QUESTION - 6

VC dimension of Guassian Naive Bayes $\rightarrow$ is atleast <u>three</u>

[NAIVE BAYES ASSUMPTION]

$\downarrow$

$$P(X|Y=0) = \prod P(x_i|Y=0)$$

$$P(X|Y=1) = \prod P(x_i|Y=1)$$

$$P(Y=1|X) = \frac{P(Y=1)P(X|Y=1)}{P(Y=1)P(X|Y=1) + P(Y=0)P(X|Y=0)}$$

$$= \frac{1}{1 + \exp\left(\ln\left(\frac{P(Y=0)P(X|Y=0)}{P(Y=1)P(X|Y=1)}\right)\right)}$$

$$= \frac{1}{1 + \exp\left(\ln\left(\frac{P(Y=0)}{P(Y=1)}\right) + \sum_i \ln\frac{P(x_i|Y=0)}{P(x_i|Y=1)}\right)}$$

[Use assumption of GUASSIANS]

$$= \frac{1}{1 + \exp\left(\underbrace{\ln\frac{1-\pi}{\pi}}_{\boxed{b}} + \underbrace{\sum_i \ln\frac{P(x_i|Y=0)}{P(x_i|Y=1)}}\right)}$$

$\hookrightarrow$ if this can be expressed as some $\boxed{W^Tx}$

$$\Rightarrow \sum_i \ln\frac{P(x_i|Y=0)}{P(x_i|Y=1)} = \sum_i \ln \frac{\cancel{\frac{1}{\sqrt{2\pi\sigma_i^2}}}\exp\left(-\frac{(x_i-\mu_{i0})^2}{2\sigma_i^2}\right)}{\cancel{\frac{1}{\sqrt{2\pi\sigma_i^2}}}\exp\left(-\frac{(x_i-\mu_{i1})^2}{2\sigma_i^2}\right)}$$

$$\Rightarrow \sum_i \ln\left(\exp\left(\frac{(x_i-\mu_{i1})^2 - (x_i-\mu_{i0})^2}{2\sigma_i^2}\right)\right)$$

$$\Rightarrow \sum_i \left[\frac{\mu_{i1}^2 - \mu_{i0}^2}{2\sigma_i^2} + x_i\frac{\mu_{i0}-\mu_{i1}}{\sigma_i^2}\right]$$

Substituting <u>back</u> gives $P(Y=1|X) = \dfrac{1}{1+\exp(b + W^Tx)}$

[SAME FORM AS <u>LOGISTIC REGRESSION</u>]

LINEAR SEPARATOR Can exist $\Rightarrow VC(H) \geq 3$