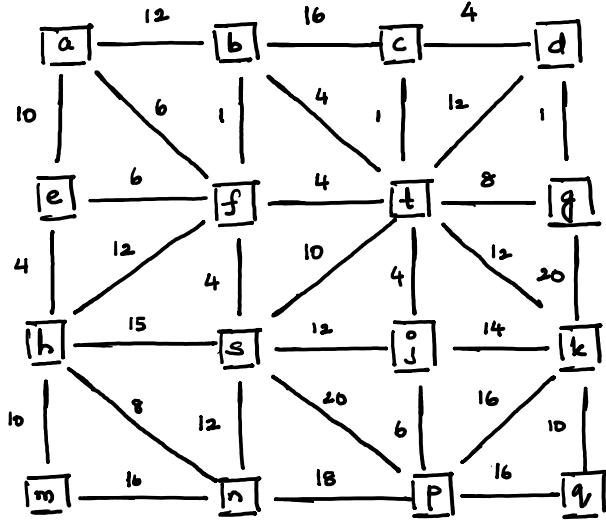


Maximum flows if edge capacities are given by $\min[u_{ij}, \lambda]$

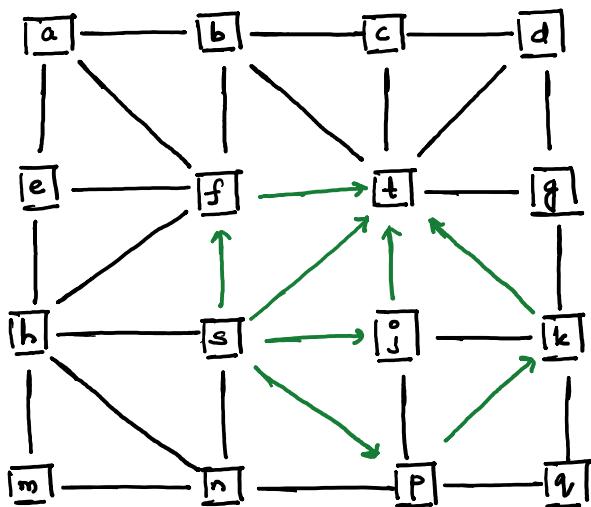
- a) Find maximum surviving flow
- b) Find maximum q -path flow where $q=2$



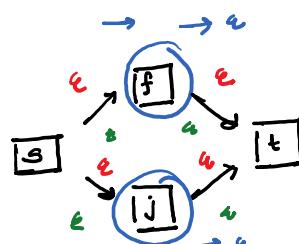
From HW1 when $\lambda = \infty$

$F(\infty)$: Max flow with current $u_{ij} = \boxed{48}$

$F(\epsilon)$: Max flow with $u_{ij} = \epsilon + \underline{(c_{ij})\epsilon}$
for small enough ϵ .

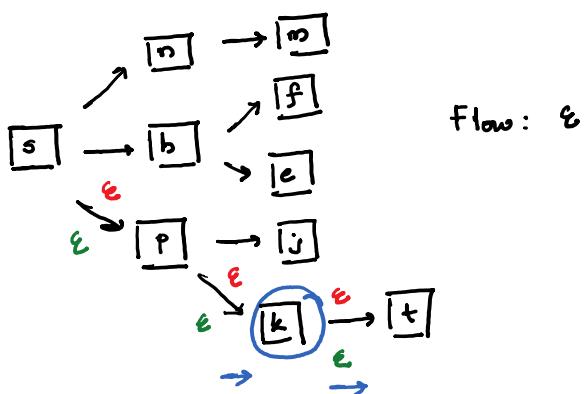


Using MKM method for finding maxflow



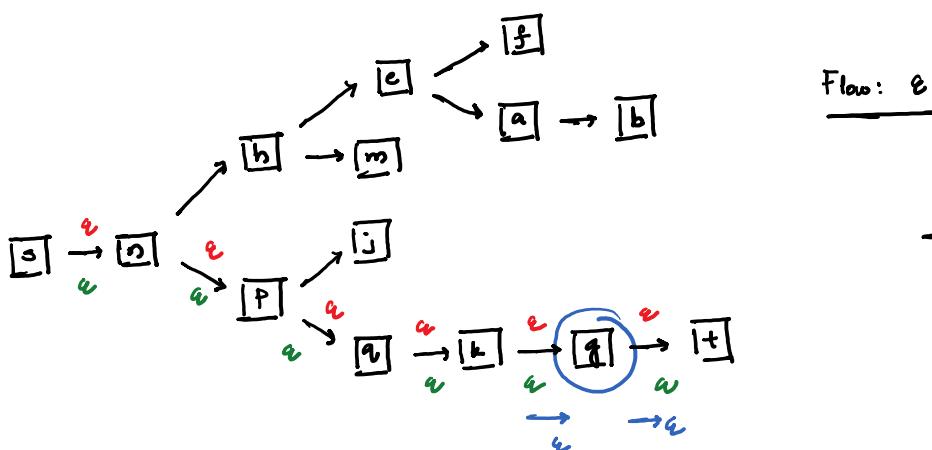
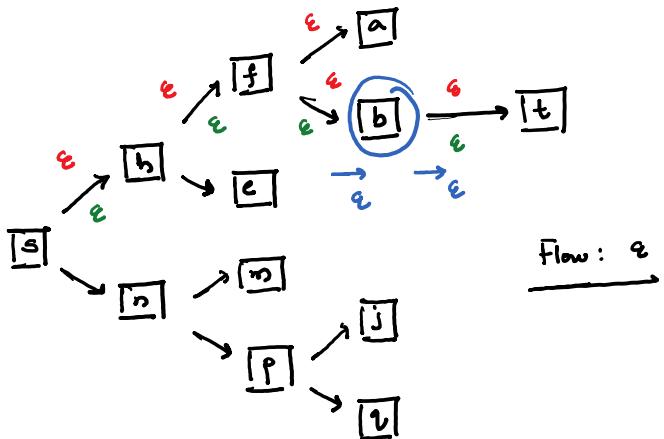
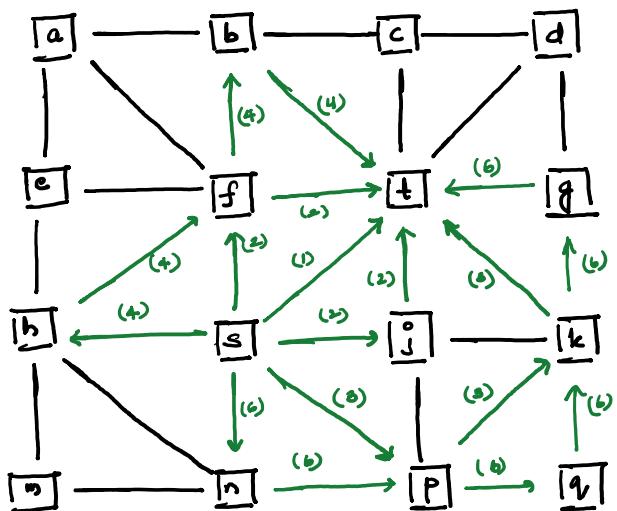
$\curvearrowright [s-t]$ edge

Flow: $\epsilon + 2\epsilon$



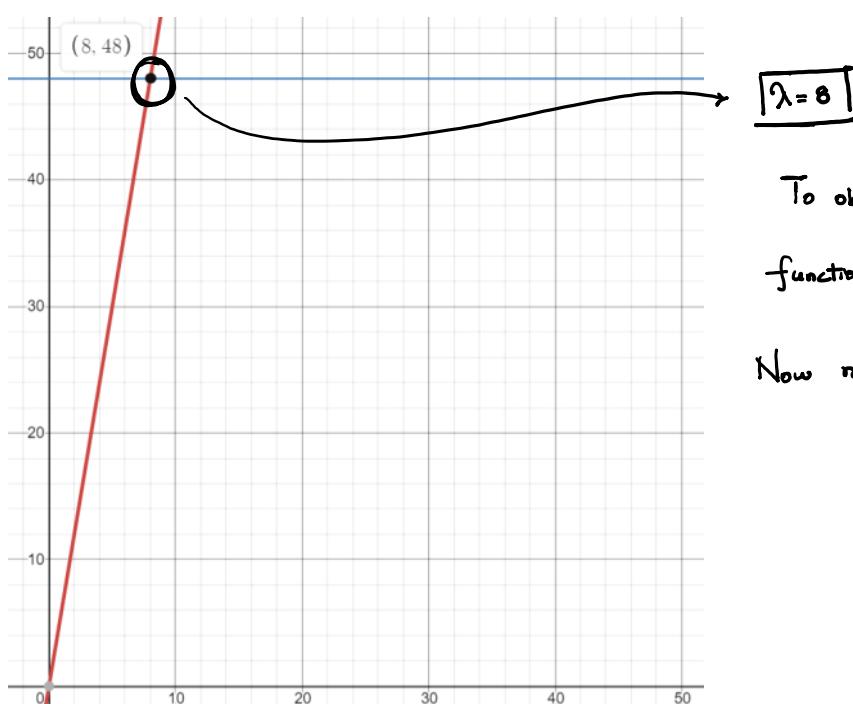
Flow: ϵ

4



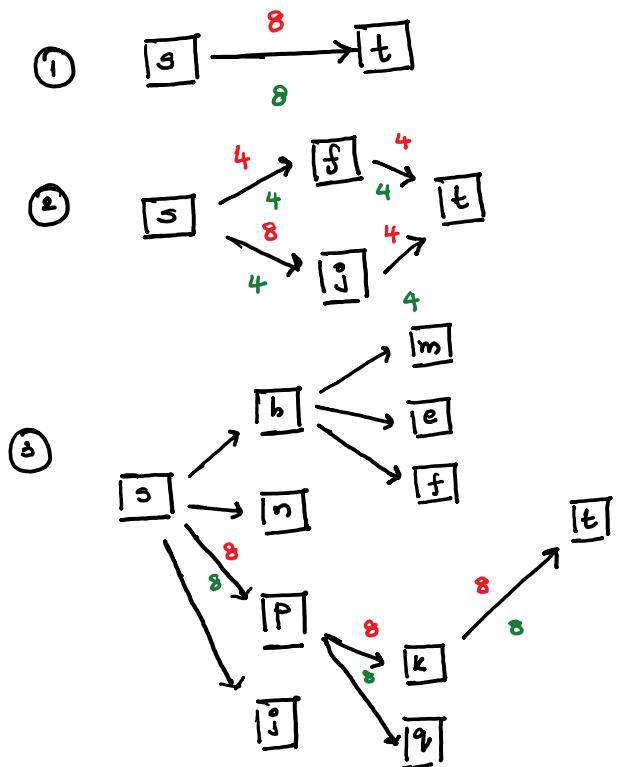
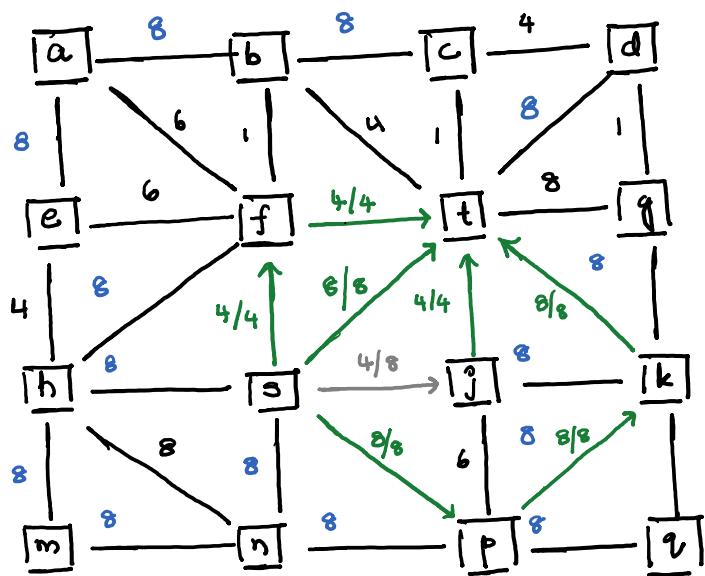
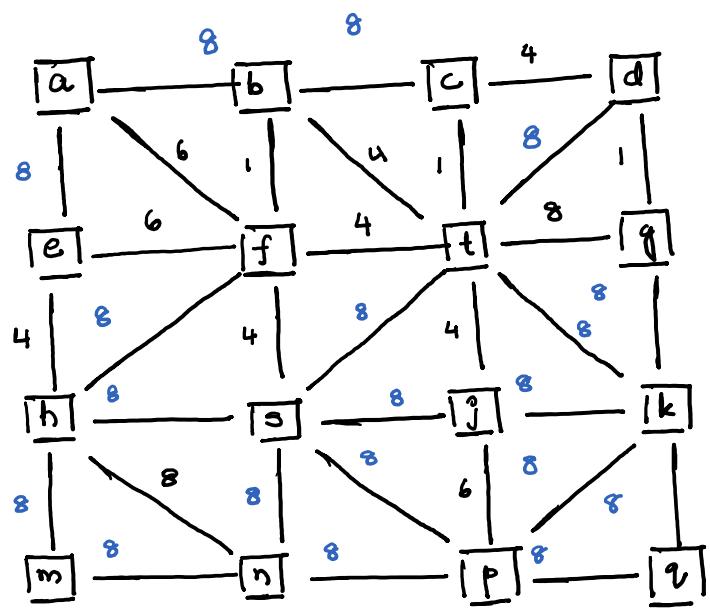
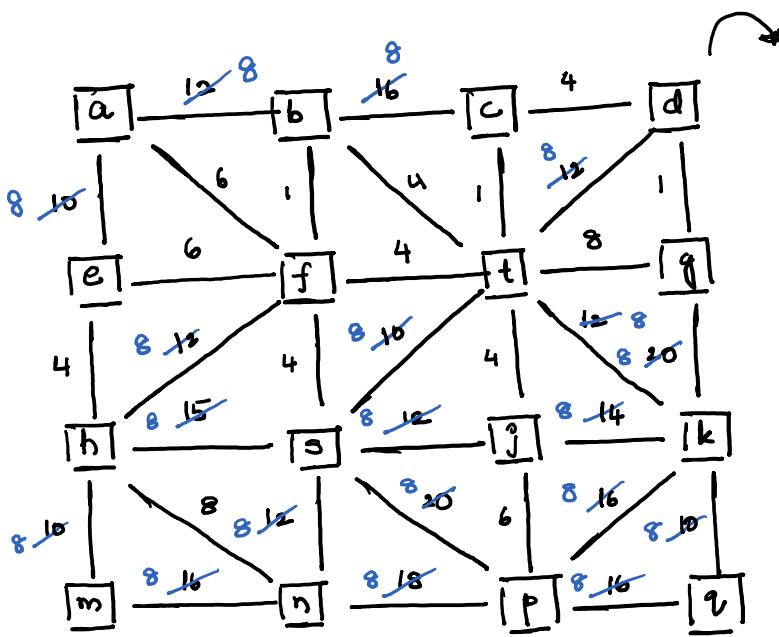
$$\text{Total flow} : 4\epsilon + 2\epsilon = 6\epsilon$$

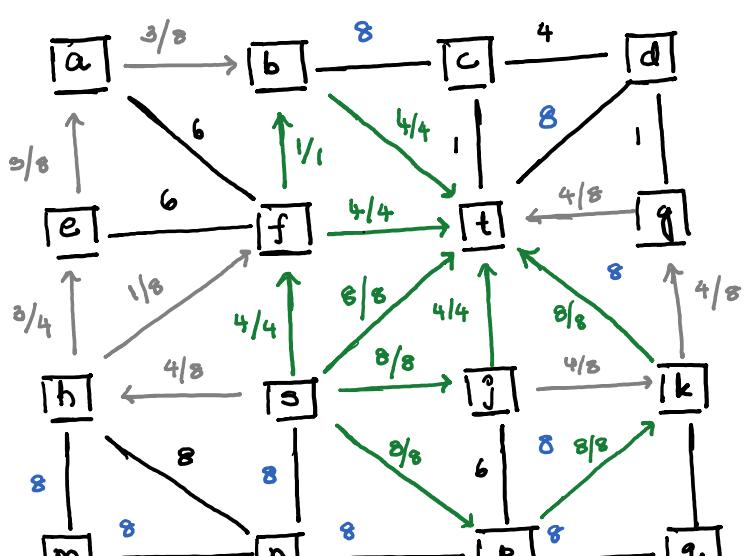
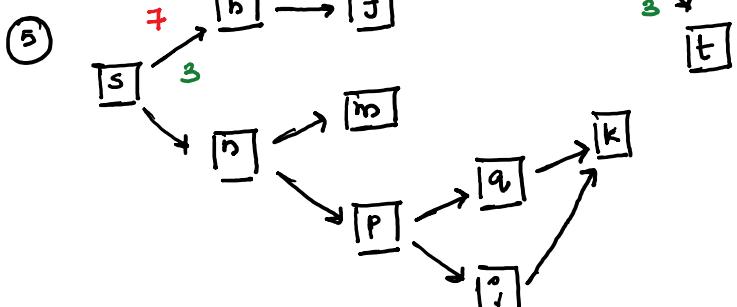
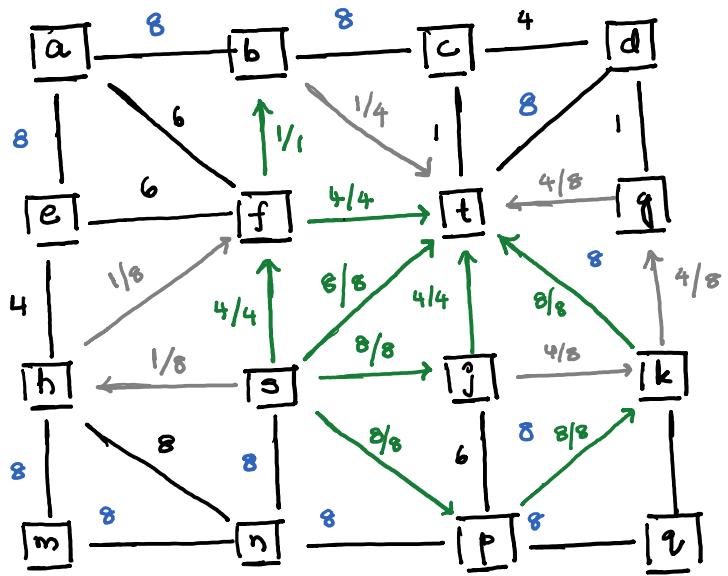
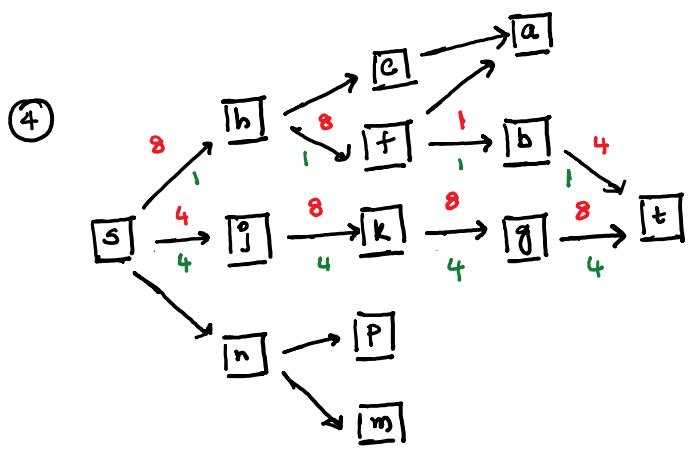
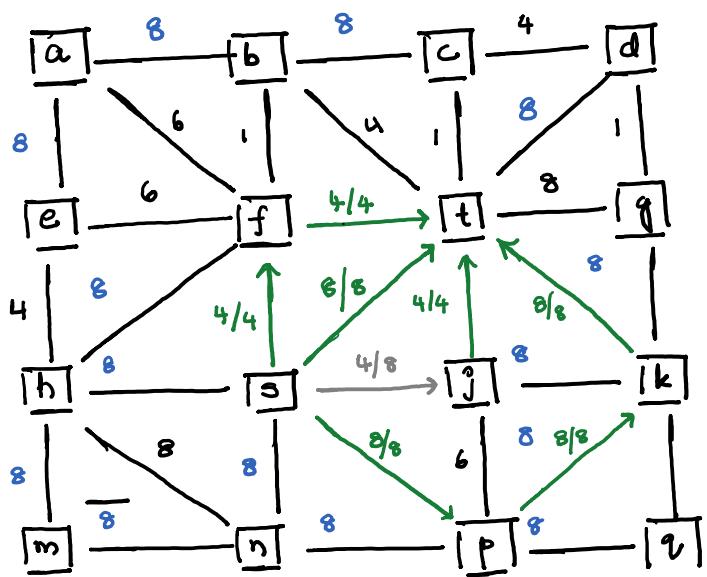
Plotting $F(\lambda)$ curve with values of $\lambda = \infty, \lambda = 0$ gives.

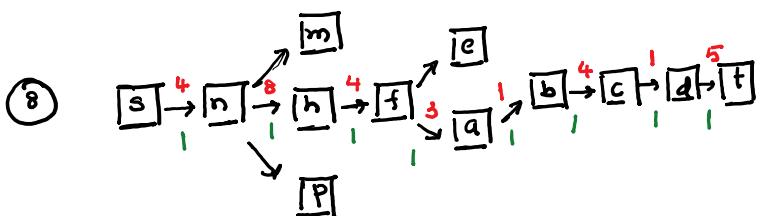
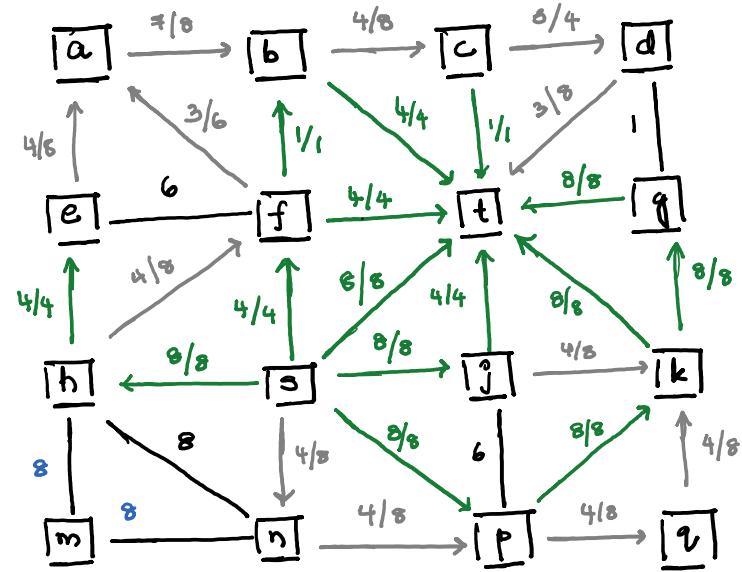
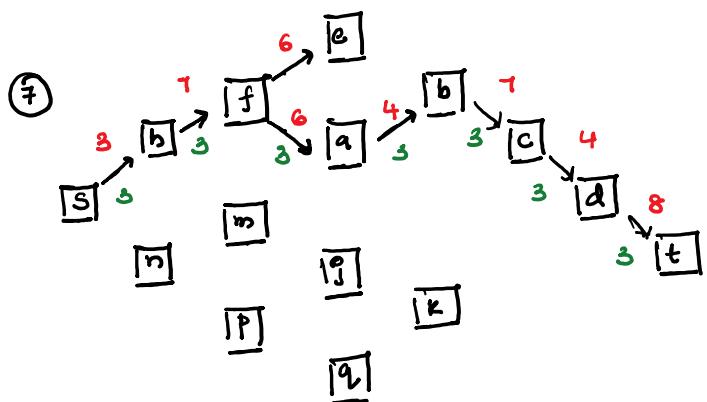
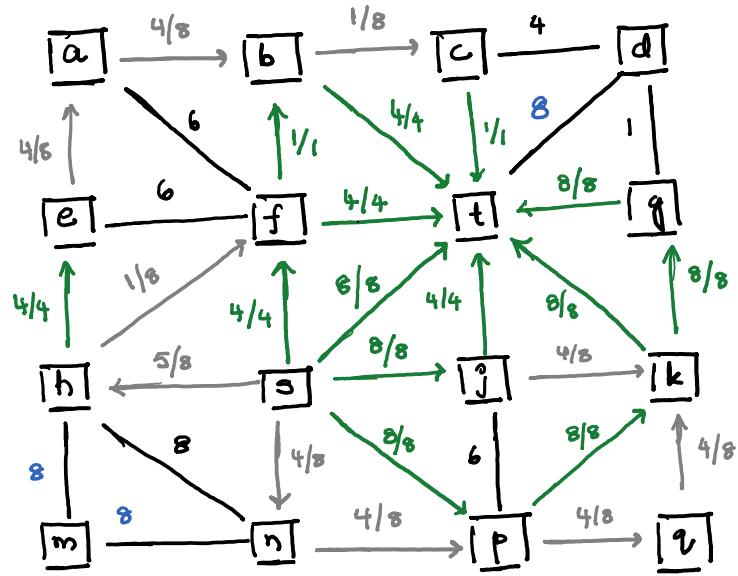


Now min [$w_{ij}, 8$]

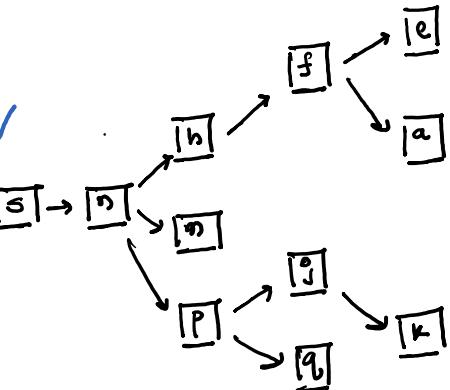
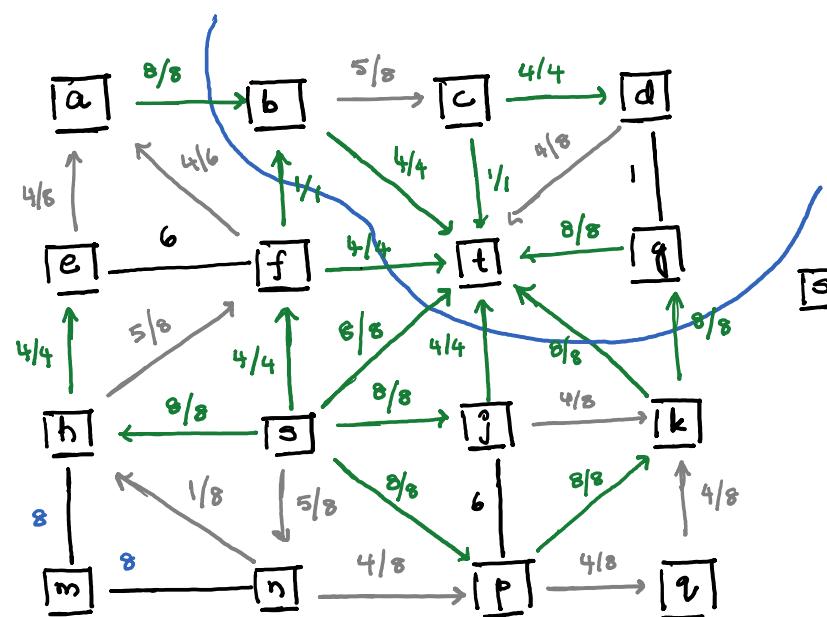
MAX FLOW for min[u, 8]







MIN CUT :



(b, c, d, t, q)

$$\text{Flow obtained : } \underline{1+3} + \underline{1+4+3} + \underline{1+4} + \underline{8+8+8} = 24 + 8 + 9 = \boxed{41}$$

No. of edges in min cut with $\lambda = 8 \Rightarrow \underbrace{(a,b), (s,t), (r,t), (k,q)}_6 \rightarrow \boxed{41} \leftarrow \boxed{\text{Slope}}$

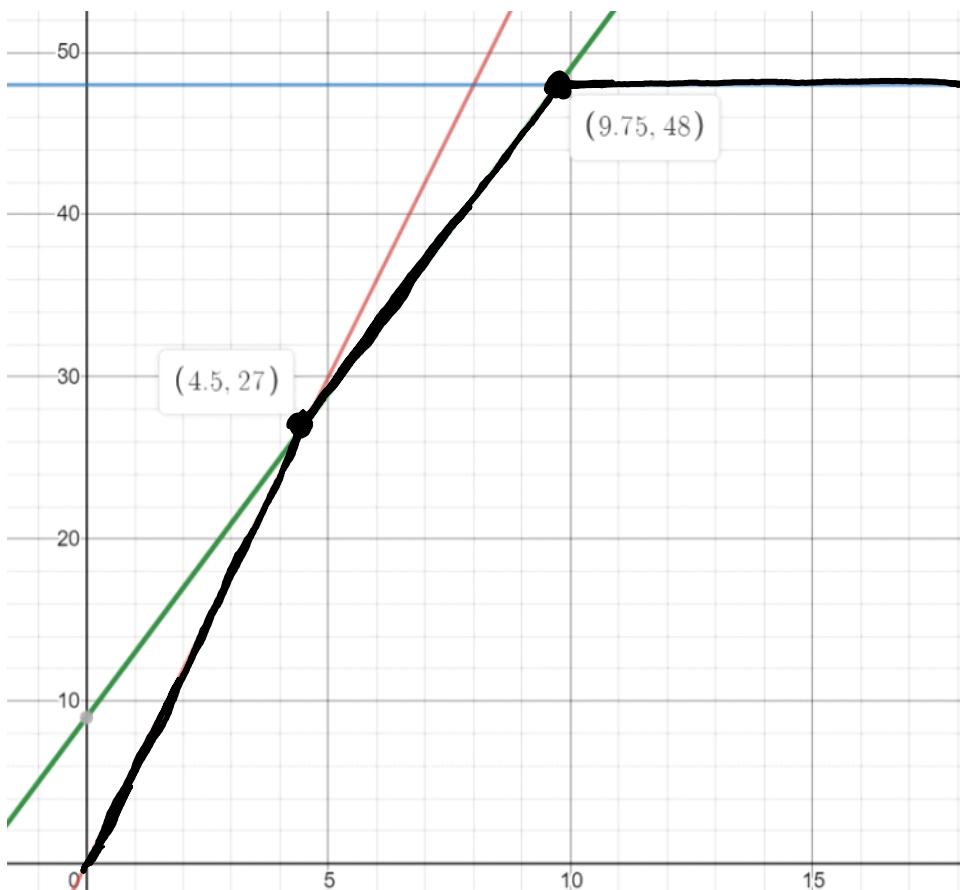
Equations of line : $y = 4x + b$

$$\left. \begin{array}{l} \\ \end{array} \right\} \quad \begin{array}{l} \\ y = 4x + 9 \end{array}$$

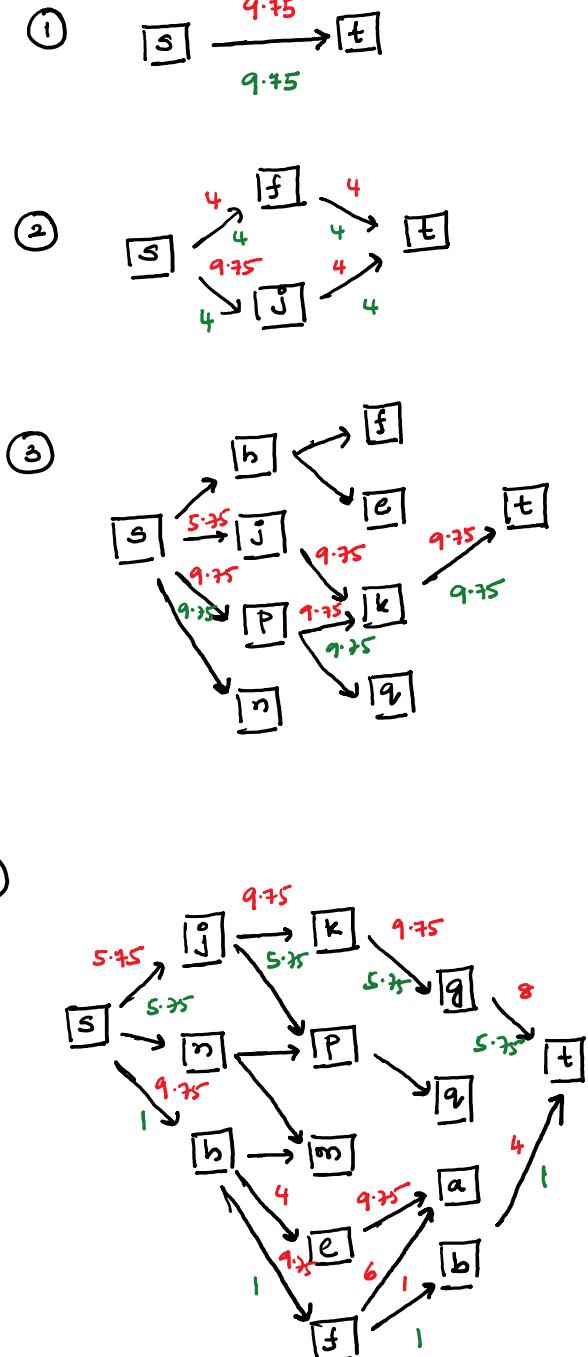
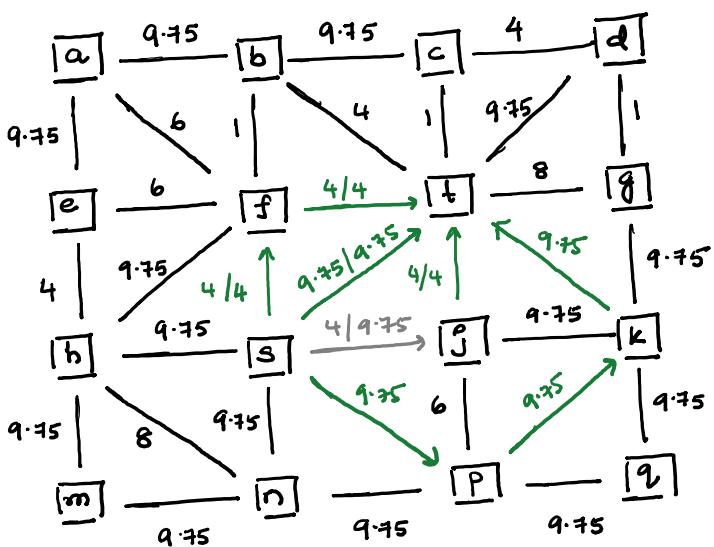
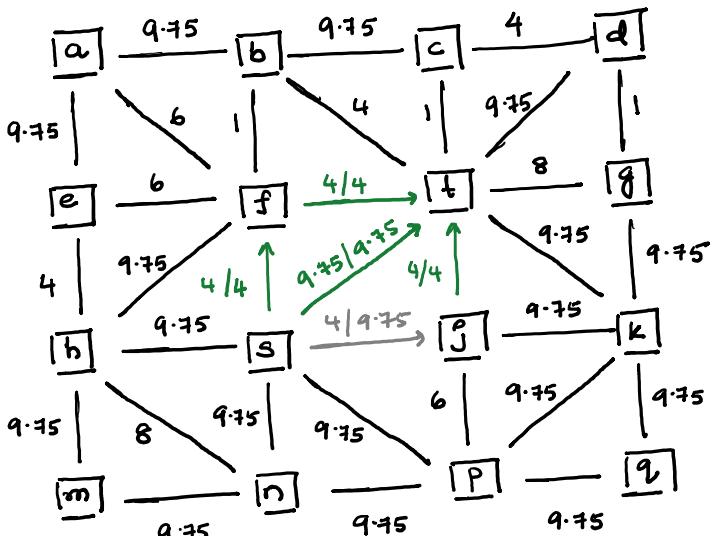
$$\textcircled{C} \quad x = 8, \quad y = 41 \Rightarrow b = 9$$

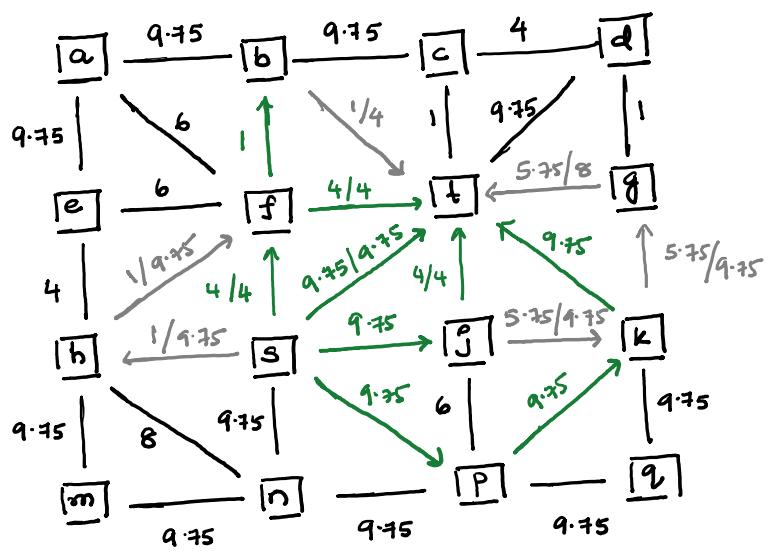
Point of intersection @ $y = 6x \rightarrow 6x = 4x + 9 \Rightarrow x = \frac{9}{2} = \underline{4.5}$

Point of intersection @ $y = 4x \Rightarrow 4x + 9 = 48 \rightarrow x = \frac{39}{4} = \underline{9.75}$

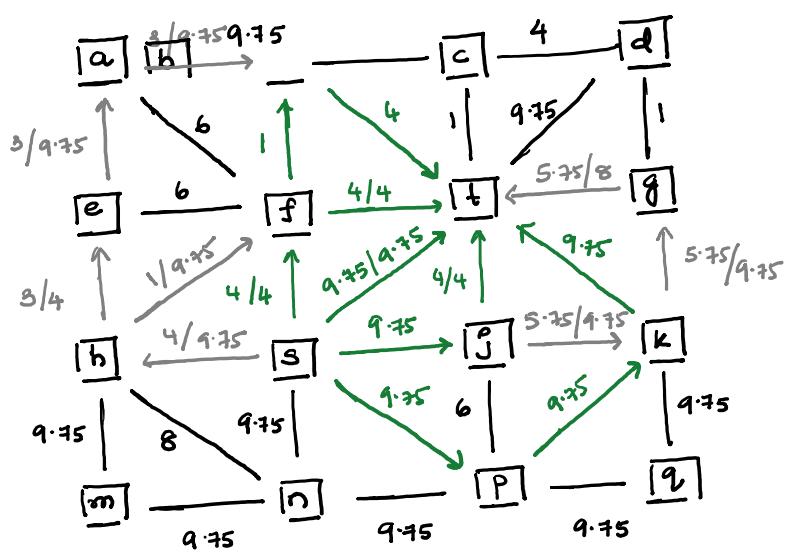
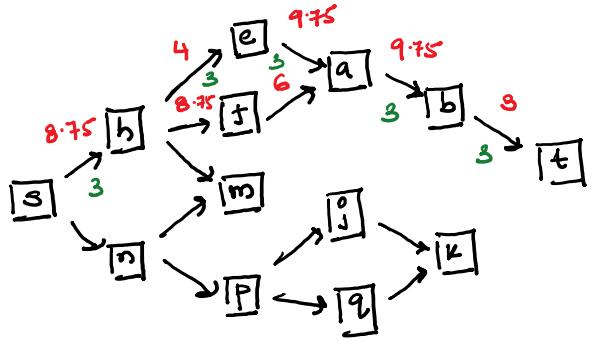


MAX FLOW for min[u, 9.75]

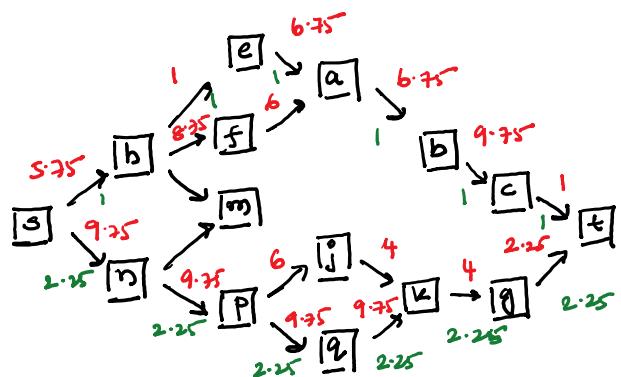


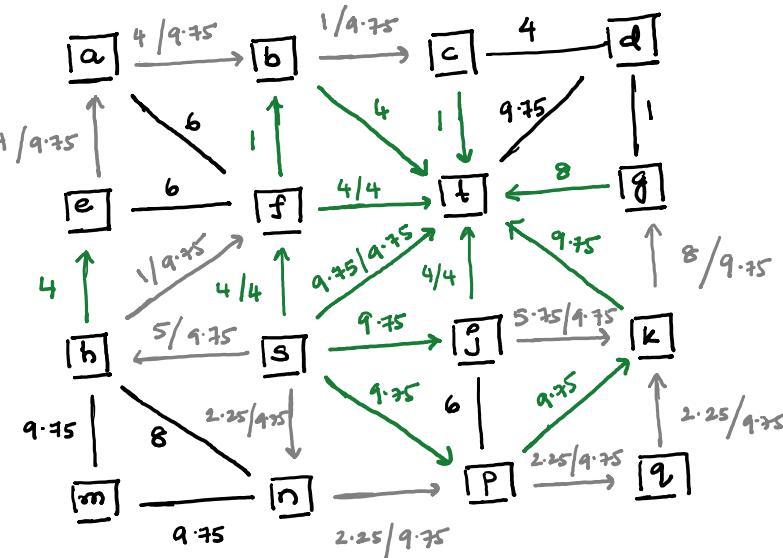


⑤

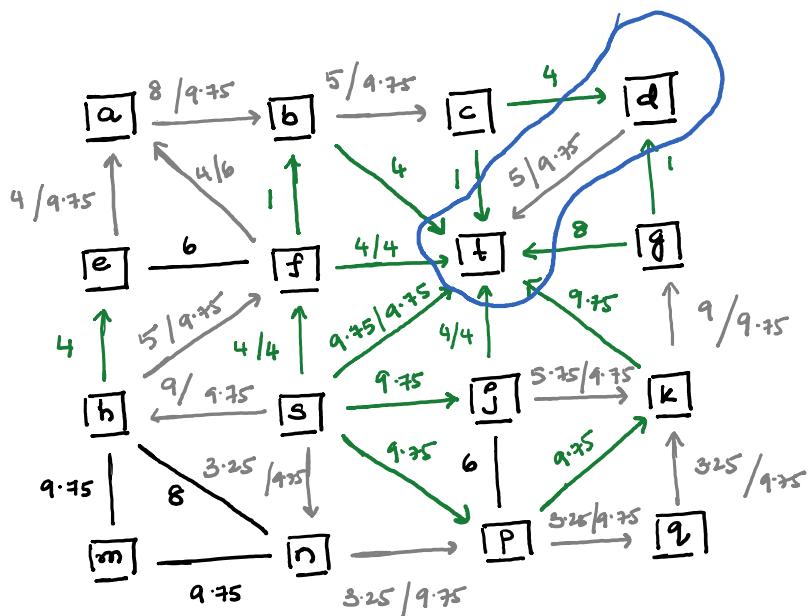
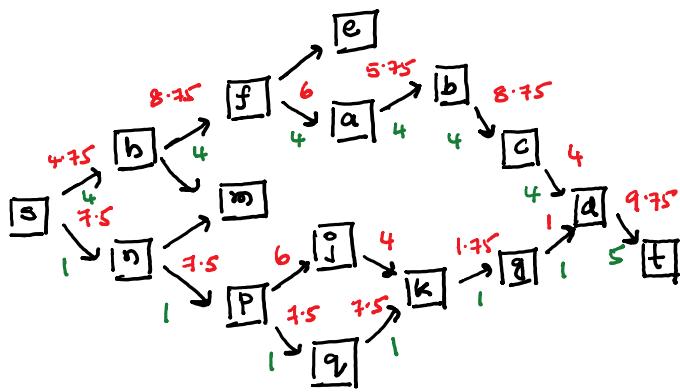


⑥



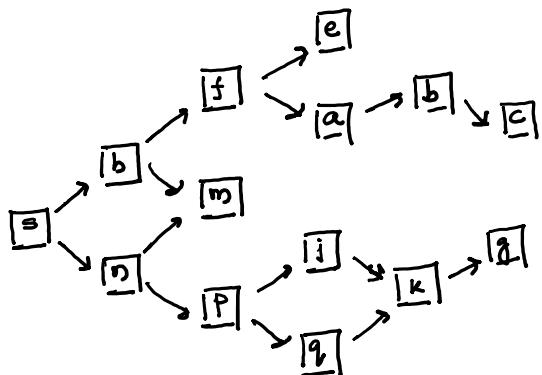


⑦



⑧

Min-cut :



Flow : 45.5

No. of edges with $\lambda = 9.75 \Rightarrow (s, t), (k, t)$

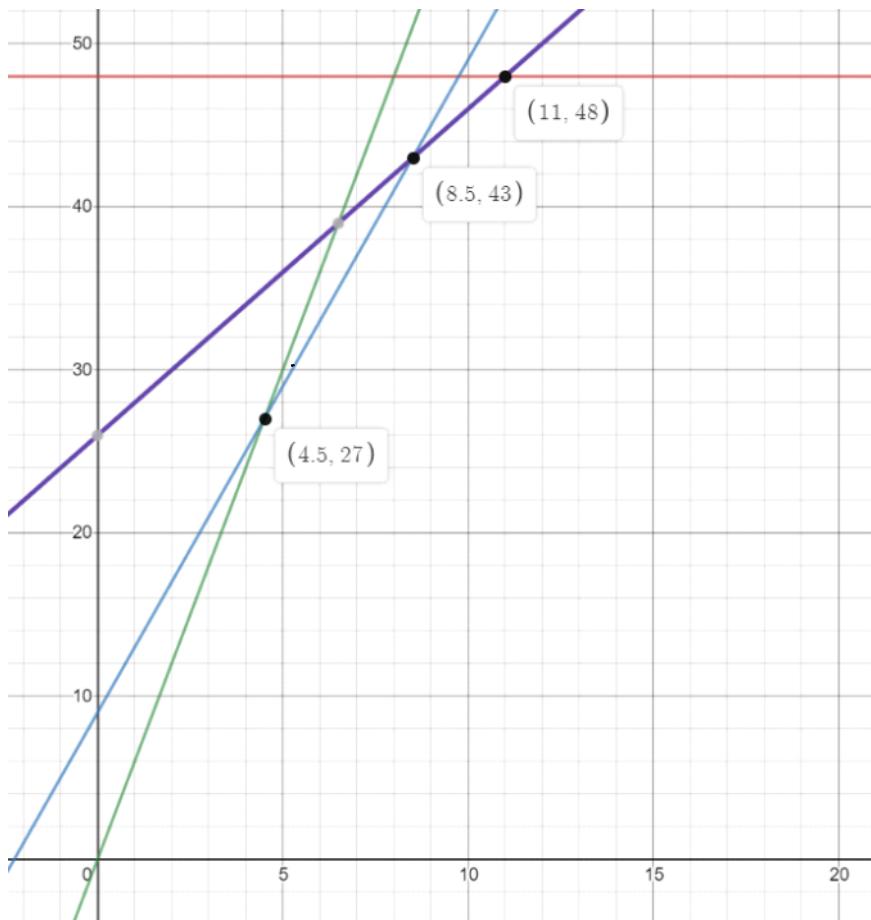
$$\Rightarrow y = 2x + b \quad @ \quad x = 9.75 \Rightarrow \underline{\text{Flow} = 45.5}$$

$$45.5 = 2(9.75) + b \Rightarrow \boxed{b = 26}$$

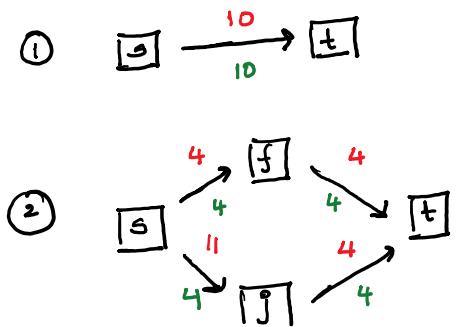
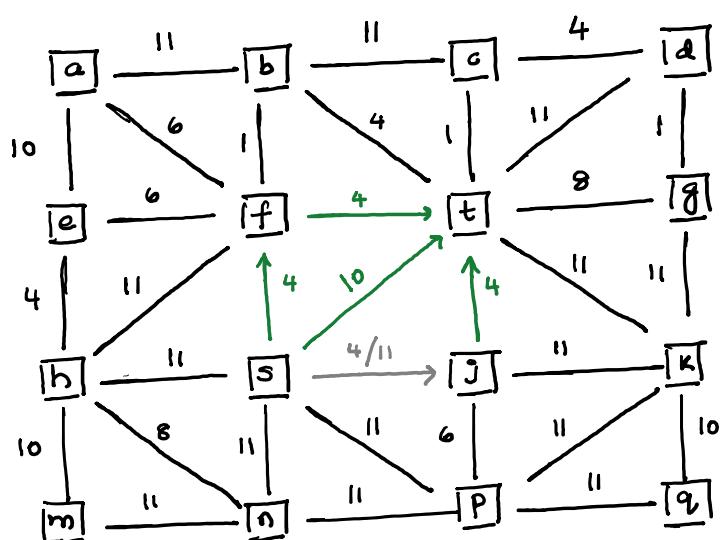
Points of intersection : $y = 48 \Rightarrow 2x + 26 = 48 \Rightarrow \boxed{x = 11}$

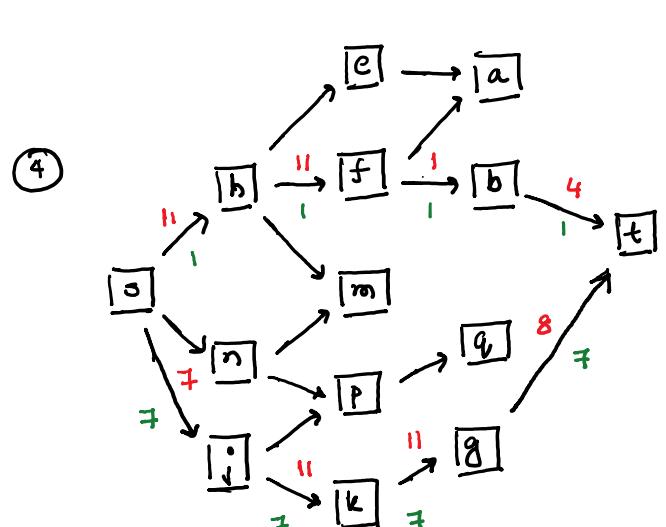
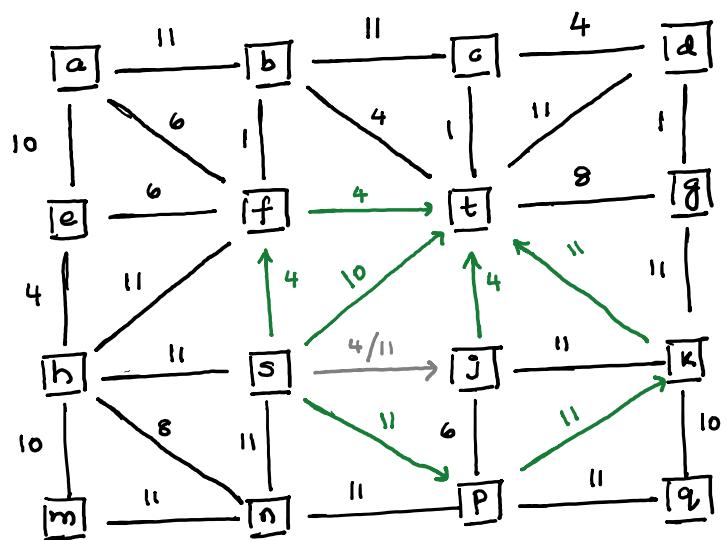
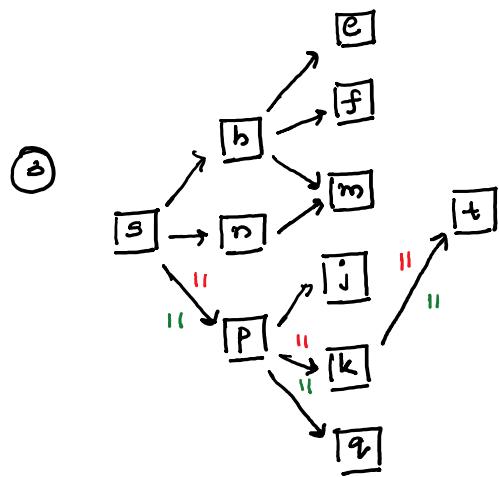
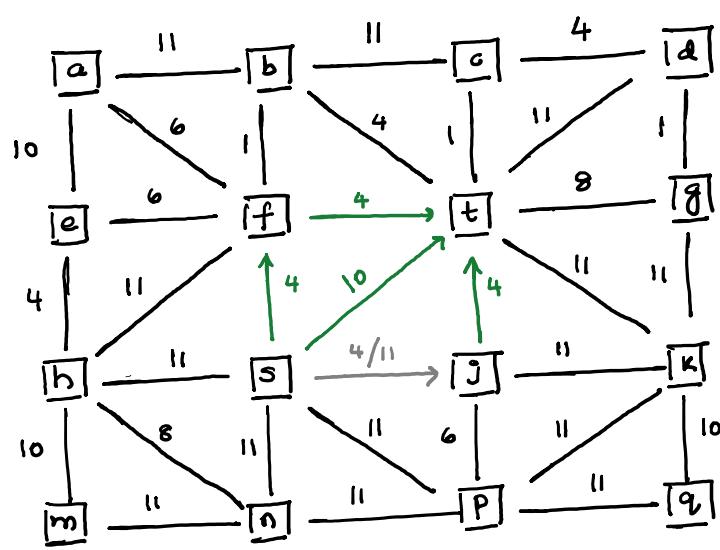
$$4x + 9 = 2x + 26 \Rightarrow 2x = 17 \Rightarrow \boxed{x = 8.5}$$

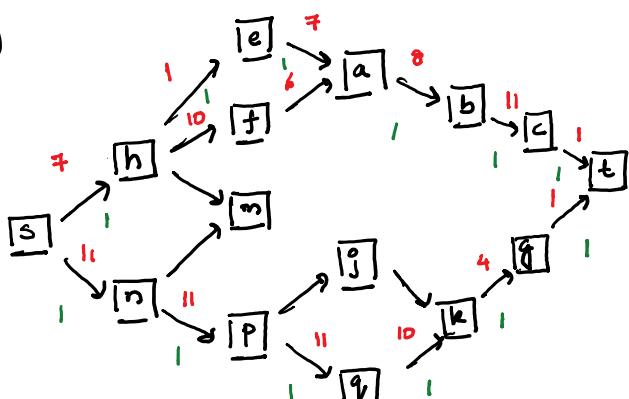
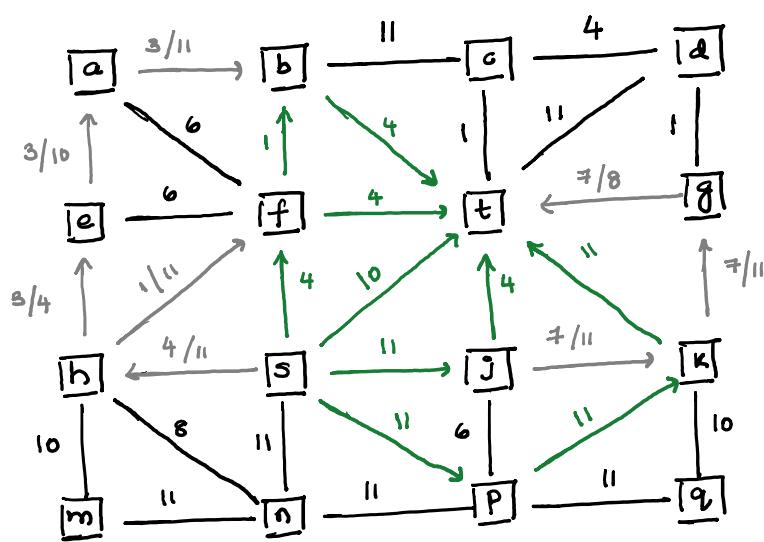
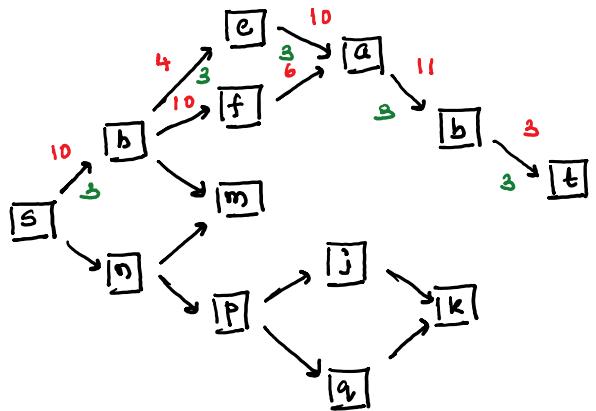
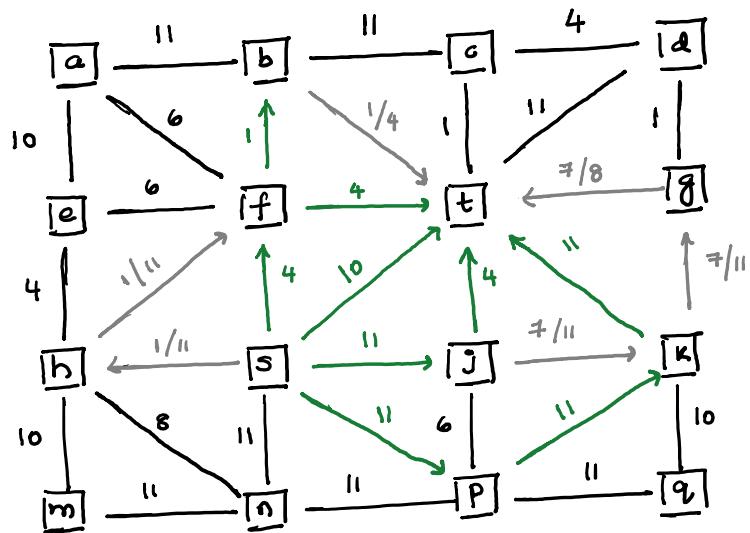
MAX FLOW for $\min[u, 11]$

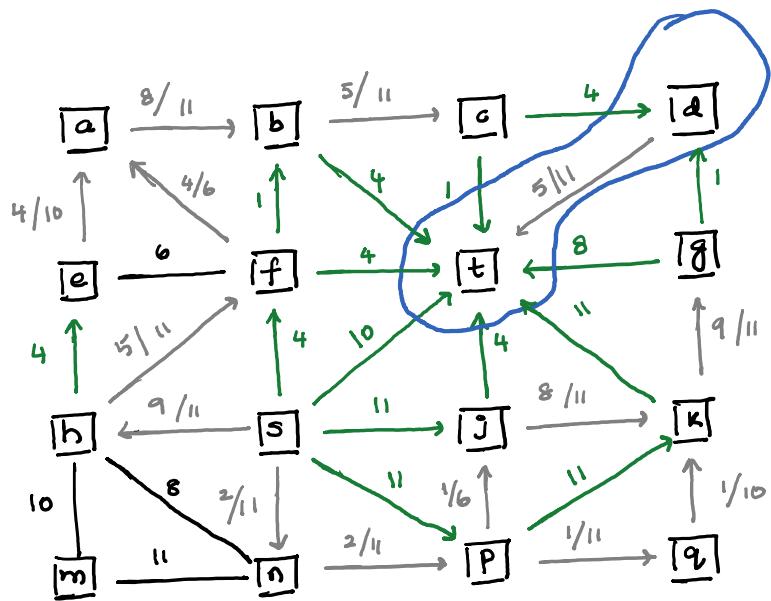
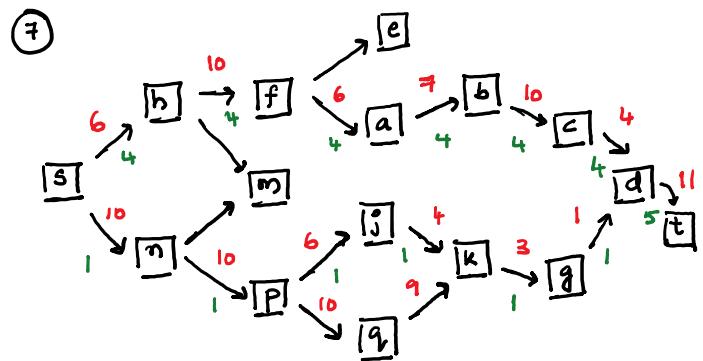
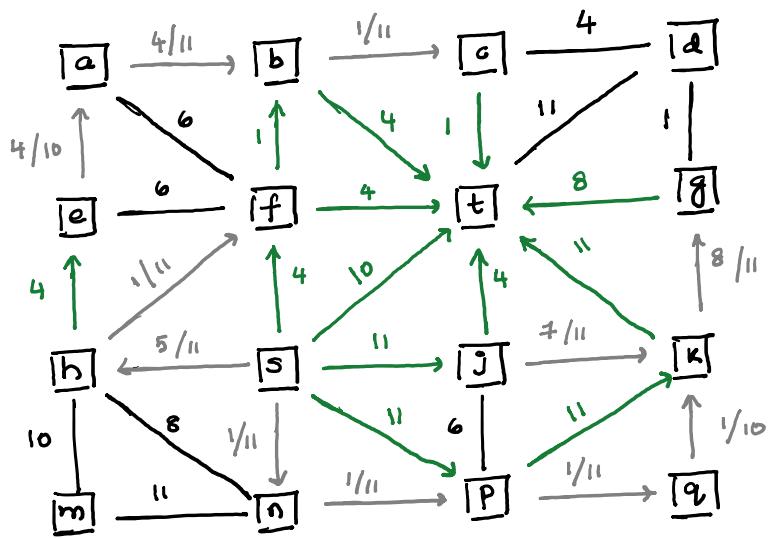


Consider $\min [u_{ij}, 11]$









$$y = x + b \Rightarrow @ z=11, \text{Flow: } 47$$

$$\Rightarrow \boxed{b = 36}$$

Points of intersection :

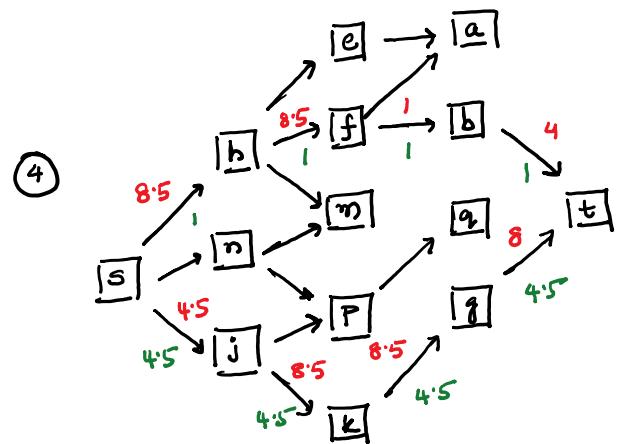
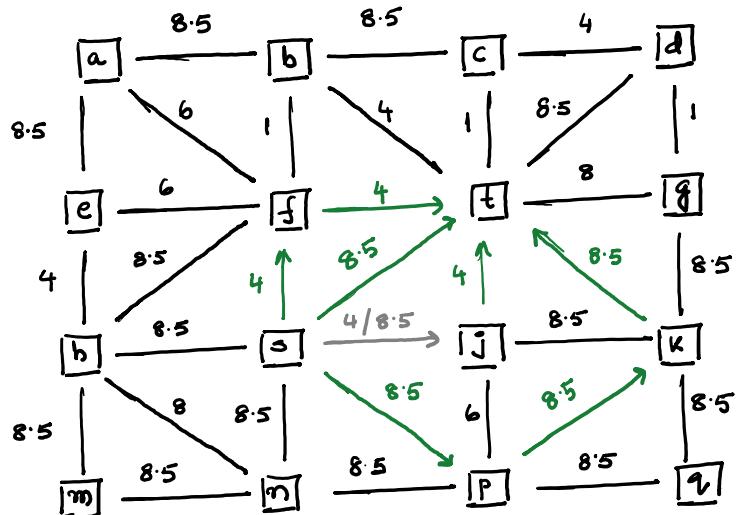
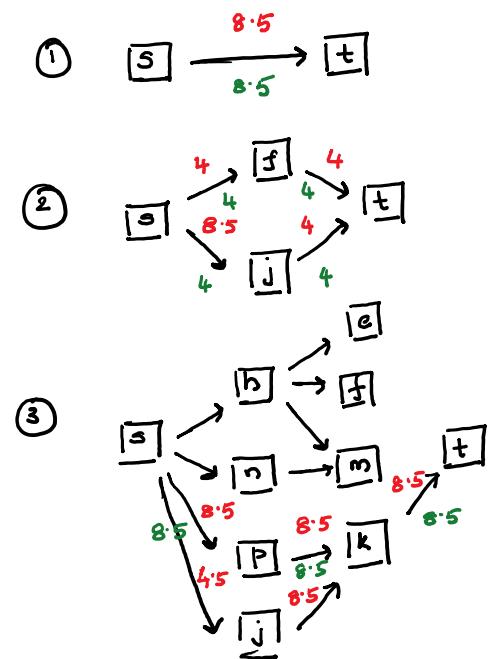
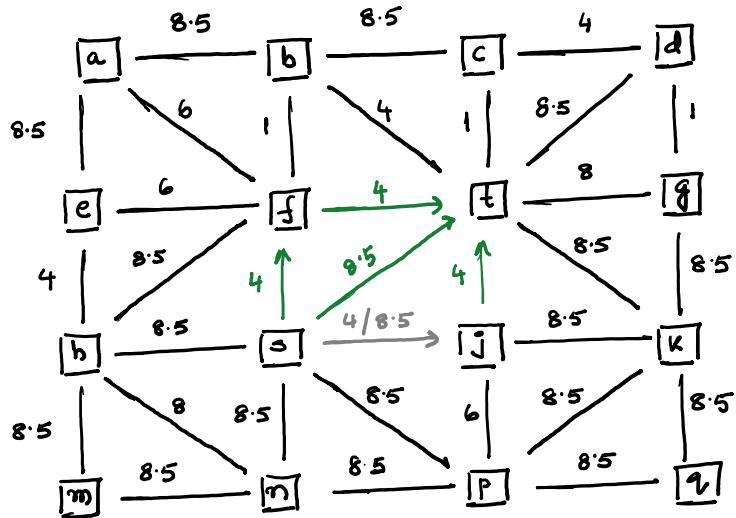
No. of edges with $\lambda = 11$: (k, t)

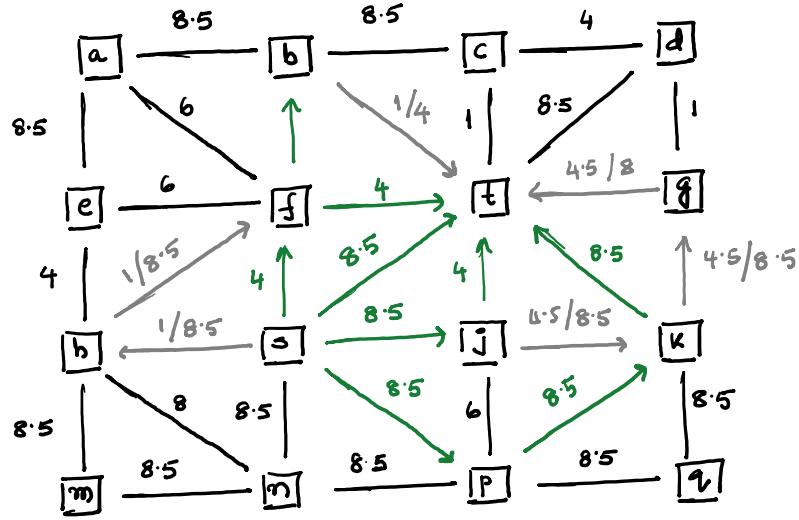
Flow : 47

$$\textcircled{1} \underset{\lambda=12}{\Leftrightarrow} F(\infty)$$

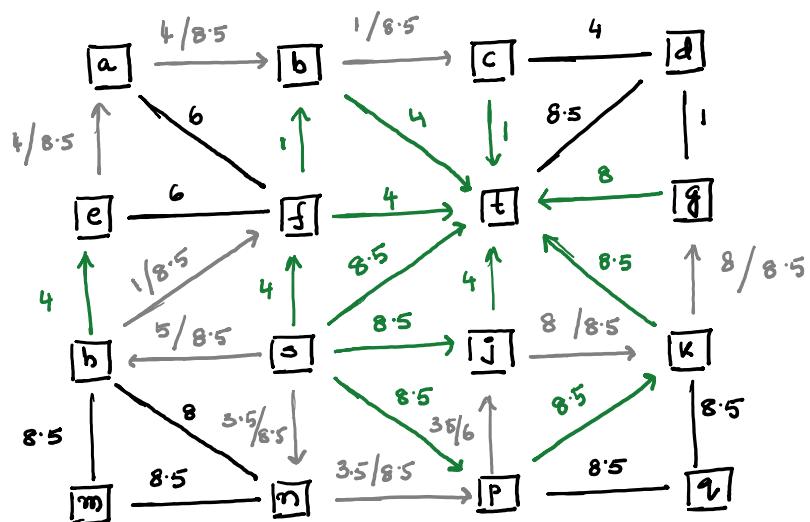
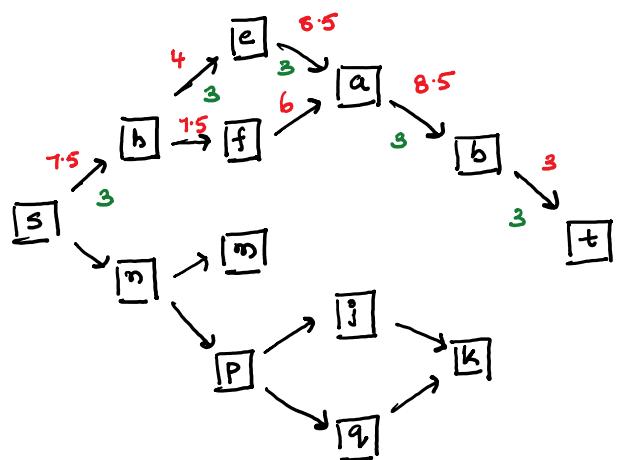
$$2x+26 = x+36 \Rightarrow \boxed{x=10}$$

MAX FLOW for min[u, 8.5]

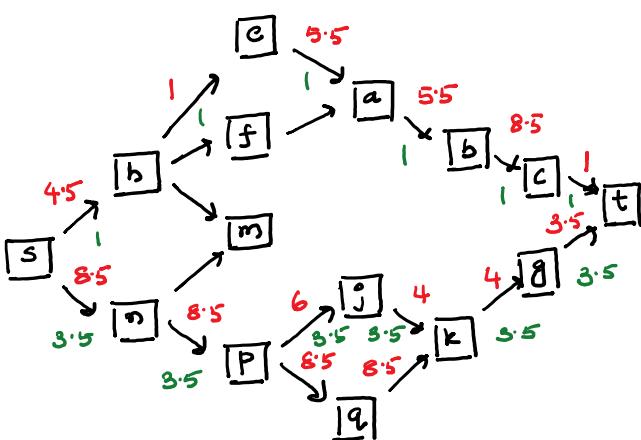


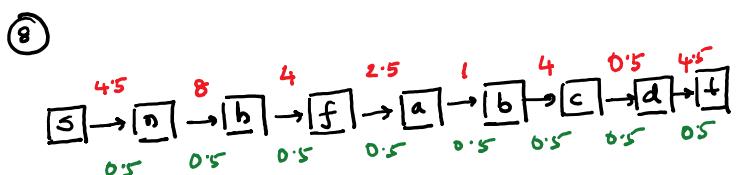
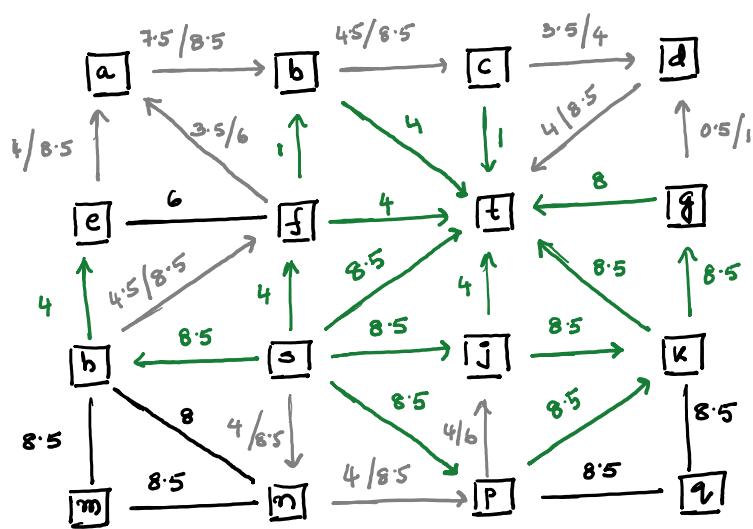
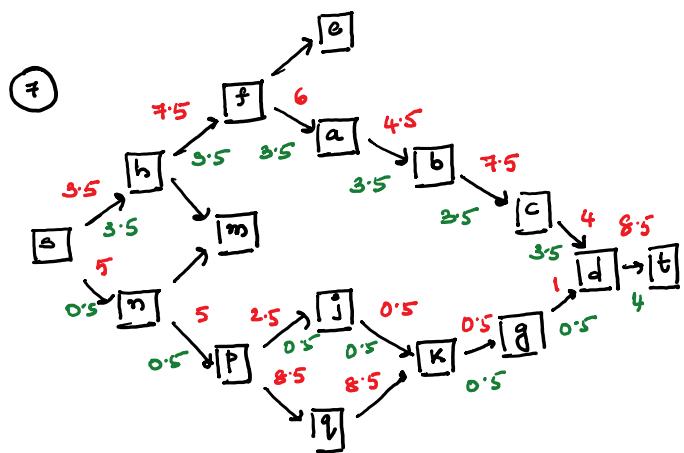
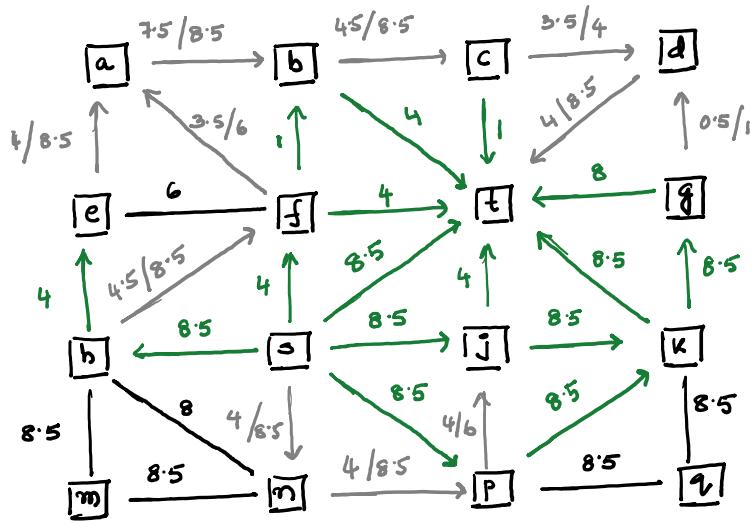


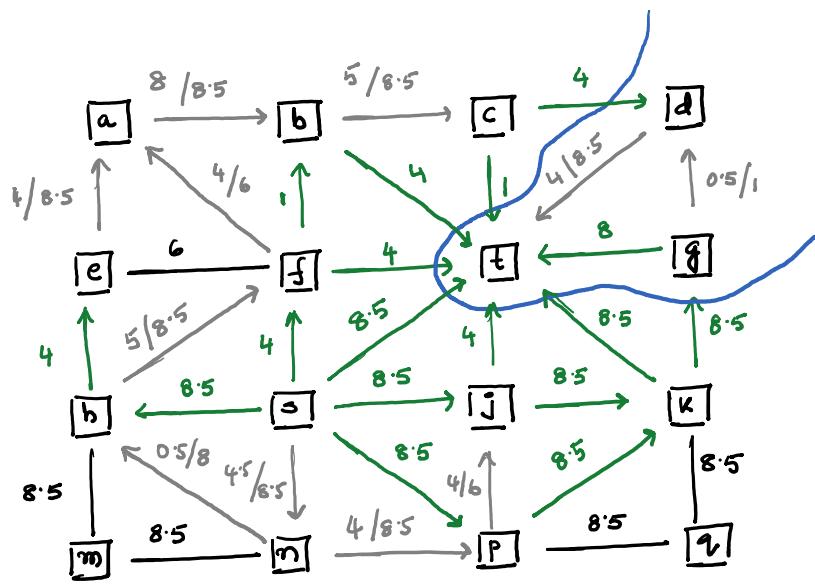
⑤



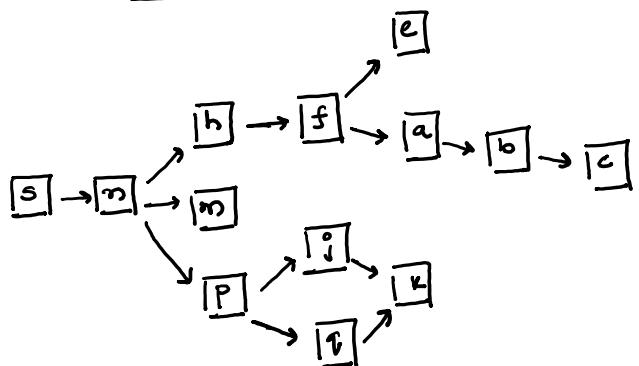
⑥







Min-cut :



No. of edges with $\lambda = 8.5$

$$(s,t), (k,t), (k,g) \Rightarrow 3$$

$$\text{Flow: } 42.5 \quad | \quad 0.5 + 4 + 3.5 + 1 + 3 + 4.5 + 1 + 8.5 \\ + 4 + 4 + 8.5 = 42.5$$

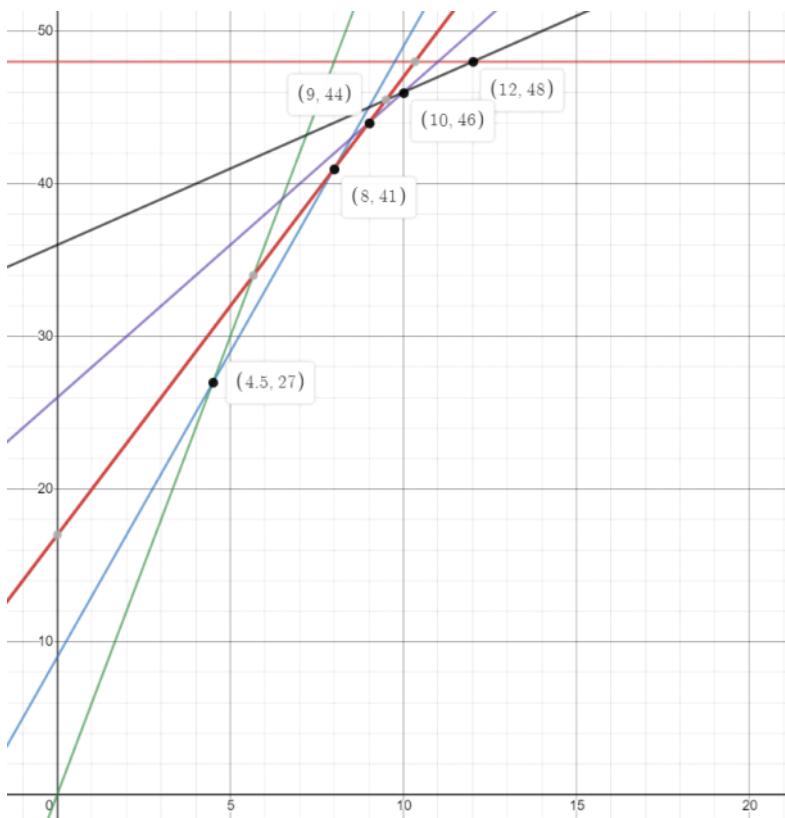
$$y = 3x + b \quad @ \quad x = 8.5 \quad \text{Flow} = 42.5$$

$$\Rightarrow \boxed{b = 17}$$

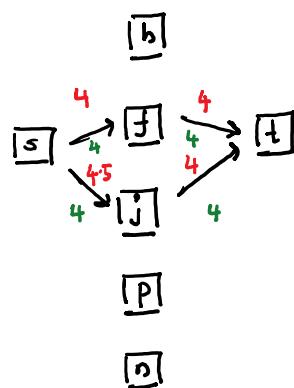
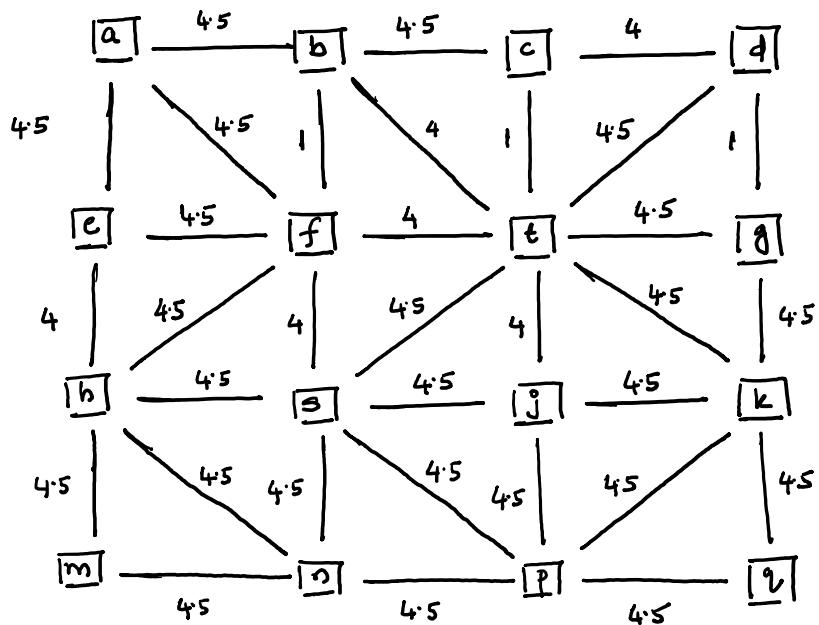
Points of intersection :

$$2x + 2b = 3x + 17 \Rightarrow \boxed{x = 9}$$

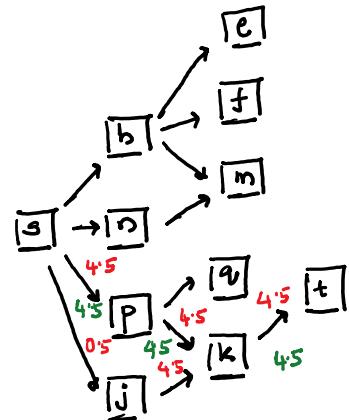
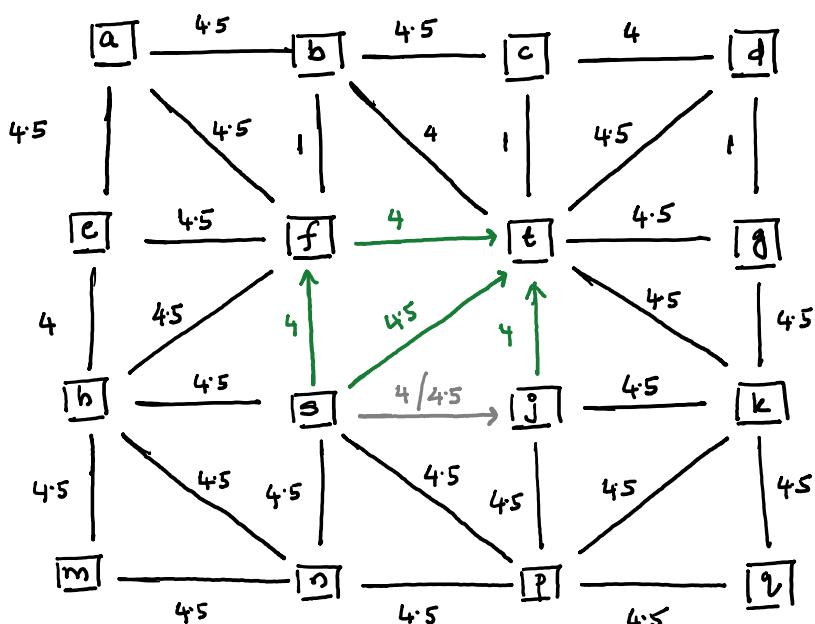
$$4x + 9 = 3x + 17 \Rightarrow \boxed{x = 8}$$



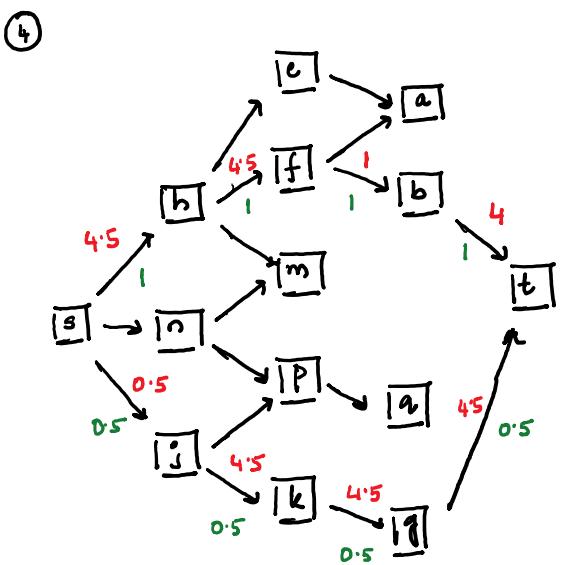
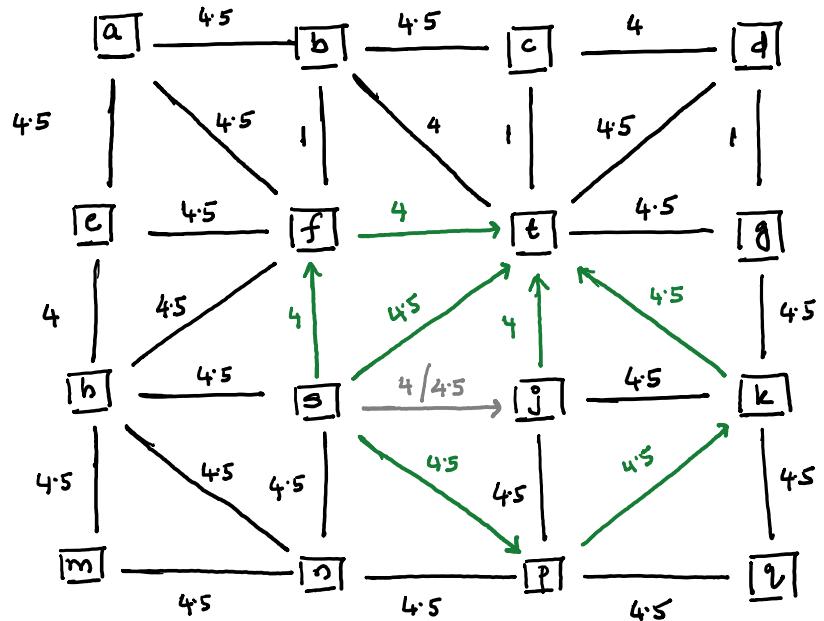
MAX FLOW for min[u, 4.5]



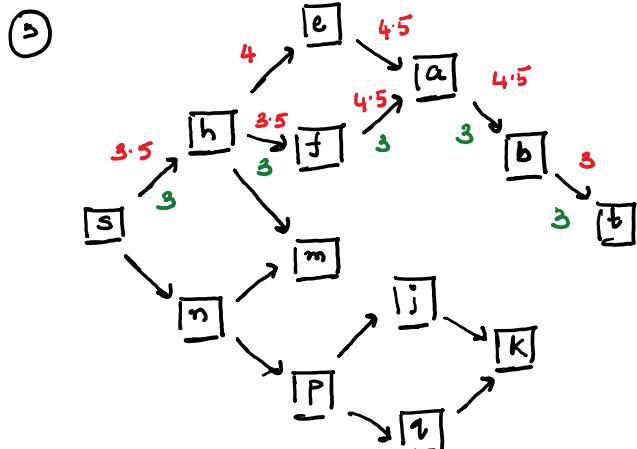
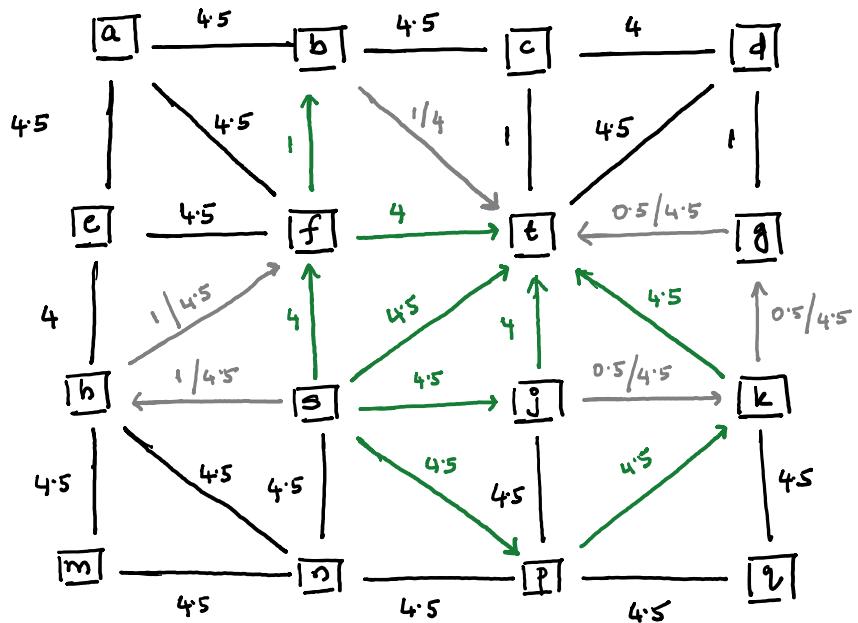
$$\text{Flow: } 4.5 + 8 = \underline{12.5}$$

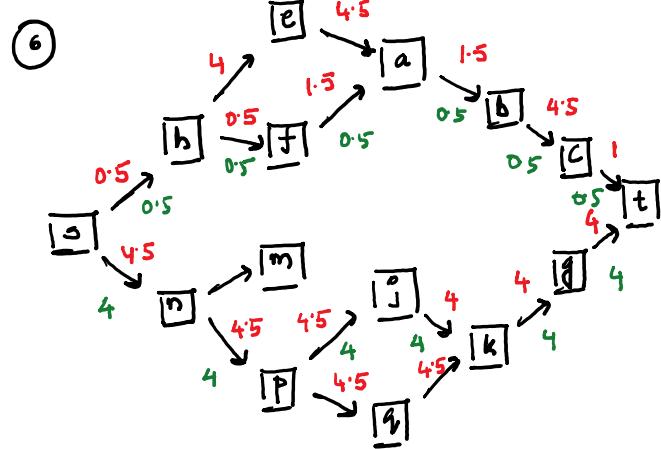
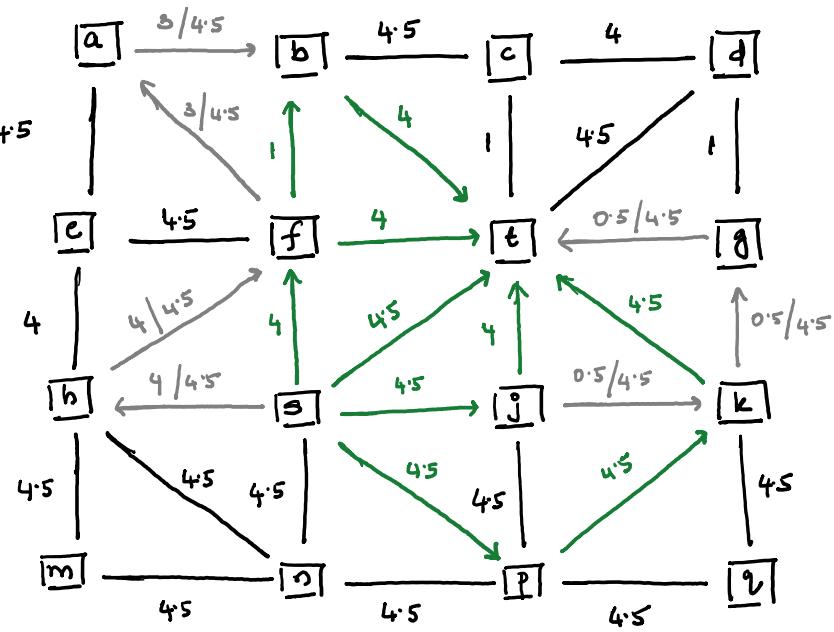


$$\text{Flow: } 12.5 + 4.5 = \underline{17}$$

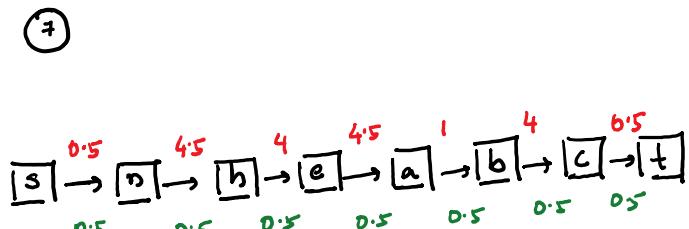
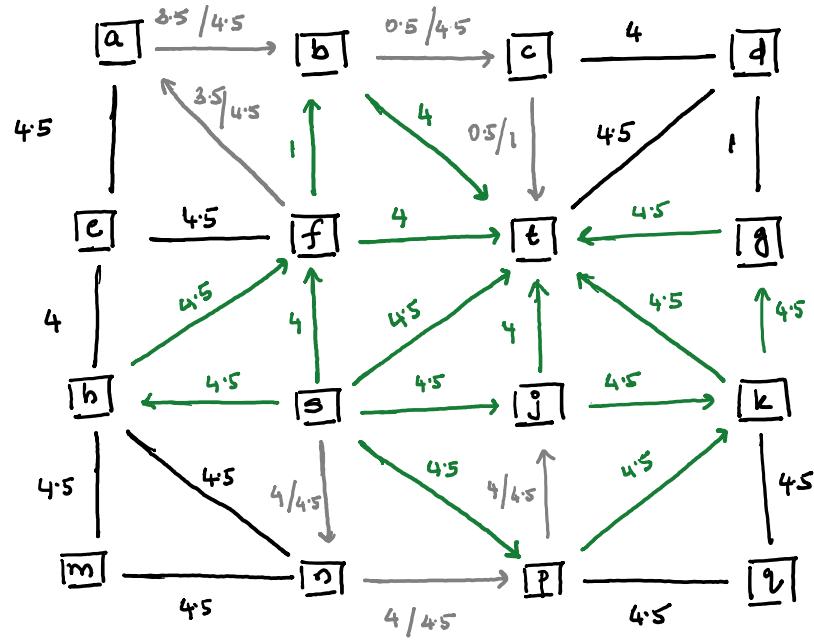


$$\text{Flow: } 17 + 1.5 = \underline{18.5}$$

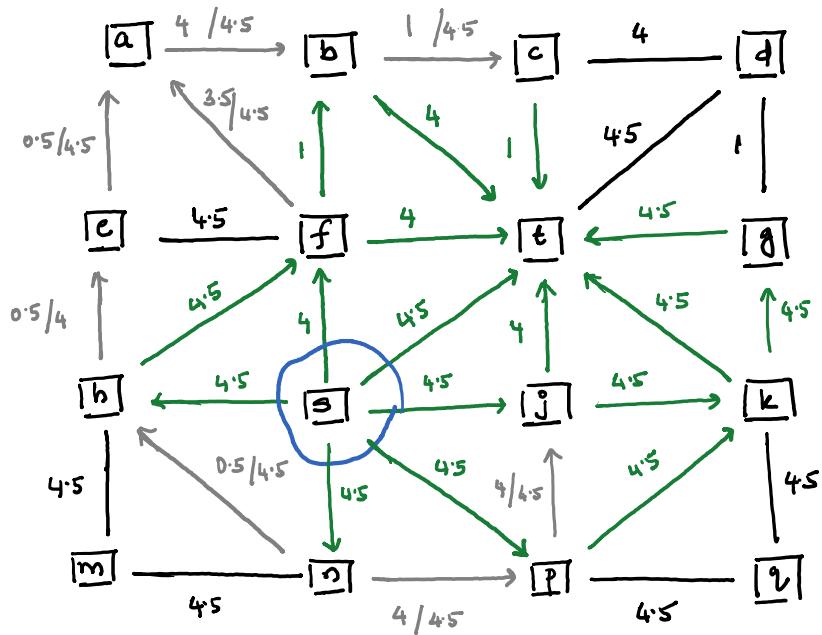




$$\underline{\text{Floor:}} \quad 21.5 + 4 + 0.5 = \underline{26}$$



Flow : 26.5



Total flow: 26.5

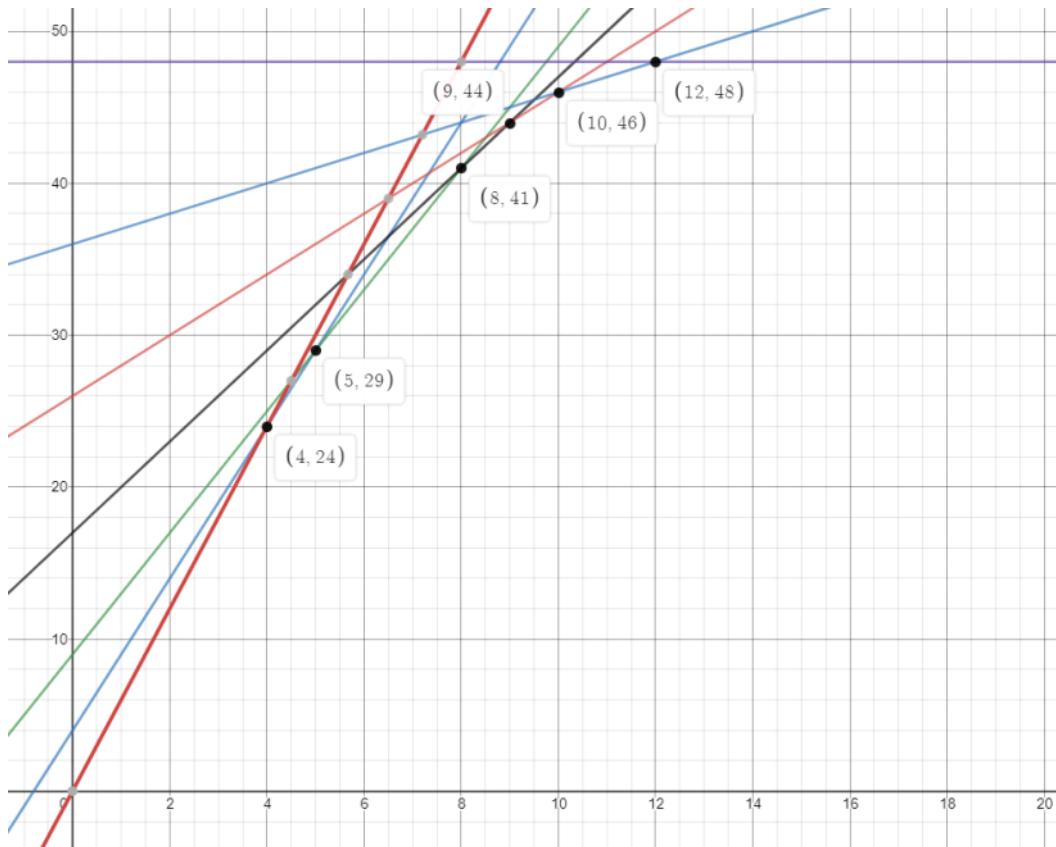
edges with $\lambda = 4.5$ (5)

Finding points of intersection

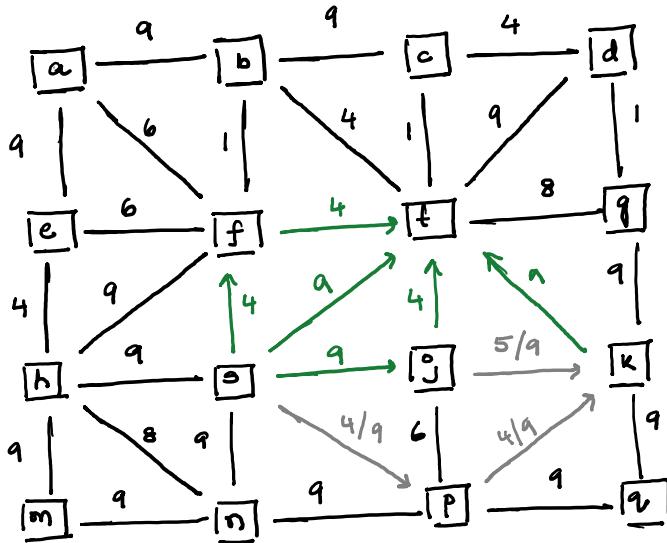
$$@ x=4.5, y = 5x + b \Rightarrow y = 26.5$$

$$\Rightarrow 6x = 5x + 4 \Rightarrow \boxed{x=4}$$

$$\Rightarrow b = 26.5 - 5(4.5) = \boxed{4} \Rightarrow 5x + 4 = 4x + 9 \Rightarrow \boxed{x=5}$$



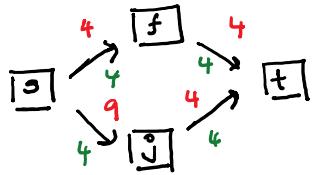
MAX FLOW for min[u, 9]



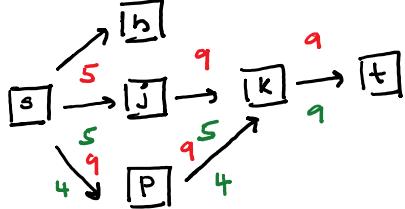
①



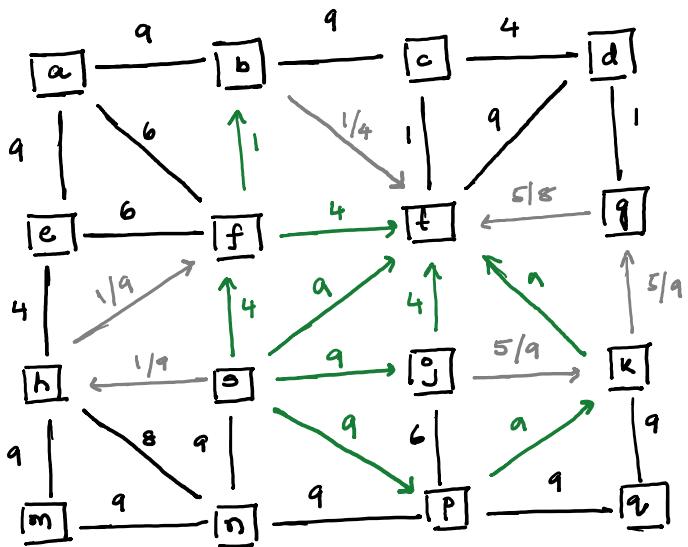
②



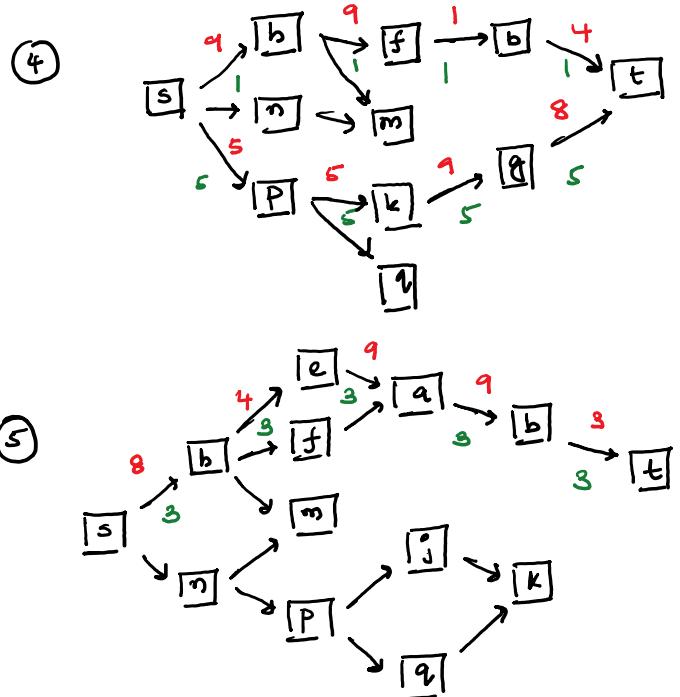
③

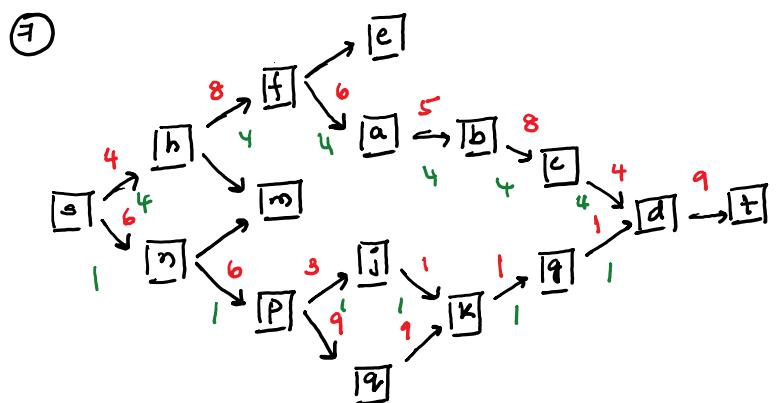
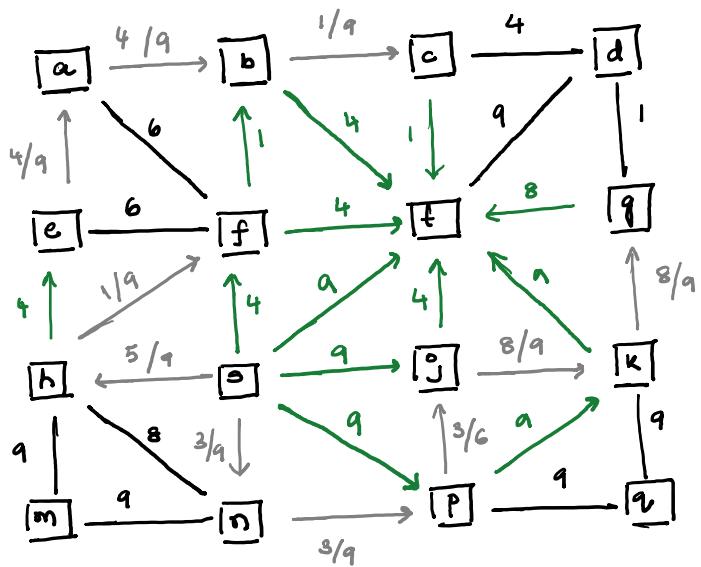
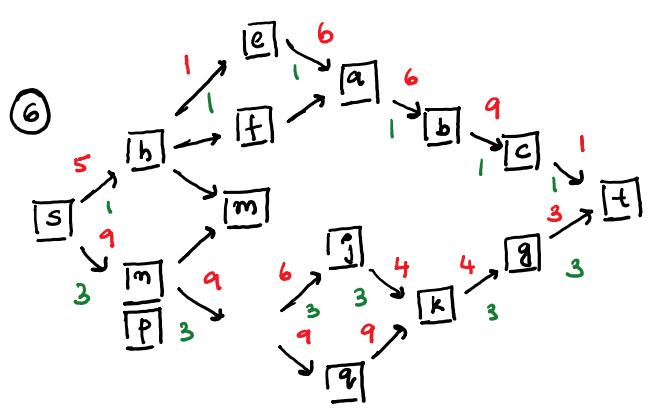
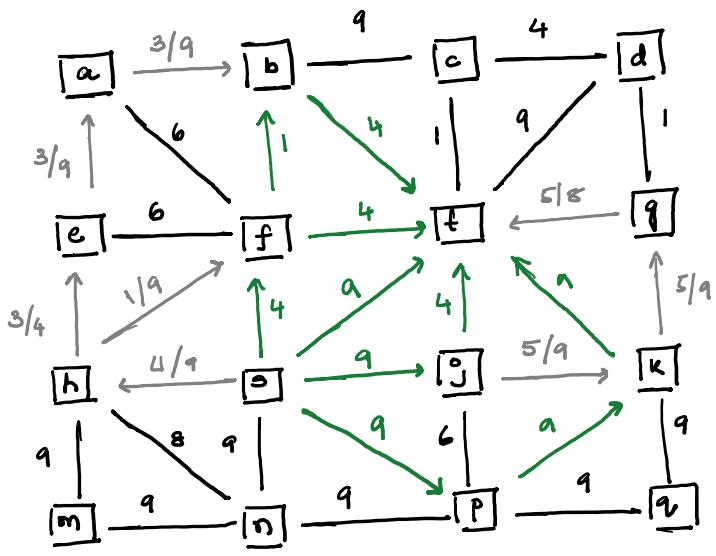


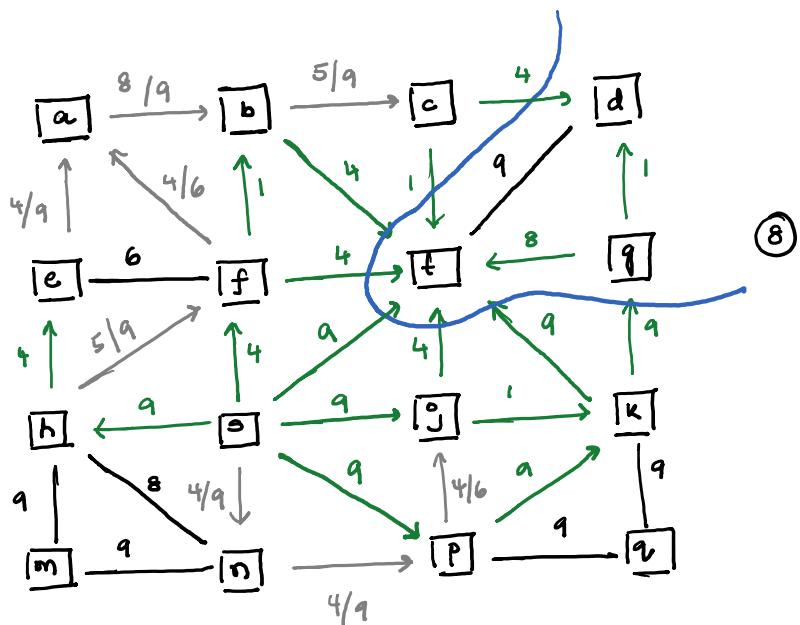
④



⑤







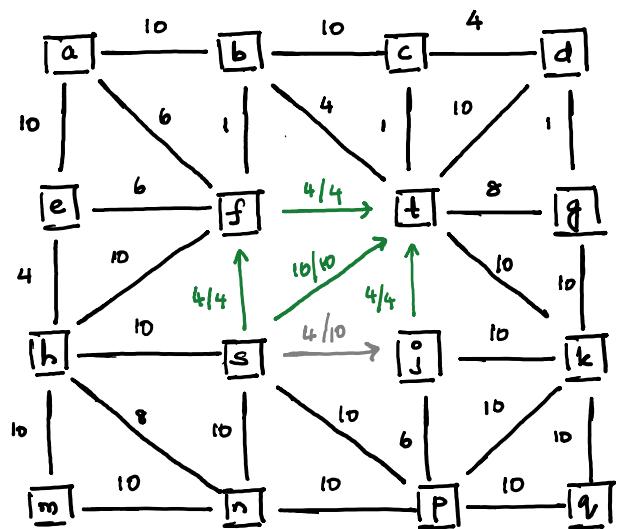
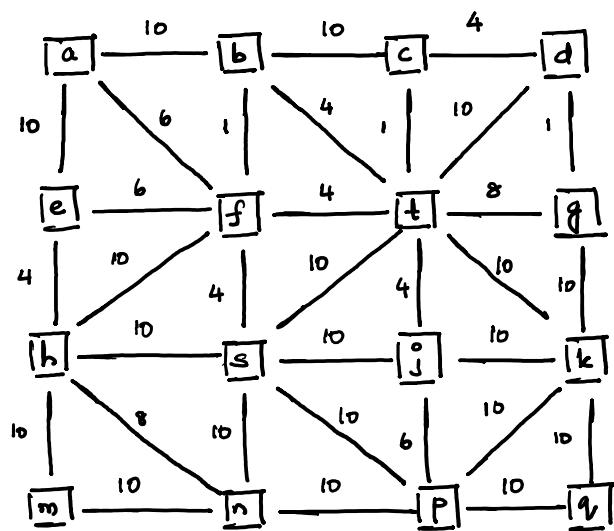
Current flow @ $\lambda = 9$

in graph \Rightarrow 44

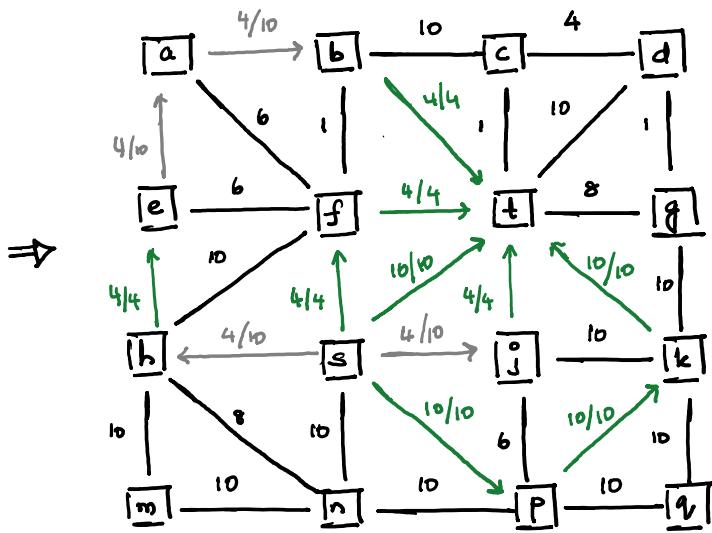
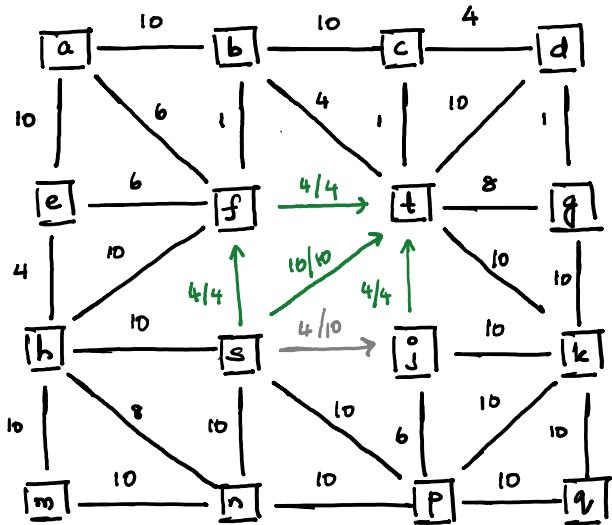
Flow : 44

MAX FLOW for min[u, 10]

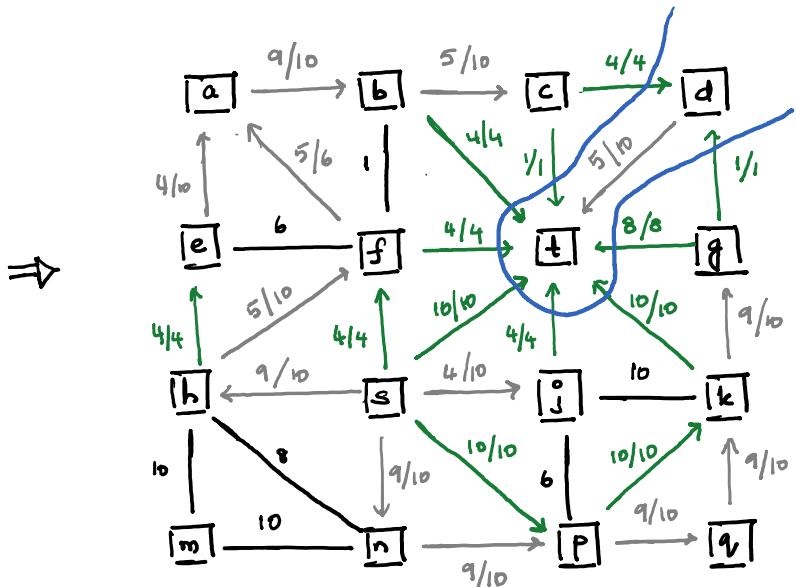
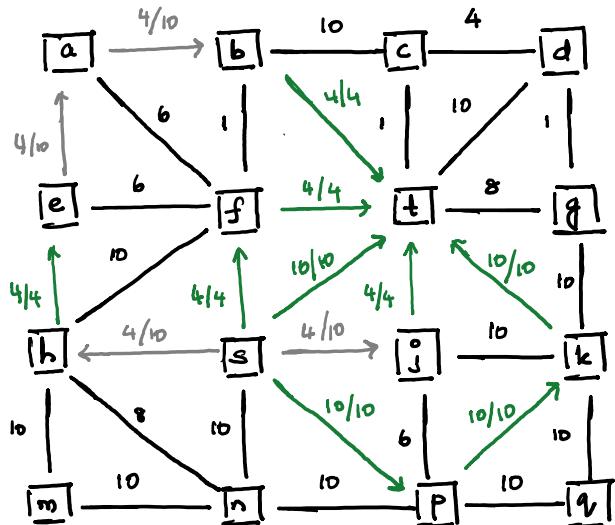
Consider graph with $\min [u_{ij}, 10]$



$$[10 + 4 + 4]$$



$[10 + 4]$



$[9 + 5]$

Max flow: $18 + 14 + 14 = 46$.

Min cut: $\{(s, j, n, b, p, q, k, g, f, c, a, b, c), (d, t)\}$

No. of edges in min-cut with flow = 10: $\boxed{2}$

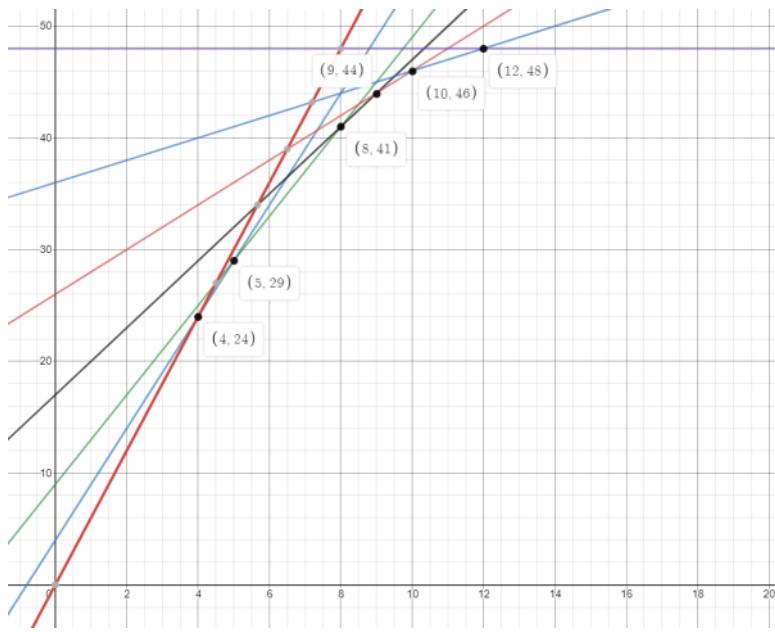
As the slope jump for piece wise linear functions for lambda values of 4, 5, 12, 10 is 1.
It is implicit that there shall be no change in curve.

But we still verify the max flow associated with the following lambda values :

Following are the flow values obtained in each individual case:

1. Lambda = 4 --> Max flow : 24 ---> Min cut : $(s, V \setminus \{s\})$ --> Edges {6}
2. Lambda = 5 --> Max flow : 29 ---> Min-cut : $(s, V \setminus \{s\})$ --> Edges : {5}
3. Lambda = 12 --> Max flow : 48 --> Min cut : $(V \setminus \{d,t\}, \{d,t\})$

It can be observed that there is no change in the values of the lambda curve.

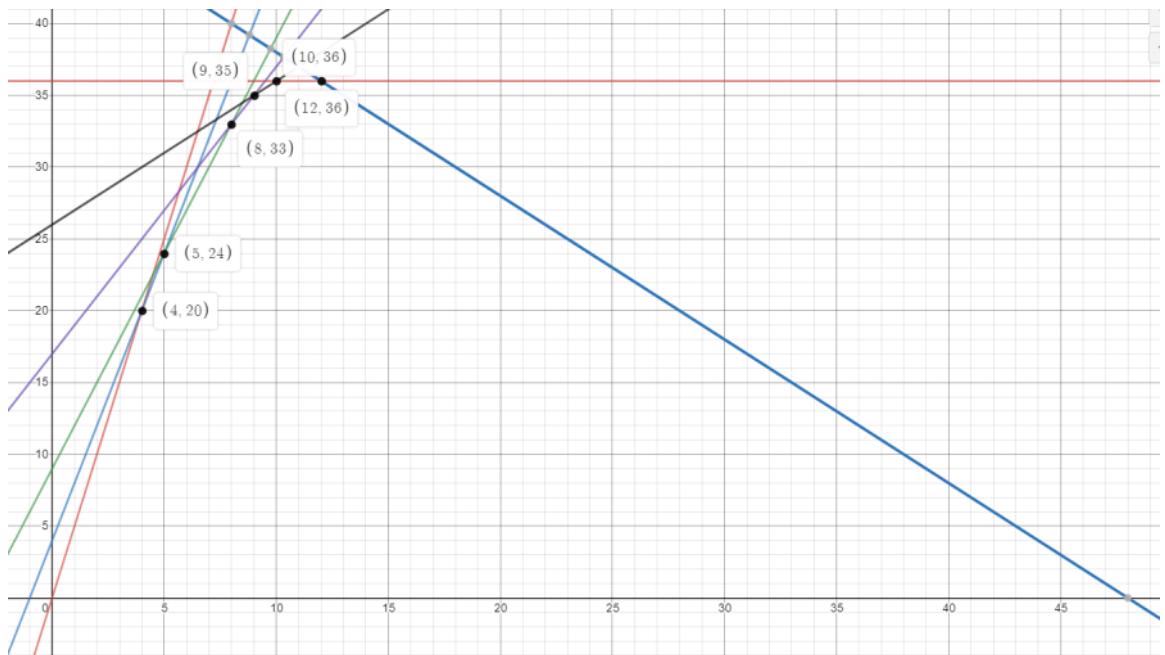


Piece wise linear function ($F(\lambda)$) :

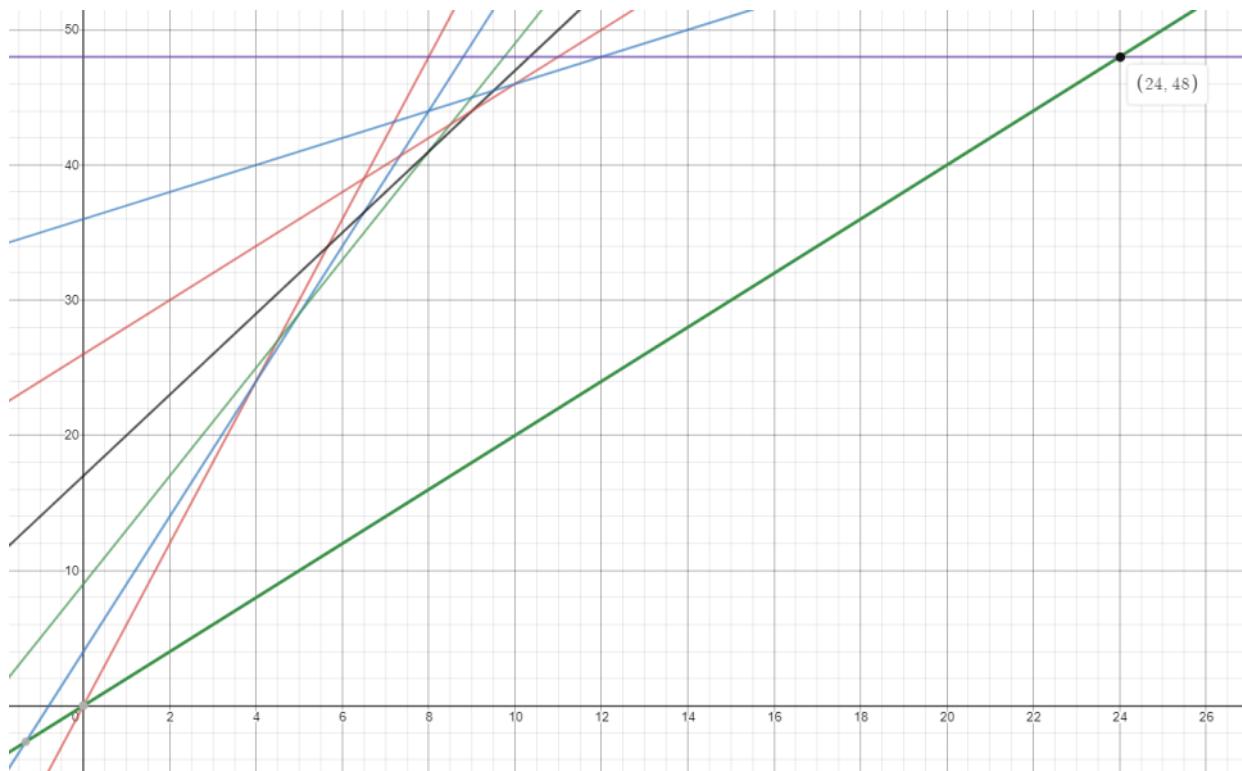
- ① 6λ
- ② $5\lambda + 4$
- ③ $4\lambda + 9$
- ④ $3\lambda + 17$
- ⑤ $2\lambda + 26$
- ⑥ $\lambda + 36$
- ⑦ 48

To find maximum surviving flow: $F(\lambda) - \lambda$ [curve] \rightarrow Equations shall be

- ① 5λ
- ② $4\lambda + 4$
- ③ $3\lambda + 9$
- ④ $2\lambda + 17$
- ⑤ $\lambda + 26$
- ⑥ 36
- ⑦ $48 - \lambda$

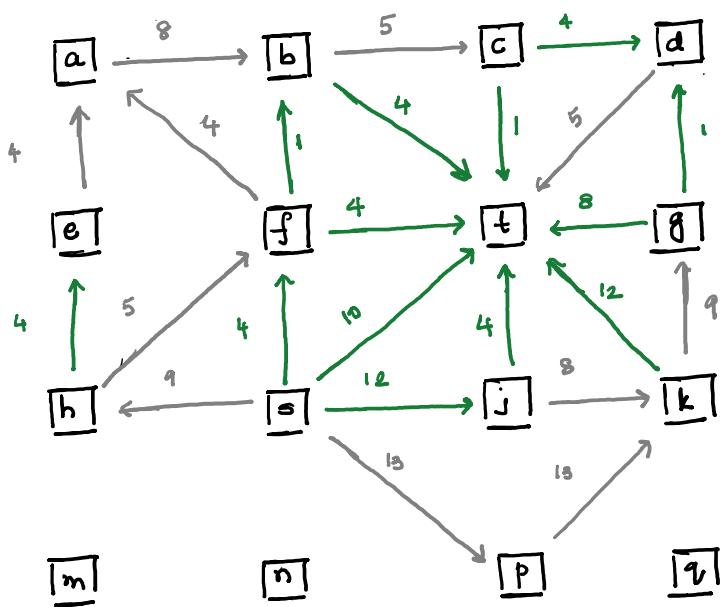


For maximum q -path flows $\underline{q=2}$



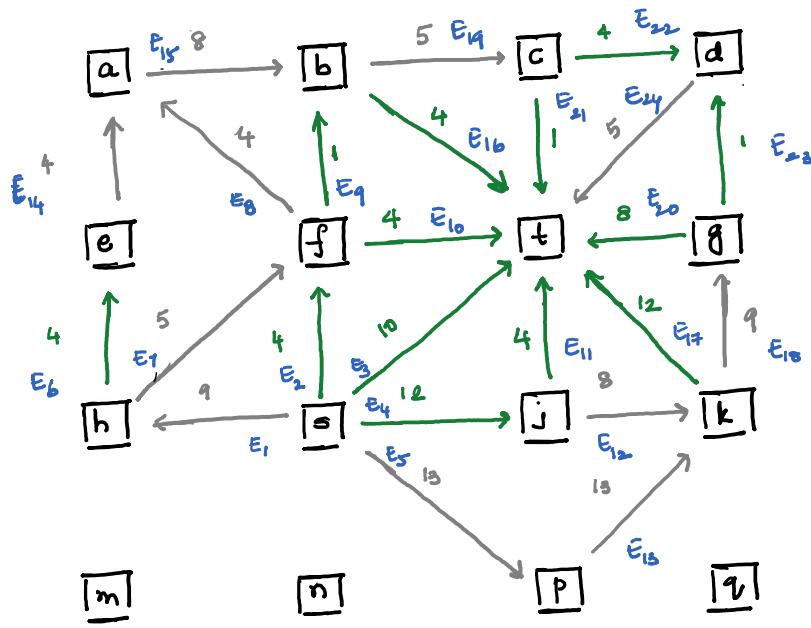
$$\lambda^* = 24 \rightarrow \text{Consider } \min [w_{ij}, 24]$$

Flow with this \leftarrow Equivalent to flow with $\underline{\lambda=\infty}$ (As all edge capacities ≤ 24)



Considering only
positive flows

To find the q -path decomposition. We use Kishimoto's work



Satisfies

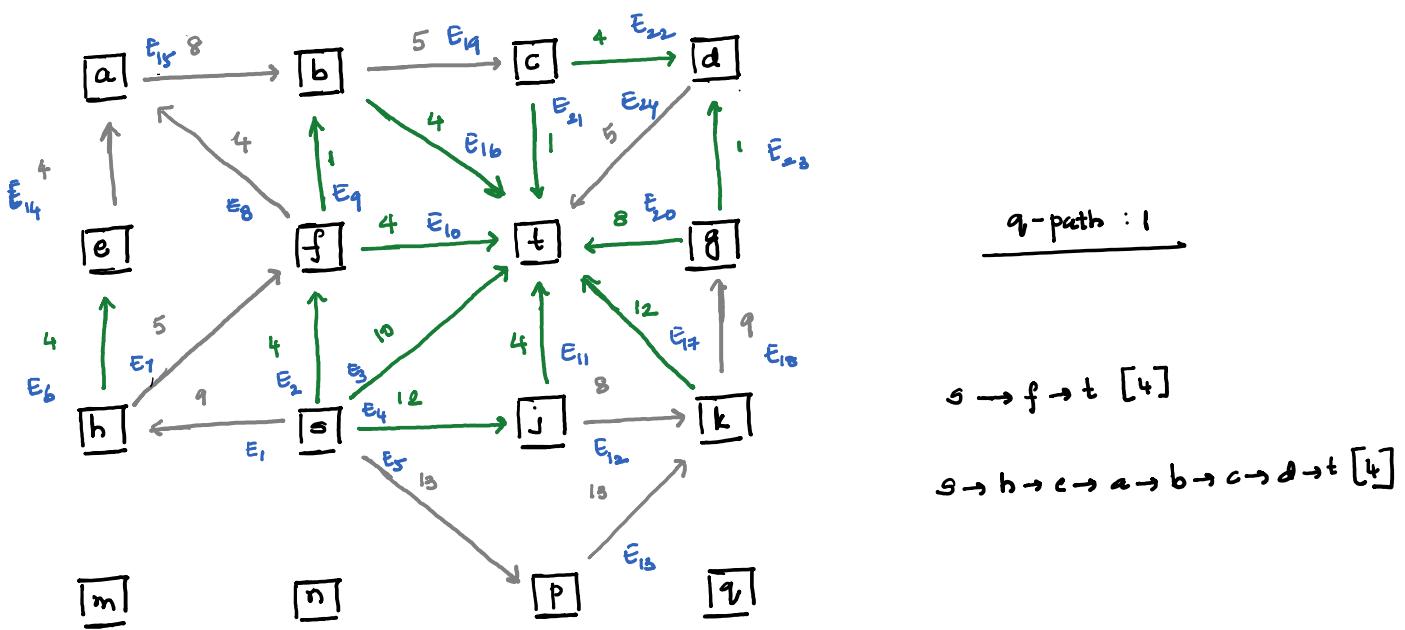
$$\forall (i, j) \in E \quad 0 \leq f_{i,j} \leq u_{i,j}$$

$$f_{i,j} \leq F/2 \leq 24$$

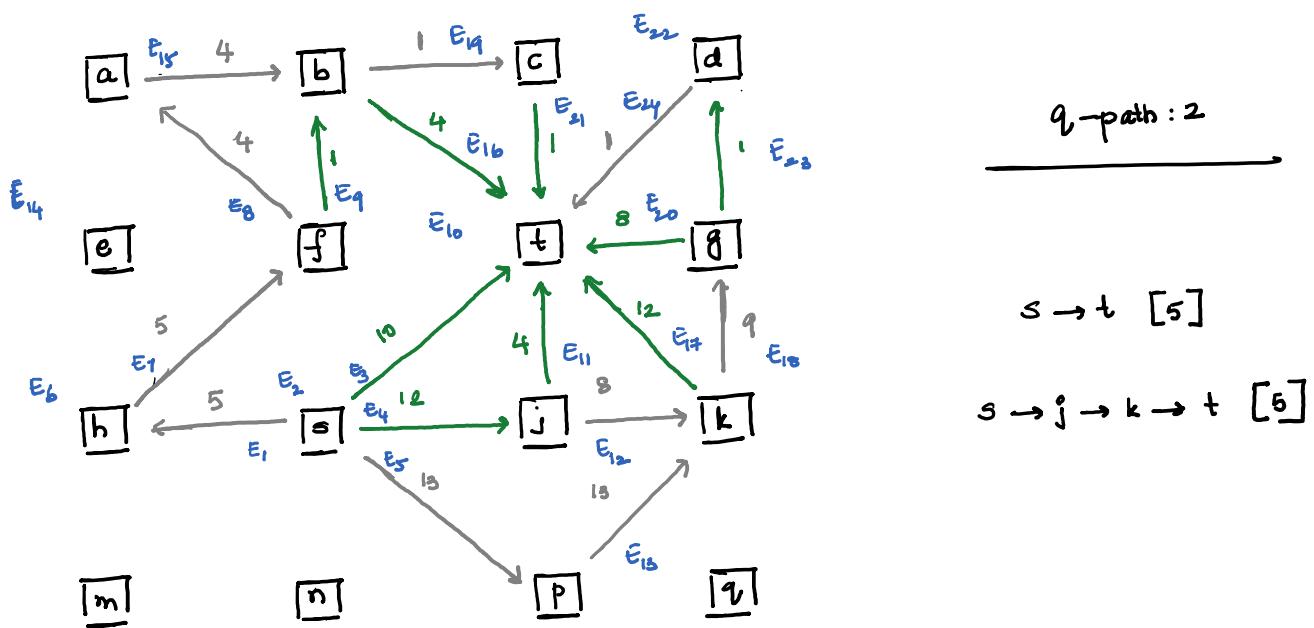
It has 24 edges that carry positive flows and $\underline{q}=2$

So, we create 26×26 table with 24 rows corresponding to edges

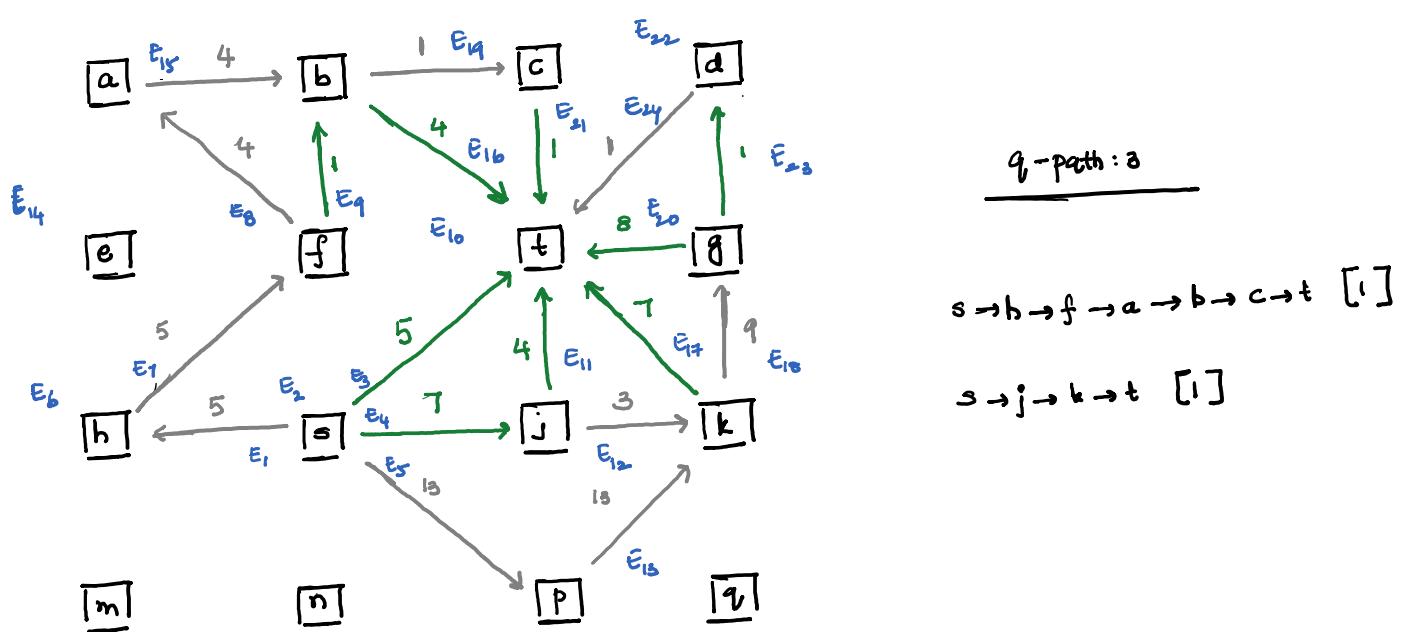
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2	a1	9		5	6	4																						24	
3	a2		4	5	6	9																						24	
4	U1	15					4	5																				24	
5	U2	20									4																	24	
6	U3	14			12							4	8													10		24	
7	U4			11										13														24	
8	U5					20								4														24	
9	U6														20			4										24	
10	U7						19	4	1																			24	
11	U8						20											4										24	
12	U9						23												1									24	
13	U10										20																4	24	
14	U11										20																4	24	
15	U12											16								8								24	
16	U13											11								4	9							24	
17	U14												20	4														24	
18	U15												16	3														24	
19	U16													20													4	24	
20	U17														12												12	24	
21	U18														15			8										24	
22	U19															19	1	4										24	
23	U20																16										8	24	
24	U21																	23									1	24	
25	U22																		20									24	
26	U23																		23	1								24	
27	U24																			19	1	4						24	
28		24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24		
29																													



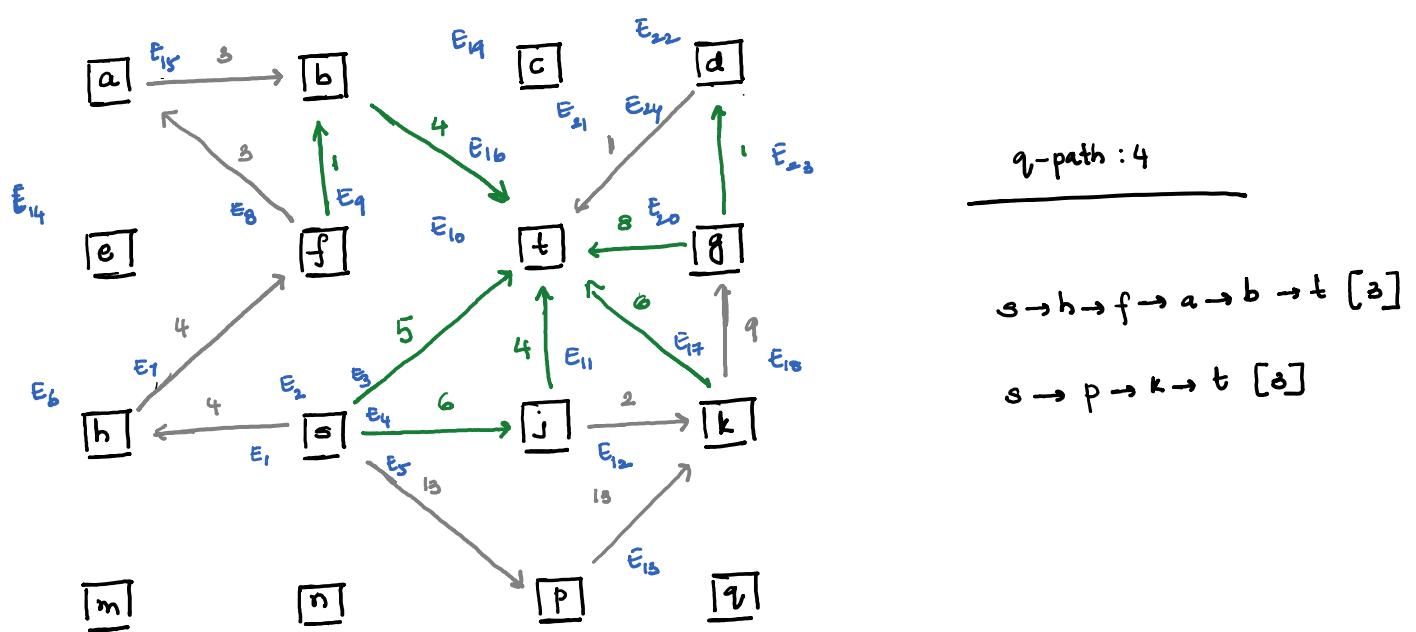
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2	a1	5		5	6	4																					20	20					
3	a2			5	6	9																					20	20					
4	U1	15						5																			20	20					
5	U2		20																								20	20					
6	U3			10																							10	20					
7	U4				8														4	8							20	20					
8	U5					7														13							20	20					
9	U6						20																				20	20					
10	U7							15	4	1																	20	20					
11	U8							16													4						20	20					
12	U9								19													1					20	20					
13	U10									20																	20	20					
14	U11										16																4	20					
15	U12											12															20	20					
16	U13												7									4	9				20	20					
17	U14													20													20	20					
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22	U19																	19		1									20				
23	U20																		12										8	20			
24	U21																		19										1	20			
25	U22																			20										20			
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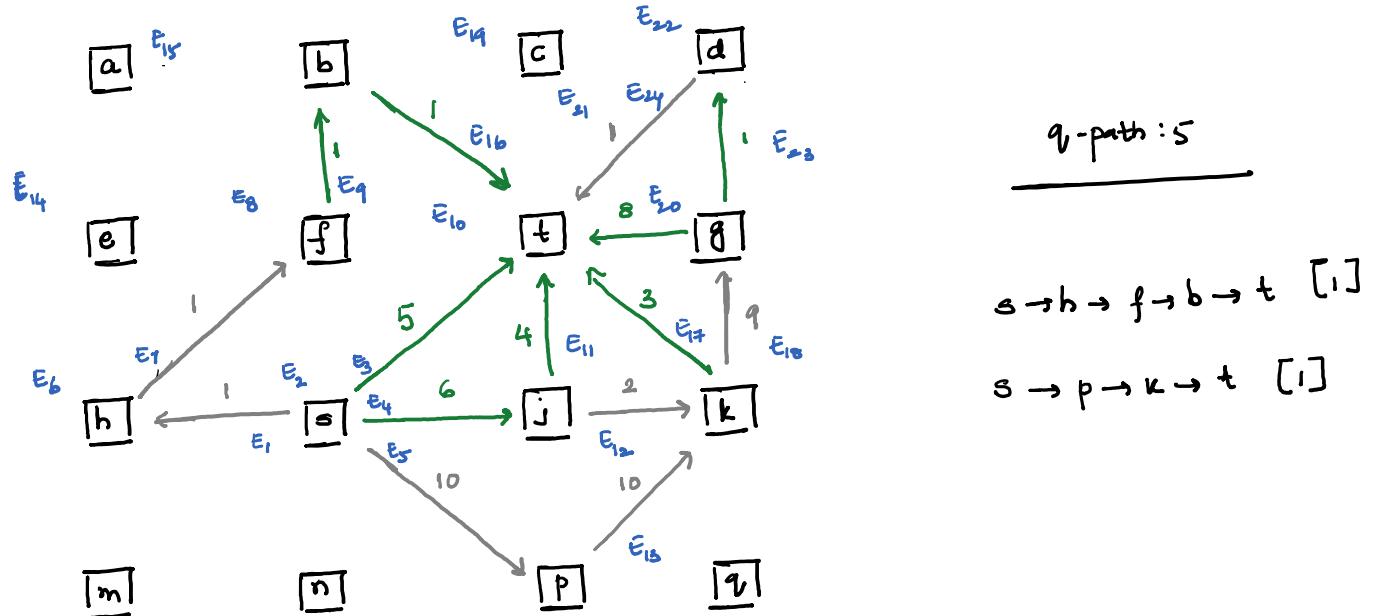
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3	a2		5	1	9																							15	
4	U1	10						5																				15	
5	U2	15																										15	
6	U3		10																								5	15	
7	U4		8										4	3														15	
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10	U7				10	4	1																					15	
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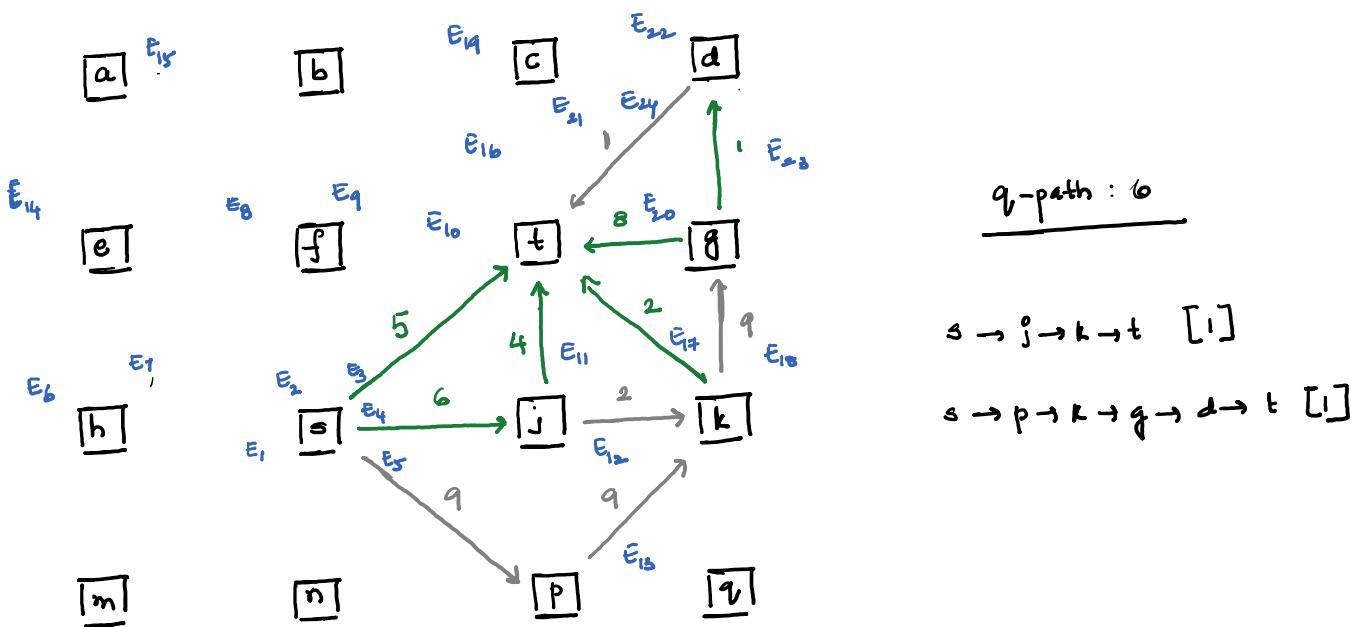
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2	a1	4			6	4																					14		
3	a2		5		9																						14		
4	U1	10						4																			14		
5	U2		14																								14		
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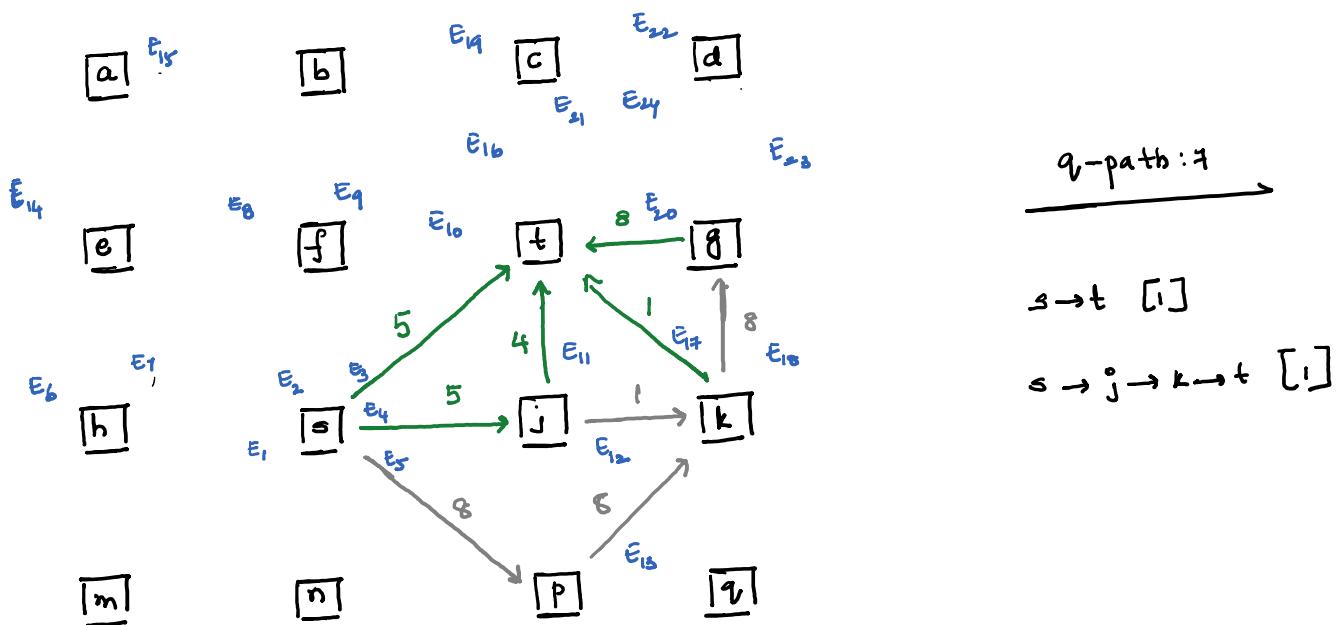
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4	U1	10							1																			11	
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6	U3	6																										5	
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8	U5		1									10															11		
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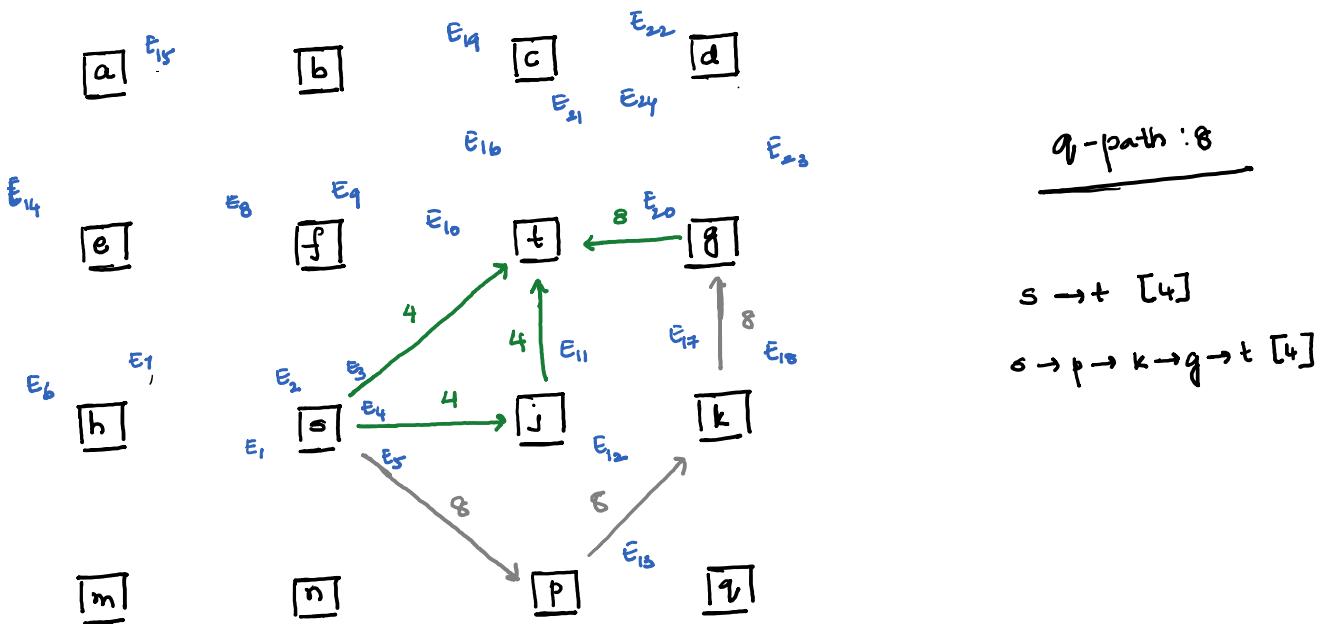
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4	U1	10																										10	
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29																													



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	
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2	a1				5	4																					9			
3	a2			5		4																					9			
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5	U2		9																									9		
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29																														



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC
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3	a2				4		4																					8	
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29																													



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC
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2	a1					4																						4	
3	a2						4																					4	
4	U1	4																										4	
5	U2		4																									4	
6	U3			4																								4	
7	U4				4												4											4	
8	U5					4												4										4	
9	U6						4												4									4	
10	U7							4																				4	
11	U8								4																			4	
12	U9									4																		4	
13	U10										4																	4	
14	U11											4																4	
15	U12												4															4	
16	U13													4														4	
17	U14														4													4	
18	U15															4												4	
19	U16																4											4	
20	U17																	4										4	
21	U18																		4									4	
22	U19																			4								4	
23	U20																				4							4	
24	U21																					4						4	
25	U22																						4					4	
26	U23																							4				4	
27	U24																								4			4	
28		4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
29																													

