Gabor wavelets from COVID-19 waves

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We have seen how Gabor wavelets arise from the Uncertainty Principle. In this section, we use the method of filtering and cross correlation of signals from COVID-19 infection waves, and observe the presence of Gabor wavelets in the cyclical signals. An understanding of the nature of these representations is crucial to prediction of future infection waves of the pandemic.

The 1-Dimension Gabor wavelet is a Gaussian-modulated complex exponential described by the equation:

$$f(x) = \exp\big[-(x-x_0)^2/a^2\big] \exp\big[-ik_0(x-x_0)\big] \tag{1}$$

A property of Gabor wavelets is locality, which means that as the distance from center x_0 increases the function is exponentially suppressed. The parameter a controls the rate of this exponential decay and k_0 controls the modulation.

The Fourier transform of a Gabor wavelet $F(k) = \mathcal{F}[f(x)]$ is also a Gabor wavelet.

$$F(k) = \exp\big[-(k-k_0)^2/a^2\big] \exp\big[-ix_0(k-k_0)\big] \tag{2}$$

The COVID-19 pandemic caused by the novel coronavirus SARS-CoV-2 has impacted livelihoods of people all over the world. The virus has a spike protein S that attacks the human Angiotensin-converting-enzyme-2 (hACE2 for short) receptor that lines the surface of airways. Our objective is to study the COVID-19 infections and deaths time series and derive insights on the correlations between the two, across select countries such as India, USA, UK and Brazil.

We collect the time series data of daily infections and daily deaths for over 470 days in the period Feb 1, 2020 to May 15, 2021. Since there is noise in reporting the daily data, we perform a 7-day moving average smoothing of all the series. This smoothes small fluctuation arising out of human reporting errors, while keeps the

cyclicality in the data intact. We then perform the exercise of trend filtering using the D_3 Filter, with a penalty of $\lambda = 1000$. We find necessary level of smoothing of infections and death series, that have polynomial wave-like trends and smaller cyclical fluctuations. The smoothed series and cyclical components are shown in Figures 1 and 2.

All countries have experienced at least 2 waves at the time of analysis. Brazil's infections appears to have waves with large wavelengths and small amplitudes. The number of daily deaths also peaks with 2 waves for 3 countries while deaths in US peaks thrice. Different countries have had peaks of infections and deaths at different periods, nonetheless in the US and UK the waves appear to have hit roughly around the same time, albeit with different magnitudes.

We further investigate the lags between the infection and death waves as well as the lags between infection waves of different countries, using cross correlations of cyclical components. This is plotted in Figures 3 and 4 respectively. The cross correlation plot between D_3-filtered cycles for daily infections and deaths in India shows a peak at 18 days. This implies that deaths in India lag infections by 18 days.
The cross correlation between infections in India versus other countries picks out the lag between the major waves. India lags by UK and US by over a 100 days in terms of severity of cases. The lag between India and Brazil however is only 15 days. We also interestingly observe that for all countries the spectrum appears to that could be modelled by a Gabor wavelet. There exists small sinusoidal wavelets enveloped by a Gaussian wave.

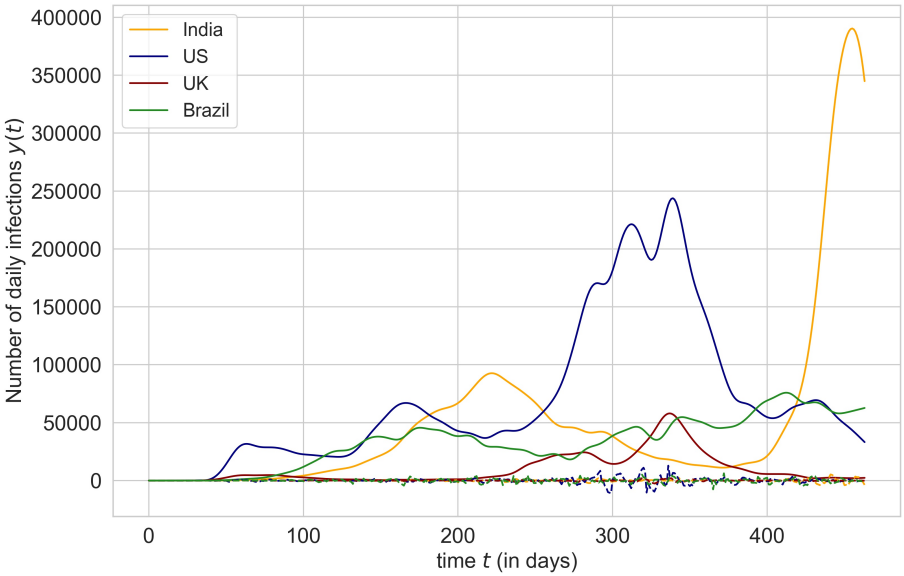

Figure 1: D_3-filtered trends and cycles of daily infections across countries. Cyclical components are denoted by dotted lines.

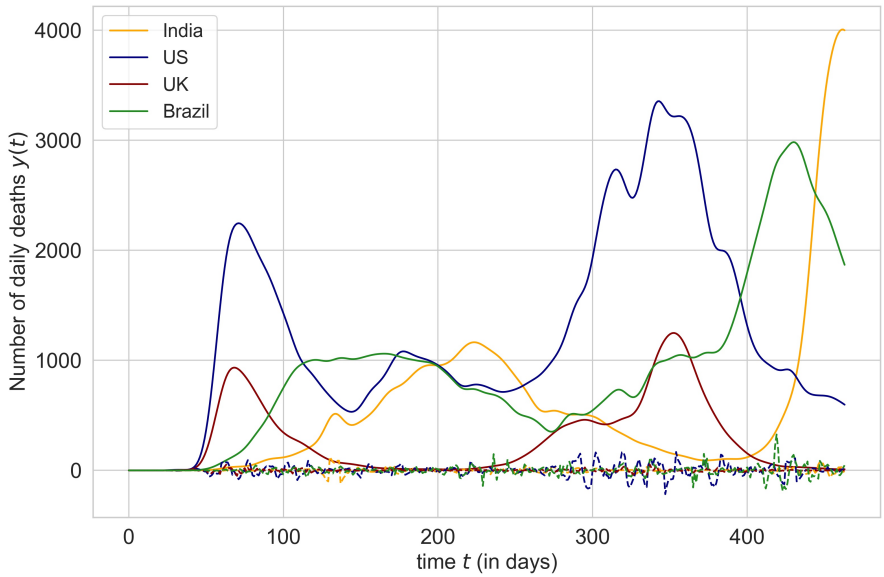

Figure 2: D_3-filtered trends and cycles of daily deaths across countries. Cyclical components are denoted by dotted lines.

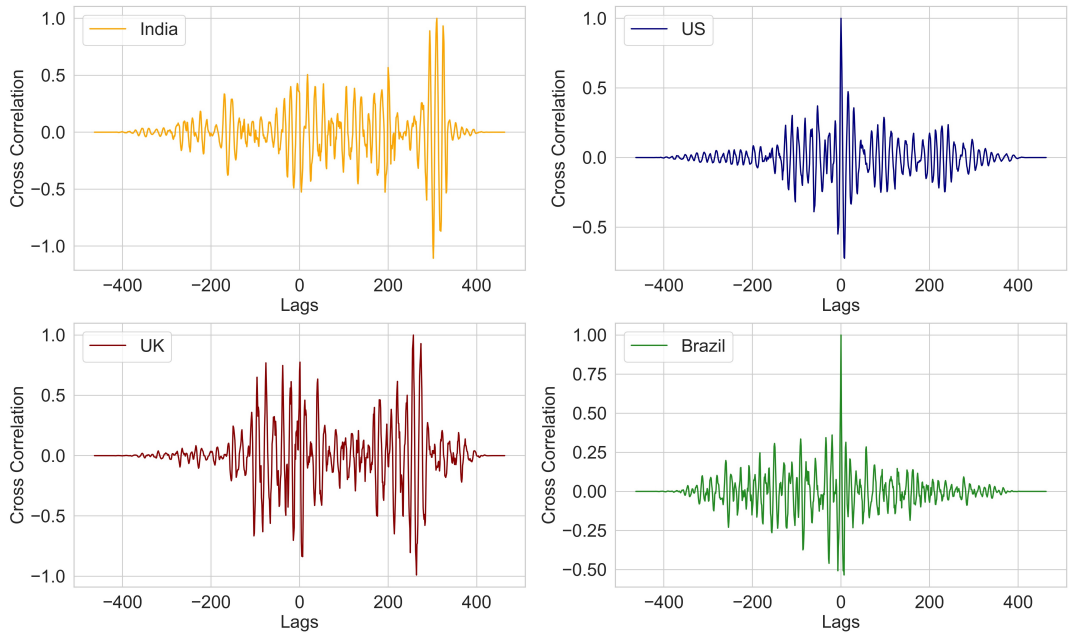

Figure 3: Cross correlation between D_3-filtered cycles of daily infections and daily deaths for different countries.

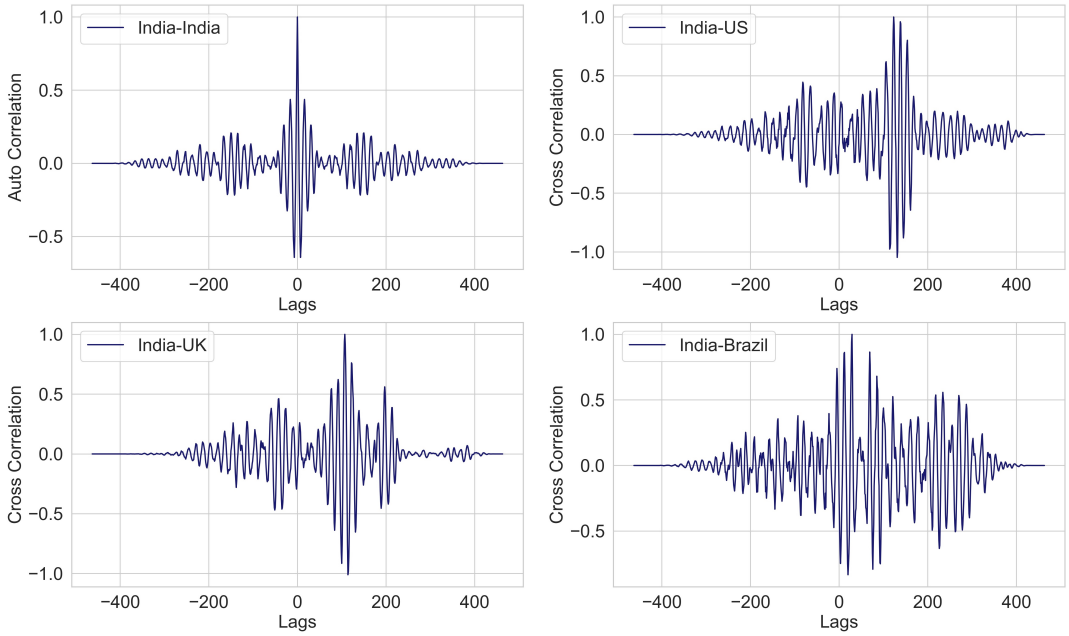

Figure 4: Cross correlation between D_3-filtered cycles of daily infections between different countries.