Preimage attack on TCS_SHA-3

Research Paper Review

Group 3

Ishita Gupta   Sri Rajitha
Rohith Krishna   Yashraj Varma
Aditya Kedar Tata   Akshat Mehra
Roopchand Parise   Parikshit Powani

Madras School of Economics

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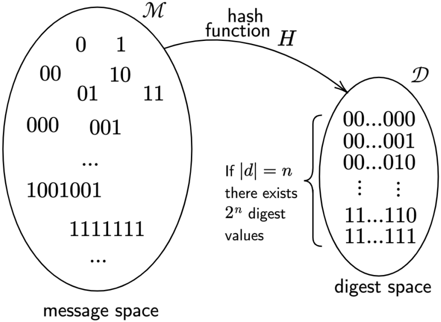
Outline

1. Introduction to Hash Functions
2. Construction of TCS_SHA-3
3. Compression Function in TCS_SHA-3
4. Poor Diffusion & Differential input characteristic
5. Algorithm for Preimage attack

Hash Functions

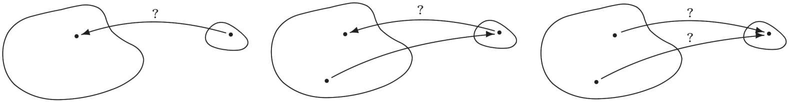
- A cryptographic hash function takes a message M of arbitrary length and outputs a bit string h of fixed length.
- Hash functions are used in:
  - Password Protection, Message Authentication, ...
  - Digital Signatures, Public Key Cryptography, ...



Properties of good hash functions

A cryptographic hash function must satisfy certain conditions:

- **Preimage Resistance** : Given the hash h of an unknown message, it should be computationally infeasible to find preimage M such that $H(M) = h$.
- **Second Preimage Resistance** : Given a particular message M and its hash $H(M)$, it should be computationally infeasible to find another distinct message M' having the same hash, $H(M) = H(M')$, such that $M \neq M'$.
- **Collision Resistance**: It is computationally infeasible to find any M and $M' \in \mathcal{M}$, such that $H(M) = H(M')$, and $M \neq M'$.



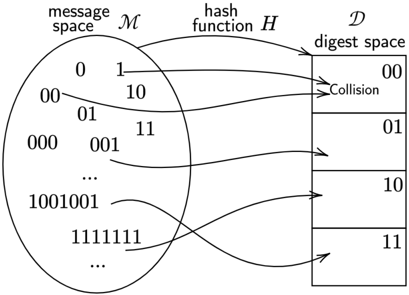
(a) Preimage (b) Second Preimage (c) Collision

Preimage and Second Preimage Resistance

- A preimage refers to a message $M \in \mathcal{M}$ that maps to a known hash $h \in \mathcal{D}$. The basic underlying assumption here is that there exists at least one message that hashes to the given value.
- One way of finding preimages is through a *exhaustive search*, wherein we hash random messages until the given hash value is reached. For a *n-bit* hash value, the number of random messages that must be tried is at least 2^n.
- A second preimage is a message that hashes to the same value as a randomly selected first preimage. The basic assumption here is that the attacker possesses the hash value of the first preimage.
- Thus second preimage resistance points out that given a randomly selected preimage, it is computationally infeasible to find another message that has an identical hash value.

Collision Resistance

- A collision refers to the case when a pair of distinct messages $M, M' \in \mathcal{M}$ have the same hash h.
- This follows from the pigeon-hole principle that if p items are put in q containers with $p > q$, then there must be at least one container with more than 1 item (thus collision occurs).
- Since the message space is larger than digest space, in fact $\mathcal{D} \subseteq \mathcal{M}$, collision is bound to occur.
- The challenge for the designer is to ensure that the adversary would find it infeasible to find a collision.



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Introduction to TCS_SHA-3

- TCS_SHA-3 is a family of four cryptographic hash functions proposed by Vijayarangan of the Tata Consultancy Services (TCS).
- It is a product of the E-Security group of the TCS Innovation Labs, Hyderabad, India and is covered by a US patent (US 2009/0262925).
- It produces a digest size of 224, 256, 384 and 512 bits. .
- TCS SHA-3-d denotes the member that produces d-bit digests.
- The design of TCS_SHA-3 deviates from the general model such that the standard compression function is replaced by a bijective function. The function uses a linear feedback shift register (LFSR) and a T-function.
- The design goals of TCS_SHA-3 include preventing hash collisions and providing a secure hash function.

Contributions of the paper

The paper establishes the inability of TCS_SHA-3 to meet the design goals of 'preventing hash collisions' and 'providing a secure hash function' by demonstrating the following attacks:

1. A second pre-image attack that requires negligible time and negligible memory for nearly guaranteed success.
2. A first pre-image attack on the TCS_SHA-3-d that requires $O(2^{27}.d)$ time and negligible memory.
3. A second pre-image attack that also requires negligible time and negligible memory for nearly guaranteed success on a strengthened variant of TCS_SHA-3.

Notations

- $\mathcal{M}$: Message Space
- $\mathcal{D}$: Digest Space
- M: An arbitrary length bit string, $M \in \mathcal{M}$
- H: A hash function
- h: A fixed length bit string called hash value or digest.
- d: Length of a block. $d \in \{224, 256, 384, 512\}$
- k: Number of partitioned message blocks $= \lceil |M|/d \rceil$
- F: A bijective function
- M^*: Partitioned message blocks after padding.
- c: a $d-$bit constant.

Notations

- LSB: Least significant bit
- MSB: Most significant bit
- $\Gamma_i(\omega)$: i^{th} 32-bit word ($i = 0$ denotes the least significant word of $d-$bit ω
- $|x|$: length of x in bits
- $x_{(i)}$: i^{th} bit ($i = 0$ denotes the LSB) of x
- $x||y$: concatenation of two 32-bit words, x and y
- $\oplus$: XOR (exclusive OR)
- α_i: 32-bit words - blocks of the original message M
- z_l^j: The output at the end of the l^{th} step of the j^{th} round.

Specification of TCS_SHA-3

- The **digest length**, for TCS_SHA-3 are $d \in \{224, 256, 384, 512\}$ as specified earlier.
- The **padding rule** for TCS_SHA-3 can be defined as follows: for any $k \geq 1$,

$$M_k \to M_k^* := \begin{cases} M_k \oplus IV & \text{if } |M_k| < d, \\ M_k & \text{if } |M_k| = d \end{cases}$$

- Here, $k = \lceil \frac{|M|}{d} \rceil$ where k is the **number of partitioned blocks** and the ceiling function denotes that if M is not a multiple of d then we will choose the next integer following M/d.
- The **IV rule** for TCS_SHA-3 is,

$$IV = 1||\{0\}^{d-1}$$

Example, if $d = 224$, the IV Rule is $1\{0\}^{223}$ which can be written as,

$$1\,\underbrace{000\ldots00}_{223 \text{ zeroes}}$$

Thus, forming a 224 bit string.

Construction of TCS_SHA-3

Each round in TCS_SHA-3 has k steps where k is the number of blocks in the message (after padding).

- **Round 1:**
  - Step 1: $k \geq 1$ Choose a constant c of length d' where $\{d' \leq d\}$. Perform $c \oplus M_1$ which becomes the input for the bijective function F.

    $$c \oplus M_1^* \xrightarrow{\text{input}} F \implies F(c \oplus M_1^*) \text{ is the output.}$$

  - Step $2 - k$: For each step i, $2 \leq i \leq k-1$, use recursion $z_i^1 = F(z_{i-1}^1 \oplus M_i)$. Thus, for step k, we will have,

    $$z_k^1 = F(z_{k-1}^1 \oplus M_k^*)$$

- **Round 2:** The number of steps in Round 2 is s which may or may not be equal to k.
  - Step 1: We XOR c, a $d-$ bit constant with the output of the previous round z_k^1. Thus, after applying the bijective function F on this input, the final output for this step will be,

    $$F(c \oplus z_k^1)$$

- Step $2 - s$: For each step i, $2 \leq i \leq s$, we have,

$$z_i^2 = F(z_{i-1}^2 \oplus z_k^1)$$

  z_l^2 denotes the output of step l, $1 \leq l \leq s$, of round 2 and z_{i-1}^2 is the output for the previous step within round 2.

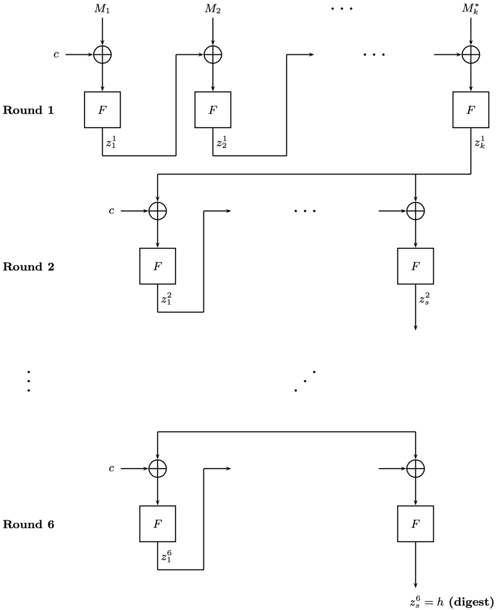
- **Round 3-6:** The number of steps in these rounds is again s. Similar to round 2, the output of the previous round is XORed with the output of the previous step.
  - Step 1: Constant c is XORed with the output of previous round z_s^{j-1}. This forms the input to the bijective function F. A $d-$bit string, $F(c \oplus z_s^{j-1})$ is generated as output.
  - Step $2 - s$: These steps are given by the following recursion,

$$z_i^j = F(z_{i-1}^j \oplus z_s^{j-1}), \qquad z_1^j = F(c \oplus z_s^{j-1})$$

  z_l^j denotes the output of step l of round j.

- The $d-$bit digest h is simply the final output z_s^6.

Construction of TCS_SHA-3



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Compression Function F

- The compression funtion in TCS_SHA-3 is bijective.
  $F : \{0,1\}^d \to \{0,1\}^d$ i.e. it has the same input and output space.

$$F(\alpha) = \lambda \text{ with } |\alpha| = |\lambda| = d$$

- We require a d-bit input α and must ensure a d-bit output λ. Algorithm follows these 5 steps :
  1. **Partition**: $\alpha \to \alpha_1 || \alpha_2 || ... || \alpha_{d/32}$ such that $|\alpha_i| = 32$ for all $1 \leq i \leq d/32$ where α is a d bit input.
  2. **Shuffling**: $\alpha_i \to \beta_i,$ for all $1 \leq i \leq d/32$, such that

$$\beta_{i(j)} = \begin{cases} \alpha_{i(j/2)} & \text{if } 2|j \\ \alpha_{i(16+(j-1)/2)} & \text{otherwise;} \end{cases}$$

  3. **T-function** : $\beta_i \to \gamma_i$ where $\gamma_i = 2\beta_i^2 + \beta_i \mod 2^{32}$, for all $1 \leq i \leq d/32$
  4. **LFSR**: $\gamma_i \to \lambda_i$, for all $1 \leq i \leq 32$, such that $|\lambda_i| = 32$
  5. **Concatenate**: $\lambda := \lambda_1 || \lambda_2 || ... || \lambda_{d/32}$

Partition Function

- The given message is divided into a sequence of equal sized words and the total number of words is m.
- The size of each word is chosen to be 32 bits, where $m = \{7, 8, 12, 16\}$ depending upon the digest length, $d = \{224, 256, 384, 512\}$ bits.
- And $\alpha \rightarrow \alpha_1 || \alpha_2 || \dots || \alpha_{d/32}$ such that $|\alpha_i| = 32$ for all $1 \leq i \leq d/32$ where α is a d-bit input.

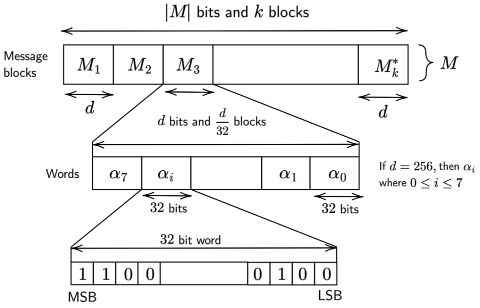


Figure: Partitioning the Message

Shuffle Function

- Shuffling of bits helps to improve the diffusion of the input across different parts of system.
- Shuffle function S, shuffles the bits in the 32-bit words i.e., within a partition of the message.
- The shuffling procedure used is an *outer perfect shuffle*, which means the outer (end) bits remain in the outer positions.
- $\alpha_i \to \beta_i,$ for all $1 \leq i \leq d/32$, such that

$$\beta_{i(j)} = \begin{cases} \alpha_{i\left(\frac{j}{2}\right)} & \text{if } 2|j \\ \alpha_{i\left(16+\frac{j-1}{2}\right)} & \text{otherwise} \end{cases}$$

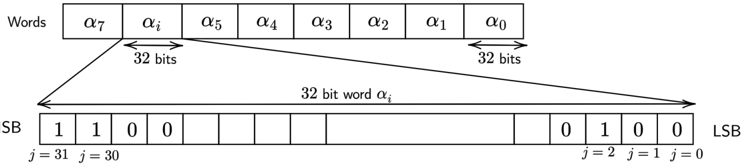


Figure: Word wise shuffle

Shuffle Function

- Suppose $j = 4$, then

$$j \bmod 2 = 0 \implies \beta_{3(4)} = \alpha_{3(4/2)} = \alpha_{3(2)}$$

- Suppose $j = 7$, then

$$j \bmod 2 \neq 0 \implies \beta_{3(7)} = \alpha_{3(16+(7-1)/2)} = \alpha_{3(19)}$$

- The following table shows where each bit is mapped to:

α_i	0	1	2	3	4	5	6	7
β_i	0	16	1	17	2	18	3	19
α_i	8	9	10	11	12	13	14	15
β_i	4	20	5	21	6	22	7	23
α_i	16	17	18	19	20	21	22	23
β_i	8	24	9	25	10	26	11	27
α_i	24	25	26	27	28	29	30	31
β_i	12	28	13	29	14	30	15	31

Table: Shuffle Function mapping of bit-positions.

T Function

- T function is a bijective mapping that updates every bit of the state.
- Each bit of the state is updated by a linear combination of the same bit and a function of a subset of its less significant bits.
- If every single less significant bit is included in the update of every bit in the state, such a T-function is referred to as *triangular*.
- The output of shuffle function is passed into the T-function which performs an invertible mapping containing all the 2^n possible states on a single cycle for any word size n.
- T function: $\beta_i \to \gamma_i$ where

$$\gamma_i = 2\beta_i^2 + \beta_i \text{ mod } 2^{32}, \quad \forall \;\; 1 \leq i \leq d/32$$

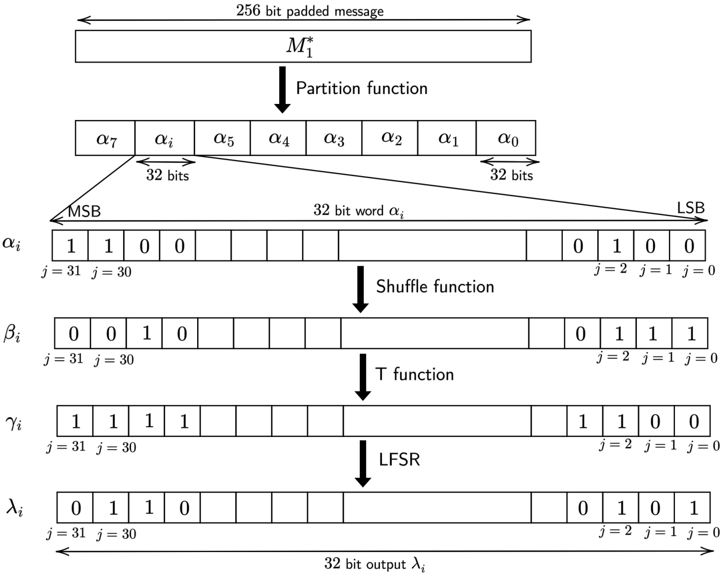
LFSR function

- The Linear Feedback Shift Register uses the connection polynomial,

$$\begin{aligned} f(x) &= x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} \\ &\quad + x^{10} + x^{8} + x^{7} + x^{5} + x^{4} + x^{2} + x + 1 \end{aligned}$$

- LFSR is an irreducible polynomial of degree 32 with period $2^{32} - 1$.
- Each time the 32-bit input is executed for different number of rounds. Thus, even if the input bits are identical, the output will appear random.
- After applying the T-function, the LFSR function is applied on the output of the T-function γ_i.
- LFSR function: $\gamma_i \rightarrow \lambda_i$, for all $1 \leq i \leq 32$, such that $|\lambda_i| = 32$
- Finally we concatenate the $\lambda_i's$ to obtain a d-bit output λ where $\lambda = \lambda_1||\lambda_2||...||\lambda_{d/32}$

Schematic of Bijective function F



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Diffusion in Block-Cipher based Hash Functions

- Diffusion in a block cipher means that flipping 1 bit of the plaintext, should ideally flip the bits of the ciphertext with probability 1/2. (Shannon)
- In other words a single bit flip in the plaintext should result in approximately half the ciphertext bits flipped.
- When it is exactly half, it is called the Strict Avalanche Criterion (as named by Feistel).
- MD5 has good diffusion properties. A single change in input string results in significantly different digests.

Message	Hash Value
0000	4a7d1ed414474e4033ac29ccb8653d9b
0001	25bbdcd06c32d477f7fa1c3e4a91b032
0010	fc1198178c3594bfdda3ca2996eb65cb
0011	ae2bac2e4b4da805d01b2952d7e35ba4

Table: MD5 hash values in hexadecimal

First Preimage attack on TCS_SHA-3-256

- Consider the TCS_SHA-3 variant with $d = 256$ and $k = 1$ (single block), that is message of size $|M| \leq d$.
- Since hash functions are often used to store user passwords, this assumption is fairly practical.
- It can be shown that TCS_SHA-3-256 has poor diffusion property.
- That is a flip of 1-bit of message does not result in approximately 128 bits of digest flipped.
- Thus weakness is exploited by and the result of a differential characteristic of the input on the hash is explored.
- It is found that ideal worst case complexity of $O(2^{256})$ is reduced drastically to $O(2^{35})$. Likewise for digest size $d = 512$, the worst case complexity is $O(2^{36})$.

Exploiting Poor Diffusion

- For $k = 1$ (single block input), the bijective function F first partitions the message into $d/32 = 256/32 = 8$ words, each of size 32-bits.
- For each 32-bit word α_i input to function F we get a pseudo-random 32-bit output λ_i.
- The output λ_i depends only on α_i and not on any other $\alpha_u, u \neq i$.
- Thus the last 32-bits of the hash depends only on the least significant 32-bit word of M_1^*.

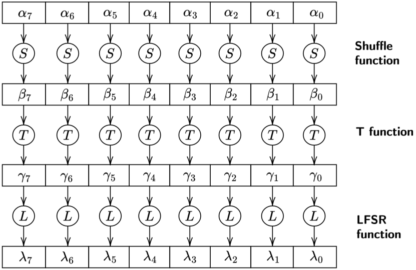

Figure: Diffusion in a 32-bit word is confined to that word alone.

Differential Characteristic of input

- Instead of exhaustively searching the 256-bit message space with 2^{256} elements for matching hash value, one can now search the 32-bit word space with 2^{32} elements for matching corresponding word in the hash.
- In general, the complexity for d-bit hash variant reduces from $O(2^d)$ to $O(2^{32})$, for a specified 32-bit word of the hash and for $k = 1$.
- Let $\Gamma_i(M_1^*)$ denote an arbitrary 32-bit word in the message M_1^*.
- Under exhaustive search, we start with 32-bit word $000\ldots000$ producing hash h_i^a and $000\ldots001$ producing hash h_i^b respectively.
- Difference in the input $\Delta\Gamma_i(M_1^*)$ is now 1-bit. Say this produces a corresponding difference in hash of $\Delta h_i = h_i^b - h_i^a$.
- From the poor diffusion property, exhaustive search of 32-bit word is expected to uncover the portion of M_1^*, α_i corresponding to hash portion h_i.

Differential Characteristic of input

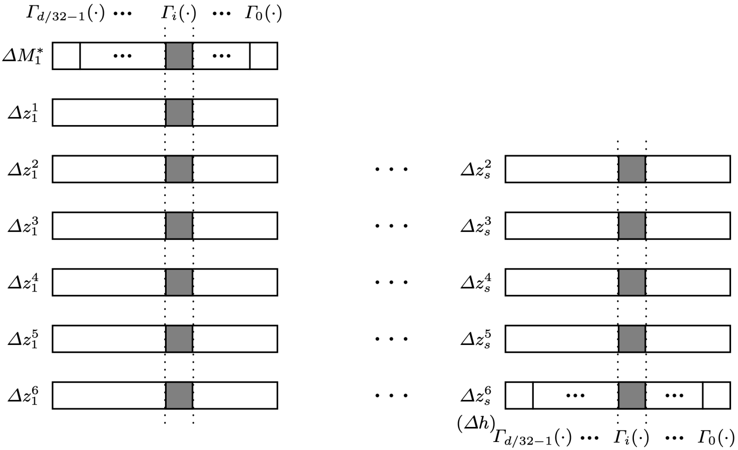

Figure: Differences in input $\Delta\Gamma_i(.)$ propagates only through 6 rounds and is confined to the chosen word i alone.

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Algorithm for M_1 recovery

Algorithm 2 Recovering M_1 from h when $k = 1$

Require: Whether $|M_1| = d$ or $|M_1| < d$
Ensure: d-bit output M_1

for $i = 0 \to d/32 - 1$ **do**
  for $j = \{0_2\}^{32} \to \{1_2\}^{32}$ **do**
    $\ell \leftarrow \{0_2\}^{32(d/32-i-1)} \| j \| \{0_2\}^{32i}$;
    Compute $\hbar :=$ TCS_SHA-3-$d(\ell)$;
    if $\mathit{\Gamma}_i(\hbar) = \mathit{\Gamma}_i(h)$ **then**
      $\mathit{\Gamma}_i(M_1^*) \leftarrow j$;
      break;
    else
      $j \leftarrow j + 1$;
  $i \leftarrow i + 1$;
Compute $M_1^* = \mathit{\Gamma}_{d/32-1}(M_1^*) \| \mathit{\Gamma}_{d/32-2}(M_1^*) \| \ldots \| \mathit{\Gamma}_0(M_1^*)$;
if $|M_1| < d$ **then**
  Output $M_1 = M_1^* \oplus$ IV;
else
  Output $M_1 = M^*$;

Algorithm for M_1 recovery

We fix $k = 1$, that is a single message block. We also require if $|M| = d$ or $|M| < d$, in order to reverse the IV rule.

Line 1. Loop i across each of 8 words for a 256 bit block. This takes time 8 units. Or in the general case $O(d/32)$.

Line 2. Loop j across each of the 2^{32} elements for the chosen 32-bit word.

Line 3. Compute 256-bit string l by prefixing and suffixing required number of zeros in all positions other than the chosen word.

$$l \leftarrow \{0^{32(\frac{d}{32}-1-i)}\}||j||\{0\}^{32i}$$

								$j \in \{0,1\}^{32}$
$i = 0$	00..00	00..00	00..00	00..00	00..00	00..00	00..00	j
$i = 1$	00..00	00..00	00..00	00..00	00..00	00..00	j	00..00
				$\vdots$				
$i = 6$	00..00	j	00..00	00..00	00..00	00..00	00..00	00..00
$i = 7$	j	00..00	00..00	00..00	00..00	00..00	00..00	00..00

Algorithm for M_1 recovery

Line 4 : Compute the hash $\hbar$ produced by l.

Line 5^{+}: If the corresponding 32-bit word of the hash $\Gamma_i(h)$ matches with the computed hash $\Gamma_i(\hbar)$, then assign j (32-bit word) to the corresponding part of the message $\Gamma_i(M_1^*)$. Else increment and go to the next 32-bit element (in loop 2).

Line 10: Increment i to the next word. Exit loop once search over all 8 words are complete.

Line 11: Concatenate all $\Gamma_i(M_1^*)$ to obtain 256-bit M_1^*.

$$M_1^* = \Gamma_{d/32-1}(M_1^*)||\Gamma_{d/32-2}(M_1^*)||\ldots||\Gamma_1(M_1^*)||\Gamma_0(M_1^*)$$

Line 12: If $|M| < d$, then the original message would have been padded using the IV rule. To reverse this, we XOR the obtained M_1^* with IV, that is

$$M_1 = \begin{cases} M_1^* \oplus IV & \text{if } |M_1| < d \\ M_1^* & \text{if } |M_1| = d \end{cases}$$

How many searches?

Consider the following scenario of a 8-bit message (to be split into multiple words.) The corresponding hash h is also 8 bits long (say).

M	00000000
h	10000110

Start with 00000000 and search all 2^8 possible elements in $\mathcal{M}$ such that $H(M) = h$.

256 searches!

M	0000	0000
h	1000	0110

Now, diffusion is confined to the chosen 4-bit space.
Start with 0000 and search all 2^4 elements in $\mathcal{M}$.
Repeat for second word $\implies$ $2^4 \times 2$

32 searches!

M	00	00	00	00
h	10	00	01	10

Now, diffusion is confined to the chosen 2-bit space.
Start with 00 and search all 2^2 elements in $\mathcal{M}$.
Repeat for other 3 words $\implies$ $2^2 \times 4$

16 searches!

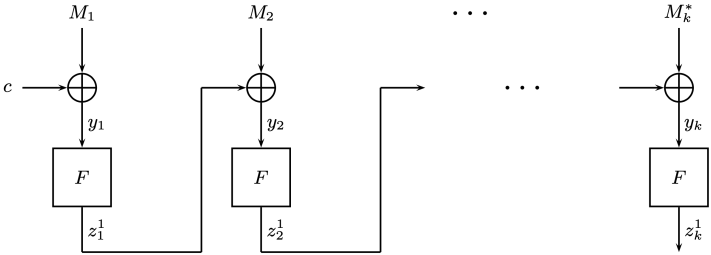
Reduction in Computational Complexity

- As seen previously, had there been good diffusion of bits across all 256-bit length of the block size, recovery would take $O(2^{256})$ time. And in general it would take $O(2^d)$ time.
- The reduction in complexity stems from the fact that diffusion is limited to 32-bit words.
- The time taken to run the two loops is as follows. The first loop runs over $d/32$ values. The second loop runs over 2^{32} elements. The total run time complexity is therefore:

$$O\left(\frac{d}{32} \times 2^{32}\right) = O\left(\frac{d}{2^5} \times 2^{32}\right) = O\Big(d \cdot 2^{27}\Big)$$

- For $d = 512$, it takes $O(2^{36})$ time and for $d = 256$, it takes $O(2^{35})$ time.

Second pre-image attack



- Let the message $M = M_1||M_2||M_3||....||M_k^*$ where $k \geq 2$.
- Let another message be $M' = M_1'||M_2'||M_3||....||M_k^*$ where M_1 and M_2 are not equal to M_1' and M_2', but the successive blocks are identical.
- If $y_2' = y_2$, we see that the outputs are identical, i.e. $h = h'$

What does $y = y'$ entail?

- The implication of $y = y'$ is as follows. We know that $y_2 = F(M_1 \oplus c) \oplus M_2$ and $y_2' = F(M_1' \oplus c) \oplus M_2'$
- The condition $y_2' = y_2$ therefore implies:

$$F(M_1 \oplus c) \oplus M_2 = F(M_1' \oplus c) \oplus M_2'$$

- If we assume the following forms of M_2' and M_1', that $y_2 = y_2'$ follows.

$$\begin{aligned} M_2' &= F(M_1' \oplus c) \oplus M_1' \\ M_1' &= F(M_1 \oplus c) \oplus M_2 \end{aligned}$$

- From the RHS of the $y_2 = y_2'$ condition we have:

$$\begin{aligned} F(M_1' \oplus c) \oplus M_2' &= \underbrace{F(M_1' \oplus c) \oplus \Big[F(M_1' \oplus c)}\oplus M_1'\Big] \\ &= 0 \oplus M_1' = M_1' \\ &= F(M_1 \oplus c) \oplus M_2 \end{aligned}$$

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