PROBLEM-3 A

(5 points) Compute the probability that: a1 = 1, a2 = 1, a3 = 1, b1 = 0, b2 = 0, b3 = 0, b4 = 0, b5 = 0

Based on bayes net structure we have.

$$P(a_1, a_2, a_3, b_1, b_2, b_3, b_4, b_5)$$

$$= P(a_1)\, P(a_2)\, P(b_1 \mid a_1, a_2)\, P(b_2 \mid b_1)\, P(a_3)\, P(b_3 \mid a_3, b_2)\, P(b_4 \mid b_3)\, P(b_5 \mid b_4)$$

$$\Rightarrow P(a_1 = 1, a_2 = 1, a_3 = 1, b_1 = 0, b_2 = 0, b_3 = 0, b_4 = 0, b_5 = 0)$$

$$= P(a_1 = 1)\, P(a_2 = 1)\, P(b_1 = 0 \mid a_1 = 1, a_2 = 1)\, P(b_2 = 0 \mid b_1 = 0)\, P(a_3 = 1)\, P(b_3 = 0 \mid a_3 = 1, b_2 = 0)$$

$$P(b_4 = 0 \mid b_3 = 0)\, P(b_5 = 0 \mid b_4 = 0)$$

$$= (0.7)(0.8)\, [1 - (0.9)]\, [1 - (0.6)]\, (0.9)\, [1 - (0.8)]\, [1 - (0.1)]\, [1 - 0]$$

$$= (0.7)(0.8)(0.1)(0.4)(0.9)(0.2)(0.9)(1)$$

$$= \underline{0.0036288}$$

PROBLEM 3B

(15 points) Compute the probability that b5 = 1

Given bayes network is **a polytree.**

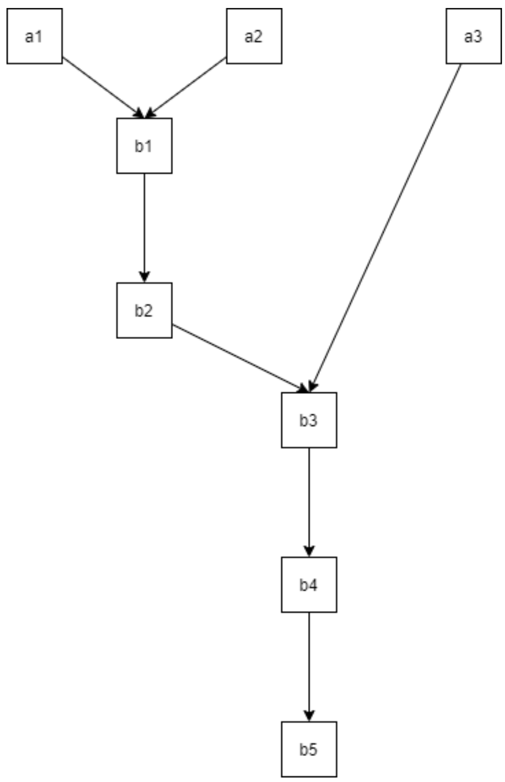
We can use the
1. Possible worlds approach
2. Variable elimination approach

Considering the possible worlds approach we find the probabilities :

a1	P(a1)
0	0.3
1	0.7

a2	P(a2)
0	0.2
1	0.8

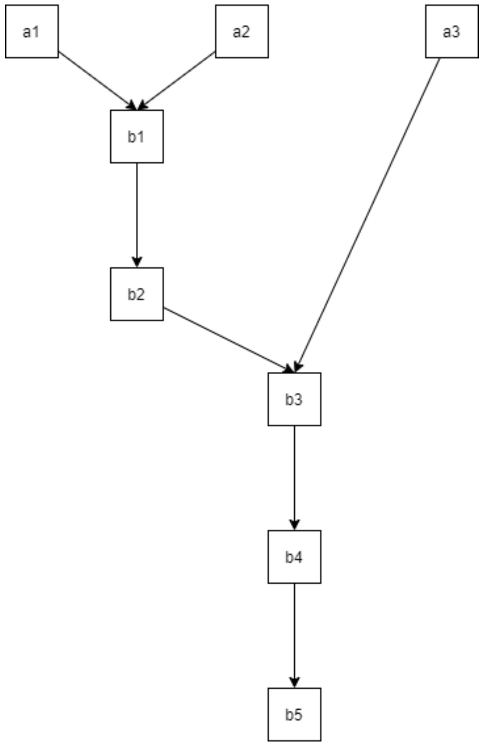
a3	P(a3)
0	0.1
1	0.9



a1	a2	P(b1)	P(W)
0	0	0.2	0.3*0.2 = 0.06
0	1	0.6	0.3*0.8 = 0.24
1	0	0.7	0.7*0.2 = 0.14
1	1	0.9	0.7*0.8 = 0.56

b1	P(b1)
0	1-0.758 = 0.242
1	0.2*0.06 + 0.6*0.24 + 0.7*0.14 + 0.9*0.56 = 0.758

b1	P(b1)
0	0.242
1	0.758



b1	P(b1)
0	0.242
1	0.758

b1	P(b2)	P(W)
0	0.6	0.242
1	0.8	0.758

b2	P(b2)
0	1-0.7516 = 0.2484
1	0.758*0.8 + 0.242*0.6 = 0.7516

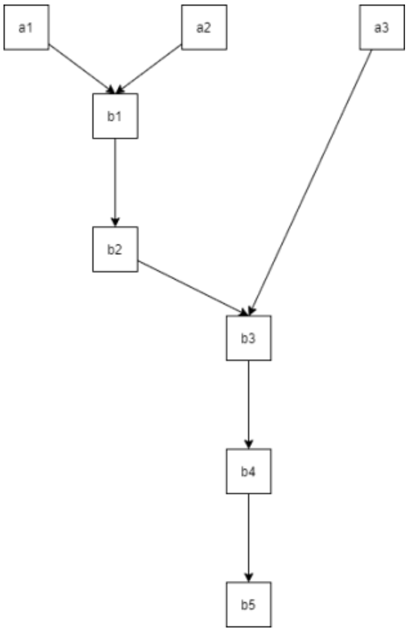
a3	P(a3)
0	0.1
1	0.9

b2	P(b2)
0	0.2484
1	0.7516

a3	b2	P(b3)	P(W)
0	0	0	0.1*0.2484 = 0.0248
0	1	0.7	0.1*0.7516 = 0.0752
1	0	0.8	0.9*0.2484 = 0.2236
1	1	1	0.9*0.7516 = 0.6764

b3	P(b3)
0	1-0.9079 = 0.0921
1	0.7*0.0752 + 0.8*0.2236 + 0.6764 = 0.9079

b3	P(b3)
0	0.0921
1	0.9079



b3	P(b3)
0	0.0921
1	0.9079

b3	P(b4)	P(W)
0	0.1	0.0921
1	0.7	0.9079

b4	P(b4)
0	1 - 0.6447 = 0.3553
1	0.7*0.9079 + 0.1*0.0921 = 0.6447

b4	P(b4)
0	0.3553
1	0.6447

b4	P(b5)	P(W)
0	0	0.3553
1	1	0.6447

b5	**P(b5)**
0	**0.3553**
1	**0.6447**

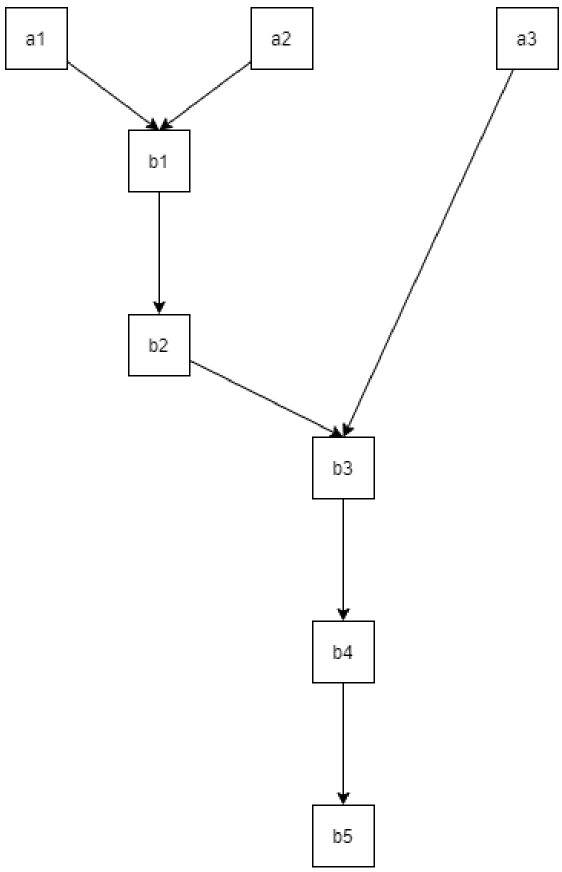
PROBLEM 3C

Probability that b5=1 given a1 = 1, a2 = 1, a3 = 1, b1=0, b2=0, b3=0

By the conditional independence

We can write P(b5=1| a1 = 1, a2 = 1, a3 = 1, b1=0, b2=0, b3=0) = P(b5= 1 | b3=0)

Now we need to find P(b5=1 | b3=0)



b3	P(b4)	P(W)
0	0.1	1
1	0.7	0

b4	P(b4)
0	1 - 0.1 = 0.9
1	0.1*1 = 0.1

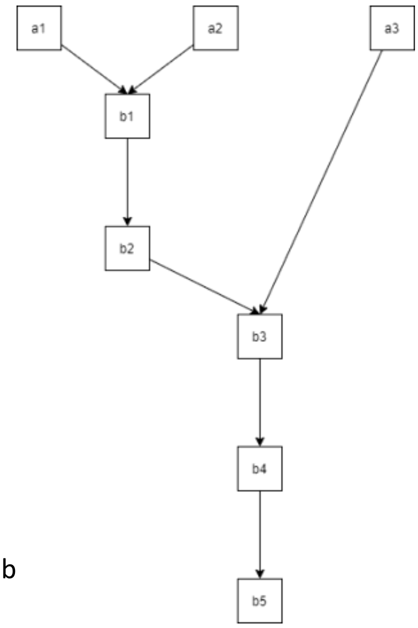
b4	P(b5)	P(W)
0	0	0.9
1	1	0.1

b5	**P(b5)**
0	**0.9**
1	**0.1**

PROBLEM 3D

(5 points) Compute the probability that b3=0 given that: b5=1

Compute P(b3=0 | b5=1)



Evidence below the query :
So, we use Bayes' Rule to find the solution

P(b3=0 | b5=1) = P(b5=1|b3=0) P(b3=0) / P(b5=1)

We already calculated **P(b5=1|b3=0) as 0.1**
From subpart 2 we have :

b3	P(b3)
0	0.0921
1	0.9079

P(b5=1) can be written as P(b5=1|b3=0) P(b3=0) + P(b5=1|b3=1) P(b

So we have :

P(b3=0|b5=1) = (0.1*0.0921)/(0.1*0.0921 + 0.9079*P(b5=1|b3=1))

b3	P(b4)	P(W)
0	0.1	0
1	0.7	1

b4	P(b4)
0	1 - 0.7 = 0.3
1	0.7*1 = 0.7

b4	P(b5)	P(W)
0	0	0.3
1	1	0.7

b5	**P(b5)**
0	**0.3**
1	**0.7**

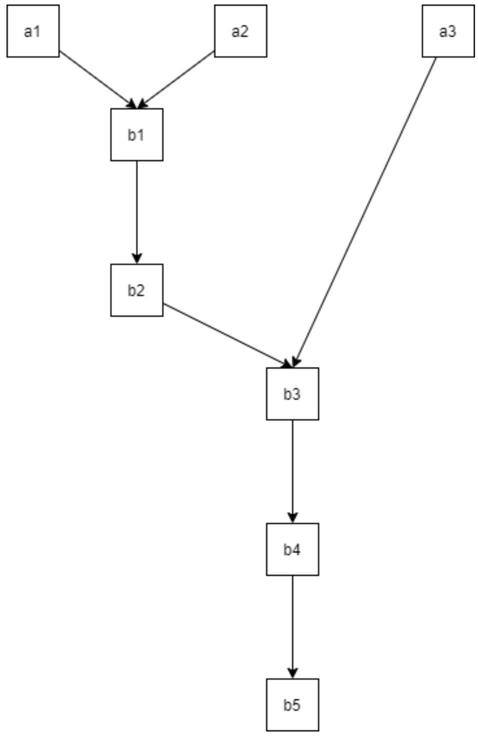
P(b3=0|b5=1) = (0.1*0.0921)/(0.1*0.0921 + 0.9079*P(b5=1|b3=1))
= (0.1*0.0921)/(0.1*0.0921 + 0.9079*0.7) = **0.0143**

PROBLEM 3E

(20 points) The CPT in node a3 is changed to:

where the value of x is unknown. What values of x would make it make it more likely that b5 happened than that b5 did not happen?

Here we are not given values for CPT of a3 and asked to assume them to be x



a3	P(a3)
0	1-x
1	x

b2	P(b2)
0	0.2484
1	0.7516

a3	b2	P(b3)	P(W)
0	0	0	(1-x)*0.2484
0	1	0.7	(1-x)*0.7516
1	0	0.8	x*0.2484
1	1	1	x*0.7516

b3	P(b3)
0	0.47388 - x*0.4242
1	x*0.7516 + x*0.2484*0.8 + (1-x)*0.7516*0.7 = x*0.7516 + x*0.19872 + (1-x)*0.52612 = = x*0.4242 + 0.52612

x*0.4242 + 0.52612 <= 1
=> x*0.4242 <= 0.47388
=> x <= 1.11 [Always]

b3	P(b3)
0	1-m
1	m

b3	P(b4)	P(W)
0	0.1	1-m
1	0.7	m

Where m = x*0.4242 + 0.52612

b4	P(b4)
0	0.9-0.6*m
1	0.7*m + 0.1*(1-m) = 0.1+0.6*m

b4	P(b5)	P(W)
0	0	0.9-0.6*m
1	1	0.1+0.6*m

0.1+0.6*m <= 1
=> m <= 1.5
=> x*0.4242 + 0.52612 <= 1.5
=> x*0.4242 <= 0.97388
=> x <= 2.29 [Always]

b5	P(W)
0	0.9-0.6m
1	0.1+0.6m

P(b5=1) > P(b5=0)

0.1+0.6m > 0.9-0.6m
=> 1.2 m > 0.8
=> m > 0.667
=> x*0.4242 + 0.52612 > 0.667
=> x*0.4242 > 0.14088
=> 0.33191 < x <= 1