Fundamentals of Bonds

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Lectures on Fixed Income Models by Sunil Parameswaran

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Lecture 1 - Introduction to Bonds

Introduction

- **Bonds and debentures are known as fixed income securities**. They are a type of *financial claims or securities*.
- The idea of a *claim* is this: as long as your money is with an entity you have a claim against that entity. The entity we refer to in this course is usually the government or a corporate. Thus, bonds, equity shares, preferred shares etc. are types of financial claims.

> **Who issues the claim?** *It is always the entity taking the money.*
> **Who holds the claim?** *It is the party giving the money.*

- The claim is an asset for the holder, for they own something of value. While the claim is a liability for the issuer, for they owe something of value.
- Note. If we add up all financial assets and liabilities, it will amount to zero.
- Difference between debt and equity is this: If you buy an equity share, you become a part owner of the business. However, if you buy a debt security, you do not become a part owner. It is merely an IOU.
    - Debt is a promise to pay interest at periodic intervals, and,

- To repay the principal at a pre-specified maturity date.
- Usually, **debt has a finite life span**. Perpetual debt is possible, but rare. However, equity is always a perpetuity. Why? Because a company that is going concern is assumed to meet all its financial obligations when they are due.
- In ancient Europe, there were securities by the name Consols (consolidated stock). These were government debt securities that were perpetuities.
- **Interest payments are contractual obligations**. And the borrowers are required to pay these irrespective of their financial performance.
- **Interest payments are to be made before any dividends are paid out to equity holders**. In the preference ladder, bond-holders are higher than equity holder and hence usually have lesser returns than equity holders who take up greater risk.
- In the event of liquidation, the claims of debt holders is to be settled first. Only then can equity holders be paid.

> Bonds are called fixed income securities. Why? Consider a company issuing bond saying it will pay 6% interest. This company cannot say that business is dull and offer 4% interest. Likewise, the investor (bond-holder) cannot say that business is booming. Therefore, offer me 8%. All returns are contractually agree upon and fixed.

- Bonds and debentures are fixed income securities because the interest rate is set at the onset of the due period and failure to pay is tantamount to default.

> Returns for bonds is not a function of profitability of the firm.

Bonds and debentures

- Bonds may be secured or unsecured. Secured means that a specific asset has been earmarked as collateral. This means that if the company gets into difficulties, then the asset can be sold to pay the bondholders. Whereas, in an unsecured debt, no asset has been earmarked. The holders can only hope that the issuer has enough assets and liquidity at the time of payment.
- Secured debt holders are prioritized in the standpoint of payments at the time of liquidity. Thus, secured debt holders > unsecured debt holders > equity holders.
- In the US, unsecured debt is called a bond and secured debt is called a debenture. These are used interchangeably in India.
- Worldwide, government securities are called bonds. Government debentures (wherein

a government gives a collateral) is unheard of. Also, the only security that a government can issue is a bond.
- Governments do not give collateral. But municipal bonds can give collateral. Suppose a new bridge is build across a river and that is financed using bonds, then a usage toll collected is used to pay off the bond-holders. These are called revenue bonds where specific revenue stream is given as collateral.

Negotiable debt securities

- Debt securities may be **negotiable** or **non-negotiable**.

  Negotiable securities can be traded in the market. Some debt securities are negotiable, while others are not.

  - A Fixed Deposit receipt or a National Savings Certificate are non-negotiable.
  - Say you have an FD with SBI for Rs.200000. You cannot endorse this receipt and transfer it to someone who you owe money. Instead you must close the account and withdraw the cash and pay the person who you owe.
  - Likewise, National Savings Certificate are used for tax saving purposes. At maturity, you get principal + interest. Before maturity, if you need money, you may pledge the NSC and borrow. But you cannot transfer the NSC - it is non-negotiable.
  - However, many corporate and government bonds are negotiable. They can be traded.

Plain Vanilla vs Bells & Whistles

- The most basic form of a security is known as Plain Vanilla version. Any security which is something more than plain vanilla is said to have Bells & Whistlesattached.
  - Say, Face value = 1000, interest = 8%pa, payable for 5 years. - this is plain vanilla with no additional features.
  - However, say, Face value - 1000, payable in 5 equal annual installments. And interest is 8% on the outstanding principal at the start of the year. - this is Bells and Whistles attached.
  - Say, the bond can be converted after two years to equity shares, this again has Bells & Whistles attached.

Floating Rate and Variable Rate bonds

- Fixed rate bonds have a fixed interest rate for the life of the security. For example, if I issue a 5-year bond of 8% pa on a face value of Rs.1000 I will get Rs. 80 every year for 5 years.
- But suppose I say, interest is MIBOR + 1%, and
    - if MIBOR is 3%, the first payment will be 4% of 1000, that is Rs.40
    - if 6 months later, MIBOR is 5%, the second payment will be 6% of 1000, that is Rs. 60.
    - Higher the benchmark rate (MIBOR here), higher is the interest rate and vice-versa.
- If the benchmark is short term (with maturity less than 1 year) then it is a floating rate bond. Example, Interest is 3 month MIBOR + 1% is a floating rate bond.
- However, if the benchmark is long-term then it is a variable rate bond. Example, Interest is the 10 year GSec rate + 1%, it is a variable rate bond.

Bonds with embedded options

- Convertible bonds can be converted into shares of a stock.
- Callable bonds can be prematurely recalled by the issuer. Before maturity, the issuer can say, take the principal and return the bond (claim).
- Putable bonds can prematurely surrendered by the holders. Before maturity, the holders can say, take the bond and give me the principal.
- Callable, putable and convertible are all bonds with Bells and Whistles.

Terminology and Definitions

- **Face value** is also called par value, redemption value, maturity value or principal. A bond must have a face value.
    - It is the amount payable by the issuer to the last holder at maturity. Why last holder? Because the holder can trade the security - negotiable security.
    - It is also the amount on which the periodic interest payments are calculated.
- **Term to maturity,** tenor, tenure - all mean the following.
    - It is the length of time after which the principal has to be paid back to the last holder.
    - It is the length of time for which periodic interest payments have to be made.
    - OTM - original term to maturity - this is the maturity at the time of issue - static - doesn't change.

    - ATM - actual time to maturity - right now, what is the term to maturity - decreases as each day passes and you get closer to maturity.
- **Coupon** is the periodic interest payable by the issuer of the bond. In earlier days, bonds would come with a booklet of coupons. These were like post-dated cheques. Every 6 months, one had to tear the coupon and claim the interest. These days, the RBI takes the account number and does an electronic transfer. Even today, there are bonds that comes with coupons. Most bonds pay coupons semiannually; Eurobonds pay annually.
- Difference between a **registered bond** and a **bearer bond**.
    - Bearer bond is like a currency note. It belongs to whoever has it. There is no record of ownership.
    - Registered bond have a record of who own how many bonds at that point in time.
    - To collect the coupon on a bearer bond, the holder has to present a coupon from the booklet.

> Let c be the coupon rate and M be the face value. Then the Rupee or Dollar value of the periodic interest payment to be paid by the borrower is the coupon rate and is given by $\text{coupon} = c \times M$.

- **Yield to Maturity (YTM)** - is the rate of return for an investor who buys at the prevailing price and holds it to maturity. It is similar to the IRR for a project. You invest money and take money back - as we see in a project.
- To get the YTM, the conditions to be satisfied are:
    - The bond must be held till maturity.
    - All cash flows received prior to maturity must be reinvested at the YTM till maturity.
- At any point of time YTM may be less than, equal to or greater than the coupon rate.

> **Descartes' Rule of Signs** The maximum number of real positive IRRs is equal to the number of sign changes.

- Pure Cash flows involve only a single sign change. A mixed cash flow has multiple sign changes.
- A pure cash flow will have only one IRR. Mixed cash flows may have upto 3 positive IRRs or more depending on the number of sign changes.
- Pure and Mixed Cash Flows care also called normal and non-normal cash flows respectively.

Time	Pure Cash Flow	Mixed Cash Flow
0	-10000	-10000
1	2000	2000
2	3000	3000
3	4000	-4000
4	5000	5000

YTM is the solution to a nonlinear equation. → Thus they are solved iteratively. **Bonds involve only pure cash flows** → Thus only one real positive YTM exists.

- If there are n cash flows, the YTM is the solution to a polynomial of degree n. So it will have n roots and therefore will have to be solved iteratively.
- If there are only 2 cash flows, then we have a quadratic equation, whose solution gives the YTM.
- For a single cash flow, YTM can be easily solved by simple algebraic manipulation.

Value of a bond

- The value of a bond cannot be just its face value. It cannot also just be the face value plus the sum of all coupon payments. Why? Because of time value of money.
- A holder of a bond gets a stream of contractually promised payments: the coupons and at maturity, the face value. **The value of the bond is the value of the stream of cash flows discounted and added.**

Price vs. Yield

Price vs. Yield is a hen and egg story. Because, when I go to the market, I have a required yield in mind. On that basis, I will quote a price. If I get my price or a better price, I will trade. Once I trade at a price, I can work back and compute my yield. Knowing price enables you to calculate yield and vice versa. A subsequent section would discuss extensively, the relation between bond price and yield.

Bond valuation

A bond will pay identical coupons every period, and the face value at maturity. The periodic cash flows constitute an annuity. The terminal face value is a lump sum payment.

Note: An annuity is a series of identical cash flows at equally spaced intervals. Insurance premiums, rent for apartments, salaries etc. are annuities.

Consider a bond with annual coupon rate of c. We know that semiannually, the coupon rate is $c/2$. Let it pay a semi-annual coupon of $C/2$ dollars and have a face value (fv) of M dollars. The relation $C = c \times M$ holds. Assume that there are N coupons left and we are standing on a coupon date. (Today morning, we have just received a coupon and the next coupon is exactly 6 months aw.) Let y be the required annual yield. (Yield is the rate of return required or YTM). This means that the semi-annual yield is $y/2$.

The present value of the coupon stream is given by:

$$\text{PV(coupons)} = \frac{C/2}{y/2}\left[1 - \frac{1}{(1+\frac{y}{2})^N}\right] \tag{1}$$

$$\text{PV(coupons)} = \frac{(c/2) \times M}{y}\left[1 - \frac{1}{(1+\frac{y}{2})^N}\right] \tag{2}$$

If M is the face value(fv) then the present value of the face value at maturity is given by:

$$\text{PV(face-value)} = \frac{M}{(1+\frac{y}{2})^N} \tag{3}$$

Thus, the price of the bond is given by the sum of present values (PV) of coupon stream and face value.

$$\text{Bond Price} = \frac{c \times M}{y}\left[1 - \frac{1}{(1+\frac{y}{2})^N}\right] + \frac{M}{(1+\frac{y}{2})^N} \tag{4}$$

> The coupons are paid semi-annually and so we discount them semi-annually. However the face value is paid lump sum. Why cannot we discount it annually?
> *Because, the method of discounting must be consistent for all the cash flows. You cannot discount coupons semi-annually and face value annually.*

Example. IBM has issued a bond with fv = 1000, coupon is 8% pa to be paid on July 15 and Jan 15 every year (semi-annually). Today is 15 July 2013. The bond matures on 15 Jan 2033. The required yield (YTM) is 10% pa. What is the bond price?

Solution. The number of periods, N is 39 coupon-receiving months in 19.5 years, that is 39 coupons. Simply plug these values into the above formula:

$$\text{PV} = \frac{0.04 \times 1000}{0.05}\left[1 - \frac{1}{(1+\frac{0.1}{2})^{39}}\right] + \frac{1000}{(1+\frac{0.1}{2})^{39}} \approx 830 \tag{5}$$

One can solve this problem in Excel using the `PV` function, since we are standing on a coupon data. The parameters for this function are `PV(rate, nper, pmt, [fv], [type])`. Here, `rate = 5%` , which is 10% pa YTM divided by 2, to obtain semi-annual yield rate. `nper = 39` , which is 19.5 years times 2. We specify cash inflows as negative. Thus, `pmt = -40` , which is semi-annual coupon rate of face value, 4% of 1000 and finally, the face value `fv = -1000` .

Thus, `PV(0.05, 39, -40, -1000) = 829.83` . That is, the price of the bond is $829.83.

PV function in excel

If you are on a coupon date, you can use the `PV` function in Excel to calculate the bond price. The `PV` function has the parameters: `PV(rate, nper, pmt, [fv], [type])`

`rate` - periodic discount rate, YTM y (yield rate per period: check if the coupon is semi-annual or annual. If semi-annual coupons, then divide the per-annum YTM by 2, that is $y/2$.

`nper` - number of payment periods, N. For semi-annual coupons, this is 2 times number of years, $2 \times N$.

`pmt` - payment made each period (coupon amount), $C = c \times M$. Note. Cashflows in one direction are positive while the other direction are negative. So if you give these inflows as negative, `PV` function returns a positive number.

`[fv]` - face value; optional. This is the last payment that is made at the end, corresponding to the second term in the above equation. If omitted, this means a zero face value. Make sure to be consistent with the sign convention of cashflow.

`[type]` - an optional variable to denote when the payments are due, where 0 denotes end of the period [default], while 1 denotes the beginning of the period.

Par, discount and premium bonds

- If the price of a bond is less than its face value, it is called a discount bond. If the price is more than the face value, it is called a premium bond. If the price is equal to the face value, it is called a par bond.
    - Par bond: yield = coupon; bond price = face value.
    - Premium bond: yield < coupon; bond price > face value.
    - Discount bond: yield > coupon; bond price < face value.
- *Why is the bond at discount? Rather, why is the bond priced lesser than its face value?* This is because the yield is more than the coupon rate. Yield is the return demanded by the market, while coupon is the return provided by the issuer. So, if the market wants a higher return, it will bring down the price. **Remember, price and yield are inversely related.**

Effect of change in face value on bond price

Let c be the annual coupon rate paid by the issuer, y be the per annum yield rate required by the market (also called YTM), and let M be the face value of the bond. If the coupon is paid semi-annually by an amount $C = c \times M$, the semi-annual coupon rate turns out to be $c/2$ and this each coupon amount in the annuity is $C/2$. Likewise, the semi-annual yield is $y/2$ and we note that the face value is also discounted at this rate to maintain consistency. The bond price P is given by the expression,

$$P = \frac{c \times M}{y}\left[1 - \frac{1}{(1+\frac{y}{2})^N}\right] + \frac{M}{(1+\frac{y}{2})^N} \tag{6}$$

Keeping c, y, N constant, we differentiate the bond price P, with respect to face value M of the bond,

$$\frac{\partial P}{\partial M} = \frac{c}{y}\left[1 - \frac{1}{(1+\frac{y}{2})^N}\right] + \frac{1}{(1+\frac{y}{2})^N} > 0 \tag{7}$$

We find that **keeping c, y constant, a higher face value implies a higher bond price.**

Effect of change in coupon rate on bond price

Likewise, keeping other factors fixed, the derivative of P with respect to c is also positive number,

$$\frac{\partial P}{\partial c} = \frac{M}{y}\left[1 - \frac{1}{(1+\frac{y}{2})^N}\right] > 0 \tag{8}$$

Thus, **for a fixed yield and face value, a higher coupon rate implies a higher bond price.**

Effect of change in period till maturity on bond price

Further, keeping everything else fixed, we are interested to find the change in bond price as it matures, i.e. with respect to time to maturity. Now, N is a discrete quantity that can only increase in units of 1. The bond prices P_N and P_{N+1} between two similar bonds with N and $N+1$ coupon payments remaining till maturity are given by,

$$P_N = \frac{c \times M}{y}\left[1 - \frac{1}{(1+\frac{y}{2})^N}\right] + \frac{M}{(1+\frac{y}{2})^N} \tag{9}$$

$$P_{N+1} = \frac{c \times M}{y}\left[1 - \frac{1}{(1+\frac{y}{2})^{N+1}}\right] + \frac{M}{(1+\frac{y}{2})^{N+1}} \tag{10}$$

From the difference between the two bond prices $\Delta P = P_{N+1} - P_N$, we have,

$$\Delta P = \frac{cM}{y\left(1+\frac{y}{2}\right)^{N+1}}\left[1 + \frac{y}{2} - 1\right] + \frac{M}{\left(1+\frac{y}{2}\right)^{N+1}}\left[1 - 1 - \frac{y}{2}\right] \tag{11}$$

$$\Delta P = P_{N+1} - P_N = \frac{M}{\left(1+\frac{y}{2}\right)^{N+1}}\left[\frac{c}{2} - \frac{y}{2}\right] = \frac{M(c-y)}{2\left(1+\frac{y}{2}\right)^{N+1}} \tag{12}$$

Thus the effect of change time to maturity on prices of two otherwise similar bonds depends on the nature of c and y. This exercise motivates the pull-to-par effect described in the next section.

- For a par bond, there would be no change in prices even as one approaches the date of maturity. This shown in the following example.
- However, for a discount bond $(y > c)$, the change in prices is negative on incremental increase in time of maturity. Consequently as you approach date of maturity, N

decreases and the value of the price goes up to its par value at maturity.
- For the premium bond $(y < c)$, the change in prices is positive on incremental increase in time of maturity. Thus as we approach maturity, the bond price falls to its par value.

Effect of change in yield on bond price

Keeping all other factors fixed, we consider the derivative of P with respect to YTM y.

$$\frac{\partial P}{\partial y} = \frac{-cM}{y^2}\left[1 - \frac{1 + \frac{y}{2}(N+1)}{(1+\frac{y}{2})^{N+1}}\right] - \frac{MN}{2(1+\frac{y}{2})^{N+1}} \tag{13}$$

Note that the second term in the above expression is negative. Let $r = y/2$. Expand the term $(1+r)^{N+1}$ using the Taylor's series and truncate the series after the second term. We can deduce,

$$1 + r(N+1) < (1+r)^{N+1} \implies \frac{1 + r(N+1)}{(1+r)^{N+1}} < 1 \tag{14}$$

$$\frac{1 + \frac{y}{2}(N+1)}{(1+\frac{y}{2})^{N+1}} < 1 \implies \frac{\partial P}{\partial y} < 0 \tag{15}$$

Keeping everything else same, a higher YTM means a lower bond price. Since, by definition, the price of a bond is the the present value of its expected cash flows, an increase in yield leads to a decrease in the present value of expected cash flows and therefore, (as seen from the above expression) leads to a decrease in bond price.

> The price vs. yield curve is convex and decreasing. The price of a bond changes over time because of:
>
> - perceived credit risk of issuer changes.
> - approach towards maturity (for a discount or premium bonds only).
> - change in market interest rates, or the yield.

Pull to Par Effect

Let us keep the YTM fixed and move from one coupon date to the next. We know that as the number of periods to maturity decreases, the price of a discount bond $y > c$ increases, while that of a premium bond $(c > y)$ increases. A changes in period of maturity does not affect the bond price. This is called the **pull to par effect.**

Pull to par effect

- par bonds would continue to trade at par.
- premium bonds will steadily decline in price, and
- discount bonds will steadily increase in price.
- At maturity, all bonds must trade at par. *Why? Because the coupon rate equals the yield rate at maturity, and the price of the bond equals its face value.*

Rationale. As we go from one coupon date to the next, one coupon drops out. This reduces the price. But the face value is discounted for one period less. This pulls up the price. For discount bonds, the second effect dominates, for premium bonds the first effect dominates, while for par bonds both effects cancel each other.

$$P = \underbrace{\frac{c \times M}{y}\left[1 - \frac{1}{(1+\frac{y}{2})^N}\right]}_{\text{as } (N \to N-1), \text{ one coupon reduced. } \textbf{Effect 1}} + \underbrace{\frac{M}{(1+\frac{y}{2})^N}}_{M \text{ discounted for one period less. } \textbf{Effect 2}} \tag{16}$$

fv = 1000	discount bond	par bond	premium bond
time	c = 6%, y=8%	c = 6%, y=6%	c = 6%, y=4%
5	955.48	1,000.00	1,047.13
4	963.70	1,000.00	1,038.08
3	972.25	1,000.00	1,028.84
2	981.14	1,000.00	1,019.42
1	990.38	1,000.00	1,009.80
0	1,000.00	1,000.00	1,000.00

Zero Coupon Bonds

- Zero coupon bonds do not pay any coupons. They always trade at a discount, and can never trade at a premium. If you buy and hold till maturity, you will get back the face value. The difference between the face value and price is the interest for the investor.
- You can either discount the fv annually at 10% or semi-annually at 5%. Since it doesn't give any coupon, one would assume that it doesn't make any difference. However, the market follows the second approach: discounting ZCB at semi-annual interest rates.

> **The market discounts ZCBs semi-annually. Why?**
>
> **Rationale.** If I give you and ask you to invest: you have a choice to invest between coupon bonds and ZCBs. You cannot make comparisons unless the discounting technique is common. Since plain vanilla coupon bonds are discounted semi-annually, so are ZCBs.

Example. Microsoft issues a ZCB with 5 years to maturity and face value of $10000. What should be the bond price if a yield of 10% is required?

A yield of 10% pa corresponds to 5% semi-annually, number of periods is 5 times 2 equals 10 and the face value is 10000. Plugging in the `PV` function, we get `PV(5%,10,0,-10000) = 6,139.13`

Alternatively, discount the face value of $10000 at 5% semi-annual interest rate for 10 periods.

$$P = \frac{10000}{(1+0.05)^{10}} = \$6139.13 \tag{17}$$

- ZCB will never trade at premium. Before maturity date the ZCB will trade in a discount and on maturity date, it will trade at par.
- *If held to maturity, a ZCB will always give rise to a **capital gain**.* However, *if sold prior to maturity it could either lead to a capital gain or a capital loss*.

Example. Suppose I buy a ZCB with a face value of 1000 and YTM is 8%. The price is `PV(0.04,10,,-1000) = 675.56`. Now, 6 months later, the YTM is 8.2%. This has a price `PV(0.041,9,,-1000) = 696.54`. What is 6 months later, YTM is 8.6%. In this case price is `PV(0.045,9,,-1000) = 672.90`.

> Capital Gain: Selling Price > Purchase Price
> Capital Loss: Selling Price < Purchase Price

- Bullish speculators expect prices to rise. In the bond market, they expect yield to fall. Since for a given maturity, ZCBs have the highest price sensitivity, bullish speculators like ZCBs.
- Insurance companies and pension funds like such ZCBs because they can lock in the rate of return for a long period.
- YTM assumes that all coupons are reinvested at the YTM. In the case of a ZCB, the holder need not worry about reinvestment. *Why?* Because there is no coupon present to reinvest. So, to get the YTM of a ZCB, simply buy and hold till maturity - that gives an assured return. This is why such long term investors prefer such bonds.

Lecture 2 - Auctions, Callable and Puttable Bonds

Computing YTM using `RATE` function

The relationship between price and yield is inverse and non-linear. Consider the following example where the bond price is computed as the present value of its cash flows.

Example. Given face value `fv = 1000`, time to maturity is 7.5 years, coupon rate is 8% pa paid semi-annually and the expected YTM is 9% pa. *Determine the bond price*.

The price of the bond is calculated using the `PV` function. We know that semi-annual coupon payment is 4% of 1000 that is `pmt = -40`, the number of periods is twice the number of years `nper = 15` and the semi-annual YTM is `rate = 4.5%`. The price of the bond is `PV(0.045,15,-40,-1000) = 946.30`.

Now *suppose this were a zero coupon bond, what would be its price?*

Intuition. A ZCB pays no coupons and is always traded in discount. That is YTM is always greater than coupon rate. The price is given by,

$$P = \frac{c \times M}{y}\left[1 - \frac{1}{(1+\frac{y}{2})^N}\right] + \frac{M}{(1+\frac{y}{2})^N} \tag{18}$$

Setting $c = 0$ for a ZCB, the first term vanishes because there are no coupon streams to be discounted. Price of the ZCB is solely the face value at maturity M discounted by the semi-annual YTM $y/2$. Thus, this ZCB is priced at `PV(0.045,15,0,-1000) = 516.72`.

Suppose the original coupon-bond had a price of 958, what is the YTM?

Intuitively one can argue that this number must be below 9%. We know that when YTM is 9% the price was 946.3 Pricing the bond in excess of 946.3 would lead to a lesser yield, owing to the inverse relationship.

Given all other parameters, the yield can be calculated using the `RATE` function on Excel. By hand, one would have to solve the non-linear bond price equation analytically to obtain the yield as roots of the polynomial equation. For this bond, we know `nper = 15`, coupon amount `c = -40`, present value is the price of the bond `pv = 958` and the face value `fv = -1000`. Note that PV is of opposite sign relative to both coupon and face value. Thus, `RATE(15,-40,958,-1000) = 4.39%`. This is the semi-annual interest rate and is multiplied by 2 to obtain the annual YTM of `2 * RATE(15,-40,958,-1000) = 8.78%`

If the bond were priced at 915, what happens to the YTM?

In this case, pricing the bond in deficit of 946.3 (value at 9% yield) would lead to an increase in YTM beyond 9%. We compute annual YTM as `2 * RATE(15,-40,915,-1000) = 7.80%`

Treasury Securities

- In the US, Ministry of Finance is called the Treasury Department. Hence Federal Government Debt Securities are called Treasury Securities.
- The US Treasury issues: **T-bills, T-notes and T-bonds**
  - T-bills have a maturity of less than or equal to 1 year.
  - T-notes have a maturity between 1 to 10 years.
  - T-bonds have a maturity between 10 to 30 years.
- The US government as well as the GoI issue debt with a maximum maturity of 30 years.
- Treasury securities are backed by federal government of the issuing nation. Therefore, they are devoid of credit risk or risk of default.

> **Why are Treasury securities free of default risk?**

> Federal Government default is highly unlikely because it can:
> a. print money whenever it wants.
> b. raise taxes whenever it wants.

- So *for any maturity, if a government security is available it will usually have the lowest yield*. During the 2008 crisis there was a time when rates quoted by London Banks were lower than T-bills for the same maturity. The perception was that private banks were more credit-worthy. But this was a highly unusual scenario.
- The yield on such securities is a benchmark for setting rates on other kinds of debt.
- **Why does the government issue debt?**
    - Because it has a fiscal deficit, which is when the revenue for the year is less than the expenditure and these expenses have to be financed.
    - Or if it has debt accumulated in previous years (due to deficits) and the interests have to be paid (coupon payments) now.
    - Or it has to pay past debt (face value) that are currently maturing.
- The US Treasury is the largest issue of debt and its market is the most liquid bond market in the world.

> Money market securities by definition have an OTM (original term to maturity) of less that 1 year. Since ATM is the current period remaining until maturity, this too would be less than 1 year for money market securities.
>
> Money market securities must be debt securities. And not equity because it has no maturity.

- T-bills have an OTM of 1 year or less, and thus come under the category of money market securities. T-bills are zero-coupon bonds and hence discount securities.
- T-notes and T-bonds are sold at face value and pay periodic interest. These are capital market instruments and consequently have OTM exceeding 1 year at the time of issue.
    - T-notes have maturities ranging from 1-10 years.
    - T-bonds have maturities ranging in excess of 10, and up to 30 years.

Re-opening of an existing issue

- An issue may be followed later by a further issue, with the same time remaining to maturity and the same coupon. The issuance of further tranches is termed as **re-opening.**

- Say 6 months ago, the treasury issued 10 year bonds with a coupon of 4%. Now it issues 9.5 year bonds with coupon of 4%. It is adding to the pool that already exists and makes it more liquid. This is termed as re-opening of an existing issue.

Primary Dealers

- A dealer who is authorized to deal directly with the Central Bank of a country is called a **primary dealer.**
- In India, primary dealers are authorized to deal directly with the RBI. Likewise, PDs are authorized to deal directly with the NY Fed. In India we have one RBI for the country with multiple admin offices. In the US, the Fed is a collection of 12 member banks known by the cities where it is located - NY Fed, Boston Fed etc. The key of these 12 Feds is the NY Fed, because any decisions that are taken have to be implemented by the NY Fed as NYC is the largest market in the world.
- In India like RBI has the MPC (Monetary Policy Committee), the US has the FOMC (Federal Open Market Committee). All decisions taken by the FOMC have to be implemented by the NY Fed. So the President of the NY Fed is a permanent member of the 7-member FOMC.

Treasury Auctions

- Treasury sells bids, notes and bonds by way of a competitive auction process. Most treasury securities are bought by primary dealers. Individual investors submit non-competitive bids and participate on a much smaller scale.
- For new issues, we have yield-based auctions. Bidders indicate the minimum yield sought and the quantity. For re-openings, we have price-based options. Bidders indicate the maximum price they are prepared to pay and the quantity.
- Obviously price and yield are inversely related. So, bidders will indicate a maximum price or a minmum yield.
- Bids may be competitive or non-competitive:
    - **Competitive bidders** indicate price and quantity or yield and quantity.
    - **Non-competitive bidders** indicate only quantity. Any price or yield is acceptable to them - they are price-takers not setters.
- Primary dealers who bid on behalf of their clients usually submit large competitive bids. These bids indicate the maximum price the bidder is prepared to pay if it is a price-based auction. Or if it is a yield-based auction, the bids indicate the minimum yield the bidder is prepared to accept.

- In India we have a quota for non-competitive bidders. In the US, a non-competitive bidder cannot ask for more than $5million face value. Compared to the size of a typical issue this is very small in scale. So, in the US, non-competitive bids are fully accepted. The quantity is subtracted from the total amount on offer and the balance is sold to competitive bidders.

> - **Competitive bidders** - indicate price & quantity or yield & quantity.
> - **Non-competitive bidders** - indicate only price
>
> The treasury will net out the total amount of the non-competitive bids and the balance will be allocated to competitive bidders.

- There are two ways in which securities can be allotted in auctions:
  - **Multiple price/yield auctions** - French auctions
  - **Uniform price/yield auctions** - Dutch auctions
- In a yield based auction, bids will be arranged in ascending order, and priority goes to the party who wants lower yield.
- In a price based auction, bids will be arranged in descending order and priority goes to the party who is willing to pay a higher price.
- In a multiple price/yield auction, each successful bidder pays a price corresponding to their bid.
- In a uniform price/yield auctions, each successful bidder pays a price corresponding to the market-clearing bid. Since 1999 the US treasury has been conducting only uniform yield auctions.
- Since in a price-based auction bids are arranged in descending order people willing to pay a higher price will not object.
- In a yield based auction, since bids are arranged in ascending order of yield people willing to accept lower yields will not object.
- The highest yield that is accepted is called the stop yield or high yield or **market clearning yield.**
- The ratio of total demand (bids received) to the total amount offered is called the **bid to cover ratio**. Higher the ratio stronger is the auction.

> **On auction types**
>
> - **Yield-based auction** - bids arranged in ascending order. Lower yield prioritized.

- **Price-based auction** - bids arranged in descending order. Higher price prioritized.
- **Multiple p/y or French auctions** - each successful bidder pays a price corresponding to their bid.
- **Uniform p/y or Dutch auctions** - each successful bidder pays a price corresponding to the market-clearing bid.
- **Market clearning yield** - highest yield that is accepted.
- **Bid to cover ratio** - ratio of total demand (bids received) to the total amount offered by the issuer.

Illustration of auctions

Let us assume that the Treasury is offering $25 billion worth of T-bonds. If $2 billion worth of non-competitive bids have been received, they are fully accepted and the remainng $23 billion worth of bonds are available to competitive bidders. Say there exists 6 bidders with the following bids.

Bidder	Bid Yield (in %)	Bid Amount (in billion $)	Aggregate Amount (in billion $)
Alpha	5.37	3	3
Beta	5.372	5	8
Gamma	5.373	4	12
Delta	5.375	8	20
Charlie	5.375	12	32
Tango	5.38	3	35

Since the bids are arranged in ascending order of yield, this is a yield-based auction. The priority goes to the bidder with the lowest bid. Of $23 billion, Alpha gets their $3 billion, Beta gets $5 billion and Gamma gets $4 billion. Now, the Treasury has to allocate the remaining $11 billion and Delta and Charlie have quoted the same yield of 5.375 with an aggregate demand of $20 billion. Thus, allocation must be done pro-rata.

We allocate 11/20 = 55% to each bidder at this yield. Thus 55% of $8 billion = $4.4 billion will go to Delta and 55% of $12 billion = $6.6 billion will go to Charlie.

People who bid a higher yield will get nothing and are said to be shut out of the auction. However if you are shut out you can always buy from the secondary market after auction.

The market clearing yield which is the highest yield accepted is 5.375%. The treasury is offering $25 billion while the total demand is 2 + 35 = $37 billion. So the bid-to-cover ratio is 37/25 = 1.48 - higher this value, the more successful the auction.

Suppose if the total bids are $200 billion then the bid to cover ratio is 8:1. This auction would be considered more successful than the previous auction.

Zero Coupon Treasury Securities

- The US does not issue Zero coupon T-notes or T-bonds. However, using T-notes and T-bonds, synthetic zero coupon securities can be created.

  > **Coupon Stripping**
  >
  > - To do this, take a large quantity of a T-note or T-bond and separate all cash flows from each other. (Strip the cash flow)
  > - Sell the entitlement to each cashflow separately. This is called **coupon stripping.**

- For instance, say we have a bond with a face value 1000, 8% coupon pa payable semi-annually and a time to maturity of 2 years. My cash flows are:

Period	Cash Flow	Sell off the CF to:
6 month	40	Person A
12 month	40	Person B
18 month	40	Person C
24 month	40 + 1000	Person D + Person E

- We have used a plain vanilla security to create 5 aritificial zero coupon securities. Each cash flow is a ZCB maturing at that point in time. For instance the cash flow with a face value of 40 matures in 18 months and is sold to person C. This is called **coupon stripping.**

- Earlier, investment banks such as Merrill Lynch, Morgan Stanley etc. would buy large quantities of Treasury securities, strip the cash flows and sell them as a zero coupon bond. Such issues are called **trademarks**, since the investment bank have its own name to such securities.
- The issue of trademarks have now ceased because the Treasury has now started collaborating with Primary dealers (such as investment banks) to create such securities.
- Such ZCBs are known as **STRIPS**, an abbreviation for Separate Trading of Registered Interest and Principal of Securities. These are not issued or sold by the Treasury and the market is made by investment banks.
- If the price of the mother bond is different from the sum of prices of the baby bond then there is an arbitrage profit.
- What is the motivation to create such products like STRIPS or trademarks?
    - Because arbitrage profits can be made. If the coupon security (or the mother bond) is purchased at a lower price than that could be obtained by each cash flow (zero-coupons sold by SPVs) separately, then arbitrage profits could be made.

> **Example. (Profits out of trademarks)**
> Merrill buys 500 million USD T-bonds. It will transfer it to an SPV (special purpose vehicle). The SPV will own the bonds. It has been set up exclusively to issue STRIPS (or trademarks). It cannot sell the bonds nor pledge the bonds and borrow. It cannot also create derivatives based on these bonds. The SPV is therefore a single purpose dedicated Trust whose only is to own the bonds and pay off the cash flows when they arise.
> *How does Merrill make money then?* If the cost of the mother bond is lower than the sum of prices received for the trademarks then Merrill makes money.

Coupon Stripping and SPVs

- Coupon stripping is a case of financial engineering - that is creating a risk-return profile that is not otherwise available.
- An investment bank would buy a large quantity of Treasury security and these would be placed with an SPV, which is a single-purpose dedicated trust.
    - The SPV has the powers to own the bonds and collect payments.
    - It cannot sell or lend the bonds, not can it write options on bonds.

    - It cannot use them as collateral for borrowing.
    - However, the SPV is empowered to issue ZCBs where each security represents the ownership of a single cash flow from the mother bond.
- For instance, assume that 100MM USD of 15 year bonds with a coupon of 8% are placed with the SPV. The SPV can issue 6m, 12m, 18m extending upto 14.5 year ZCBs with a total face value of 10$MM USD.
- The Treasury launched this program in 1985 to facilitate the stripping of designated securities. *What can be stripped?*
    - All new T-bonds and notes with a maturity of 10 years or more are eligible to be stripped.
    - Furthermore, the ZCBs created in the process are direct obligations of the US government.
    - An entity cannot buy directly buy STRIPS from the Treasury. One has to approach an Investment bank. An I-Bank which owns the mother bond can request the Fed to replace it with a set of baby bonds so that each can be traded.
- A dealer who owns a bond or a note can ask the FRB where it is held
    - In order to replace it with an equivalent set of STRIPS representing each payment as a separate security.
    - Each of these securities can be traded independently of others.
- After some time (in 1987), the Treasury allowed dealers to reverse the process. It said that if you can strip securities, you can also reconstitute them. That is, if an I-Bank has all the baby-bonds, it can request the Fed to replace it with the mother bond. This is called a **reconstitution.**

> **STRIPS reconstitution**
> If a dealer owns STRIPS representing all the coupon and the principal payments of a bond, then it can request the Fed to convert these holdings into a single position in the correspinding bond. This is called STRIPS reconsitution.

Callable bonds

- **Callable bonds** can be prematurely retired by the issuer. Thus, the issuer has the call option and hence must pay the premium.
- The issuer buys back the bond from the holders before maturity by paying the face value.

- **Price of a callable will be lower than that of a comparable plain vanilla**. *Why?* Because, the callable can be recalled by the issuer any time before the maturity and the investor is exposed to a cash flow uncertainty. They could potentially suffer a reinvestment risk.
- Consequently, buyers of callable bonds demand a higher yield. The uncertainty in the cash flow again requires them to demand a higher rate of return (YTM) on the bond.
- **At the time of issue, a callable carries a higher coupon than a comparable plain vanilla.** This is because, the issuers (being buyers of the call) must pay the premium to investors who take up cash flow uncertainty. They must promise a higher coupon rate.
- Post-issue, a callable with a given coupon would have a lower price than the plain vanilla with the same coupon,
- Callables expose the holders to timing risk. They do not know when the bond will be called. They do not know how many coupons they will get or when their face value will be repaid.
- *When will a callable bond be recalled?* When the interest rates or required yields are falling the call is exercised.
    - The issuer can call back the bonds and offer fresh bonds at lower coupons.
    - In this scenario, the holders would have to reinvest their money in a falling interest rate environment. This risk suffered by the investors of a callable is called the **reinvestment risk.**
    - It favours the borrower and works against the lender. This is why investors demand a higher YTM.

Discrete and continuously callable bonds

- A bond may be discretely or continuously callable.
- Discretely callable bond may be recalled only at certain pre-specified dates. For example on the coupon dates over a period of time. If not called today, then it cannot be called by the issuer for the next 6 months.
- Continuously callable bond may be called at any time after it becomes callable.

> **In options language,** we have
>
> - discretely callable - like Bermudan options
> - contiuously callable - like American options

> European options can be execised only at maturity, while American options can be exercised on any date. A Bermudan option has multiple exercise date cannot be exercised on any date other than these.

Deferred Callable Bonds

- Suppose the issuer says, *20 year bond that can be called any time after tomorrow.* This bond might not have any takers. Because, they offer no protection to the lender. These bonds that can be called at any time are called **freely callable bonds.**
- Say the issuer says, *No matter what happens to interest rates it cannot be called during the first 5 years.* There is a lock-in period of 5-years where the holder's interest are protected.
- This lock-in period where bonds cannot be recalled is called **call protection period** and such bonds offering some protection are called **deferred callable bonds.**
- In practice, when a bond is recalled, the issuer will pay the lender a call premium, of usually 6m or 1yr coupons. This call premium acts as a sweetener (and make these callables attractive to potential investors.)

Reinvestment Risk & Price Compression

- The biggest risk faced by a lender to a callable bond is **reinvestment risk.**
    - The bond can be called by the issuer when the market rates are low. Issuer can then use it to issue fresh bonds at a lower coupon rate.
    - Consequently, the holder would have to invest the proceeds at a lower rate of interest.
- *When does a plain vanilla price appreciate?* This happens when the yields are falling.
    - However in declining yield environment the callable may be recalled.
    - If recalled the issuer would pay a predecided amount (as premium).
    - In a falling rate environment, the price appreciation of a callable will be less than that of a plain vanilla. This is called **price compression.**
- Price compression happens because, the price appreciation potential of a callable in a declining interest rate environment is limited. The market will increasingly expect the bond to be redeemed at the call price as the rates fall.
- *Given the reinvestment risk and price compression, why would any investor want to hold a callable?* Because, there could be sufficient compensation in the form of higher yield, in order to take up this risk.

> **Reinvestment risk.** The risk faced by holder of a callable of having to reinvest proceeds at a declining interest rate environment when the issuer recalls the bond.
>
> **Price Compression.** The fact that in a falling rate environment, price appreciation of callables would be less than that of a comparable plain vanilla.

Puttable bonds

- Puttable bonds can be prematurely surrendered by holders. The holder has the option in this case and must pay the premium.
- **A puttable will have a higher price than a comparable plain vanilla.** Because, the holder is having an option and he must pay a premium for this. Consequently, they hav lower yield.
- **A puttable can be issued with a lower coupon,** initially, when compared to an equivalent plain vanilla.
- Subsequently a puttable will carry a higher price than a plain vanilla of the same coupon rate.
- *When are puttables exercised?* When interest rates are rising, the holders can return the bonds and buy fresh bonds that offer higher coupons.
- Puttables can be usually put back at par value. So the par value becomes the floor for the value of the bond in a rising yield environment.
    - Rationale: The holders can always surrender the bonds at par value. They will never sell the puttables at a lower price. So par value becomes the **floor price** for puttables in a rising yield environment.

Convertible bonds

- **Convertible bonds** grant the holder the right to conver the bond into a predetermined number of equity shares.
- They can be thought of as a plain vanilla corporate bond with a call option to buy the common stock of the issuer.
- The number of shares receivable on conversion is called the **conversion ratio.**
- Conversion privilege may extend for all or only a portion of the bond's life.
    - **Conversion ratio may decline over time**. More the period after issue not put, one could potentially get lesser number of shares.

- **Conversion ratio is adjusted proportionately for stock splits and dividends (bonus shares).**
- It may be the case that 20 year bond may be convertible for all its life or only for the first 5 years.
- It may also be the case that if it is converted before 3 years the conversion ratio is 40, 3-5 years then CR = 32 and after 5 years only 25.
- So longer the holder waits to convert the greater is the price paid per share.

Example. **Adjustment for stock splits.**

ABC corporation has issued a convertible bond of face value 1000 and conversion ratio of 40. Then, conversion price = 1000/40 = $25.

If the current market price of the share is $48, a lot of people may convert. Now if the company announced a 4:1 stock split. Then if the conversion ratio stays unadjusted at 40, then this corresponds to a stock price of $48/4 = $12. In this scenario holders would be unwilling to purchase the shares for $25, when it sells for $12 in the market.

Thus the conversion ratio must be adjusted so that it becomes 40 times 4, CR = 160. Then the conversion price would be $1000/160 = $6.25. Once again, holders may prefer to buy shares.

- **Conversion value** of a convertible is the value of the shares if conversion occurs immediately.

  Conversion value = Current Share Price X Conversion Ratio

- The minimum price of a convertible bond is the greater of:
  - value of shares if the conversion occurs immediately (**conversion value of the bond**).
  - its value as a bond without conversion option (**straight value of the bond**).
- To estimate the straight value we need the required yield on a non-convertible bond with the same credit rating and similar investment characteristics. [Identical in plain vanilla except for the conversion feature.]
- To rule out arbitrage, the price of convertible must be greater than or equal to the maximum of conversion and face value.

$$\text{Convertible price} \geq \max(\text{conversion value}, \text{straight value}) \tag{19}$$

Example. **Conversion value vs. straight value.**

> ABC corporation has issued a convertible bond of face value 1000, maturity 10 years, conversion ratio (CR) of 40, coupon rate 8%, current market price (CMP) of $900 and current share price (CSP) of $20. Assume YTM of comparable straight bond is 10% pa
>
> Since CSP = 20 and CR = 40, the conversion value = 20 x 40 = $800. The price of the comparable stright bond is `PV(0.05, 20, -40, -1000) = $875.38`. One can also use the function `40*PVIFA(5,20) + 1000*PVIF(5,20)` to get the same straight value.
>
> To rule out arbitrage the price of the convertible must be greated than or equal to the higher of the conversion and straight value.
>
> Thus the minimum price should be $875.38. Our value was $900, so there is no arbitrage opportunity.

Lecture 3 - Risk in the Bond Market

Risk inherent in bonds

- **Risk is the possibility of loss arising due to the uncertainty regarding the outcome of a transaction.**
    - All bonds are exposed to one or more sources of risk
    - There should be at least 2 outcomes and at least one must lead to a loss.
- Buy a security with payoffs: 20% return with probability 0.4, 30% return with 0.4 chance and 40% return with 0.2 probability. There are multiple outcomes, but no possibility of a loss - hence this is not really risky.
- Buy a security and you are guaranteed to lose 10%. There is a loss but no there is no uncertainty - hence not risky, but a ridiculous option.
- Buy a security with payoffs: -20% return with probability 0.4, 30% return with 0.4 chance and 40% return with 0.2 probability. This is risky.

Credit Risk

- **Credit or default risk is taken by the holder of the bond. The risk that the issuer may not pay coupons and/or face value as promised.**
- Except for Federal Government backed Treasury Securities, all other securities carry default risk (of varying degrees).

- The Fed cannot default because it can print money or raise taxes at any time.
- At the time of issue, the issuer provides details of the issuer's credit-worthiness and the risk factors are mentioned in the **Offer Document** or **Prospectus.**
- Like equity debt also comes with a Prospectus. But if a prospectus is available, it does not mean that everyone will understand. So, we have credit-rating agencies to evaluate the riskiness of debt.
- We have credit rating agencies: Moody's, S&P, Fitch etc. and in India we have CRISIL, CARE, ICRA, and so on.
- **Investment grade bonds** are rated: AAA, AA, A and BBB. Below BBB, the bonds are called **non-investment, speculative or junk bonds.**
- Junk bonds are highly risky and thus carry higher coupons than investment grade bonds.
- There are two kinds of junk bonds: Original Issue Junk and Fallen Angels:
    - Original Issue junk - At the time of issue, the issuer is a dabba company.
    - Fallen Angels - these were investment grade originally, but have been reclassified as junk. A fallen angels will have low coupons and high yields.

Liquidity Risk

- Liquidity means the availability of many buyers and sellers. Buyers can locate sellers easily and vice-versa. Because, time is also money, it it takes time to locate a counter-party, there is a cost involved.
- Illiquid markets are also known as thin markets.
- **Liquidity risk taken up by the investor is the possibility that the market might be illiquid (unavailability of buyers and sellers).**
- Dealers carry inventories. So they will carry two prices: a bid and an ask. A purchase at the bid followed by a sale at the ask is called a round trip transaction, because the inventory reverts to the original level. The profit per round-trip transaction is the **bid-ask spread.**
- If the dealer is doing a lot of round trip transactions he can afford a small spread. If volumes are high, margins can be low.

> **Illiquid Markets**
>
> - In illiquid markets, buyers will have to offer a large premium over the fair value, whereas sellers will have to accept large discounts at the time of sale.

> - Illiquid markets are characterized by large bid ask spreads. (Because trades will be few and far between).

Interest Rate Risk

- Interest rate or yield is the key variable of interest in debt markets. **Interest rate risk is the risk that the rates may move in an adverse fashion.**
- Interest rate risk impacts fixed income securities in 2 ways.
  - **Reinvestment risk - risk that market rates of interest may decline before a coupon is received**.
  - All bonds expose the holders to reinvestment risk except for ZCBs. For ZCBs pay no coupons to reinvest in the first place. Reinvestment risk is higher in case of callables. To get the YTM of a ZCB, just hold it to maturity.
  - **Market/ Price risk - risk that the interest rates may be higher than anticipated, in which case the bond will have to be sold at a lower than anticipated price.**
  - If I sell the bonds before maturity, there is the risk that price could be lower than anticipated. If I hold till maturity, I will get the face value. There is the risk that YTMs at the time of sale may be higher than anticipated (which means the price will be lower than anticipated).
  - Two risks work in the opposite directions. Reinvestment risk hits the holders if rates fall. Market risk hits the holders if rates rise.

Inflation Risk

- Inflation is the erosion in purchasing power of money. Most bonds promise fixed cash flows in dollar terms.
- **Inflation risk is the risk that the purchasing power may have eroded more than expected by the time the cash flow from the bond is received.**
- Higher inflation will reduce the effective or real interest rate. The interest rate in monetary terms is called the nominal rate. The real rate is adjusted for changes in purchasing power.

> **Fisher's equation**
>
> $(1+R) = (1+r)(1+\pi)$

> where R is the nominal rate, r is the real rate
> and π is the inflation rate.

- If inflation is higher than anticipated the actual real rate may be lower than the expected real rate and may even be negative.
- Investors want a 3% real rate and think that inflation will be 5%. They will demand $(1+0.03)\times(1+0.05)=1.0815$. Assume there is no default risk, then they will get 8.15%. However, actual inflation turns out to be 7.5% and actual real rate would be 0.6% since, $\frac{1+0.0815}{1+0.075}=1.006$.
- The forecasted value is called the ex-ante rate. The rate obtained is called the ex-post rate. The ex-post rate may be lower than the ex-ante rate and may even be negative.

Indexed Bonds

- **These are bonds whose coupons are linked to and varies with a price index**. Price indices are a barometer of changes in the purchasing power of a currency. If inflation is high, so will be the index level and hence the cash flows will be high. If index is low, the cash flows will be low.
- **P-linkers** or pricinpal linkers and **C-linkers** or coupon linkers.
  - In a P-linker, the coupon rate remains constant. The prinicipal is adjusted using the price index.
  - Example, say coupon rate is 5% over a face value of 1000. If the index is 100 today, then the adjusted principal is 1000 and the coupon is 50. However if tomorrow if the index rises to 108, then the face value is adjusted by the factor of (108/100) times 1000 to get 1080. The coupon now is 5% of 1080 which is 54.

period	index	face value (adj.)	coupon @ 5% of fv
0	100	1000	50
1	108	1080	54
2	112.5	1125	56.25

    So if inflation is high, then the holder will get higher cash flows.
  - In a C-linker, the principal remains constant. The inflation rate is added to the real rate to determine the coupon.

- For example, consider the same scenario as before. Since in period 1, the inflation rises by 8% and is added to the coupon rate of 5%. Thus the coupon is 13% of face value, which yields 130.

period	index	face value	inflation	coupon @ 5% of fv
0	100	1000	0	50
1	108	1000	8%	130
2	112.5	1000	12.50%	175

- Sometimes there may be deflation (reduction in price levels). If because of deflation, the adjusted principal of the P-linker falls below the original, then the usual practice is to pay back the original principal.
- Similarly, if because of deflation, the coupon of the C-linker becomes negative, then it will be set to 0%.

Timing Risk

- For plain vanilla bonds, it there is no default, there is no uncertainty regarding the cash flows, you will get the coupons every six months and get back the principal on the scheduled maturity date.
- In case of Callable bonds, the holder does not know how many coupons they are going to get, when the face value will be repaid. This is called timing risk. Because of this, the YTM of the callable will be higher than the YTM of a comparable plain vanilla bond.
- Thus holders of callable bonds will demand a premium for bearing this risk. This is why callable bonds trade at a lower price than comparable plain vanilla bonds.

Foreign Exchange risk

- This risk arises when the cash flow from a bond are denominated in a foreign currency. If the foreign currency depreciates in value with respect to the home country, the returns will be lower than anticipated.
- If you buy a domestic bond there is no Forex risk. But if the cash flows from the bond are denominated in a foreign currency and the home currency appreciates there is a risk.
- Say, I buy a bond paying a coupon of $10. The exchange rate was 72 USD-INR. So I

expected Rs.720. But in 6months, the Rupee rallied to 68 USD-INR, I will get only Rs.680. This is foreign exchange risk.

Types of risk associated with bonds

1. **Risk** is the possibility of loss arising due to the uncertainty regarding the outcome of a transaction.
2. **Credit/default risk**, taken by the holder of the bond is that the issuer may not pay coupons and/or face value as promised.
3. **Liquidity risk** taken up by the investor is the possibility that the market might be illiquid (unavailability of buyers and sellers).
4. **Interest rate risk** is the risk that the rates may move in an adverse fashion. Two types (opposite direction):

- **Reinvestment risk** - that market rates of interest may decline before a coupon is received, in which case investor has to reinvest at a lower rate.
- **Market/Price risk** - that the interest rates may be higher than anticipated, in which case the bond will have to be sold at a lower than anticipated price.

5. **Inflation risk** is the risk that the purchasing power may have eroded more than expected by the time the cash flow from the bond is received.
6. **Timing risk** is taken up by holders of Callable bonds who are uncertain about the number of coupon payments and time at which face value would be repaid.
7. **Foreign exchange risk** arises when the cash flow from a bond are denominated in a foreign currency. If the foreign currency depreciates in value with respect to the home country, the returns will be lower than anticipated.

Valuation in between coupon dates

- Thus far we have assumed that we have on a coupon date and have priced the bonds as the present value of all the cash flows using the Excel function `PV(rate, nper, pmt, [fv], [type])` or the formula:

$$P = \frac{c \times M}{y}\left[1 - \frac{1}{(1+\frac{y}{2})^N}\right] + \frac{M}{(1+\frac{y}{2})^N} \tag{20}$$

- *What happens when we trade between coupons?* Say, when the scenario is such: interval: 0-1: less than 6 months and subsequent intervals of time - 6 months each.
- There is no unique approach for computing the fractional first semi-annual period. In practise, we have day-count conventions, which can vary from market to market, and in a given market, can vary from product to product.

Deriving the expression for valuation between coupon dates

Consider the following variables:

y - yield to maturity, in percentage per annum.
N - number of coupons left in the life of the bond.
k - time until receipt of first coupon expressed as a fraction of six months. On a coupon date we have $k = 1$. Otherwise, $k < 1$.
$P_{i,t}$ - clean price of bond i in the spot market at time t.
P_d - dirty price
AI_{i,t_1,t_s} - accrued interest on bond i from t_1 (last coupon date) till t_s (the settlement date).

If today is the settlement date (period 0), which does not coincide with an actual coupon date, then first value the bond at the next coupon date (period 1) and then discount the value back to the actual settlement date. If we receive a coupon of $C/2$ at period 1, there are $N-1$ remaining coupons left with the value of these cash flows given by,

$$\text{PV(N-1 coupons)} = \frac{C/2}{y/2}\left[1 - \frac{1}{(1+y/2)^{N-1}}\right] \tag{21}$$

The present value of the face value is

$$\text{PV(face value)} = \frac{M}{(1+y/2)^{N-1}} \tag{22}$$

The sum the above plus the single coupon received at period 1, would give the value of the bond at period 1 (first coupon date after the settlement date).

$$\underset{\text{at period 1}}{\text{Bond Value}} = \frac{C}{2} + \frac{C/2}{y/2}\left[1 - \frac{1}{(1+y/2)^{N-1}}\right] + \frac{M}{(1+y/2)^{N-1}} \tag{23}$$

$$\underset{\text{at period 1}}{\text{Bond Value}} = \frac{C/2}{y/2}\left[\frac{(1+y/2)^{N}-1}{(1+y/2)^{N-1}}\right] + \frac{M}{(1+y/2)^{N-1}} \tag{24}$$

We now discount this amount back to period 0, to get the price of the bond.

$$P_d = \frac{cM/2}{(1+y/2)^k}\left[\frac{(1+y/2)^N - 1}{\frac{y}{2}(1+y/2)^{N-1}}\right] + \frac{M}{(1+y/2)^{N-1+k}} \tag{25}$$

The above expression is the **market method** for bond valuation used by Wall Street traders. The **treasury method** uses simple interest in discounting the fractional first period. That is,

$$P_d = \frac{cM/2}{(1+k\frac{y}{2})}\left[\frac{(1+y/2)^N - 1}{\frac{y}{2}(1+y/2)^{N-1}}\right] + \frac{M}{(1+k\frac{y}{2})(1+y/2)^{N-1}} \tag{26}$$

Dirty Price and Accrued Interest

Let the bond pays a coupon at t_1 and the next coupon date is t_2. If we buy the bond on t_s (*ex-dividend date*) such that $t_1 < t_s < t_2$, then between t_1 and t_s the bond trades *cum dividend*. This means that the holder of the bond between t_1 and t_s is entitled to part of the coupon paid on t_2.

But in both the methods, the entire coupon paid on t_2 is discounted back through the fractional period to the date at which it was bought. Thus, valuing the bond by discounting it as the PV of all cash flows, through the fractional first period by including the entire coupon amount received after purchasing the bond results in its ***dirty price.***

It includes the compensation made to the seller for parting with the entirety of the next coupon (on date t_2) despite having held the coupon for the period between last coupon date t_1 and settlement date t_s. This compensation received by the seller of the bond factored into the bond'd dirty price is called ***accrued interest.***

$$AI = \frac{cM}{2}\left(\frac{t_s - t_1}{t_2 - t_1}\right) \tag{27}$$

$$AI = \frac{cM}{2}\left(\frac{\texttt{Settlement date - Last coupon date}}{\texttt{Next coupon date - Last coupon date}}\right) \tag{28}$$

Bonds are quoted in ***clean prices***, which is bereft of accreued interest. We obtain the clean price by subtracting accrued interest from the dirty price.

$$P_{i,t_s} = P_d - AI_{i,t_1,t_s} \tag{29}$$

$$\texttt{Clean Price = Dirty Price - Accrued Interest} \tag{30}$$

Actual/Actual method

- This is used for T-bonds in the US and for corporate bonds in India. The settlement date is the date of valuation of the bond. Let's consider:
  - Settlement Date: 24 September 2020
  - Maturity Date: 15 December 2040
- The first step is to obtain the **Semi-annual Anniversary Dates** or SADs. In this case, since maturity date is also a coupon date, 15-Dec. is a SAD. The other SAD would be 6 months prior to this: 15-Jun.
- We need to measure the gap between SADs and also that between settlement date and nearest SAD. That is to measure the gap between 24-Sep & 15-Dec and 15-Jun and 15-Dec.

  > **Note.**Either include the starting date or the ending date, but not both. For ease of counting, just include the ending and not starting date.

- Between 24-Sep and 15-Dec, gap = 6+31+30+15 = 82 days
  Between 15-Jun and 15-Dec, gap = 15+31+31+30+31+30+15 = 183 days. The fractional first period is 82/183 = 0.44809.
- If you count the number of days between 2 dates that are 6 months apart, there are only 4 possibilities: 181, 182, 183, or 184.

> **Actual/Actual Method**
>
> - Calculate the actual number of days between the date of valuation and the next coupon date. Include the next coupon date but not the starting date (or vice versa).
>   - Call the period between settlement date and SAD as N_1.
> - Calculate the actual number of days between the preceding coupon date and the next coupon date, i.e, the period between semi-annual anniversary dates (SADs).
>   - Call this period between consecutive SADs as N_2.
> - The next coupon is then k periods away where $k = N_1/N_2 < 1$.

Example. There is a Treasury bond with a face value of $1,000. The coupon rate is 8% per annum, paid on a semi-annual basis. The coupon dates are 15 July and 15 January. The maturity date is 15 January 2033 and today is 15 September 2013. Let the YTM be 10% pa.

Method 1 (Using PV function). Settlement Date: 15-Sept; Maturity Date: 15-Jan; SADs: 15-Jul and 15-Jan. The gaps between SADs (N_2) and that between settlement date and nearest SAD (N_1) is measured.

Between settlement and SAD: 15-Sep to 15-Jan = 15+31+30+31+15 = 122.
Between SADs: 15-Jul to 15-Jan = 16+31+30+31+30+31+15 = 184.
Thus $N_1 = 122$ and $N_2 = 184$. Thus $k = N_1/N_2 = 122/184 = 0.66304$.

Rationale. We want to value the bond at time 0 when the interval between 0 and 1 is less than 6 months. We first go to time 1 and compute the sum of that day's coupon, the remain coupons and the face value. Since we will be on a coupon date at period 1, we may use the `PV` function to compute the value of the remaining coupons plus face value. Note. Remember to add that day's coupon. Bring this sum back to zero.

We are on 15-Sep and the next coupon is on 15-Jan. The gap is 122 days. We go to 15-Jan (period 1) and calculate the bond value on that date. On 15-Jan we have `40 + PV(0.05,38,-40,-1000) = $871.32`. Next, we bring this amount back for the fractional period, by simple discounting at the semi-annual YTM:

$$P = \frac{871.32}{(1.05)^{0.66304}} = \$843.584 \tag{31}$$

Method 2. (Using PRICE function and adding accrued interest)

We have the parameters: `settlement = date(2013,09,15)`, `maturity = date(2033,01,15)` the annual coupon rate `rate=8%` and YTM `yld = 1%`. Since we obtain all of face value, `redemption = 100` and `freq = 2` and `basis = 1` for actual/actual. Thus to get the price, we have:

Price = `10*PRICE(date(2013,9,15),date(2033,1,15),8%,10%,100,2,1)`
(Clean) Price = $830.1067 (using the `PRICE` function)

However, this price does not include the accrued interest. We have to add the compensation to the seller for the fraction of the coupon (between 15 July 2013 and 15 September 2013).

Of 184 days, 122 are over, thus accured interest is calculated for 62 days,

$$AI = 40 \times \frac{62}{184} = 13.4783 \tag{32}$$

Adding accrued interest with the clean price, we get the dirty price of the bond.

$$P_d = P_c + AI = 830.1067 + 13.4783 = 843.585 \tag{33}$$

30/360 Method

Actual/Actual method is used for Treasury bonds in the US. For corporate bonds in the US, the 30/360 NASD method is used. In this method, we treat the year to comprise of 360 days, hence $N_2 = 180$ always, as opposed to 181,182, 183 or 184 in the Act/Act method.

In this method everything remains the same but for the computation of the fractional first period. The fractional period also has months with only 30 days in all.

We determine the period N_1 as: `360*(year2-year1) +30*(month2 - month2) + (day2 - day2)`. The accrued interest period is 180 minus the fractional first period.

- If day1 = 31 then set day1 = 30.
- If day1 is the last day of February, then set day1 = 30
- If day1 = 30 or has been set equal to 30, then if day2 = 31, set day2 = 30.

30/360 European Method

This convention is used in India for Government securities. In this convention, if day2 = 31, then it is always set equal to 30. The additional rules are: If day1 = 31 then set day1 = 30. If day2 = 31 then set day2 = 30.

> **`PRICE` function in Excel**
>
> The parameters of the `PRICE` function are (in order): `PRICE(settlement, maturity, rate, yld, redemption, frequency, [basis])`
>
> - `settlement` - settlement date entered as `DATE(yyyy,mm,dd)`
> - `maturity` - maturity date entered as `DATE(yyyy,mm,dd)`
> - `rate` - annual coupon rate c
> - `yld` - annual yield-to-maturity y
> - `redemption` - % of par that is payable when the bond matures. When it is 100% it means that $100 is the corresponding face value. So if the face value is say 1000, multiply this entire `PRICE()` expression by 10 to obtain the actual price.
> - `frequency` - frequency of coupon payment. Value of 2 for semi-annual coupons.

- `[basis]` - optional entry - day-count convention where:
  0 - 30/360 NASD
  1 - Actual/Actual
  2 - Actual/360
  3 - Actual/365
  4 - Actual/360 E

Exercises

1. **Which of these statements is true:**

- [x] If the face value of a bond, keeping other variables constant, is doubled the price will double
- [ ] If the coupon rate is doubled, keeping other variables constant, the price will double
- [ ] If the face value is doubled and the coupon rate is reduced by 50% the bond price will remain unchanged
- [ ] All of the above

- **True.** If the face value of a bond doubles, all the cash flows will double both coupon and face value. Therefore, bond price will double.
- **False**. If coupon rate is doubled, all coupon payments double, but face value will be unchanged. Thus all cash flows will not double. Hence the bond price will increase but will not double.
- **False.** If the face value is doubled and the coupon rate is reduced to half, then the coupons will be unchanged, but the face value will increase. Thus the bond price will increase.

2. **Which of these statements is true:**

- [ ] A par value bond will continue to trade at par on every subsequent coupon date if the YTM remains constant
- [ ] A premium bond will steadily decline in price as we go from coupon date to the next if the YTM remains constant

- [ ] A discount bond will steadily increase in price as we go from one coupon date to the next if the YTM remains constant
- [x] All of the above

All of the above are True.

- If YTM remains constant then a par bond will trade at par on every coupon date.
- A premium bond will steaduly decline in price and approach the par value.
- A discount bond will steadily increase in price and approach the par value.
- This is the pull to par effect. At maturity all bonds must trade at par.

3. **Which of these statements is true:**

- [ ] At maturity all coupon paying bonds will trade at par
- [ ] At maturity zero coupon bonds will trade at par
- [ ] At maturity floating rate bonds will trade at par
- [x] All of the above

All of the above are True.

4. **Which of these is a money market security:**

- [x] T-bills
- [ ] T-notes
- [ ] T-bonds
- [ ] None of the above

T-bills are money-market securities. They have an OTM of 1 year or less. Capital market securities like T-bonds and T-notes have an OTM of greater than 1 year. So, medium and long-term bonds, preference shares and equities are a part of the capital market.

5. **Which of these statements is true:**

- [ ] In a price based auction bids are arranged in descending order
- [ ] In a yield based auction bids are arranged in ascending order
- [ ] In both price based and yield based auctions there is usually pro-rata allocation at the market clearing bid
- [x] All of the above

All of the above are True.

Bids are arranged in descending order in a price based auction and in ascending order in a yield based auction - lower yield means higher price. Usually, there is pro-rata allocation at the market-clearing bid because there may not be enough bonds to satisfy the demand

6. **Which of these statements is true:**

- [ ] Debentures are unsecured debt
- [ ] Debentures provide leverage
- [ ] Debentures provide a tax shield
- [x] All of the above

All of the above are True.

7. **Which of these statements is true:**

- [ ] The YTM is a solution to a non-linear equation
- [ ] Bonds will have only one real positive YTM
- [ ] Higher the default risk the higher will be the YTM
- [x] All of the above

All of the above are True

True. YTM is the solution to a non-linear equation.

True. Bonds give rise to pure cash flows. There is a single sign change. Therefore from *Descartes rule* all bonds will have a single YTM.

True. Higher the default risk keeping all other variables the same, lower the price of higher the yield.

8. **Which of these statements is true:**

- [x] A zero coupon bond will always give rise to a capital gain if held to maturity
- [ ] A zero coupon bond cannot give rise to a capital gain if traded before maturity
- [ ] Holders of zero coupon bond face reinvestment risk
- [ ] None of the above

A zero coupon bond will always give rise to a capital gain if held to maturity.

9. **Which of these statements is true:**

- [ ] Callable bonds may be recalled if rates decline significantly
- [ ] Callable bonds will carry higher coupons than otherwise similar plain vanilla bonds
- [ ] Callable bonds may offer a call premium if recalled
- [x] All of the above

All of the above are True.

10. **The conversion value of a bond is Rs 875 and its straight value is Rs 920. Which of these statements is true if arbitrage is to be ruled out:**

- [ ] The bond must trade below 875
- [ ] The bond must trade above 875 but less than 920
- [x] The bond must trade above 920
- [ ] Arbitrage cannot be ruled out no matter what the price is

The bond must be traded above $920.

Conversion value is 875 and straight value is 920. The price of convertible must be greater than or equal to the maximum of conversion and face value.

$$\text{Convertible price} \geq \max(\text{conversion value}, \text{straight value}) \tag{34}$$

So to rule out arbitrage the bond must be traded above 920.

Arbitrage opportunities

- Suppose bond is priced at 840.
  Buy the bond at 840 and immediately convert. We will get 875. There is an arbitrage profit of 875.
- Suppose the bond price is 900.
  - Buy the convertible and short sell the equivalent plain vanilla. So at the outset, you will get -900 plus 920 (straight value) - we get 20 at the beginning.
  - Then every 6 months the convertible will give a coupon of C which has to be paid on the short position. SO next cash flow on all coupon dates is zero.
  - At maturity, the convertible will pay face value 1000 which will be just sufficient to close out the short position. So net cash flow is zero.
  - Thus the initial inflow of 20 is an arbitrage profit.
- So price cannot be less than 875, not can it be between 875 and 920. So it must be at least 920 to rule out arbitrage.

11. **Bond insurance provides protection against:**

- [x] Default risk
- [ ] Liquidity risk
- [ ] Inflation risk
- [ ] All of the above

Insurance gives you protection against Default Risk. And no other risk.

12. **Which of these statements is true:**

- [ ] A floating rate bond's coupon will fluctuate from day to day
- [ ] The coupon of an indexed bond will fluctuate from day to day
- [x] The YTM of all bonds will fluctuate from day to day
- [ ] None of the above

The YTM of all bonds will fluctuate from day to day.

Coupons will usually be reset once in 6 months (on a coupon date) or even less frequently. On a floating rate housing loan, the rate is typically reset once a year in India. However, the YTM on an actively traded bond will keep fluctuating just like the share price.

13. **Which of these bond related parameters is not determined by the issuer:**

- [ ] Face value
- [ ] Coupon
- [ ] Time to maturity
- [x] Yield to Maturity

YTM is determined by the market. The others are determined by the issuer.

14. **Which of these statements is true:**

- [ ] Zero coupon bonds are not negotiable
- [ ] Zero coupon bonds are devoid of price risk
- [x] The price of a zero coupon bond is determined by discounting on a semi-annual basis
- [ ] All of the above

False. ZCB can be negotiated.

False. ZCB have no reinvestment risk, because they pay no coupons. But they have price risk. If the holder sells before maturity and the market yield rise, then he can have a capital loss.

True. Although ZCB has a single cash inflow, it is discounted on a semi-annual basis. This is because the investor has a choice between coupon paying bonds and ZCB. We cannot make a comparison unless the discounting technique is common. Since coupon-paying bonds are usually discounted on a semi-annual basis, so are ZCBs.

15. **The time to maturity of a bond is increased by six months and the price rises. Which of these statements is true:**

- [x] It is a premium bond
- [ ] It is a discount bond
- [ ] It is a par bond
- [ ] It is a zero coupon bond

It is a premium bond. For a premium bonds, $y < c$, thus, $\Delta P = P_{N+1} - P_N > 0$. That is, increase in maturity period, leads to increase in prices.

Rationale

Keeping everything else fixed, we are interested to find the change in bond price as it matures, i.e. with respect to time to maturity. Now, N is a discrete quantity that can only increase in units of 1. The bond prices P_N and P_{N+1} between two similar bonds with N and $N+1$ coupon payments remaining till maturity are given by,

$$P_N = \frac{c \times M}{y}\left[1 - \frac{1}{(1+\frac{y}{2})^N}\right] + \frac{M}{(1+\frac{y}{2})^N} \tag{35}$$

$$P_{N+1} = \frac{c \times M}{y}\left[1 - \frac{1}{(1+\frac{y}{2})^{N+1}}\right] + \frac{M}{(1+\frac{y}{2})^{N+1}} \tag{36}$$

From the difference between the two bond prices $\Delta P = P_{N+1} - P_N$, we have,

$$\Delta P = \frac{cM}{y\left(1+\frac{y}{2}\right)^{N+1}}\left[1 + \frac{y}{2} - 1\right] + \frac{M}{\left(1+\frac{y}{2}\right)^{N+1}}\left[1 - 1 - \frac{y}{2}\right] \tag{37}$$

$$\Delta P = P_{N+1} - P_N = \frac{M}{\left(1+\frac{y}{2}\right)^{N+1}}\left[\frac{c}{2} - \frac{y}{2}\right] = \frac{M(c-y)}{2\left(1+\frac{y}{2}\right)^{N+1}} \tag{38}$$

Thus the effect of change time to maturity on prices of two otherwise similar bonds depends on the nature of c and y. This exercise motivates the pull-to-par effect described in the next section.

- For a par bond, there would be no change in prices even as one approaches the date of maturity. This shown in the following example.
- However, for a discount bond $(y > c)$, the change in prices is negative on incremental increase in time of maturity. Consequently as you approach date of maturity, N decreases and the value of the price goes up to its par value at maturity.
- For the premium bond $(y < c)$, the change in prices is positive on incremental increase in time of maturity. Thus as we approach maturity, the bond price falls to its par value.

16. **The share price of a company is 40. The conversion ratio of a convertible bond issued by it is 25. A plain vanilla bond that is comparable in all other respects is trading at par. Which of these statements is true:**

- [ ] The convertible may be trading at a discount
- [ ] The YTM of the convertible may be greater than its coupon
- [x] The YTM of the convertible will be less than or equal to its coupon
- [ ] The YTM must be equal to the coupon

The YTM of the convertible will be less than or equal to its coupon.

We know that, Conversion value = Current Share Price X Conversion Ratio. Thus conversion value = 40 times 25 = 1000.
It's straight value is also 1000. So, the convertible must trade for atleast 1000. So, it must be a par or a premium bond, meaning, YTM must be less than or equal to the coupon rate $y \leq c$.

17. **A person buys a bond with a current yield that is higher than its coupon rate. He then reinvests every coupon at the coupon rate till maturity. Which of these statements is true:**

- [ ] The ex-post realized compound yield will be equal to the YTM
- [x] The ex-post realized compound yield will be less than the YTM

- [ ] The ex-post realized compound yield will be greater than the YTM
- [ ] The ex-post realized compound yield may be greater than or less than the YTM

The ex-post realized compound yield will be less than the YTM.

Current yield (which is the annual coupon amount/current price) is higher than the coupon rate. It means that price is less than the face value, which means that the coupon is less than the YTM.

If you reinvest at the YTM, realized compound yield, RCY = YTM. However, if you invest at the coupon rate, RCY < YTM.

18. **A bond denominated in dollars is subject to foreign exchange risk. Which of these statements is true?**

- [ ] It cannot be a Treasury bond
- [x] The investor whose perspective we are taking is based outside the US
- [ ] Its YTM will be greater than its coupon
- [ ] None of the above

The investor whose perspective we are taking is based outside the US.

It is a Treasury Bond. And only a foreign investor of this bond would face exchange rate risk.

19. **A bond is subject to price risk but not to reinvestment risk. Which of these statements must be true?**

- [ ] It is a Treasury security
- [x] It is a zero-coupon security
- [ ] It may be a coupon paying bond which is trading at a substantial discount
- [ ] None of the above

It is a zero-coupon security. It has no reinvestment risk because it pays no coupons. However it has price risk. In case the holder sells the bond and the yields rise, there is a capital loss.

20. **Consider an Indian investor who buys US T-bills. He is exposed to:**

- [ ] Interest rate risk
- [ ] Liquidity risk
- [ ] Foreign exchange risk
- [x] All of the above

All of the above are True.

21. **Consider an American investor who buys US T-bills. He is exposed to:**

- [ ] Credit risk
- [x] Interest rate risk
- [ ] Foreign exchange risk
- [ ] None of the above

Interest rate risk. He is not subject to credit risk for the US government does not default. He is a US national and hence is not subject to forex risk. However interest rates can fluctuate.

22. **If a plain vanilla bond is held to maturity, which of these statements is false:**

- [ ] There is no price risk
- [ ] There is no liquidity risk
- [x] There is no reinvestment risk
- [ ] None of the above

"There is no reinvestment risk" - This is false. Coupons can be reinvested.

23. **A plain vanilla bond has a low coupon and a high YTM. Which of these statements is most likely to be true:**

- [ ] It may be an original issue junk bond
- [x] It may be a fallen angel
- [ ] It may trade at a premium
- [ ] None of the above

It may be a fallen angel.

At issue it was very credit-worthy, hence low coupons. But now it has become junk, so the YTM is high.

24. **A person buys a plain vanilla bond, holds it to maturity, and reinvests each intermediate coupon at a different rate of interest. Which of these statements is true:**

- [x] His realized compound yield may be equal to the YTM
- [ ] His realized compound yield must be equal to the YTM
- [ ] His realized compound yield cannot be equal to the YTM
- [ ] The realized compound yield cannot be computed

His realized compound yield may be equal to the YTM. It need not necessarily be equal to YTM. And there is no reason for it not be equal to YTM. And obviously, it can be computed.

25. **Which of these plain vanilla bonds is subject to reinvestment risk:**

- [ ] A par bond
- [ ] A premium bond

- [ ] A discount bond
- [x] All of the above

All of the above are True.

26. **A bond pays an annual coupon of 8% per annum and has 9 years to maturity. Which of these statements is true if the face value is 1,000 and the YTM is 8% per annum:**

- [ ] The bond will trade at par
- [ ] The contribution of the face value to the price is the same as that of the coupons
- [ ] The current yield will be equal to the coupon rate
- [x] All of the above

All of the above are True.

The rule of 72. The number of years to double a cash flow is 72 divided by the number of periods left. Thus, 9 years to maturity and the rate is 8%. So it will take 72/8 = 9 years. Thus, in 9 years, the PV of the face value will double and become 1000. So today, PV of the face value is 500. Since it is a par bond, the PV of the coupon is also 500.

27. **Which of these bonds gives an option to the issuer:**

- [x] Callable bonds
- [ ] Convertible bonds
- [ ] Putable bonds
- [ ] Exchangeable bonds

Callable bonds. For the others the option lies with the holder.

28. **Bond A is a plain vanilla bond, while Bond B is a convertible bond that is**

identical in all other respects. Bond C is identical too except for the fact that it is putable. Which of these statements is true:

- [x] A will have the lowest price
- [ ] A will have the highest price
- [ ] The price of B is an average of the prices of A and C
- [ ] None of the above

A will have the lowest price.

Both convertible (B) and puttable (C) give the option to the holder and will have a higher price than plain vanilla (A).

29. **Which of these statements is true about a STRIP:**

- [ ] It is a zero coupon security
- [ ] It is devoid of reinvestment risk
- [ ] It will always give rise to a capital gain if held to maturity
- [x] All of the above

All of the above are True.

30. **In a yield based auction the coupon is set equal to the stop yield. Which of these statements is true:**

- [x] If it is a uniform yield auction all the securities will be issued at par
- [ ] If it is a discriminatory yield auction all the securities will be issued at par
- [ ] The securities cannot be callable in nature
- [ ] None of the above

If it is a uniform yield auction all the securities will be issued at par.

The coupon rate is set equal to the market clearing yield. Thus the coupon and YTM are equal and the bond will be issued at par in a uniform auction.

Had it been a *discriminatory auction*, and the coupon is set equal to the market-clearing yield, people who bid the market clearing yield would get it at par, while other people who have bid at a lower yield would get it at a lower yield.