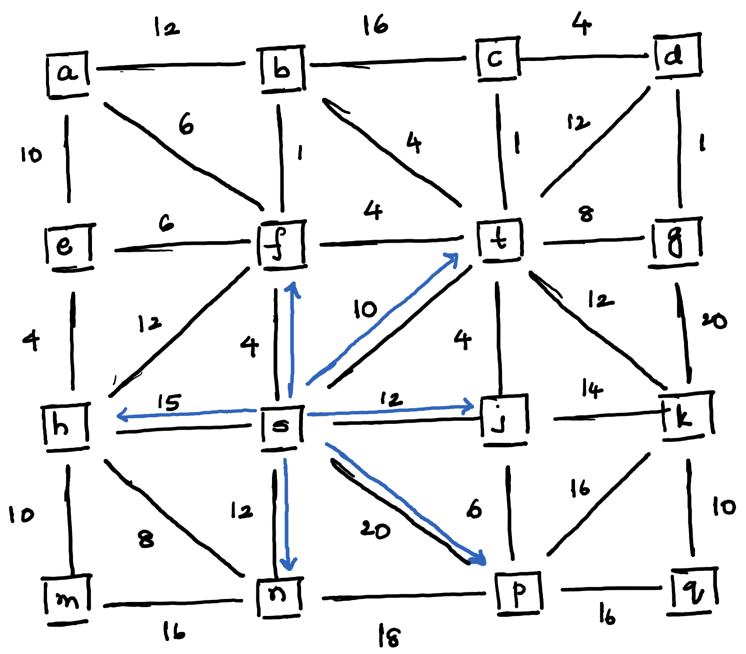
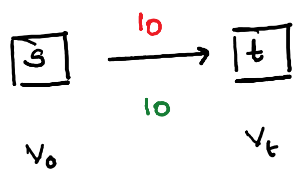
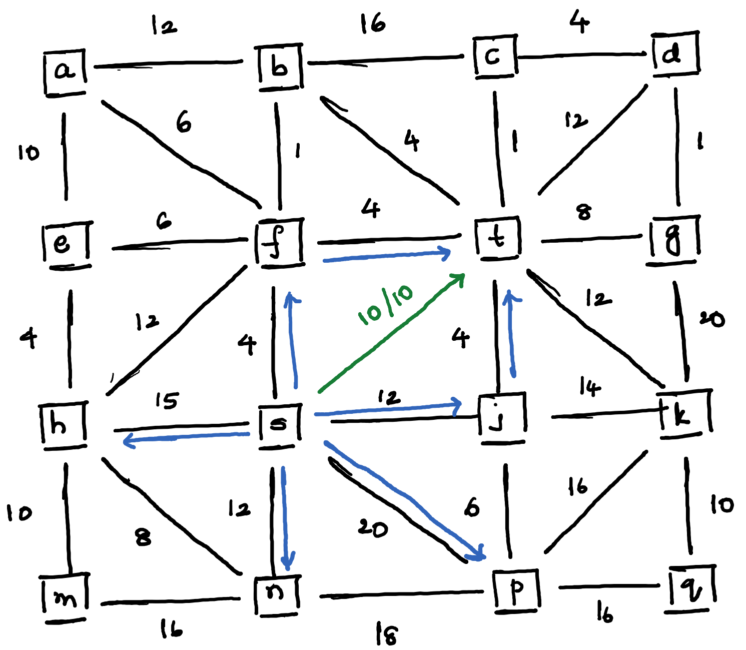
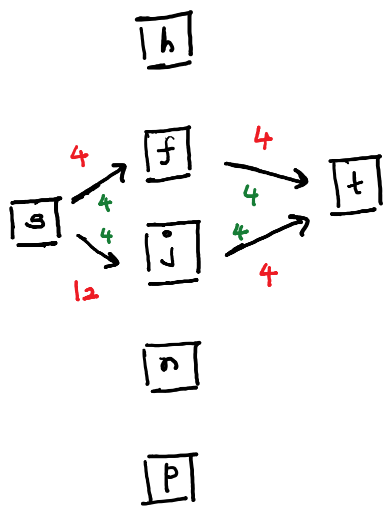
1-(i) : Dinic's Algorithm :



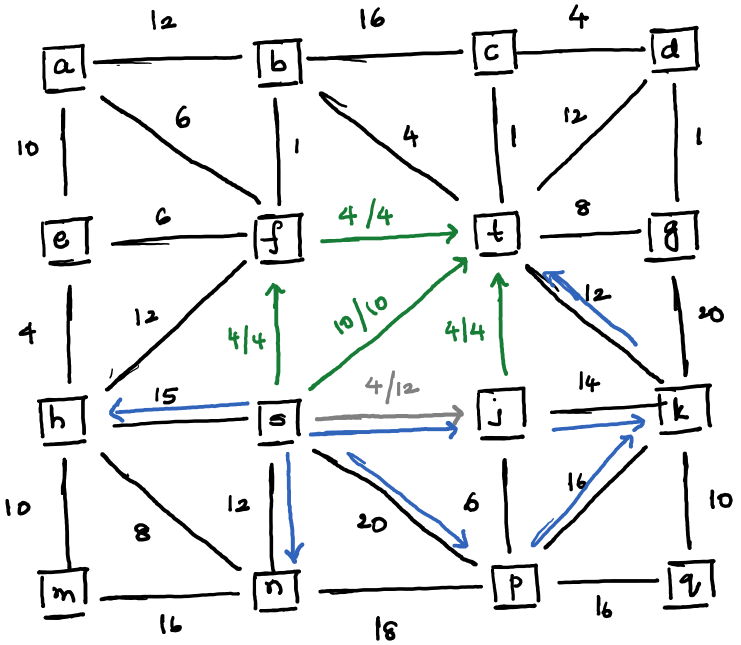


Flow augmented = $\boxed{10}$

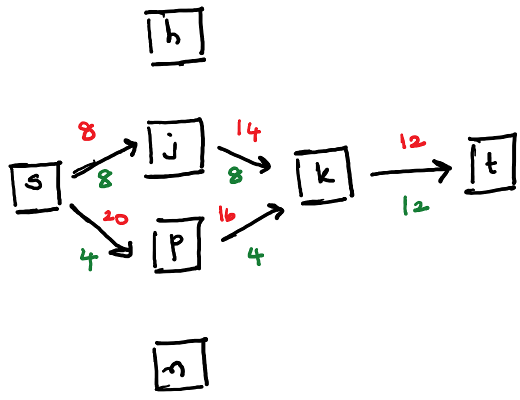




Flow augmented = 4+4 = $\boxed{8}$

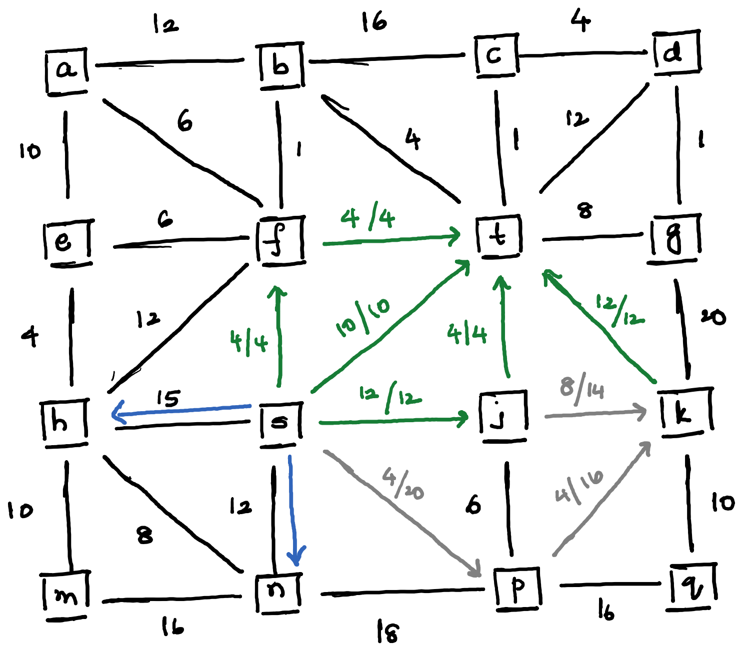


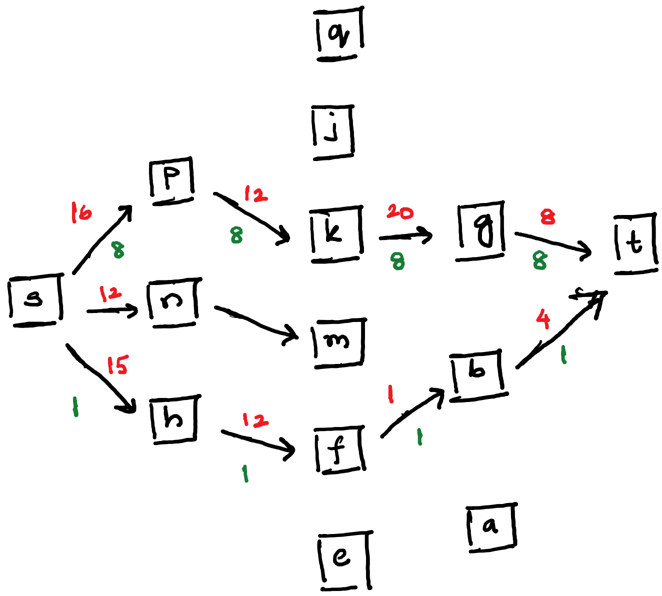
3



Flow augmented = 8 + 4 = $\boxed{12}$

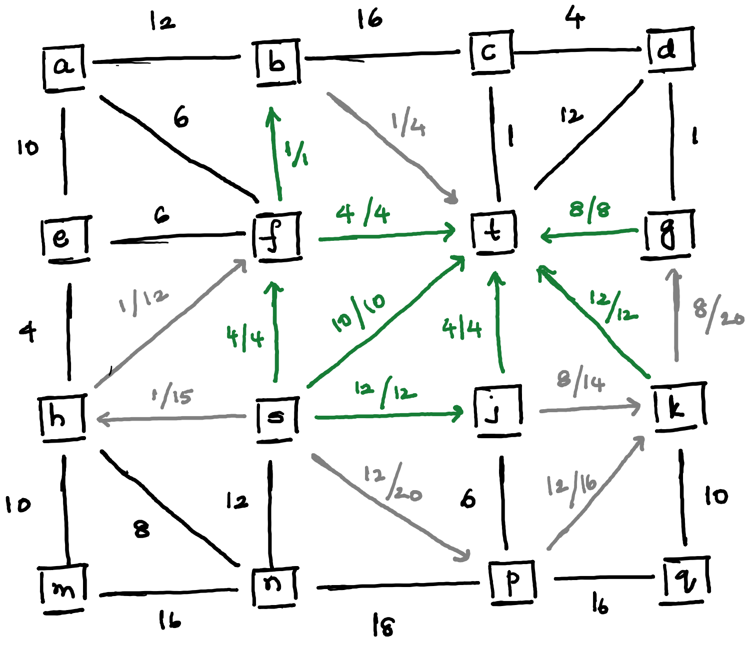
4

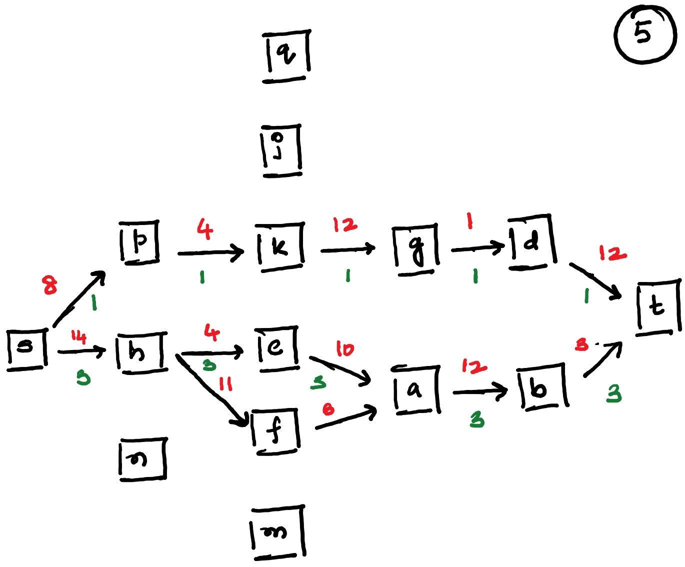




Flow augmented = 8 + 1 = $\boxed{9}$

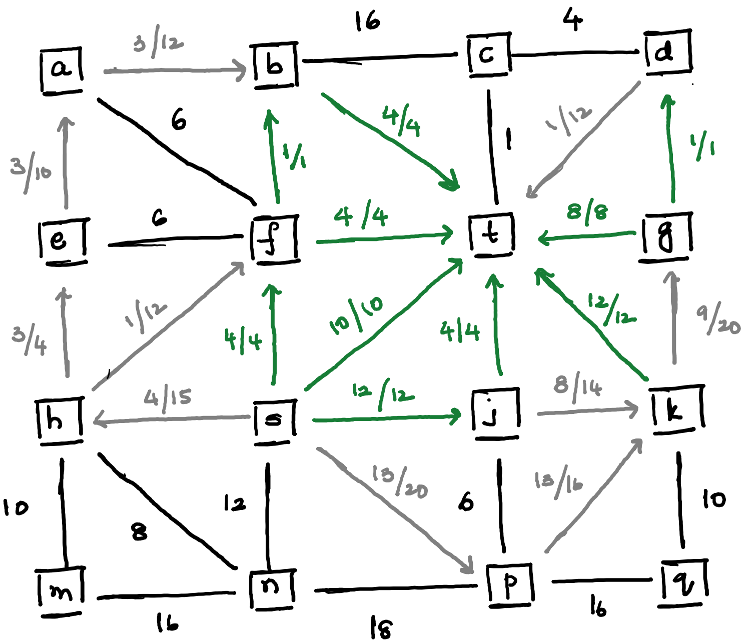
5

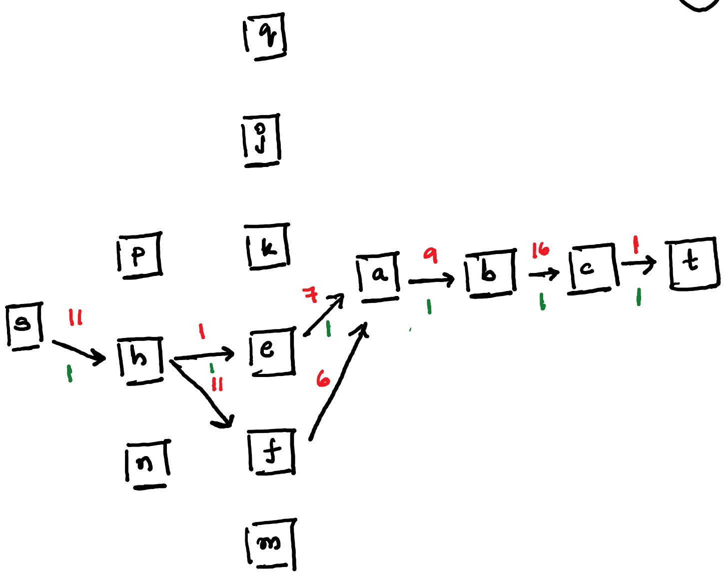




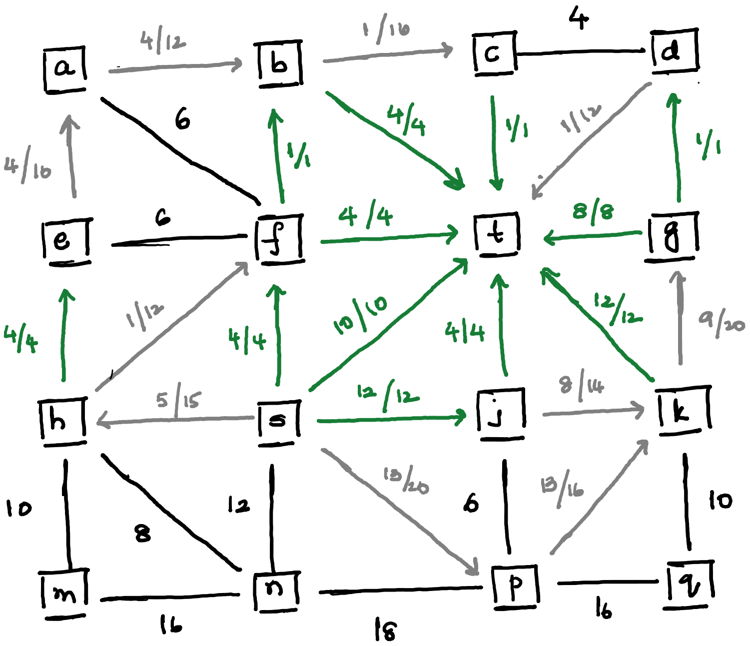
Flow augmented = 3 + 1 = $\boxed{4}$

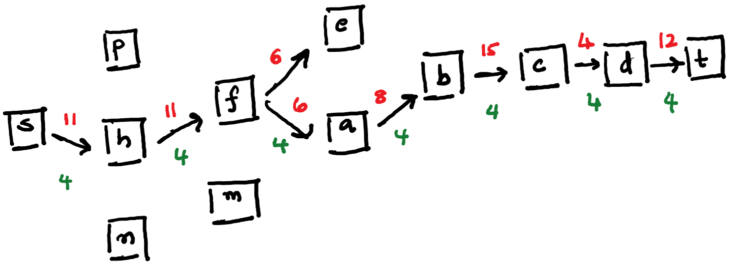
6



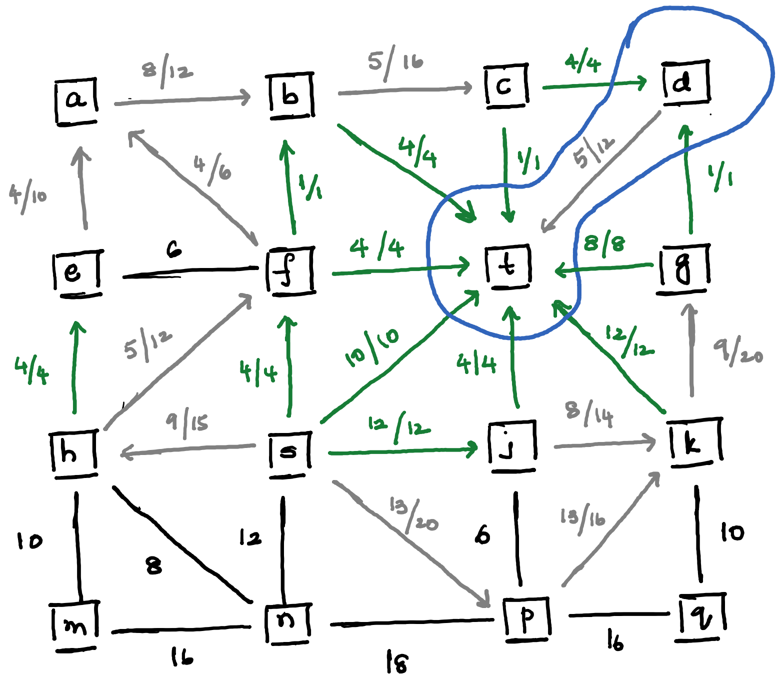


Flow augmented = $\boxed{1}$

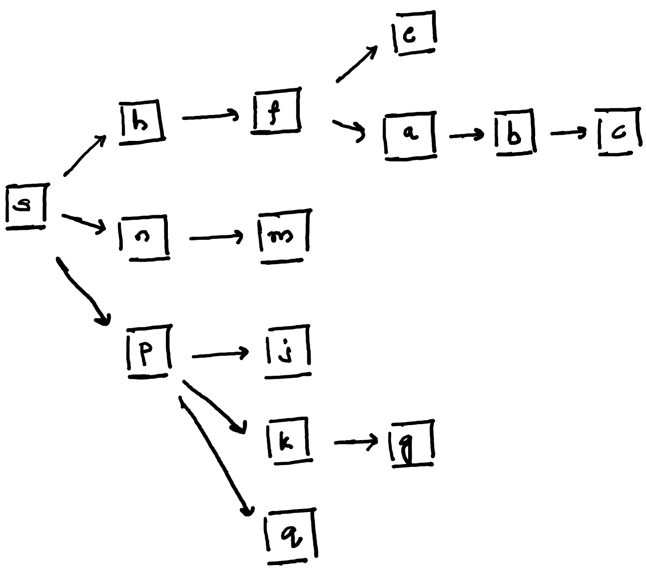




Flow augmented = $\boxed{4}$



For finding the min cut



There does not exist a path from s to t

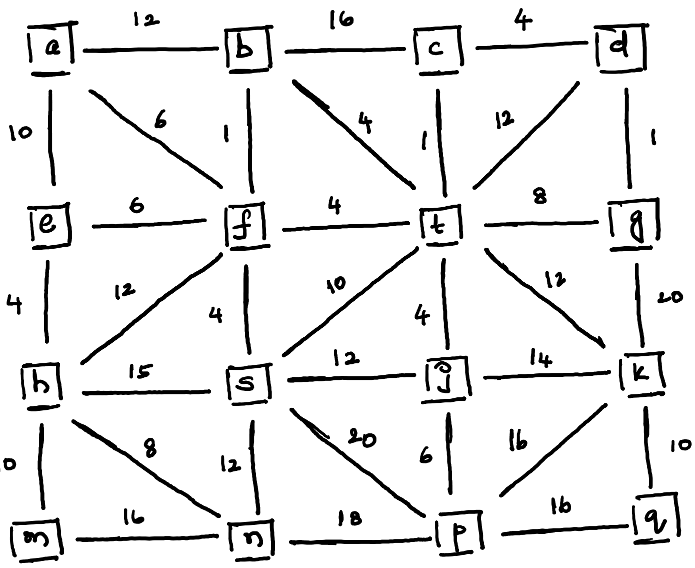
Reached max flow

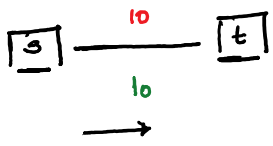
$\boxed{\text{Flow: } 48, \ \text{Cut: } (V - \{d, t\}, \{d, t\})}$

1-(ii) : Using MKM method

Depth in LN

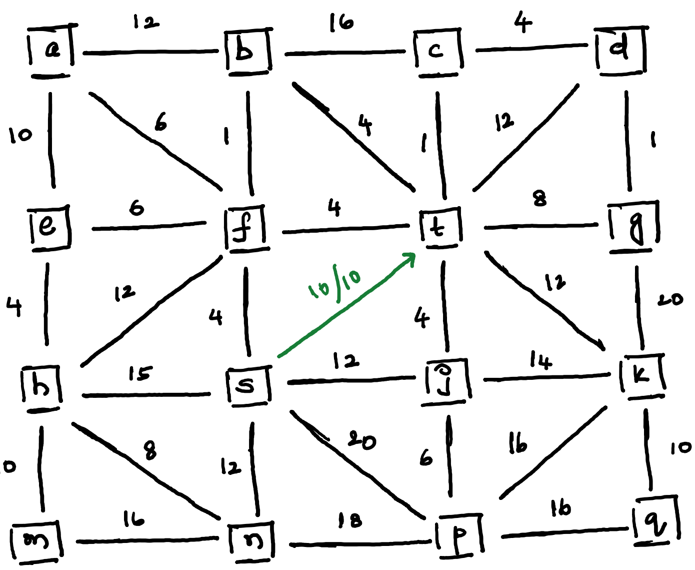
1



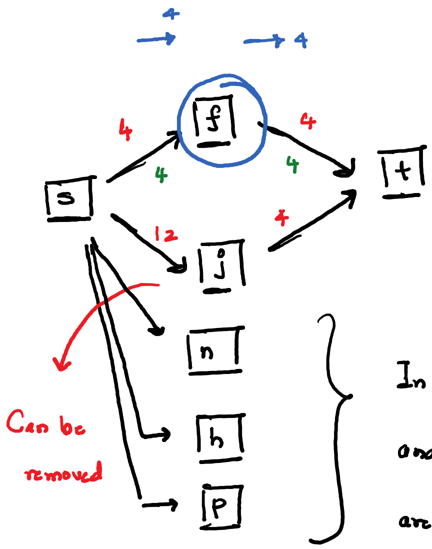


s is saturated

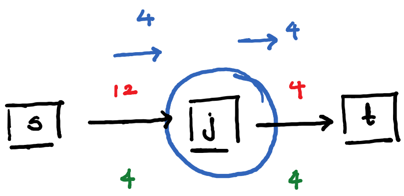
Flow augmented = $\boxed{10}$



2



In subsequent analysis such nodes are NOT CONSIDERED

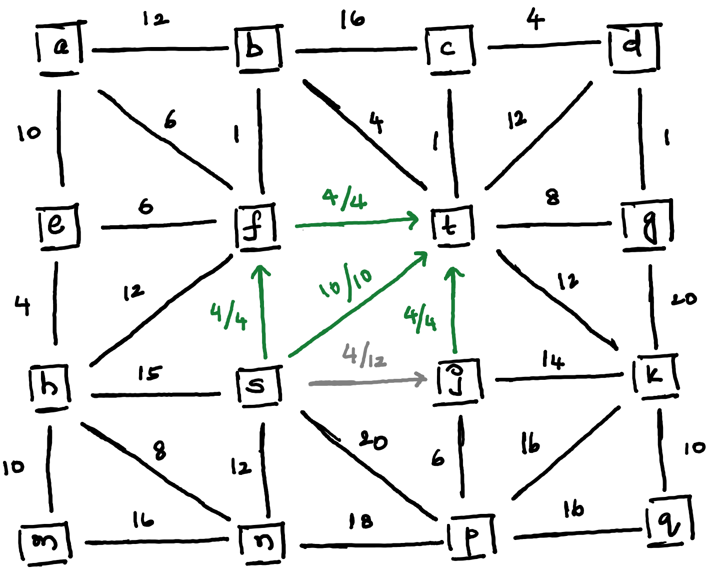


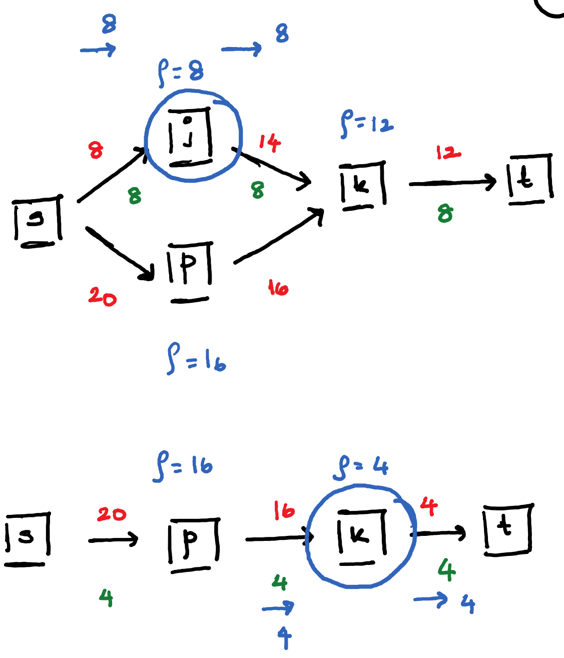
Flow augmented $= 4 + 4 = \boxed{8}$

Here in MKM method we remove nodes with 0 potential and their edges

Remove edges with 0 residual capacities

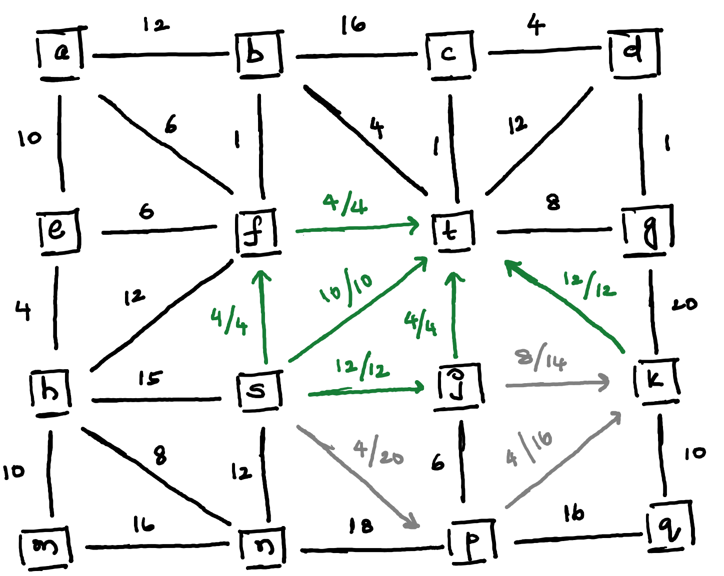
3

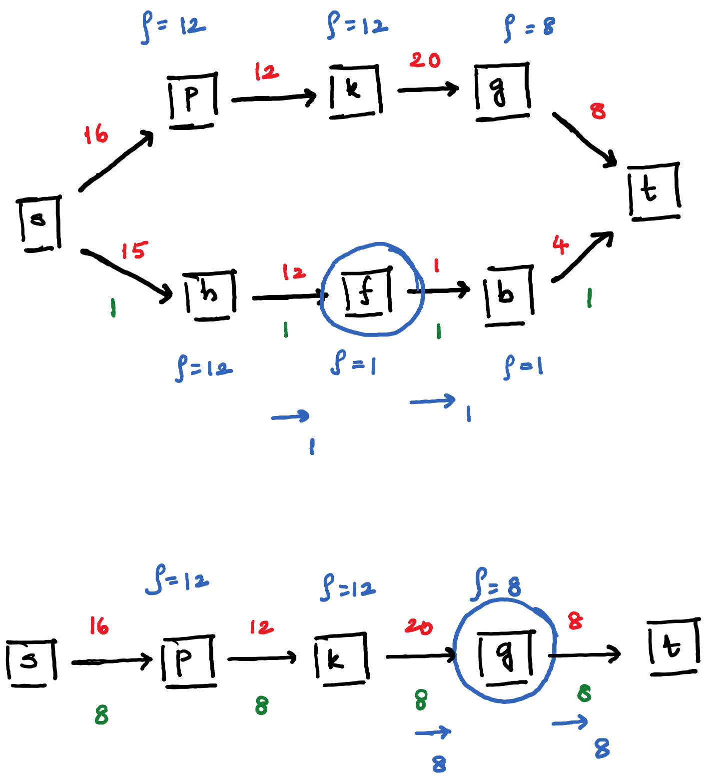




Flow augmented : $8 + 4 = \boxed{12}$

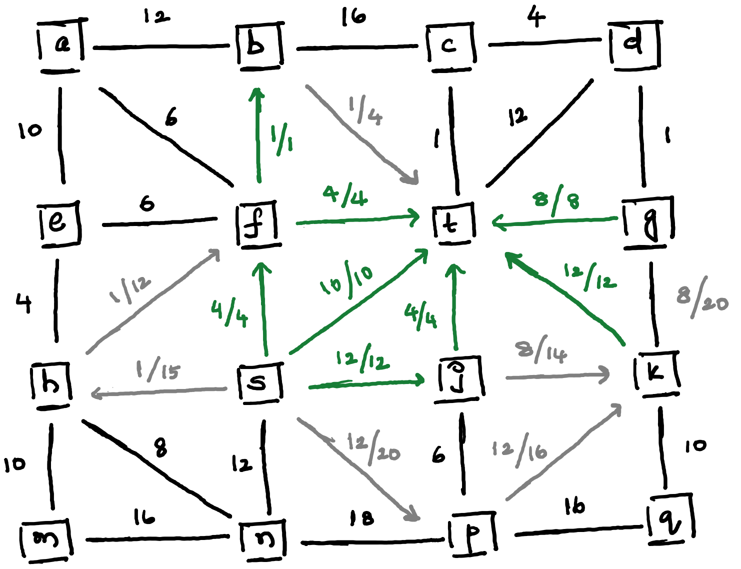
4

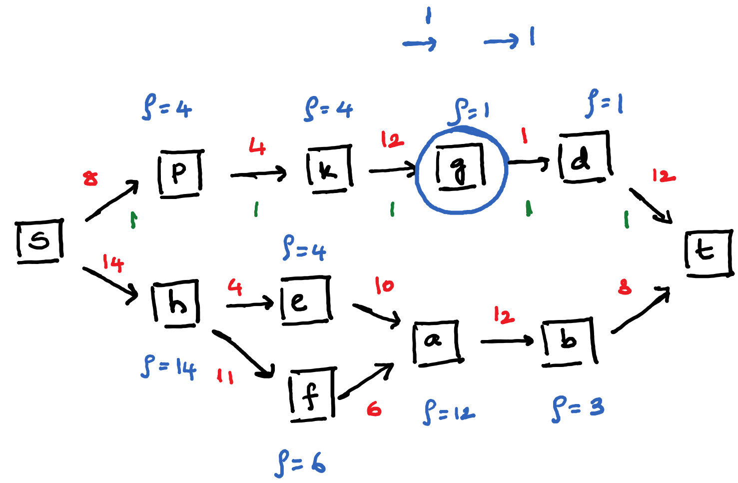


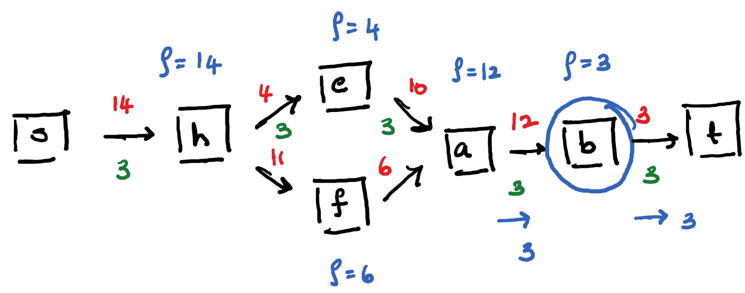


Flow augmented : $1 + 8 = \boxed{9}$

5

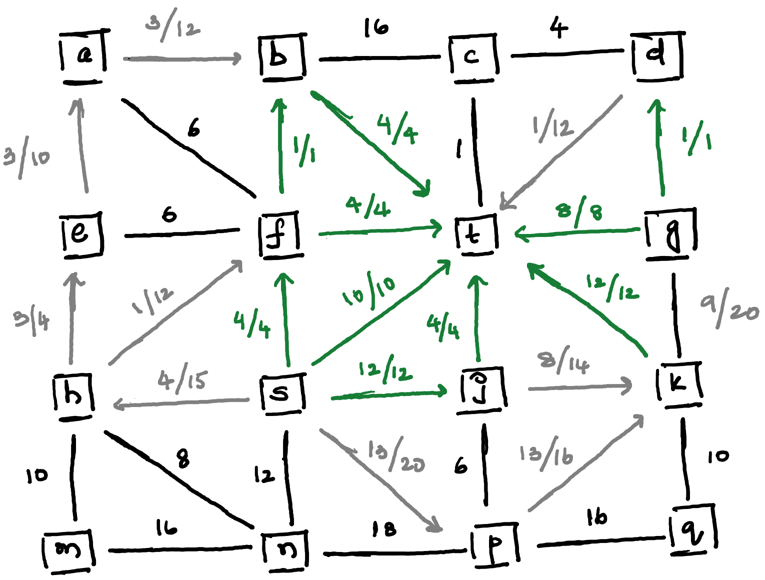


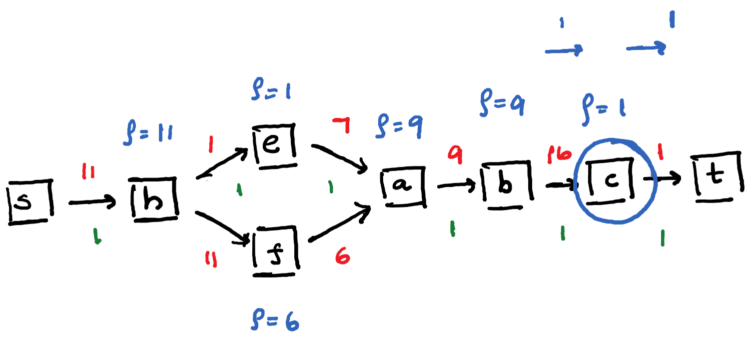




Flow augmented : 3+1 = $\boxed{4}$

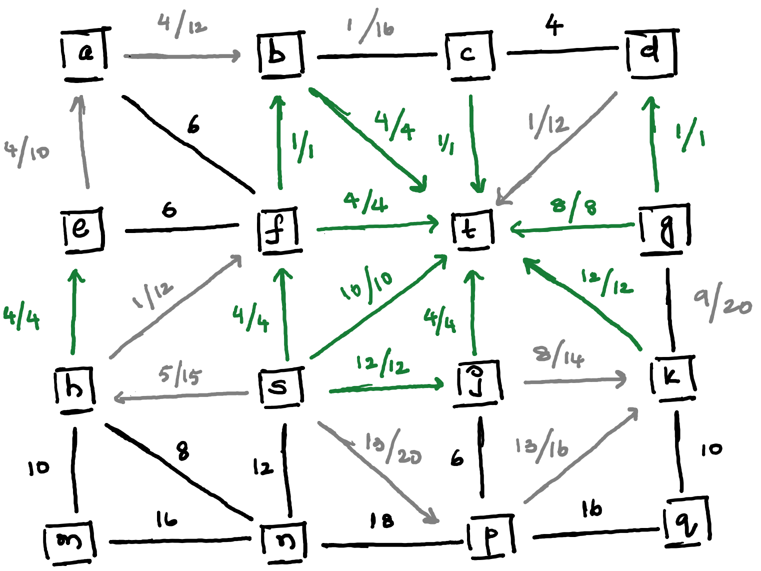
6

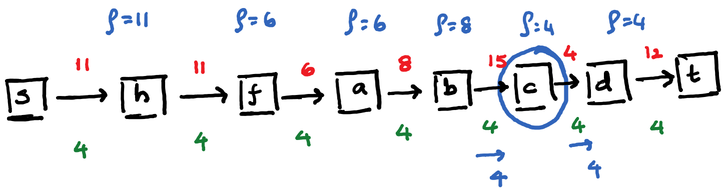




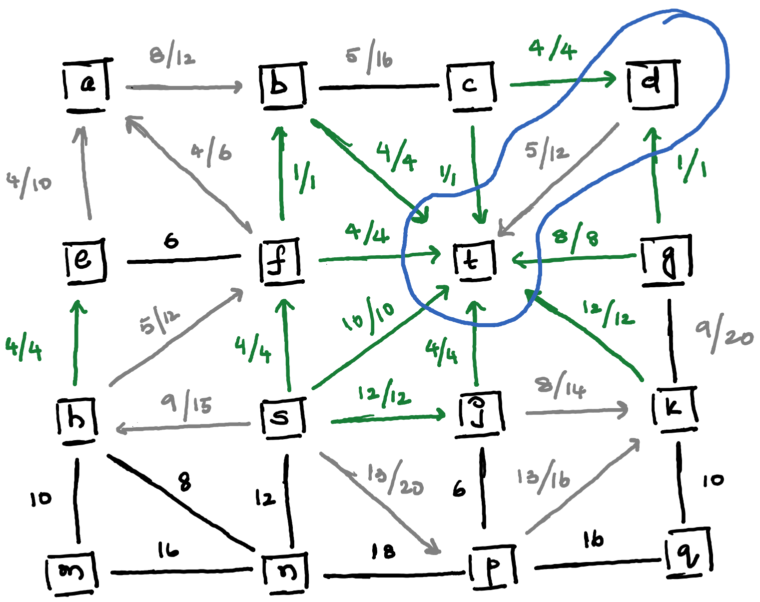
Flow augmented : $\boxed{1}$

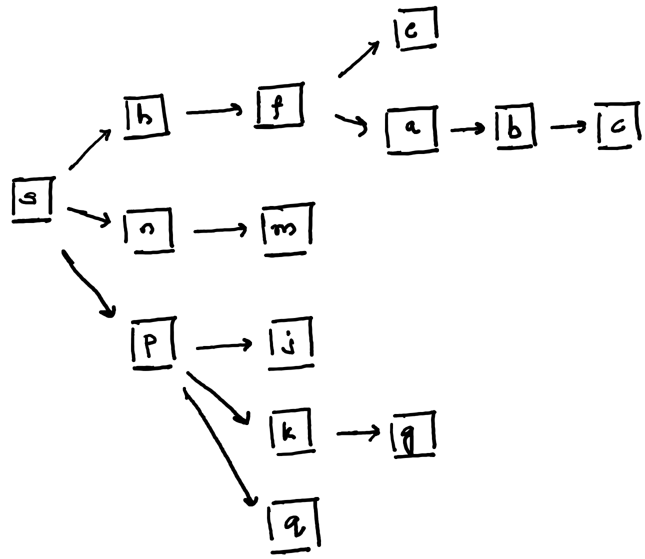
7





Flow augmented : $\boxed{4}$





Total flow: $4+1+4+9+12+10+8$

$= \boxed{48}$

Cut: $(V - \{d,t\}, \{d,t\})$

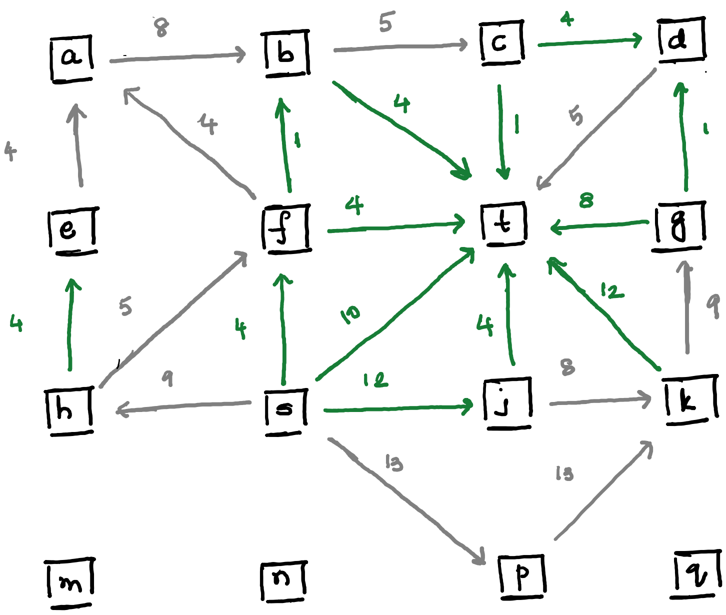
(or)

$(\{s,h,n,p,f,m,j,k,q,e,a,b,c,g\}, \{d,t\})$

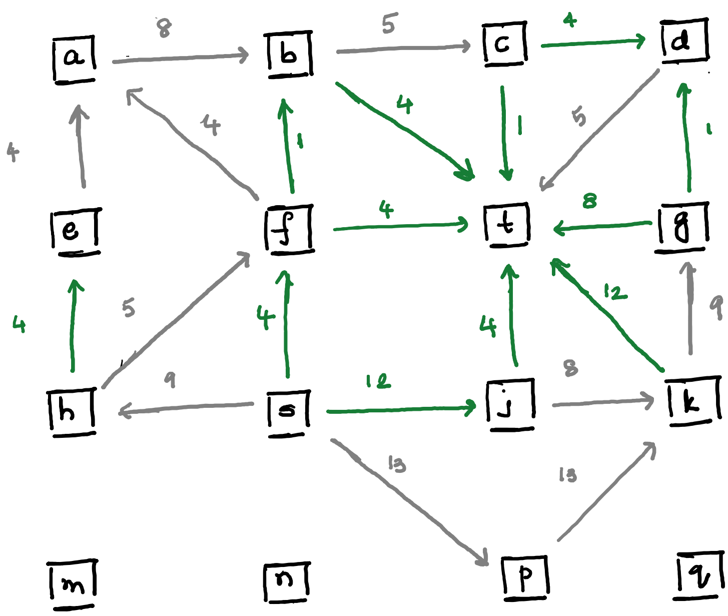
Decomposition of flow into chain flows:

[Node formulation — Path formulation]

It can be easily noticed that. In Dinic's algorithm paths generated in the process of saturation of layered network can be considered as path flows.

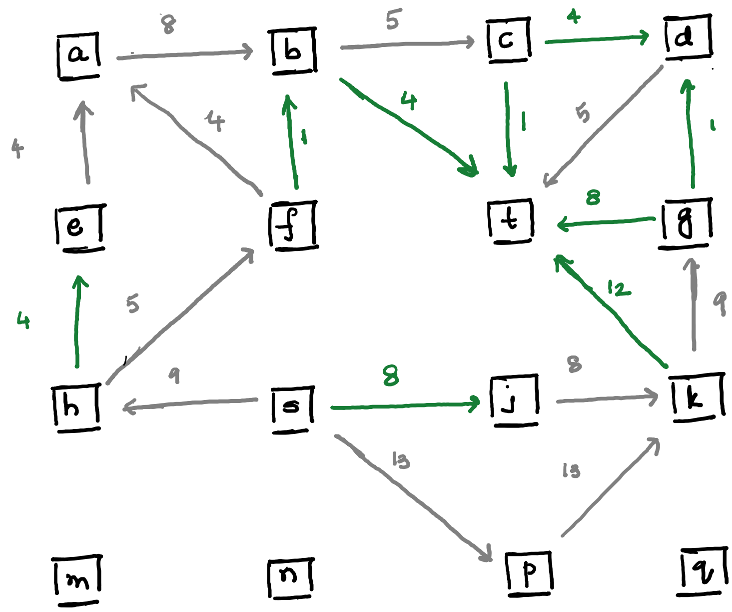


Path P_1 : $s \to t$ [10]

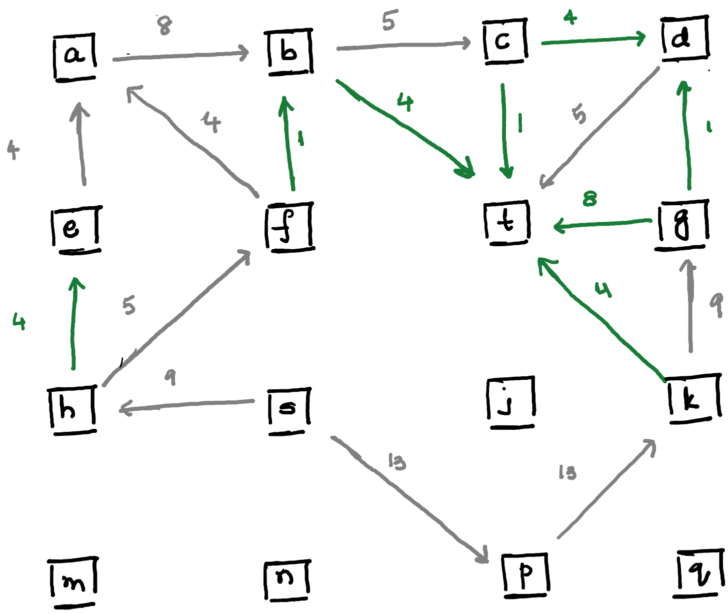


Path P_2 : $s \to f \to t$ [4]

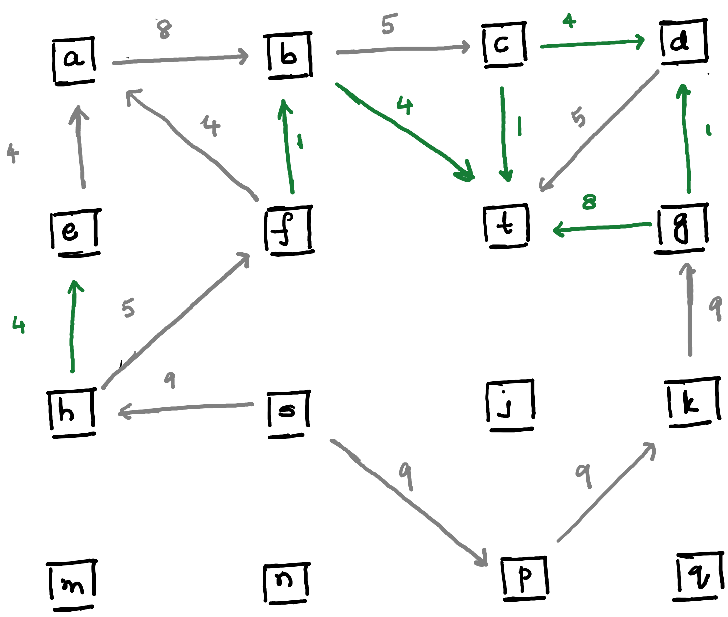
P_3 : $s \to j \to t$ [4]



Path P_4 : $s \to j \to k \to t$ [8]

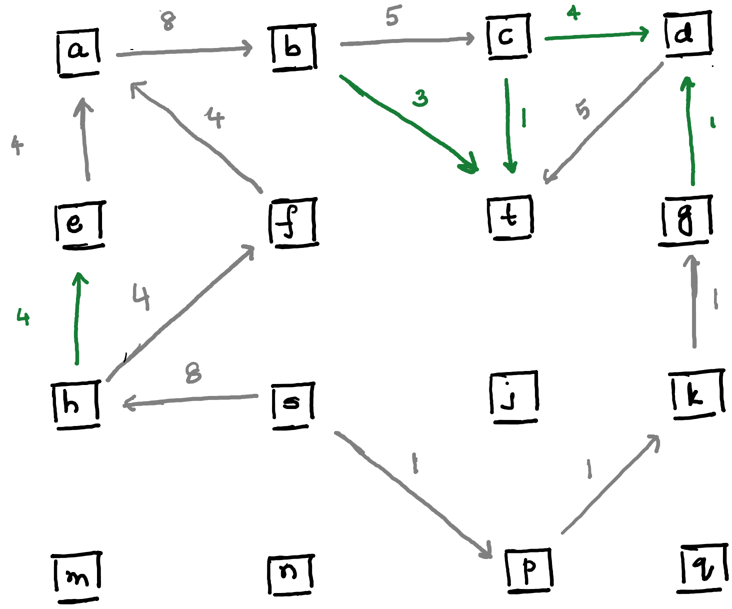


Path P_5 : $s \to p \to k \to t$ [4]



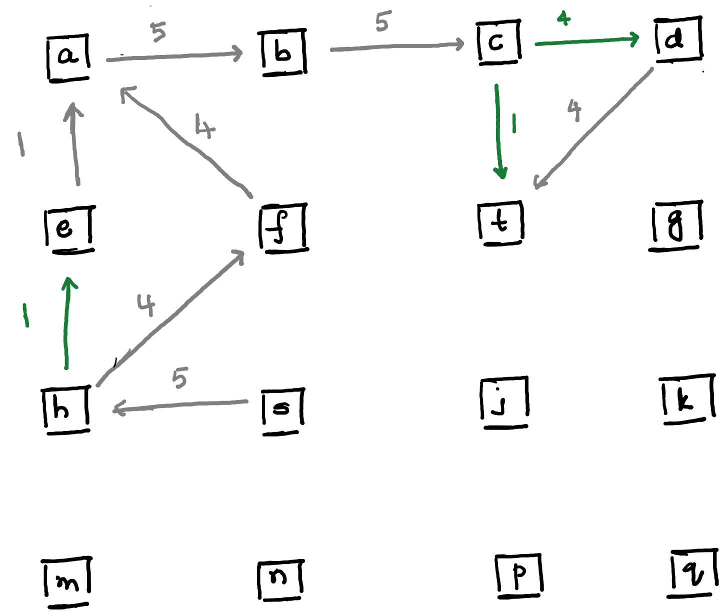
Path P_6 : $s \to p \to k \to g \to t$ [8]

P_7 : $s \to h \to f \to b \to t$ [1]



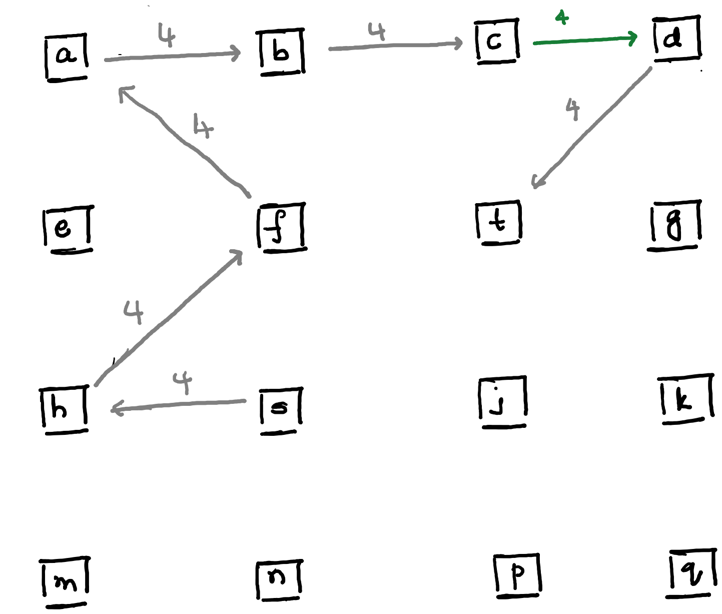
Path P_8 : s → p → k → g → d → t [1]

Path P_9 : s → h → e → a → b → t [3]



Path P_{10} :

s → h → e → a → b → c → t [1]



Path P_{11} :

s → h → f → a → b → c → d → t [4]

$\boxed{a}$	$\boxed{b}$	$\boxed{c}$	$\boxed{d}$
$\boxed{e}$	$\boxed{f}$	$\boxed{t}$	$\boxed{g}$
$\boxed{h}$	$\boxed{s}$	$\boxed{j}$	$\boxed{k}$
$\boxed{m}$	$\boxed{n}$	$\boxed{p}$	$\boxed{q}$

s, t are disconnected now

Finished decomposition into paths

So, paths formed are

P_1 : $s \to t$ [10]

P_2 : $s \to j \to t$ [4]

P_3 : $s \to f \to t$ [4]

P_4 : $s \to j \to k \to t$ [8]

P_5 : $s \to p \to k \to t$ [4]

P_6 : $s \to p \to k \to g \to t$ [8]

P_7 : $s \to h \to f \to b \to t$ [1]

P_8 : $s \to p \to k \to g \to d \to t$ [1]

P_9 : $s \to h \to e \to a \to b \to t$ [3]

P_{10} : $s \to h \to e \to a \to b \to c \to t$ [1]

P_{11} : $s \to h \to f \to a \to b \to c \to d \to t$ [4]

Total flow: $\boxed{48}$ ✓