CS 6364 HW 4

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1 QUALIFYING UNCERTAINITY

Problem 13.21 from the Textbook at page 509. (It should start with (Adapted from Pearl (1988)). Suppose you are a witness to a nightime. . . .
(a) 10 points
(b) 10 points

1.1 a)

<u>SOLUTION</u>

Random variable information :

1. B : Taxi was blue
2. LB : Taxi looked blue

Given reliability information :

$$P(LB|B) = 0.75, P(\neg LB|\neg B) = 0.75$$

Given hint : Distinguish between proposition that taxi is blue and proposition that it appears blue

We need to know the probability that taxi was blue given that it looked blue.

$$P(B|LB) = \alpha P(LB|B)P(B) = 0.75\alpha P(B) \;\; where \;\; \alpha = Normalization \;\; constant$$

$$P(\neg B|LB) = \alpha P(LB|)P() = 0.25\alpha(1 - P(B))$$

Here we are not presented with enough information reach a conclusion.
Any information about prior probabilities shall suffice.

1.2 b)

<u>SOLUTION</u>

Given 9 out of 10 are green.

$$P(B) = 1 - 0.9 = 0.1$$

Thus giving us the following values:

$$P(B|LB) = 0.75 * 0.1\alpha = 0.075\alpha$$
$$P(\neg B|LB) = 0.25 * 0.9\alpha = 0.225\alpha$$
$$\alpha = \frac{1}{0.075 + 0.225} = \frac{10}{3} = 3.33$$

$$P(B|LB) = 0.75 * 0.1\alpha = 0.075\alpha = 0.25$$
$$P(\neg B|LB) = 0.25 * 0.9\alpha = 0.225\alpha = 0.75$$

2 NAIVE BAYESIAN REASONING

Consider a traveler that wants to climb the Everest. He gets to Nepal in summer and also finds an experienced guide. Use Naive Bayesian reasoning to decide if the traveler will climb to 1000 ft from the top of the Everest based (15 points) on the following information:

1. 10% of all climbers get to 1000 ft from the top of the Everest.
2. Among all travelers who get to 1000 ft from the top of the Everest, 90% went to Nepal in summer and 80% used an experienced guide.
3. 50% of climbers that cannot get to 1000 ft from the top of the Everest went to Nepal in summer and 30% were able to find an experienced guide.

Explain your conclusion. (10 points)

<u>SOLUTION</u>

Random variables :

1. C : a boolean variable for can be a climbers who get to 1000ft from top of the everest
2. N : a boolean variable for went to Nepal in summer
3. G : a boolean variable for usage of experienced guide

Given information

$$P(C) = 0.1$$
$$P(N|C) = 0.9, P(G|C) = 0.8$$
$$P(N|\neg C) = 0.5, P(G|\neg C) = 0.3$$

To answer : $P(C|N,G)$

We assume that going to Nepal and finding an experienced guide are conditionally independent given climber gets to 1000ft from top of the everest.

$$P(C|N,W) = \frac{P(N,W|C)P(C)}{P(N,W)}$$

$$P(N,W|C) = P(N|C)P(W|C)$$

$$P(C|N,W) = \frac{P(N|C)P(W|C)P(C)}{P(N,W)}$$

$$P(C|N,W) = \frac{(0.9)(0.8)(0.1)}{P(N,W)}$$

$$P(C|N,W) = \frac{(0.072)}{P(N,W)}$$

$$But \quad P(N,W) = P(N,W|C)P(C) + P(N,W|\neg C)P(\neg C)$$

$$P(N,W|\neg C)P(\neg C) = P(N|\neg C)P(W|\neg C)P(\neg C)$$

$$P(N,W|\neg C)P(\neg C) = P(N|\neg C)P(W|\neg C)P(\neg C)$$

$$P(N,W|\neg C)P(\neg C) = (0.5)(0.3)(0.9) = 0.135$$

$$\implies P(N,W) = 0.135 + 0.072 = 0.207$$

$$\implies P(C|N,W) = \frac{(0.072)}{0.207} = 0.347$$

PROBLEM-3 A

(5 points) Compute the probability that: a1 = 1, a2 = 1, a3 = 1, b1 = 0, b2 = 0, b3 = 0, b4 = 0, b5 = 0

Based on bayes net structure we have.

$$P(a_1, a_2, a_3, b_1, b_2, b_3, b_4, b_5)$$

$$= P(a_1)\, P(a_2)\, P(b_1 \mid a_1, a_2)\, P(b_2 \mid b_1)\, P(a_3)\, P(b_3 \mid a_3, b_2)\, P(b_4 \mid b_3)\, P(b_5 \mid b_4)$$

$$\Rightarrow P(a_1=1, a_2=1, a_3=1, b_1=0, b_2=0, b_3=0, b_4=0, b_5=0)$$

$$= P(a_1=1)\, P(a_2=1)\, P(b_1=0 \mid a_1=1, a_2=1)\, P(b_2=0 \mid b_1=0)\, P(a_3=1)\, P(b_3=0 \mid a_3=1, b_2=0)$$

$$P(b_4=0 \mid b_3=0)\, P(b_5=0 \mid b_4=0)$$

$$= (0.7)(0.8)\,[1-(0.9)]\,[1-(0.6)]\,(0.9)\,[1-(0.8)]\,[1-(0.1)]\,[1-0]$$

$$= (0.7)(0.8)(0.1)(0.4)(0.9)(0.2)(0.9)(1)$$

$$= \underline{0.0036288}$$

PROBLEM 3B

(15 points) Compute the probability that b5 = 1

Given bayes network is **a polytree.**

We can use the
1. Possible worlds approach
2. Variable elimination approach

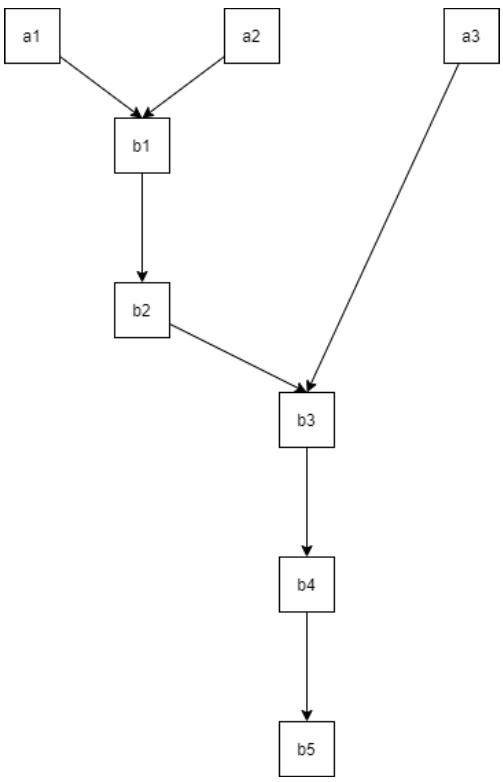
Considering the possible worlds approach we find the probabilities :

a1	P(a1)
0	0.3
1	0.7

a2	P(a2)
0	0.2
1	0.8

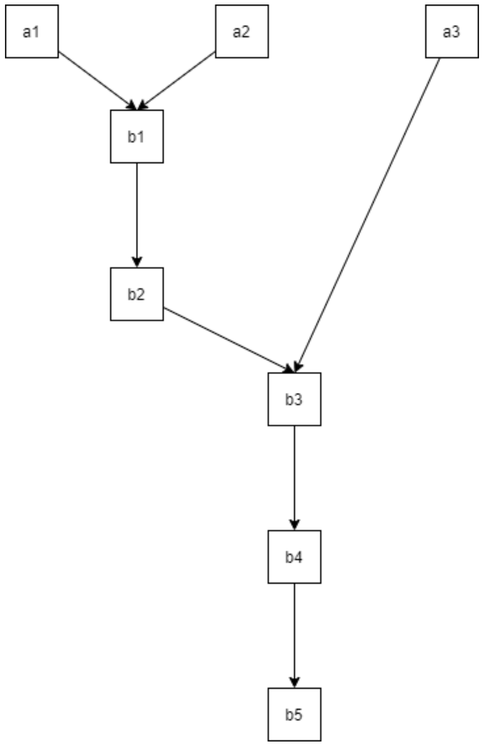
a3	P(a3)
0	0.1
1	0.9

a1	a2	P(b1)	P(W)
0	0	0.2	0.3*0.2 = 0.06
0	1	0.6	0.3*0.8 = 0.24
1	0	0.7	0.7*0.2 = 0.14
1	1	0.9	0.7*0.8 = 0.56



b1	P(b1)
0	1-0.758 = 0.242
1	0.2*0.06 + 0.6*0.24 + 0.7*0.14 + 0.9*0.56 = 0.758

b1	P(b1)
0	0.242
1	0.758



b1	P(b1)
0	0.242
1	0.758

b1	P(b2)	P(W)
0	0.6	0.242
1	0.8	0.758

b2	P(b2)
0	1-0.7516 = 0.2484
1	0.758*0.8 + 0.242*0.6 = 0.7516

a3	P(a3)
0	0.1
1	0.9

b2	P(b2)
0	0.2484
1	0.7516

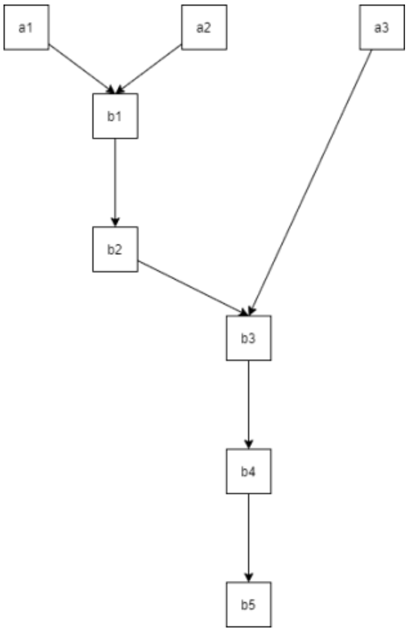
a3	b2	P(b3)	P(W)
0	0	0	0.1*0.2484 = 0.0248
0	1	0.7	0.1*0.7516 = 0.0752
1	0	0.8	0.9*0.2484 = 0.2236
1	1	1	0.9*0.7516 = 0.6764

b3	P(b3)
0	1-0.9079 = 0.0921
1	0.7*0.0752 + 0.8*0.2236 + 0.6764 = 0.9079

b3	P(b3)
0	0.0921
1	0.9079

b3	P(b3)
0	0.0921
1	0.9079

b3	P(b4)	P(W)
0	0.1	0.0921
1	0.7	0.9079



b4	P(b4)
0	1 - 0.6447 = 0.3553
1	0.7*0.9079 + 0.1*0.0921 = 0.6447

b4	P(b4)
0	0.3553
1	0.6447

b4	P(b5)	P(W)
0	0	0.3553
1	1	0.6447

b5	**P(b5)**
0	**0.3553**
1	**0.6447**

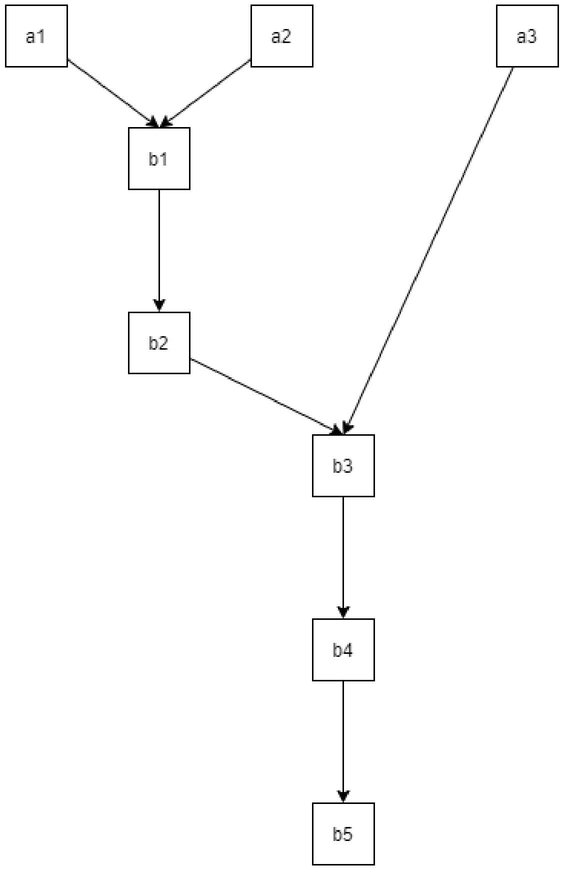
PROBLEM 3C

Probability that b5=1 given a1 = 1, a2 = 1, a3 = 1, b1=0, b2=0, b3=0

By the conditional independence

We can write P(b5=1| a1 = 1, a2 = 1, a3 = 1, b1=0, b2=0, b3=0) = P(b5=1 | b3=0)

Now we need to find P(b5=1 | b3=0)



b3	P(b4)	P(W)
0	0.1	1
1	0.7	0

b4	P(b4)
0	1 - 0.1 = 0.9
1	0.1*1 = 0.1

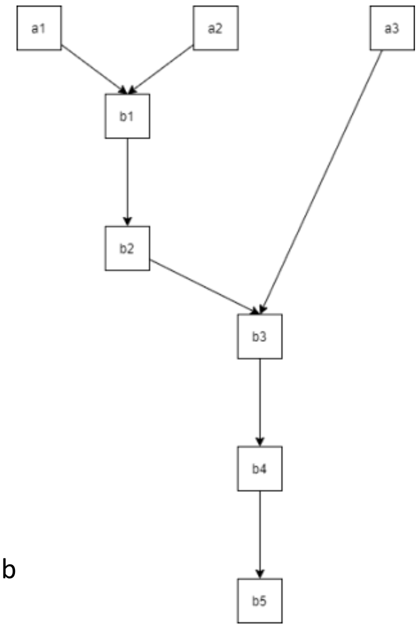
b4	P(b5)	P(W)
0	0	0.9
1	1	0.1

b5	**P(b5)**
0	**0.9**
1	**0.1**

PROBLEM 3D

(5 points) Compute the probability that b3=0 given that: b5=1

Compute P(b3=0 | b5=1)



Evidence below the query :
So, we use Bayes' Rule to find the solution

P(b3=0 | b5=1) = P(b5=1|b3=0) P(b3=0) / P(b5=1)

We already calculated **P(b5=1|b3=0) as 0.1**
From subpart 2 we have :

b3	P(b3)
0	0.0921
1	0.9079

P(b5=1) can be written as P(b5=1|b3=0) P(b3=0) + P(b5=1|b3=1) P(b

So we have :

P(b3=0|b5=1) = (0.1*0.0921)/(0.1*0.0921 + 0.9079*P(b5=1|b3=1))

b3	P(b4)	P(W)
0	0.1	0
1	0.7	1

b4	P(b4)
0	1 - 0.7 = 0.3
1	0.7*1 = 0.7

b4	P(b5)	P(W)
0	0	0.3
1	1	0.7

b5	**P(b5)**
0	**0.3**
1	**0.7**

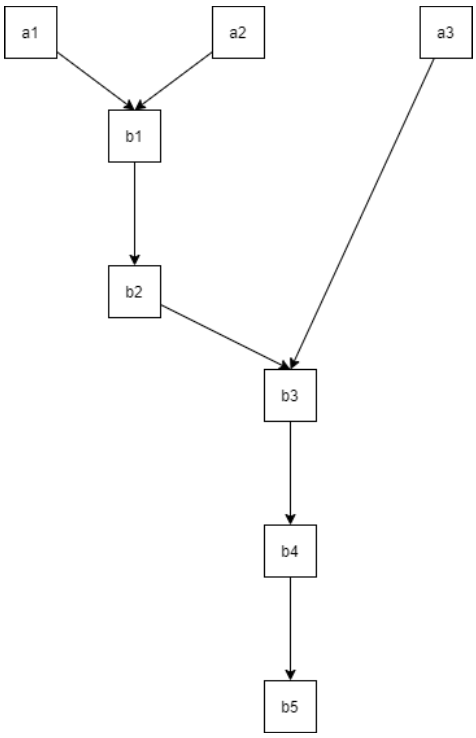
P(b3=0|b5=1) = (0.1*0.0921)/(0.1*0.0921 + 0.9079*P(b5=1|b3=1))
= (0.1*0.0921)/(0.1*0.0921 + 0.9079*0.7) = **0.0143**

PROBLEM 3E

(20 points) The CPT in node a3 is changed to:

where the value of x is unknown. What values of x would make it make it more likely that b5 happened than that b5 did not happen?

Here we are not given values for CPT of a3 and asked to assume them to be x



a3	P(a3)
0	1-x
1	x

b2	P(b2)
0	0.2484
1	0.7516

a3	b2	P(b3)	P(W)
0	0	0	(1-x)*0.2484
0	1	0.7	(1-x)*0.7516
1	0	0.8	x*0.2484
1	1	1	x*0.7516

b3	P(b3)
0	0.47388 - x*0.4242
1	x*0.7516 + x*0.2484*0.8 + (1-x)*0.7516*0.7 = x*0.7516 + x*0.19872 + (1-x)*0.52612 = = x*0.4242 + 0.52612

x*0.4242 + 0.52612 <= 1
=> x*0.4242 <= 0.47388
=> x <= 1.11 [Always]

b3	P(b3)
0	1-m
1	m

b3	P(b4)	P(W)
0	0.1	1-m
1	0.7	m

Where m = x*0.4242 + 0.52612

b4	P(b4)
0	0.9-0.6*m
1	0.7*m + 0.1*(1-m) = 0.1+0.6*m

b4	P(b5)	P(W)
0	0	0.9-0.6*m
1	1	0.1+0.6*m

0.1+0.6*m <= 1
=> m <= 1.5
=> x*0.4242 + 0.52612 <= 1.5
=> x*0.4242 <= 0.97388
=> x <= 2.29 [Always]

b5	P(W)
0	0.9-0.6m
1	0.1+0.6m

P(b5=1) > P(b5=0)

0.1+0.6m > 0.9-0.6m
=> 1.2 m > 0.8
=> m > 0.667
=> x*0.4242 + 0.52612 > 0.667
=> x*0.4242 > 0.14088
=> 0.33191 < x <= 1