File Ref.No.Ac.C1/Faculty of Science/2021

COCHIN UNIVERSITY OF SCIENCE AND TECHNOLOGY

(Abstract)

Faculty of Science - Department of Mathematics - M.Sc Mathematics - Outcome Based Syllabus (OBE) - Approved - Orders issued.

ACADEMIC C SECTION

No.CUSAT/AC(C).C1/3927/2021 Dated,KOCHI-22,04.11.2021

Read:-Item No.I (i) of the minutes of the meeting of the Academic Council held on 28.07.2021.

ORDER

The Academic Council at its meeting held on 28.07.2021, vide minutes item read above, considered along with the recommendation of the Standing Committee, the minutes of the online meeting of the Board of Studies in Mathematics and resolved to approve the Outcome Based Syllabus (OBE) of M.Sc Mathematics offered at the Department of Mathematics, with effect from 2021 admission as in Appendix.

Orders are issued accordingly.

Dr. Meera V *
Registrar

To:

1. Dr.K Girish Kumar, Dean of Faculty of Science & Professor, Department of Applied Chemistry, CUSAT, Kochi-22.
2. Dr. Sasi Gopalan, Chairman, Board of Studies in Mathematics & Professor, Department of Mathematics, CUSAT, Kochi-22.
3. The Head, Department of Mathematics, CUSAT, Kochi-22.
4. PS to V.C/PS to PVC/PA to Registrar.
5. The Controller of Examinations/ The Director, Academic Admissions/ JR (Exams)/ DR (Exams/Academic Admissions).
6. Exam B/D/E/P/Y sections/ Academic A/C sections.
7. Day File/Stock File/File Copy.

* This is a computer generated document. Hence no signature is required.

Course Structure of MSC Mathematics 2021 admissions onwards

Semester I

Course Code	Name of the Course	Credits	Pre-Requisites
MAM2101	Linear Algebra	4	
MAM2102	Real Analysis	4	
MAM2103	Measure and Integration	4	
MAM2104	Groups and Rings	4	
MAM2105	Computational Mathematical Laboratory	4	
	VIVA VOCE	0	
Total Credits		20	

Semester II

Course Code	Name of the Course	Credits	Pre-Requisites
MAM2201	Fields and Modules	4	MAM 2104
MAM2202	Functional Analysis	4	MAM 2101-2103
MAM2203	Complex Analysis	4	MAM 2102
MAM2204	Topology I	4	MAM 2102
MAM2205	Functions of Several variables & Geometry	4	MAM 2101-2102
	VIVA VOCE	0	
Total Credits		20	

Semester III

Course Code	Name of the Course	Credits	Pre-Requisites
MAM2301	Operator Theory	4	MAM 2202
MAM2302	Topology II	4	MAM 2204
MAM2303	Ordinary Differential Equations & Integral Equations	4	
MAM2304	Probability Theory	4	MAM 2101-2103
	Elective I	3	
	VIVA VOCE	0	
Total Credits		19	

Semester IV

Course Code	Name of the Course	Credits	Pre-Requisites
MAM2401	Partial Differential Equations & Variational Problems	4	MAM 2303
	Elective II	4	
	Elective III	4	
	Elective IV	4	
	Elective V	4	
	*Project (Optional)	8*	
	VIVA VOCE	1	
Total Minimum Credits		21	
Minimum Credits for Pass		80	

*Project is optional to the students. The students opt for project shall start the work immediately after the second semester. The project is equivalent to two Electives in the fourth semester.

List of Elective Courses

1. MAM 2305 : TOPICS IN APPLIED MATHEMATICS
   (Inter-departmental elective)
2. MAM 2402 : WAVELETS
3. MAM 2403 : OPTIMIZATION & MATHEMATICAL METHODS FOR DEEP LEARNING
4. MAM 2404 : COMMUTATIVE ALGEBRA
5. MAM 2405 : GRAPH THEORY
6. MAM 2406 : ADVANCED LINEAR ALGEBRA
7. MAM 2407 : DISCRETE FRAMELETS
8. MAM 2408 : HARMONIC ANALYSIS
9. MAM 2409 : INTEGRAL TRANSFORMS
10. MAM 2410 : FUNCTIONS OF SEVERAL VARIABLES
11. MAM 2411 : ADVANCED SPECTRAL THEORY

12. MAM 2412 : BANACH ALGEBRAS AND SPECTRAL THEORY
13. MAM 2413 : NUMBER THEORY
14. MAM 2414 : REPRESENTATION THEORY OF FINITE GROUPS
15. MAM 2415 : ALGEBRAIC TOPOLOGY
16. MAM 2416 : DIFFERENTIAL GEOMETRY

Syllabus for MSC 2021 Admissions Onwards

MAM 2101 LINEAR ALGEBRA

Credits: 4

Objective: This course starts with the notion of vector spaces. Finite-dimensional vector spaces and maps between them preserving the structure are objects of study. The dual of a vector space also forms a major part of the study, especially with the study of the adjoint map. Studying the important multi-linear maps, like the Determinant map, form an important part of the course. Finally, the important primary decompositions of the vector space concerning a linear transformation is studied. This also helps to understand the extra symmetry in the representation of the matrices.

Learning Outcomes: After the completion of this course, the student should be able to

1. have a clear understanding of vector spaces, linear transformations, coordinates and the representation of transformation by matrices.

2. have a knowledge of the dual space of a vector space and importantly we also introduce the notion of the adjoint of a linear map which acts between the dual spaces.

3. understand the important generalization of the notion of linear maps to more than one variable. In particular the multi-linear Determinant map and its important properties are studied in details.

4. achieve ideas on the advanced topics like annihilating polynomials, simultaneous triangulation and diagonalization and direct sum decomposition.

5. have knowledge on primary decompositions associated with subspaces or with respect to a given operator.

UNIT 1: Review of system of linear equations and their solution set, Vector spaces, Subspaces, Bases and dimensions, Coordinates, Summary of row equivalence, Linear Transformations, The Algebra of Linear transformations, Isomorphism, Representation of Transformations by matrices.

UNIT 2: Linear functionals, The double Dual, The Transpose of a Linear Transformation, Inner product spaces, Linear functionals and Adjoints. (Sections 3.1, 3.2, 3.3 and Sections 8.1, 8.2, 8.3 from Hoffman and Kunze)

UNIT 3: Bilinear forms, Symmetric forms: Orthogonality, The geometry associated to a positive form, Hermitian forms (Chapter 7 Sections 1, 2, 3, 4 from Artin), Determinants-Commutative rings, Determinant functions, Permutations and the Uniqueness of determinants.

(Sections 5.1, 5.2, 5.3 from Hoffman and Kunze)

UNIT 4: Characteristic Values, Annihilating polynomials, Invariant subspaces, Simultaneous Triangulation, Simultaneous Diagonalization, Direct-Sum Decompositions, Invariant Direct Sums, The Primary Decomposition Theorem. (Chapter 6 of Hoffman and Kunze)

UNIT 5: The Rational and Jordan Forms- Cyclic Subspaces and Annihilators, Cyclic Decompositions and the Rational Form, The Jordan Form. (Sections 7.1, 7.2, 7.3 from Hoffman and Kunze)

Text Books:

1. Kenneth Hoffman and Ray Kunze *Linear Algebra*, Second Edition, PHI (1975).

2. M. Artin, Algebra, Prentice-Hall, (1991)

References:-

1. M. Artin, *Algebra*, Prentice-Hall, (1991).

2. Serge Lang, *Introduction to Linear Algebra*, Second Editon, Springer (1997).

3. K.T Leung, *Linear Algebra and Geometry*, Hong Kong University Press, (1974).

4. S.Kumaresan, *Linear Algebra: A Geometric Approach*, Fist Edition PHI Learning (2009).

5. Sheldon Axler, *Linear Algebra Done Right*, Second Edition, Springer, (1997).

6. Richard Kaye and Robert Wilson, *Linear Algebra*, Oxford University Press, (1998).

Syllabus for MSC 2021 Admissions Onwards

MAM 2102 REAL ANALYSIS

Credits: 4

Objective: This course starts with the structure of Real Numbers. This course is planned to introduce the notions Metric Spaces, Continuity, Uniform continuity, Differentiation, Riemann-Steiltjes integration, Fundamental theorem of Calculus, Convergence of sequence of functions, Uniform convergence, Stone-Weierstrass Theorem and Power series.

Learning Outcomes: This course is planned to build up calculus and other important notions on the set of real numbers. After the completion of this course, the student should able to be familiar with Metric Spaces, Continuity, Uniform continuity, Differentiation, Riemann-Steiltjes integration, Fundamental theorem of Calculus, Convergence of sequence of functions, Uniform convergence, Stone-Weierstrass Theorem and Power series.

UNIT 1: Metric Spaces; Definition and examples, open and closed sets in metric space, compactness, Connectedness, Continuity, Uniform continuity, discontinuity.(Chapter 2 and 4)

UNIT 2: Derivative: Derivatives and continuity, L Hospital Rules, Mean-Value theorem, Derivatives of vector-valued functions.(Chapter 5)

UNIT 3: The Riemann-Steiltjes integrals, Fundamental theorem of Calculus, Differentiation under integral signs, integration under vector valued function, rectifiable curves. (Chapter 6)

UNIT 4: Sequences and series of functions: Uniform convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation. (Chapter 7, sections upto 7.18)

UNIT 5: Equicontinuous families of functions, Stone-Weierstrass Theorem, Power series. (Chapter 7; sections upto 7.18-7.33, Chapter 8; sections up to 8.5)

Text Book: Walter Rudin, Principles of Mathematical analysis, 3rd edition, McGraw-Hill Higher Education (1976).

References:-

1. Terence Tao, Analysis I and II, Third Edition, Springer 2016.

2. N.L Carothers, Real Analysis, Wiley 2000.

3. Halsey L. Royden, Real Analysis, Prentice Hall, Upper Saddle River, NJ, (1988).

4. Tom M. Apostol, Mathematical Analysis, Addison-Wesley, Reading, MA, (1974).

5. A. K. Sharma, Real Analysis, Discovery publishing house Pvt. Lts., New Delhi, (2008).

6. D Somasundaram and B. Choudhary, A first course in mathematical analysis, Narosa, Oxford, London,(1996).

7. S Kumaresan, Topology of Metric Space, Alpha Science international Ltd, Harrow, UK, (2005)

8. K. A. Ross, Elementary Analysis; Theory of Calculus, Springer-Verlag,(2013).

Syllabus for MSC 2021 Admissions Onwards

MAM 2103 MEASURE AND INTEGRATION

Credits: 4

Objective: One of the objectives of measure theory is to make platform for developing tools for a new method of integration of functions that are not Riemann integrable. Apart from studying the Lebesgue measure and integration, this course introduces the concept of general measure spaces and the integration in this setting also.

Learning Outcomes: After the completion of this course, the student should be able to

1. be familiar with Lebesgue measure, General measure spaces.

2. be familiar with the new tools of integration of measurable functions.

Pre-requisites: Familiarity with complex numbers and basic calculus, Geometric ideas of school level.

UNIT 1: The Axiom of Choice, Zorn's Lemma, Lebesgue Outer measure, Measurable sets and Lebesgue measure, Non measurable sets (Chapter 2 and relevant sections of Preliminaries of the text)

UNIT 2: Lebesgue measurable functions: Litlewood's Three Principles, The Riemann Integral, The Lebesgue Integral (Chapters 3 and 4 of the text, upto section 4.3)

UNIT 3: The General Lebesgue Integral, Continuity of Integration, Convergence in Measure, Characterizations of Riemann and Lebesgue integrability, Differentiation of monotone functions, Lebesgue's theorem, Functions of bounded variations: Jordan's Theorem (avoid proofs of Vitali Covering lemma and Lebesgue's theorem). (Section 4.4-4.5, 5.2-5.3 and 6.1-6.3 of the text)

UNIT 4: Differentiation of an integral, Absolute continuity, Convex Functions, The L^p spaces, Minkowski and Hölder inequalities, (Section 6.4-6.6 and 7.1-7.2 of the text)

UNIT 5: Completeness of L^p spaces, Approximation and Separability, The Riesz Representation for the Dual of L^p spaces (Section 7.3-7.4 and 8.1 of the text)

Text Book: H L Royden, P. M. Fitzpatrick, Real Analysis, Fourth Edition (2009), PHI

References:-

1. I K Rana, An Introduction to Measure and Integration, Narosa Publishing Company.

2. P R Halmos, Measure Theory, GTM , Springer Verlag.

3. T.W. Gamelin, Complex Analysis, Springer.

4. R.G. Bartle, The elements of Integration (1966) John Wiley & Sons, Delhi,(2006)

5. K B. Athreya and S N Lahiri:,Measure theory, Hindustan Book Agency, New Delhi.

6. Thamban Nair, Measure and Integration: A First Course, CRC Press, 2019.

7. Terence Tao: An Introduction to Measure Theory,Graduate Studies in Mathematics,Vol 126 AMS.

8. S. Kesavan Measure and Integration, Hindustan Book Agency, Springer (TRIM 77).

Syllabus for MSC 2021 Admissions Onwards

MAM 2104 - GROUPS AND RINGS

Credits: 4

Objective: This course starts with the basic algebraic structure Group, and studies various aspects of groups. It also covers another mathematical structure Rings and various types of rings.

Learning Outcomes: After the completion of this course, the student should be able to

1. have a working knowledge of the concepts such as definition of a group, order of a finite group and order of an element.
2. have a clear understanding of different types of subgroups such as normal subgroups, cyclic subgroups, and understand the structure of the structure of these subgroups
3. will be able to understand the mathematical concepts such as permutation groups, factor groups, group homomorphisms etc.
4. will have knowledge on advanced topics such as Sylows theorem and should be able to apply this result.
5. will be able to understand other mathematical structures such as rings and various classes of rings, their sub structures ideals, and their homomorphisms.

UNIT 1: Introduction to Groups: Basic Axioms and Examples, Dihedral Groups, Symmetric Groups, Matrix Groups, The Quaternion Group, Homomorphisms and Isomorphisms, Subgroups: Definitions and Examples, Centralizers and Normalizers, Stabilizers and Kernels, Cyclic groups, Groups generated by subsets of a Group.

UNIT 2: Quotient Groups and Homomorphisms: Quotient Groups, homomorphisms, Langanges Theorem, The Isomorphism Theorems, Composition Series and Holder Program, Transpositions and Alternating Group

UNIT 3: Group Actions: Group actions and permutation representations, Cayleys Theorem, Orbits, Counting Lemma, Class Equation, Automorphisms, Sylow Theorems, Simplicity of A_n.

UNIT 4: Rings : Definition, Examples, Rings of Continuous Functions, Matrix Rings, Polynomial Rings, Power series rings, Laurent Rings, Boolean Rings, Direct Products, Several Variables, Characteristic of a ring.

Ideals: Definitions, Maximal ideals, generators, basic properties of ideals, algebra of ideals, quotient rings, ideals in quotient rings, local rings.

UNIT 5: Homomorphisms of rings: Definitions and basic properties, fundamental theorems, endomorphism rings, field of fractions, prime fields.

Factorisation in domains: Division in domains, Euclidean domain, Principal ideal domains, factorisation domains, Unique factorisation domains, Eisensteins criterion.

Text Books:

1. Abstract Algebra - D.S. Dummit and R.M. Foote, 3rd Edition, Publisher: Wiley.

2. Rings and Modules - C. Musili, Second revised edition, Narosa Publishing House.

References:-

1. A First Course in Abstract Algebra - J.B. Fraleigh, 7th Edition, Publisher - Pearson

2. Algebra - M. Artin, Second Edition, Publisher - Pearson

3. Contemporary Abstract Algebra - J. A. Gallian, 4th Edition, Publisher - Narosa

4. Topics in Algebra - I.N. Herstein, Second Edition, Publisher - Wiley Student Edition.

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MAM 2105 COMPUTATIONAL MATHEMATICAL LABORATORY

Credits: 4

Objective: This course starts with the review of Numerical methods for differentiation and integration, and simple models of Partial differential equations. This course is planned to introduce the basics of mathematical documention setting using LATEX. Introduction of programming using Python for solving Mathematical problems arising in various fields, that are covered in the Msc curriculum.

Learning Outcomes: After the completion of this course, the student should be able to Be familiar with the skill to prepare mathematical documnets in LATEX and python programming techniques which are focused to be applied in mathematical problems.

UNIT 1: Introduction to LATEX Documentation setting, Standard document classes, Bibtex, standard environments, Macros, Table of contents, Bibliography styles, tables, Pstricks, Multiline math displays (Texts 1, 2)

UNIT 2: Introduction to programming with Python, Fundamentals, Data types, Functions, Pointers and string handling, Class, File handling, Programming Excercises from Linear Algebra, Number Theory, Numerical Approximations, Differential Equations. (Texts 3, 4, 5 , 6)

UNIT 3: Matplotlib, Numpy, and Scipy Exercises. (Texts 7, 3)

UNIT 4: Introduction to SageMath, Symbolic Calculus, Linear Algebra using SageMath, SageTex Package, Graphics, Combinatorics, Graph Theory (Text 8).

UNIT 5: Coding Theory using SageMath, Standard Rings and Fields (Text 8)

References:-

1. George *Grätzer*, *Math into LATEX an Introduction to LATEX and AMS-LATEX*, Birkhauser Boston, (1996).

2. Donald. E. Knuth, *Computers & Type setting*, Addison-Wesley, (1986).

3. Hans Petter Langtangen, *A Primer on Scientific Programming with Python*, Third Edition, Springer (2012).

4. John M. Zelle, *Python Programming: An Introduction to Computer Science*, (2002).

5. Steven Lott, *Functional Python Programming*, Packt Publishing Ltd, (2015).

6. Jody. S. Ginther Start here: Python programming made simple for the Beginner.

7. John Hunter, Darren Dale, Eric Firing, Michael Droettboom, *Matplotlib Release 1.4.3*.

8. William Stein, *SAGE Reference Manual Release 2007.10.29*.

NB: A Lab Report type-setted in LaTeX by the student has to be submitted at the end of the semester.

Syllabus for MSC 2021 Admissions Onwards

MAM 2201 FIELDS AND MODULES

Credits: 4

Objective: This course starts with the advanced topics in Group theory. It also covers other mathematical structures Modules and Fields.

Learning Outcomes: After the completion of this course, the student should be able to

1. have a working knowledge of the advanced concepts of group theory such as direct products, semi-direct products.
2. should be able to classify the groups of small orders using the advanced concepts such as semi-direct products and direct products.
3. understand the concept of algebraic structures called modules and recognize various types of modules.
4. use the terminology and concepts of Field theory and apply those in a problem-solving approach.
5. to apply the group-theoretic information to deduce results about fields and polynomials.

UNIT 1: Direct and Semi-direct Products and Abelian Groups: Direct products, Fundamental theorem of finitely generated abelian groups, Groups of small order, Recognizing direct products, Semi-direct Products.

UNIT 2: $p-$groups, nilpotent groups, solvable groups, applications in groups of medium order, free groups.

UNIT 3: Modules: Definitions and Examples, direct sums, free modules, vector spaces, quotient modules, homomorphisms, simple modules, modules over PIDs.

UNIT 4: Fields: Irreducible polynomials, Classical Formulas, Splitting Fields, Finite fields, The Galois group, roots of unity, solvability by radicals.

UNIT 5: Fields: Independence of characters, Galois extensions, The fundamental theorem of Galois theory, Applications.

Text Books:

1. Abstract Algebra - D.S. Dummit and R.M. Foote, 3rd Edition, Publisher: Wiley.
2. Rings and Modules - C. Musili, Second revised edition, Narosa Publishing House.
3. Galois Theory - J. Rotman, Second Edition, Springer International Edition.

References:-

1. A First Course in Abstract Algebra - J.B. Fraleigh, 7th Edition, Publisher - Pearson
2. Algebra - M. Artin, Second Edition, Publisher - Pearson
3. Contemporary Abstract Algebra - J. A. Gallian, 4th Edition, Publisher - Narosa Publishing
4. Topics in Algebra - I.N. Herstein, Second Edition, Publisher - Wiley Student Edition

Syllabus for MSC 2021 Admissions Onwards

MAM 2202 FUNCTIONAL ANALYSIS

Credits: 4

Objective: This is the first part of the series of 2 courses taught in the second and third semester on Functional Analysis. In the first part, we cover important structures used in analysis like Banach spaces, Hilbert spaces and operators acting on them. The foundation results are discussed in this part.

Learning Outcomes: After the completion of this course, the student should be able to be familiar with the concepts of Banach spaces, Hilbert spaces and operators acting on them.

Pre-requisites:

1. A first course in linear algebra
2. Basic real analysis and topology

UNIT 1: Review of Linear Spaces and Linear Maps, Metric Spaces and Continuous Functions, Lebesgue Measure and integration on R. (Chapter I, Section 2, 3, and 4; excluding the proofs of 2.1, 2.3, 3.4, 3.5, 3.9 and 3.10).

UNIT 2: Normed Spaces, Continuity of Linear Maps, Hahn-Banach Theorems (Chapter II, Section 5, 6, 7; upto Theorem 7.11).

UNIT 3: Banach Spaces., Uniform Boundedness Principle, Closed Graph and Open Mapping Theorem, Bounded Inverse Theorem. (Chapter III, Section 8, 9 upto Theorem 9.4, Section 10).

UNIT 4: Bounded Inverse Theorem, Inner Product Spaces, Orthonormal Sets. (Chapter III: Section 11, Chapter VI: Section 21, 22)

UNIT 5: Duals and Transpose. Duals of $L^p([a,b])$ and $C([a,b])$. (Chapter IV, Section 13, 14; upto Theorem 14.5).

Text Book: Balmohan V. Limaye, *Functional Analysis*, Revised Second Edition, New Age International Publishers, 1996 (Reprint 2013)

References:-

1. Courant, R. and D. Hilbert, Methods of Mathematical Physics, vol. I, Interscience, Newyork (1953).
2. Dunford N. and T. Schwartz, Linear Operators, Part I, Interscience, Newyork (1958).
3. E. Kreyzig, Introduction to Function Analysis with Applications, Addison Wesley.
4. Rudin W., Real and Complex Analysis, 3rd edition, McGraw-Hill, Newyork (1986).

5. Rudin W., Functional Analysis, 2nd edition, McGraw-Hill, Newyork (1991).

6. Reed, M. and B. Simon, Methods of Mathematical Physics, vol. II, Academic Press, Newyork (1975).

7. Rajendra Bhatia, Notes on Functional Analysis, Texts and Readings in Mathematics, Hindusthan Book Agency, New Delhi(2009).

8. G. F. Simmons, Introduction to Topology and Modern Analysi,s TMH.

9. M. Thamban Nair, Functional Analysis; A first course, PHI Learning Pvt. Ltd (2001).

MAM 2203 COMPLEX ANALYSIS

Credits: 4

Objective: This course starts with the review of complex functions which will be followed by the Classical theory of analytic functions. This will involve some of the classical theorems in the subject such as Cauchys integral formula and its general forms.

Learning Outcomes: After the completion of this course, the student should be able to

1. be familiar with the Conformal mapping, Linear transformations, Analytic functions and the classical results in this regard.

2. use the results like residue theorems to compute integrals and apply to various fields.

Pre-requisites: Familiarity with complex numbers and basic calculus, Geometric ideas of school level.

UNIT 1: The field of complex numbers, The complex plane, Polar representations and roots of complex numbers, Lines and half planes in complex plane, The extended plane and its spherical representations, Power series, Analytic functions and Analytic functions as mapping and Mobius transformations. [Chapter - I (Sections - 2,3,4,5,6), Chapter - III (Sections - 1,2,3)]

UNIT 2: Riemann-Stieltjes integrals, Power series representation of analytic functions, Zeros of an analytic function and The index of a closed curve [Chapter - IV (Sections - 1,2,3,4)].

UNIT 3: Cauchys Theorem and Integral Formula, The homotopic version of Cauchys Theorem and simple connectivity, Counting zeros; the Open Mapping Theorem and Goursats Theorem [Chapter - IV (Sections - 5,6,7,8)].

UNIT 4: Classification of singularities, Residues and The Argument Principle [Chapter - V (Sections - 1,2,3)].

UNIT 5: The Maximum Principle, Schwarzs Lemma, Convex functions and Hadamards Three Circles Theorem and Phragmen-Lindelof Theorem [Chapter - VI (Sections - 1,2,3,4)].

Text Book: J.B. Conway, Functions of One Complex Variable (2^{nd} Edition), Springer 1973.

References:-

1. L.V. Ahlfors, Complex Analysis (Third Edition) Mc-Graw Hill International (1979)

2. Milnor, Dynamics in One Complex Variable (3rd ed.), Princeton U. Press.

3. T.W. Gamelin, Complex Analysis, Springer

4. H. A. Priestley: Introduction to Complex Analysis, Oxford University Press.

5. J.H. Mathews and R.W. Howell: Complex Analysis for Mathematics and Engineering, Jones & Bartlett Learning.

Syllabus for MSC 2021 Admissions Onwards

MAM 2204 TOPOLOGY I

Credits: 4

Objective: Topology is essentially the study of surfaces in which normally non geometric properties are studied. This course introduces the basic concepts of topology and standard properties such as compactness connectedness, separation axioms.

Learning Outcomes: On completion of this course, the student should be able to

1. understand topological properties
2. understand the connection of topology with other branches of mathematics
3. apply topological properties to prove theorems.

Pre-requisites: Basic ideas of Set Theory, Basic concepts of Real Analysis and Metric Spaces.

UNIT 1: Topological Spaces: Logical warm up, Motivation for topology, Definition of topological spaces, examples, Bases and Sub bases, Subspaces. (Chapter 3 & 4 of Text 1)

UNIT 2: Basic Concepts: Closed sets and Closure, Neighbourhoods, Interior and Accumulation Points, Continuity and Related Concepts, Making functions continuous and Quotient Spaces (Chapter 5 of Text 1)

UNIT 3: Spaces with special properties: Smallness conditions on a space, Connectedness, Locally connectedness and paths. (Chapter 6 of Text 1)

UNIT 4: Separation axioms: Hierarchy of separation axioms, Compactness and separation axioms, Urysohn's characterization of normality, Tietze extension Theorem. (Chapter 7 of Text 1)

UNIT 5: Product and Coproducts: The Cartesian product of family of sets, product topology, productive properties, Embedding Lemma, Embedding theorem and Urysohns Metrization Theorem. (Relevant sections of Chapter 8 & 9 of Text 1)

Text Book: K.D. Joshi: Introduction to General Topology (Revised Edn.), New Age International (P) Ltd., New Delhi, Revised printing in 1984.

References:-

1. G.F. Simmons: Introduction to Topology and Modern Analysis; McGraw-Hill International Student Edn.; 1963
2. J. Dugundji: Topology; Prentice Hall of India; 1975
3. J. R. Munkers; Topology (Second Edition) PHI, 2009.

4. M. Gemignani: Elementary Topology; Addison Wesley Pub Co Reading Mass; 1971

5. M.A. Armstrong: Basic Topology; Springer- Verlag New York; 1983

6. M.G. Murdeshwar: General Topology (2nd Edn.); Wiley Eastern Ltd; 1990

7. S. Willard: General Topology; Addison Wesley Pub Co., Reading Mass; 1976

8. John Gilbert Hocking and Gail S. Young, Topology (Revised Edition), Dover Publications, (1988).

MAM 2205 FUNCTIONS OF SEVERAL VARIABLES AND GEOMETRY

Credits: 4

Objective: In the first module, the students will be introduced to multivariable functions in Euclidean spaces and the notion of differentiation. The second module is aimed to apply the notions of multi-variable differentiation and associated local properties to regular curves and surfaces. Differentiable manifolds are introduced in the third module. In the fourth module the notions of geometry are introduced. The Riemannian metric structure on a differentiable manifold is introduced for conceptual clarity. The first fundamental form on regular surfaces is introduced first, after which comes orientation and the Gauss map. The Gauss map for regular surfaces is studied in details culminating in the concept of Gaussian curvature along with applications of the Gauss-Bonnet theorem. Finally the standard concepts in geometry of parallel transport, geodesics and the exponential map are also studied.

Learning Outcomes: After completion of this course, the students shall learn

1. Have a clear understanding about continuity and differentiability of functions of several variables and their applications.

2. Application of these concepts to regular curves and surfaces in Euclidean spaces.

3. Develop understanding of tangent planes to regular surfaces and then differentiable manifolds are introduced.

4. Different examples of manifolds, the concept of orientation and vector fields on such manifolds are studied.

5. Riemannian structure on a differentiable manifold is introduced which makes the study of geometry on regular surfaces in $\mathbb{R}^3$ more clear conceptually.

6. Special emphasis is laid on the study of the Gauss map culminating with the Gaussian curvature for regular surfaces in $\mathbb{R}^3$. Gauss-Bonnet theorem and its applications are studied in details.

7. Other important geometric concepts that are studied include the first fundamental form, parallel transport, geodesics and the exponential map.

Pre-requisites:

1. Basic real analysis and Linear Algebra

UNIT 1: Norm and inner product, subsets of Euclidean spaces, functions and continuity, (Differentiation in several variables), Basic definitions, basic theorems, partial derivatives, derivatives. (Relevant sections from chapters 1, 2 of textbook 1)

UNIT 2: Inverse functions, Implicit functions (Chapter 2 of textbook 1), Regular curves, The local theory of curves parametrised by arc length, The local canonical form, Regular surfaces, Change of parameters, The tangent plane (Sections 1.3, 1.5, 1.6, 2.2, 2.3, 2.4 of textbook 2).

UNIT 3: Introduction to differentiable manifolds, tangent space of differentiable manifolds, Immersions and embeddings, other examples, Orientation, vector fields, brackets, topology of manifolds (Chapter 0 of textbook 3).

UNIT 4: Introduction to Riemannian metrics, Riemannian metrics (Chapter 1 of textbook 3), The first fundamental form (Area), Orientation of Surfaces, The definition of the Gauss map and its fundamental properties, The Gauss map in local coordinates. (Sections 2.5, 2.6, 3.2, 3.3 of textbook 2).

UNIT 5: The Gauss theorem and the equations of compatibility, Parallel transport, Geodesics, The Gauss-Bonnet theorem and its applications, The exponential map, Geodesic polar coordinates. (Sections 4.3, 4.4, 4.5, 4.6 of textbook 2).

Text Book:

1. Michael Spivak: *Calculus on Manifolds A modern approach to classical theorems of advanced calculus*, Addison-Wesley Publishing house, 1965.

2. Manfredo P. Do Carmo: *Differential geometry of curves and surfaces*, Dover Publications, Second edition, 2016.

3. Manfredo P. Do Carmo: *Riemannian Geometry*, Birkhauser, 1993.

References:-

1. Andrew Pressley: Elementary Differential Geometry, Springer, 2000.

2. Theodore Shifrin: Differential Geometry: A first course in curves and surfaces, 2016.

MAM 2301 OPERATOR THEORY

Credits: 4

Objective: This is the second part of the series of 2 courses taught in the second and third semester on Functional Analysis. In the second part, we focus on compact operators on Banach spaces, Hilbert spaces and their spectral properties.

Learning Outcomes: After the completion of this course, the student should be able to be familiar with the spectral theory of compact self-adjoint operators and its applications.

Pre-requisites:

1. A first course in functional analysis
2. Basic real analysis and topology

UNIT 1: Spectrum of a Bounded Operator, Weak and Weak* Convergence, Reflexivity. (Chapter III, Section 12, Chapter IV, Section 15, upto Theorem 15.5, Chapter IV: Section 16 excluding the proof of Theorem 16.5).

UNIT 2: Compact Linear Maps, Spectrum of a Compact Linear Map. (Chapter V, Section 17, 18).

UNIT 3: Fredholm Alternative, Approximate Solutions, Normal, Unitary and Self-Adjoint Operators (Chapter V, Section 19, 20, upto Theorem 20.4, Chapter VII: Section 26).

UNIT 4: Approximation and Optimization, Projection and Riesz Representation Theorems. Bounded Operators and Adjoints. (Chapter VI: Section 23, 24, 25)

UNIT 5: Spectrum and Numerical Range, Compact Self-adjoint Operators, Sturm-Liouville Problems. (Chapter VII, Section 28, Appendix C).

Text Book: Balmohan V. Limaye, *Functional Analysis*, Revised Second Edition, New Age International Publishers, 1996 (Reprint 2013)

References:-

1. Courant, R. and D. Hilbert, Methods of Mathematical Physics, vol. I, Interscience, Newyork (1953).
2. Dunford N. and T. Schwartz, Linear Operators, Part I, Interscience, Newyork (1958).
3. E. Kreyzig, Introduction to Function Analysis with Applications, Addison Wesley.
4. Rudin W., Real and Complex Analysis, 3rd edition, McGraw-Hill, Newyork (1986).

5. Rudin W., Functional Analysis, 2nd edition, McGraw-Hill, Newyork (1991).

6. Reed, M. and B. Simon, Methods of Mathematical Physics, vol. II, Academic Press, Newyork (1975).

7. Rajendra Bhatia, Notes on Functional Analysis, Texts and Readings in Mathematics, Hindusthan Book Agency, New Delhi(2009).

8. G. F. Simmons, Introduction to Topology and Modern Analysi,s TMH.

9. M. Thamban Nair, Functional Analysis; A first course, PHI Learning Pvt. Ltd (2001).

Syllabus for MSC 2021 Admissions Onwards

MAM 2302 TOPOLOGY II

Credits: 4

Objective: With this course, the students will have a sound introductory knowledge of the topics in Algebraic topology. The first module is important to understand the topology of non-metric spaces. From second module onwards the student is gradually introduced to the important category of topological spaces and subsequently the algebraic machinery like simplicity homology and fundamental groups for their study. The course ends with a rigorous understanding of covering spaces.

Learning Outcomes: After completion of this course, the students shall learn

1. About nets and filters, the generalisation of sequences for topologies that are no more defined by a metric.
2. The important geometric objects like complexes and Polyhedra and different identification spaces whose topology is studied.
3. The definition of simplicial homology groups and their application to compute the homology groups for certain important spaces.
4. The fundamental group and the Van Kampen theorem with examples.
5. Covering spaces their properties along with their classification.

UNIT 1: Nets and Filters: Definition and convergence of Nets, Topolgy and convergence of Nets, Filters and their convergence, Ultra filters (Tychnoffs theorem) (Relevant Sections from text 1)

UNIT 2: Geometric Complexes and Polyhedra: Introduction. Examples, Geometric Complexes and Polyhedra, Orientation of geometric complexes. **Simplicial Homology Groups:**Chains, cycles, Boundaries and homology groups, Examples of homology groups, The structure of homology groups, (Sections 1.1 to 1.4, Sections 2.1 to 2.3 from text 2)

UNIT 3: Simplicial Homology Groups (Contd.): The Euler Poincares Theorem,Pseudomanifoldsand the homology groups of S_n. **Simplicial Approximation:**Introduction, Simplicial approximation, Induced homomorphisms on the Homology groups, The Brouwer fixed point theorem and related results (Sections 2.4, 2.5, and Sections 3.1 to 3.4 from text 2)

UNIT 4: The Fundamental Group: Introduction, Homotopic Paths and the Fundamental Group, The Covering Homotopy Property for S1, Examples of Fundamental Groups. (Sections 4.1 to 4.4 from text 2)

UNIT 5: Covering Spaces: The Definition and Some Examples, Basic Properties of Covering Spaces, Classification of Covering Spaces, Universal Covering Spaces, Applications (Sections 5.1 to 5.5 of text 2)

Text Books:

1. K.D. Joshi: Introduction to General Topology (Revised Edn.),New Age International(P) Ltd., New Delhi, 1983.
2. F.H. Croom: Basic Concepts of Algebraic Topology, Springer, 1978

References:-

1. Allen Hatcher: Algebraic Topology, Cambridge University Press, 2002
2. C.T.C. Wall: A Geometric Introduction to Topology,Addison-WesleyPub. Co. Reading Mass, 1972
3. Eilenberg S, Steenrod N.: Foundations of Algebraic Topology, Princeton Univ. Press, 1952.
4. J. R. Munkers: Elements Of Algebraic Topology, Perseus Books, Reading Mass, 1993, CRC, 2018.
5. J. R. Munkers: Topology (Second Edition) PHI, 2009.
6. Massey W.S.: Algebraic Topology : An Introduction, Springer Verlag NY, 1977
7. S.T. Hu: Homology Theory,Holden-Day,1965

MAM 2303 Ordinary Differential Equations & Integral Equations

Credits: 4

Objective: This course starts with the review of Ordinary differential equations. Course aims to build an understanding of the classical models in terms of ordinary differential equations and pave the foundations for the study of Integral equations.

Learning Outcomes: Students will be able to understand the popular mathematical models of real life problems in terms of ordinary differential equations and Integral equations.

UNIT 1: Oscillations and the Sturm Separation Theorem, The Sturm Comparison Theorem, Series solutions of First order equations, Second order Linear Equations, Gausss Hyper Geometric Equation. (Chapter 4, Section 24, 25. Chapter 5, sections 27, 28, 29, 30, 31.)

UNIT 2: Legendre Polynomials, Properties of Legendre Polynomials, Bessel Polynomials, Properties of Bessel Polynomials. (Chapter 8, sections 44, 45, 46, 47.)

UNIT 3: Systems, Nonlinear equations: Autonomous systems, The Phase Plane and its Phenomena, Types of Critical points. Stability, Critical points and Stability for Linear Systems. (Review Chapter 10, Chapter 11, Sections 58, 59,60)

UNIT 4: Method of successive approximations, Picard's Theorem, Integral Equations with separable kernels, Fredholm Integral Equations, Method of successive approximations. (Chapter 13, sections 68, 69 of text 1, Chapter 2 and 3 of the text 2.)

UNIT 5: The Fredholm Method of Solution, Fredholm's Theorems, Applications to Ordinary Differential Equations. (Chapters 4, 5 of the text 2)

Text Books:

1. George F. Simmons, *Differential Equations with Applications and Historical Notes*, Tata McGraw-Hill, Third Editon 2003.
2. Ram P. Kanwal, *Linear Integral Equations*, Second Edition, Springer Science+Business Media, LLC, (1997).

References:-

1. Peter J. Collins, *Differential and Integral Equations*, Oxford University Press,(2006).
2. Carmen Chicone, *Ordinary Differential Equations with Applications*, Springer (2006).
3. Linear Integral Equations
4. Michael D. Greenberg, *Ordinary Differential Equations*, Wiley (2012).

5. Michael E. Taylor, *Introduction to Differential Equations*, AMS (2011).

6. Vladimir I. Arnol'd, *Ordinary Differential Equations*, Springer (1992).

7. Earl A. Coddington, *An Introduction to Ordinary Differential Equations*, Dover Publications, New york, (1961).

Syllabus for MSC 2021 Admissions Onwards

MAM 2304: Probability Theory

Credits: 4

Objective: This course starts with the introduction to probability theory following different probability distributions. The connection between probability theory and measures are also discussed in this course. This will involve some of the classical theorems in the subject such as central limit theorem and law of large numbers.

Learning Outcomes: After the completion of this course, the student should be able to

1. be familiar with the concepts of probability theory and classical results.
2. use the terminology and concepts of probability theory and apply those in a problem-solving approach.

Pre-requisites:

1. A first course in measure theory.
2. Basic real analysis and topology.

UNIT 1: Recalling Probability: Sample Space, events and probability, Independence and conditioning, Discrete random variables, The branching process, Borels strong law of large numbers (Chapter 1)

UNIT 2: Integration: Measurability and measure, The Lebesgue integral, The other big theorems (Chapter 2)

UNIT 3: Probability and Expectation: From integral to expectation, Gaussian vectors, Conditional expectation (Chapter 3)

UNIT 4: Convergences Almost-sure convergences, Two other types of convergence, Zero-one laws (Chapter 4, section 4.1-4.3)

UNIT 5: Convergence continued: Convergence in distribution and in variation, Central Limit Theorem, The hierarchy of convergences (Chapter 4, section 4.4-4.6)

Text. Pierre Bremaud, Probability Theory and Stochastic Processes, Springer 2020.

References:-

1. S.R. Athreya, V.S. Sunder: Measure and Probability, University Press (India) Pvt. Ltd. (2008).
2. Sidney I Resnick: A Probability Path, Birkhauser 2005 Edition

3. A.K. Basu: Probability Theory, Prentice Hall, India, 2002.

4. W. Feller: An Introduction to Probability Theory and Its Applications.

Syllabus for MSC 2021 Admissions Onwards

MAM 2401 PARTIAL DIFFERENTIAL EQUATIONS & VARIATIONAL CALCULUS.

Credits: 4

Objective: This course starts with simple models of Partial differential equations which will be followed by the analytic and algebraic study of PDEs. This will involve some of the classical models in the subject: diffusion equations and wave equations. Towards the end of the course students will get an idea of variational calculus.

Learning Outcomes: After the completion of this course, the student should be able to be familiar with the concepts of classical models of diffusion and wave phenomena. Able to use the terminology and concepts of PDE's and apply those in a problem-solving approach.

UNIT 1: Classification of First-Order Equations, Construction of a First-Order Equation, Geometrical Interpretation of a First-Order Equation, Method of Characteristics and General Solutions, Canonical Forms of First-Order Linear Equations, Method of Separation of Variables (Chapter 2 of Text 1).

UNIT 2: The Vibrating String, The Vibrating Membrane, Waves in an Elastic Medium, Conduction of Heat in Solids, Second-Order Equations in Two Independent Variables, Canonical Forms, Equations with Constant Coefficients, The Cauchy Problem, Charpit's method. (Chapter 3, sections 3.2-3.5, Chapter 4 of Text 1, Sections 5.1-5.4.).

UNIT 3: Eigenvalue Problems and Special Functions, Sturm–Liouville Systems, Eigenfunction Expansions, Completeness and Parseval's Equality, Bessel's Equation and Bessel's Function (Sections 8.1-8.6 of the Text 1).

UNIT 4: Variation and its properties, Euler equation, Functionals involving higher order derivatives, Functionals involving partial derivatives, Variational problems with movable boundaries. (Chapter 1, 2 of text 2).

UNIT 5: Sufficiency condition for an extremum, Variational problems with constrained extrema, isoperimetric problems, Direct methods, Eulers method of finite differences, Ritz method. (Chapter 3, 4, 5 of text 2).

Text 1. Tyn Myint-U, Lokenath Debnath *Linear Partial Differential Equations for scientists and Engineers*, Fourth Edition, Birkhauser (2007).

Text 2. Lev D. Elsgolc, *Calculus of Variations*, Dover publications, Inc. (2007.)

References:-

1. Walter A. Strauss, *Partial Differential Equations an Introduction*, John Wiley, (1992).

2. Ravi P. Agarwal, Donal O'Regan, *Ordinary and Partial Differential Equations With Special Functions, Fourier Series, and Boundary Value Problems*, Springer-Verlag (2009).

3. Fritz. John, *Partial Differential Equations*, Fourth Edition, Springer (2009).

4. G. Evans, I. Blackedge and P.Yardley, *Analytic Methods for Partial Differential Equations*, Springer (1999).

5. Ian N. Sneddon, *Elements of Partial Differential Equations*, McGraw Hill (1983).

MAM 2305 TOPICS IN APPLIED MATHEMATICS

Credits: 3

Objective: To Learn important Mathematical Tools applicable in Science and Technology.

Learning Outcomes: After the completion of this course, the student should able to be

1. familiar with the necessary mathematical tools that are used in science and technology.

2. failiar with the popular transforms of Laplace and Fourier and their applications to various fields.

3. familiar with the popular mathematical models like vibating string, Heat conduction etc. and its solution using transforms.

4. familiar with the necessary machinery in complex function theory.

UNIT 1: Second order Linear ODEs, Homogeneous Linear ODEs of Second Order, Homogeneous Linear ODEs with Constant Coefficients, Euler-Cauchy Equations.

UNIT 2: Laplace Transform, Linearity, First Shifting Theorem (s-Shifting), Transforms of Derivatives and Integrals ODEs, Unit Step Function (Heaviside Function), Second Shifting Theorem (t-Shifting)

UNIT 3: Fourier Series, Arbitrary Period, Even and Odd Functions, Half-Range Expansions, Forced Oscillations, Fourier Integral, Fourier Cosine and Sine Transforms, Fourier Transform.

UNIT 4: Basic Concepts of PDEs, Modeling: Vibrating String, Wave Equation, Modeling: Heat Flow from a Body in Space, Heat Equation

UNIT 5: Complex Numbers: Preliminary requirements, limits, Continuity, Cauchy-Reimann equations, Complex Integration, Line Integral in the complex plane, Cauchy's Integral Theorem, Cauchy's Integral formula, Derivatives of Analytic functions, Laurent Series, Singularities and zeros, Residue Integration method, Residue Integration of real Integrals.

Text Book: Advanced Engineering Mathematics, Erwin Kreyszig, 10^{th} edition, JOHN WILEY & SONS, INC.2011. (Chapter 2, Section 2.1-2.3, and 2.5, Chapter 6, Section 6.1-6.4, Chapter 11, Section 11.1-11.3, 11.7,11.8, Chapter 12, Section 12.1-12.6, Chapter 14, Section 14.1-14.4, Chapter 16, Section 16.1-16.4.)

References:-

1. Advanced Engineering Mathematics, C.Ray Wylie, Louis. C. Barrett, 6th edition, McGraw Hill Publishing, 1998.

2. Advanced Engineering Mathematics, K.A Stroud, 5th edition, Palgrave Macmillain, 2003.

3. Advanced Engineering Mathematics, Michael Greenberg, 2nd edition, Prentice Hall, 1998.

4. Advanced Engineering Mathematics, Dennis. G.Zill, Warren S.Wright, 4th edition, 2011.

Syllabus for MSC 2021 Admissions Onwards

MAM 2402 WAVELETS

Credits: 4

Objective: This course starts with the structure of $\mathbb{C}^n$. This course is planned to introduce the Wavelets as an extension to the idea of Fourier's method in Linear algebraic perpsective.

Learning Outcomes: After the completion of this course, the student should be able to be familiar with Multi-resolution analysis and its applications in different contexts like the space of periodic functions, non-periodic functions and on the space of square integrable functions on the real line.

UNIT 1: The Discrete Fourier Transform, Translation-Invariant Linear Transformations, First Stage Construction of Wavelets on $\mathbb{Z}_N$ (Chapter 2, Chapter 3, Sections 2.1, 2.2, 3.1)

UNIT 2: Construction of Wavelets on $\mathbb{Z}_N$: Iteration step, Examples and Applications, $l^2(\mathbb{Z})$ (Chapter 3, Sections 3.2, 3.3, Chapter 4, Section 4.1)

UNIT 3: Complete Orthonormal Sets in Hilbert Spaces, $L^2([-\pi, \pi))$ and Fourier Series, The Fourier Transform and Convolution on $l^2(\mathbb{Z})$ (Chapter 4, Sections 4.2, 4.3, 4.4, 4.5)

UNIT 4: First-Stage Wavelets on $\mathbb{Z}$, The Iteration step for Wavelets on $\mathbb{Z}$, Implementation and Examples. (Chapter 4, Sections 4.6, 4.7, Chapter 5, Section 5.1,)

UNIT 5: $L^2(\mathbb{R})$ and approximate Identities, The Fourier Transform on $\mathbb{R}$, Multiresolution Analysis and Wavelets, Construction of MRA (Chapter 5, Sections 5.2, 5.3, 5.4)

Text Book: Michael W. Frazier, An Introduction to Wavelets Through Linear Algebra, Springer-Verlag New York, (1999).

References:-

1. Charles K. Chui, *An Introduction to Wavelets*, Academic (1992).

2. Ingrid Daubechies, *Ten Lectures on Wavelets*, SIAM, (1992).

3. K.R Unni, *Wavelets, Frames and Wavelet Bases in L^P Lecture notes*, Bhopal (1997).

4. Stephane Mallat, *A Wavelet Tour Of Signal Processing*, Academic Press (1999).

5. Don Hong, Jianzhong Wang, Robert Gardner, *Real Analysis with an Introduction to Wavelets*, Elsevier Academic Press (2005).

6. Yves Meyer, *Wavelets and Operators*, Cambridge University Press (1992).

7. John. J Beneditto, Michael W. Frazier *Wavelets-Mathematics and Applications*, CRC, (1994).

8. Eugenio Hernandez, Guido L. Weiss, *First course on wavelets*, CRC, (1996).

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MAM 2403 Optimization and Mathematical Methods for Deep Learning

Credits: 4

Objective: The objective of this course is

1. to introduce optimization and discuss mathematical methods behind deep learning process.
2. to give an insight on the mathematical tools and techniques required for modelling real life problems related to deep learning process.
3. to provide the basis idea of convex analysis.

Learning Outcomes: After the completion of this course, the student will

1. learn about the different types of optimization problems.
2. acquire knowledge on mathematical models in Deep Learning process through linear algebra, Basic Probability theory, Optimization techniques, Neural Network and Fuzzy Logic and fuzzy systems.
3. understand the basic concepts behind convex optimization such as convex sets, convex functions, differentiable and non-differentiable convex functions, optimum of convex function, necessary and sufficient optimality conditions for constrained and unconstrained optimization problem.

UNIT 1: (Different types of optimization) (Text 4) Discrete and Continuous Optimization, Constrained and Unconstrained Optimization, Deterministic and Stochastic Optimization, Optimization with none, one or many objectives. Some Illustrative Examples: Optimal Control Problems, Electrical Networks, Water Resources Management, Stochastic Resource Allocation, Location of Facilities. (Chapter 1)

UNIT 2: (Fuzzy Sets & Systems) (Text 3) Basic Definition and Terminology, Set-theoretic Operations, Member Function, Formulation and Parameterization, Fuzzy rules and fuzzy Reasoning, Extension Principal and Fuzzy Relations, Fuzzy if-then Rules, Fuzzy Inference Systems, Implementation using MATLAB.

UNIT 3: (Basics in Deep Learning) (Text 2) History of Deep Learning, Mc Culloch Pitts Neuron, Perceptron, Perceptron Learning Algorithm, MLP, Sigmoid Neurons, Gradient Descent, Feedforward Neural Networks, Back propagation Algorithm; Implementation using MATLAB.

UNIT 4: (Some Learning Strategies and Modern Trends in Deep Learning) (Text 1 & 2) Learning Theory, Supervised and Unsupervised Learning; Regression- Ordinary Least Squares, Polynomial Regression, Classification- Nearest Neighbour Model; Linear Discriminant Analysis, Quadratic Discriminant Analysis; Clustering- Dimensionality Reduction, K- Means Clustering. (Text 1) Convolution Neural Network (CNN), Recurrent Neural network (RNN), Auto encoders. (Text 2)

UNIT 5: (Basics of Convex Analysis) (Text 4) Convex sets, Convex hull, Closure and interior of a set, Weierstrass theorem, Separation and support of sets, Convex cones and polarity. (Chapter 2: 2.1-2.5) Convex functions and basic properties, Subgradients of convex functions, Differentiable convex functions, minima and maxima of convex functions. (Chapter 3: 3.1-3.4) The Fritz john and Karush-Kuhn-Tucker Optimality Conditions (without proofs). (Chapter 4: 4.1-4.3)

Text Books:

1. T. M. Mitchell, Machine Learning, McGraw Hill, 2017.
2. Deep Learning, Ian Goodfellow and YoshuaBengio and Aaron Courville, MIT Press, 2016.
3. Bo Yuan, George J. Klir, Fuzzy Sets and Fuzzy Logic:Theory and Applications, Pearson Publishers, Second Edition 2015.
4. M. S. Bazaraa, Hanif D. Sherali, C. M. Shetty; Nonlinear Programming theory and algorithms. John Wiley & Sons, Inc. Second Edition 2004.

References:-

1. S.N. Shivnandam, Principle of soft computing, Wiley.
2. Timothy J.Ross, Fuzzy logic with Engineering Applications, McGraw-Hills
3. Jack M. Zurada, Introduction to Artificial Neural Network System JAico Publication.
4. Simon Haykins, Neural Network- A Comprehensive Foudation.
5. Hans-Jurgen Zimmermann, Fuzzy Set Theory - and its Applications Second -Revised Edition 2013, Springer Netherlands.
6. Raos Linear Statistical Inference and its Applications" published by Wiley ISBN: 978-0-471-21875-3.
7. Press, Teukolsky, Vetterling, & Flannery Numerical Recipes: the Art of Scientic Computing" published by Cambridge University Press, ISBN: 978-0-521-88068-8.

8. Horn & Johnsons Matrix Analysis" published by Cambridge University Press, ISBN: 978-0-521-54823-6.

9. Golub & Van Loans Matrix Computations", by Hopkins ISBN-13: 978-0801854149.

10. Ross A First Course in Probability", published by Pearson, ISBN-13: 978-0321794772.

11. Christopher Bishops Pattern Recognition and Machine Learning" published by Springer, ISBN: 978-0-387-31073-2.

12. R. T. Rockafellar Convex Analysis published by Princeton University Press, 1970.

Syllabus for MSC 2021 Admissions Onwards

MAM 2404 COMMUTATIVE ALGEBRA

Objective: This course is an advanced course in algebra. This course discusses the theory of commutative rings. These rings are of fundamental significance in Mathematics because of its applications to other topics such as algebraic number theory, algebraic geometry and many other advanced topics in mathematics.

Learning Outcomes: After the completion of this course, the student should be able to

1. understand the basic definitions concerning different classes of commutative rings, elements in commutative rings, and ideals in commutative rings.
2. know the theory of modules, including the tensor product of modules and algebras, and localisation.
3. know the theory of primary decomposition of ideals in a commutative rings.
4. know the theory of integral dependance and integral extensions.
5. know the definition and examples of Noetherian and Artinian rings.

UNIT 1: Rings and ideals: review of ideals in quotient rings; prime and maximal ideals, prime ideals under quotient, existence of maximal ideals; operations on ideals (sum, product, quotient and radical); Chinese Remainder theorem; nilradical and Jacobson radical; extension and contraction of ideals under ring homomorphisms; prime avoidance.

UNIT 2: Free modules; Projective Modules; Tensor Product of Modules and Algebras; Flat, Faithfully Flat and Finitely Presented Modules; Shanuels Lemma.

UNIT 3: Localisation and local rings, universal property of localisation, extended and contracted ideals and prime ideals under localisation, localisation and quotients, exactness property.

UNIT 4: Nagata's criterion for UFD and applications; equivalence of PID and one-dimensional UFD. Associated Primes and Primary Decomposition.

UNIT 5: Integral dependence, Going-up theorem, Integral Extensions: integral closure, Going-down theorem, Valuation rings, Chain Conditions. Definition and examples of Noetherian rings and Artinian rings.

Text Book: M.F. Atiyah and I.G. Macdonald, Introduction to commutative algebra, Addison-Wesley (1969).

References:-

1. R.Y. Sharp: Steps in commutative algebra, LMS Student Texts (19), Cambridge Univ. Press (1995).

2. D. Eisenbud: Commutative algebra with a view toward algebraic geometry GTM (150), Springer-Verlag (1995).

3. H. Matsumura: Commutative ring theory, Cambridge Studies in Advanced Mathematics No. 8, Cambridge University Press (1980).

4. N.S. Gopalakrishnan: Commutative Algebra (Second Edition), Universities Press (2016).

5. Miles Reid: Undergraduate Commutative Algebra , Cambridge University Press (1995).

Syllabus for MSC 2021 Admissions Onwards

MAM 2405 - GRAPH THEORY

Credits: 4

Objective: The course introduce the concept of automorphism of simple graphs, graph operators, graph parameters and some interesting graph classes

Learning Outcomes: After the completion of this course, the student should be able to

1. Understand the basic concepts of graph theory
2. Have a clear picture of graph operators, graph parameters and graph classes
3. Build graph models of real life problems
4. Apply graph theoretic tools to solve problems.

UNIT 1: Basic Concepts, Degree of Vertices, Automorphism of a Simple Graph, Line Graphs, Operation on Graphs, Directed Graphs, Tournaments (Chapter 1: Sec. 1.1 - 1.12, Chapter 2: Sec. 2.1 - 2.3)

UNIT 2: Connectivity, Vertex Cuts and Edge Cuts, Connectivity and Edge Connectivity, Blocks, Trees, Definition, Characterization, Centers, Cayleys Formula, Applications (Chapter 3:Sec.3.1 3.4 (Theorem 3.4.3 omitted), Chapter 4: Sec. 4.1 - 4.5, 4.7)

UNIT 3: Independent sets, Vertex coverings, Edge Independent sets, Matchings, Factors, Matching in Bipartile Graphs, Eulerian Graphs, Hamiltonian Graphs, Hamilton Cycles in Line Graphs, 2-Factorable Graphs (Chapter 5: Sec. 5.1 - 5.5, Chapter 6: Sec. 6.1 - 6.3, 6.5 - 6.6)

UNIT 4: Graph Colorings, Critical Graphs, Brooks Theorem, Triangle Free Graphs, Edge Colorings, Chromatic Polynomials, Perfect Graphs, Triangulated Graphs, Interval Graphs (Chapter 7: Sec. 7.1 - 7.2, 7.3, 7.3.1, 7.5 7.6, 7.9, Chapter 9: Sec. 9.1 9.4)

UNIT 5: Planar and nonplanar graphs, Eulers Formula, Dual, Four Color Theorem and Five Color Theorem, Kuratowskis Theorem (without proof), Hamilton Plane graphs, Domination, Bounds, Independent Domination and Irredundance (Chapter8: Sec. 8.1 8.8, Chapter 10: Sec.10.1 10.3. 10.5)

Text Book: R. Balakrishnan, K. Ranganathan: A Text book of Graph Theory (Second Edition), Springer 2012.

References:-

1. D. B. West: Introduction to Graph Theory, 2nd ed. Prentice Hall, New Jersey (2011)
2. F. Harary: Graph Theory, Addison Wesley Publishing Company, Inc. (1969).

3. M. C. Golumbic: Algorithmic Graph Theory and Perfect Graphs, Academic Press, New York (1980)

4. Teresa W. Haynes, S. T. Hedetneimi, P. J. Slater: Fundamentals of Domination in Graphs, Marcel Dekker, New York (1998)

MAM 2406 ADVANCED LINEAR ALGEBRA

Credits: 4

Objective: This course starts with the review of linear algebra, which will be followed by the factorisation and triangulation theorems. This will also discuss canonical forms and eigenvalue inequalities and inclusions for hermitian matrices. Some important results in linear algebra are discussed here which are not done in the core courses on this subject. This will benefit students wants to pursue research in the areas like Functional Analysis, Spectral theory, Stochastic models, Numerical linear algebra, etc.

Learning Outcomes: After the completion of this course, the students will be familiar with the advanced concepts of linear algebra and matrix analysis. It is expected to develop the skills to deal with advanced techniques in estimating eigenvalues, singular values, etc.

Pre-requisites:

1. A basic course in linear algebra and matrix theory.
2. Normed spaces and basic analysis.

UNIT 1: Review of Linear Algebra: Eigenvalues, Algebraic and geometric multiplicity, Special types of matrices, Change of basis, etc.

UNIT 2: Unitary matrices and QR factorization, Unitary similarity, Triangulation theorems and consequences, Singular Value Decomposition (SVD).

UNIT 3: Jordan canonical form and its consequences, minimal polynomial, Triangular factorization.

UNIT 4: Hermitian matrices, Eigenvalue inequalities, diagonalization.

UNIT 5: Matrix norms, Condition numbers, Gersgorin discs, Eigenvalue perturbation theorems.

Text Book: Roger A Horn, Charles R Johnson, Matrix Analysis, Second Edn., Cambridge University Press, 2013.

References:-

1. M. Artin, Algebra, Prentice-Hall, (1991).
2. Serge Lang, Introduction to Linear Algebra, Second Edition, Springer (1997).
3. K.T Leung, Linear Algebra and Geometry, Hong Kong University Press, (1974).
4. Kenneth Hoff man and Ray Kunze Linear Algebra, Second Edition, PHI (1975)

5. Sheldon Axler, Linear Algebra Done Right, Second Edition, Springer, (1997).

Syllabus for MSC 2021 Admissions Onwards

MAM 2407 DISCRETE FRAMELETS

Credits: 4

Objective: Course is aimed to introduce the basic tools for applications using Discrete Framelets. Students will get knowlege in analysing signals and images using finite filters. This course will pave the necessary foundations to study numerical solutions of partial differential equations and some insights into computer aided geometric design.

Learning Outcomes: After the completion of this course, the student should be able to

1. Understand the subject in a signal processing perspective with the help of finite filters.

2. Be familiar with filterbank theory for signal analysis.

3. Understand the multilevel framelet decomposition of signals in bounded intervals.

UNIT 1: Discrete Framelet Transform, Perfect reconstruction of discrete framelet transforms, One-Level Standard Discrete Framelet Transforms, Perfect Reconstruction of Discrete Framelet Transforms, Some Examples of Wavelet or Framelet Filter Banks. (Section 1.1 of text.)

UNIT 2: Sparsity of Discrete Framelet transforms, Convolution and Transition Operators on Polynomial Spaces, Subdivision Operator on Polynomial Spaces, Linear-Phase Moments and Symmetry Property of Filters, An Example. (Section 1.2 of text.)

UNIT 3: Multilevel Discrete Framelet Transforms and Stability, Multilevel Discrete Framelet Transforms, Stability of Multilevel Discrete Framelet Transforms, Discrete Affine Systems in $\ell^2(\mathbb{Z})$, Nonstationary and Undecimated Discrete Framelet Transforms (Section 1.3 of text.)

UNIT 4: Oblique extension principle, OEP-Based Tight Framelet Filter Banks, OEP-Based Filter Banks with One Pair of High-Pass Filters, OEP-Based Multilevel Discrete Framelet Transforms. (Section 1.4 of text.)

UNIT 5: Discrete Framelet Transforms for signals on bounded Intervals, Boundary Effect in a Standard Discrete Framelet Transform, Discrete Framelet Transforms Using Periodic Extension, Discrete Framelet Transforms Using Symmetric Extension, Symmetric Extension for Filter Banks Without Symmetry, Discrete Framelet Transforms Implemented in the Frequency Domain. (Section 1.5 and 1.6 of text.)

Text. Bin Han, Framelets and Wavelets Algorithms, Analysis and Applications, Birkhauser 2017.

References:-

1. Ole Christensen, Frames and Bases An Introductory Course, Birkhauser, 2008.

2. Ole Christensen, Frames and Riesz Bases, Birkhauser, 2008.

3. Christopher Heil, A Basis Theory Primer, Citeseer, 1998.

4. Yves Meyer, Wavelets and Operators, CUP, England, 1992.

5. Ingrid Daubechies, Ten Lectures on Wavelets, SIAM, Philadelphia, 1992.

Syllabus for MSC 2021 Admissions Onwards

MAM 2408 HARMONIC ANALYSIS

Credits: 4

Objective: This course starts with the review of Measure theory. This course is planned to introduce the basics of Topolgical groups and measure and Intergration on Locally compact groups.

Learning Outcomes: After the completion of this course, the student should be able to Be familiar with the formulation of Measure and integration on Locally comopact groups and representations of Compact groups.

UNIT 1: Topological groups, Haar Measure, Modular Functions, Convolutions (Sections 2.1, 2.2, 2.3, 2.4, 2.5)

UNIT 2: Homogeneous spaces, Unitary Representations, Representation of a group and its group algebra (Sections 2.6, 2.7, 2.8, 3.1, 3.2)

UNIT 3: Functions of positive type, The Dual group, The Fourier transform, The Pontrjagin Duality theorem (Sections 3.3, 3.4, 4.1, 4.2, 4.3)

UNIT 4: Representations of Locally Compact Abelian Groups, Closed ideals, Spectral synthesis, Bohr Compactification(Sections 4.4, 4.5, 4.6, 4.7, 4.8)

UNIT 5: Representations of Compact Groups, The Peter-Weyl Theorem, Fourier Analysis on Compact Groups. (Sections 5.1, 5.2, 5.3, 5.4, 5.5)

Text Book: Folland, G.B., *A Course in Abstract Harmonic Analysis*, CRC Press, (1995).

References:-

1. Hewitt, E and Ross K., *Abstract Harmonic Analysis* Vol.1 Springer (1979).

2. Gaal, S.A., *Linear Analysis and Representation Theory*, Dover (2010).

3. Asim O. Barut and Ryszard Raczka, *Theory of Group Representations*, second revised edition, Polish scientific publishers (1980).

4. Groenchenig, K., *Foundations of time frequency analysis*, Birkhauser Boston (2001).

Syllabus for MSC 2021 Admissions Onwards

MAM 2409 INTEGRAL TRANSFORMS

Credits: 4

Objective: This course starts with Fourier Transforms in detail. This course is planned to introduce the basics of Integral Transforms and its applications in various fields.

Learning Outcomes: After the completion of this course, the student should be able to be familiar with popular integral transforms and its applications.

UNIT 1: Integral Transforms, The Fourier Integral Formulas, Fourier Transforms of generalised functions, Basic Properties of Fourier Transforms, Z-transforms (Sections 1.1, 1.2, 2.1, 2.2, 2.3, 2.4, 2.5 and Chapter 12)

UNIT 2: Poisson's Summation formula, The Shannon Sampling Theorem, Gibbs Phenomenon, Heisenbergs' Uncertainty Principle, Applications of Fourier Transform to ODE, Laplace Transforms and their basic properties. (Sections 2.6, 2.7, 2.8, 2.9, 2.10, 3.1, 3.2, 3.3, 3.4)

UNIT 3: Convolution Theorem and the properties of convolution, Differentiation and Integration of Laplace transforms, The Inverse Laplace Transforms, Tauberian theorems and Watson's Lemma, Applications of Laplace transforms, Evaluation of Definite Integrals, Applications of Joint Laplace and Fourier Transform. (Sections 3.5, 3.6, 3.7, 3.8, 3.9, 4.1, 4.2, 4.3, 4.6, 4.8)

UNIT 4: Finite Fourier Sine and Cosine transforms, Basic properties and Applications, Finite Lapace Transforms, Tauberian Theorems. (Chapter 10, 11)

UNIT 5: Hilbert Transform and its basic properties, Hilbert transform in the complex plane, applications of Hilbert Transform, Asymptotic expansion of One sided Hilbert Transform. (Sections 9.1, 9.2, 9.3, 9.4, 9.5, 9.6)

Text Book: Lokenath Debnath, Dambaru Bhatta *Integral Transforms and their Applications*, second edition, Taylor and Francis, (2007).

References:-

1. Frederick W. King, *Hilbert Transforms*, CRC (2009).
2. Larry C. Andrews, Bhimsen K. Shivmaoggi *Integral Transforms for Engineers*, (1999).
3. Ian N. Sneddon, *The Fourier Transforms*, Dover Publishers (1995).
4. Joel L.Schiff, *Laplace Transforms: Theory and Applications*, second revised edition, Springer (1980).
5. B.Davies, *The Integral Transforms and their applications*, Springer-Verlag (1978).
6. Ian N. Sneddon, *The Use of Integral Transforms*, McGraw-Hill (1972).

Syllabus for MSC 2021 Admissions Onwards

MAM 2410 FUNCTIONS OF SEVERAL VARIABLES

Credits: 4

Objective: This course starts with the structure of $\mathbb{R}^n$. This course is planned to introduce the Differential calculus on the finite dimensional Euclidean Space and Integration on $\mathbb{R}^n$.

Learning Outcomes: After the completion of this course, the student should be able to be familiar with Differentiation and Intgration on $\mathbb{R}^n$.

UNIT 1: Multivariable Differential Calculus, Directional Derivatives and continuity, Total Derivative, The Jacobian matrix, Matrix form of the chain rule, Taylor formula for functions from $\mathbb{R}^n$ to $\mathbb{R}$ (Chapter 12)

UNIT 2: Implicit Functions and Extremum problems, functions with nonzero Jacobian determinant, Inverse function theorem, Implicit funtion theorem, Extrema of real-valued functions of several variables, Extremum problems with side conditions(Chapter 13)

UNIT 3: Multiple Riemann Integrals, The measure of a bounded interval in $\mathbb{R}^n$, Riemann Integral of a bounded funtion on a compact interval in $\mathbb{R}^n$, Lebesgue criterion for the existence of a multiple Riemann integral. (Chapter 14, Sections 14.1, 14.2, 14.3, 14.4, 14.5)

UNIT 4: Jordan Measurable sets in $\mathbb{R}^n$, Multiple Integration over Jordan-measurable sets, Step functions and their integrals, Fubini's reduction thorem for the double integral of a step function. (Chapter 14, 15 Sections 14.6, 14.7, 14.8, 14.9, 14.10, 15.1,15.2,15.3,15.4, 15.5)

UNIT 5: Multiple Lebesgue Integrals, Fubini's reduction theorem for double integrals ,Tonelli-Hobson test for integrability The transformation formula for multiple integrals(Chapter 15, Sections 15.6, 15.7, 15.7, 15.8, 15.9, 15.10, 15.11, 15.12, 15.13)

Text Book: Tom M. Apostol, *Mathematical Analysis*, Second Edition, Addison-Wesley 1974.

References:-

1. Serge Lang, *Calculus Of Several Variables*, Addison-Wesley Publications, (1973).

2. C.H. Edwards Jr.,*Advanced Calculus of Several Variables*, Academic Press New York, (1973).

3. Rudin W., Real and Complex Analysis, 3rd edition, McGraw-Hill, New York (1986).

4. Rudin W., Functional Analysis, 2nd edition, McGraw-Hill, New York (1991).

5. D Somasundaram and B. Choudhary, A first course in mathematical analysis, Narosa, Oxford, London, (1996).

6. K. A. Ross, Elementary Analysis; Theory of Calculus, Springer-Verlag, 2013.

Syllabus for MSC 2021 Admissions Onwards

MAM 2411 ADVANCED SPECTRAL THEORY

Credits: 4

Objective: This course starts with the review of Spectral Theory of Linear Operators in Normed Spaces. The idea of this course is to cover various classifications of spectrum and finally present the spectral theorem for bounded self-adjoint operators. Applications to quantum mechanics is also done.

Learning Outcomes: After the completion of this course, the student should be able to be familiar with the properties and applications of spectrum and spectral theorem.

Pre-requisites:

1. Functional Analysis, Basic Analysis.

2. Linear Algebra.

UNIT 1: Review of Spectral Theory of Linear Operators in Normed Spaces; Properties of Resolventa and Spectrum, Use of Complex Analysis in Spectral Theory. (Chapter 7)

UNIT 2: Spectral Properties of Bounded Self-adjoint Operators; Positive Operators, Spectral Family. (Chapter 9, Section 9.1 to 9.7)

UNIT 3: Spectral Theorem for Bounded Self-adjoint Operators, Properties of Spectral Family. (Chapter 9, Section 9.8 to 9.11)

UNIT 4: Unbounded Linear Operators in Hilbert Spaces; Spectral Representation of Unitary Operators, Spectral Representation of Self-Adjoint Operators (Unbounded). (Chapter 10)

UNIT 5: Unbounded Linear Operators in Quantum Mechanics. (Chapter 11)

Text Book: E. Kreyzig, Introduction to Functional Analysis with Applications, Addison Wesley.

References:-

1. Courant, R. and D. Hilbert, Methods of Mathematical Physics, vol. I, Interscience, Newyork (1953).

2. Dunford N. and T. Schwartz, Linear Operators, Part I, Interscience, Newyork (1958).

3. Rudin W., Real and Complex Analysis, 3rd edition, McGraw-Hill, Newyork (1986).

4. Rudin W., Functional Analysis, 2nd edition, McGraw-Hill, Newyork (1991).

5. Reed, M. and B. Simon, Methods of Mathematical Physics, vol. II, Academic Press, Newyork, (1975).

6. Rajendra Bhatia, Notes on Functional Analysis, Texts and Readings in Mathematics, Hindusthan Book Agency, New Delhi (2009).

7. G. F. Simmons, Introduction to Topology and Modern Analysi,s TMH.

8. M. Thamban Nair, Functional Analysis; A first course, PHI Learning Pvt. Ltd. (2001).

Syllabus for MSC 2021 Admissions Onwards

MAM 2412 BANACH ALGEBRAS AND SPECTRAL THEORY

Credits: 4

Objective: This course introduces the notion of Banach Algebras. The theory of commutative Banach algebras are discussed in detail. Also, the spectral theory of bounded and unbounded operators on Hilbert spaces are discussed.

Learning Outcome: After completing the course, the student is expected to become familiar with the fundamental concepts and applications of Banach Algebras and Spectral Theory.

Prerequisites: A first course in Functional Analysis, Complex Analysis, Linear Algebra, Topology and Measure Theory is needed. The core courses taught in the first three semesters of the M.Sc. program will do the purpose.

UNIT 1: Banach Algebras: Introduction, Complex homomorphisms, Basic properties of Spectra, Symbolic Calculus, Invariant subspace theorem. (Chapter 10 of Text Book)

UNIT 2: Commutative Banach Algebras: Ideals and homomorphisms, Gelfand Transforms, Involutions, Positive functionals. (Chapter 11 of Text Book)

UNIT 3: Bounded Operators on a Hilbert Space: A commutativity theorem, Resolutions of the identity, The spectral theorem, Positive operators, An ergodic theorem. (Chapter 12 of Text Book)

UNIT 4: Unbounded Operators: Symmetric operators, The Cayley transform, Resolutions of the identity. (Chapter 13 of Text Book)

UNIT 5: Unbounded Operators (Contd.): The Spectral Theorem, Semigroup of Operators. (Chapter 13 of Text Book)

Text Book: Rudin, Walter. Functional Analysis. Second Edition. International Series in Pure and Applied Mathematics. McGraw-Hill, Inc., New York, 1991.

References:-

1. Takesaki, M. Theory of Operator Algebras I. Reprint of the first (1979) edition. Encyclopaedia of Mathematical Sciences, 124. Operator Algebras and Non-commutative Geometry, 5. Springer- Verlag, Berlin, 2002.

2. Arveson, William. An Invitation to C*-algebras. Graduate Texts in Mathematics, No. 39. Springer-Verlag, New York-Heidelberg, 1976.

3. Douglas, Ronald G. Banach Algebras Techniques in Operator Theory. Second Edition. Graduate Texts in Mathematics, 179. Springer-Verlag, New York, 1998.

MAM 2413 NUMBER THEORY

Credits: 4

Objective: This course starts with the review of theory of numbers which will be followed by the divisibility and prime. This will involve some of the classical theory in the subject such as congruences, the Chinese remainder theorem, quadratic reciprocity law, Arithmetic functions and diophantine equations.

Learning Outcomes: After the completion of this course, the student should be able to be familiar with divisibility, primes, congruences, the Chinese remainder theorem, quadratic reciprocity law, Arithmetic functions and diophantine equations.

UNIT 1: Introduction to Numbers, Divisibility, Primes, [Chapter - 1 (Sections - 1.1,1.2,1.3)]

UNIT 2: Congruences, Solutions to congruences, The Chinese remainder theorem. [Chapter - 2 (Sections - 2.1,2.2,2.3)]

UNIT 3: Quadratic residues, Quadratic reciprocity, The Jacobi symbol. [Chapter - 3 (Sections - 3.1,3.2,3.3)]

UNIT 4: Greatest integer function, Arithmetic functions, The Mobius inversion formula. [Chapter - 4 (Sections 4.1, 4.2, 4.3)]

UNIT 5: The equation $ax + by = c$, Simultaneous equations, Pythagorean triangles, Assorted examples. [Chapter - 5 (Sections 5.1,5.2,5.3,5.4)]

Text Book: I. Niven, H.S. Zuckerman and H.L. Montgomery, An Introduction to the Theory of Numbers, 4th Ed., Wiley, New York, (1980).

References:-

1. W.W. Adams and L.J. Goldstein, Introduction to the Theory of Numbers, 3rd ed., Wiley Eastern, (1972).

2. A. Baker, A Concise Introduction to the Theory of Numbers, Cambridge University Press, Cambridge, (1984).

3. K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory, 2nd ed., Springer-Verlag, Berlin, (1990).

4. T.M. Apostol, An Introduction to Analytic Number Theory, Springer-Verlag, (1976).

MAM 2414 REPRESENTATION THEORY OF FINITE GROUPS

Credits: 4

Objective: To introduce the facinating theory of representations to the learner. Group representation theory will be discussed in detail through FG- Modules. To discuss the irreducible representations which are the building blocks of representations in detail. Chacter of a representation is a beautiful idea which is playing a vital role in the study of representations, here we discuss the character table of a group in detrail and construct the character table which will in fact replace the group itself.

Learning Outcome: The leaner must have gained a proper understanding of the idea of group representations such as permutation representation, linear representations. The leaner will be capable of constructing the character table of some interesting class of groups.

UNIT 1: Vector spaces, Modules, FG- modules, Group representations, Group algebras and homomorphisms. (Sections 1 to 7 of the text.)

UNIT 2: Maschke's theorem, Schur's lemma, Irreducibility (Sections 8 to 11 of the text.)

UNIT 3: Conjugacy classes, Character, Irreducibility, Inner product, Character table, Normal subgroups and lifted characters. (Sections 12 to 17 of the text.)

UNIT 4: Elementary charactertables, Tensor products, Restriction to subgroup, Induced modules and characters. (Sections 18 to 21 of the text.)

UNIT 5: Properties of character tables. Permutation chracters. (Sections 24 and 29 of the text.)

Text Book: Gordon James and Martin Liebeck, Representation and Characters of Groups, Cambridge University Press, Second Edition, 2001.

References:-

1. Willim Fulton, Joe Harries, Representation theory, A first course, 191 Springer Verlag, ISBN 81-8128-134-9.

2. David S Dummit, Richard M. Foot, Abstract Algebra , Third edition, John Wiley & Sons, Inc. 2004.

3. Walter Ledermann, Introduction to group characters, Second edition, Cambridge University Press, 2008. ISBN 978-0-521-33781-6.

Syllabus for MSC 2021 Admissions Onwards

MAM 2415 ALGEBRAIC TOPOLOGY

Credits: 4

Objective: At the end of the course the students will have the necessary introduction to the subject of Algebraic topology. The algebraic notions of the fundamental group of a space and that of homology and even cohomology theories is covered in the course. All the important topological constructions and concepts conducive for the algebraic study are also studied with enough examples.

Learning Outcomes: At the completion of the course, students will be comfortable with the necessary topological concepts and constructions like attaching spaces, suspension, excision, homotopy and deformation retraction among others. Along with the study of the fundamental group and classification of covering spaces, the students will also work with the homology and cohomology theories, which will serve as an important application of their course in module theory.

UNIT 1: Homotopy and homotopy type, Cell complexes, Operations on spaces, Two criteria for homotopy equivalence, the homotopy extension property. (Chapter 0 of Hatcher)

UNIT 2: Applications of Van Kampen's theorem, Covering spaces, lifting properties, Universal cover and classification of covering spaces, Deck transformations and properly discontinuous actions. (chapter 1 of Hatcher)

UNIT 3: Delta-complexes and Simplicial homology, Singular homology, Homotopy Invariance, Exact sequences and excision, Equivalence of simplicial and singular homology. (Chapter 2 of Hatcher)

UNIT 4: Cellular homology (with special emphasis on CW-complexes), Mayer-Vietoris sequences, Homology with coefficients, the formal viewpoint of homology theories (briefly) (Chapter 2 of Hatcher)

UNIT 5: The definition of cohomology groups, The Universal Coefficient theorem, computation of cohomology of spaces, Relative groups and the long exact sequence of a pair of spaces (X, A), Cup product and the Cohomology ring structure, Kunneth formula for product of spaces, Poincare duality. (Chapter 3 of Hatcher)

Text Book: Algebraic Topology, Allen Hatcher.

References:-

1. Lecture notes in Algebraic Topology, James F. Davis, Paul Kirk.

MAM 2416 DIFFERENTIAL GEOMETRY

Credits: 4

Objective: The course is aimed to introduce the popular tools to perform a study of geometry with the help of calculus on an n-dimensional surface. Develop the notion of curvature of parametric surfaces with the idea of, vector fields along a parametrized curve on the surface. Towards the end of the course, students will get all the necessary foundations to study Riemannian Geometry.

Learning Outcomes: After the completion of this course, the student should be able to

1. be familiar with the concepts vector fields, tangent space, surfaces and its orientations.
2. get introduced to the spherical image of surfaces, geodesics, Weingarten map, and curvature of surfaces.
3. understand local equivalence of surfaces and parametrized surfaces.
4. obtain sound knowledge in rigid motions, congruence, isometries and results related these.

Pre-requisites: Linear Algebra, Multivariate Calculus, and Differential Equations.

UNIT 1: Graphs and level sets, Vector fields, Tangent spaces, Surfaces, Vector Fields on Surfaces; Orientation, Gauss map.

UNIT 2: Geodesics, Parallel Transport, Weingarten Map, Curvature of Plane Curves.

UNIT 3: Arc lengths, Line integrals, Curvature of surfaces

UNIT 4: Parametrized surfaces, Local equivalence of surfaces and parametrized surfaces.

UNIT 5: Differentiable manifolds, Introduction, Tangent space, Immersions and embeddings; examples, Other Examples of manifolds, Orientation, Vector fields, brackets, Topology of manifolds. (Chapter 0 of the text 2)

Texts:

1. J.A. Thorpe: Elementary Topics in Differential Geometry, Springer-Verlag [Chapters 1 -12, 14, 15, 22, 23]
2. Manfredo Perdigao do Carmo, Riemannian Geometry, Birkhauser 1993.

References:-

1. L. M. Woodward, J. Bolton, A First Course in Differential Geometry: Surfaces in Euclidean Space, Cambridge university press, 2019.

2. Edouard Goursat, A Course in Mathematical Analysis, Vol. 1, Forgotten Books, 2012.

3. Andrew Pressley, Elementary Differential Geometry, second edition, Springer 2010.

4. Dirk J. Struik, Lectures on Classical Differential Geometry, Dover publications Inc. 1988.

5. Kreyszig, Introduction to Differential Geometry and Reimannian Geometry, University of Toronto Press, 1968.