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CS 6364.002 – Artificial Intelligence

FINAL EXAM

Fall 2020
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Instructions: Do not communicate with anyone in any shape or form. This is an independent exam. Do not delete any problem formulation, just attach your answer in the space provided. If the problem is deleted and you send only the answer, you shall receive ZERO points.

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Problem 1 Uncertainty (10 points)

Given the joint probability table:

	rain		*¬rain*	
	sprinkle	*¬sprinkle*	*sprinkle*	*¬sprinkle*
Grasswet	0.1	0.02	0.023	0.007
¬Grasswet	0.09	0.06	0.14	0.56

Compute:

1. $P(\neg Grasswet)$ (**2 points**)
2. $P(rain|Grasswet)$ (**3 points**)
3. $P(Grasswet|rain \vee sprinkle)$ (**5 points**)

1. $$P(\neg Grasswet) = \sum_{r,s} P(\neg Grasswet, r, s)$$

	rain		*¬rain*	
	sprinkle	*¬sprinkle*	*sprinkle*	*¬sprinkle*
Grasswet	0.1	0.02	0.023	0.007
¬Grasswet	0.09	0.06	0.14	0.56

$$= 0.09 + 0.06 + 0.14 + 0.56 = \boxed{0.85}$$

2. $$P(rain | Grasswet) = \frac{P(Grasswet, rain)}{P(Grasswet)} = \frac{\sum_{s} P(Grasswet, rain, s)}{\sum_{r,s} P(Grasswet, r, s)}$$

	rain		*¬rain*	
	sprinkle	*¬sprinkle*	*sprinkle*	*¬sprinkle*
Grasswet	0.1	0.02	0.023	0.007
¬Grasswet	0.09	0.06	0.14	0.56

$$= \frac{0.1 + 0.02}{0.1 + 0.02 + 0.023 + 0.007} = \frac{0.12}{0.15} = \boxed{0.8}$$

	rain		¬rain	
	sprinkle	¬sprinkle	sprinkle	¬sprinkle
Grasswet	0.1	0.02	0.023	0.007
¬Grasswet	0.09	0.06	0.14	0.56

3. $$P(\text{Grasswet} \mid \text{rain} \lor \text{sprinkle}) = \frac{P(\text{Grasswet} \land (\text{rain} \lor \text{sprinkle}))}{P(\text{rain} \lor \text{sprinkle})}$$

$$= \frac{P(g,r,s) + P(g,r,\neg s) + P(g,\neg r,s)}{P(g,r,s) + P(\neg g,r,s) + P(g,r,\neg s) + P(\neg g,r,\neg s) + P(g,\neg r,s) + P(\neg g,\neg r,s)}$$

$$= \frac{0.1 + 0.02 + 0.023}{0.1 + 0.02 + 0.023 + 0.09 + 0.06 + 0.14}$$

$$= \frac{0.143}{0.433} = \boxed{0.330254}$$

Problem 2 First Order Logic Representations (30 points)

(a) (**15 points**) Represent in First-Order Logic (FOL) the following sentences, using the semantics of the predicates provided for each sentence:

1. *Every artist is inspired by a painting.*
   Predicates:
   artist(x)- x is artist;
   inspire(x,y): x is inspired by y;
   painting(z): z is a painting.

2. *All musicians in an orchestra follow the instructions of their conductor.*
   Predicates:
   musician(x)- x is musician;
   inOchestra(x,y): x is in orchestra y;
   conductor(z,y): z is conductor of orchestra y;
   follows(x,y): x follows the instructions of y;

3. *In each concert, every musician plays some instrument and sings some song and no one gets bored.*
   Predicates: *concert(c)* – c is a concert;
   musician(m) – m is a musician;
   musicianInConcert(m,c) – m is a musician participating in concert c;
   playsInstrument(m,i) – m plays instrument i;
   singsSong(m,s) – m sings song s;
   getBored(m) – m is getting bored;

ANSWERS

1. *Every artist is inspired by a painting.*

$$\forall x \left[artist(x) \Rightarrow \exists y \left[painting(y) \land inspire(x,y) \right] \right]$$

2. *All musicians in an orchestra follow the instructions of their conductor.*

$$\forall x \forall y \left[\text{musician}(x) \wedge \text{inOrchestra}(x, y) \Rightarrow \exists z \left[\text{conductor}(z, y) \wedge \text{follows}(x, z) \right] \right]$$

3. *In each concert, every musician plays some instrument and sings some song and no one gets bored.*

$$\forall c \forall m \left[\text{concert}(c) \wedge \text{musician}(m) \wedge \text{musicianInConcert}(m, c) \Rightarrow \exists i, s \left[\text{playsInstrument}(m, i) \wedge \text{singsSong}(m, s) \wedge \neg \text{getBored}(m) \right] \right]$$

(b). Generate the Conjunctive Normal Form (CNF) for the FOL expressions obtained in (a). Specify at each step of the conversion in CNF what you are doing!

1. Every artist is inspired by a painting

$$\forall x \, [\, artist(x) \Rightarrow \exists y \, [\, painting(y) \wedge inspire(x, y) \,]\,]$$

STEP-1 : Eliminate implications

$$\forall x \, [\, \neg artist(x) \vee \exists y \, [\, painting(y) \wedge inspire(x, y) \,]\,]$$

STEP-2 : Move ¬ inwards.

$$\forall x \, [\, \neg artist(x) \vee \exists y \, [\, painting(y) \wedge inspire(x, y) \,]\,]$$

STEP-3 : Standardize variables

$$\forall x \, [\, \neg artist(x) \vee \exists y \, [\, painting(y) \wedge inspire(x, y) \,]\,]$$

STEP-4 : Skolemize

$$\forall x \, [\, \neg artist(x) \vee [\, painting(f(x)) \wedge inspire(x, f(x)) \,]\,]$$

STEP-5 : Drop universal quantifiers

$$\neg artist(x) \vee [\, painting(f(x)) \wedge inspire(x, f(x)) \,]$$

STEP-6 : Distribute ∨ over ∧

$$[\, \neg artist(x) \vee painting(f(x)) \,] \wedge [\, \neg artist(x) \vee inspire(x, f(x)) \,]$$

2. All musicians in an orchestra follow the instructions of their conductor

$$\forall x \forall y \left[musician(x) \wedge inOrchestra(x,y) \Rightarrow \exists z \left[conductor(z,y) \wedge follows(x,z) \right] \right]$$

STEP-1 : Eliminate implications

$$\forall x \forall y \left[\neg \left(musician(x) \wedge inOrchestra(x,y) \right) \vee \exists z \left[conductor(z,y) \wedge follows(x,z) \right] \right]$$

STEP-2 : Move $\neg$ inwards

$$\forall x \forall y \left[\left(\neg musician(x) \vee \neg inOrchestra(x,y) \right) \vee \exists z \left[conductor(z,y) \wedge follows(x,z) \right] \right]$$

STEP-3 : Standardize variables

$$\forall x \forall y \left[\left(\neg musician(x) \vee \neg inOrchestra(x,y) \right) \vee \exists \underset{\substack{\downarrow \\ f(x,y)}}{z} \left[conductor(z,y) \wedge follows(x,z) \right] \right]$$

STEP-4 : Skolemize

$$\forall x \forall y \left[\left(\neg musician(x) \vee \neg inOrchestra(x,y) \right) \vee \left(conductor(f(x,y),y) \wedge follows(x, f(x,y)) \right) \right]$$

STEP-5 : Drop universal quantifiers

$$\left(\neg musician(x) \vee \neg inOrchestra(x,y) \right) \vee \left(conductor(f(x,y),y) \wedge follows(x, f(x,y)) \right)$$

STEP-6 : Distribute $\vee$ over $\wedge$

$$\left[\neg musician(x) \vee \neg inOrchestra(x,y) \vee conductor(f(x,y),y) \right] \wedge$$
$$\left[\neg musician(x) \vee \neg inOrchestra(x,y) \vee follows(x, f(x,y)) \right]$$

3. In each concert, every musician plays some instrument and sings some song and no one gets bored.

$$\forall c\, \forall m \left[concert(c) \land musician(m) \land musicianInConcert(m,c) \Rightarrow \exists i,s \left[playsInstrument(m,i) \land singsSong(m,s) \land \neg getBored(m) \right]\right]$$

STEP-1 : Eliminate implications

$$\forall c\, \forall m \left[\neg \left(concert(c) \land musician(m) \land musicianInConcert(m,c) \right) \lor \exists i,s \left[playsInstrument(m,i) \land singsSong(m,s) \land \neg getBored(m) \right]\right]$$

STEP-2 : Move ¬ inwards.

$$\forall c\, \forall m \left[\left(\neg concert(c) \lor \neg musician(m) \lor \neg musicianInConcert(m,c) \right) \lor \exists i,s \left[playsInstrument(m,i) \land singsSong(m,s) \land \neg getBored(m) \right]\right]$$

STEP-3 : Standardize variables

$$\forall c\, \forall m \left[\left(\neg concert(c) \lor \neg musician(m) \lor \neg musicianInConcert(m,c) \right) \lor \exists i,s \left[playsInstrument(m,i) \land singsSong(m,s) \land \neg getBored(m) \right]\right]$$

STEP-4 : Skolemize

$$\forall c\, \forall m \Big[\big(\neg concert(c) \vee \neg musician(m) \vee \neg musicianInConcert(m,c)\big) \vee$$

$$\exists i,s \big[playsInstrument(m,i) \wedge singsSong(m,s) \wedge \neg getBored(m) \big]\Big]$$

$f_1(c,m)$ $f_2(c,m)$

$$\forall c\, \forall m \Big[\big(\neg concert(c) \vee \neg musician(m) \vee \neg musicianInConcert(m,c)\big) \vee$$

$$\big(playsInstrument(m, f_1(c,m)) \wedge singsSong(m, f_2(c,m)) \wedge \neg getBored(m)\big)\Big]$$

STEP-5 : Drop universal quantifiers

$$\big(\neg concert(c) \vee \neg musician(m) \vee \neg musicianInConcert(m,c)\big) \vee$$

$$\big(playsInstrument(m, f_1(c,m)) \wedge singsSong(m, f_2(c,m)) \wedge \neg getBored(m)\big)$$

STEP-6 : Distribute $\vee$ over $\wedge$

$$\big[\neg concert(c) \vee \neg musician(m) \vee \neg musicianInConcert(m,c) \vee playsInstrument(m, f_1(c,m))\big]$$

$$\wedge \big[\neg concert(c) \vee \neg musician(m) \vee \neg musicianInConcert(m,c) \vee singsSong(m, f_2(c,m))\big]$$

$$\wedge \big[\neg concert(c) \vee \neg musician(m) \vee \neg musicianInConcert(m,c) \vee \neg getBored(m)\big]$$

Problem 3 (20 points) Inference in First Order Logic

You are given the following description:

"If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned"

1. Transform the text in First-Order Logic (FOL) **(4 points).**
2. Convert each axiom in Conjunctive Normal Form (CNF) and produce a knowledge base (KB) containing all the clauses derived from the CNF. **(6 points).**
3. Given the KB, can you prove that a unicorn is mythical? How about magical? Or horned? Use refutation to prove each of your answers and show the entire proof. **(10 points)**

1.

Unicorn(x) : x is a unicorn. [This can also be avoided as we deal with only unicorns here. adding it makes it more general]

Mythical (x) : x is mythical

Immortal (x) : x is immortal

Mortal (x) : x is mortal.

Mammal (x) : x is mammal.

Horned (x) : x is horned

Magical (x) : x is magical.

S1 : $\forall x \ \text{Unicorn}(x) \wedge \text{Mythical}(x) \Rightarrow \text{Immortal}(x)$

S2 : $\forall x \ \text{Unicorn}(x) \wedge \neg \text{Mythical}(x) \Rightarrow \text{Mortal}(x) \wedge \text{Mammal}(x)$

S3 : $\forall x \ \text{Unicorn}(x) \wedge (\text{Immortal}(x) \vee \text{Mammal}(x)) \Rightarrow \text{Horned}(x)$

S4 : $\forall x \ \text{Unicorn}(x) \wedge \text{Horned}(x) \Rightarrow \text{Magical}(x)$

Common sense knowledge

S5 : $\forall x \ \text{Mortal}(x) \Rightarrow \neg \text{Immortal}(x)$

2. Converting each axiom into CNF

i) $\forall x \; Unicorn(x) \wedge Mythical(x) \Rightarrow Immortal(x)$

STEP-1 : (Eliminate implications) $\forall x \; \neg(Unicorn(x) \wedge Mythical(x)) \vee Immortal(x)$

STEP-2 : (Move $\neg$ inwards) $\forall x \; \neg Unicorn(x) \vee \neg Mythical(x) \vee Immortal(x)$

STEP-3 : (Standardize variable) $\forall x_1 \; \neg Unicorn(x_1) \vee \neg Mythical(x_1) \vee Immortal(x_1)$

STEP-4 : (Skolemize, drop universal quantifiers, distribute $\vee$ over $\wedge$) $\neg Unicorn(x_1) \vee \neg Mythical(x_1) \vee Immortal(x_1) \longrightarrow C_1$

ii) $\forall x \; Unicorn(x) \wedge \neg Mythical(x) \Rightarrow Mortal(x) \wedge Mammal(x)$

STEP-1 : (Eliminate implications) $\forall x \; \neg(Unicorn(x) \wedge \neg Mythical(x)) \vee (Mortal(x) \wedge Mammal(x))$

STEP-2 : (Move $\neg$ inwards) $\forall x \; (\neg Unicorn(x) \vee Mythical(x)) \vee (Mortal(x) \wedge Mammal(x))$

STEP-3 : (Standardize variables) $\forall x_2 \; (\neg Unicorn(x_2) \vee Mythical(x_2)) \vee (Mortal(x_2) \wedge Mammal(x_2))$

STEP-4 : (Skolemize, drop universal) $(\neg Unicorn(x_2) \vee Mythical(x_2)) \vee (Mortal(x_2) \wedge Mammal(x_2))$

STEP-5 : (Distribute $\vee$ over $\wedge$) $[\neg Unicorn(x_2) \vee Mythical(x_2) \vee Mortal(x_2)] \longrightarrow C_2 \; \wedge$

$[\neg Unicorn(x_2) \vee Mythical(x_2) \vee Mammal(x_2)] \longrightarrow C_3$

iii) $\forall x \; Unicorn(x) \wedge (Immortal(x) \vee Mammal(x)) \Rightarrow Horned(x)$

STEP-1 : (Eliminate implications) : $\forall x \; \neg \big(Unicorn(x) \wedge (Immortal(x) \vee Mammal(x)) \big) \vee Horned(x)$

STEP-2 : (Move $\neg$ inwards) : $\forall x \; [\neg Unicorn(x) \vee \neg (Immortal(x) \vee Mammal(x))] \vee Horned(x)$

$$\forall x \; [\neg Unicorn(x) \vee (\neg Immortal(x) \wedge \neg Mammal(x))] \vee Horned(x)$$

STEP-3 : (Standardize variables) :

$$\forall x_3 \; [\neg Unicorn(x_3) \vee (\neg Immortal(x_3) \wedge \neg Mammal(x_3))] \vee Horned(x_3)$$

STEP-4 : (Skolemize, drop universal quantifiers)

$$[\neg Unicorn(x_3) \vee (\neg Immortal(x_3) \wedge \neg Mammal(x_3))] \vee Horned(x_3)$$

STEP-5 : Distribute $\vee$ over $\wedge$

$$[\neg Unicorn(x_3) \vee Horned(x_3) \vee \neg Immortal(x_3)] \wedge \quad C_4$$

$$[\neg Unicorn(x_3) \vee Horned(x_3) \vee \neg Mammal(x_3)] \quad C_5$$

iv) $\forall x \; Unicorn(x) \wedge Horned(x) \Rightarrow Magical(x)$

STEP-1 : (Eliminate implications) $\forall x \; \neg (Unicorn(x) \wedge Horned(x)) \vee Magical(x)$

STEP-2 : (Move $\neg$ inwards) $\forall x \; \neg Unicorn(x) \vee \neg Horned(x) \vee Magical(x)$

STEP-3 : (Standardize variables) $\forall x_4 \; \neg Unicorn(x_4) \vee \neg Horned(x_4) \vee Magical(x_4)$

STEP-4 : (Skolemize, drop universal quantifiers, distribute $\vee$ over $\wedge$)

$$\neg \text{Unicorn}(x_4) \vee \neg \text{Horned}(x_4) \vee \text{Magical}(x_4) \longrightarrow C_6$$

v) Common sense knowledge : $\forall x \ \text{Mortal}(x) \Rightarrow \text{Immortal}(x)$

STEP-1 (Eliminate implications) : $\forall x \ \neg \text{Mortal}(x) \vee \text{Immortal}(x)$

STEP-2 (Move $\neg$ inwards, standardize variables)

$$\forall x_5 \ \neg \text{Mortal}(x_5) \vee \text{Immortal}(x_5)$$

STEP-3 (Skolemize, drop universal quantifiers, distribute $\vee$ over $\wedge$)

$$\neg \text{Mortal}(x_5) \vee \text{Immortal}(x_5) \longrightarrow C_7$$

3. Given the KB, can you prove that a unicorn is mythical? How about magical? Or horned? Use refutation to prove each of your answers and show the entire proof. **(10 points)**

i) A unicorn is horned.

Q_1 : $\forall x\ Unicorn(x) \Rightarrow Horned(x)$

$\neg Q_1$: $\neg(\forall x\ Unicorn(x) \Rightarrow Horned(x))$

Conversion to CNF :

STEP-1 (Eliminate implications) : $\neg(\forall x\ \neg Unicorn(x) \vee Horned(x))$

STEP-2 (Move $\neg$ inwards) : $\exists x\ \neg(\neg Unicorn(x) \vee Horned(x))$

$\exists x\ Unicorn(x) \wedge \neg Horned(x)$

STEP-3 (Standardize variables) $\exists x_6\ Unicorn(x_6) \wedge \neg Horned(x_6)$

STEP-4 (Skolemize) $Unicorn(U_1) \wedge \neg Horned(U_1)$

STEP-5 (Drop universal quantifier and distribute) : $\boxed{Q_{11} : Unicorn(U_1)}$ $\boxed{Q_{12} : \neg Horned(U)}$ Added to KB

Consider C1 & Q_{11} : For $\theta : \{x_1 / v_1\}$ they resolve into Predicate : Unicorn

Y_1 : $\neg$ Mythical (v_1) $\vee$ Immortal (v_1)

Consider C_3 & Q_{11} : For $\theta : \{x_2 / v_1\}$ they resolve into Predicate : Unicorn

Y_2 : Mythical (v_1) $\vee$ Mammal (v_1)

Consider C_4 & Q_{11} : For $\theta : \{x_3 / v_1\}$ they resolve into Predicate : Unicorn

Y_3 : $\neg$ Immortal (v_1) $\vee$ Horned (v_1)

Consider C_5 & Q_{11} : For $\theta : \{x_3 / v_1\}$ they resolve into Predicate : Unicorn

Y_4 : $\neg$ Mammal (v_1) $\vee$ Horned (v_1)

Consider Y_1 & Y_2 : For $\theta : \{ \}$ they resolve into Predicate : Mythical

Y_5 : Immortal (v_1) $\vee$ Mammal (v_1)

Consider Y_5 & Y_3 : For $\theta : \{ \}$ they resolve into Predicate : Immortal

Y_6 : Mammal (v_1) $\vee$ Horned (v_1)

Consider Y_6 & Y_4 : For $\theta : \{ \}$ they resolve into Predicate : Mammal

Y_7 : Horned (v_1)

Consider Y_7 & Q_{22} : For $\theta : \{ \}$ they resolve into Predicate : Horned.

$\boxed{\text{NIL}}$

ii) A unicorn is magical.

$Q_2 : \forall x \ Unicorn(x) \Rightarrow Magical(x)$

$\neg Q_2 : \neg (\forall x \ Unicorn(x) \Rightarrow Magical(x))$

Conversion to CNF :

STEP-1 : (Eliminate implications) : $\neg (\forall x \ \neg Unicorn(x) \vee Magical(x))$

STEP-2 : (Move $\neg$ inwards) : $\exists x \ (Unicorn(x) \wedge \neg Magical(x))$

STEP-3 : (Standardize, skolemize) : $Unicorn(c_2) \wedge \neg Magical(c_2)$

STEP-4 : Drop universal quantifiers : $\boxed{Q_{21} : Unicorn(c_2)}$

↳ Distribute $\vee$ over $\wedge$ $\boxed{Q_{22} : \neg Magical(c_2)}$

Consider C1 & Q_{11} : For θ : $\{ x_1 / U_2 \}$ they resolve into Predicate : Unicorn

Y_1 : $\neg$ Mythical (U_2) $\vee$ Immortal (U_2)

Consider C_3 & Q_{11} : For θ : $\{ x_2 / U_2 \}$ they resolve into Predicate : Unicorn

Y_2 : Mythical (U_2) $\vee$ Mammal (U_2)

Consider C_4 & Q_{11} : For θ : $\{ x_3 / U_2 \}$ they resolve into Predicate : Unicorn

Y_3 : $\neg$ Immortal (U_2) $\vee$ Horned (U_2)

Consider C_5 & Q_{11} : For θ : $\{ x_3 / U_2 \}$ they resolve into Predicate : Unicorn

Y_4 : $\neg$ Mammal (U_2) $\vee$ Horned (U_2)

Consider C_6 & Q_{11} : For θ : $\{ x_4 / U_2 \}$ they resolve into Predicate : Unicorn

Y_5 : $\neg$ Horned (U_2) $\vee$ Magical (U_2)

Consider Y_1 & Y_2 : For θ : $\{ \}$ they resolve into Predicate : Mythical

Y_6 : Immortal (U_2) $\vee$ Mammal (U_2)

Consider Y_6 & Y_3 : For θ : $\{ \}$ they resolve into Predicate : Immortal

Y_7 : Mammal (U_2) $\vee$ Horned (U_2)

Consider Y_7 & Y_4 : For θ : $\{ \}$ they resolve into Predicate : Mammal

Y_8 : Horned (U_2)

Consider Y_8 & Y_5 : For substitution $\theta : \{ \}$ they resolve into Predicate : Horned

Y_9 : Magical (U_2)

Consider Y_9 & Q_{22} : For $\theta : \{ \}$ they resolve into Predicate : Magical.

$\boxed{\text{NIL}}$

(iii) A unicorn is mythical.

$Q_3 : \forall x \ Unicorn(x) \Rightarrow Mythical(x)$

$\neg Q_3 : \neg (\forall x \ Unicorn(x) \Rightarrow Mythical(x))$

<u>Conversion to CNF :</u>

<u>STEP-1</u> (Eliminate all implications) : $\neg (\forall x \ \neg Unicorn(x) \vee Mythical(x))$

<u>STEP-2</u> (Move $\neg$ inwards) : $\exists x \ \neg (\neg Unicorn(x) \vee Mythical(x))$

$\exists x \ Unicorn(x) \wedge \neg Mythical(x)$

<u>STEP-3</u> (Standardize, skolemize) : $Unicorn(c_3) \wedge \neg Mythical(c_3)$

<u>STEP-4</u> (Drop universal quantifiers, distribute $\vee$ over $\wedge$) :

$Q_{31} : Unicorn(c_3)$

$Q_{32} : \neg Mythical(c_3)$

} ~~Adding~~ them to <u>KB</u>

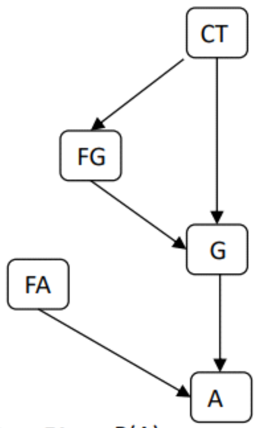
We see that for <u>no pair</u> of resolvents they resolve into $\boxed{NIL}$

$\Rightarrow$ It is not possible to prove a unicorn is <u>mythical</u>

Problem 3 (20 points) Probabilistic Reasoning

In a nuclear power plant, there is an alarm that senses when a temperature gauge exceeds a given threshold. The gauge measures the temperature of the core. Consider the Boolean random variables A (alarm sounds), FA (alarm is faulty), FG (gauge is faulty), G (gauge is reading) and CT (the core temperature is too high).

The following Bayesian network represents this domain:



P(CT)
m

CT	P(FG)
T	*n*
F	*q*

CT	FG	P(G)
T	T	*y*
F	T	*1-y*
T	F	*x*
F	F	*1-x*

P(FA)
fa

G	FA	P(A)
T	T	*0*
F	T	*0*
T	F	*0*
F	F	*1*

1. (5 points) Compute the probability that A=1, FA=0, FG=0 and CT=1.
2. (10 points) Compute the probability that the core temperature is too high when the alarm sounds, and the alarm and the gauge work well, i.e. P(CT|A, ¬FA, ¬FG).
3. (5 points) Decide whether A and FA have any effect on CT given G and explain why you reached that decision.

1) Given evidence : $A=1$, $FA=0$, $FG=0$, $CT=1$

Based on bayes network given above

$$P(CT, FG, G, A, FA) = P(CT)\, P(FG \mid CT)\, P(G \mid FG, CT)\, P(FA)\, P(A \mid G, FA)$$

$\Rightarrow$ Based on given evidence

$$P(CT=1, FG=0, A=1, FA=0) = \sum_{g} P(CT=1, FG=0, G, A=1, FA=0)$$

$$= P(CT=1)\, P(FG=0 \mid CT=1)\, P(G=0 \mid FG=0, CT=1)\, P(FA=0)\, P(A=1 \mid G=0, FA=0) +$$

$$P(CT=1)\, P(FG=0 \mid CT=1)\, P(G=1 \mid FG=0, CT=1)\, P(FA=0)\, P(A=1 \mid G=1, FA=0)$$

$$= (m)(1-n)(1-z)(1-fa)(1) + (m)(1-n)\cancelto{0}{(z)}(1-fa)(0)$$

$$= m(1-n)(1-z)(1-fa)$$

2) $$P(CT \mid a, \neg fa, \neg fg) = \frac{P(CT, a, \neg fa, \neg fg)}{P(a, \neg fa, \neg fg)} = \frac{\sum_{G} P(CT, a, G, \neg fa, \neg fg)}{\sum_{CT, G} P(CT, a, G, \neg fa, \neg fg)}$$

$$\sum_{g} P(CT, a, g, \neg fa, \neg fg) = \sum_{g} P(CT)\, P(\neg fg \mid CT)\, P(g \mid \neg fg, CT)\, P(\neg fa)\, P(a \mid g, \neg fa)$$

$$= P(CT)\, P(\neg fg \mid CT)\, P(\neg fa) \sum_{g} \underbrace{P(g \mid \neg fg, CT)}\, \underbrace{P(a \mid g, \neg fa)}$$

Using the method of variable elimination [factors to be considered]

$$P(CT)\,P(\neg fg \mid CT)\,P(\neg fa) \sum_{g} \underbrace{P(g \mid \neg fg, CT)}_{f_2(G,CT)}\, \underbrace{P(a \mid g, \neg fa)}_{f_1(G)}$$

$f_1(G) \rightarrow$

G	$f_1(G)$
T	0
F	1

G	CT	$f_2(G,CT)$
T	T	x
T	F	$1-x$
F	T	$1-x$
F	F	x

Considering [POINTWISE PRODUCT]

G	CT	$f_3(G,CT)$
T	T	0
T	F	0
F	T	$1-x$
F	F	x

Summing out G gives $f_4(CT)$

CT	$f_4(CT)$
T	$1-x$
F	x

$$\Rightarrow P(CT)\,P(\neg fg \mid CT)\,P(\neg fa) \sum_{g} f_3(g, CT)$$

$$= \underbrace{P(CT)}\ \underbrace{P(\neg fg \mid CT)}\ P(\neg fa)\ f_4(CT)$$

CT	$f_5(CT)$
T	m
F	$1-m$

CT	$f_6(CT)$
T	$1-n$
F	$1-q$

$(1-fa)$

Considering pointwise product of $f_5(CT), f_6(CT), f_4(CT)$ gives

CT	$f_7(CT)$
T	$m(1-n)(1-x)(1-fa)$
F	$(1-m)(1-q)(x)(1-fa)$

$$\Rightarrow \sum_{CT} f_7(CT) = m(1-n)(1-x)(1-fa) + (1-m)(1-q)x(1-fa)$$

$$\therefore P(CT \mid a, \neg fa, \neg fg) = \frac{m(1-n)(1-x)\cancel{(1-fa)}}{m(1-n)(1-x)\cancel{(1-fa)} + (1-m)(1-q)x\cancel{(1-fa)}}$$

$$= \frac{m(1-n)(1-x)}{m(1-n)(1-x) + (1-m)(1-q)x}$$

3) Effect of A and FA on CT given G

No, They do not have any effect on CT given G

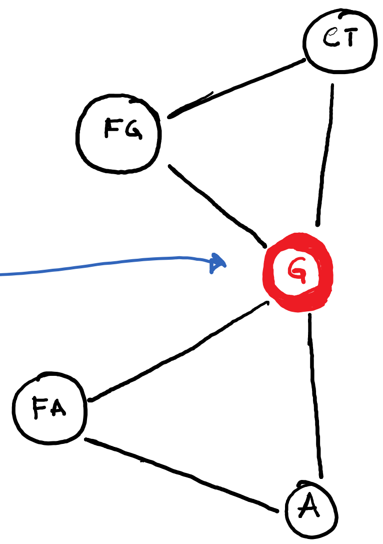
A, FA are D-SEPARATED from CT given G

Following the procedure for checking d-separation from slides

Considering the moral graph for the given graph

→ Replace all directed links to undirected links produces

Observing node G blocks all the paths from FA, A to CT



Thus we can say that G d-separates FA, A and CT

So, we can say that A is conditionally independent of CT given G

and thus A, FA have no effect on CT given G

Problem 4 (20 points) Inference in Propositional Logic

If your knowledge base (HB) contains the following Horn clauses:

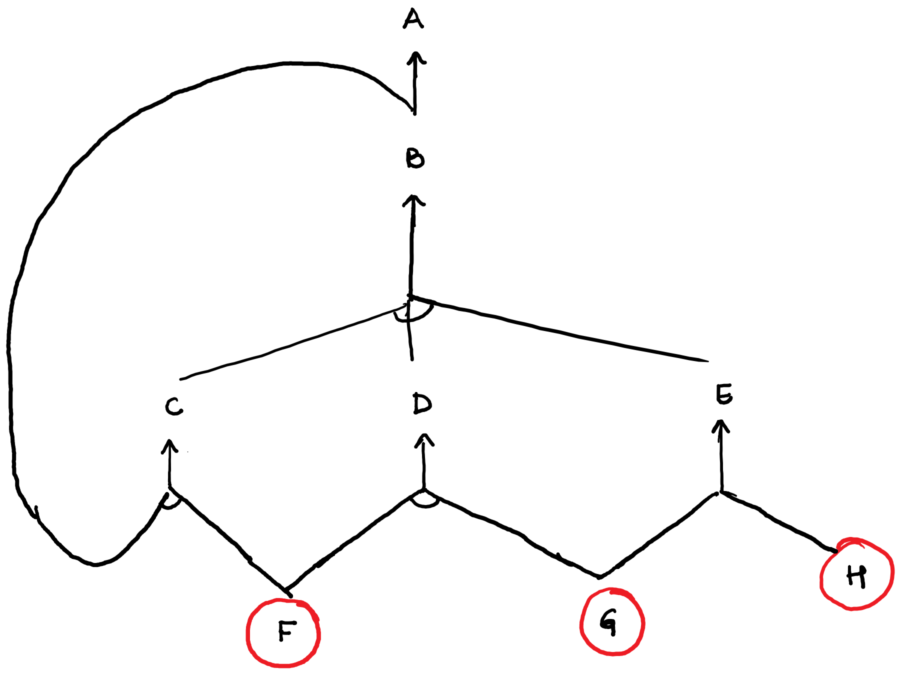
1. $B \Rightarrow A$
2. $C \wedge D \wedge E \Rightarrow B$
3. $B \wedge F \Rightarrow C$
4. $F \wedge G \Rightarrow D$
5. $G \wedge H \Rightarrow E$
6. F
7. G
8. H

Use backward chaining to prove A.

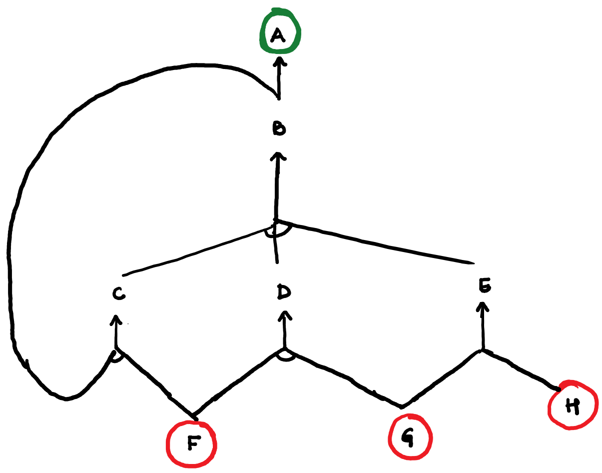
i. Draw the AND=OR graph for this KB (**5 points**).

ii. Show each step of your proof using backward chaining (**15 points**).

1) AND - OR GRAPH :



2) Using backward chaining to prove A

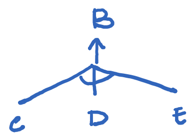


STEP-1 : Assume Q_1 : A is true.

According to S1, B must be true.

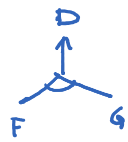
STEP-2 : Assume B is True

According to S2, $C \wedge D \wedge E$ must be true.



STEP-3 : Assume D is True

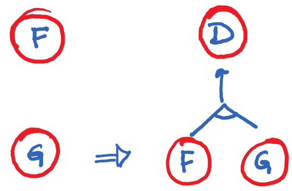
According to S4, $F \wedge G$ must be true



STEP-4 : From S6, we have evidence that F is true

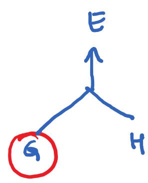
STEP-5 : From S7, we have evidence that G is true

From step-4, 5 we can conclude that $F \wedge G$ is true and thus D is true

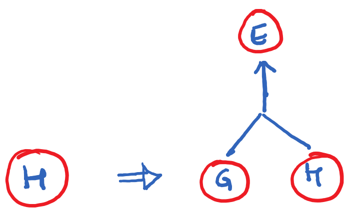


STEP-6 : Assume E is true

According to S5, $G \wedge H$ must be true



STEP-7 : From S8, we have evidence that H is true

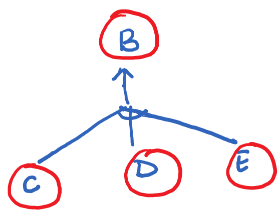


From step-7, 5 we can conclude that $H \wedge G$ is true and thus E is true

STEP-8 : From step-4, we can conclude that C is true

STEP-9 : From step-8, 7, 5 we conclude that $C \wedge D \wedge E$ is true

and thus B is true



STEP-10 : From step-9 we have B is true and thus we can conclude our

query A is true