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Introduction To Measurement Systems

Measurement : Measurement is the process of comparing an unknown quantity with an accepted standard quantity.

Static characteristics

1. Accuracy : The degree of exactness or closeness of a measurement compared to the expected or desired value.
2. Resolution : The smallest change in a measured variable to which an instrument responds.
3. Precision : A measure of the consistency of repeatibility of measurement.
4. Error : The deviation of the true value from the desired value.

It can be expressed in two ways.

i. Absolute error
ii. Percentage of error

5. Sensitivity : The ratio of the change in output (or) response to a change in input (or) measured value.

$$S = \frac{\Delta\, o/p}{\Delta\, i/p}$$

Error

i. Absolute error: It may be defined as difference between the expected value of the variable and the measured value of the variable.

$$\text{error} = \text{expected value} - \text{measured value}$$

$$e = Y_n - X_n$$

where Y_n = expected value, X_n = measured value.

ii. Percentage of error: It is the ratio of the absolute error and expected value.

$$\%\ \text{error} = \frac{\text{absolute error}}{\text{expected value}} \times 100$$

$$\%\ e = \frac{Y_n - X_n}{Y_n} \times 100$$

→ The error is more frequency expressed as an accuracy rather than error

$$\text{Accuracy, } A = 1 - \left| \frac{Y_n - X_n}{Y_n} \right|$$

→ Accuracy is expressed as % of accuracy

$$a = 100\% - \%\ \text{error}$$

(or) $a = A \times 100$

Problems

① The expected value of the voltmeter across a resistor is 80 volts. However the measurement gives a value of 79 volts. Calculate:

a) Absolute error
b) % error
c) Relative accuracy
d) % of accuracy

Sol:- $Y_n = 80$, $X_n = 79$

a) $e = Y_n - X_n = 80 - 79 = 1$

b) $\% e = \frac{Y_n - X_n}{Y_n} \times 100 = \frac{1}{80} \times 100 = 1.25\%$

c) Relative accuracy $= 1 - \left|\frac{Y_n - X_n}{Y_n}\right| = 1 - \frac{1}{80} = \frac{79}{80} = 0.9875$

d) % accuracy $= 0.9875 \times 100 = 98.75\%$

② The expected value of the current through a resistor is 20 mA. However the measurement gives current value of 18 mA. Calculate:

a) Absolute error
b) % error
c) Relative accuracy
d) % accuracy

Sol:- a) $e = Y_n - X_n = 20 - 18 = 2\,mA$

b) $\% e = \frac{Y_n - X_n}{Y_n} \times 100 = \frac{2}{20} \times 100 = 10\%$

c) Relative accuracy $= 1 - \left|\frac{Y_n - X_n}{Y_n}\right| = 1 - \frac{1}{10} = \frac{9}{10} = 0.9$

d) % accuracy = 0.9 × 100 = 90%

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Systematic error: A constant uniform deviation of the operation of an instrument is known as systematic error.

It is of three types:

i. Instrumental errors (wear & tear over a period of time)

ii. Environmental error (temp, pressure, humidity)

iii. Observational error

→ parallax error (instruments where needle is used while taking readings) (earlier, not in syllabus)

Gross error: It is due to human errors. ~~the~~ It can be avoided by taking more number of reading (min -3 readings).

Q. A voltmeter having a sensitivity of 1 kΩ/V is connected across an unknown resistance in series with a mA. The voltmeter reads 80 volts on 150 V scale. When the milli-ammeter reads 10mA. Calculate:

a) apparent resistance of the unknown resistor

b) actual resistance of the unknown resistor

c) error due to the loading effect of the voltmeter

Sol:- a) The total ckt resis, $R_T = \frac{V}{I} = \frac{80}{10 \times 10^{-3}} = 8 K\Omega$

b) The voltmeter resis, $R_V = 1000 \times 150 V = 15 \times 10^{4} \Omega$

$= 150 K\Omega$

$$\boxed{\text{Actual resistance} = \frac{R_V R_T}{R_V - R_T}}$$

$$= \frac{150 \cdot 8}{150 - 8}$$

$$= 8.45\ k\Omega$$

c) error due to loading effect $= \frac{Y_n - X_n}{Y_n} \times 100$

$$= \frac{8.45 - 8}{8.45} \times 100$$

$$= 0.053 \times 100$$

$$= 5.3\ \%$$

Q. In the above problem, if the milli-ammeter reads 600mA and the voltmeter reads 30V on scale of 150 Volts Calculate:

a) apparent resistance of the unknown resistor

b) actual resistance of the unknown resistor

c) error due to the loading effect.

Sol:- a) $R_T = \frac{V}{I} = \frac{30}{600 \times 10^{-3}} = 50\ \Omega$

b) $R_V = \cancel{10}\ 150\ k\Omega$

Actual resistance $= \frac{150 \cdot 50 \times 10^3}{150 \times 10^3 - 50} = \cancel{50.0167}\ 50.0167\ \Omega$

c) % error $= \frac{50.016 - 50}{50.016} \times 100 = 0.031\ \%$

→ It can be understood that when voltmeter ~~also~~ is connected across a resistor of high value then the error will also be more when compared to situation when voltmeter is connected across resistor of less value.

Statistical Analysis

Arithmetic mean ($\bar{x}$):

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots x_n}{n} \rightarrow ①$$

$$\bar{x} = \frac{\sum_{n=1}^{n} x_n}{n} \qquad n = \text{no. of terms}$$

Defn:

The most ~~prob~~ probable value of a measured variable is the arithmetic mean of the no. of readings taken.

→ ~~The~~ arithmetic mean of 'n' measurements at a specific count of the variable 'x' is given by formulae in equation ①.

Deviation from mean

→ It is the departure of the given reading from the arithmetic mean of the group of readings

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Average Deviation: It is defined as the sum of the absolute values of deviation divided by the number of deviations.

$$D_{av} = \frac{|d_1| + |d_2| + \dots |d_n|}{n}$$

$$D_{av} = \frac{\sum_{n=1}^{n} |d_n|}{n}$$

→ It indicates the precision of the instrument.

$$D_{av} \propto \frac{1}{\text{precision}}$$

Standard Deviation:

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + \dots d_n^2}{n}}$$

$$\sigma = \sqrt{\frac{\sum_{n=1}^{n} d_n^2}{n}}$$

→ It is also called as root mean square.
→ It gives the over-all performance of the instrument.
→ It is ~~inverss~~ inversely proportional to ~~pre~~ precision. i.e. reduction in standard deviation indicates improvement of the measurement.

Problems:

Q. For the following given data calculate

a) Arithmetic mean b) Deviation of each value

c) Algebraic sum of the deviations.

d) Average Deviation e) Standard Deviation

$x_1 = 49.7$, $x_2 = 50.1$, $x_3 = 50.2$, $x_4 = 49.6$, $x_5 = 49.7$

Sol:- a) Arithmetic mean $= \frac{\Sigma x_i}{n} = \frac{249.3}{5} = 49.86$

b) $D_1 = |x_1 - \bar{x}| = |49.7 - 49.86| = -0.16$

$D_2 = \cancel{|50.1 - 49.7| = 0.24}$ $50.1 - 49.86 = 0.24$

$D_3 = \cancel{|49.6 - 49.7| = 0.34}$ $50.2 - 49.86 = 0.34$

$D_4 = 49.6 - 49.86 = -0.26$

$D_5 = 49.7 - 49.86 = -0.16$

c) Sum of deviations $= -0.16 + 0.24 + 0.34 - 0.26 - 0.16 = 0$

d) Average deviation $= \frac{0.16 + 0.24 + 0.34 + 0.26 + 0.16}{5}$

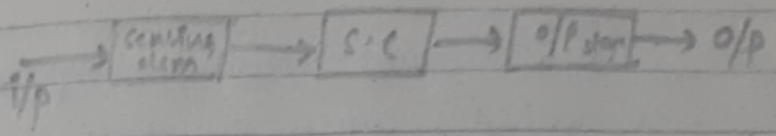
$= 0.232$

e) $\sigma = \frac{(0.16)^2 + (0.24)^2 + (0.34)^2 + (0.26)^2 + (0.16)^2}{5}$

$= \frac{0.292}{5}$

Dynamic characteristics

~~I/P~~ → ~~It~~ It is changing w.r.t time.



→ I/P ~~com~~ signals are usually step, linear (or) sinusodial

Factors affecting:

1. speed of response
2. lag
3. Fidelity
4. Dynamic error

Fidelity: It is the degree to which an instrument ~~responds~~ indicates the changes in the measured variable without dynamic error.

→ Fidelity is same as resolution, but for static it is resolution for dynamic it is fidelity, change in name only.

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Limiting errors

The economical production of any instrument requires the proper choice of material, design and skill.

The manufacture gaurentees certain accuracy; componants are gaurenteed to be within a certain percentage of the rated value. Therefore the manufacturer has to specify the deviations from the nominal value of a particular quantity.

The limits of these deviations from the specified values are defined as limiting errors

General equation relating i/p and o/p

$x_0 \rightarrow$ o/p quantity

$x_i \rightarrow$ i/p quantity

$$a_n \frac{d^n x_0}{dt^n} + a_{n-1} \frac{d^{n-1} x_0}{dt^{n-1}} + \dots a_1 \frac{dx_0}{dt} + a_0 x_0$$

$$= b_m \frac{d^m x_i}{dt^m} + b_{m-1} \frac{d^{m-1} x_i}{dt^{m-1}} + \dots b_1 \frac{dx_i}{dt} + b_0 x_i \rightarrow (1)$$

Except a_0 & b_0 all are '0'

$$a_0 x_0 = b_0 x_i$$

Any system which obey's this is named as zero order system.

$$x_0 = \left(\frac{b_0}{a_0}\right) x_i$$

$$\frac{b_0}{a_0} = k = \text{static sensitivity}$$

Ex:- potentiometer

First-Order System:

In Equation ①

Except a_1, a_0, b_0 all are zero's

$$a_1 \frac{dx_0}{dt} + a_0 x_0 = b_0 x_1 \longrightarrow ②$$

eq ② $\div a_0$

$$\frac{a_1}{a_0} \frac{dx_0}{dt} + x_0 = \frac{b_0}{a_0} x_i$$

$$\tau \frac{dx_0}{dt} + x_0 = k x_i$$

$$(\tau D + 1) x_0 = k x_i$$

$$\frac{x_0}{x_i} = \frac{k}{\tau D + 1}$$

Ex:- Relation b/w i/p and o/p in mercury in glass thermometer is first order system.

Second - order Instrument:

$$a_{n-2}\frac{d^{n-2}x_0}{dt^{n-2}} + \dots \; a_2\frac{d^2x_0}{dt^2} + a_1\frac{dx_0}{dt} + a_0x_0 = b_0x_i$$

except a_2, a_1, a_0 & b_0 all other parameters are zero.

$$a_2x_0'' + a_1x_0' + a_0x_0 = b_0x_i$$

Taking L.T on both sides

$$a_2s^2x_0(s) + a_1sx_0(s) + a_0x_0(s) = b_0x_i(s)$$

$$\frac{x_0(s)}{x_i(s)} = \left(\frac{D^2}{\omega_n^2} + \frac{2\xi D}{\omega_n} + 1\right)x_0 = Kx_i$$

$$\omega_n = \sqrt{\frac{a_0}{a_2}} = \text{undamped natural frequency}$$

$$2\xi = \frac{a_0}{\sqrt{a_0a_2}} = \text{damping ratio}$$

(zeta)

$$k = \frac{b_0}{a_0} = \text{static sensitivity}$$

Ex:- Spring balance

Standards:

1. International standard
2. Primary standards
3. Secondary standard
4. Working standard

International standard: They are agreements on common technical approaches that are used world-wide.