Linear Regression

1. Linear Regression

```python
class LinearRegression:
    """
    This class implements Linear Regression using both
    Batch Gradient Descent and Stochastic Gradient descent.
    Class attributes:
         W         : current set of weights
         W_arr     : list of weights at each iteration
         Cost  : current cost
         cost_arr : list of costs at each iteration
    """
    def init_weights(self, s):
        """
        This method initializes the weight matrix
        as a column vector with shape = (X rows+1, 1)
        """
        np.random.seed(2)
        self.W = np.random.randn(s[1],1)
        self.W_arr = []
        self.cost_arr = []
        self.cost = 0

    def get_cost(self, X, y, W):
        """
        This function returns the cost with the given set of weights
        using the formula
```

$$J = \frac{1}{2m}\sum_{i=0}^{m-1}\left(h_{\omega}(x^{i}) - y^{i}\right)^{2}$$

```python
        """
        total_cost = sum(np.square(np.matmul(X,W)-y.reshape(-1,1)))[0]
        return (0.5/X.shape[0])*total_cost

    def add_bias(self, X):
        """
        This function adds bias (a column of ones) to the feature vector X.
        """
        bias = np.ones((X.shape[0],1))
        return np.concatenate((bias,X), axis=1)
    def get_h_i(self, X, i, W):
        """
        This function returns the hypothesis of ith feature vector
        with the given weights W.
```

$$h_{w}(x^{i}) = \sum_{j=0}^{n} w_{j}{x^{i}}_{j} = x^{i}w$$

```python
        """

        h_i = 0
        h_i = np.matmul(X[i].reshape(1,-1),W)
        return h_i[0][0]
```

```python
def batch_grad_descent(self, X, y, alpha, max_iter):
    """
    This function implements the Batch Gradient Descent algorithm.
    It runs for multiple iterations until either the weights converge or
    iterations reach max_iter. At each iteration the weights are updated using
    the following rule
          repeat until convergence{
               w_j^{t+1} = w_j^t - (α/m) Σ_{i=1}^{m} (h_w(x^i) - y^i) x_j^i
          }
    """
    W_new = self.W.copy()
    for iteration in range(max_iter):
        temp = np.matmul(X,self.W) - y.reshape(-1,1)
        for j in range(X.shape[1]):
            W_new[j][0] = self.W[j][0] - (alpha/X.shape[0])*(sum(temp*X[:,j:j+1])[0])
        self.W = W_new.copy()
        self.cost_arr.append(self.get_cost(X, y, self.W))
        self.W_arr.append(self.W)
    return W_new
```

```python
def stochastic_grad_descent(self, X, y, alpha, max_iter):
    """
    This function implements the Stochastic Gradient Descent algorithm.
    It runs for multiple iterations until either the weights converge or
    iterations reach max_iter. Weights are updated for every row of the
     training set.

          repeat until convergence{
               randomly shuffle the feature matrix rows
               for each feature vector x^i {
                    update all weights j -> 0 to n+1
                         w_j^{t+1} = w_j^t - α(h_w(x^i) - y^i) x_j^i
               }
          }
    """

    mat = np.concatenate((X,y.reshape(-1,1)), axis=1)
    for iteration in range(max_iter):
        W_new = self.W.copy()
        np.random.shuffle(mat)
        X = mat[:,0:3]
        y = mat[:,3]
        for i in range(X.shape[0]):
            temp = np.matmul(X[i,:],self.W) - y[i]
            for j in range(X.shape[1]):
                W_new[j][0] = self.W[j][0] - (alpha)*(temp[0]*X[i,j])
            self.W = W_new.copy()
        self.cost_arr.append(self.get_cost(X, y, self.W))
        self.W_arr.append(self.W)
    return self.W
```

```python
    def train(self, X, y, alpha, max_iter=100, option="batch"):
        """

        This function initiates the training process.
        It runs batch gradient descent by default and can also run
        Stochastic gradient descent if the argument is passed.

        returns the cost list which has costs at every training iteration.
        """
        # adding bias column to feature matrix X.
        X = self.add_bias(X)
        self.init_weights(X.shape)
        if option=="batch":
            self.batch_grad_descent(X,y,alpha,max_iter)
        elif option=="stochastic":
            self.stochastic_grad_descent(X,y,alpha,max_iter)
        self.cost = self.cost_arr[-1]
        return self.cost_arr

    def test(self,X,W=""):
        """
        This function takes a feature matrix as test data and
        predicts the target values using the trained weights.

        returns the predicted target values.
        """
        if W=="":W = self.W

        X = self.add_bias(X)
        y_pred = np.ones(X.shape[0])
        for i in range(X.shape[0]):
            for j in range(X.shape[1]):
                y_pred[i] += X[i][j]*W[j][0]
        return y_pred
```

```python
if __name__ == "__main__":
    model = LinearRegression()

    # data input
    data = pd.read_csv("./data.csv", header=None)
    X = data.loc[:,0:1].values
    y = data.loc[:,2].values

    # data preprocessing (Normal scaling)
    mscaler = NormalScaler()
    mscaler.fit(X[:,0])
    X[:,0] = mscaler.transform(X[:,0])
    mscaler.fit(X[:,1])
    X[:,1] = mscaler.transform(X[:,1])

    # Training the model by choosing alpha and max_iter values.
    # gradient descent algorithm can be set as either 'batch' or 'stochastic'
    # in this function call.
    arr = model.train(X,y,0.19,250,"batch")

    print("weights: ",model.W)
    print("Total Cost: ",model.cost)
```

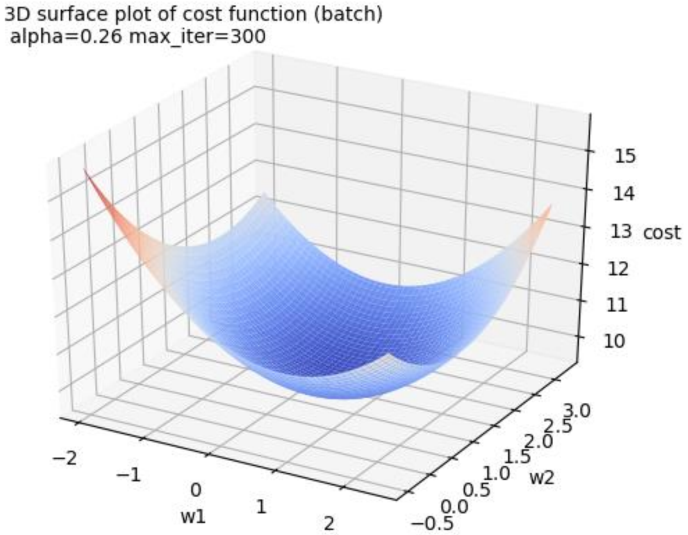
```python
# visualization of cost function.
W_arr = np.array(model.W_arr)
res = 100


xx = np.linspace(np.min(W_arr[:,1])-10, np.max(W_arr[:,1])+10, res)
yy = np.linspace(np.min(W_arr[:,2])-10, np.max(W_arr[:,2])+10, res)
minw0 = W_arr[-1][0][0]

r = np.ndarray((res,res))
s = np.ndarray((res,res))
z = np.ndarray((res,res))

for i in range(res):
    for j in range(res):
        z[i][j] = model.get_cost(model.add_bias(X), y, np.array([minw0,xx[i],yy[j]]).reshape(-1,1))
        r[i][j] = xx[i]
        s[i][j] = yy[j]

# 3d surface plot of cost function and learning curve
ax = plt.axes(projection='3d')
ax.plot_surface(r, s, z,cmap='coolwarm')
ax.plot(W_arr[:,1], W_arr[:,2], model.cost_arr,c='red')
plt.show()

# 2d contour plot of cost function
plt.title("2d contour plot of cost function")
plt.contour(r,s,z.reshape(res,res),levels=25)
plt.scatter(W_arr[:,1].ravel(),W_arr[:,2].ravel(),c=model.cost_arr)
plt.show()

# 2d line plot of cost vs iteration
plt.plot(model.cost_arr)
plt.show()
```

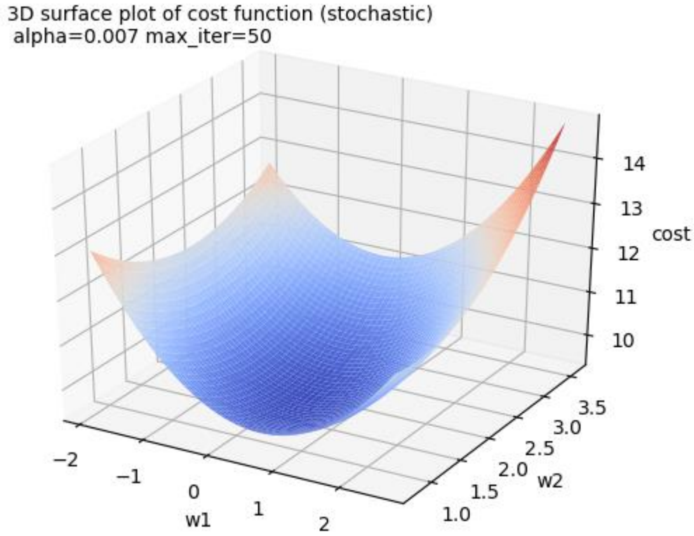
Results:

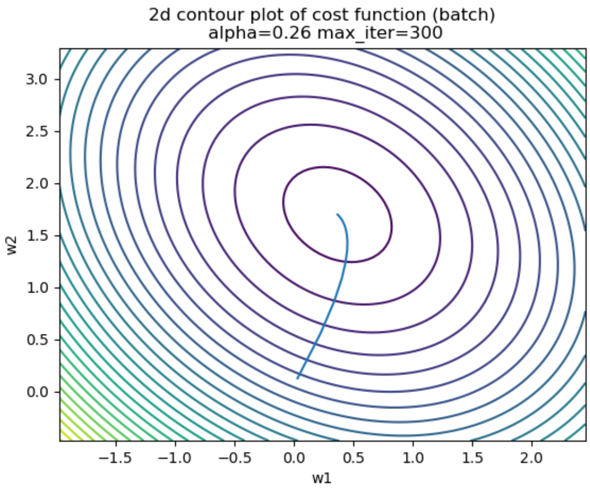
```
Batch Gradient Descent

weights:
    w0 = 14.9047221
    w1 = 0.36752656
    w2 = 1.6965344

Total Cost:  9.403235190130703
```

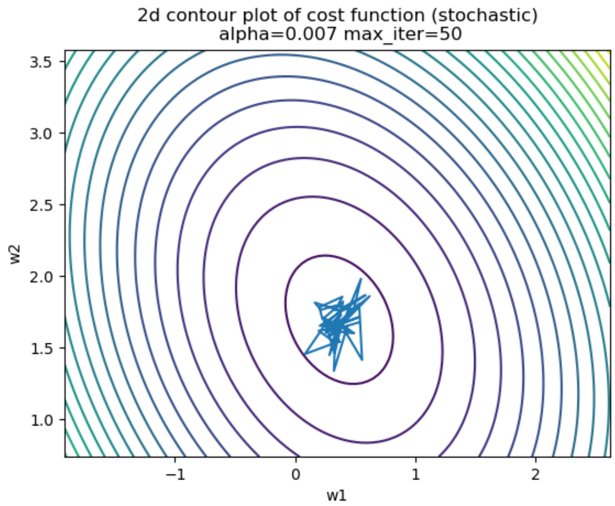
```
Stochastic Gradient Descent

weights:
    w0 = 14.95107574
    w1 = 0.39873559
    w2 = 1.75867644

Total Cost:  9.407225072389698
```

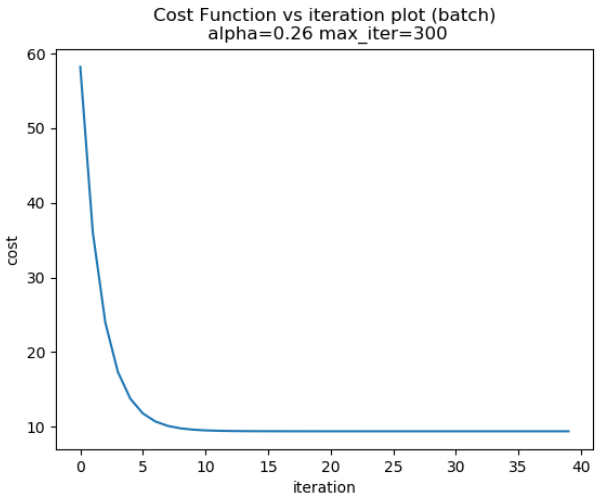
Plots:

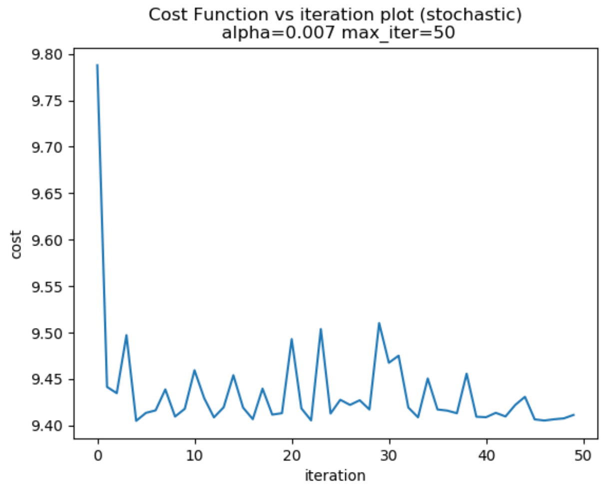

3D surface plot of cost function (batch)
alpha=0.26 max_iter=300
cost
w1
w2


3D surface plot of cost function (stochastic)
alpha=0.007 max_iter=50
cost
w1
w2


2d contour plot of cost function (batch)
alpha=0.26 max_iter=300
w1
w2


2d contour plot of cost function (stochastic)
alpha=0.007 max_iter=50
w1
w2


Cost Function vs iteration plot (batch)
alpha=0.26 max_iter=300
iteration
cost


Cost Function vs iteration plot (stochastic)
alpha=0.007 max_iter=50
iteration
cost

2. Ridge Regression

```python
def get_cost(self, X, y, W):
    """
    This function returns the cost with the given set of weights
    using the formula. Regularization term (sum of squares of weights)
    is added to the cost.
      J = 1/2m Σ_{i=0}^m (h_ω(x^i) − y^i)^2 + η/2 Σ_{i=0}^m w_j^2
     """
    reg = 0
    for i in range(1,W.shape[0]):
        reg += W[i][0]**2
    total_cost = sum(np.square(np.matmul(X,W)-y.reshape(-1,1)))[0]

    return (0.5/X.shape[0])*total_cost + 0.5*self.eta*reg
```

```python
def batch_grad_descent(self, X, y, alpha, eta, max_iter):
    """
    This function implements the Batch Gradient Descent algorithm.
    It runs for multiple iterations until either the weights converge or
    iterations reach max_iter. At each iteration the weights are updated using
    the following rule
        repeat until convergence{

                w_j^{t+1} = w_j^t (1 − ηα) − α/m Σ_{i=1}^m (h_w(x^i) − y^i) x_j^i

        }
    """
    self.eta = eta
    for _ in range(max_iter):
        W_new = np.ndarray(self.W.shape)
        for j in range(X.shape[1]):
            grad = 0
            for i in range(X.shape[0]):
                grad += (self.get_h_i(X, i, self.W) - y[i])*X[i][j]
            W_new[j][0] = self.W[j][0]*(1-eta*alpha) - (alpha/X.shape[0])*grad
        self.W = W_new.copy()
        self.cost_arr.append(self.get_cost(X, y, self.W))
        self.W_arr.append(self.W)
        if len(self.W_arr)>1:
            if sum(abs(self.W_arr[-2]-self.W_arr[-1]))<0.0001:
                break
    return W_new
```

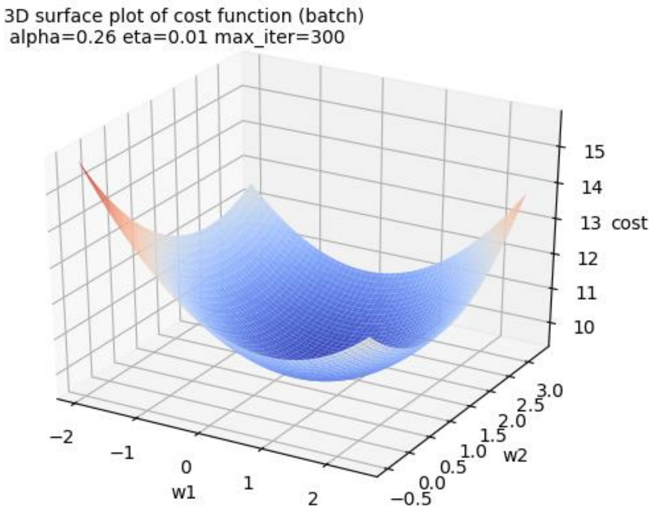
```python
    def stochastic_grad_descent(self, X, y, alpha, eta, max_iter):
        """
        This function implements the Stochastic Gradient Descent algorithm.
        It runs for multiple iterations until either the weights converge or
        iterations reach max_iter. Weights are updated for every row of the
        training set.

            repeat until convergence{
                randomly shuffle the feature matrix rows
                for each feature vector x^i {
                    update all weights j -> 0 to n+1
                     w_j^{t+1} = w_j^t (1 - ηα) - α(h_w(x^i) - y^i) x_j^i
                }
            }
        """
        mat = np.concatenate((X,y.reshape(-1,1)), axis=1)
        for _ in range(max_iter):
            W_new = self.W.copy()
            np.random.shuffle(mat)
            X = mat[:,0:3]
            y = mat[:,3]
            for i in range(X.shape[0]):
                temp = np.matmul(X[i,:],self.W) - y[i]
                for j in range(X.shape[1]):
                     W_new[j][0] = self.W[j][0]*(1-eta*alpha) - (alpha)*temp[0]*X[i,j]
                self.W = W_new.copy()
            self.cost_arr.append(self.get_cost(X, y, self.W))
            self.W_arr.append(self.W)
            if len(self.W_arr)>1:
                if sum(abs(self.W_arr[-2]-self.W_arr[-1]))<0.0001:
                    break
        return self.W
```

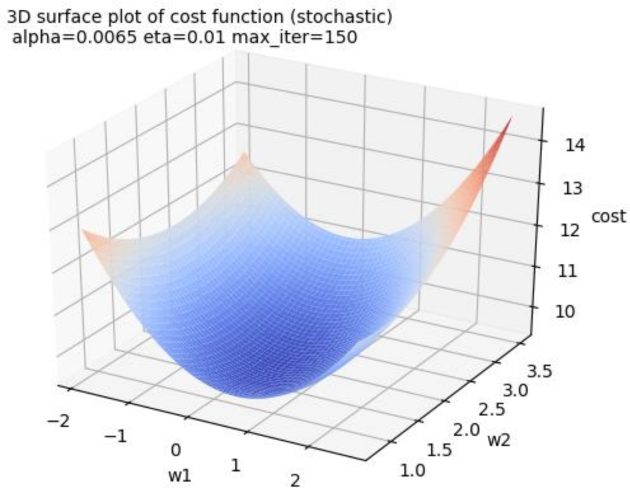
```python
if __name__ == "__main__":
    model = RidgeRegression()

    # data input
    data = pd.read_excel("./data.xlsx", header=None)
    X = data.loc[:,0:1].values
    y = data.loc[:,2].values

    # data preprocessing (MinMax scaling)
    mscaler = NormalScaler()
    mscaler.fit(X[:,0])
    X[:,0] = mscaler.transform(X[:,0])
    mscaler.fit(X[:,1])
    X[:,1] = mscaler.transform(X[:,1])

    # Training the model by choosing alpha and max_iter values.
    # gradient descent algorithm can be set as either 'batch' or 'stochastic'
    # in this function call.
    alpha = 0.1
    eta = 0.1
    max_iter = 150
    algo = 'batch'

    arr = model.train(X,y,alpha,eta,max_iter,algo)

    print("weights: ",model.W)
    print("Total Cost: ",model.cost)
```

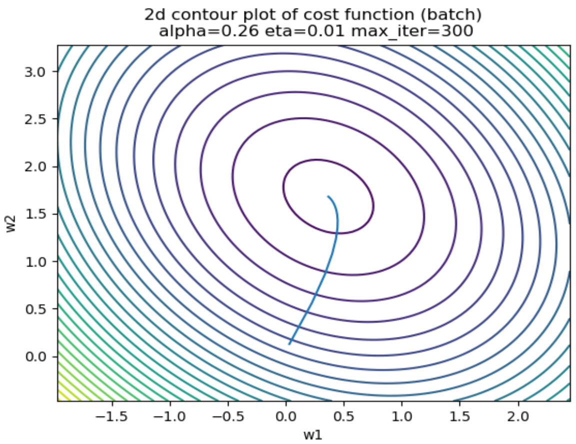
Results:

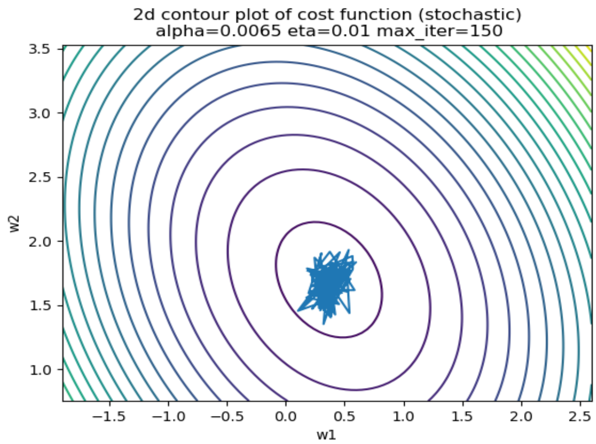
```
Batch Gradient Descent

weights:
    w0 = 14.75716074
    w1 = 0.36824758
    w2 = 1.67956428

Total Cost:  9.429059509401734
```

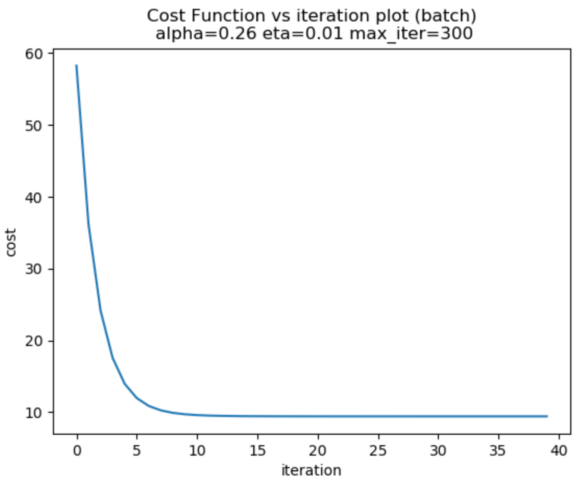
```
Stochastic Gradient Descent

weights:
    w0 = 14.8376096
    w1 = 0.18609906
    w2 = 1.76646287

Total Cost:  9.421065885290048
```

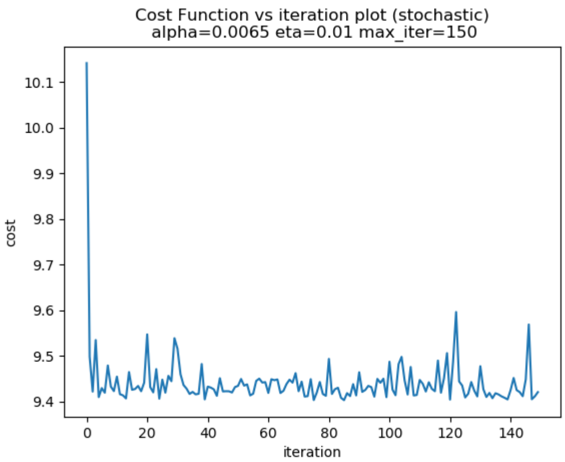












3. Least Angle Regression

```python
class LeastAngleRegression:
    """
    This class implements Linear Regression using both
    Batch Gradient Descent and Stochastic Gradient descent.
    Class attributes:
        W        : current set of weights
        W_arr    : list of weights at each iteration
        Cost     : current cost
        cost_arr : list of costs at each iteration
    """

    def get_cost(self, X, y, W):
        """
        This function returns the cost with the given set of weights
        using the formula
```

$$J = \frac{1}{2m}\sum_{i=0}^{m-1}\left(h_\omega(x^i) - y^i\right)^2 + \frac{\eta}{2}\sum_{j=1}^{m} | w_j |$$

```python
        """
        total_cost = sum(np.square(np.matmul(X,W)-y.reshape(-1,1)))[0]
        reg = 0
        for i in range(1,W.shape[0]):
            reg += abs(W[i][0])
        return (0.5/X.shape[0])*total_cost + 0.5*self.eta*reg

    def batch_grad_descent(self, X, y, alpha, eta, max_iter):
        """
        This function implements the Batch Gradient Descent algorithm.
        It runs for multiple iterations until either the weights converge or
        iterations reach max_iter. At each iteration the weights are updated using
        the following rule
            repeat until convergence{
```

$$w_j^{t+1} = w_j^t - \frac{\alpha}{m}\sum_{i=1}^{m}\left(h_w(x^i) - y^i\right)x_j^i - \frac{1}{2}\eta\,\alpha\,\mathrm{sgn}(w_j)$$

```python
            }
        """
        self.eta = eta
        for _ in range(max_iter):
            W_new = np.ndarray(self.W.shape)
            for j in range(X.shape[1]):
                grad = 0
                for i in range(X.shape[0]):
                    grad += (self.get_h_i(X, i, self.W) - y[i])*X[i][j]
                W_new[j][0] = self.W[j][0]-0.5*eta*alpha*np.sign(self.W[j][0]) - (alpha/X.shape[0])*grad
            self.W = W_new.copy()
            self.cost_arr.append(self.get_cost(X, y, self.W))
            self.W_arr.append(self.W)
            if len(self.W_arr)>1:
                if sum(abs(self.W_arr[-2]-self.W_arr[-1]))<0.0001:
                    break
        return W_new
```

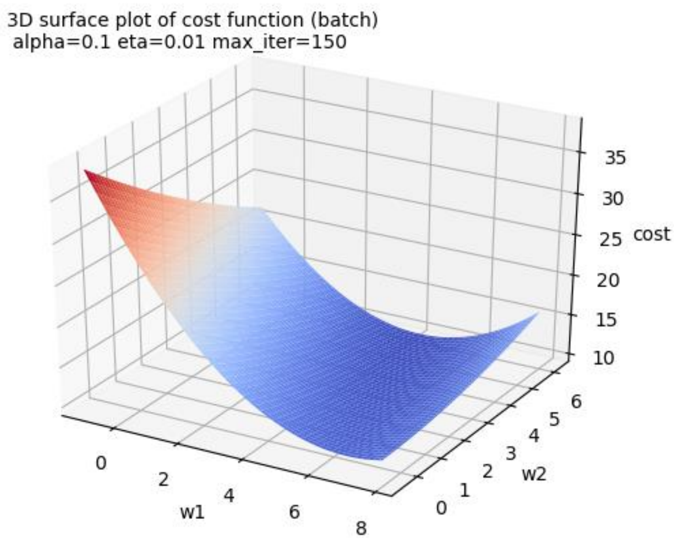
```python
    def stochastic_grad_descent(self, X, y, alpha,eta, max_iter):
        """
        This function implements the Stochastic Gradient Descent algorithm.
        It runs for multiple iterations until either the weights converge or
        iterations reach max_iter. Weights are updated for every row of the
        training set.

            repeat until convergence{
                randomly shuffle the feature matrix rows
                for each feature vector x^i {
                    update all weights j -> 0 to n+1
```
$$w_j^{t+1} = w_j^t(1-\eta\alpha) - \alpha\left(h_w(x^i) - y^i\right)x_j^i$$
```python
                }
            }
        """
        mat = np.concatenate((X,y.reshape(-1,1)), axis=1)
        for _ in range(max_iter):
            W_new = self.W.copy()
            np.random.shuffle(mat)
            X = mat[:,0:3]
            y = mat[:,3]
            for i in range(X.shape[0]):
                temp = np.matmul(X[i,:],self.W) - y[i]
                for j in range(X.shape[1]):
                    W_new[j][0] = self.W[j][0]-0.5*eta*alpha*np.sign(self.W[j][0]) - (alpha/X.shape[0])*temp*X[i,j]
                self.W = W_new.copy()
            self.cost_arr.append(self.get_cost(X, y, self.W))
            self.W_arr.append(self.W)
            if len(self.W_arr)>1:
                if sum(abs(self.W_arr[-2]-self.W_arr[-1]))<0.0001:
                    break
        return self.W
```

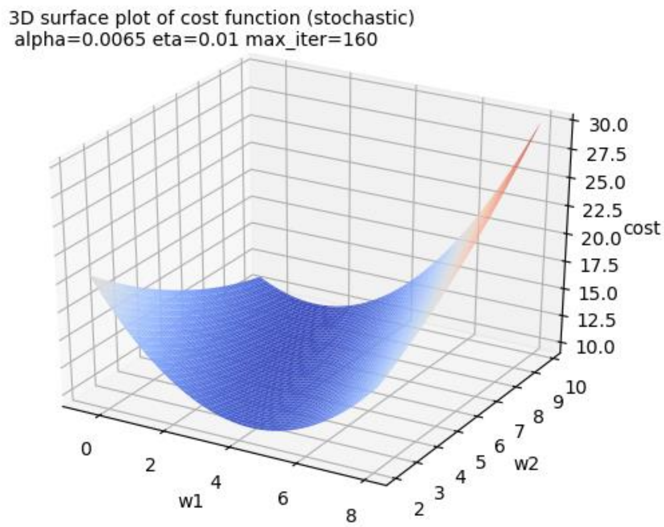
```python
if __name__ == "__main__":
    model = LeastAngleRegression()

    # data input
    data = pd.read_csv("./data.csv", header=None)
    X = data.loc[:,0:1].values
    y = data.loc[:,2].values

    # data preprocessing (MinMax scaling)
    mscaler = MinMaxScaler()
    mscaler.fit(X[:,0])
    X[:,0] = mscaler.transform(X[:,0])
    mscaler.fit(X[:,1])
    X[:,1] = mscaler.transform(X[:,1])

    # Training the model by choosing alpha and max_iter values.
    # gradient descent algorithm can be set as either ‘batch’ or ‘stochastic’
    # in this function call.
    alpha = 0.1
    eta = 0.01
    max_iter = 150
    algo = 'batch'

    arr = model.train(X,y,alpha,eta,max_iter,algo)
    print("weights: ",model.W)
    print("Total Cost: ",model.cost)
```

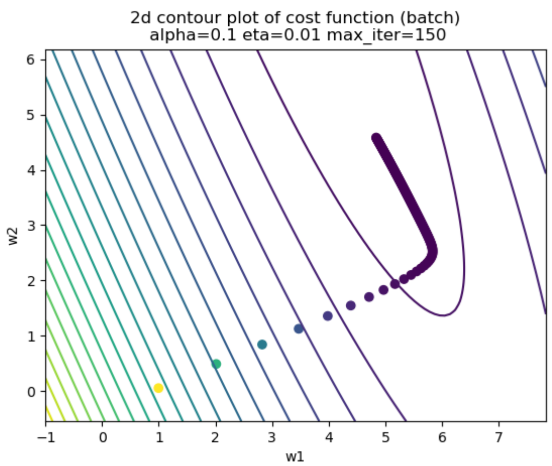
Results:

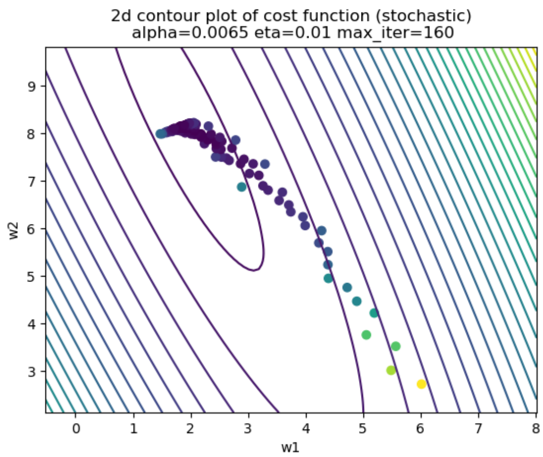
```
Batch Gradient Descent

weights:
    w0 = 8.70076276
    w1 = 8.70076276
    w2 = 4.58278454
Total Cost:  9.80262939490788
```

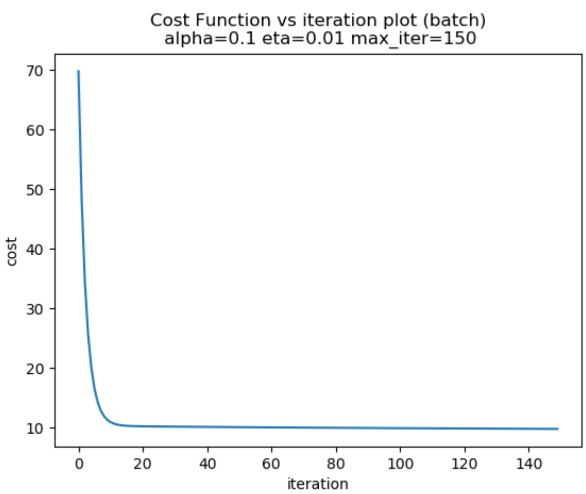
```
Stochastic Gradient Descent

weights:
    w0 = 10.11557396
    w1 = 1.71982718
    w2 = 8.10088578
Total Cost: 9.42485150686
```

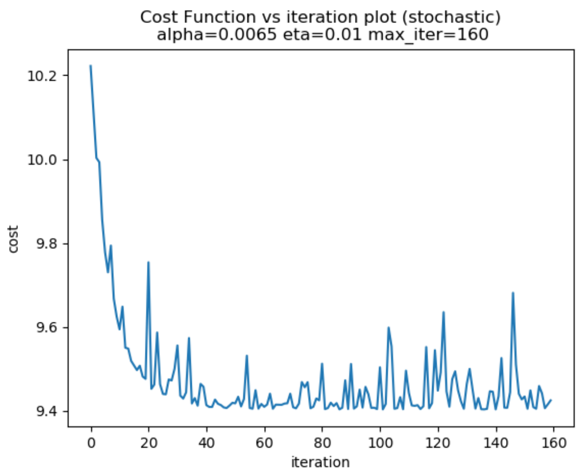












4. Vectorized Linear Regression

```
class LinearRegression_Vectorized:
    """
    This class implements Linear Regression using both
    Batch Gradient Descent and Stochastic Gradient descent.
    Class attributes:
        W        : current set of weights
        W_arr    : list of weights at each iteration
        Cost     : current cost
        cost_arr : list of costs at each iteration
    """
    def train(self, X, y):
        """
        This function uses the vectorized version of linear regression
        and obtains the optimal weights with the given feature matrix
        and target values.
```

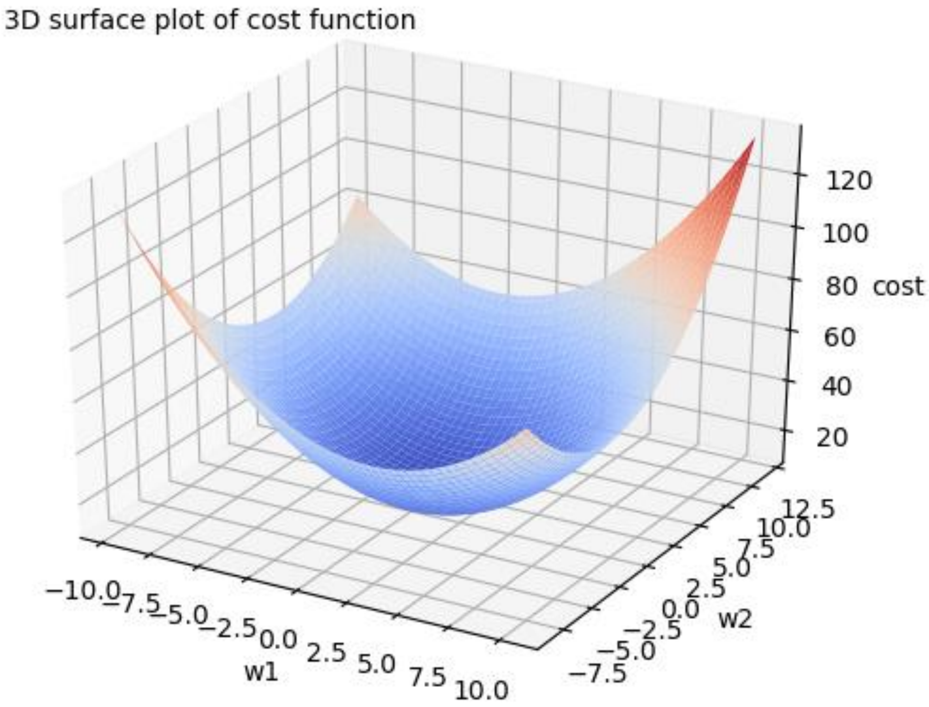
$$W = (X^TX)^{-1}X^TY$$

```
        returns the optimal weights.
        """
        X = self.add_bias(X)
        self.init_weights(X.shape)
        self.W = np.matmul(np.matmul(np.linalg.inv(np.matmul(X.T,X)), X.T), y.reshape(-1,1))
        self.cost = self.get_cost(X,y,self.W)
        return self.W

    def test(self,X):
        """
        This function takes a feature matrix as test data and
        predicts the target values using the trained weights.

        returns the predicted target values.
        """
        X = self.add_bias(X)
        return np.matmul(X,self.W)
```

```
if __name__ == "__main__":
    model = LinearRegression_Vectorized()

    # data input
    data = pd.read_excel("./data.xlsx", header=None)
    X = data.loc[:,0:1].values
    y = data.loc[:,2].values

    # data preprocessing (Normal Scaling)
    print(X[:,0].mean())
    mscaler = NormalScaler()
    mscaler.fit(X[:,0])
    X[:,0] = mscaler.transform(X[:,0])
    mscaler.fit(X[:,1])
    X[:,1] = mscaler.transform(X[:,1])

    # training the model
    arr = model.train(X,y)
    print("weights: ",model.W)
    print("Total Cost: ",model.cost)
```

Results:

```
Total Cost:  9.80262939490788
weights:

    w0 = 14.90479943

    w1 = 0.36737803

    w2 = 9.403235170782
```



Name: S Rohith

Id: 2017A7PS0034H