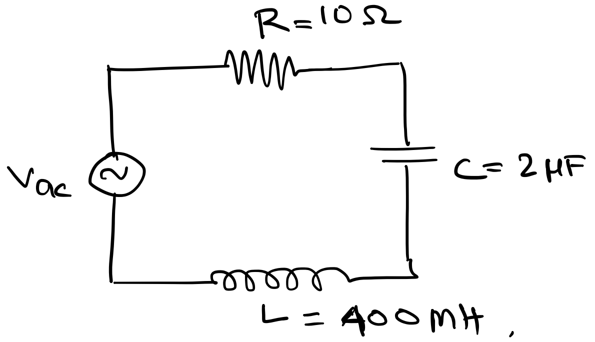
Group Problem, Physics 1302W.200, April 26, 2018

A series *RLC* circuit with $R = 10.0\ \Omega$, $L = 400$ mH, and $C = 2.00\ \mu$F is connected to an AC source of emf $\mathcal{E} = \mathcal{E}_{max} \sin \omega t$ with amplitude $\mathcal{E}_{max} = 100$ V and angular frequency $\omega = 4000\ s^{-1}$. The resulting current can be written as $I(t) = I_0 \sin(\omega t - \varphi)$. (a) Determine the maximum current I_0 and the phase shift φ for these parameters. (b) What are the magnitudes of the potential drop across each circuit element? (c) How much power is dissipated in this circuit? (d) What is the resonant frequency of the circuit, and how much power would be dissipated if the system were driven at this frequency?

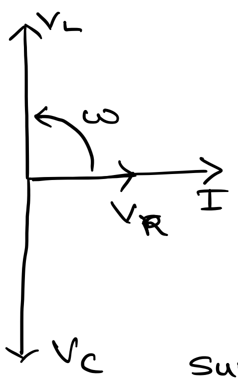


$V_{ac} = V_{max} \sin(\omega t)$ $\qquad \omega = 4000\ s^{-1}$

$V_{max} = 100V$

$= 100 \sin(4000t)$

Phasor Diagrams



V_R in phase with current

V_C lagging by $90°$

V_L preceding by $90°$

summing up vectorially

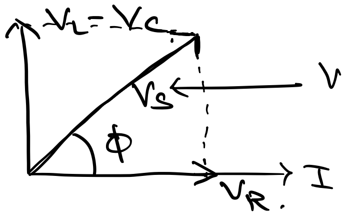
Reactance at $\boxed{\omega = 4000\ s^{-1}}$

$X_L = \omega L = 1600$

$X_C = \dfrac{1}{\omega C} = 125$

$\underline{R} = 10\ \Omega.$

$\Rightarrow$



Voltage source.

$V_s \rightarrow$ Soure Voltage

$V_c \rightarrow$ Potential difference across capacitor

$V_L \rightarrow$ " " across Inductor

So your current lags behind the source by ϕ degree

where $\tan\phi = \dfrac{V_L - V_C}{V_R}.$

look at the phasor diagram

$= \dfrac{X_L - X_C}{R}$

$Z = \sqrt{R^2 + (X_L - X_C)^2}.$

$Z = 1.48 \times 10^3\ \Omega$

$I_{max} = \dfrac{V_{max}}{Z} = 6.76 \times 10^{-2}$ A

$\Rightarrow \phi = 89.6°$

$I(t) = I_0 \sin(\omega t - \phi)$

$V_L = I(t) \cdot X_L$

from the phaser diagram.

$= 108 \sin(\omega t - \phi + 90)$

$V_C = I(t) \cdot X_C$

$= 8.45 \sin(\omega t - \phi - 90)$

$V_R = I \cdot X_R = 0.676 \sin(\omega t - \phi)$ (In phase with the current...)

(b) Power dissipated is only due to the resistor.

$$\langle P \rangle = \frac{1}{2} I_{max}^2 R.$$

$$= 2.28 \times 10^{-2}\ W.$$

$$\langle P \rangle = R \frac{\int_0^T I^2(t)\, dt}{\int_0^T dt}$$

$$= I_{max}^2 R \frac{\int_0^T \sin^2(\omega t - \phi)\, dt}{T}$$

$$\langle P \rangle = \langle I(t) \cdot V(t) \rangle$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Be Careful

$\boxed{\omega = 2\pi f}$.

$$= I_{max} V_{R_{max}} \cdot \langle \sin^2(\omega t - \phi) \rangle$$

$$= I_{max}^2 R \frac{\int_0^{\frac{2\pi}{\omega}} \sin^2(\omega t - \phi)\, dt}{\frac{2\pi}{\omega}}.$$

$$= \frac{V_{max} V_{R_{max}}}{2}$$

as well

$$= \frac{1}{2} I_{max}^2 R$$

(d) Resonance occurs at $\omega = \frac{1}{\sqrt{LC}}$. $= 1.12 \times 10^{3}\ s^{-1}$

$$\Rightarrow \quad Z = \sqrt{R^2 + \left(\frac{L}{\sqrt{LC}} - \frac{\sqrt{LC}}{C}\right)^2}$$

$$= \sqrt{R^2 + \left(\frac{LC - LC}{C\sqrt{LC}}\right)^2}$$

$$= \underline{\underline{R}}.$$

So

$$I_{max} = \frac{V_{max}}{R} = \underline{\underline{10\,A}}.$$

$$\langle P_{res} \rangle = \frac{1}{2} I_{max}^2 \cdot R = \underline{\underline{500\,W}}$$