SREE VENKATESWARA COLLEGE OF ENGINEERING

(Affiliated to Jawaharlal Nehru Technological University – Anantapur)

II B TECH II SEM, Branch:CSE-1&CE-1, 2016-2017

Subject: **Probability & Statistics (P&S)**

UNIT-I

1 a) Define Probability and explain the Axioms of Probability.

b) Probability Define Conditional Probability, Independent Event & Pair wise Independent.

c) State and Prove Addition Theorem.

d) State and Prove Multiplication theorem.

e) If A and B are Independent Events of a Sample Space S, then (I) A and B^c are Independent (II) A^c and B are Independent (III) A^c and B^c are Independent

f) State and Prove Baye's Theorem

2 a) A class consists of 6 girls and 10 boys. If a committee of 3 is chosen at random from the class find the probability that (I) 3 boys are selected (II) exactly 2 girls are selected

b) If $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(A \cap B) = \frac{1}{5}$ then find (I) $P(AUB)$ (II) $P(A^c \cap B)$ (III) $P(A \cap B^c)$ (IV) $P(A^c \cap B^c)$

c) A card is drawn from a well shuffled pack of cards. What is the probability that it is either a spade or an ace?

d) Find (I) $P\left(\frac{B}{A}\right)$ (II) $P\left(\frac{A}{B^c}\right)$ if A and B are Independent Events and $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}, P(A \cup B) = \frac{1}{2}$

e) A class has 10 boys and 5 girls. 3 students are selected at random one after another. Find the probability that (I) first two are boys and third is girl (II) first and third are of same sex and the second of opposite sex

f) Two aero planes bomb a target is succession. The probability of each correctly scoring a hit is 0.3 and 0.2 respectively. The second will bomb only if the first missed the target. Find the probability that (I) target is hit (II) both fails to score hits

g) The probability that students A, B, C, D. Solve a problem are $\frac{1}{3}, \frac{2}{5}, \frac{1}{5}$ and $\frac{1}{4}$ respectively. If all of them try to solve the problem, what is the probability that the problem is solved?

h) If the three men, the chances that a politician, a business man, academician will be appointed as a Vice-Chancellor (V.C) of a university are 0.5, 0.3, 0.2 respectively. Find probability that research is promoted by these persons, if they are appointed as V.C is 0.3, 0.7 and 0.8 respectively. (I) determine the probability that research is promoted (II) if research is promoted what is the probability that V.C is an academician

i) Suppose 5 men out of 100 and 25 women out of 10,000 are color blind. A color blind person is chosen at random what the probability is being a person is a male

j) A bolt manufacturing company has three machines A, B, C with equal capacity. A bolt is drawn at random it is found to be defective; the chance of getting a defective bolt from each machine is 1%, 2% and 1% respectively. Find the probability that the defective bolt is drawn from Machine B.

k) The Chances of A, B and C becoming the general manager of a certain company are in the ratio 4:2:3. The probabilities that the bonus scheme will be introduced in the company if A, B and C become general manager are 0.3, 0.7 and 0.8 respectively. If the bonus scheme has been introduced, what is the probability that A has been appointed as general manager?

3 a) Define Random Variable and explain types of Random Variable and also explain distribution functions.

b) Show that the variance of a random variable X is given by $V(X) = E(X^2) - [E(X)]^2$

c) Let X denotes the number of heads in a single toss of 4 fair coins. Determine (I) $P(X < 2)$ (II) $P(1 < X \leq 3)$

d) A random variable X has the following probability function:

$X = x_i$	0	1	2	3	4	5	6	7
$P(X = x_i)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find (I) k (II) $P(X < 6), P(X \geq 6)$ and $P(0 < X < 5)$ (III) if $P(X \leq k) > \frac{1}{2}$, find minimum value of X (IV) determine the distribution function of X (V) Mean (VI) Variance (VII) Standard Deviation

e) A sample of 4 items is selected from a box containing 12 items of which 5 are defective. Find expected number of defective items in a box

f) Two dices are thrown. Let 'X' assign to each point (a, b) in 'S' the maximum of its numbers i.e. $X(a, b) = Max(a, b)$. Find probability distribution 'X' is a random variable with $X(S) = \{1,2,3,4,5,6\}$ also find mean and variance of distribution

g) The p. d. f of a random variable is given by $f(x) = \begin{cases} k(1-x^2); 0 < X < 1 \\ 0; \text{Otherwise} \end{cases}$

(I) find value of K (II) find the probabilities of (a) between 0.1 & 0.2 (b) greater than 0.5 (III) mean, variance and standard deviation

4 a) the p. d . f $f(x) = k.e^{-|x|};\ -\infty < x < \infty$. Find k, mean and variance of the distribution and also find the probability that the variate lies between 0 and 4

b) The p. d. f of a random variable X is $f(x) = \begin{cases} \frac{1}{2}\sin x; 0 \le x \le \pi \\ 0; \text{elsewhere} \end{cases}$ find the mean, variance, mode and median of the distribution and also find the probability between 0 and $\frac{\pi}{2}$

c) A continuous random variable X has the distribution function $F(x) = \begin{cases} 0; x \le 1 \\ k(x-1)^4; 1 < x \le 3 \\ 1; x > 3 \end{cases}$ find k and mean

5 a) Define Binomial distribution and derive mean & variance of Binomial distribution

b) Define Poisson distribution and derive mean & variance of Poisson distribution

c) Define Gaussian (or) Normal distribution and derive mean, variance, mode & median of Normal Distribution. Explain properties of Normal Distribution

d) Prove that as $p \to 0,\ n \to \infty$ & $np = \lambda$, the Poisson distribution is approximated from the Binomial distribution

6 a) ten coins are thrown simultaneously. Find the probability of getting at least seven heads

b) If 3 of 20 tires are defective and 4 of them are randomly chosen for inspection, what is the probability that only one of the defective tires will be included?

c) The incidence of an occupational disease in an industry is such that the workers have a 20% chance of suffering from it. What is the probability that out of 6 workers chosen at random, 4 or more will suffer from the disease?

d) Determine the Binomial distribution for which the mean is 4 and variance is 3, also find mode

e) The mean and variance of a Binomial variable X with parameters n and p are 16 and 8. Find $P(X \ge 1)$ and $P(X > 2)$

f) In 256 sets of 12 tosses of a coin, in how many cases one can expect 8 heads and 4 tails

g) In 8 throws of a die 5 or 6 is considered a success. Find the mean number of success and the Standard deviation

h) In a Binomial distribution consisting of 5 independent trials. Probabilities of 1 and 2 success are 0.4096 and 0.2048 respectively. Find the parameter p of the distribution

i) Find the maximum 'n' such that the probability of getting no head in tossing a coin 'n' times is greater than 0.1

j) Assume that 50% of all engineering students are good in mathematics. Determine the probabilities that among 18 engineering students. (I) exactly 10 (II) at least 10 (III) at most 8 (IV) at least 2 and at most 9 are good in mathematics

k) Fit a Binomial distribution to the following data

X	0	1	2	3	4	5
f	2	14	20	34	22	8

7 a) A hospital switch board receives an average of 4 emergency calls in a 10 minute interval. What is the probability that (I) there are at most 2 emergency calls in a 10 minute interval (II) there are exactly 3 emergency calls in a 10 minute interval

b) A manufacturer of cotter pins knows that 5% of his product is defective. Pins are sold in boxes of 100. He guarantees that not more than 10 pins will be defective. What is the approximate probability that a box will fail to meet the guaranteed quality?

c) If a Poisson distribution is such that $\frac{3}{2}$ $P(X = 1) = P(X = 3)$. Find (I) $P(X \ge 1)$ (II) $P(X \le 3)$ (III) $P(2 \le X \le 5)$

d) Suppose 2% of the people on the average are left handed. Find (I) the probability of finding 3 or more left handed (II) the probability of finding none or one left handed

e) The distribution of typing mistakes committed by a typist is given below. Assuming the distribution to be Poisson, find the expected frequencies.

x	0	1	2	3	4	5
f	42	33	14	6	4	1

8 a) If X is a Normal variate with mean 30 and standard deviation 5. Find the probabilities that (I) $26 \leq X \leq 40$ (II) $X \geq 45$

b) The marks obtained in statistics in a certain examination found to be normally distributed. If 15% of the students ≥ 60 marks, $40\% < 30$ *marks*, find the mean and standard deviation

c) Suppose the weights of 800 male students are normally distributed with mean $\mu = 140$ pounds. Find number of students who weights are (I) between 138 and 148 pounds (II) more than 152 pounds

d) A sales tax officer has reported that the average of the 500 business that he has to deal with during a year is Rs. 36,000 with a standard deviation of 10,000. Assuming that the sales in these business are normally distributed, find (I) the number of business as the sales of which are Rs.40,000 (II) the percentage of business the sales of which are likely to range between Rs. 30,000 and Rs. 40,000

e) If X is a normal variate. Find Area (I) to the left of $Z = -1.78$ (II) to the right of $Z = -1.45$ (III) corresponding to $-0.8 \leq Z \leq 1.53$ (IV) to the left of $Z = -2.52$ and to the right of $Z = 1.83$

f) Find the mean and standard deviation of a normal distribution in which 31% of items are under 45 and 8% are over 64

UNIT-II

1 a) Find the value of finite population correction factor for n=10 and N=1000

b) What is the effect on S.E, if a sample is taken from an infinite Population of sample size is increased from 400 to 900

c) A random sample of size 100 is taken from an infinite population having the mean μ=76 (or) m=76 and variance σ^2=256 (or)$s^2 = 256$. What is probability that $\bar{X}$ will be between 75 and 78?

d) A random sample of the size 64 is taken from normal population μ=51.4,$\sigma = 68$. what is the probability that the mean of sample (I) exceed 52.9 (II) fall between 50.5 & 52.3 (III) less than 50.6

2 a) Define Hypothesis, Errors and Explain types of errors.

b) Define one tailed test, two tailed test and critical region.

c) Experience had shown that 20% of a manufactured product is of the top quality. In one day's production of 400 articles only 50 are of top quality. Test the hypothesis at 0.05level.

d) In a big city 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers?

3 a) If 80 patients are treated with an antibiotic 59 got cured. Find a 99% confidence limits to the true population of cure.

b) Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were infavour of the proposal. Test the hypothesis that proportions of men and women in the favour of the proposal are same at 5% level.

c) On the basis of their total scores, 200 candidates of a civil service examination are divided into two groups, the upper 30% and the remaining 70%. Consider the first question of the examination. Among the first group, 40 had the correct answer, where as among the second group, 80 had the correct answer. On the basis of these results, can one conclude that the first question is not good at discriminating ability of the type being examined here?

d) An oceanographer wants to check whether the depth of the ocean in a certain region is 57.4fathoms, as had previously been recorded. What can he concluded at the level of significance $\alpha = 0.05$, if readings taken at 40 random locations in the given region yielded a mean of 59.1fathoms with a S.D of 5.2fathoms.

4a) In a random sample of 60 workers, the average time taken by them to get to work is 33.8minutes with a S.D of 6.1minutes, can we reject the null hypothesis $\mu = 32.6$mminutes infavour of alternative null hypothesis $\mu > 32.6$ at $\alpha = 0.05$ level of significance.

b) The means of two large samples of sizes 1000 and 2000 members are 67.5inches and 68.0inches respectively, can the samples be regarded as drawn from the same population of S.D 2.5inches.

c) Samples of students were drawn from the two universities and from their weights in kilograms, mean and S.D are calculated and shown below. Make a large sample test to test the significance of the difference between the means.

University	Mean	S.D	Size of samples
A	55	10	400
B	57	15	100

UNIT-III

1 a) A random sample of 6 steel beams has a mean compressive strength of 58392P.S.I (Pounds per square inch) with a S.D of 648P.S.I. Use this information and the level of significance $\alpha = 0.05$ to test whether the true average compressive strength of the steel from which this sample came is 58000P.S.I. Assume normality.

b) A random sample of 10 boys had the following IQs, 70, 120, 110, 101, 88, 83, 95, 98, 107, 100 and whose Population mean IQ is 100. Test whether all samples are drawn from in the sample population mean IQ.

c) To estimate the hypothesis that the husbands are more intelligent than the wife's, and investigator took a sample of 10 couples and administrated them a test which measures the I.Q. The results are as follows:

Husband's	117	105	97	105	123	109	86	78	103	107
Wife's	106	98	87	104	116	95	90	69	108	85

Test the hypothesis with reasonable test at the level of significance 0.05.

d) Find the maximum difference that we can accept with probability 0.95 between the means of samples of sizes 10 & 12 from a normal population if their S.D's are found to be 2 and 3 respectively.

e) Scores obtained in a shooting competition by 10 soldiers before and after intensive training are given below:

Before	67	24	57	55	63	54	56	68	33	43
After	70	38	58	58	56	67	68	75	42	38

Test whether the intensive training is useful at 0.05 LOS.

2 a) two random samples gave the following results:

Sample	Size	Sample mean	Sum of squares of deviations from mean
1	10	15	90
2	12	14	108

Test whether the samples came from the same normal population.

b) The measurements of the output of two units have given the following results. Assuming that both samples have been obtained from the normal populations at 10% significant level, test whether the two populations have the same variance.

Unit-A	14.1	10.1	14.7	13.7	14.0
Unit-B	14.0	14.5	13.7	12.7	14.1

c) A fair of dice are thrown 360 times and the frequency of each sum is indicated below:

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	8	24	35	37	44	65	51	42	26	14	14

Would you say that the dice are fair on the basis of the ψ^2 test at 0.05 level of significance?

d) From the following data, find whether there is any significant liking in the habit of taking soft drinks among the categories of employees.

	Employees		
Soft drinks	Clerks	Teachers	Officers
Pepsi	10	25	65
Thumsup	15	30	65
Fanta	50	60	30

UNIT-IV

1. Define SQC & explain advantages of SQC
2. Define control chart, list out advantages of control charts
3. Explain major parts of control and also explain types of Shewart control charts
4. Explain types various causes and also define Process Control and Product control.
5. Define defectives and defect.
6. Explain 3σ control limits and also explain the difference between confidence limit and control limits.
7. Construct the control chart for mean or average or X-bar chart and Range(R) chart for the following data.

Sample No.	Sample observations				
	1	2	3	4	5
1	42	45	75	78	87
2	42	45	68	72	90
3	19	24	80	81	81
4	36	54	69	77	84
5	42	51	57	59	78
6	51	74	75	78	132
7	60	60	72	95	138
8	18	20	27	42	60
9	15	30	39	62	84
10	69	109	113	118	153
11	64	90	93	109	112
12	61	78	94	109	136

8. During an inspection of medical X-ray films, 20 operators are given 100 sheets each for inspection, and the number of defective films found by the operators is given in the following table. Construct a p-chart or control chart for Fraction defective

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of defectives	12	6	10	5	12	6	4	8	6	4

Sample No.	11	12	13	14	15	16	17	18	19	20
No. of defectives	10	12	14	5	4	6	8	10	6	6

9. The following are the figures of defectives in 22 lots of each containing 2000 rubber belts. 425,430,216,341,225,322,280,306,337,305,356,402,216,264,126,409,193,326,280,389,451,420. Construct a control chart for p-chart.

10. For setting up the control chart for end-breaks, studies were conducted on a regular basis. The results of 25 studies on the incidences of end breaks per machine hour (480 spindles hours) are given in the following table. Construct a Number of defects per hour or C-chart and offer your comments.

Sample No.	1	2	3	4	5	6	7	8	9	10	11	12	13
No. of breaks per hour	44	46	61	26	58	36	61	26	34	60	46	30	54

Sample No.	14	15	16	17	18	19	20	21	22	23	24	25
No. of breaks per hour	66	47	39	36	62	43	33	56	35	23	44	57

UNIT-V

1. Discuss queuing theory. Explain its applications
2. Explain characteristics of queuing theory.
3. Draw a General structure of a queuing system.
4. Define Pure Birth and Death Process.
5. Explain Kendall's notation for representing queuing model.
6. Discuss (M/M/1): $(\infty/FCFS)$ and (M/M/1): $(N/FIFO)$ Queueing Model and find the expected queue length in the system.
7. Derive the formula for the probability distribution density function of the waiting time distribution.
8. a) A TV repair man finds that the time spent on his jobs has an exponential distribution with mean 30minutes. He repairs sets in the order in which they arrive. The arrival of the sets is approximately Poisson with an average of 10 per an eight hour day. Find the repairman's idle time each day. How many jobs are ahead of the average set just brought in?

   b) A toll gate is operated on a freeway where cars arrive according to a Poisson distribution with mean frequency of 1.2 cars per minute. The time of completing payment follows an exponential distribution with mean of 20 seconds. Find (I) the idle time of the counter (II) average number of cars in the system (III) average number of cars in the queue (IV) average time that a car spends in the system (V) average time that a car spends in the queue (VI) the probability that a car spends more than 30 seconds in the system.

c) A self service canteen employs one cashier at its counter. 8 customers arrive per every 10 minutes on an average. The cashier can serve on average one per minute. Assuming that the arrivals are Poisson and the service time distribution is exponential, determine: (I) the average number of customers in the system (II) the average queue length (III) average time a customer spends in the system (IV) average waiting time of each customer.

d) Arrival rate of telephone calls at a telephone booth are according to Poisson distribution with an average time of 12 minutes between two consecutive call arrivals. The length of telephone calls is assumed to be exponentially distributed with mean 4 minutes. (I) find the probability that a caller arriving at the booth will have to wait (II) find the average queue length that forms from time to time (III) find the fraction of a day that the phone will be in use (IV) what is the probability that an arrival will have to wait for more than 15 minutes before the phone is free (V) the telephone company will install a second booth. When convinced that an arrival would expect to have to wait at least 5 minutes for making the call. Find the increase in flows of arrivals which will justify a second booth.

7 a) Customers arrive at a one window drive-in bank according to a Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The car space in front of the window including that for the serviced can accommodate a maximum of 3 cars. Other cars can wait outside the space. (I) what is the probability that an arriving customer can drive directly to the space in front of the window? (II) What is the probability that an arriving customer will have to wait outside the indicated space? (III) How long is an arriving customer expected to wait before starting service.

b) Barber A takes 15 minutes to complete one hair cut. Customers arrive in his shop at an average rate of one every 30 minutes. Barber B takes 25 minutes to complete one hair cut and customers arrive at his shop at an average rate of one every 50 minutes. The arrival processes are Poisson and the service times follow an exponential distribution. (I) where would you expect a bigger queue. (II) Where would you require more time waiting included to complete a haircut?

c) Consider a single server queuing system with Poisson input and exponential service time. Suppose the mean arrival rate is 3 calling units per hour with the expected service time as 0.25 hours and the maximum permissible number of calling units in the system is two. Obtain the steady state probability distribution of the number of calling units in the system and then calculate the expected number in the system.

d) A car park contains 5 cars. The arrival of cars in Poisson with a mean rate of 10 per hour. The length of time each car spends in the car park has negative exponential distribution with mean of 2 hours. How many cars are in the car park on average and wheat is the probability of a newly arriving customer finding the car park full and having to park his car elsewhere?