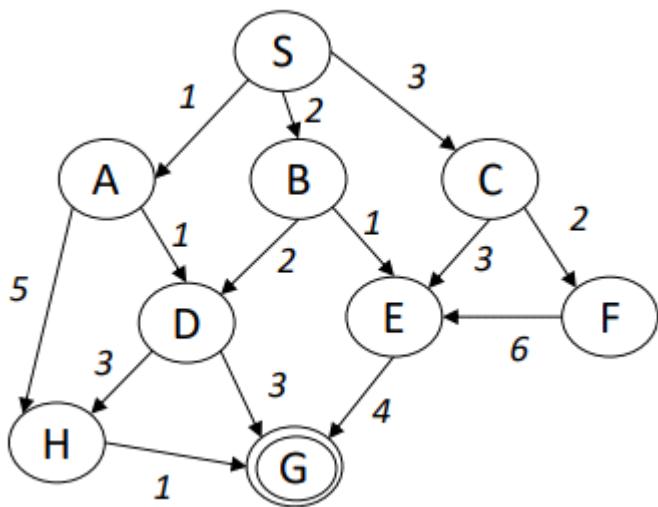


Problem 1: Consider the following search graph:



(a) Perform Uniform-Cost Search to find a solution for the search problem if the initial node is S and the goal node is G.

Provide the current node and the contents of the explored list (3 points) and the content of the frontier (6 points) at each step of the search. The format of the frontier should be: {Node1 (path-cost to Node1), Node2 (path-cost to Node2), ...}. E.g. {A(1), B(2), ...}

Indicate the solution path from S to G (3 points) as well as its cost (3 points). (TOTAL: 15 points)

Step-1 : Current node : S

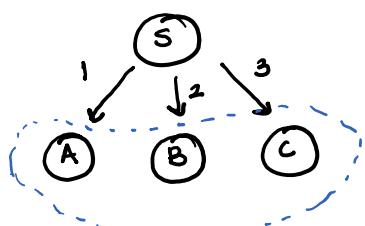
Frontier : {S(0)}

} Remove S for expansion

Explored list : {}

} Is S goal state ? No

After expanding Node S



Children: {A(1), B(2), C(3)}

Explored list : {S}

Frontier : {A(1), B(2), C(3)}

STEP-2 :

Frontier: $\{ A(1), B(2), C(3) \}$ Remove a node from frontier

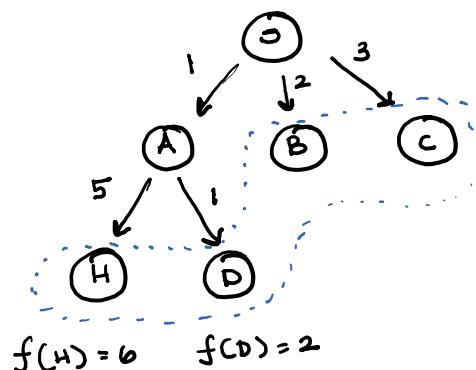
Current node: A , Is A goal state ? No

After expansion of node A

Children: $\{ D(2), H(6) \}$

Explorcd list: $\{ S, A \}$

Frontier: $\{ B(2), D(2), C(3), H(6) \}$



STEP-3 :

Remove a node for expansion from frontier

Current Node : B

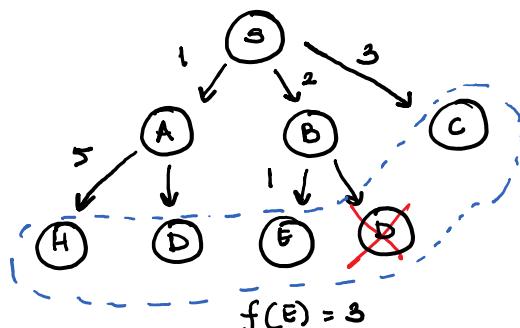
Is B a goal node ? No

After expansion of node B

Children: $\{ E(3), D(4) \}$

Explorcd list: $\{ S, A, B \}$

Frontier: $\{ D(2), C(3), E(3), H(6) \}$



STEP-4 : Remove a node from frontier for expansion

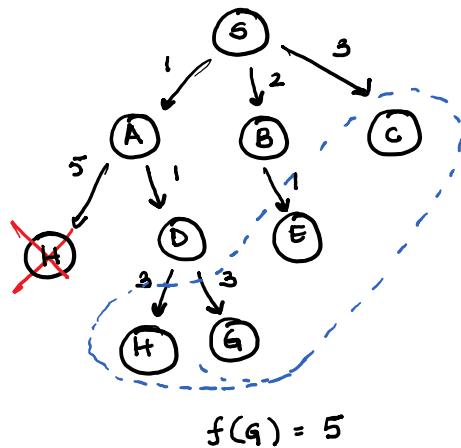
Current node : D

Is D a goal state ? No

After expansion of node D

Children : { H(5), G(5) }

Explored list : { S, A, B, D }



Frontier : { C(8), E(3), H(5), G(5) }

STEP-5 : Remove a node from frontier for expansion

Current Node : C

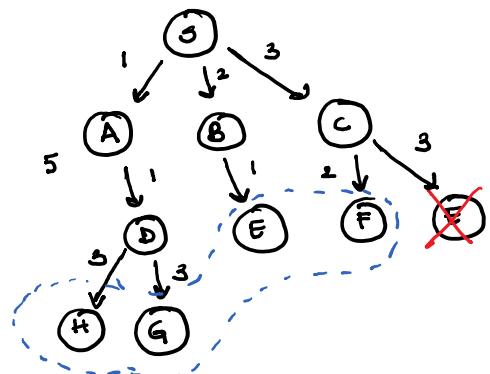
Is C a goal state ? No

After expansion of node C

Children : { F(5), E(6) }

Explored list : { S, A, B, C, D }

Frontier : { E(3), F(5), H(5), G(5) }



STEP-6 : Remove a node for expansion

Current node : E

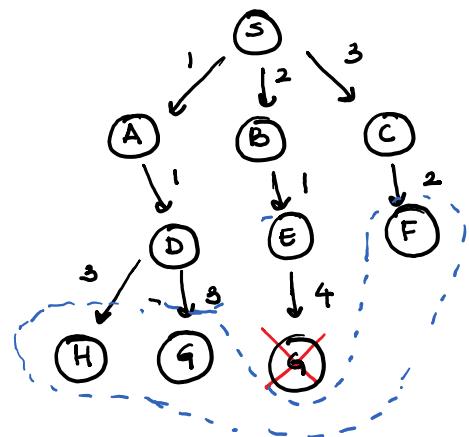
Is E a goal state ? No

After expansion of node E

Children : { G(7) }

Explored list : { S, A, B, C, D, E }

Frontier : { F(5), G(5), H(5) }



STEP-7 : Remove a node for expansion

Current node : F

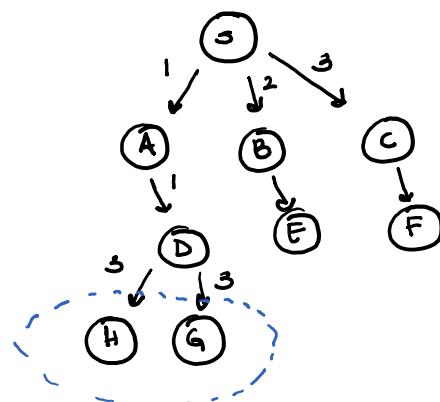
Is F a goal state ? No

After expansion of node F

Children : { E(11) }

Explored list : { S, A, B, C, D, E, F }

Frontier : { G(5), H(5) }



STEP-8 : Remove a node for expansion from frontier

Current node : q

Is q a goal state ? YES

Solution :

Father (q) = D	}	S → A → D → q
Father (D) = A		Path cost : 5
Father (A) = S		

(b) If an informed search strategy, such as A*, needs to be performed on the same graph, given knowledge about the true-cost to the goal from each node, what are all the possible values of variables a, b, c, d and e such that the heuristic is admissible? (TOTAL: 15 points)

Node	S	A	B	C	D	E	F	G	H
Heuristic Value	5	a	3	3	3	4	2	0	e
True-Cost	6	5	b	3	5	c	d	0	10

Given the heuristic is admissible

\Rightarrow For each node, it has to be an underestimator of true cost

\Rightarrow Following constraints are to be satisfied :

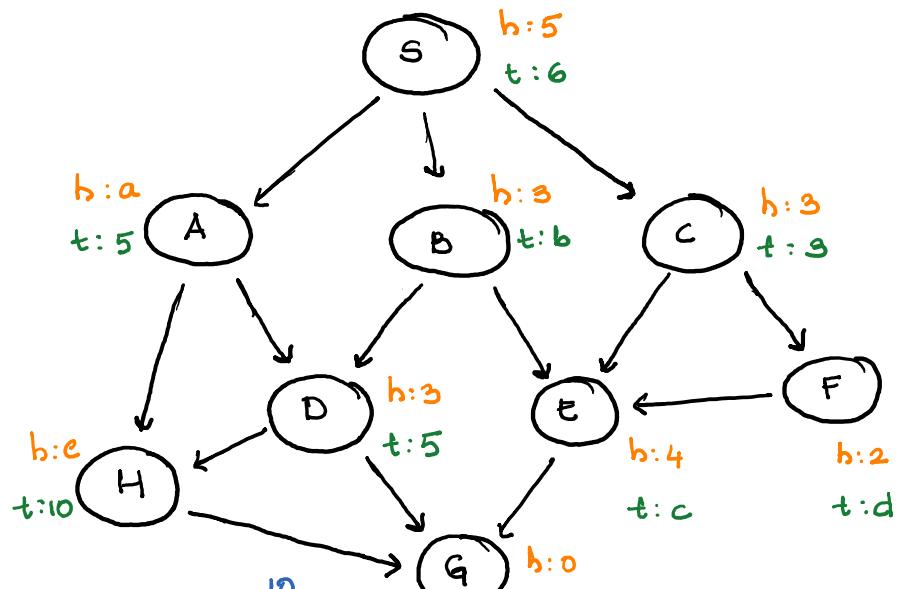
- ① $a \leq 5$
- ② $b \geq 3$
- ③ $c \geq 4$
- ④ $d \geq 2$
- ⑤ $e \leq 10$

For the same graph

As there is only one arc

in the graph that goes
from $H \rightarrow G$

{ Path cost from $[H \rightarrow G]$
 \cong True cost of H }



Similarly there is a single arc

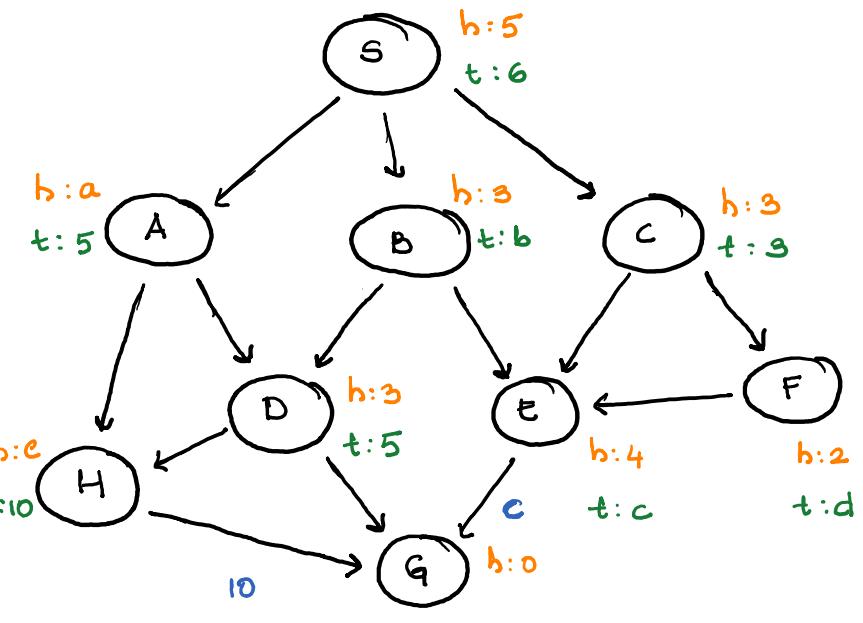
from $[E \rightarrow G]$

\therefore Path cost from $[E \rightarrow G]$

= True cost of E

E has heuristic value of $\boxed{4}$

$\Rightarrow [c \geq 4]$



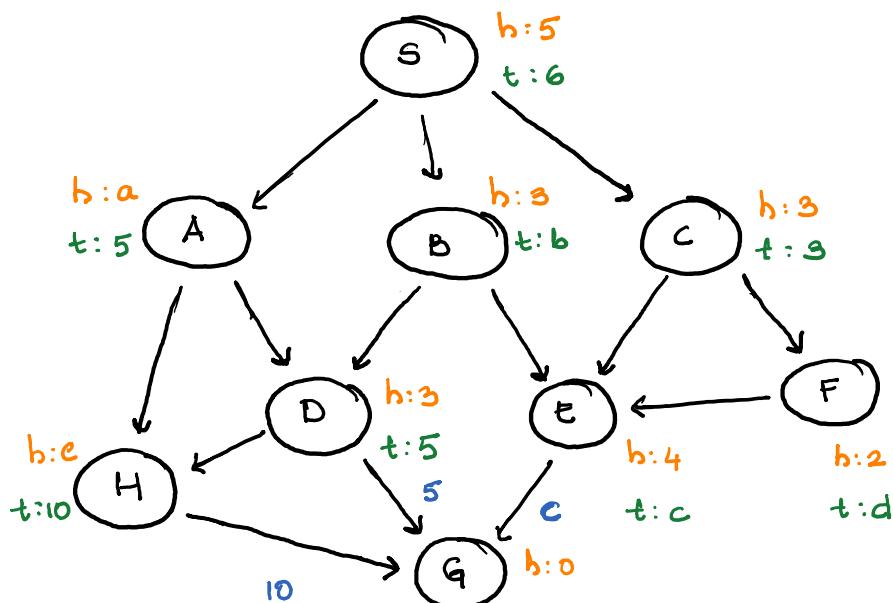
For node D

Paths from D to G

are $D \rightarrow H \rightarrow G$, $D \rightarrow G$

Given true cost for D = 5

As, $H \rightarrow G$ is 10 $9 \geq 5$

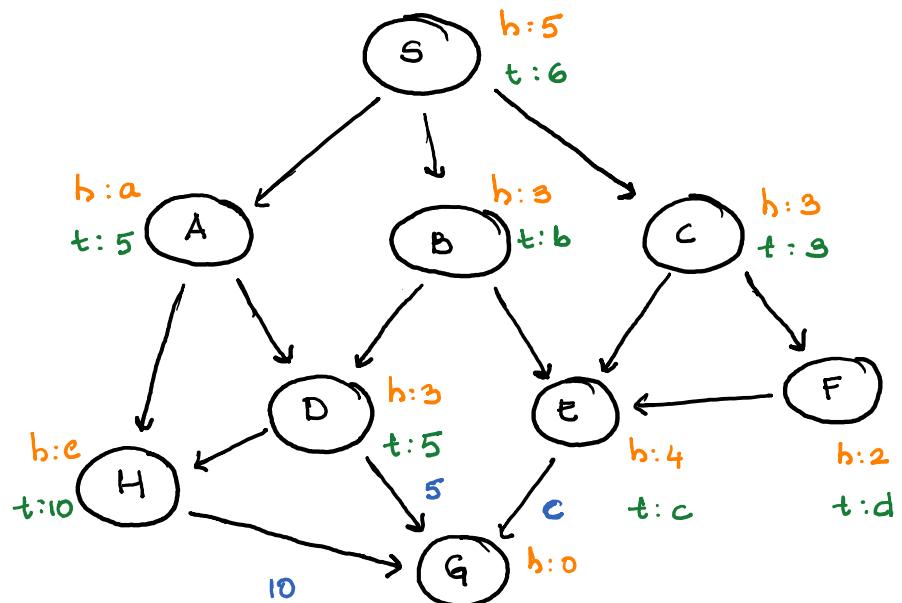


For true cost of 5 to be achievable

Path cost of $D \rightarrow G$ has to be $\boxed{5}$

There is a single path from
 $F \rightarrow G$ in the graph
 $[F \rightarrow E \rightarrow G]$

So, true cost to the goal
from node \textcircled{F} should
equal the path cost



$$\Rightarrow d = c(E, G) + c(F, E) \quad \left. \begin{array}{l} \\ \end{array} \right\} [d > c] \quad \begin{array}{l} \text{Assuming all edges} \\ \text{have positive } \underline{\text{weights}} \end{array}$$

$$= c + c(F, E)$$

Now, consider

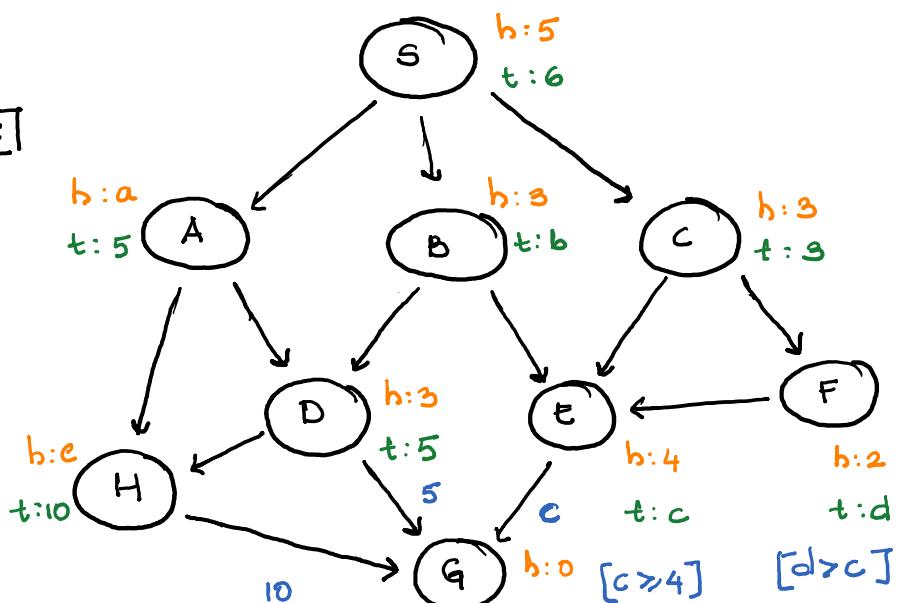
true cost to the goal from \boxed{C}

To reach G from C

Following are the paths.

$\Rightarrow C \rightarrow E \rightarrow G$

$\Rightarrow C \rightarrow F \rightarrow E \rightarrow G$



Assuming all path costs are positive

$$\text{cost}(C \rightarrow E \rightarrow G) < \text{cost}(C \rightarrow F \rightarrow E \rightarrow G)$$

Decomposing path $C \rightarrow E \rightarrow G$

true cost to reach goal from $C = \boxed{3}$

\Rightarrow Path cost $(C \rightarrow E \rightarrow G) = 3$ [As it is the least dist. path]

$$\Rightarrow c(C, E) + c(E, G) = 3$$

$$c(C, E) + c(E, G) = c + \text{cost}(C, E) \geq 4 \quad [\text{As } c \geq 4]$$

$$\Rightarrow \underline{3 \geq 4} \quad [\text{Not possible}]$$

$\therefore \exists$ no possible values for a, b, c, d, e which makes this heuristic admissible. [As it does not underestimate true cost]

(c) If an informed search needs to be performed on the same graph and the heuristics function has the following values in the graph nodes:

Node	S	A	B	C	D	E	F	G	H
Heuristic Value	5	4	5	3	3	4	2	0	1

Find if this heuristic function is consistent. (TOTAL: 15 points)

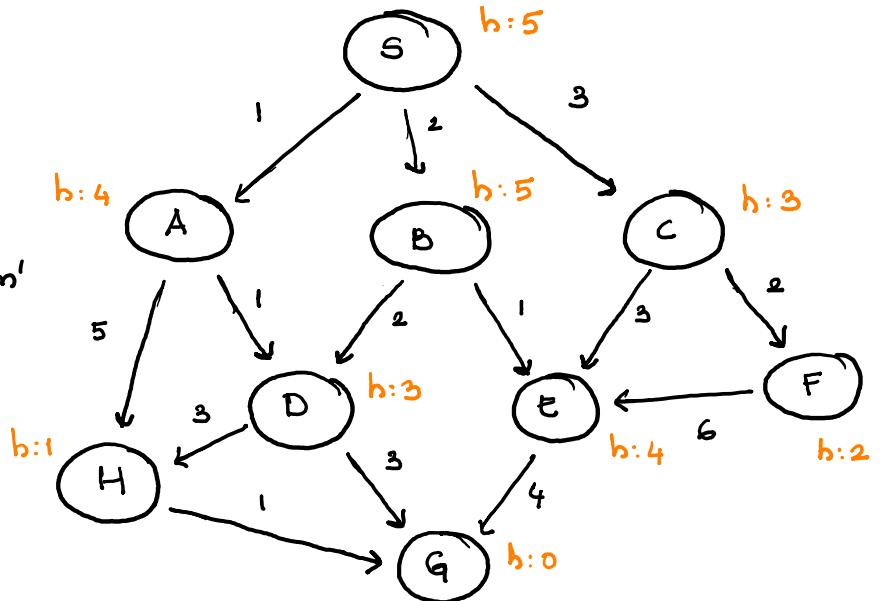
To check if a heuristic

is consistent (or) not

for every node n and successor n'

We check if

$$h(n) \leq c(n, a, n') + h(n')$$



So, considering node A

$$\text{Parent of } (A) = \{S\}$$

$$h(A) = 4, \quad c(S, A) = 1, \quad h(S) = 5$$

$$h(S) \stackrel{?}{\leq} h(A) + c(S, A) \Rightarrow 5 \stackrel{?}{\leq} 4 + 1 \quad [\text{True}]$$

Considering node B : Parent of node B = {S}

$$h(B) = 5, \quad h(S) = 5, \quad c(S, B) = 2$$

$$h(S) \stackrel{?}{\leq} c(S, B) + h(B) \Rightarrow 5 \stackrel{?}{\leq} 2 + 5 \quad [\text{True}]$$

Considering node C: Parent of C = {S}

$$h(C) = 3, c(S, C) = 3, h(S) = 5$$

$$h(S) \stackrel{?}{\leq} c(S, C) + h(C) \Rightarrow 5 \stackrel{?}{\leq} 3 + 3 \quad [\text{True}]$$

Considering node D: Parents of D = {A, B}

$$\text{i)} \quad c(A, D) = 1 \quad \text{ii)} \quad c(B, D) = 2$$

$$h(A) = 4, h(D) = 3 \quad h(B) = 5, h(D) = 3$$

$$h(A) \stackrel{?}{\leq} c(A, D) + h(D) \quad h(B) \stackrel{?}{\leq} c(B, D) + h(D)$$

$$4 \stackrel{?}{\leq} 1 + 3 \quad [\text{True}] \quad 5 \stackrel{?}{\leq} 2 + 3 \quad [\text{True}]$$

Considering node F: Parent F = {C}

$$c(C, F) = 2, h(F) = 2, h(C) = 3$$

$$h(C) \stackrel{?}{\leq} c(C, F) + h(F) \Rightarrow 3 \stackrel{?}{\leq} 2 + 2 \quad [\text{True}]$$

Considering node H: Parents of H = {A, D}

$$\text{i)} \quad c(A, H) = 5 \quad \text{ii)} \quad c(D, H) = 3$$

$$h(A) = 4, h(H) = 1, h(D) = 3$$

$$h(A) \stackrel{?}{\leq} c(A, H) + h(H) \quad h(D) \stackrel{?}{\leq} c(D, H) + h(H)$$

$$\Rightarrow 4 \stackrel{?}{\leq} 5 + 1 \quad [\text{True}] \quad 3 \stackrel{?}{\leq} 3 + 1 \quad [\text{True}]$$

Considering node E: Parents of E = {B, C, F}

$$\text{i)} c(B, E) = 1 \quad \text{ii)} c(C, E) = 3 \quad \text{iii)} c(F, E) = 6$$

$$h(B) = 5, h(E) = 4, h(C) = 3, h(F) = 2$$

$$h(B) \stackrel{?}{\leq} c(B, E) + h(E) \quad h(C) \stackrel{?}{\leq} c(C, E) + h(E) \quad h(F) \stackrel{?}{\leq} c(F, E) + h(E)$$

$$5 \stackrel{?}{\leq} 1 + 4 \quad [\text{True}] \quad 3 \stackrel{?}{\leq} 3 + 4 \quad [\text{True}] \quad 2 \stackrel{?}{\leq} 6 + 4 \quad [\text{True}]$$

Considering node G: Parents of G = {H, D, E}

$$\text{i)} c(H, G) = 1 \quad \text{ii)} c(D, G) = 3 \quad \text{iii)} c(E, G) = 4$$

$$h(H) = 1, h(D) = 3, h(E) = 4, h(G) = 0$$

$$h(H) \stackrel{?}{\leq} c(H, G) + h(G) \quad h(D) \stackrel{?}{\leq} c(D, G) + h(G) \quad h(E) \stackrel{?}{\leq} c(E, G) + h(G)$$

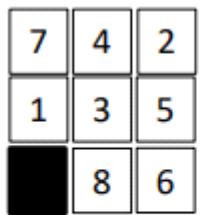
$$1 \stackrel{?}{\leq} 1 + 0 \quad [\text{True}] \quad 3 \stackrel{?}{\leq} 3 + 0 \quad [\text{True}] \quad 4 \stackrel{?}{\leq} 4 \quad [\text{True}]$$

All the arcs in the graph satisfy [TRIANGLE INEQUALITY]

So, the heuristic provided is a CONSISTENT heuristic

Problem 2: Heuristic Search:

Consider the 8-puzzle problem, with the initial state represented as I and the Goal State as G:



I



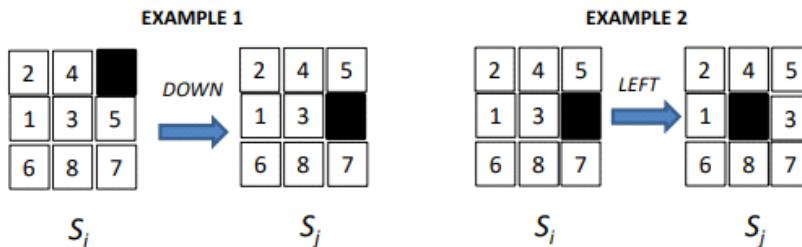
G

If the cost of sliding the empty square is equal to the sum of its neighbouring squares in the successor state and you use a *heuristic = $2 \times Manhattan_{Distance}$* , provide the details of the first 3 steps of the search using the Recursive Best-First Search (RBFS) strategy. (TOTAL: 55 points)

Hint 1: You will have to fill the values of the following table: (30 points)

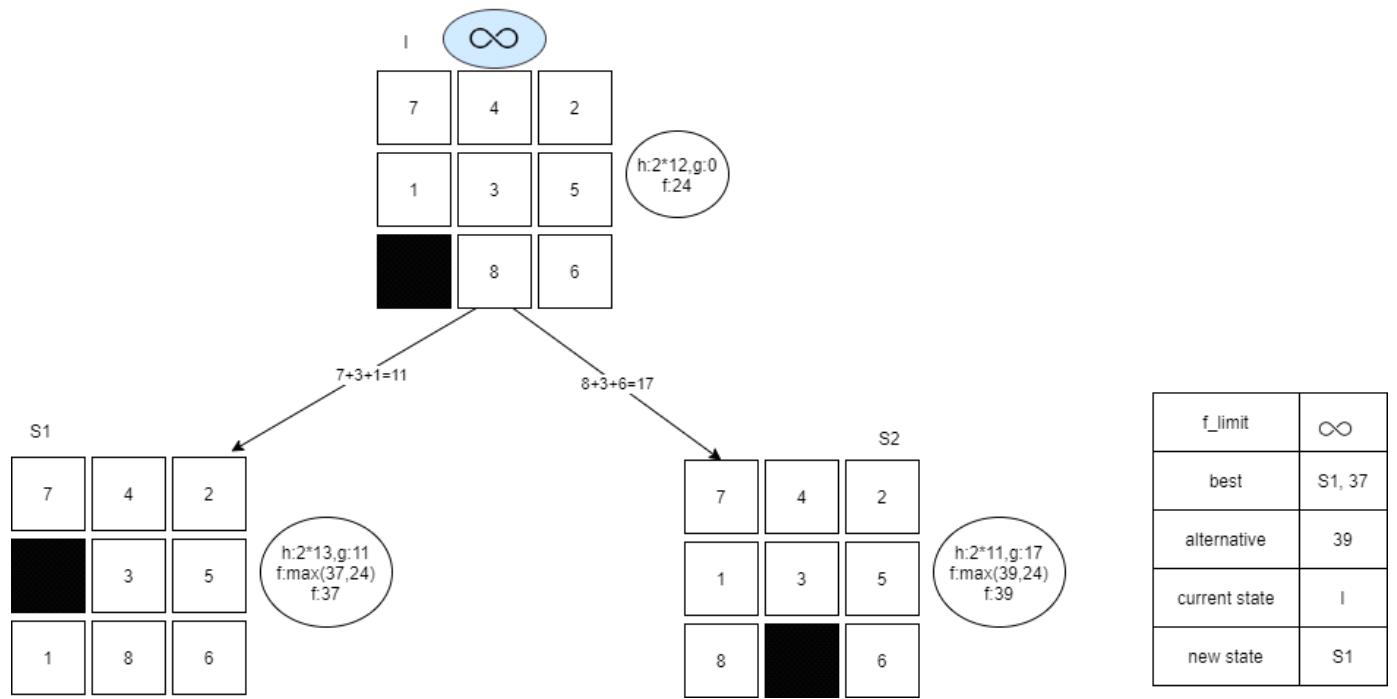
	STEP 1	STEP 2	STEP 3
F_limit	?	?	?
best	?	?	?
alternative	?	?	?
Current State	?	?	?
New State	?	?	?

Hint 2: The following two examples show how you compute the cost of getting from a state S_i to a state S_j, when using some specific action:

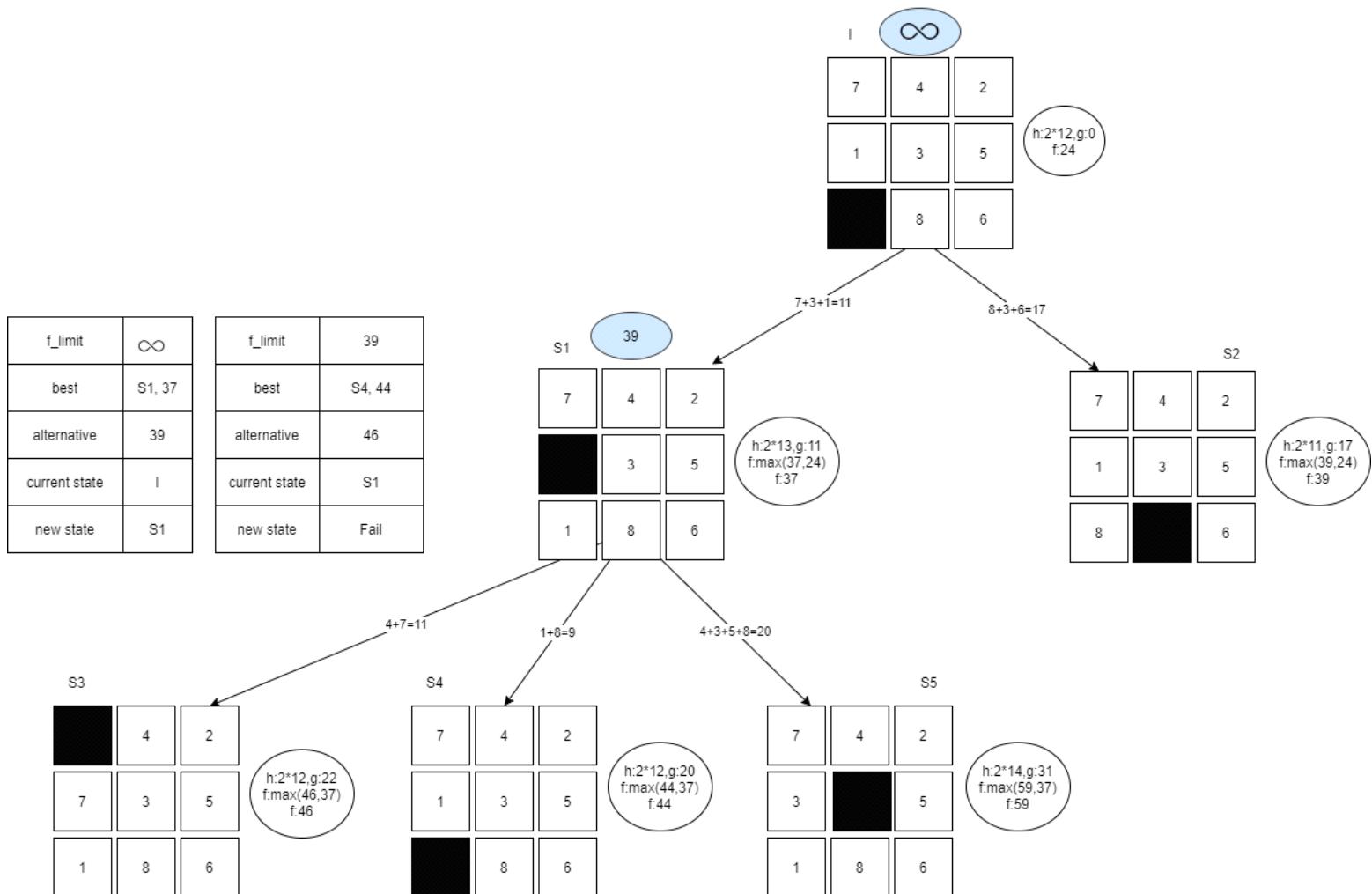


For EXAMPLE 1: $g(S_i, DOWN, S_j) = 5 + 3 + 7 = 15$. For EXAMPLE 2: $g(S_i, LEFT, S_j) = 1 + 4 + 3 + 8 = 16$. Hint 3: If you show correctly (a) all the successors of state as I as well as all successors produced in all first 3 steps and you compute correctly the cost from the initial state I to each search node as well as the heuristic value in each of the search nodes involved in the first 3 steps, you will be assigned. (25 points)

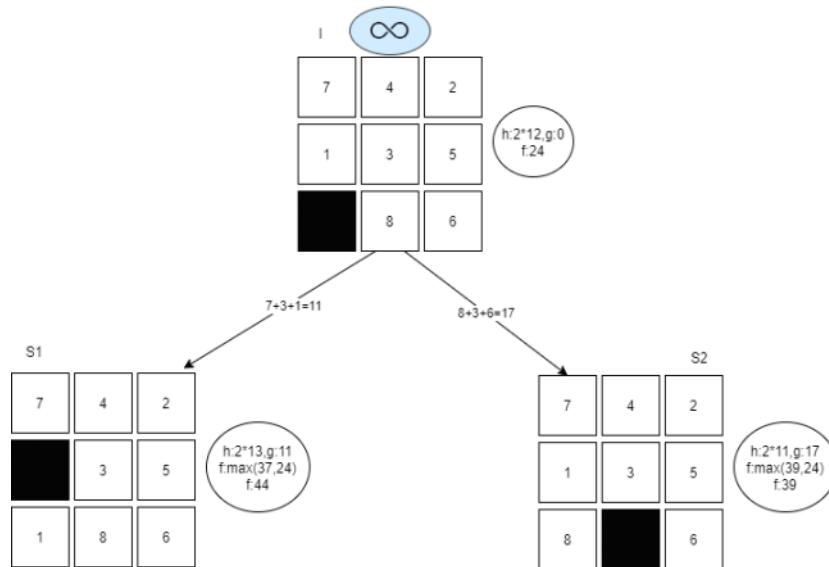
STEP - 1



STEP-2



STEP-3



f_limit	∞
best	S1, 37
alternative	39
current state	I
new state	S1

f_limit	39
best	S4, 44
alternative	46
current state	S1
new state	Fail

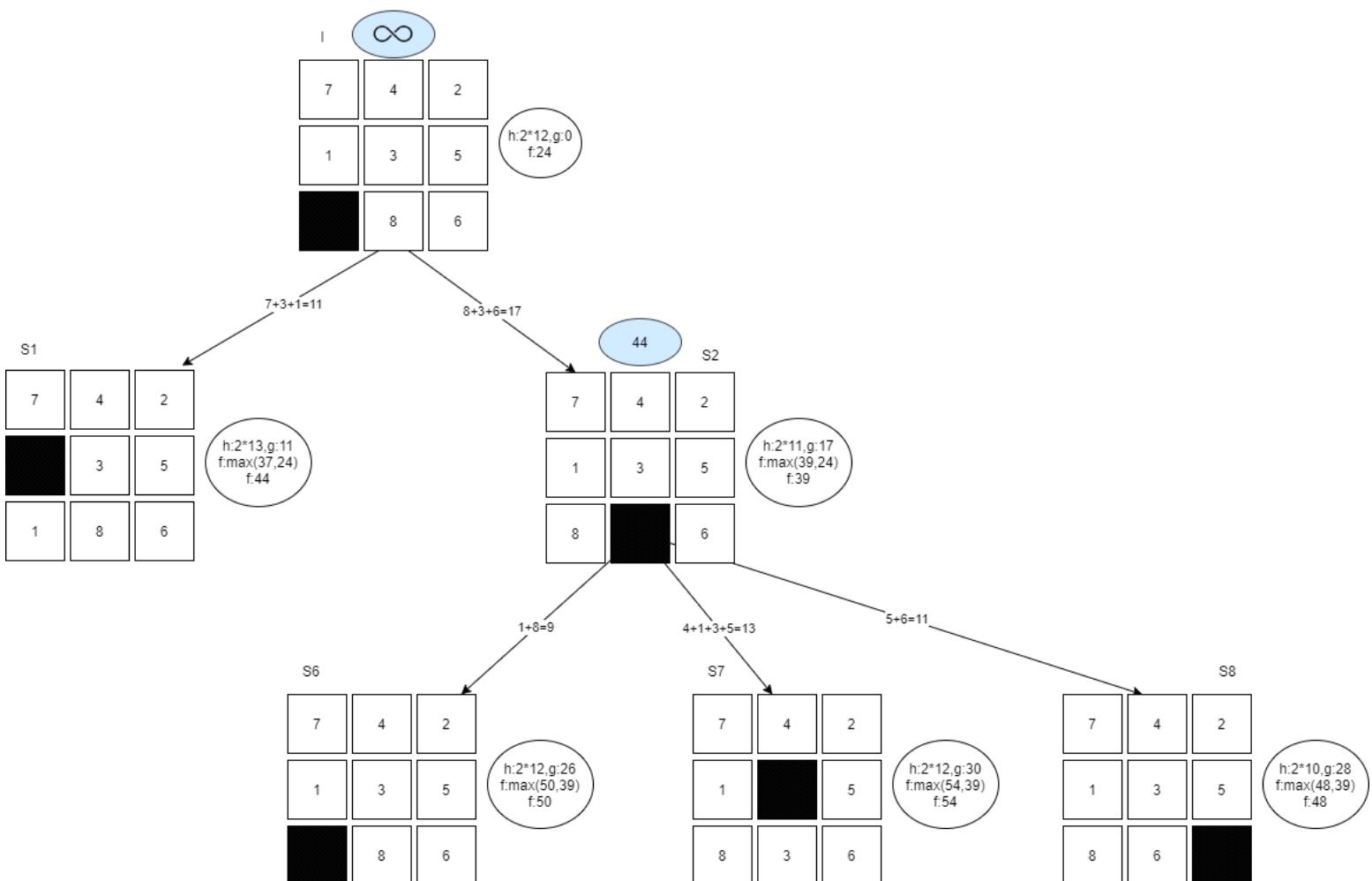
f_limit	∞
best	S2, 39
alternative	44
current state	I
new state	S2

f_limit	∞
best	S1, 37
alternative	39
current state	I
new state	S1

f_limit	39
best	S4, 44
alternative	46
current state	S1
new state	Fail

f_limit	∞
best	S2, 39
alternative	44
current state	I
new state	S2

STEP-4



f_limit	∞	f_limit	39	f_limit	∞	f_limit	44
best	S1, 37	best	S4, 44	best	S2, 39	best	S8, 48
alternative	39	alternative	46	alternative	44	alternative	50
current state	I	current state	S1	current state	I	current state	S2
new state	S1	new state	Fail	new state	S2	new state	Fail