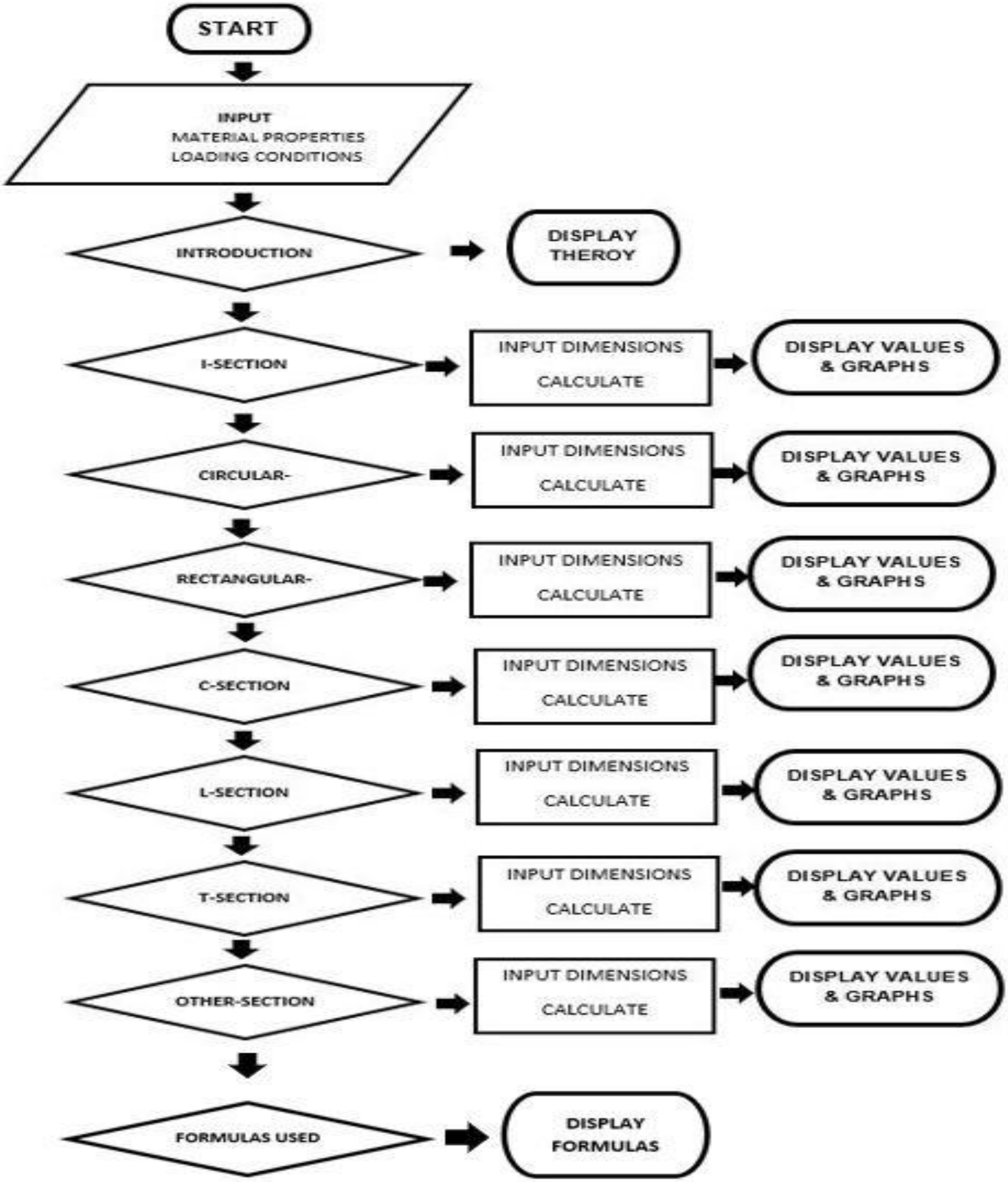
The damped natural frequency is calculated as below:

$$\omega_d = \omega_n\sqrt{1 - \zeta^2}$$

IMPORTANT FORMULAE:

Parameter	Equation
Natural angular frequency (w_n)[rad/s]	$w_n = \sqrt{\frac{k}{m}}$
Natural frequency (f_n) [Hz]	$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
Period of oscillations (T) [s]	$T = \frac{1}{f_n}$
Critical damping (c_c)	$c_c = 2mw_n$
Damping factor (c)	$c = \zeta c_c$
Damped natural frequency (w_d)	$w_d = w_n\sqrt{1-\zeta^2}$
Quality factor (Q)	$Q = \frac{1}{2\zeta}$
Logarithmic decrement	$\frac{2*\Pi}{\sqrt{(\frac{1}{\zeta^2}-1)}}$

FLOWCHART: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DEFAULT AND 3 INPUT VALUES:

For I section;

- **Default values:**

INPUT:

material = aluminum
length = 1m
damping ratio of 0.1
central mass = 0
initial velocity of 1m/s
initial displacement 0.001m
Breadth = 1cm
height = 1cm
thickness1 = 1mm
thickness2 = 1mm

OUTPUT:

Area of cross section = 2.9e-05 m^2
area moment of inertia = 4.483e-10 m^4,
stiffness = 6068 N/m
Circular natural frequency = 483.8 rad/s
Natural frequency = 73.82Hz
Period of oscillations = 0.01355 s
Critical damping = 26.16 Ns/m
Viscous damping coefficient = 2.616 Ns/m
Damped natural frequency = 481.5 rad/s
Damped natural frequency = 73.45 Hz
Logarithmic decrement = 0.6315
Quality Factor = 5

- **Value 1:**

INPUT:

material = steel
length = 2m
damping ratio of 0.2
central mass = 10 kg
initial velocity of 1m/s
initial displacement 0.002m
Breadth = 3cm
height = 3cm
thickness1 = 2mm
thickness2 = 2mm

OUTPUT:

Area of cross section = 0.000172 m^2

area moment of inertia = 2.649e-8 m^4,
stiffness = 6068 N/m
Circular natural frequency = 107.8 rad/s
Natural frequency = 17.12Hz
Period of oscillations = 0.05842 s
Critical damping = 2364 Ns/m
Viscous damping coefficient = 472.8 Ns/m
Damped natural frequency = 105.4 rad/s
Damped natural frequency = 16.77 Hz
Logarithmic decrement = 1.283
Quality Factor = 2.5

- ## Value 2:

INPUT:

material = steel
length = 3 ft
damping ratio = 0.9
central mass = 10 gram
initial velocity of 3 m/s
initial displacement 0.002 m
Breadth = 3 mm
height = 3 mm
thickness1 = 1 mm
thickness2 = 1 mm

OUTPUT:

Area of cross section = 0.000007m^2
area moment of inertia = 6.583e-12m^4,
stiffness = 330.6 N/m
Circular natural frequency = 107.9 rad/s
Natural frequency = 17.17Hz
Period of oscillations = 0.05826 s
Critical damping = 6.131 Ns/m
Viscous damping coefficient = 5.518 Ns/m
Damped natural frequency = 47.01 rad/s
Damped natural frequency = 7.482 Hz
Logarithmic decrement = 12.97
Quality Factor = 0.5556

- ## Value 3:

INPUT:

material = bronze
length = 300 cm
damping ratio = 0.01
central mass = 10 lb
initial velocity of 3 m/s
initial displacement 0.005 m

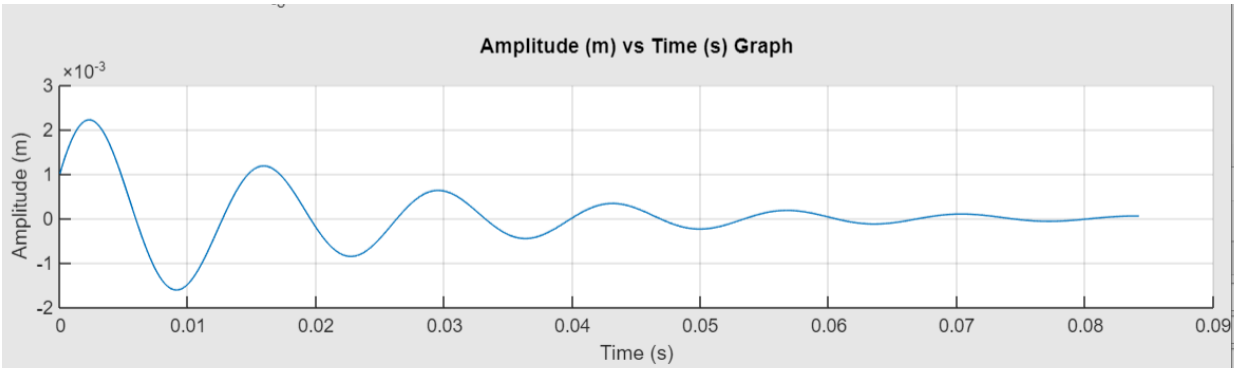
Breadth = 0.3 ft
height = 0.3 ft
thickness1 = 1 mm
thickness2 = 0.001 m

OUTPUT:

Area of cross section = 0.00018388m^2
area moment of inertia = 1.982e-10 m^4,
stiffness = 156.5 N/m
Circular natural frequency = 5.008 rad/s
Natural frequency = 0.7971 Hz
Period of oscillations = 1.255 s
Critical damping = 62.48 Ns/m
Viscous damping coefficient = 0.6248 Ns/m
Damped natural frequency = 5.008 rad/s
Damped natural frequency = 0.7971 Hz
Logarithmic decrement = 0.06283
Quality Factor = 50

GRAPHING AND ANIMATION:

GRAPHING



MATLAB has many tools which allows us to create plots from the given dataset. We made use of the *plot(X,Y)* function which allows us to plot the Amplitude vs Time graph.

The equation for the plot was taken as:

1. Underdamped

$$x(t) = \underbrace{e^{-\zeta\omega_n t}}_{\text{Exponentially decay}} \underbrace{\left[x_0 \cos(\omega_d t) + \frac{v_0 + \zeta\omega_n x_0}{\omega_d} \sin(\omega_d t)\right]}_{\text{Periodic motion}}$$

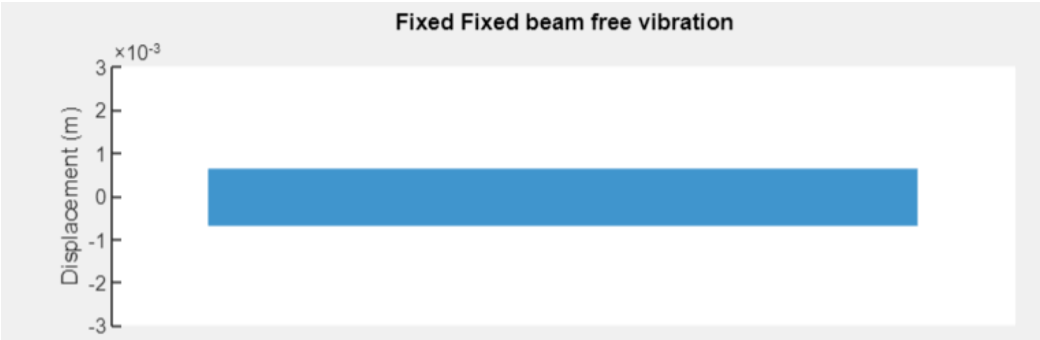
2. Critically damped

$$x(t) = (c_1 + c_2 t)e^{-\omega_n t}$$
$$\Rightarrow x(t) = e^{-\omega_n t}\left[x_0 + (v_0 + \omega_n x_0)t\right]$$

3. Over damped

$$x(t) = c_1 e^{\left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n t} + c_2 e^{\left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n t}$$
$$\Rightarrow x(t) = \frac{x_0\omega_n\left(\zeta + \sqrt{\zeta^2 - 1}\right) + v_0}{2\omega_n\sqrt{\zeta^2 - 1}} e^{\left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n t} +$$
$$\frac{-x_0\omega_n\left(\zeta - \sqrt{\zeta^2 - 1}\right) - v_0}{2\omega_n\sqrt{\zeta^2 - 1}} e^{\left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n t}$$

ANIMATION



Since MATLAB does not support custom animation generation, we innovated the animation by creating a graph that responds to the given amplitude of vibration of the given system.

It was achieved by creating a graph of a *first harmonic standing wave* whose limits were chosen so that they looked similar to a beam which is vibrating under applied conditions. The amplitude of vibration of the animation varied with the amplitude shown in the Amplitude vs Time graph.

Here is a code excerpt that contains the animation code:

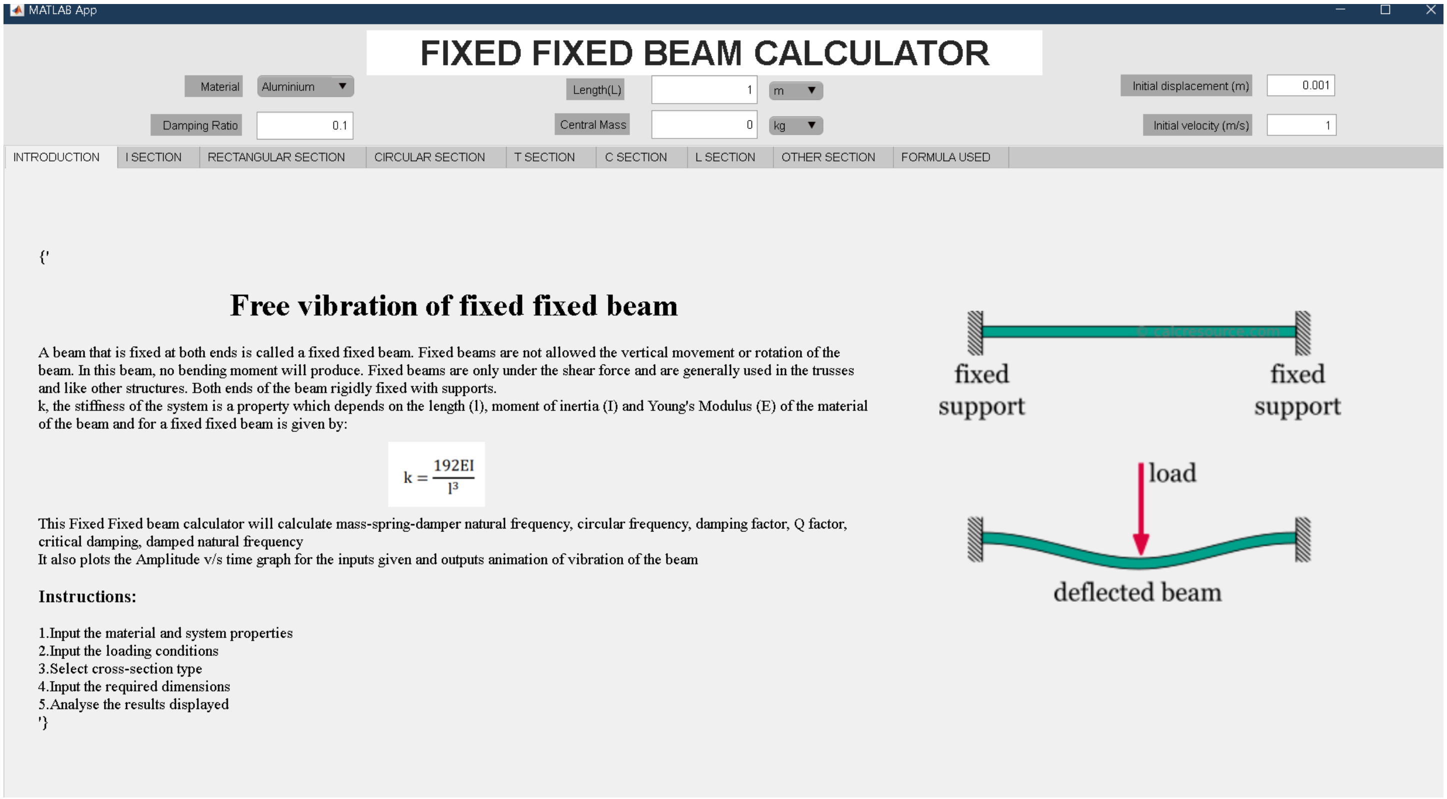
```
for j=1:incre:length(x2)
       x3(end+1)=x2(j);
 end

  x3(end+1)=0;

 x=-(pi*0.5):0.005:(pi*0.5);
 for i=1:length(x30029
     y=cos(x)*x3(i);

     plot(app.UIAxes2_2,x,y,'|','MarkerSize',25);
     app.UIAxes2_2.YLim = [-(max(x3)) (max(x3))];
     pause(0.001);

 end
```

Here, the code line *y=cos(x)*x3(i)* forms the basis of animation as it forms the underlying equation for plotting the animation. And then we make use of the for loop to vary the amplitude of the equation so that it directly represents how a beam would react subjected to the conditions taken in the calculator.

CODE TEST RUNS:

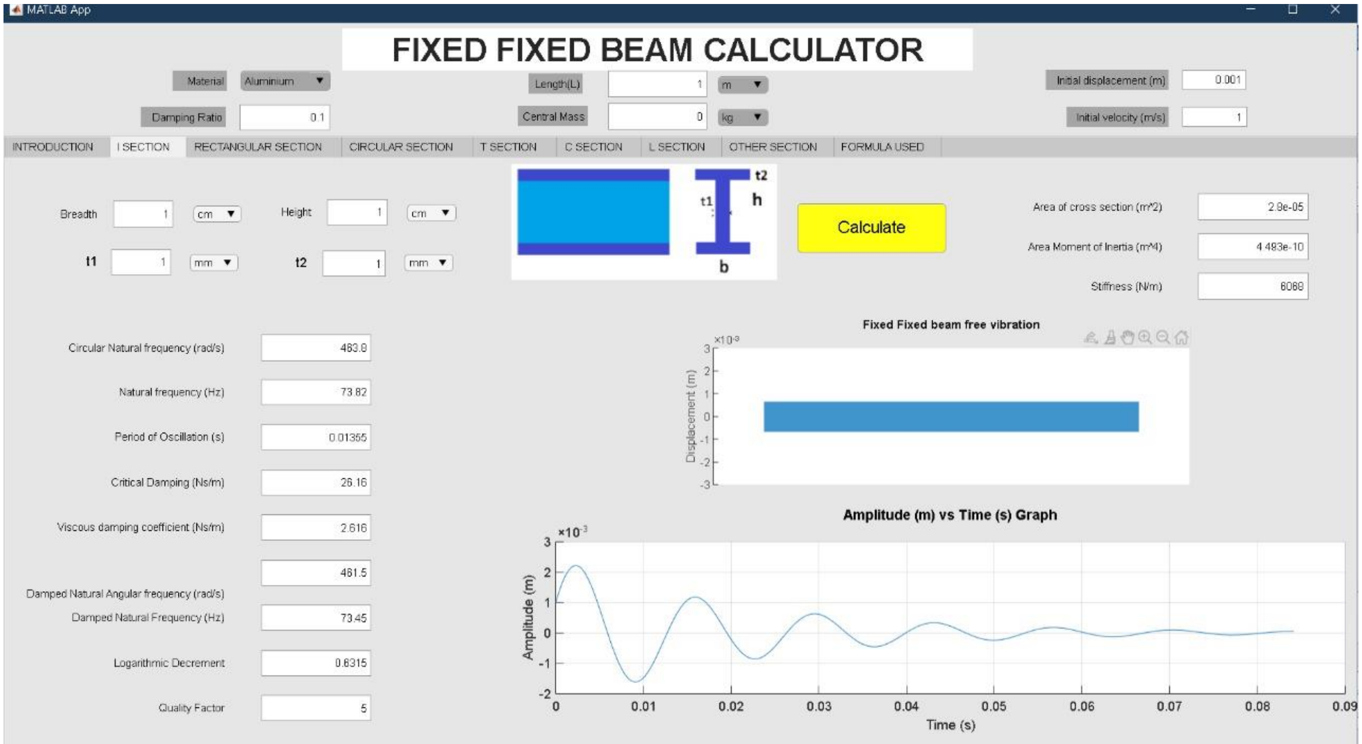
HOMEPAGE

Here is a screenshot of the homepage of our project. We have made a short and succinct explanation of the project before heading into the actual calculator.



And following are some snapshots of the calculated results along with animation for different cross sections of the type, fixed – fixed beam.

❖ *I SECTION*



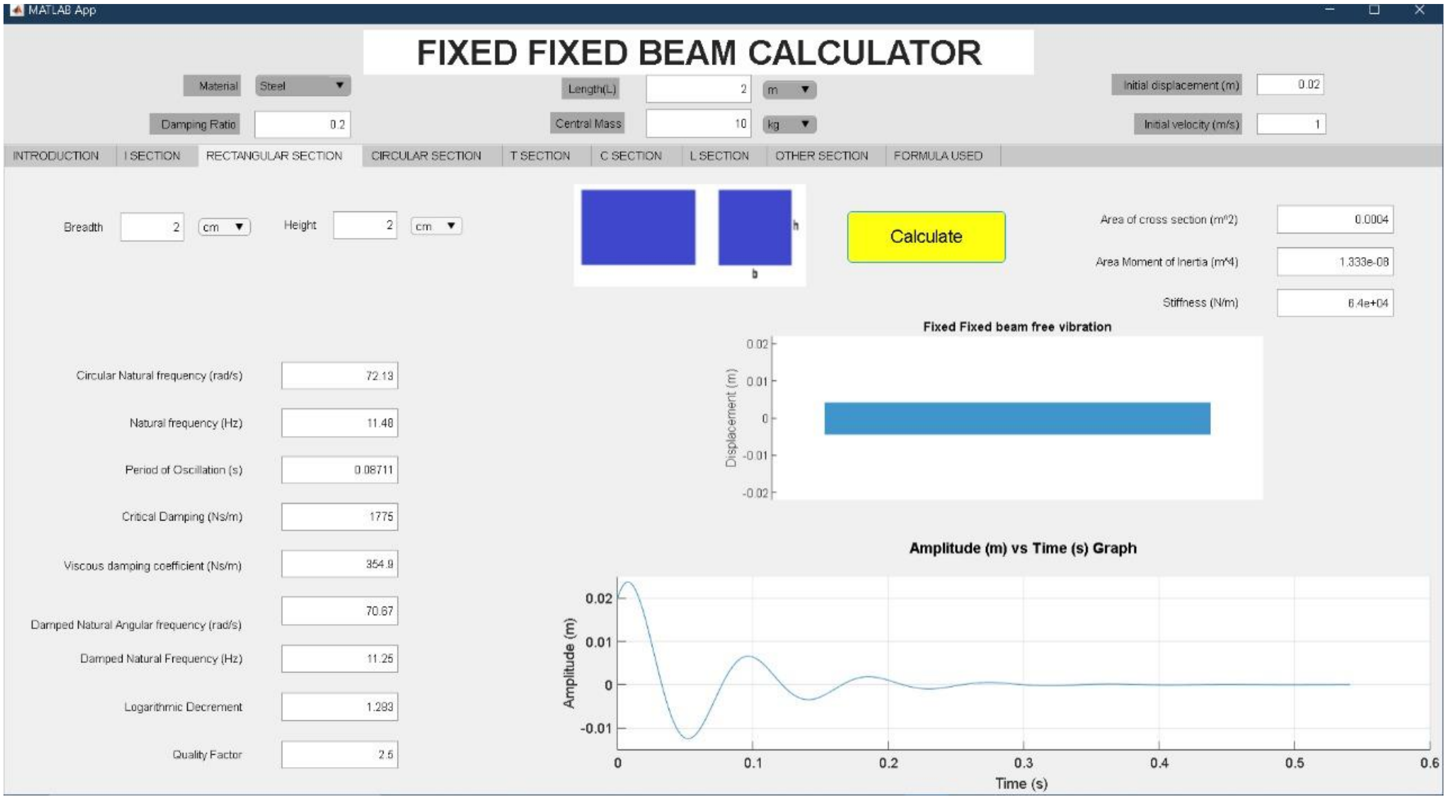
All the values to input are taken by default by the system and in the remaining sections some values were fed prior to the start.

Default values:

By default, the system takes the material aluminum having **length 1m** with an **initial displacement 0.001m** and **damping ratio of 0.1** with **no central mass** and having an **initial velocity of 1m/s**. Breadth = 1cm, height = 1cm, thickness1 = 1mm, thickness2 = 1mm. The following results have been printed.

Area of cross section = 2.9e-05 m^2, area moment of inertia = 4.483e-10 m^4, stiffness = 6068 N/m
Circular natural frequency = 483.8 rad/s
Natural frequency = 73.82Hz
Period of oscillations = 0.01355 s
Critical damping = 26.16 Ns/m
Viscous damping coefficient = 2.616 Ns/m
Damped natural frequency = 481.5 rad/s
Damped natural frequency = 73.45 Hz
Logarithmic decrement = 0.6315
Quality Factor = 5
Animation is given out and respective graph is plotted

❖ *RECTANGULAR SECTION*

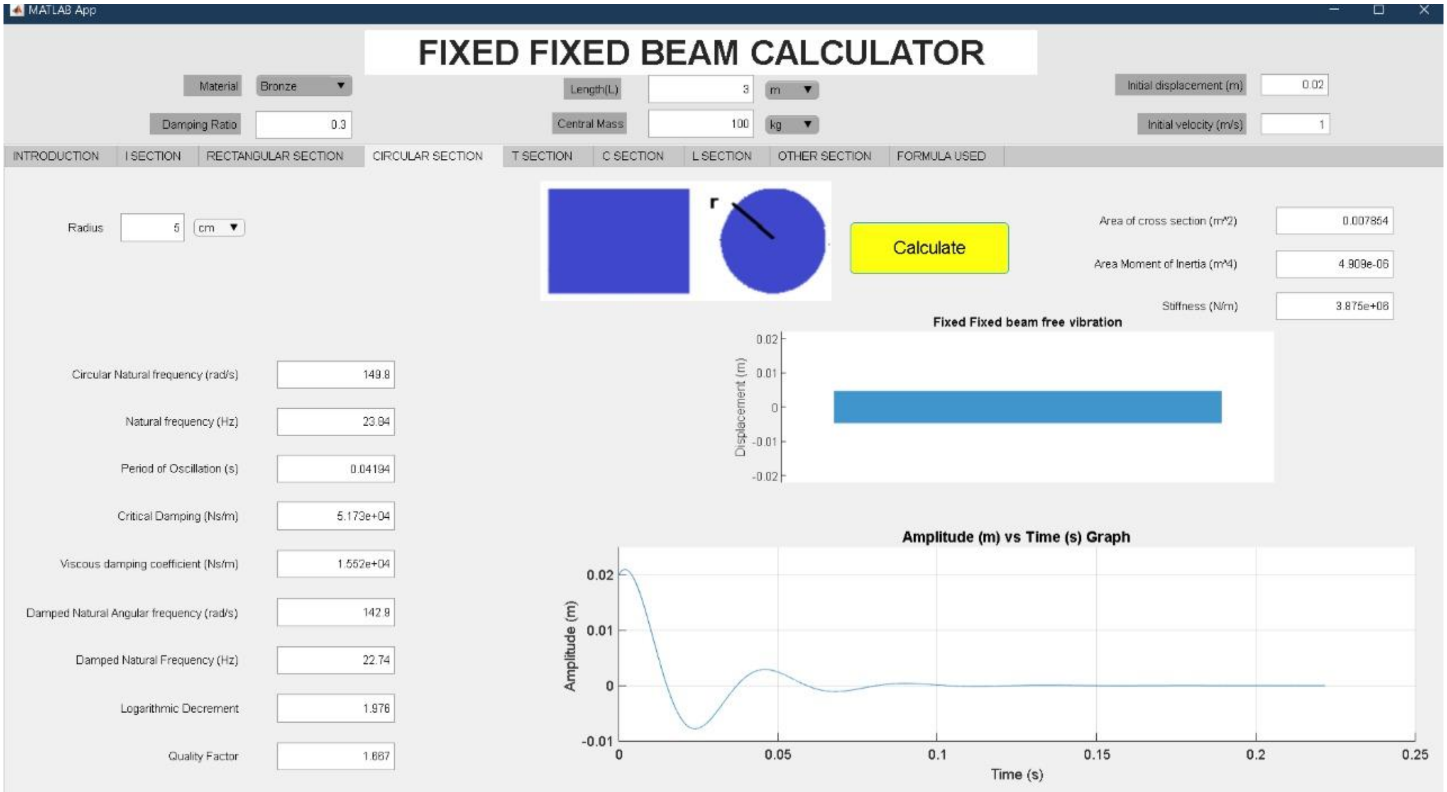


Here, the user needs to input the values and some input data was fed beforehand which it would calculate and show and of course the values can be modified as per will.

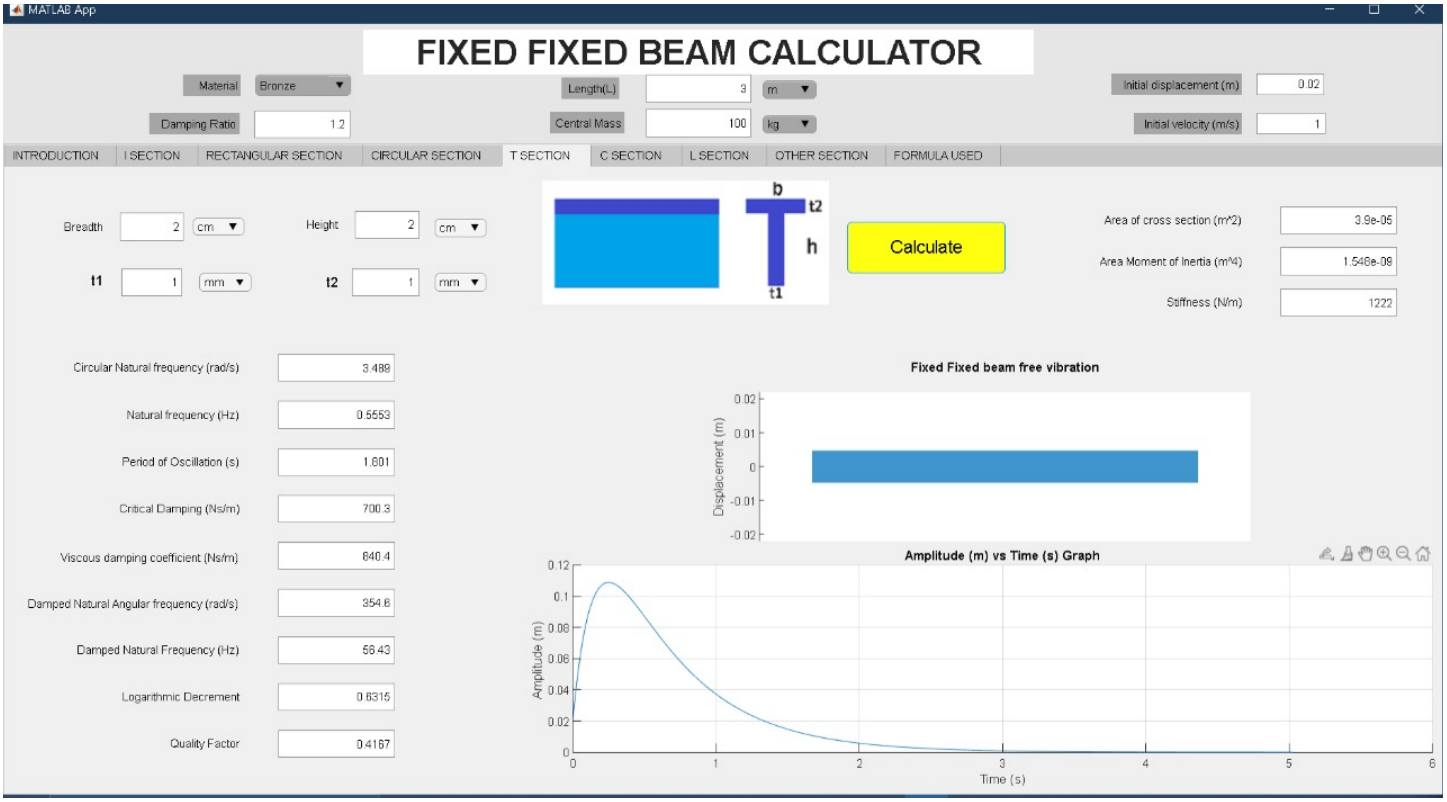
A steel beam of **length 2m** with an **initial displacement 20mm** and **damping ratio of 0.2** with **10kg central mass** and having an **initial velocity of 1m/s** is chosen, breadth = 2 cm, height = 2 cm and the following results were printed.

Circular natural frequency = 72.13 rad/s
Natural frequency = 11.48 Hz
Period of oscillations = 0.08711 s
Critical damping = 1775 Ns/m
Viscous damping coefficient = 354.9 Ns/m
Damped natural frequency = 70.67 rad/s
Damped natural frequency = 11.25 Hz
Logarithmic decrement = 1.283
Quality Factor = 2.5
Animation is given out and respective graph is plotted

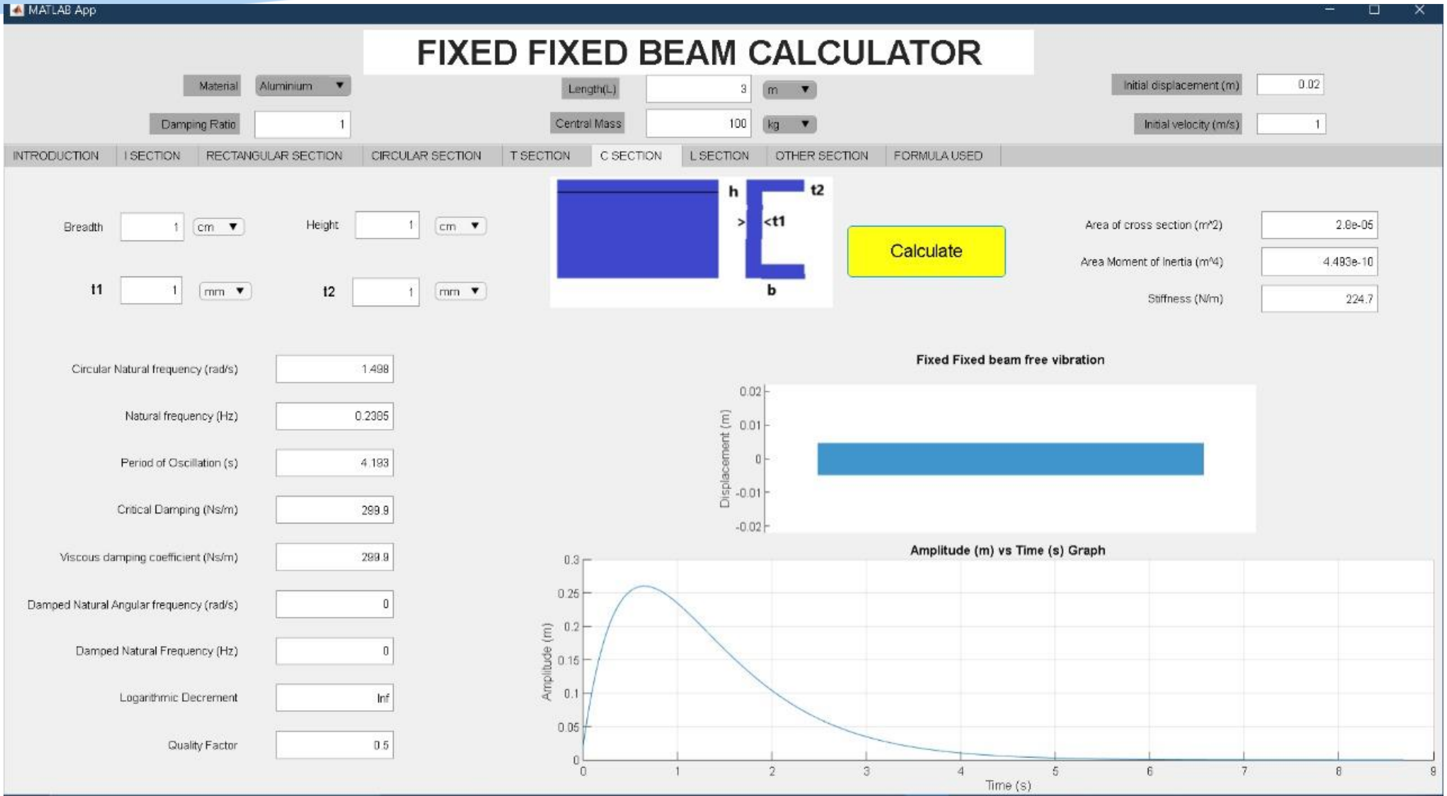
❖ CIRCULAR SECTION



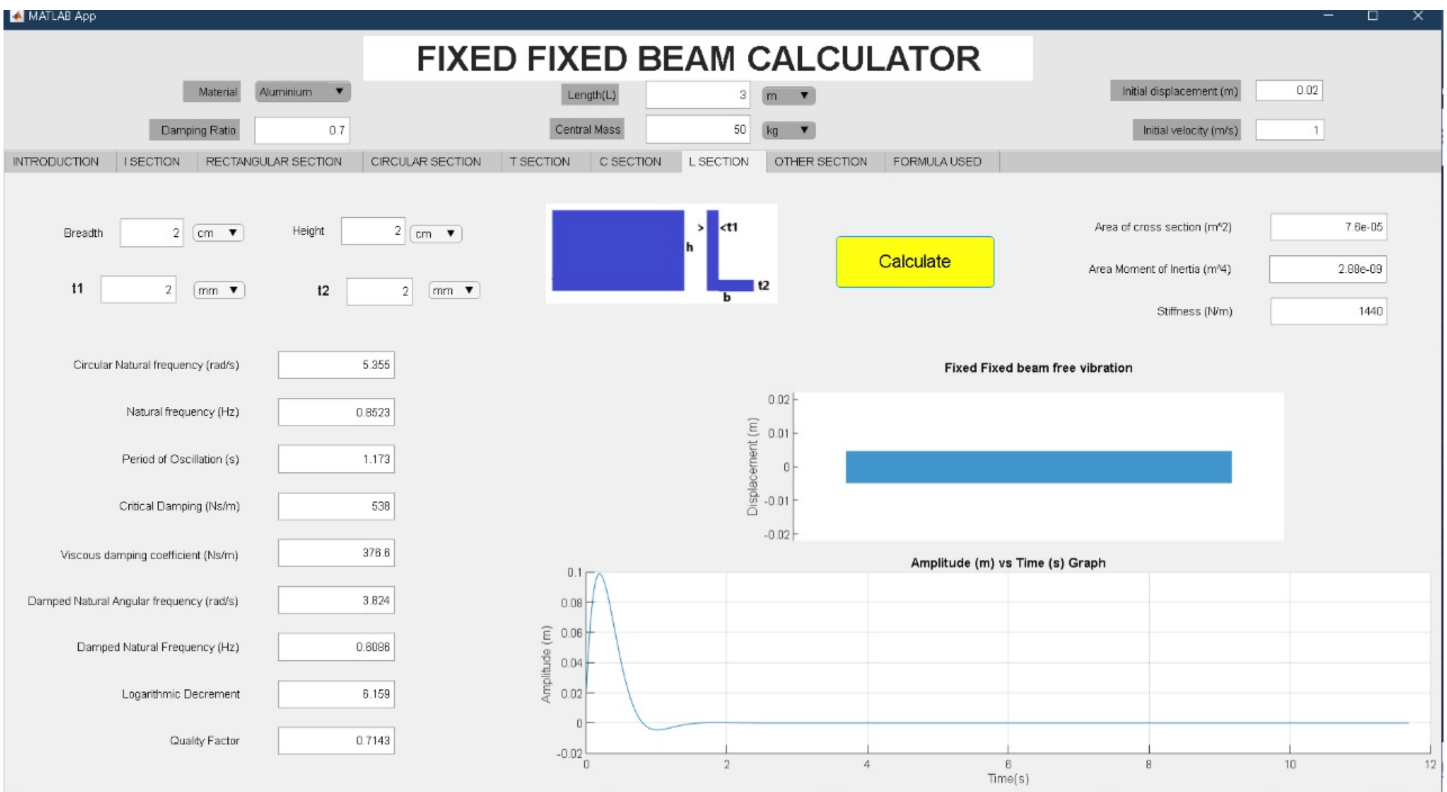
T SECTION



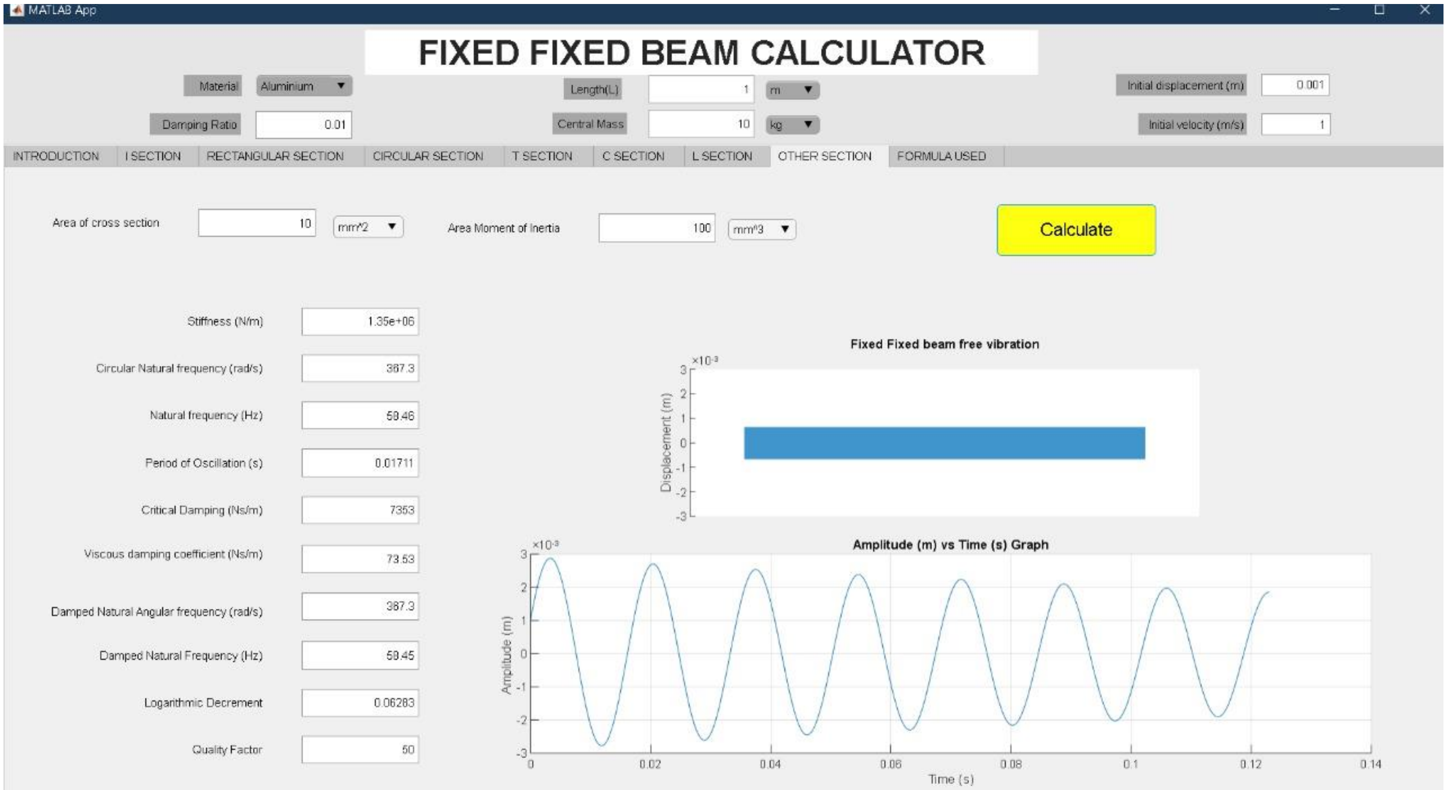
❖ *C SECTION*



❖ *L SECTION:*



❖ *OTHER SECTIONS:*



Advantages:

- The program uses widely used materials like steel, aluminum and bronze.
- The calculator is capable of calculating and displaying results of widely used cross sections like I section, T section, C section, CRICULAR section, RECTANGUALR section, L section.
- An alternative to use other cross-sections also exists.
- The calculator accepts values of different standards like m, cm, mm, kg, gram, etc.
- The calculator is incorporated with an introduction tab and a formulas tab to help and guide first time users.
- The calculator is simple to use and consists interactive animation.
- The program displays amplitude vs time graph, which can be used to further analyze the data.
- As the calculator is designed on MATLAB app designer, the live GUI code can accept multiple inputs continuously without terminating the program.

Limitations:

- The effect of temperature is not considered for calculations.
- It was assumed that the material will operate in elastic region.
- The calculator cannot be used for compound beams and beam should be of uniform mass.
- Material selection is limited to only 3 materials.
- The cross-section selection is limited to only 6 different cross section.
- The output is given only in SI units.

PROJECT CONCLUSION:

The project Free vibration of Beam as SDOF System CALCULATOR where the Boundary conditions is given as Fixed-Fixed beam. We have made use of the different concepts and formulae which have been taught to us during the coursework. Our project has involved the creation of the calculator with the help of MATLAB and has included various different features which have been mentioned in the report to provide with the required output from the given input from the user, also plot a Amplitude versus Time graph of the simulated system and also create an accurate animation of the system.

The project has been successful in analyzing and reporting the behavior of a Single degree of freedom Fixed-Fixed Beam system. We would like to hereby conclude the project by thanking our professors for imparting the valuable knowledge which was essential for the execution of this project. Our professor's way of teaching with help of many internet sources like calculator, graph plotter has helped us understand the core concepts behind the successful calculator. Mentioning the previous students' work and showing them to us really motivated us to work more and think uniquely while developing the calculator.