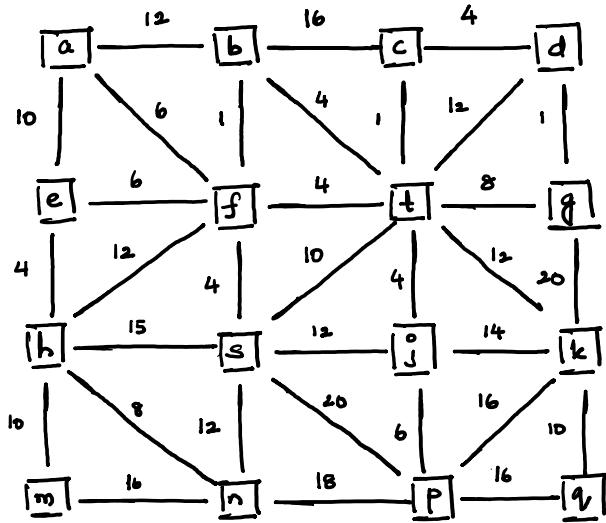


Maximum flows if edge capacities are given by $\min[u_{ij}, \lambda]$

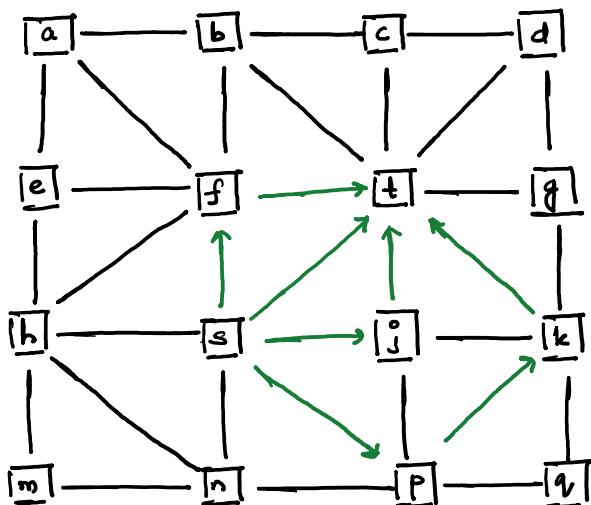
- a) Find maximum surviving flow
- b) Find maximum q -path flow where $q=2$



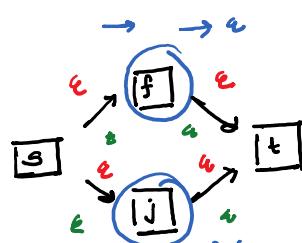
From HW1 when $\lambda = \infty$

$F(\infty)$: Max flow with current $u_{ij} = \boxed{48}$

$F(\epsilon)$: Max flow with $u_{ij} = \epsilon + \underline{(c_{ij})\epsilon}$
for small enough ϵ .

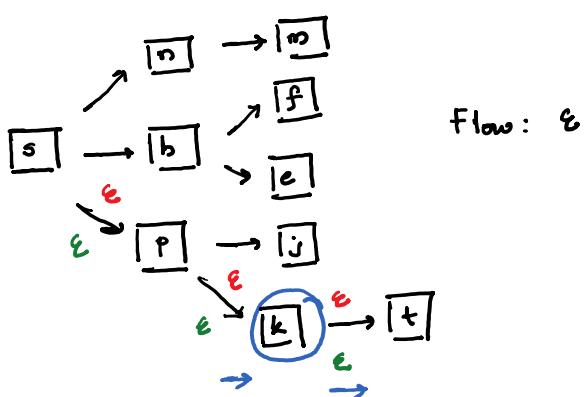


Using MKM method for finding maxflow

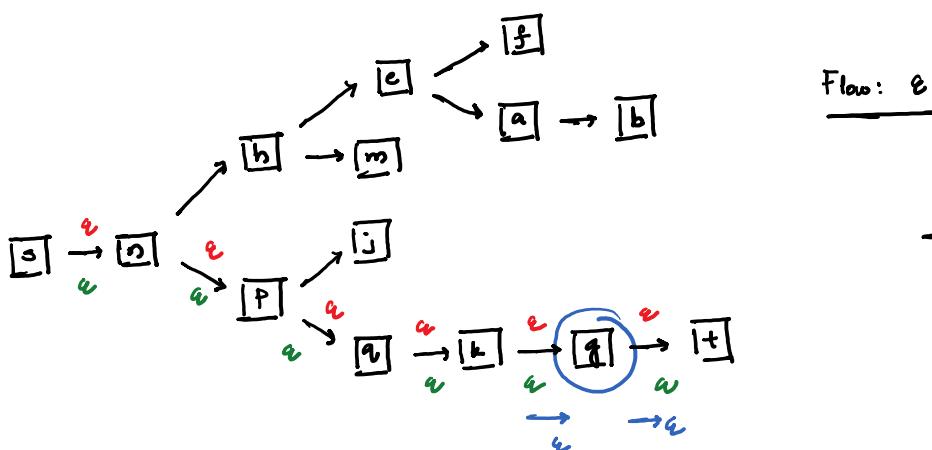
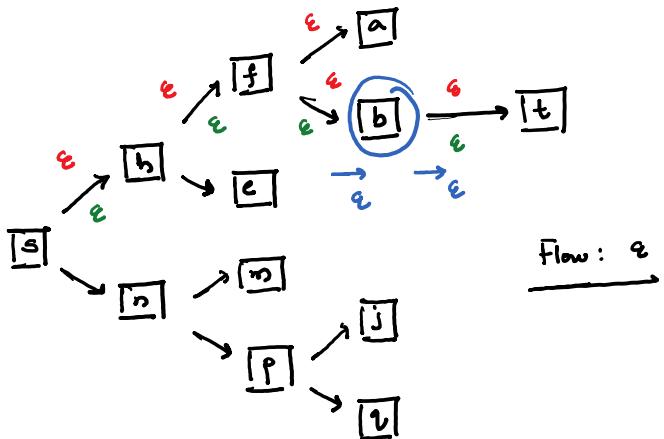
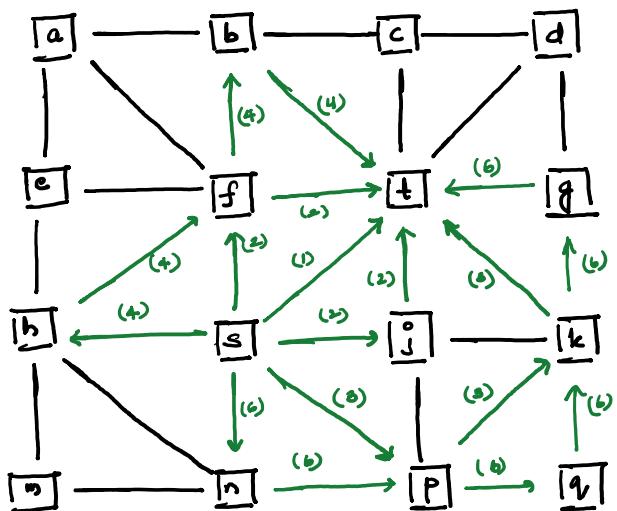


$\curvearrowleft [s-t]$ edge

Flow: $q + 2e$

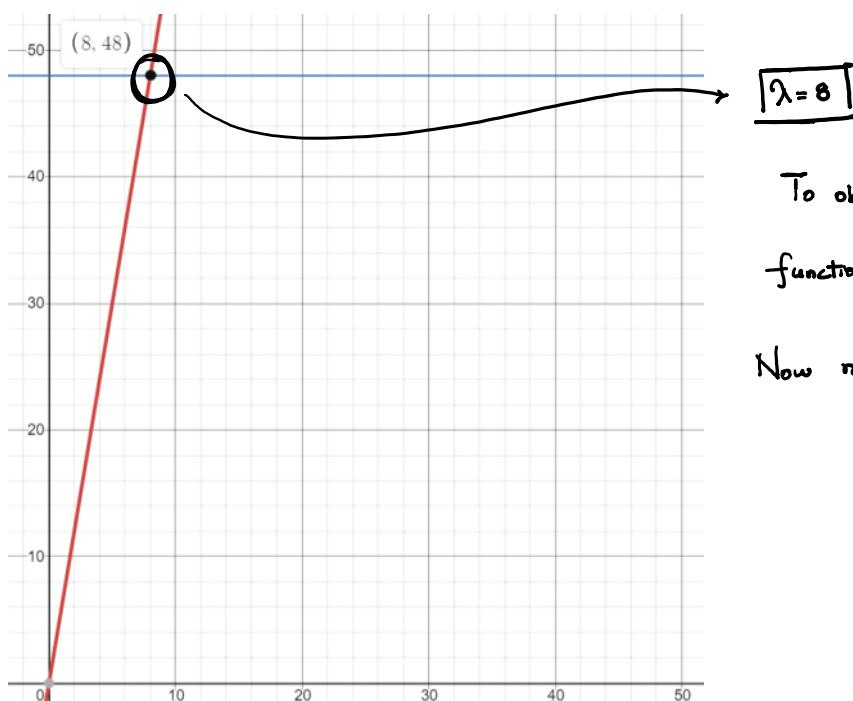


4



$$\text{Total flow} : 4\epsilon + 2\epsilon = 6\epsilon$$

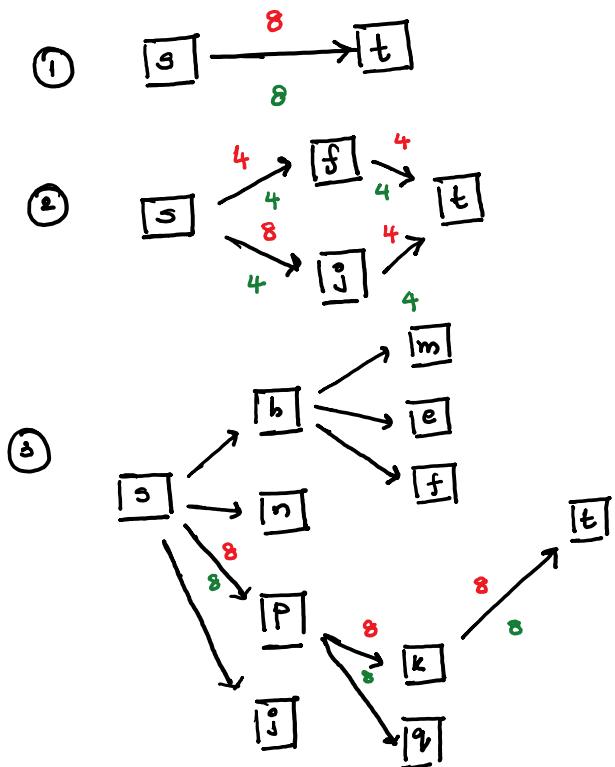
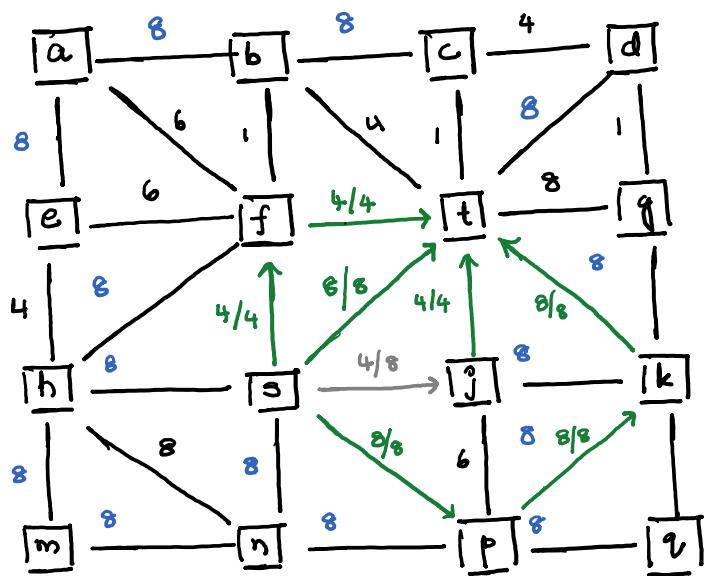
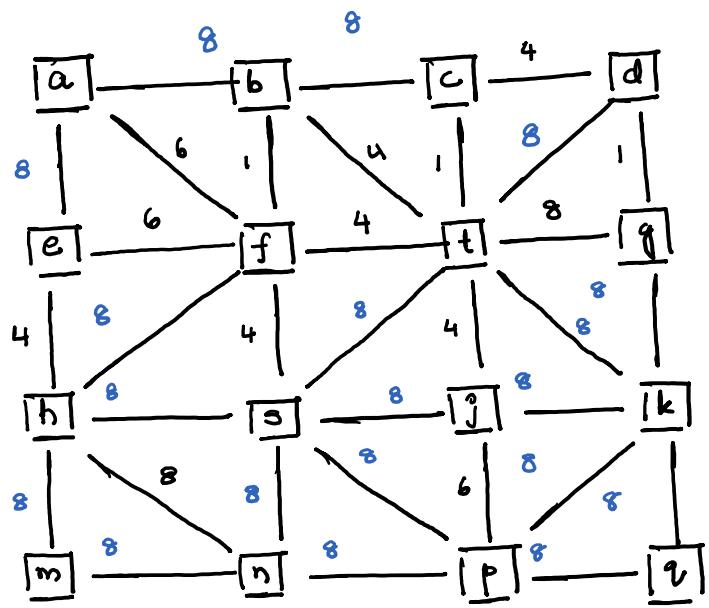
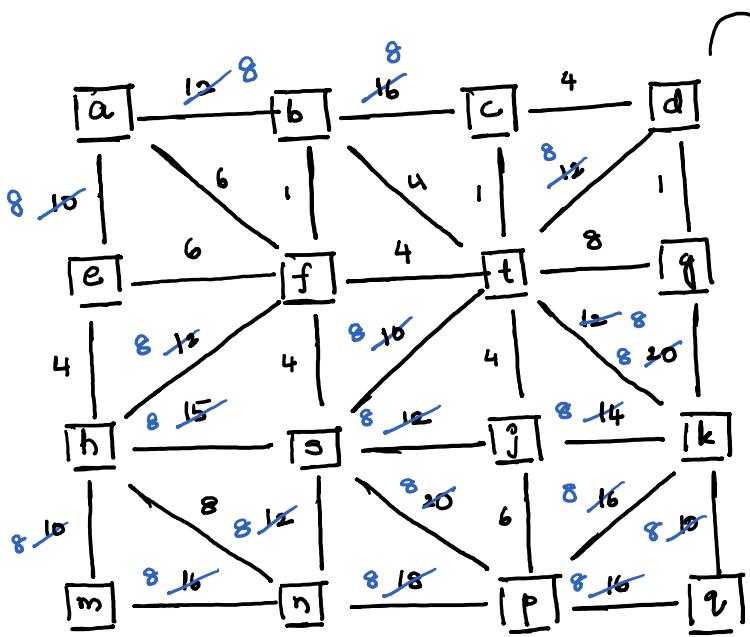
Plotting $F(\lambda)$ curve with values of $\lambda = \infty, \lambda = 0$ gives.

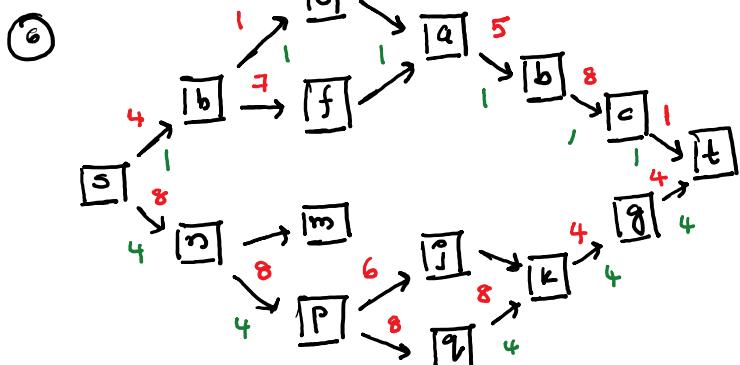
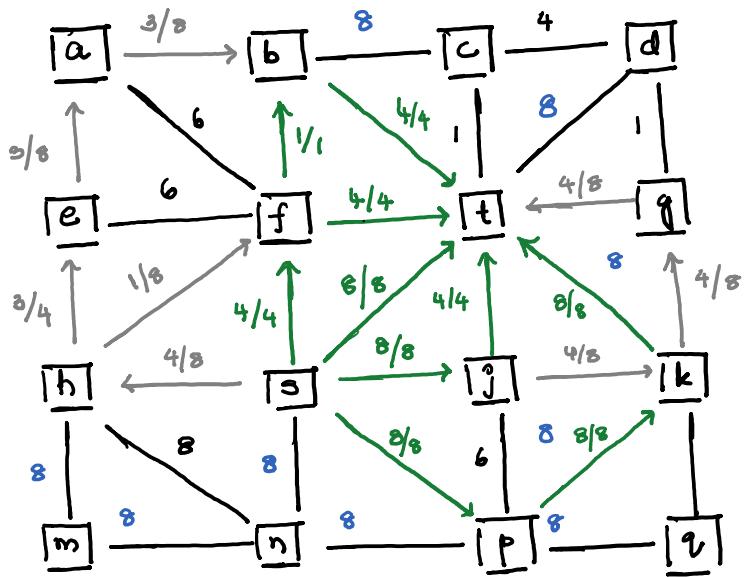
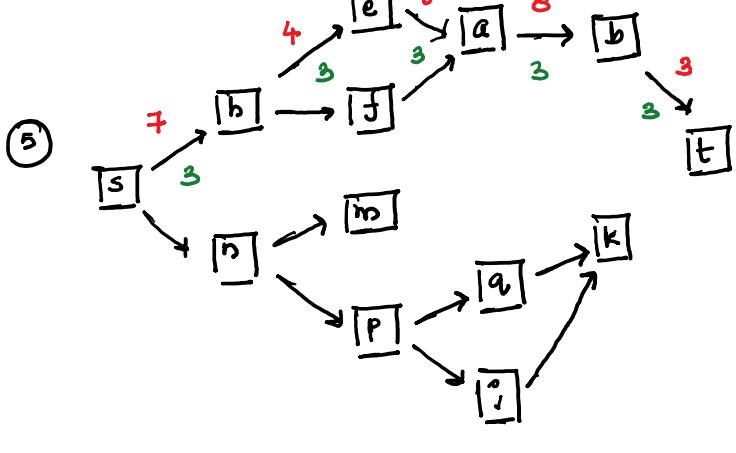
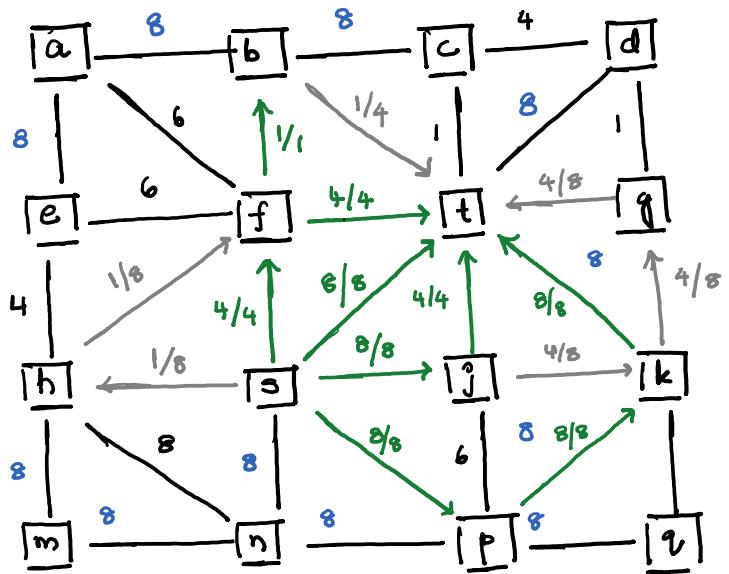
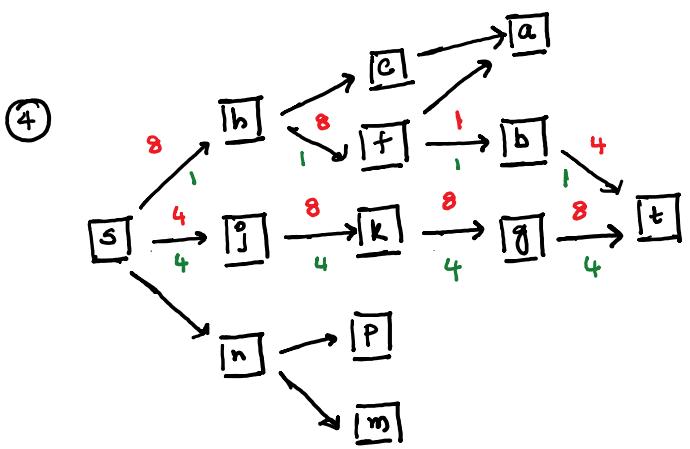
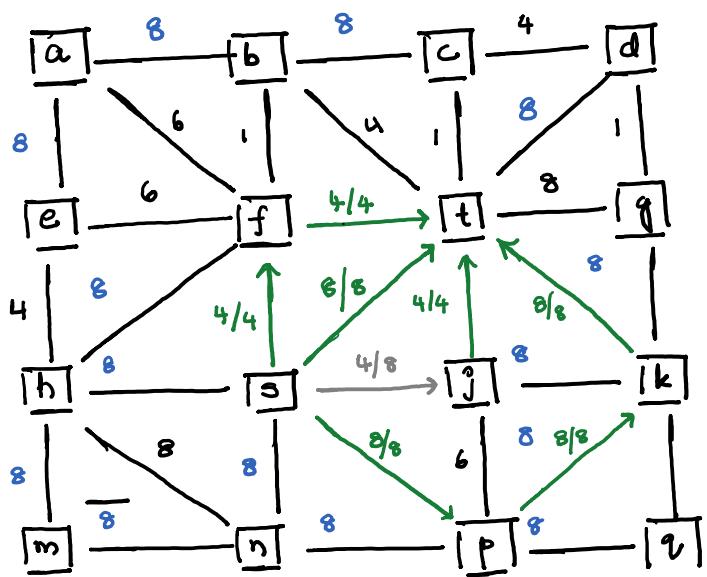


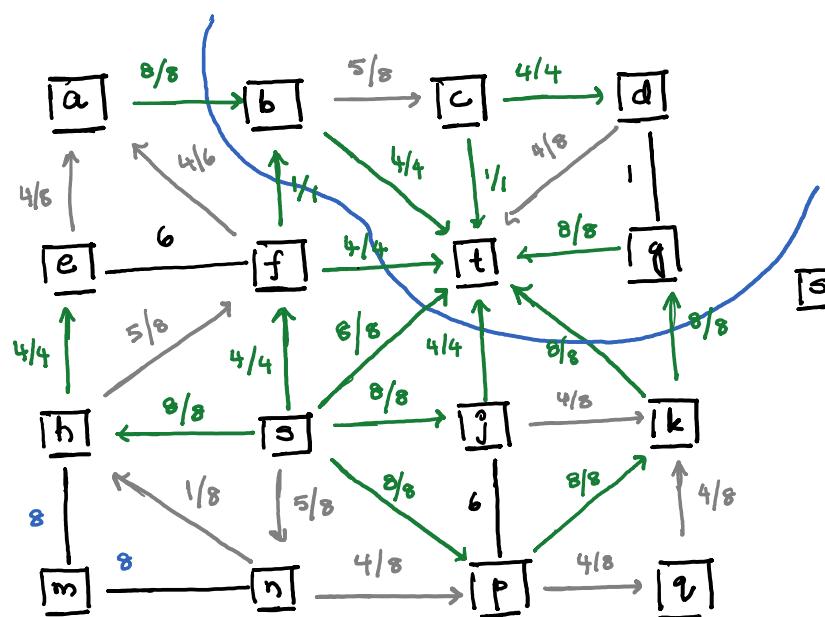
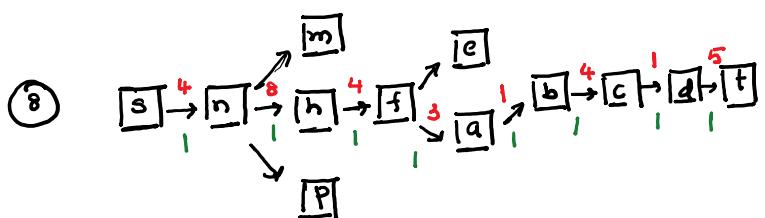
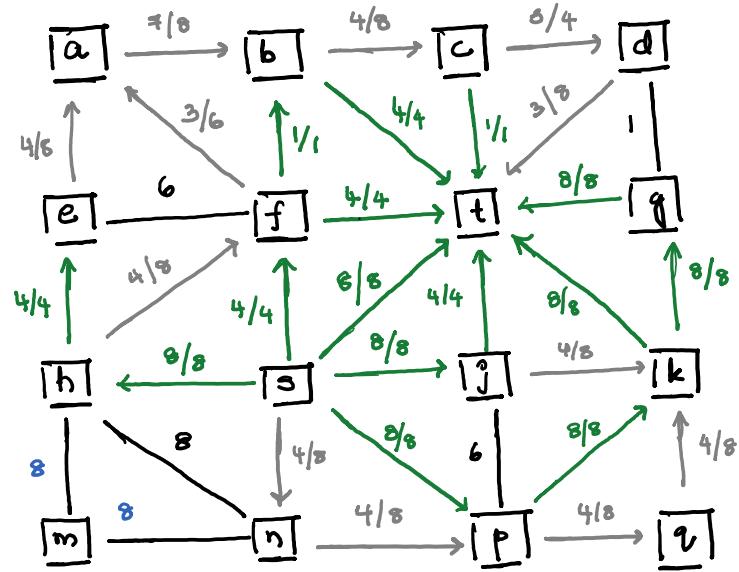
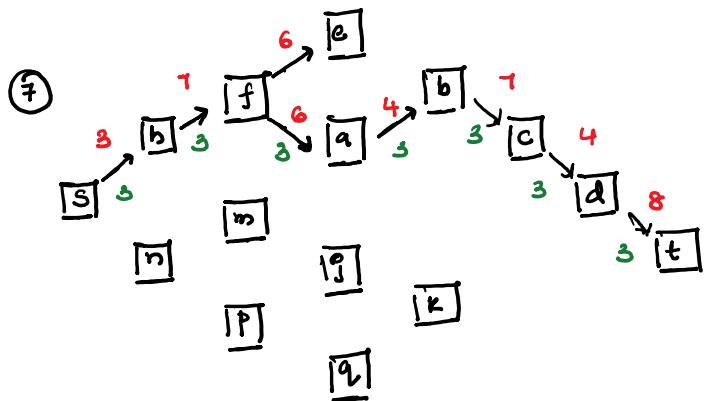
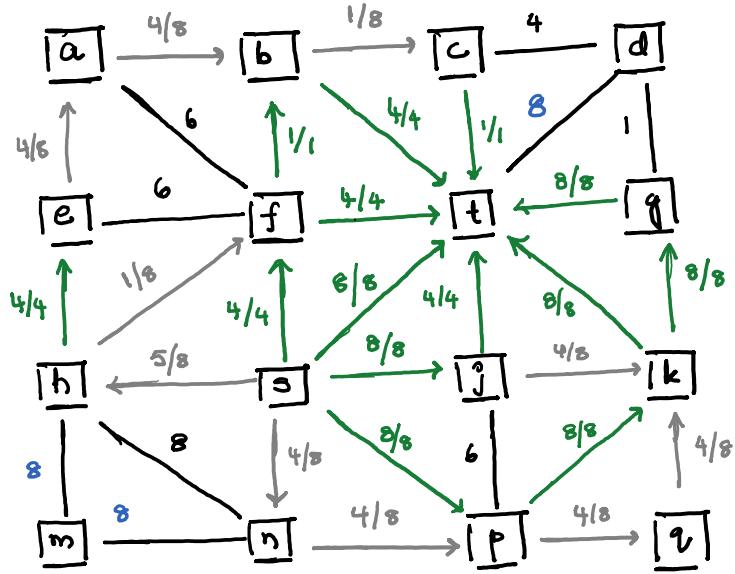
To obtain next linear functions lets use $\lambda = 8$

Now min [$w_{ij}, 8$]

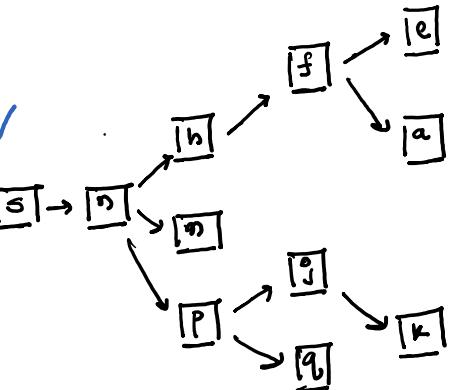
Finding max flow when capacities are $\min [u_{ij}, 8]$







M_{IN} COT :



(b, c, d, t, q)

$$\text{Flow obtained : } \underline{1+3} + \underline{1+4+3} + \underline{1+4} + \underline{8+8+8} = 24 + 8 + 9 = \boxed{41}$$

No. of edges in min cut with $\lambda = 8 \Rightarrow \underbrace{(a,b), (s,t), (r,t), (k,q)}_6 \rightarrow \boxed{41} \leftarrow \boxed{\text{Slope}}$

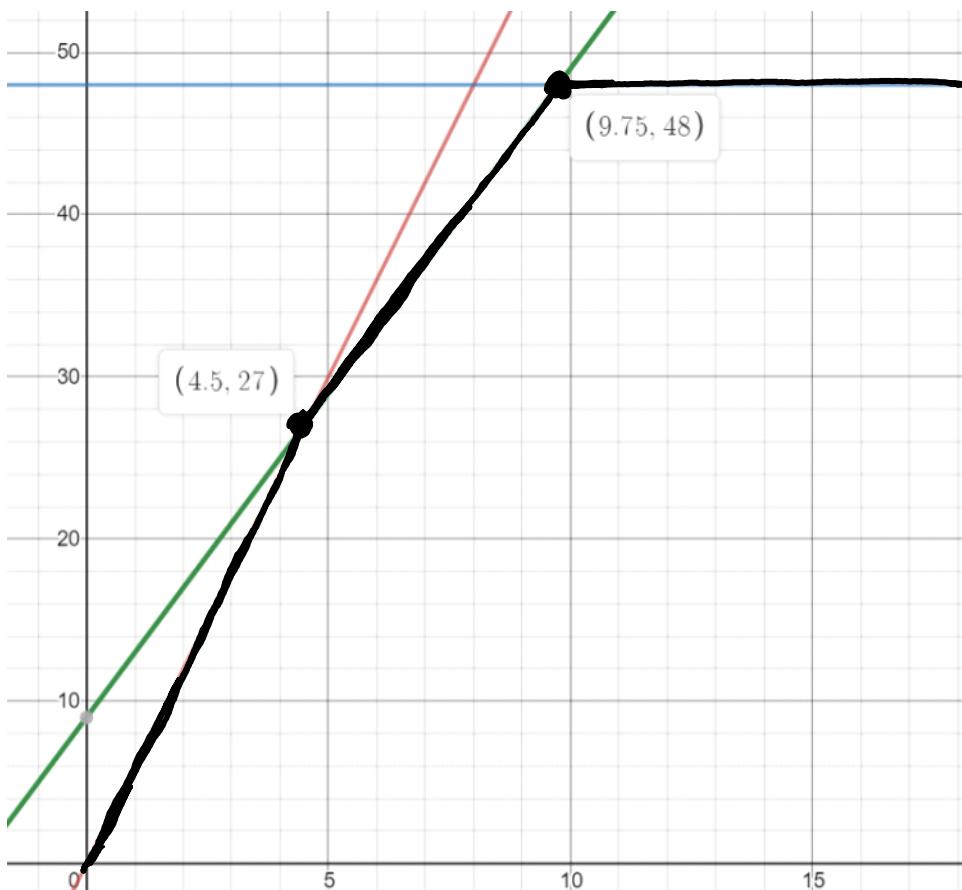
Equations of line : $y = 4x + b$

$$\left. \begin{array}{l} \\ \end{array} \right\} \quad \begin{array}{l} \\ y = 4x + 9 \end{array}$$

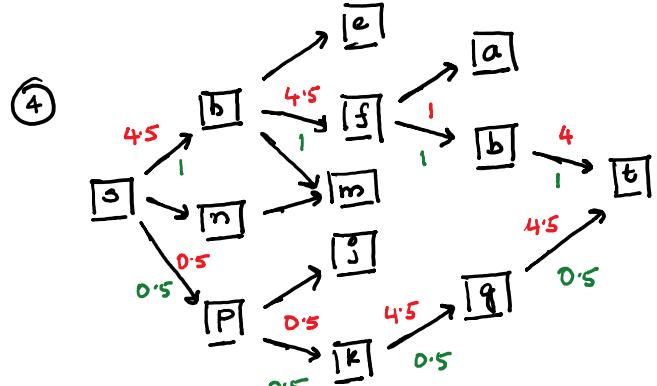
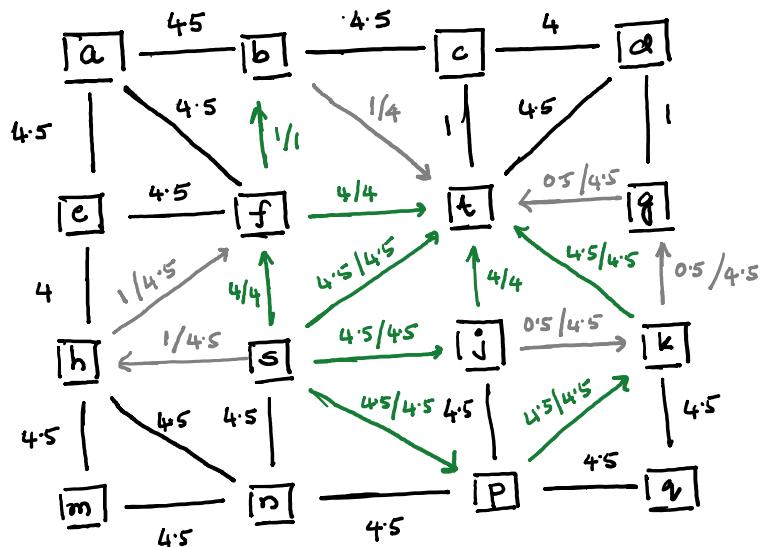
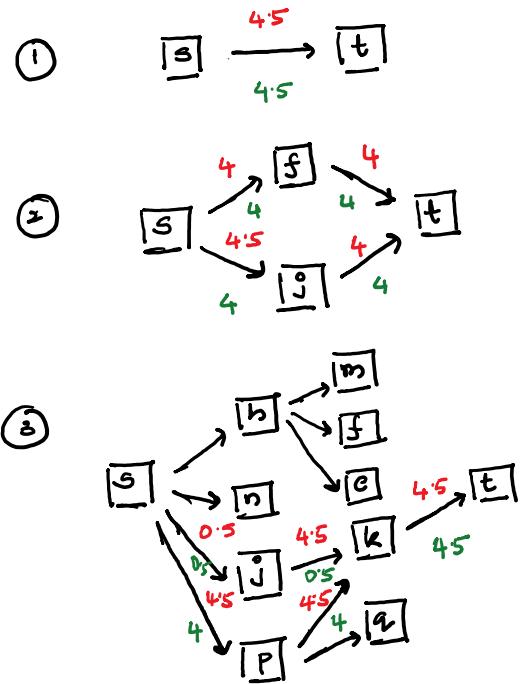
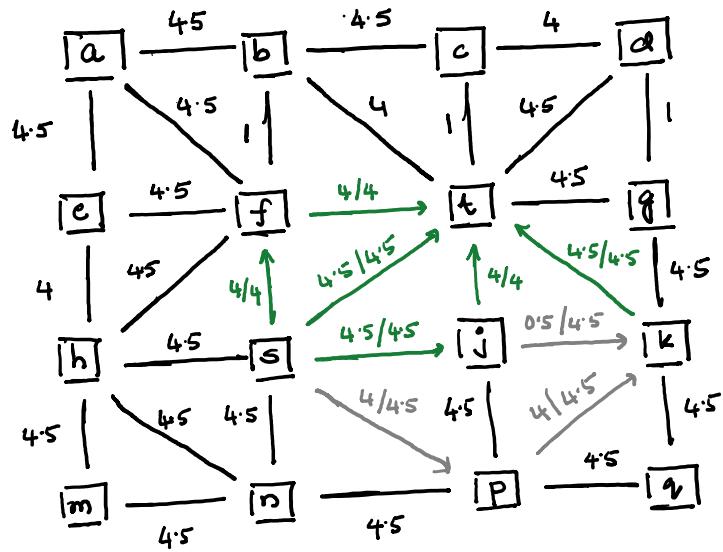
$$\textcircled{C} \quad x = 8, \quad y = 41 \Rightarrow b = 9$$

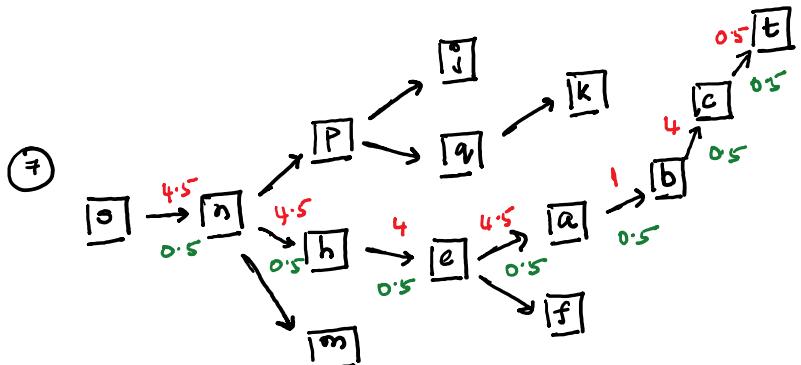
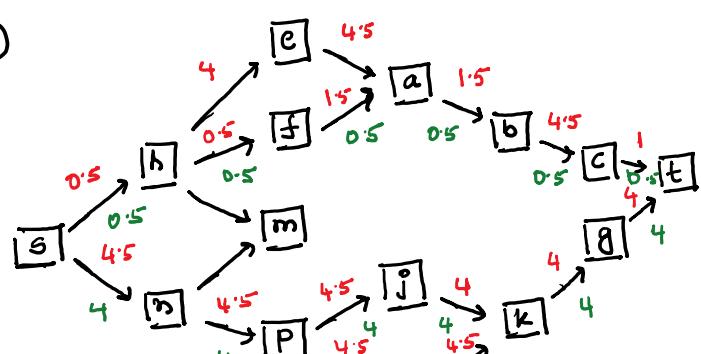
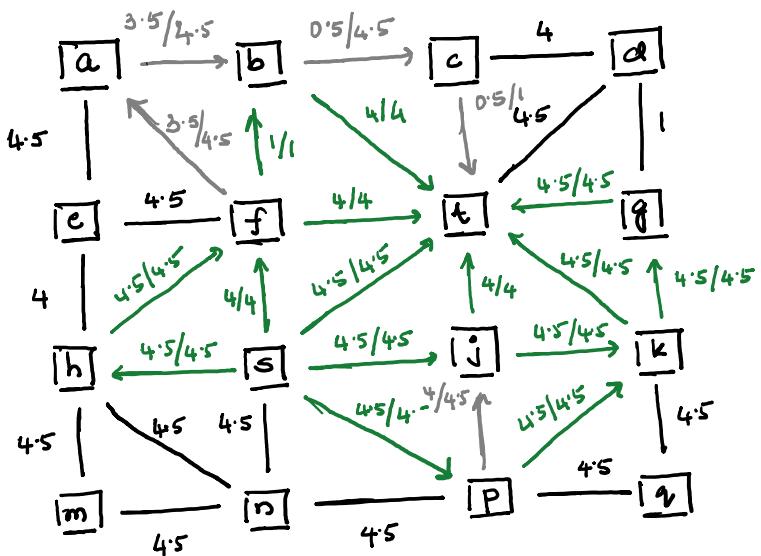
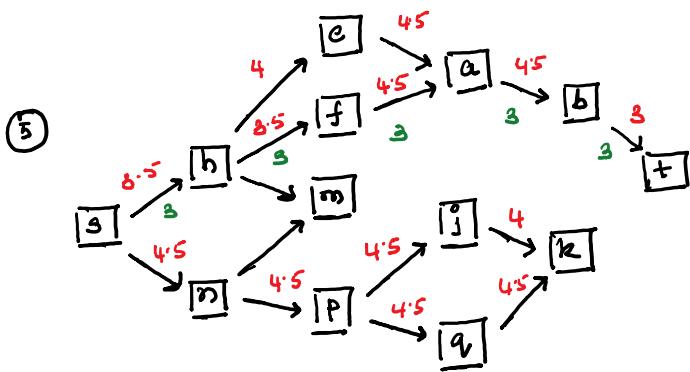
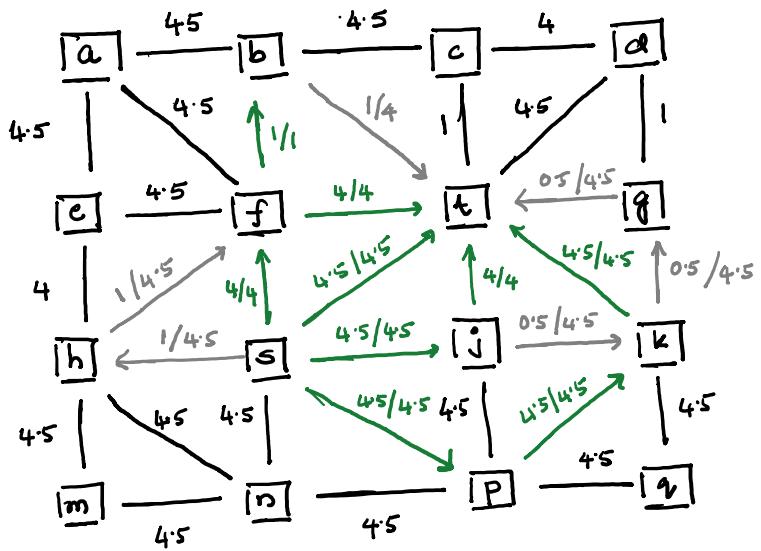
Point of intersection @ $y = 6x \rightarrow 6x = 4x + 9 \Rightarrow x = \frac{9}{2} = \underline{4.5}$

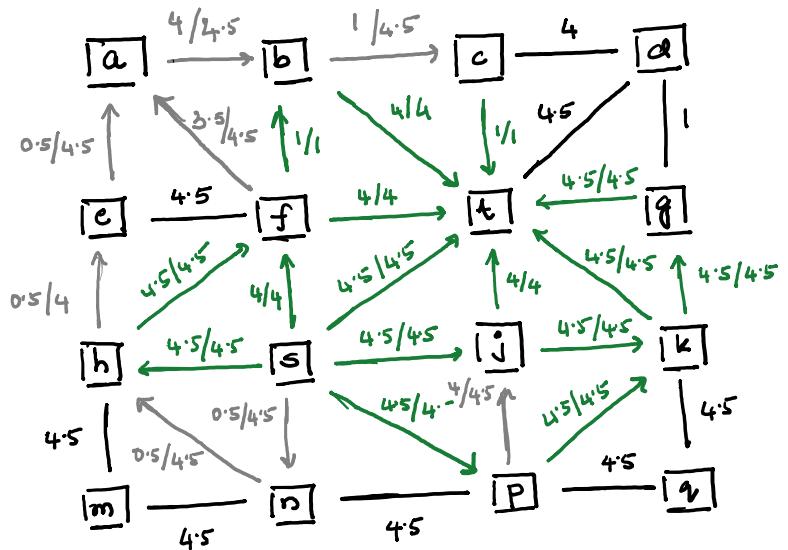
Point of intersection @ $y = 4x \Rightarrow 4x + 9 = 48 \rightarrow x = \frac{39}{4} = \underline{9.75}$



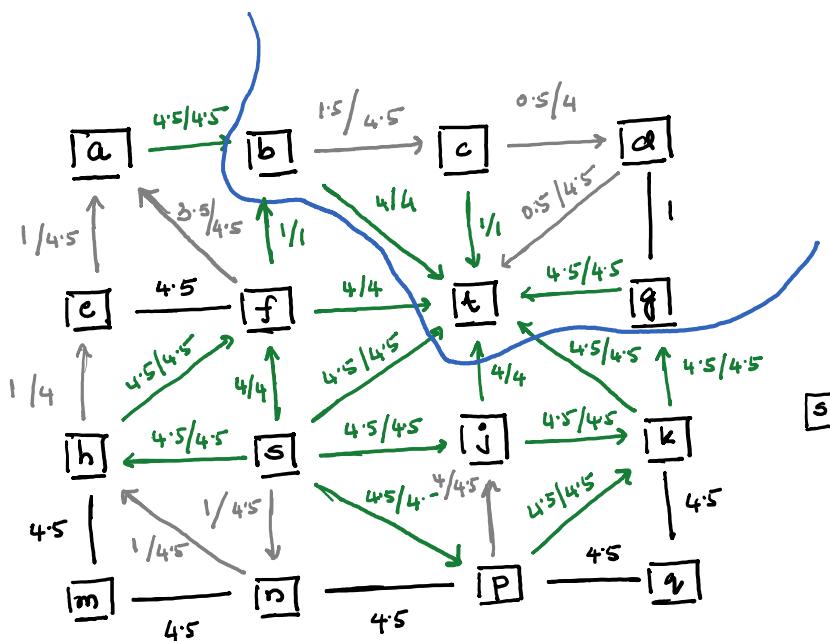
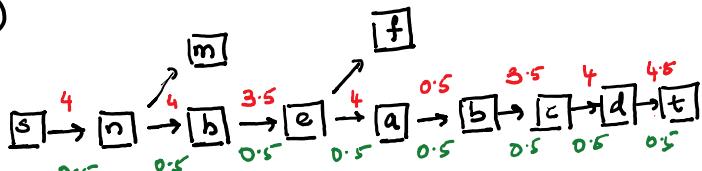
Consider the graph $\min [w_{ij}, 4.5]$



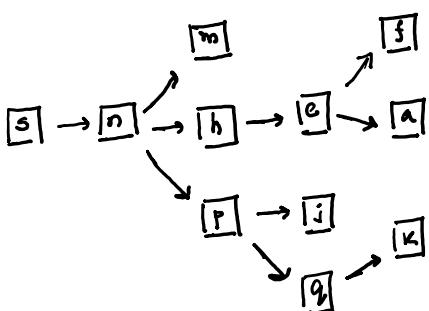




8



Min-cut :



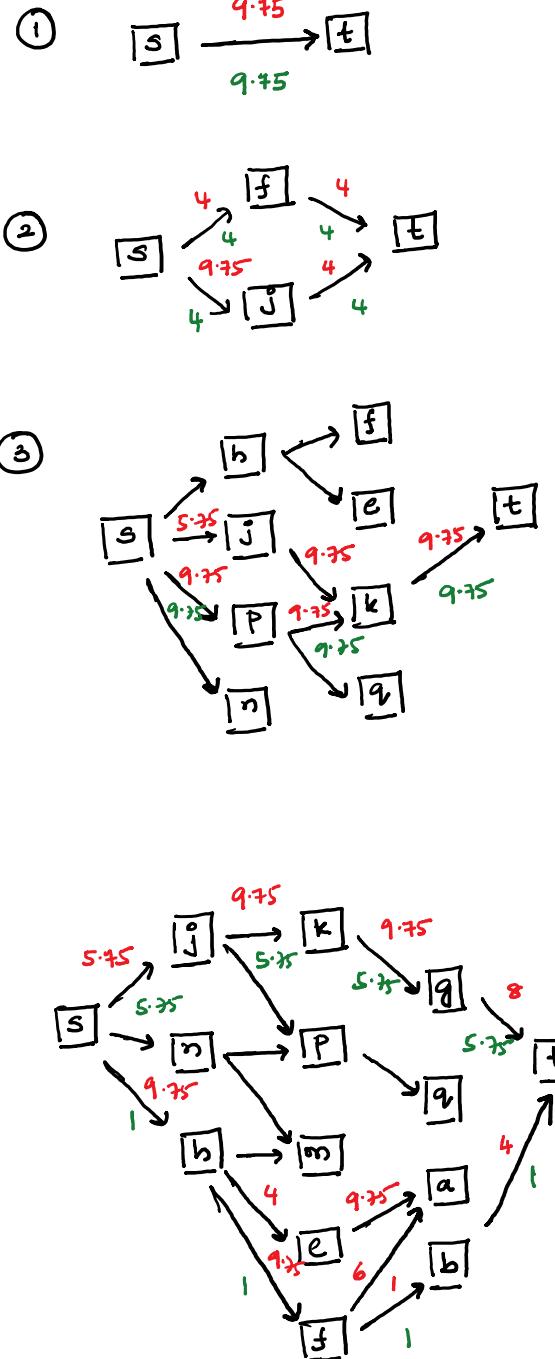
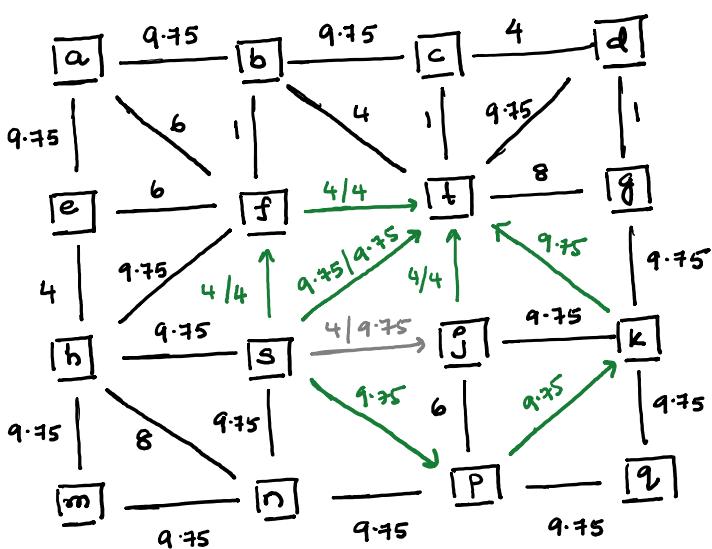
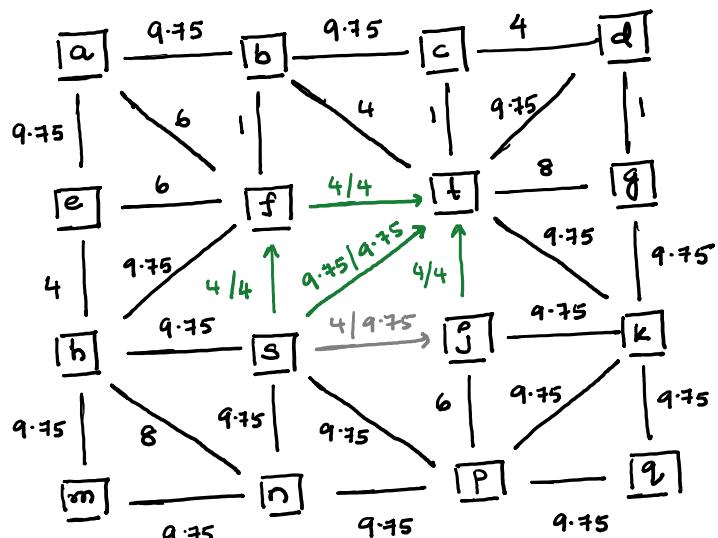
$$\text{Flow : } 27 \quad | \quad \begin{array}{r} \underline{0.5 + 0.5 + 4 + 0.5 + 3 + 0.5 + 4.5} \\ 1 + 4 + 4 + 4.5 = 27 \end{array}$$

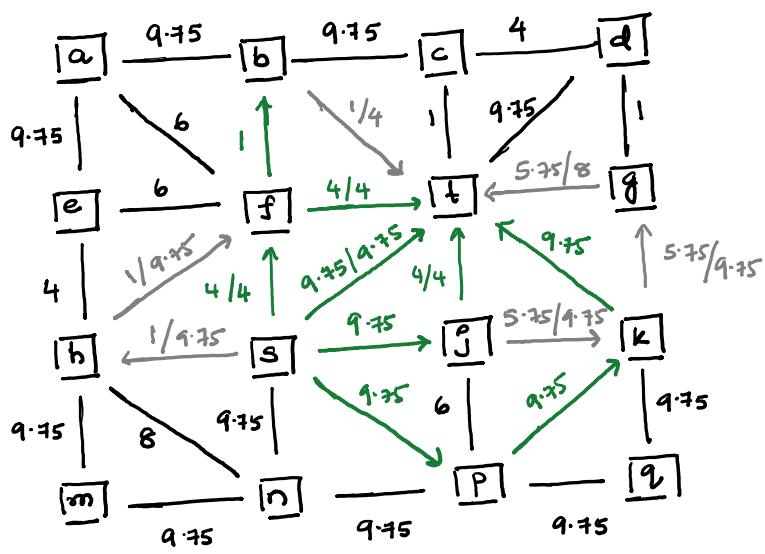
@ $Z = 4.5$ Flow = 27

$$42+9 = 4(4 \cdot 5) + 9 = 27$$

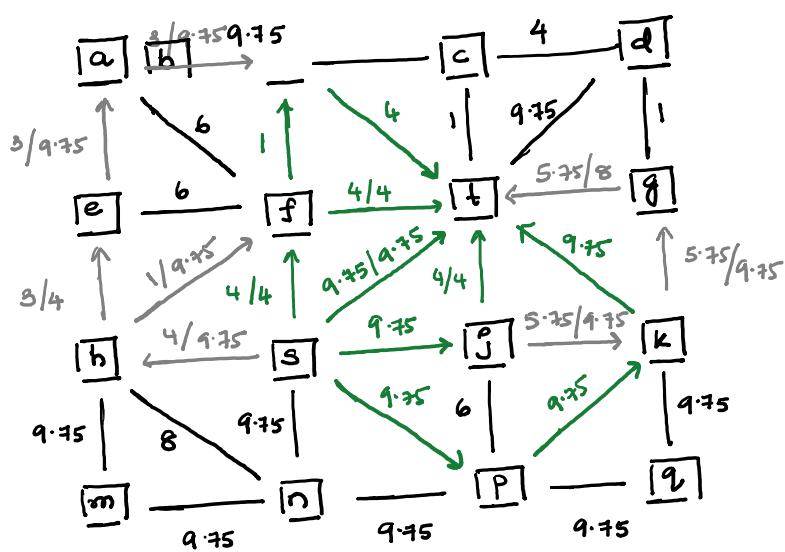
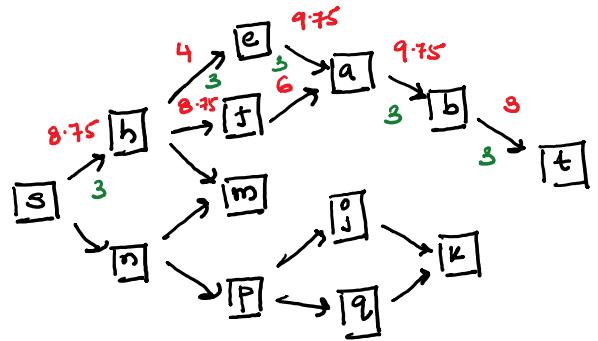
No new linear function is added

Consider $\min [u_{ij}, 9.75]$

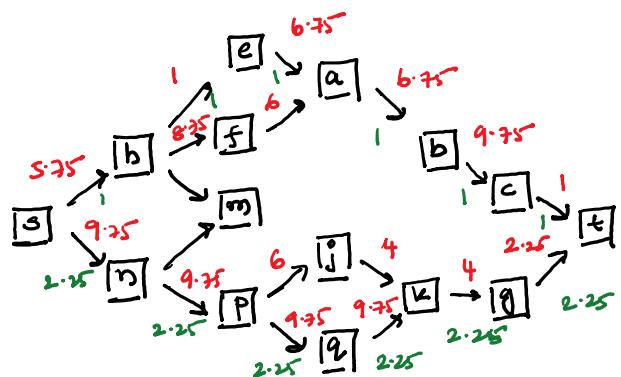


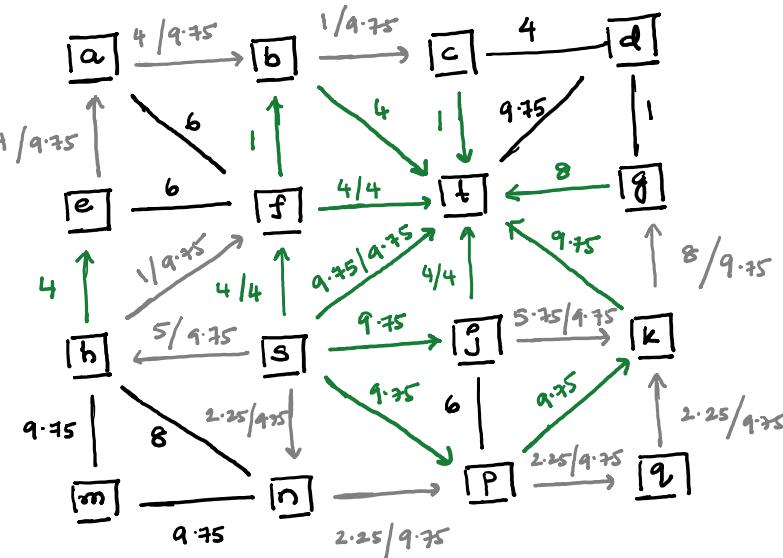


⑤

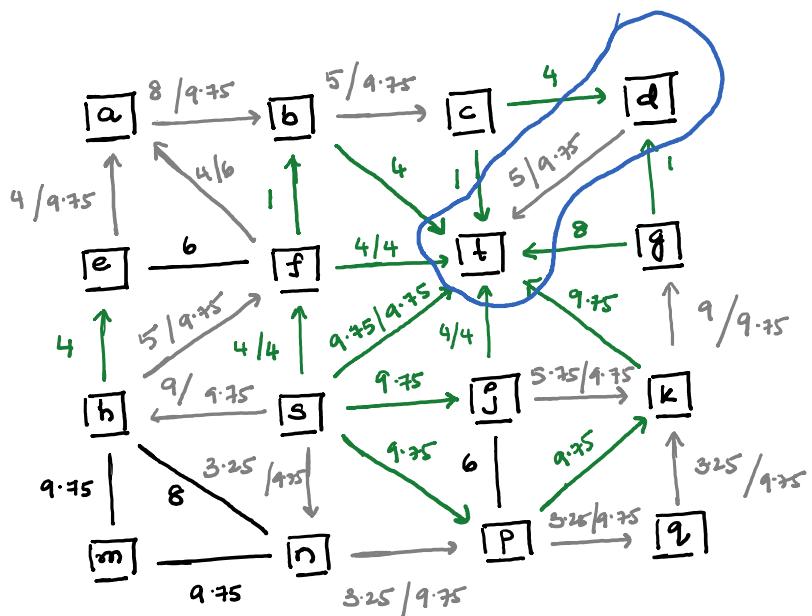
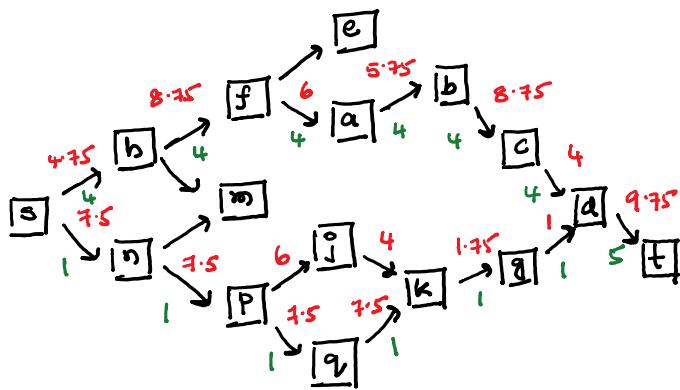


⑥



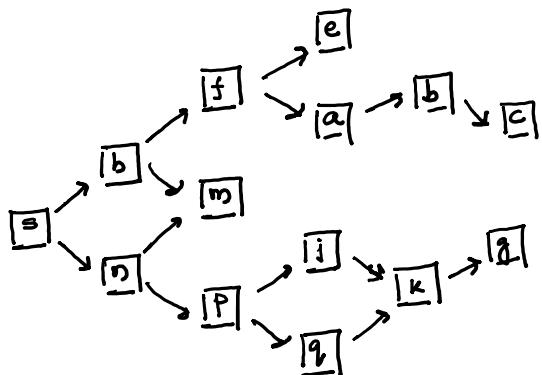


7



8

Min-cut :



Flow : 45.5

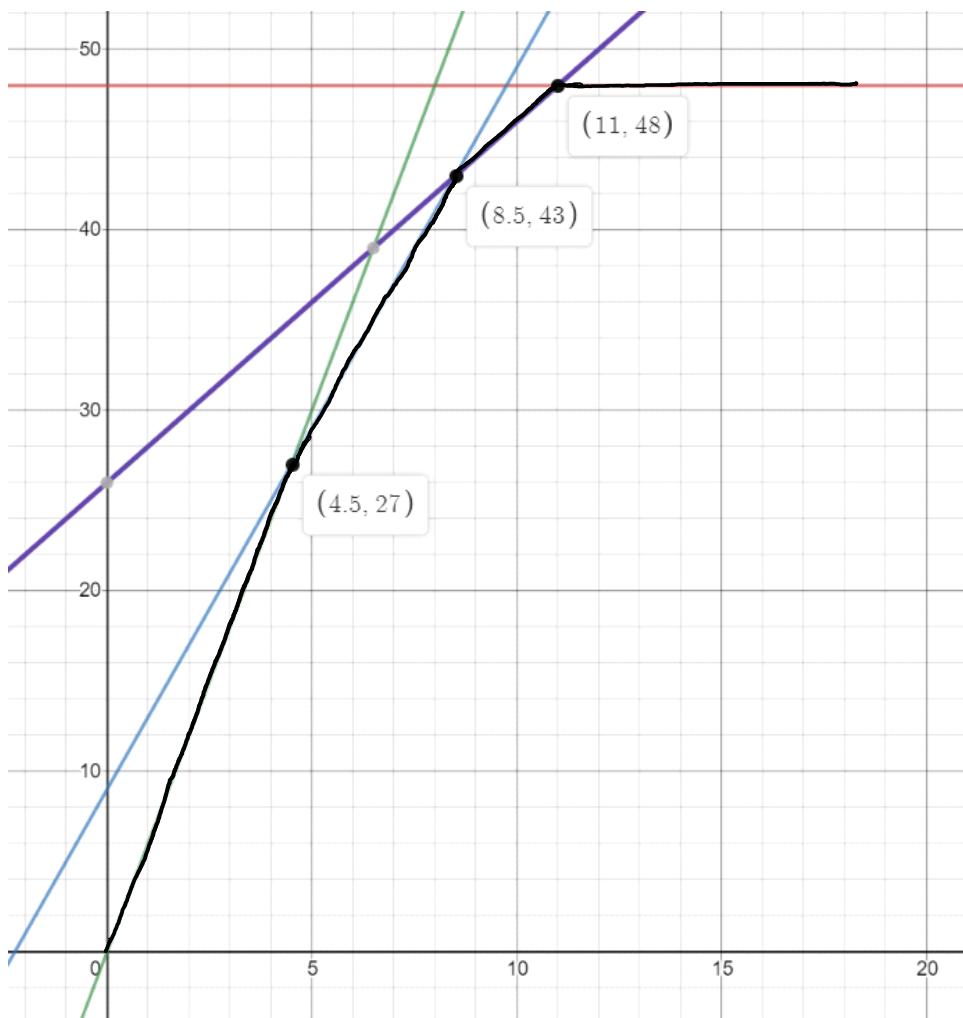
No. of edges with $\lambda = 9.75 \Rightarrow (s, t), (k, t)$

$$\Rightarrow y = 2x + b \quad @ \quad x = 9.75 \Rightarrow \underline{\text{Flow} = 45.5}$$

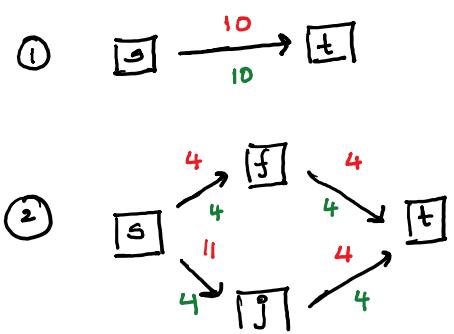
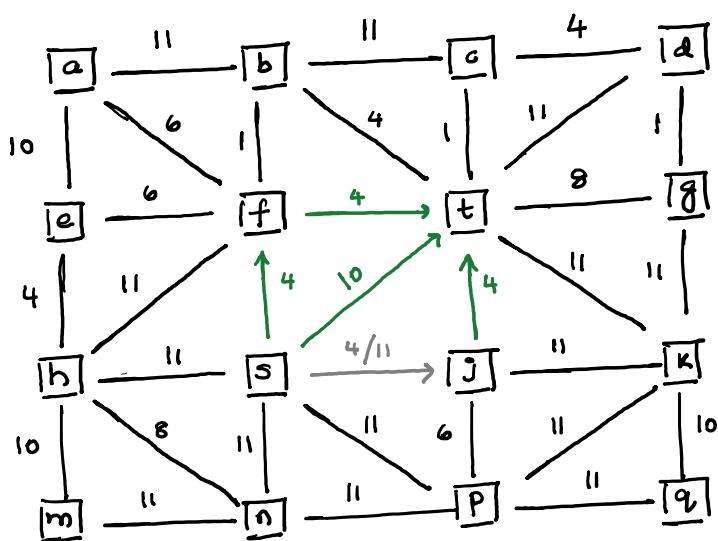
$$45.5 = 2(9.75) + b \Rightarrow \boxed{b = 26}$$

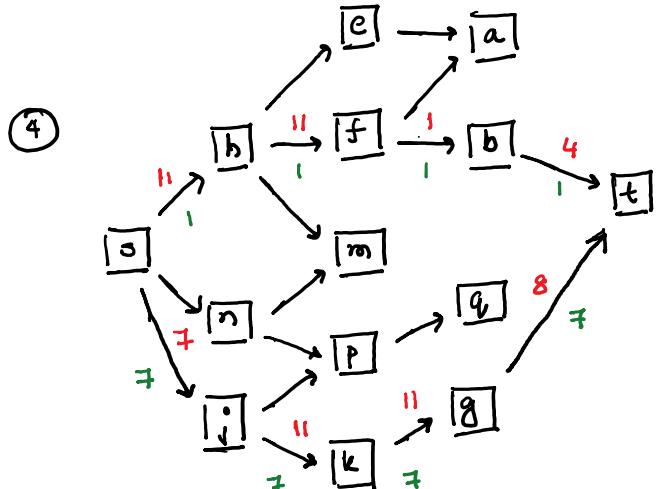
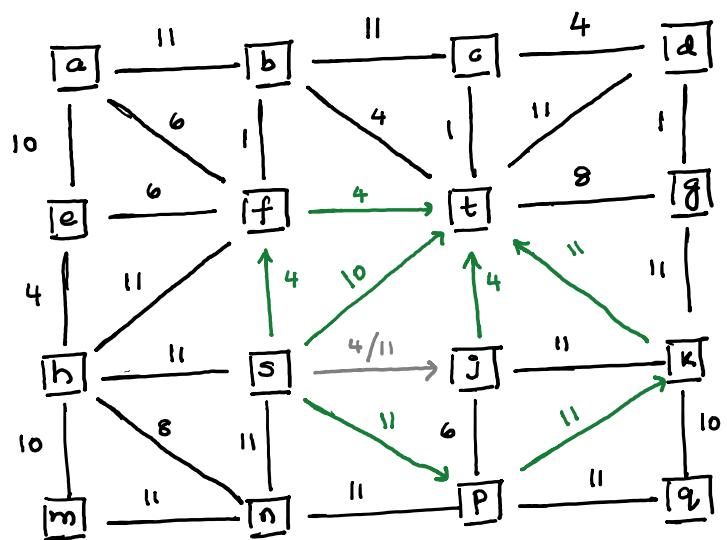
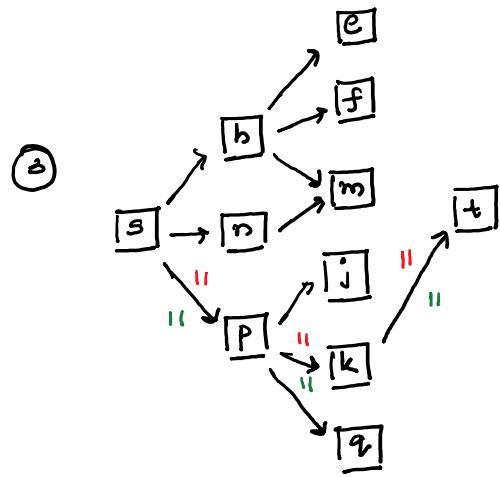
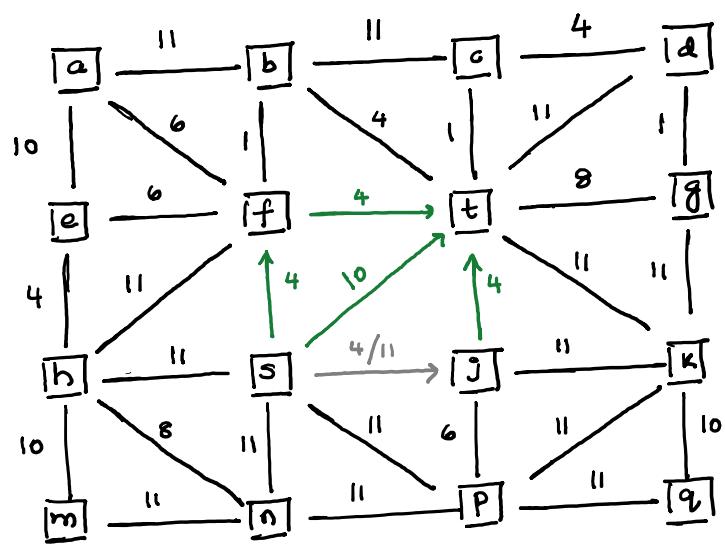
$$\text{Points of intersection: } y = 48 \Rightarrow 2x + 26 = 48 \Rightarrow \boxed{x = 11}$$

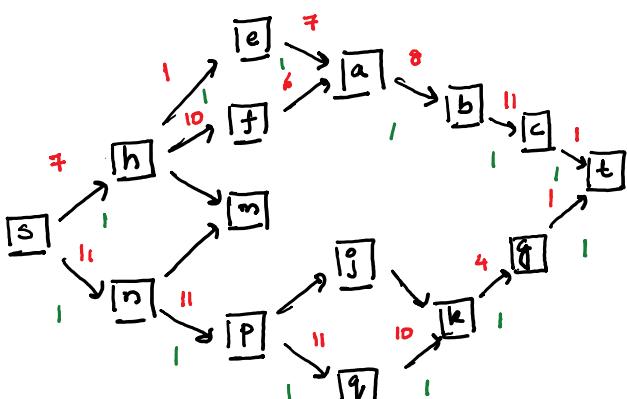
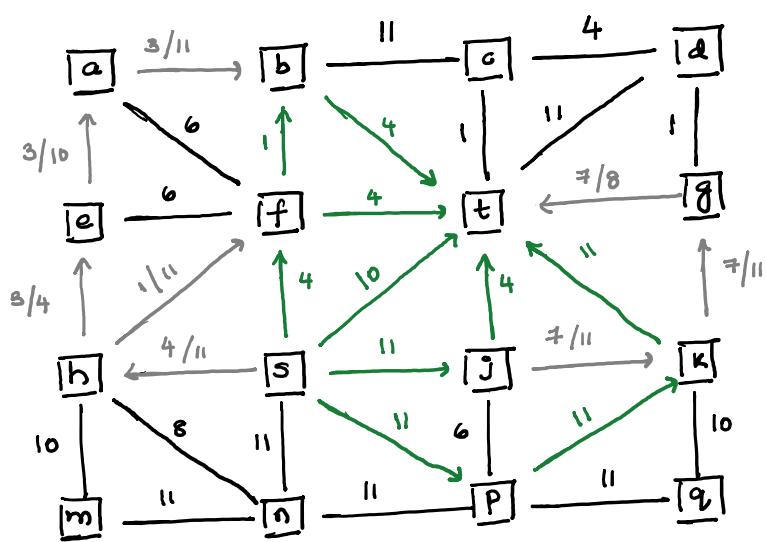
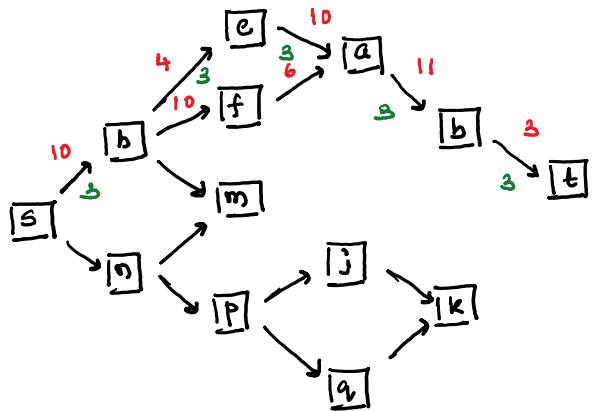
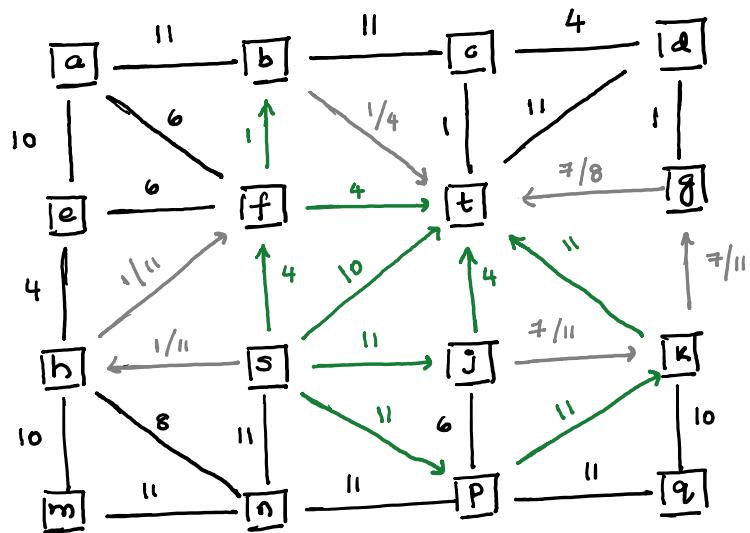
$$4x + 9 = 2x + 26 \Rightarrow 2x = 17 \Rightarrow \boxed{x = 8.5}$$

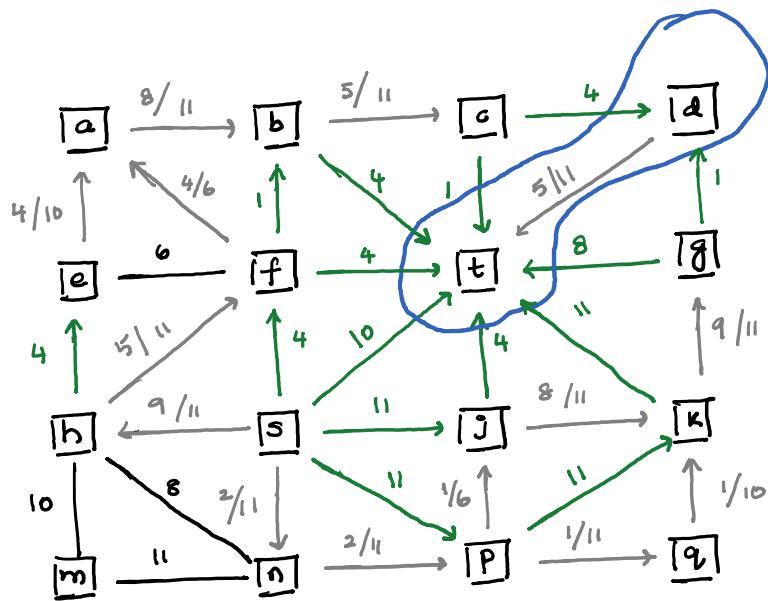
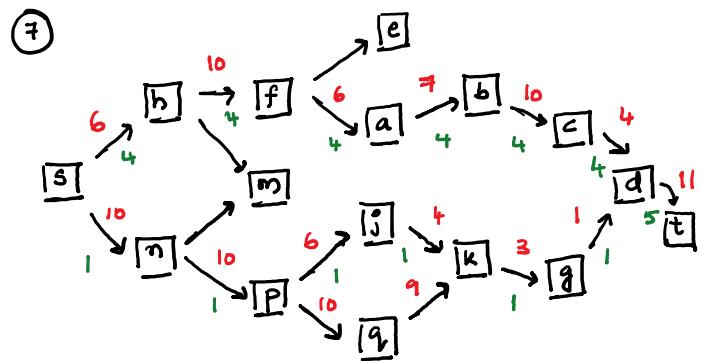
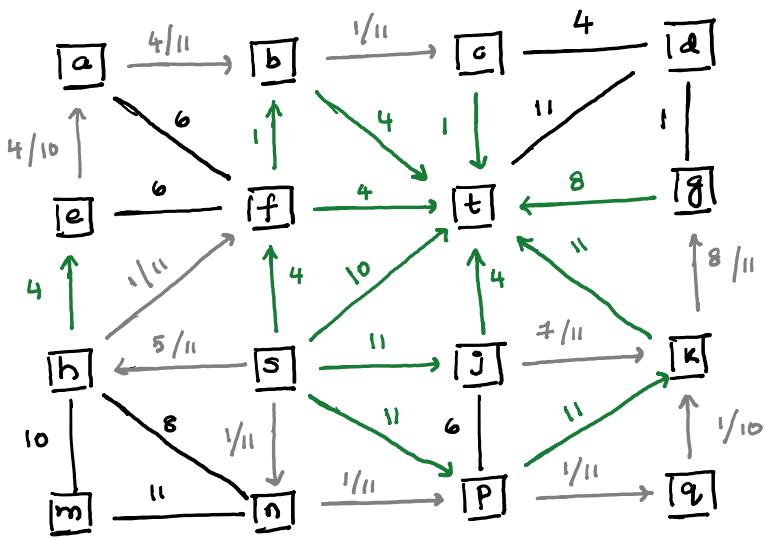


Consider $\min [u_{ij}, 11]$









$$y = x + b \Rightarrow \textcircled{2} \quad z=11, \text{ Flow: } 47$$

$$\Rightarrow \boxed{b=36}$$

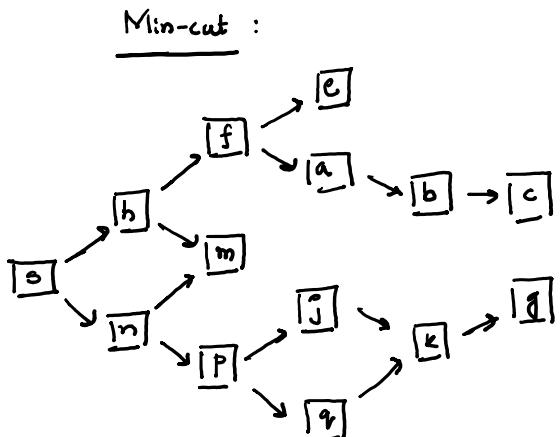
Points of intersection:

No. of edges with $\lambda=11$: (k, t)

$$48 = x+36 \Rightarrow \boxed{x=12}$$

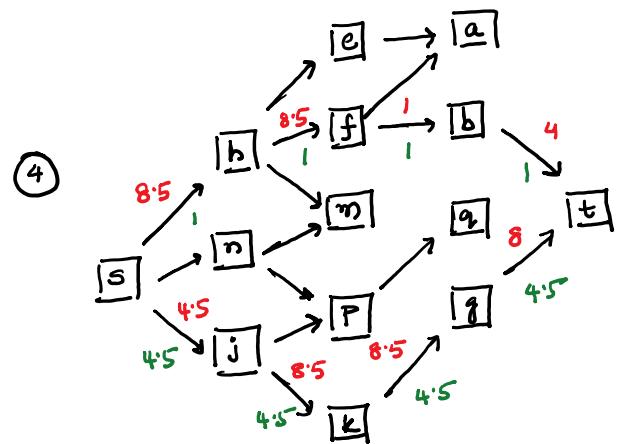
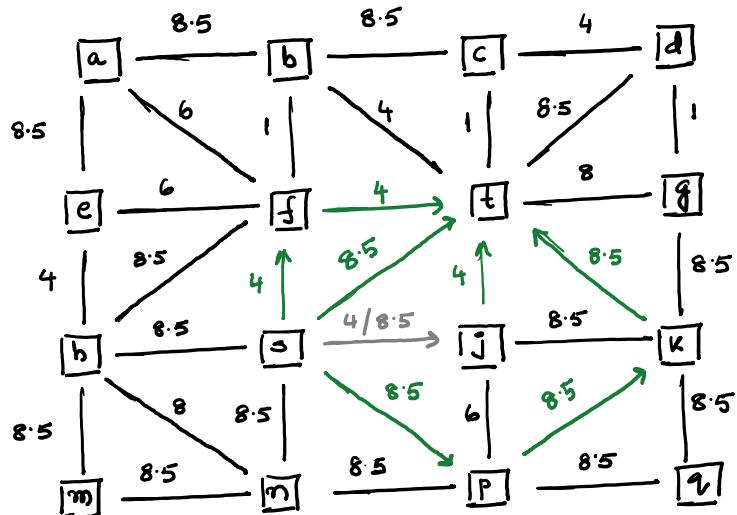
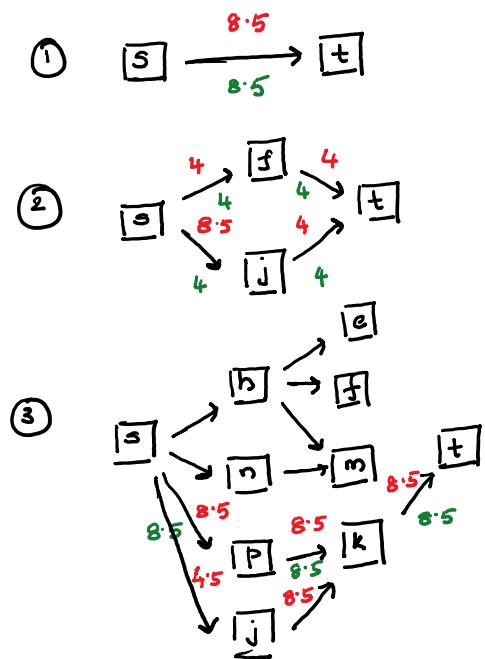
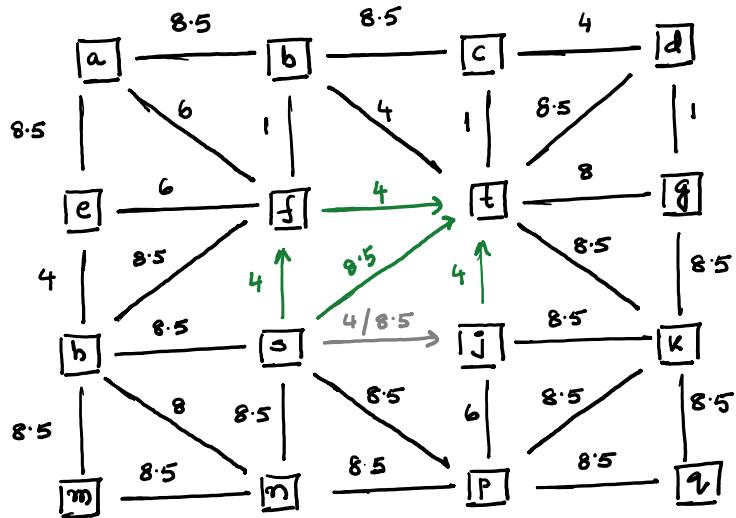
$$\underline{\textcircled{2} \quad \lambda=12 \iff F(\infty)}$$

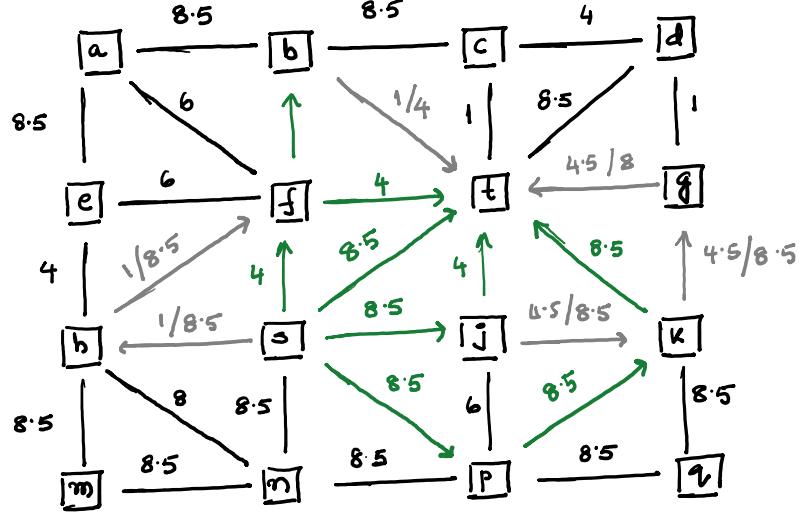
$$2x+26 = x+36 \Rightarrow \boxed{x=10}$$



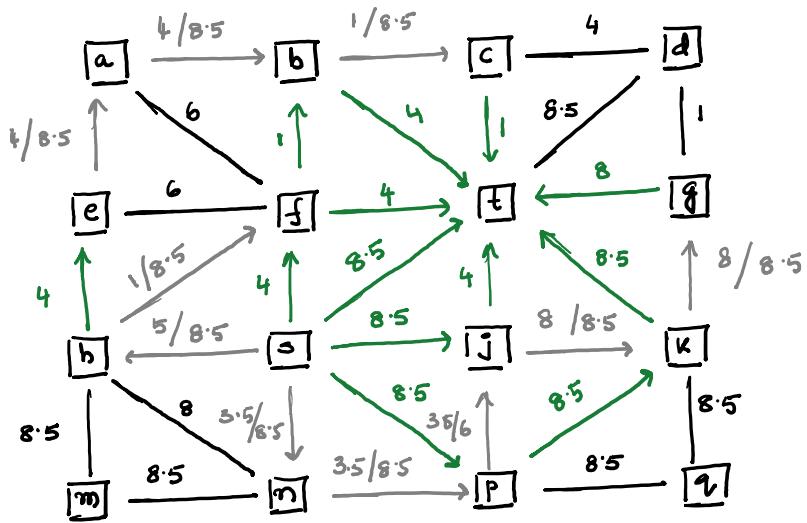
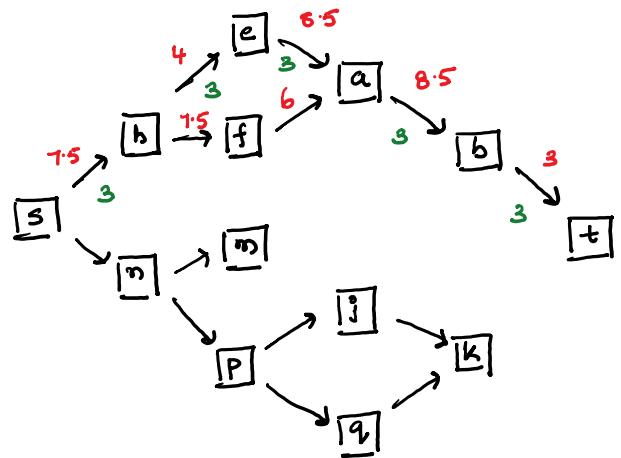
Flow : 47

Consider $\min [u_{ij}, 8.5]$

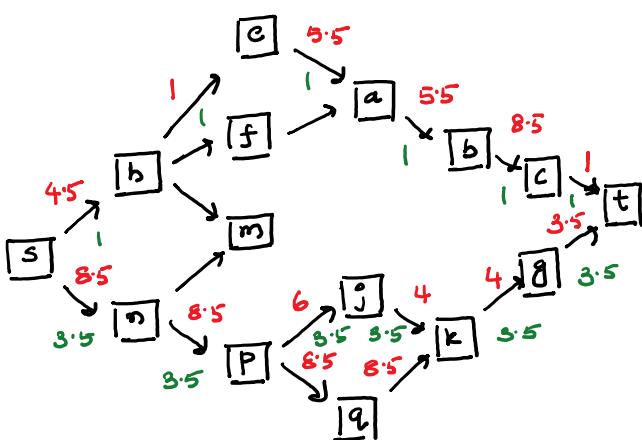


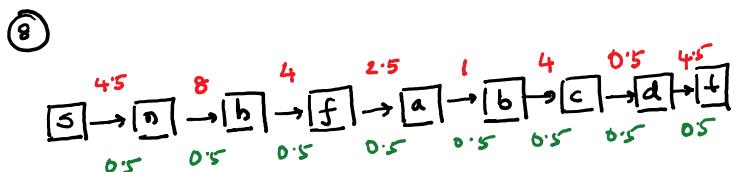
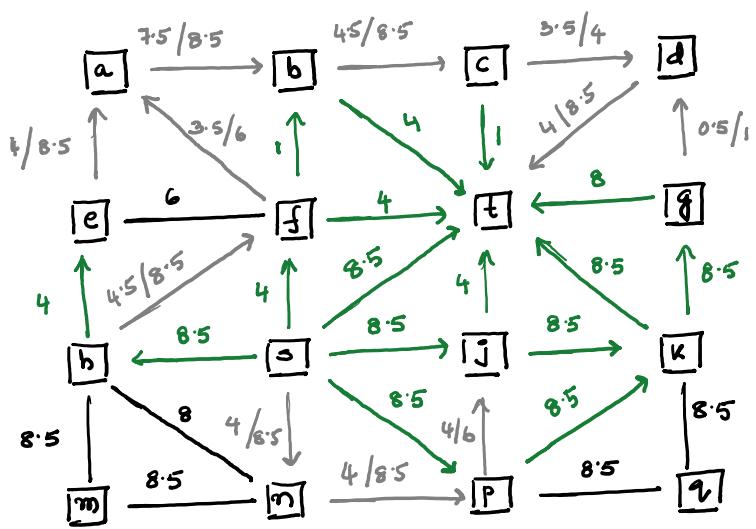
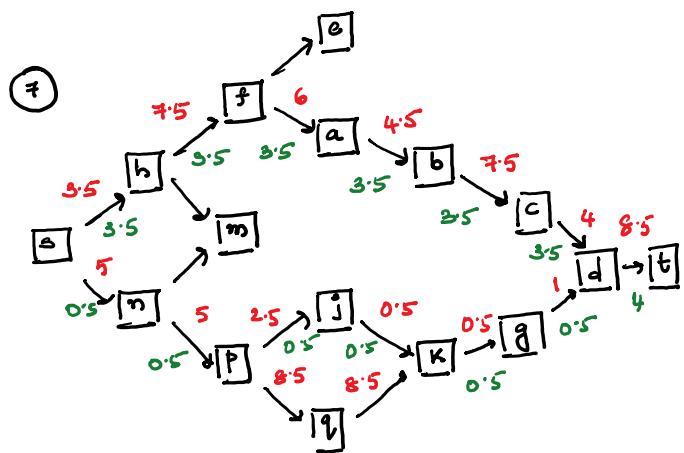
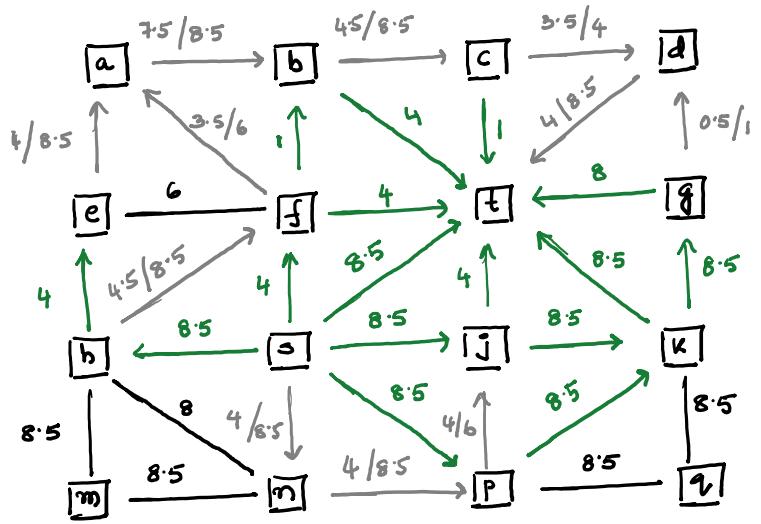


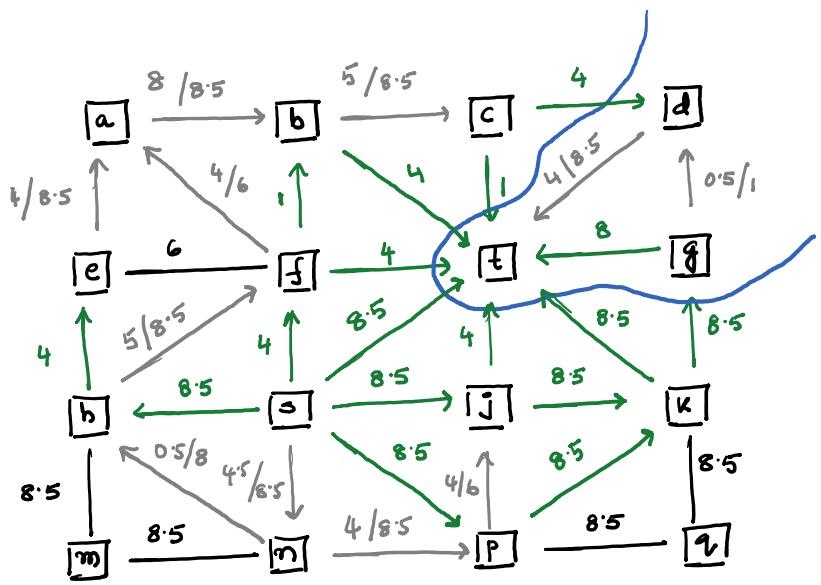
5



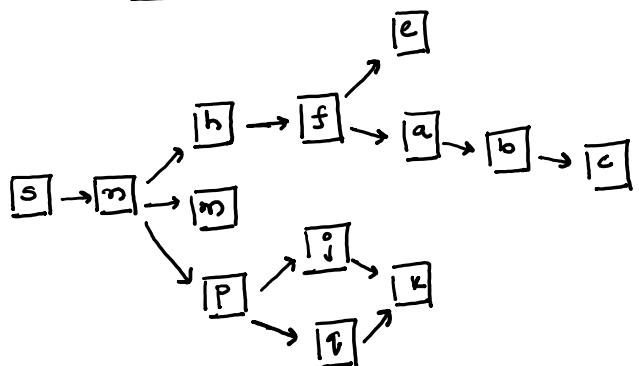
6







Min-cut :



No. of edges with $\lambda = 8.5$

$$(s,t), (k,t), (k,g) \Rightarrow 3$$

$$\text{Flow: } 42.5 \quad | \quad 0.5 + 4 + 3.5 + 1 + 3 + 4.5 + 1 + 8.5 \\ + 4 + 4 + 8.5 = 42.5$$

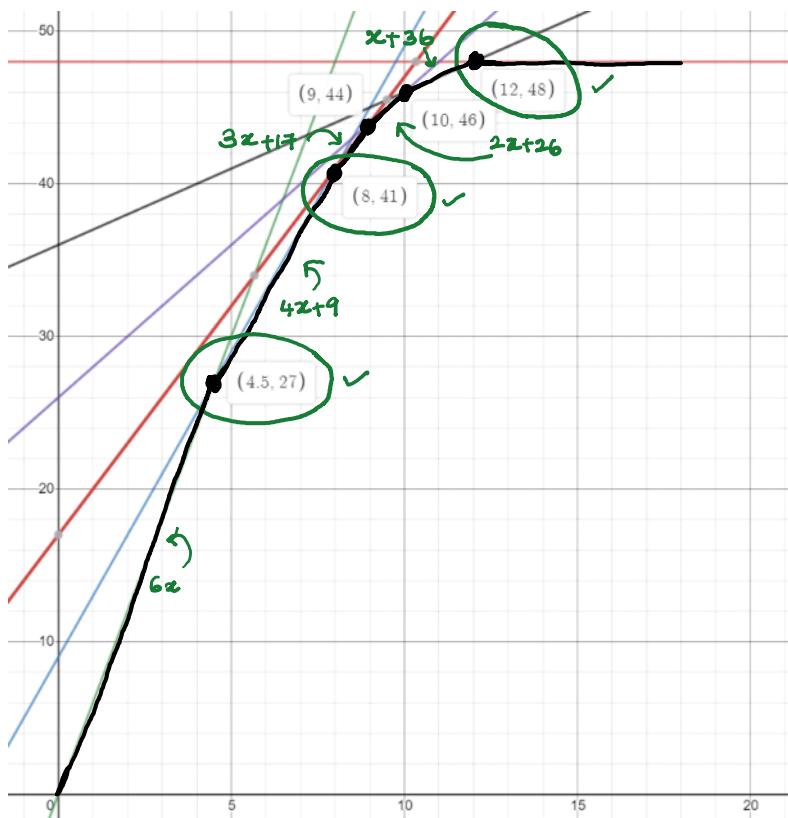
$$y = 3x + b \quad @ \quad x = 8.5 \quad \text{Flow} = 42.5$$

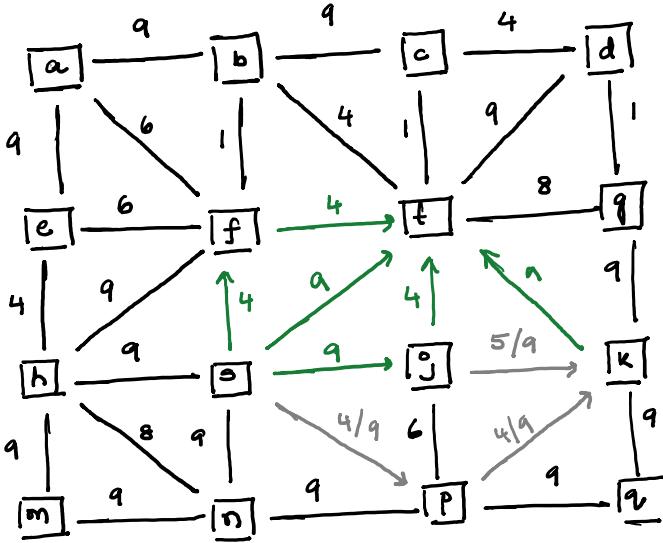
$$\Rightarrow \boxed{b = 17}$$

Points of intersection :

$$2x + 2b = 3x + 17 \Rightarrow \boxed{x = 9}$$

$$4x + 9 = 3x + 17 \Rightarrow \boxed{x = 8}$$

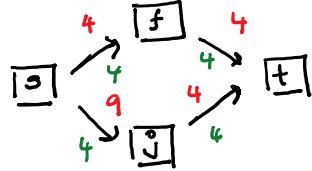




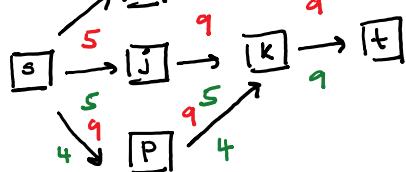
①



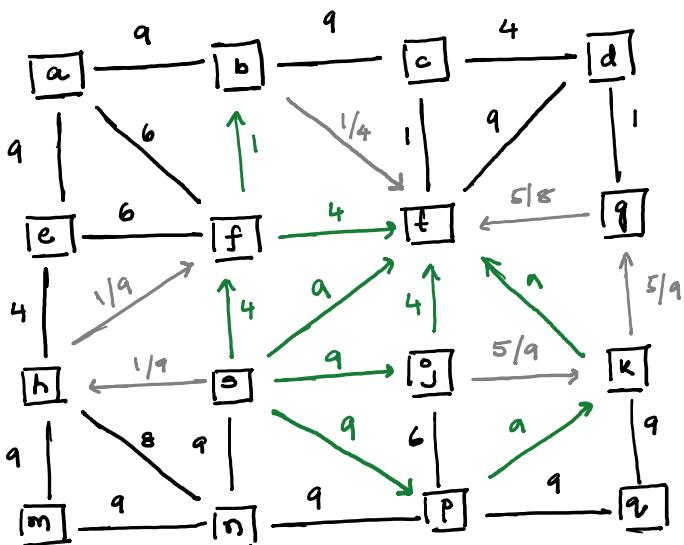
②



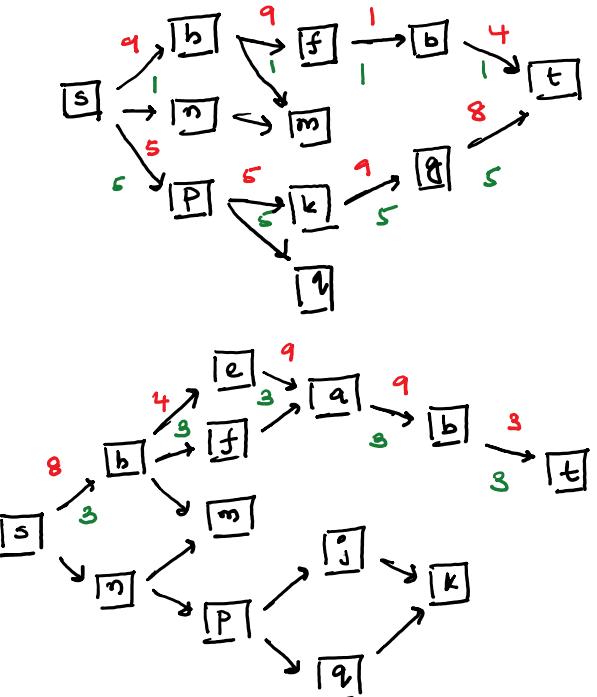
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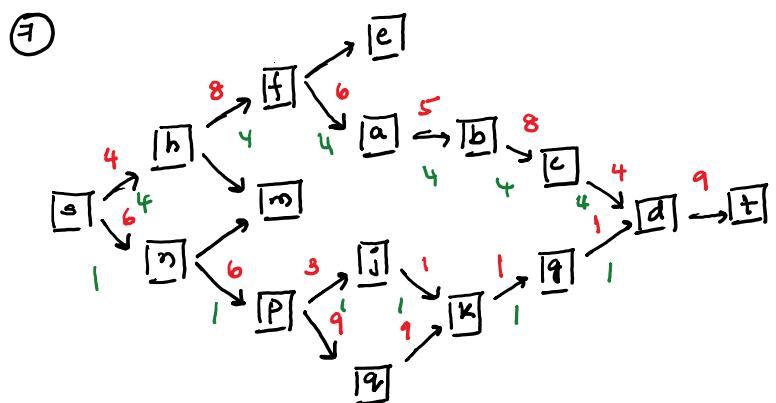
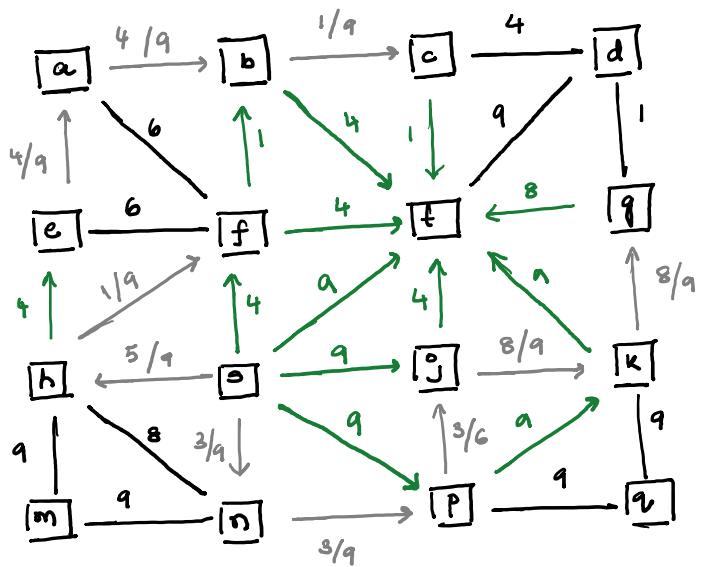
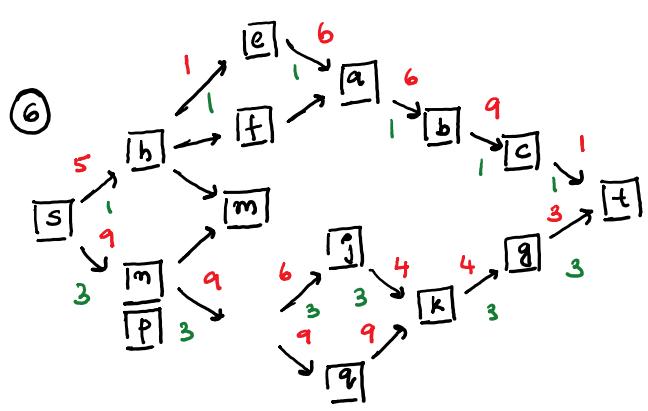
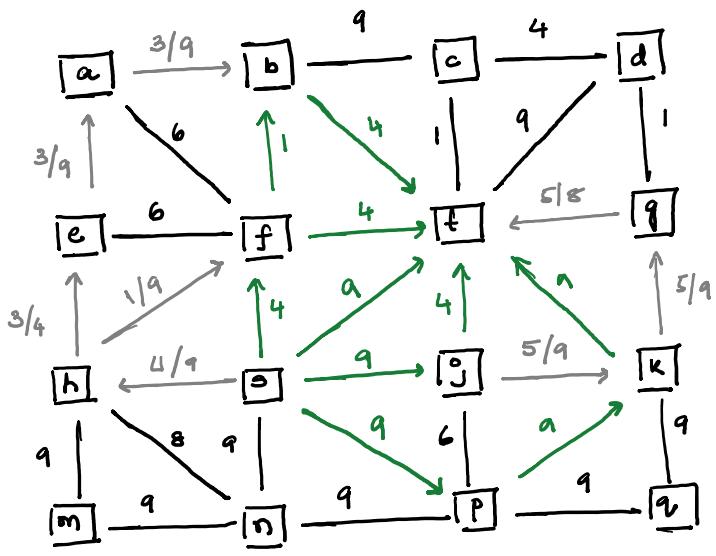


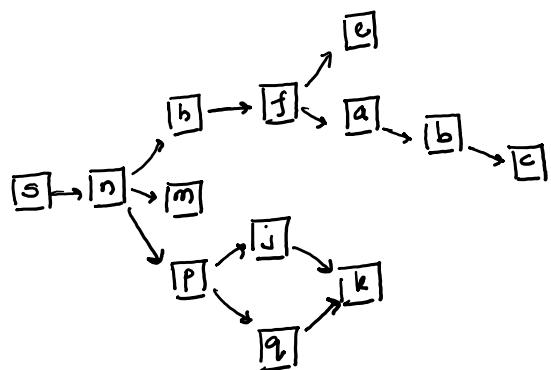
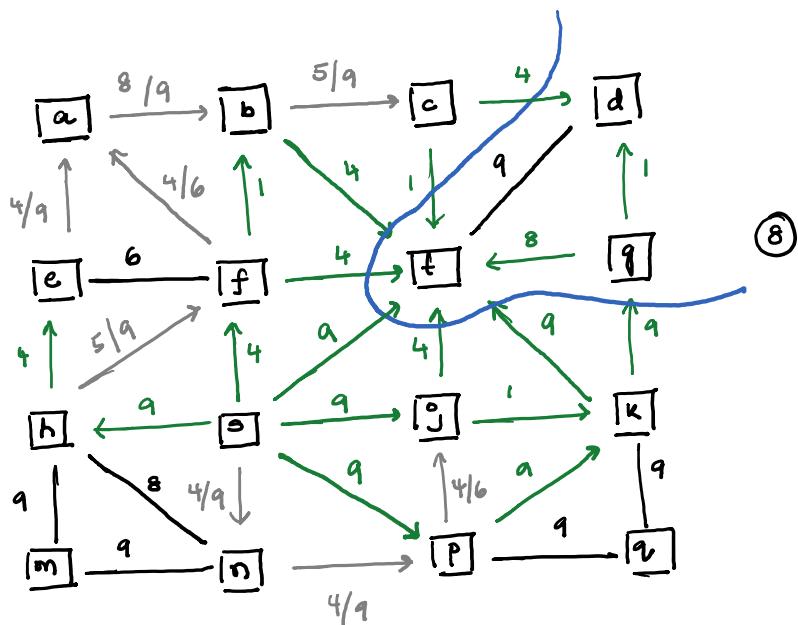
④



⑤





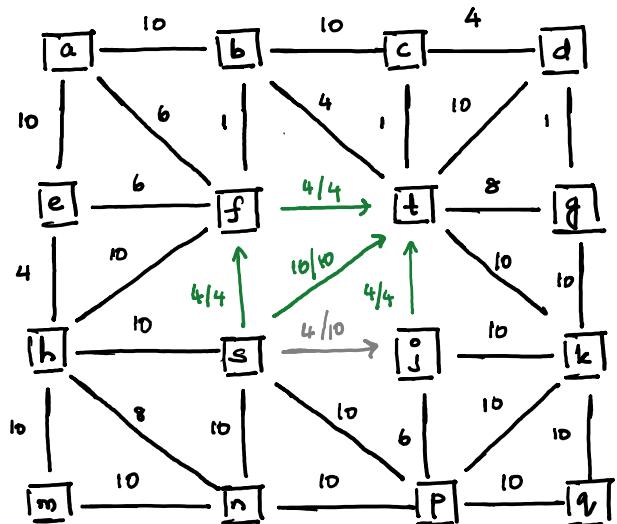
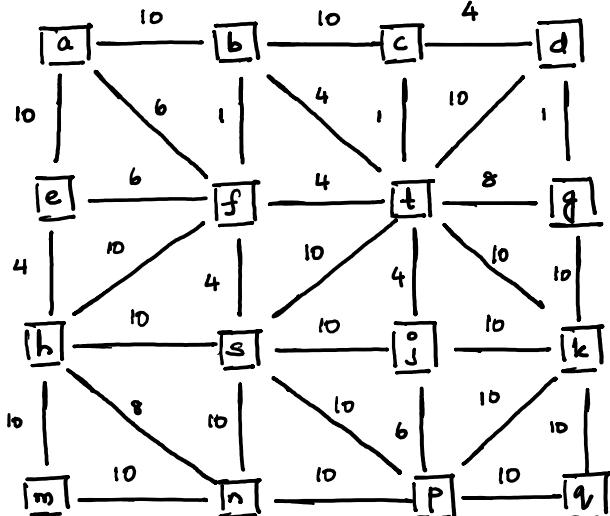


Current flow @ $\lambda=9$

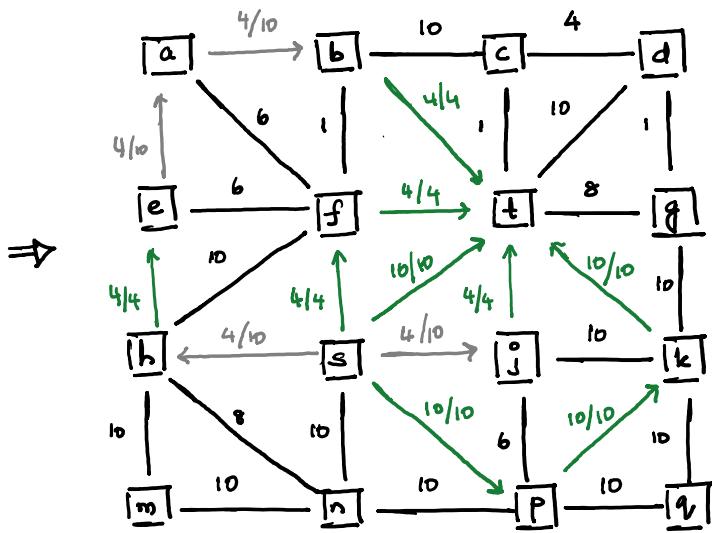
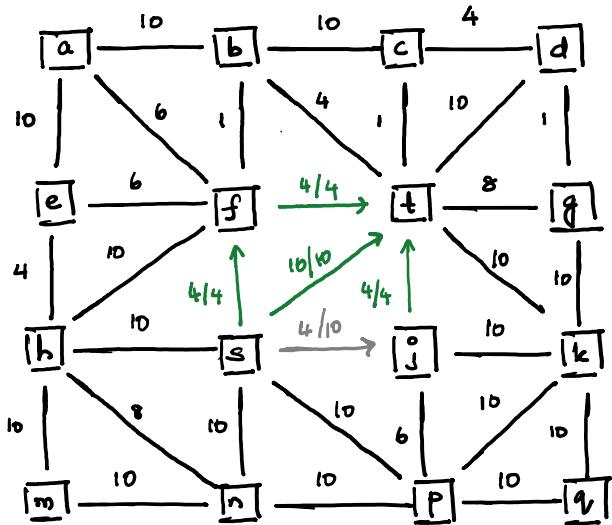
in graph $\Rightarrow 44$

Flow: 44

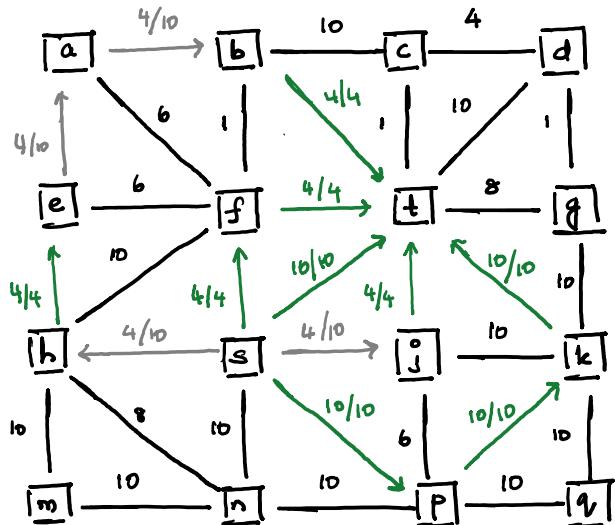
Consider graph with $\min [u_{ij}, 10]$



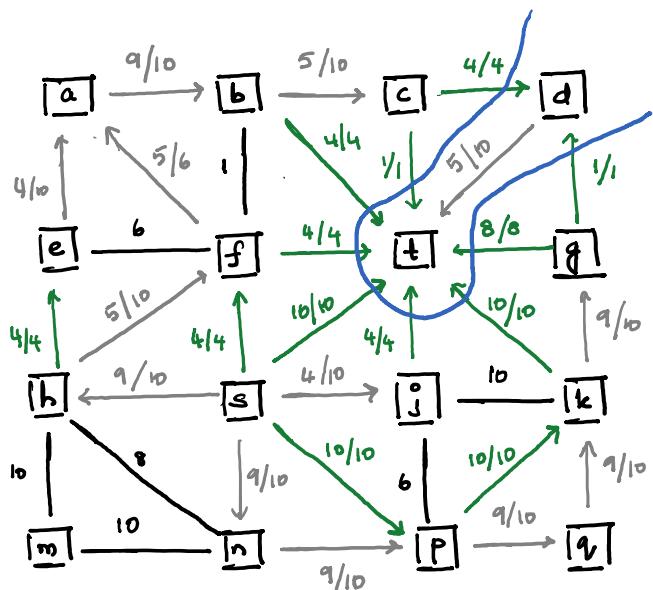
$[10 + 4 + 4]$



$[10 + 4]$



\Rightarrow

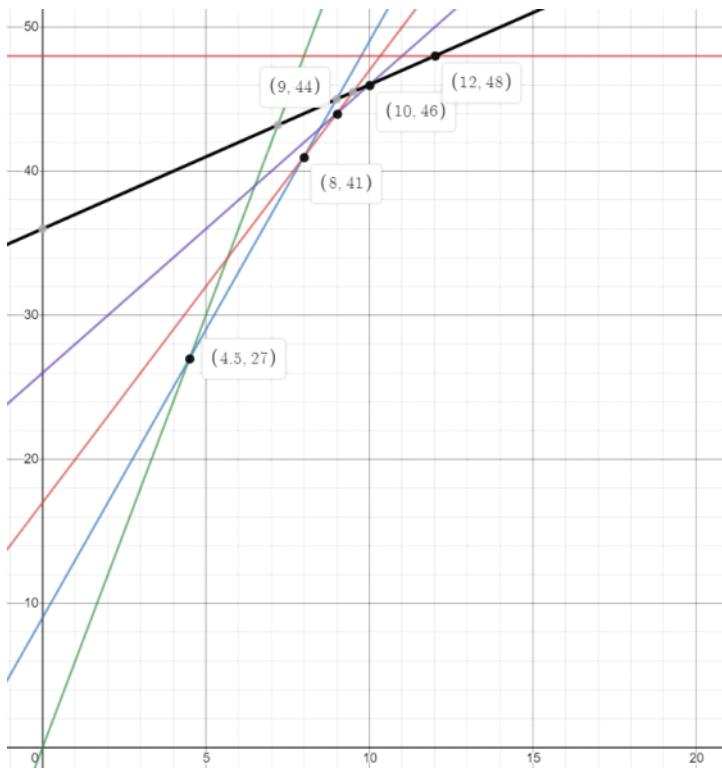


$[9 + 5]$

Max flow: $18 + 14 + 14 = 46$.

Min cut: $\{(s, j, n, b, p, q, k, g, f, c, a, b, c), (d, t)\}$

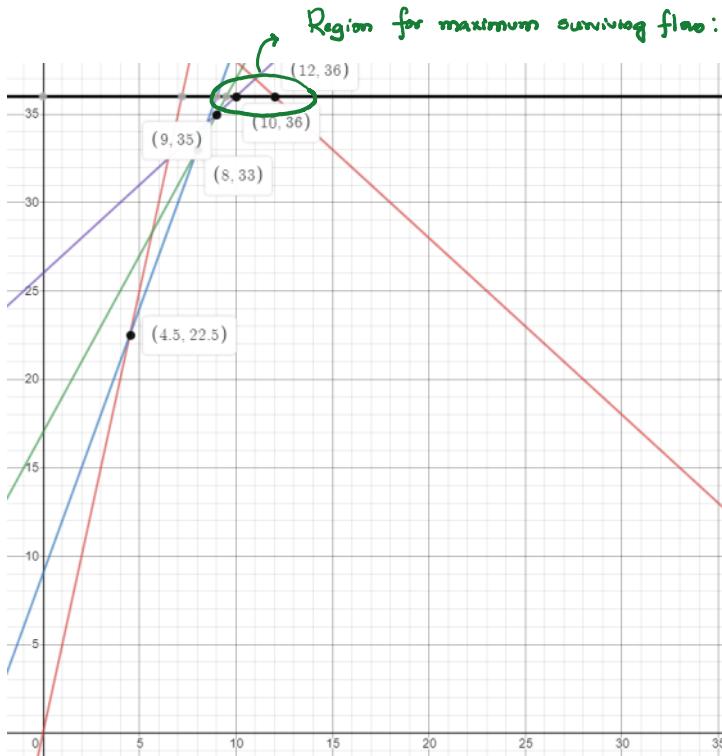
No. of edges in min-cut with flow = 10: $\boxed{2}$



Piece wise linear function ($F(x)$) :

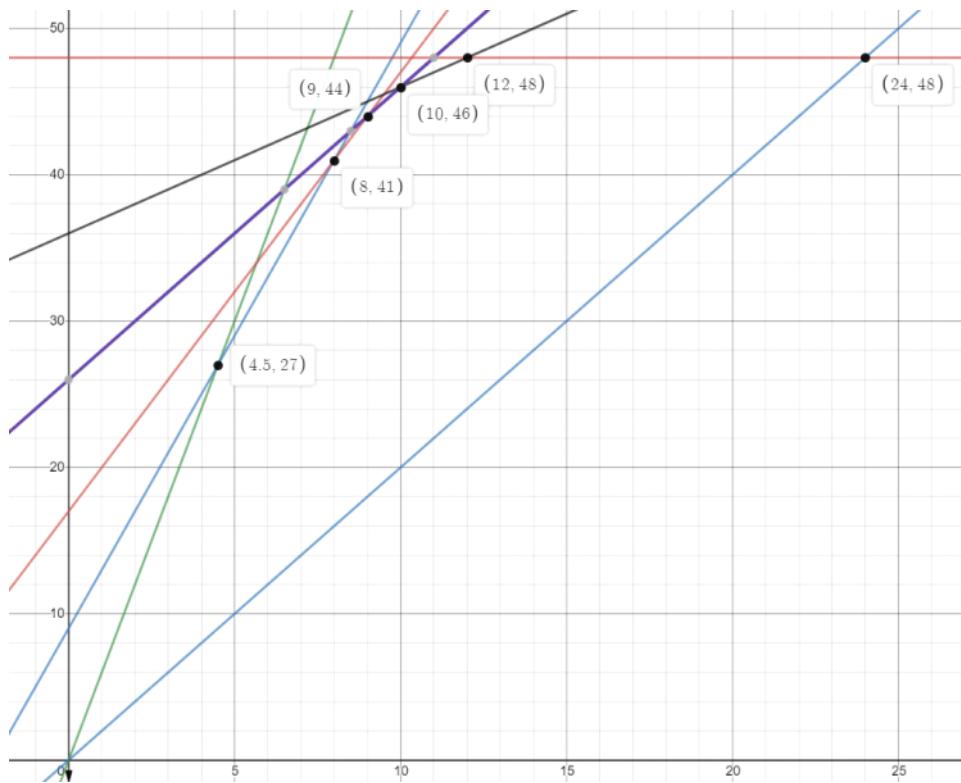
- ① $6x$
- ② $4x + 9$
- ③ $3x + 17$
- ④ $2x + 26$
- ⑤ $x + 36$
- ⑥ 48

To find maximum surviving flow: $F(x) - x$ [curve] \rightarrow Equations shall be



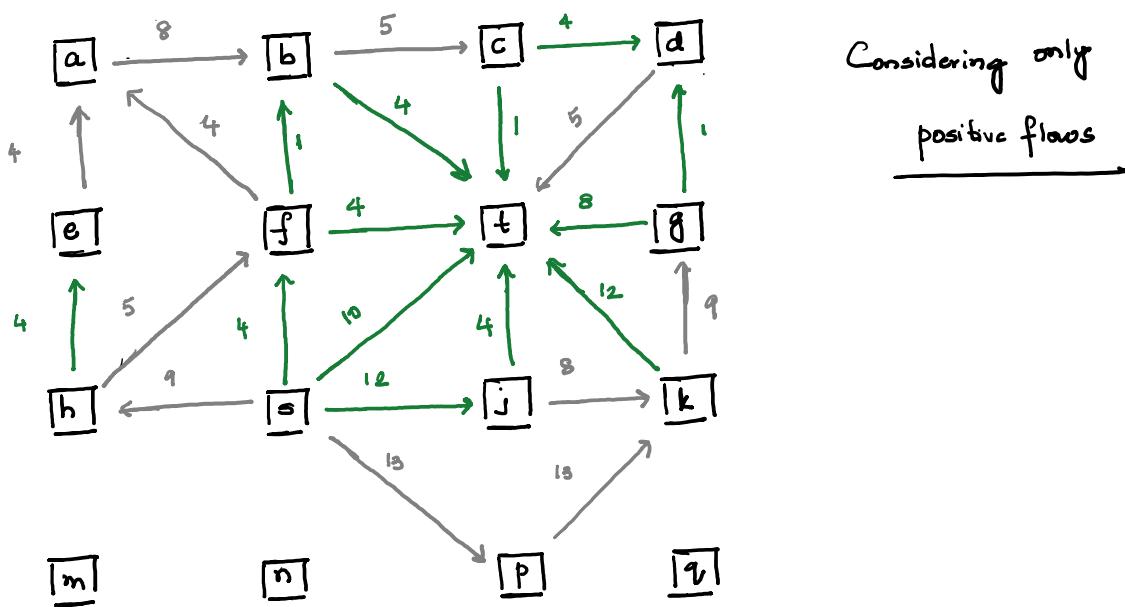
- ① $5x$
- ② $3x + 9$
- ③ $2x + 17$
- ④ $x + 26$
- ⑤ 36
- ⑥ $48 - x$

For maximum q -path flows $\underline{q=2}$

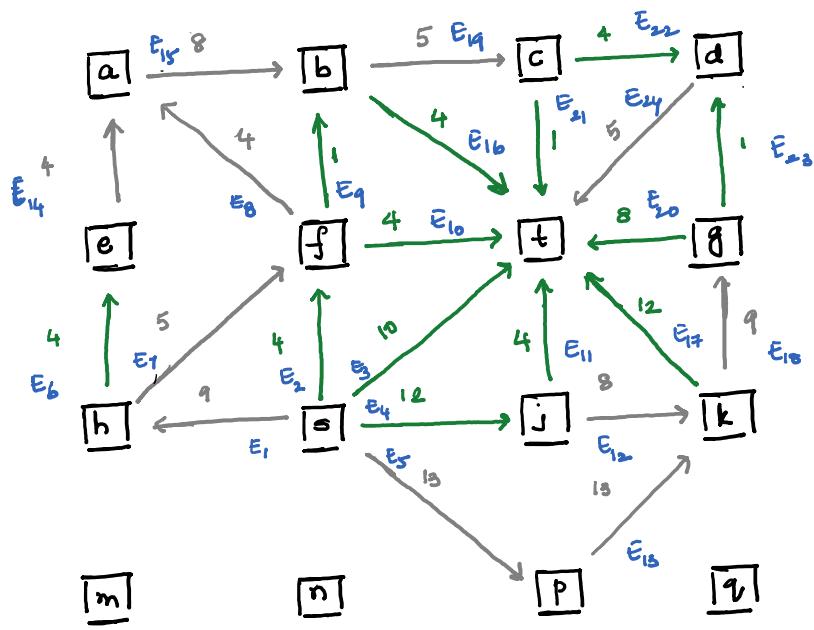


$$\lambda^* = 24 \rightarrow \text{Consider } \min [u_{ij}, 24]$$

Flow with this \leftarrow Equivalent to flow with $\underline{\lambda=\infty}$ (As all edge capacities ≤ 24)



To find the q -path decomposition. We use Kishimoto's work



Satisfies

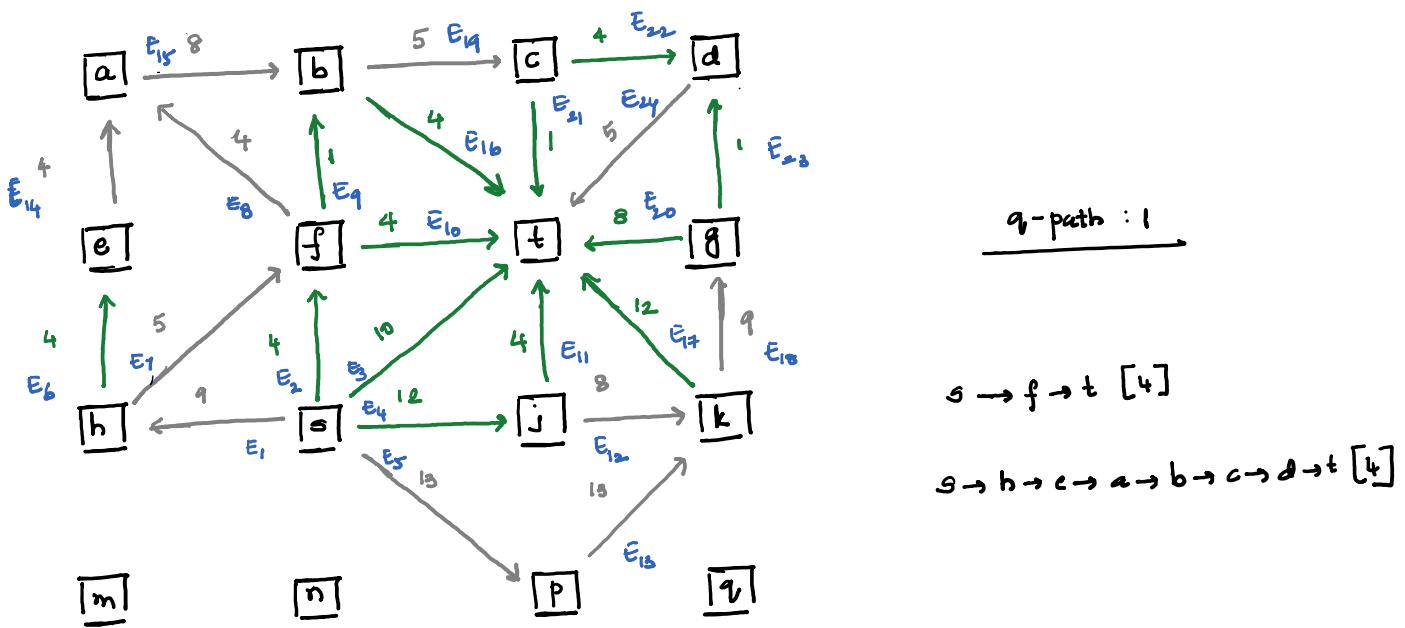
$$\forall (i, j) \in E \quad 0 \leq f_{i,j} \leq u_{i,j}$$

$$f_{i,j} \leq F/2 \leq \underline{24}$$

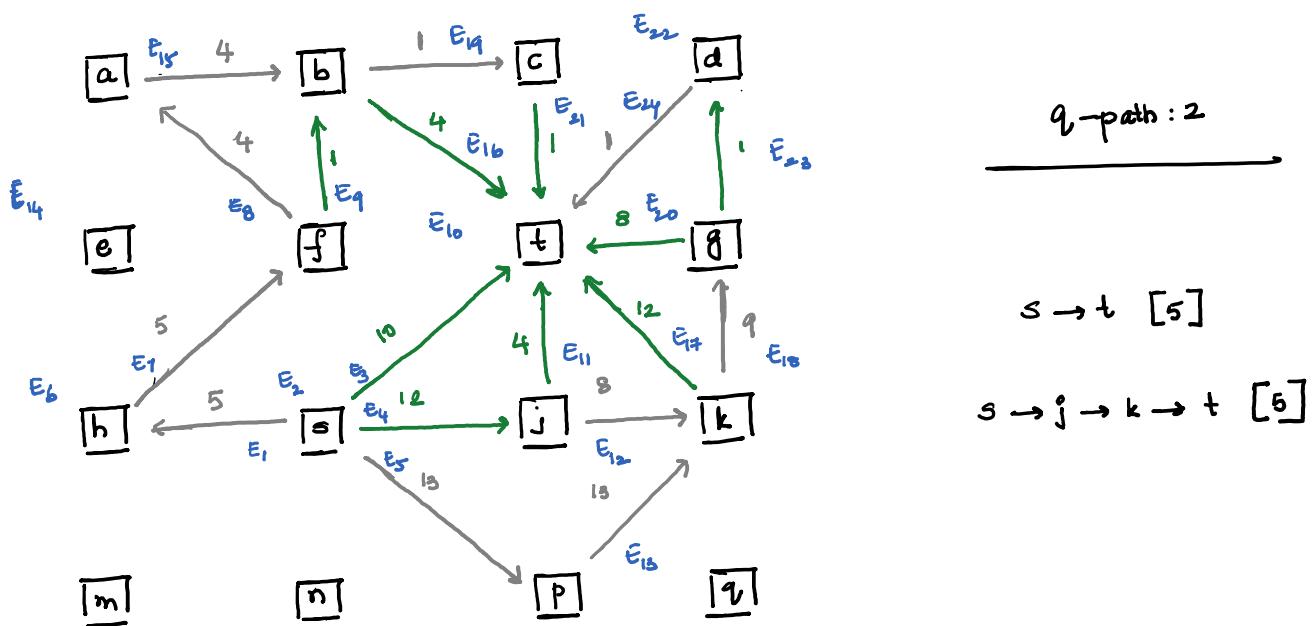
It has 24 edges that carry positive flows and $\underline{q}=2$

So, we create 26×26 table with 24 rows corresponding to edges

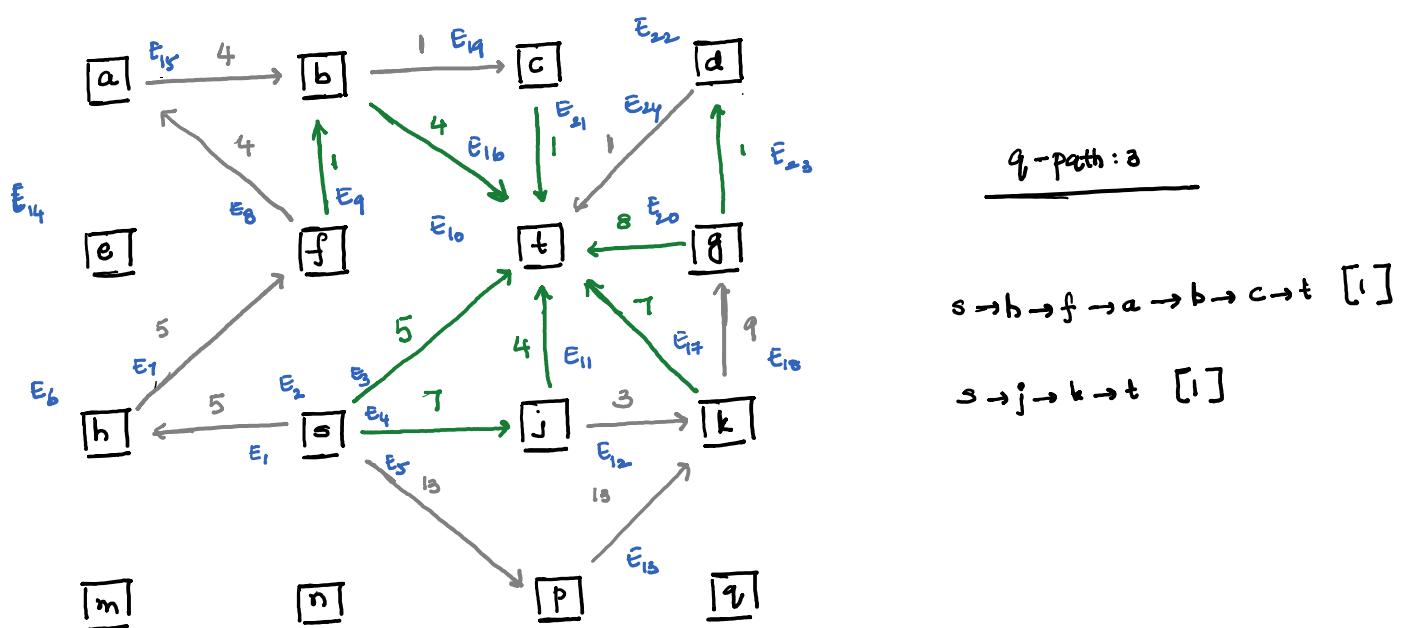
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1		W1	W2	W3	W4	W5	W6	W7	W8	W9	W10	W11	W12	W13	W14	W15	W16	W17	W18	W19	W20	W21	W22	W23	W24	b1	b2		
2	a1	9		5	6	4																						24	
3	a2		4	5	6	9																						24	
4	U1	15					4	5																				24	
5	U2	20									4																	24	
6	U3		14						12																		10	24	
7	U4			11														4	8									24	
8	U5				20													13										24	
9	U6					20												4										24	
10	U7						19	4	1																			24	
11	U8						20											4										24	
12	U9						23																					24	
13	U10							20																			4	24	
14	U11								20									16									4	24	
15	U12									11									8									24	
16	U13										20								4	9								24	
17	U14										20	4																24	
18	U15										16	3																24	
19	U16											20															4	24	
20	U17												12						15		8						12	24	
21	U18													19	1	4												24	
22	U19													16														8	24
23	U20														23	1												1	24
24	U21															20	4											24	
25	U22																19	1	4									24	
26	U23																23	1										24	
27	U24																19	1	4									24	
28		24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24		
29																													

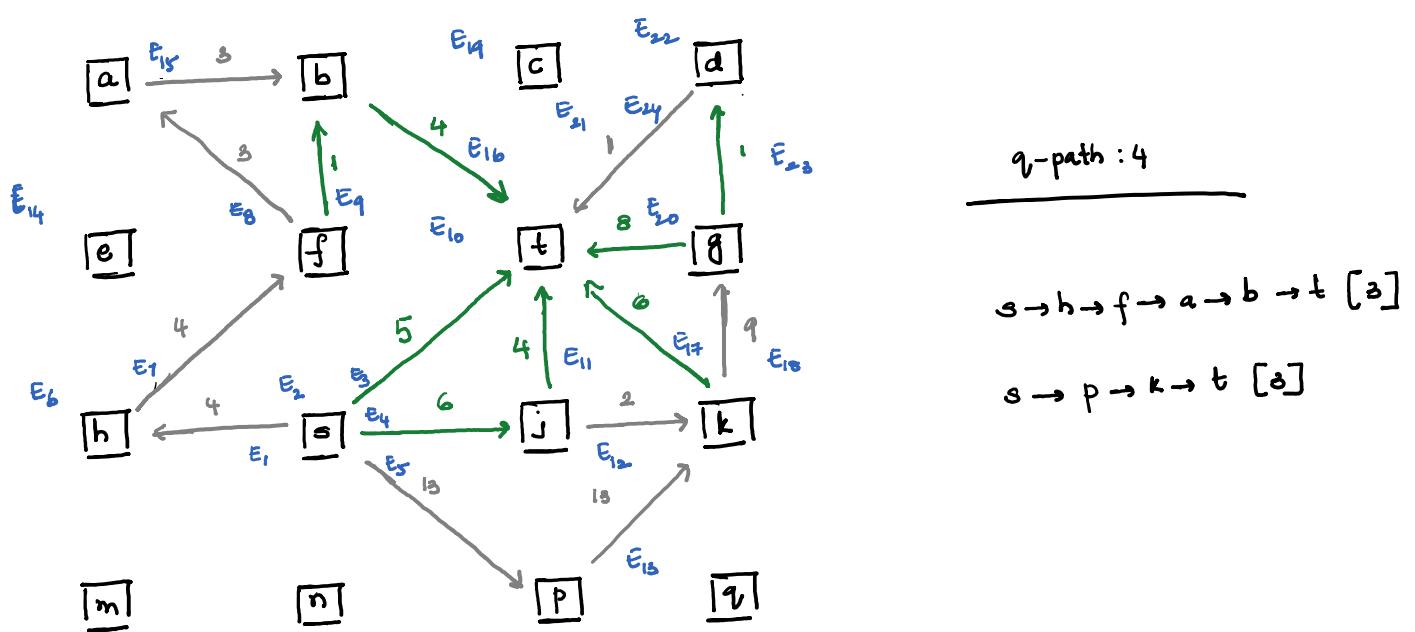


	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC				
1		W1	W2	W3	W4	W5	W6	W7	W8	W9	W10	W11	W12	W13	W14	W15	W16	W17	W18	W19	W20	W21	W22	W23	W24	b1	b2						
2	a1	5		5	6	4																					20	20					
3	a2			5	6	9																					20	20					
4	U1	15						5																			20	20					
5	U2		20																								20	20					
6	U3			10																							10	20					
7	U4				8														4	8							20	20					
8	U5					7														13							20	20					
9	U6						20																				20	20					
10	U7							15	4	1																	20	20					
11	U8							16													4						20	20					
12	U9								19													1					20	20					
13	U10									20																	20	20					
14	U11										16																4	20					
15	U12											12															20	20					
16	U13												7									4	9				20	20					
17	U14													20													20	20					
18	U15														16	3											20	20					
19	U16															16											4	20					
20	U17																8											12	20				
21	U18																	11		8									20				
22	U19																	19		1									20				
23	U20																		12										8	20			
24	U21																		19										1	20			
25	U22																			20										20			
26	U23																				19	1									20		
27	U24																				19	1									20		
28		20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20					
29																																	

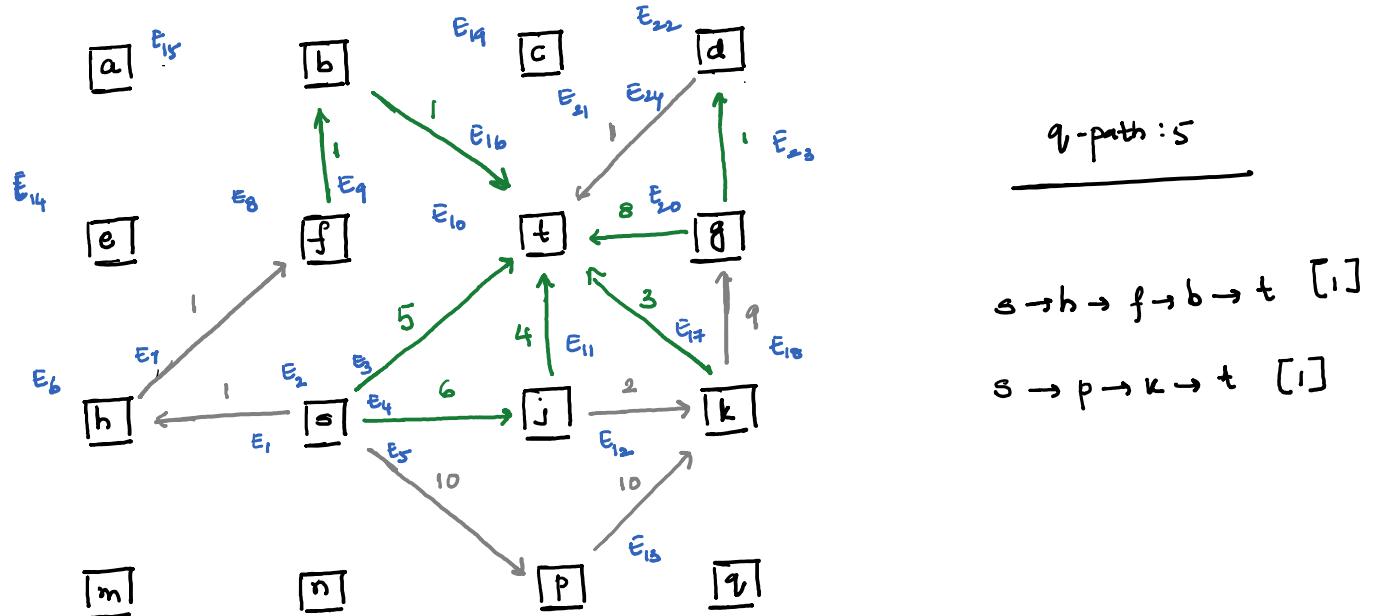


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1		W1	W2	W3	W4	W5	W6	W7	W8	W9	W10	W11	W12	W13	W14	W15	W16	W17	W18	W19	W20	W21	W22	W23	W24	b1	b2			
2	a1	5			6	4																						15		
3	a2			5	1	9																						15		
4	U1	10						5																					15	
5	U2	15																											15	
6	U3		10																								5		15	
7	U4		8										4	3															15	
8	U5			2										13															15	
9	U6			15																									15	
10	U7				10	4	1																					15		
11	U8					11														4									15	
12	U9					14														1									15	
13	U10						15																						15	
14	U11							11																					15	
15	U12								12											3									15	
16	U13									2										4	9								15	
17	U14										15																		15	
18	U15											11	3							1									15	
19	U16												11															4	15	
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22	U19																		14	1										15
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27	U24																		14	1										15
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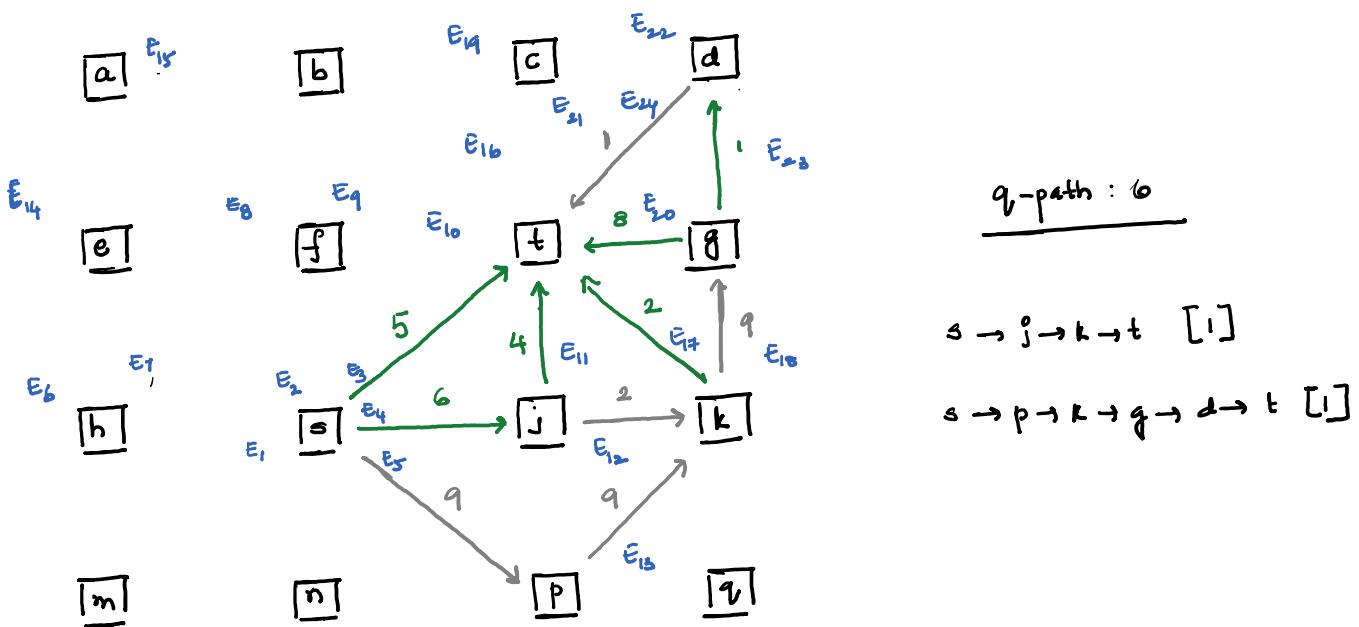




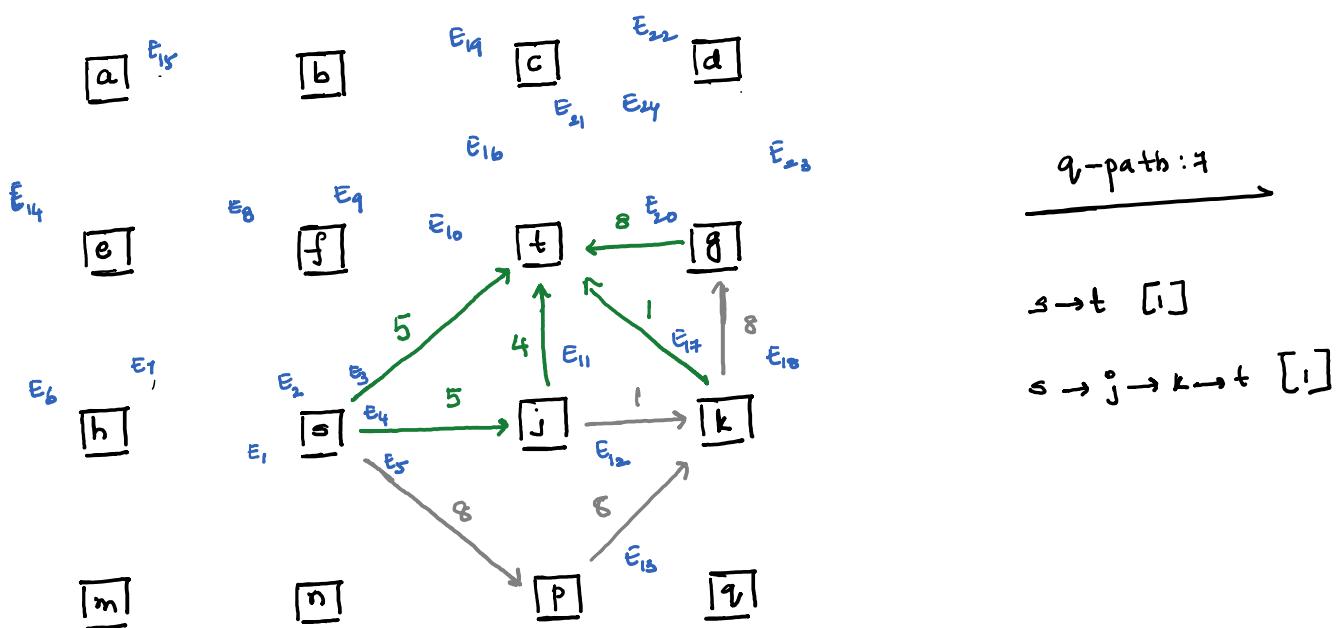
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1																													
2	a1	1				6	4																					11	
3	a2			5		6																						11	
4	U1	10							1																			11	
5	U2	11																										11	
6	U3	6																										5	
7	U4		5							4	2																11		
8	U5			1										10													11		
9	U6			11																								11	
10	U7				10				1																			11	
11	U8				11																							11	
12	U9					10													1									11	
13	U10						11																					11	
14	U11							7																				4	
15	U12								9											2								11	
16	U13									1									1	9								11	
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25	U22																		11									11	
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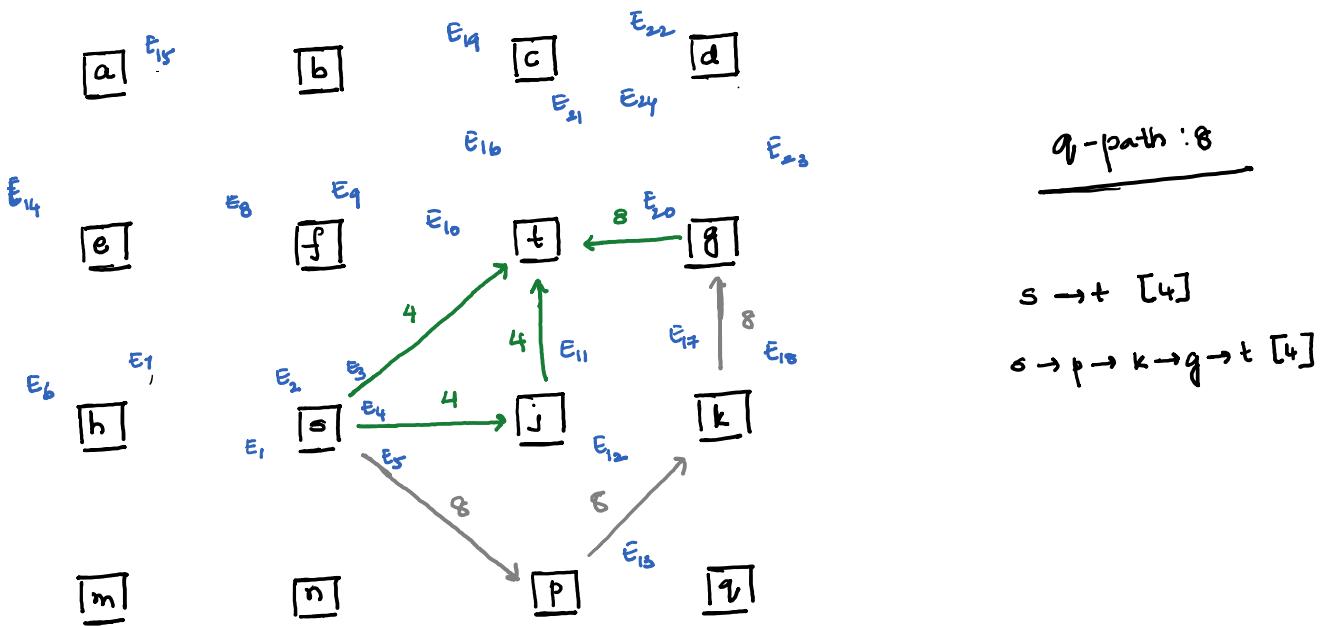
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2	a1				6	4																						10	
3	a2			5		5																						10	
4	U1	10																										10	
5	U2		10																									10	
6	U3			5																								5	
7	U4				4										4	2												10	
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11	U8						10																					10	
12	U9							10																				10	
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24	U21																					10						10	
25	U22																						10					10	
26	U23																							9	1			10	
27	U24																								9	1		10	
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29																													



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	
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2	a1				5	4																					9			
3	a2			5		4																					9			
4	U1	9																										9		
5	U2		9																									9		
6	U3		4																									9		
7	U4		4														4	1										5		
8	U5			1														8										9		
9	U6				9																							9		
10	U7				9																							9		
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29																														



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC
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2	a1				4	4																						8	
3	a2				4		4																					8	
4	U1	8																										8	
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28		8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8		
29																													



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC
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2	a1					4																						4	
3	a2						4																					4	
4	U1	4																										4	
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29																													

