Trajectory Tracking Control of an Autonomous Vehicle using Model Predictive Control and PID Controller

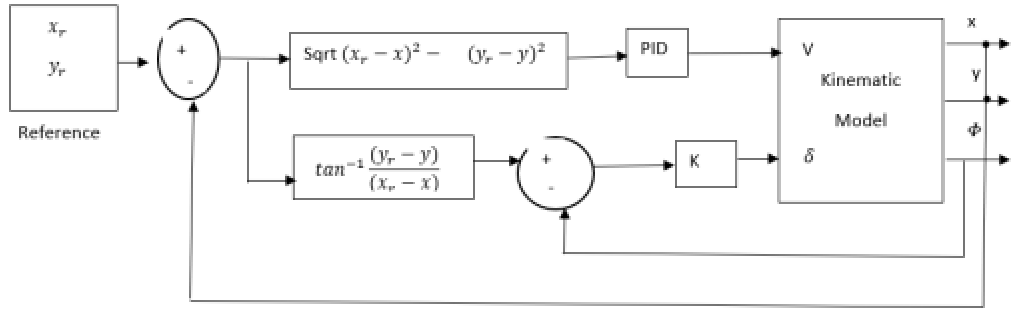
Nazrin Gurbanova UID: 120426469
Rohith Vikram Saravanan UID: 119474198

TABLE OF CONTENTS

Trajectory tracking control

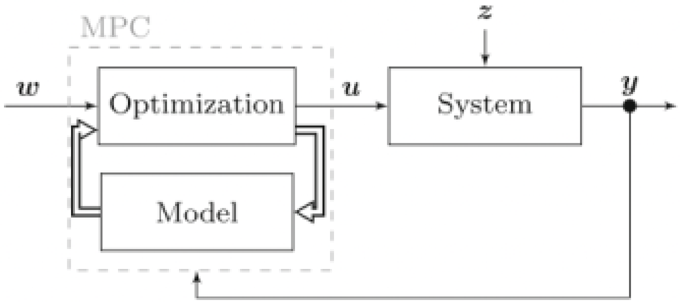
Trajectory tracking is the process of making the vehicle to follow the trajectory preplanned by the planner. This is done with the help of controllers such as pid or mpc controller. The controllers are tuned so that they take in the kinematic and dynamic positional error values and gives out control parameter to make the vehicle follow the desired trajectory path

PID Controller



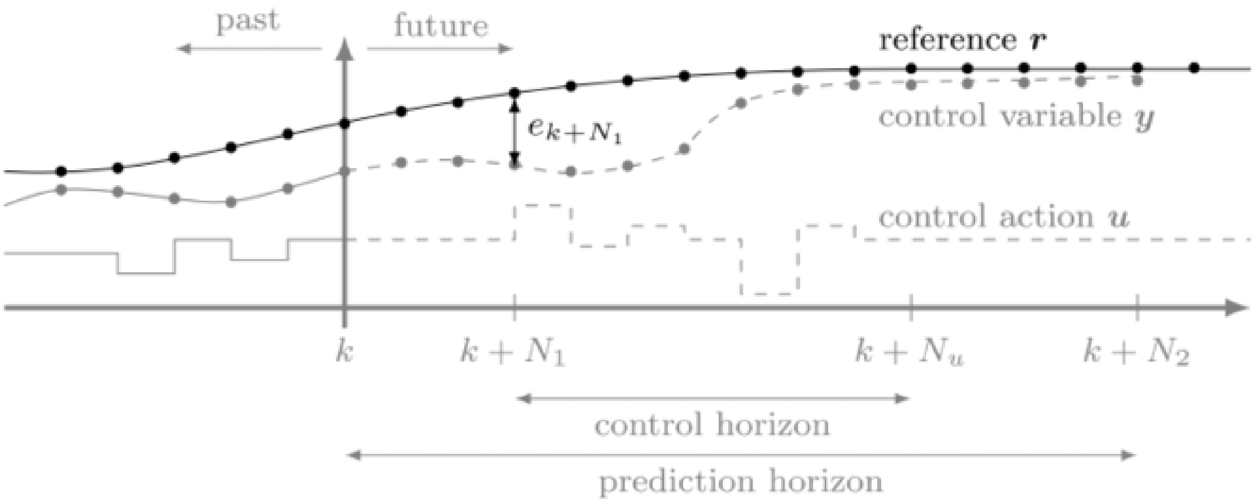
The position error is mathematically calculated and provided as input to the PID controller. The PID controller calculates the velocity value to proportionally manage the error. The calculated velocity from the PID controller is then used as the input velocity for the vehicle kinematic model,

Model Predictive Control



The output of the MPC controller is u which is considered as an optimal one. A constrained optimal solution search is solved and consequently, the best output is generated by the controller. Model Predictive Control promptly deals with constraints. The cost function is described in a manner that ensures the system's output (y) closely follows the desired reference (r) within a predefined future time horizon (N2)

Working principle



Cost function

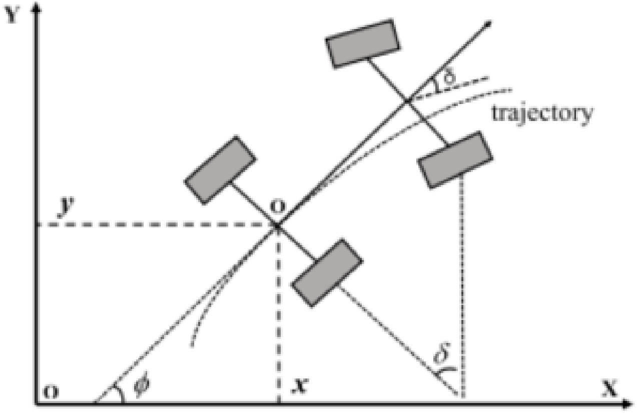
The cost function is described in a manner that ensures the system's output (y) closely follows the desired reference (r) within a predefined future time horizon (N2)

$$J(k) = \sum_{i=1}^{N_p} \left|\left|\eta(k+i|t) - \eta_{ref}(k+i|t)\right|\right|_Q^2 + \sum_{i=1}^{N_c-1} ||\Delta U(k+i|t||_R^2 + \rho\varepsilon^2 \ (3.10)$$

$\sum_{i=1}^{N_p} \left|\left|\eta(k+i|t) - \eta_{ref}(k+i|t)\right|\right|_Q^2$ the objective of minimizing the error between the predictive output points and the desired trajectory points

$\sum_{i=1}^{N_c-1} ||\Delta U(k+i|t||_R^2 + \rho\varepsilon^2$ reflects the consideration given to the size of U(t)

Vehicle Kinematics



$$\dot{x} = v\, cos\phi$$

$$\dot{y} = v\, sin\phi$$

$$\dot{\phi} = \frac{v\, tan\delta}{l}$$

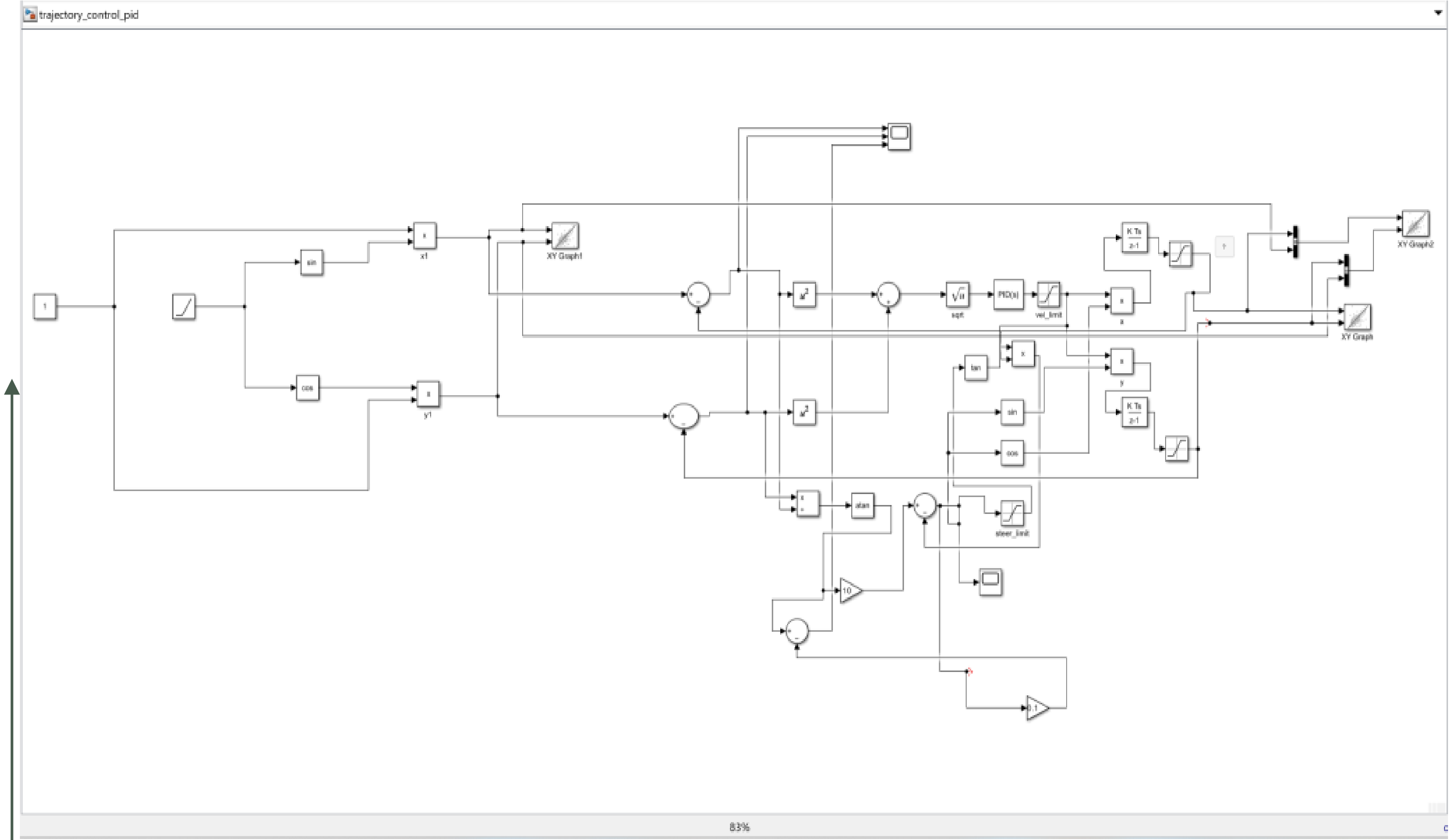
Vehicle Dynamics

The state space model:

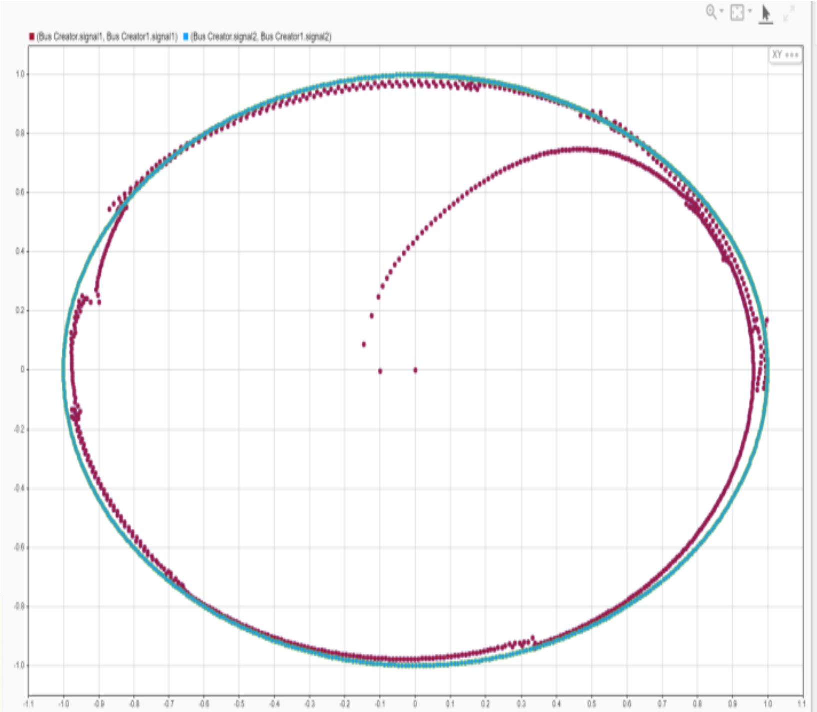
$$\frac{d}{dt}\begin{Bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2C_{\alpha f}+2C_{\alpha r}}{mV_x} & 0 & -V_x-\frac{2C_{\alpha f}\ell_f-2C_{\alpha r}\ell_r}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2\ell_f C_{\alpha f}-2\ell_r C_{\alpha r}}{I_z V_x} & 0 & -\frac{2\ell_f^2 C_{\alpha f}+2\ell_r^2 C_{\alpha r}}{I_z V_x} \end{bmatrix} + \begin{Bmatrix} 0 \\ \frac{2C_{\alpha f}}{m} \\ 0 \\ \frac{2\ell_f C_{\alpha f}}{I_z} \end{Bmatrix}\delta$$

The dynamic model of the vehicle is used in the simulation part of the trajectory tracking of the vehicle using MPC controller.

Simulation 1 : Results



Simulation 1 : Results

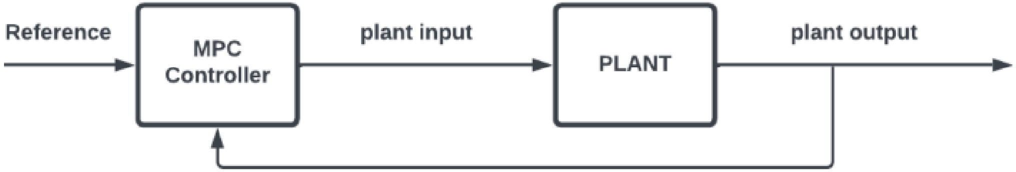


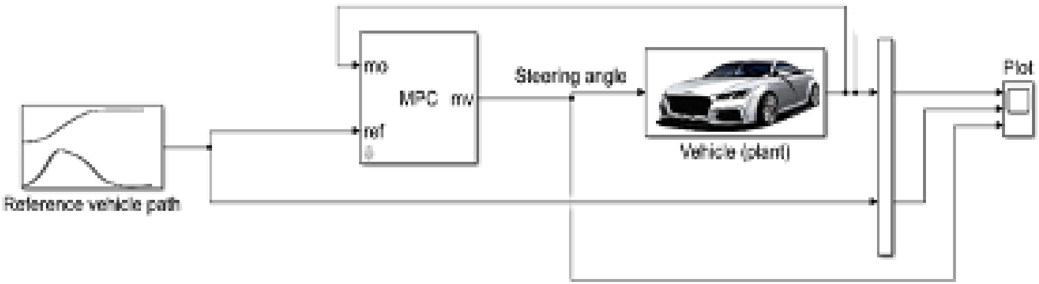
The green line corresponds to the reference trajectory, which is a unit circle with a center at (0,0). The red dots represent the actual trajectory followed by the vehicle, which is being controlled by the PID controller.

Trajectory error tracking

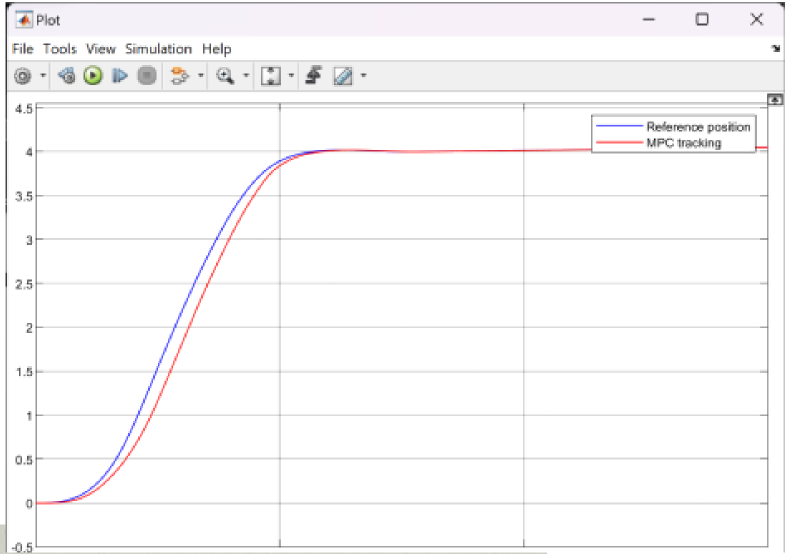
Blue curve represents the change in position error along y coordinate wrt time
Yellow curve represents the change in position error along x coordinate wrt time
Red curve represents the change in orientation error of the vehicle wrt time

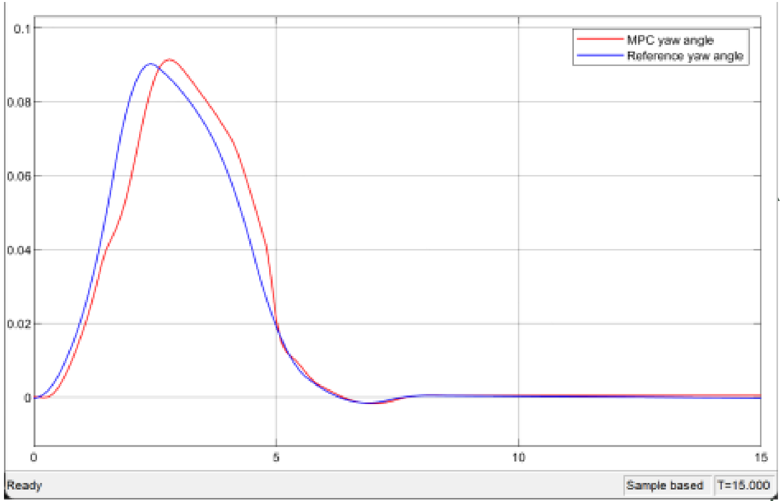
MPC simulation block diagram





Simulation 2 : Results





As can be seen, MPC manages following the paths with satisfactory performance showing a small error between. Graphs compare the reference and MPC tracking showing lateral positions and yaw angles.

Conclusion

the MPC controller does a better job at trajectory tracking when compared to the PID controller. This is because the MPC controller considers various constraints and dynamic design parameters of the vehicle to predict the system's behavior, tune the controller, and generate an output yaw angle such that the vehicle accurately follows the reference path