

Hands On 11

1) Given a dynamic table (see section 17.4) that doubles in size when it needs more space. Find the amortized runtime for inserting n elements.

a) Use the aggregate method in the aggregate method we consider the total cost across all the insertions and calculate the average (amortized) cost per insertion.

When inserting the i th element, if a resize operation is not needed the existing happens cost $O(1)$ as it involves copying the existing elements to the new table of size 2^k (k is number of resizes performed).

$$\text{Total Cost} = O(n) \cdot k \Rightarrow O(n \log n)$$

$$\text{Cost per insertion} = O(\log n)$$

$$\text{Runtime per insertion} = O(\log n)$$

$$\text{Total time is } O(n) * \log(n+1)$$

b) Accounting method:- In the accounting method, we assign each insertion a higher "amortized" cost the store "credits" that pay for future resizing costs.

Pseudo code:-

for $i = 1$ to n

if table is full

newtable = create new table

with size $2n$ copy elements from old table to new table

table = new-table

insert element i into table

initial charge = 0.

for $i = 1$ to n

charges $+= 2$

table deleted in size from
into $2m$

credits $+= m$

$$\text{total charges} = 2 * n = O(n)$$

$$\text{total ~~cost~~ credits} = m + 2m = \frac{n}{2} * m = O(n)$$

Amortized cost per insertion

$$= \text{total} / n$$

$$= O(n/n)$$

$$= O(1)$$

Runtime per insertion $O(1)$

$$\text{Total time} = O(n)$$