Unit-1

Q.1)

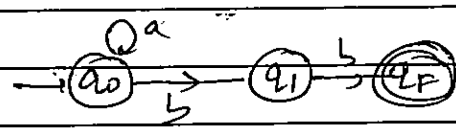
a) ~~DFA~~ - NFA

1) It stands for Non-Deterministic Finite Automata.

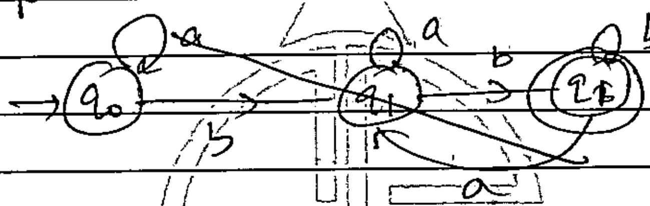
2) The transaction function is $Q \times \Sigma \rightarrow 2^Q$, where

Q = finite set of states

Σ = finite set of inputs.



3) example-

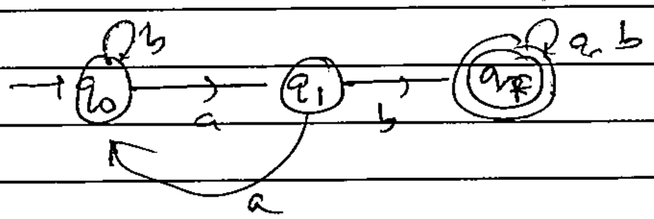


DFA -

1) It stands for Deterministic Finite Automata

2) The transaction function is $Q \times \Sigma \rightarrow Q$

3) example-



b) Mealy to Moore Machine.

From the table given we the states which are derived to different outputs for different inputs, and the states are -

$\{q_2, q_3\}$

~~$(q_2, 0) \to$~~

~~q_2 for input 0 goes to~~

q_2 state gives output 0 as well as 1 and q_3 state also gives output 0 as well as 1.

The Transitional table for the given Question is -

	a=0		a=1	
	State	output	State	output
$\to q_1$	q_1	1	q_2	0
q_2	q_4	1	q_4	1
q_3	q_2	0	q_3	1
q_4	q_3	0	q_2	1

In Moore Machine the output Column is also build.

~~The transiti~~ States are q_2, q_3 are divided into following parts -

$q_{21} \rightarrow$ gives output 1

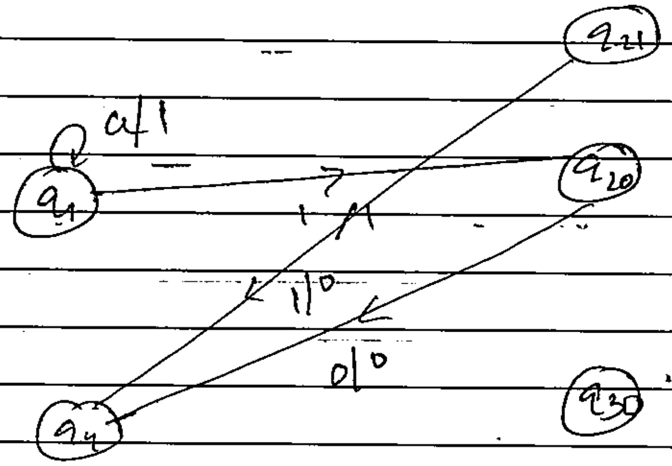
$q_{20} \rightarrow$ gives output 0

$q_{31} \rightarrow$ gives output 1

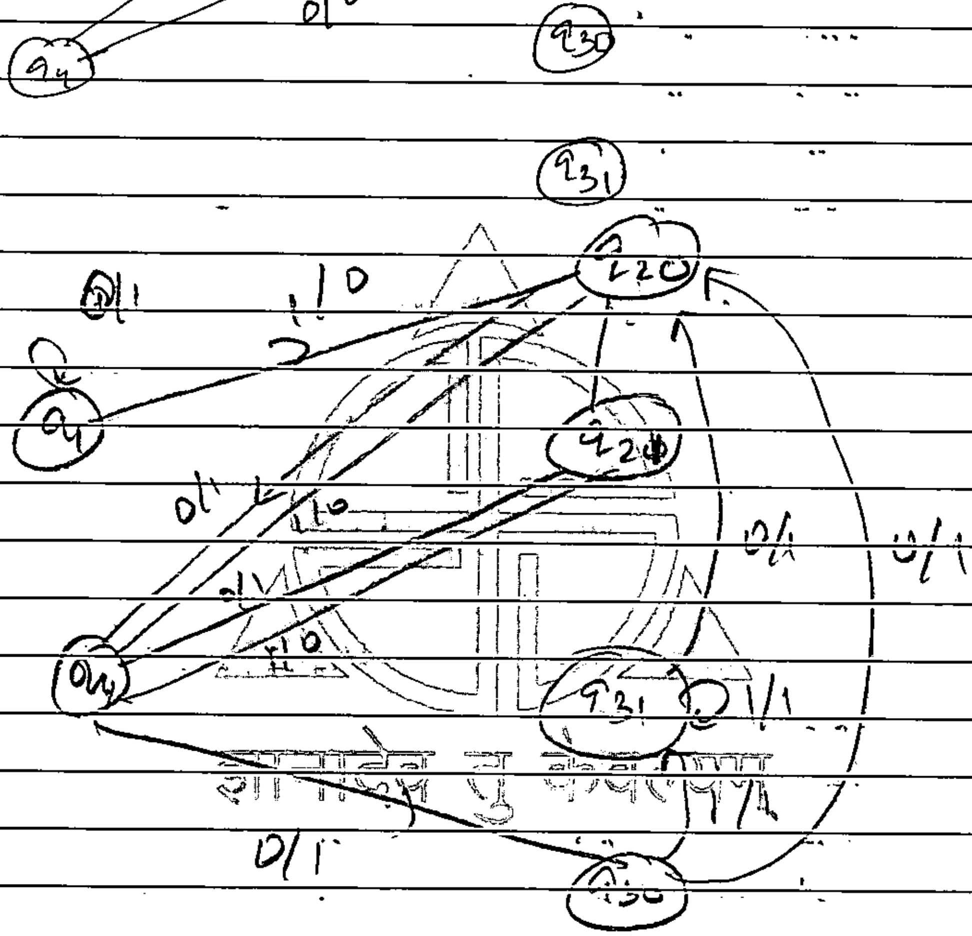
$q_{30} \rightarrow$ gives output 0.

Transitional table for the moore machine is -

	a=0 state	a=1 state	output
q_1	q_1	q_{20}	1
q_{20}	q_4	q_4	0
q_{21}	q_4	q_4	0
q_{30}	q_{20}	q_{31}	0
q_{31}	q_{20}	q_{31}	0
q_4	q_{30}	q_{21}	0



q21
a/1
q20
q1
1/1
1/0
0/0
q4



q30
q31
q20
0/1
q1
1/0
q2
0/1
1/0
0/1
0/1
0/1
q4
1/0
q31
1/1
0/1
q30

(a)

c)

Step 1: taking pairs p, q, mark cross (x) if p belongs to final state and q belongs to non-final state i.e.

$p \to F$, $q \not\to F$

	q_0	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8
q_0									
q_1									
q_2	X	X							
q_3	X	X							
q_4			X	X					
q_5	X	X			X				
q_6	X	X			X				
q_7			X	X		X	X		
q_8			X	X		X	X		

fig. 1

Final state - $\{q_2, q_3, q_5, q_6\}$

Non-final state - $\{q_0, q_1, q_4, q_7, q_8\}$

Step-2 In this step the pairs p, q on input a, b we will see whether it is marked ~~already~~ or not. If marked then cross (x) that element otherwise not i.e.

$\delta(p, a) \to$ if crossed then mark (x) ~~it cross~~ otherwise not.

$\delta(q, b) \to$ if crossed then mark (x) otherwise not

and same for $\delta(p, b)$ and $\delta(q, a)$.

	q_0	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8
q_0									
q_1	x								
q_2	x	x							
q_3	x	x	O						
q_4	x	O	x	x					
q_5	x	x	O	O	x				
q_6	x	x	O	O	x	O			
q_7	x	x	x	x	x	x	x		
q_8	x	x	x	x	x	x	x	O	

fig. 2

for (q_1, q_0)

$\delta(q_1, 0) \Rightarrow q_2$ $\quad \delta(q_1, 1) = q_4$
$\delta(q_0, 0) \Rightarrow q_1$ $\quad \delta(q_0, 1) = q_3$

(q_2, q_1) is marked ∴ mark(x) $[q_1, q_0]$

for (q_3, q_2)

$\delta(q_3, 0) = q_6$ $\quad \delta(q_3, 1) = q_7$
$\delta(q_2, 0) = q_7$ $\quad \delta(q_2, 1) = q_8$

It is not marked so ∴ leave it

for (q_0, q_4)

$\delta(q_0, 0) = q_1$ $\quad \delta(q_0, 1) = q_4$
$\delta(q_4, 0) = q_5$ $\quad \delta(q_4, 1) = q_6$
(q_4, q_6) is marked ∴ mark (x) $[q_0, q_4]$

for (q_1, q_4)

$\delta(q_1, 0) = q_2$ $\delta(q_1, 1) = q_3$
$\delta(q_4, 0) = q_5$ $\delta(q_4, 1) = q_6$

~~no~~ don't mark (q_1, q_4), leave it.

for (q_2, q_5)

$\delta(q_2, 0) = q_7$ $\delta(q_2, 1) = q_2$
$\delta(q_5, 0) = q_7$ $\delta(q_5, 1) = q_2$

don't mark $\therefore$ leave (q_2, q_5)

for (q_3, q_5)

$\delta(q_3, 0) = q_2$ $\delta(q_3, 1) = q_7$
$\delta(q_5, 0) = q_7$ $\delta(q_5, 1) = q_2$

don't mark, as it is not crossed
$\therefore$ leave it

for (q_2, q_6)

$\delta(q_2, 0) = q_7$ $\delta(q_2, 1) = q_2$
$\delta(q_6, 0) = q_7$ $\delta(q_6, 1) = q_2$

don't mark, $\therefore$ leave it.

for (q_3, q_6)

$\delta(q_3, 0) = q_2$ $\delta(q_3, 1) = q_7$
$\delta(q_6, 0) = q_7$ $\delta(q_6, 1) = q_2$

don't mark, as it is not Crossed
∴ leave it

for (q_0, q_7)

$\delta(q_0, 0) = q_1$ ∴ $\delta(q_0, 1) = q_4$
$\delta(q_7, 0) = q_7$ $\delta(q_7, 1) = q_7$
leave it

for (q_1, q_7)
$\delta(q_1, 0) = q_2$ $\delta(q_1, 1) = q_3$
$\delta(q_7, 0) = q_7$ $\delta(q_7, 1) = q_7$
mark it as (x)

for (q_4, q_7)

$\delta(q_4, 0) = q_5$ $\delta(q_4, 1) = q_6$
$\delta(q_7, 0) = q_7$ $\delta(q_7, 1) = q_7$
mark it as x

for (q_0, q_8)

$\delta(q_0, 0) = q_1$ $\delta(q_0, 1) = q_4$
$\delta(q_8, 0) = q_8$ $\delta(q_8, 1) = q_8$
don't mark, leave it

for (q_1, q_8)

$\delta(q_1, 0) = q_2 \quad \delta(q_1, 1) = q_3$
$\delta(q_8, 0) = q_8 \quad \delta(q_8, 1) = q_8$
mark (q_1, q_8) as x

for (q_4, q_8)

$\delta(q_4, 0) = q_5 \quad \delta(q_4, \cancel{1}1) = \cancel{q}\, q_6$
$\delta(q_8, 0) = q_8 \quad \delta(q_8, 1) = q_8$
~~leave it~~ ~~don't~~ mark (x) in (q_4, q_8)

for (q_7, q_8)

$\delta(q_7, 0) = q_7 \quad \delta(q_7, 1) = q_7$
$\delta(q_8, 0) = q_8 \quad \delta(q_8, 1) = q_8$
leave it don't mark (x)

~~St~~ The Fig. 2 describes the Step 2.

Now,
step 3: again iterate ~~it~~ and follow the ~~same~~ Step 2

We got (q_0, q_7) as marked (x)
(q_0, q_8) marked as (x).

The States marked as (O) are not reduced.

The new table is-

	q_0	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8
q_0									
q_1	x								
q_2	x	x							
q_3	x	x	0						
q_4	x	0	x	x					
q_5	x	x	0	0	x				
q_6	x	x	0	0	x	0			
q_7	x	x	x	x	x	x	x		
q_8	x	x	x	x	x	x	x	0	

fig. 3

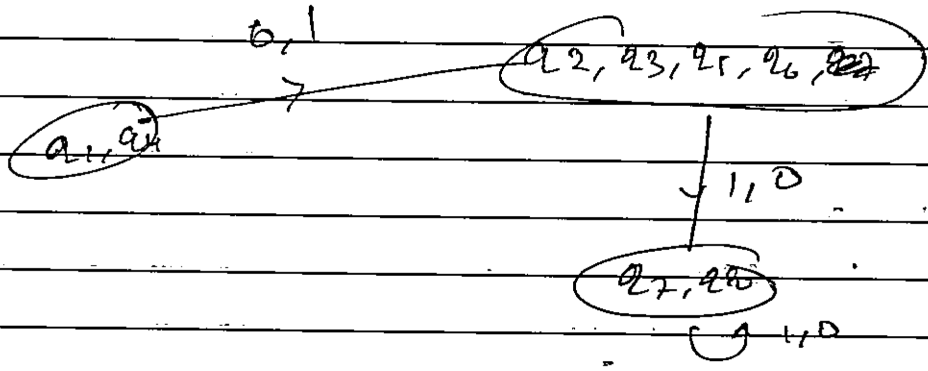
Now the states which are not reduced are -

~~(q_4, q_3)~~, (q_1, q_4), (q_2, q_5), (q_3, q_5),
(q_2, q_6), (q_3, q_6), (q_5, q_6)
(q_7, q_8)

Now after merging we get,

~~(q_4, q_3)~~, (q_1, q_4), (q_7, q_8);

(q_2, q_3, q_5, q_6, ~~q_7~~)



Unit-3

Q.3)

a)

(1)

(2) $(0+1)^* 00$

~~b)~~
d) Required string

— $w = 00110101$

~~Taking to~~

Building leftmost Derivation tree.

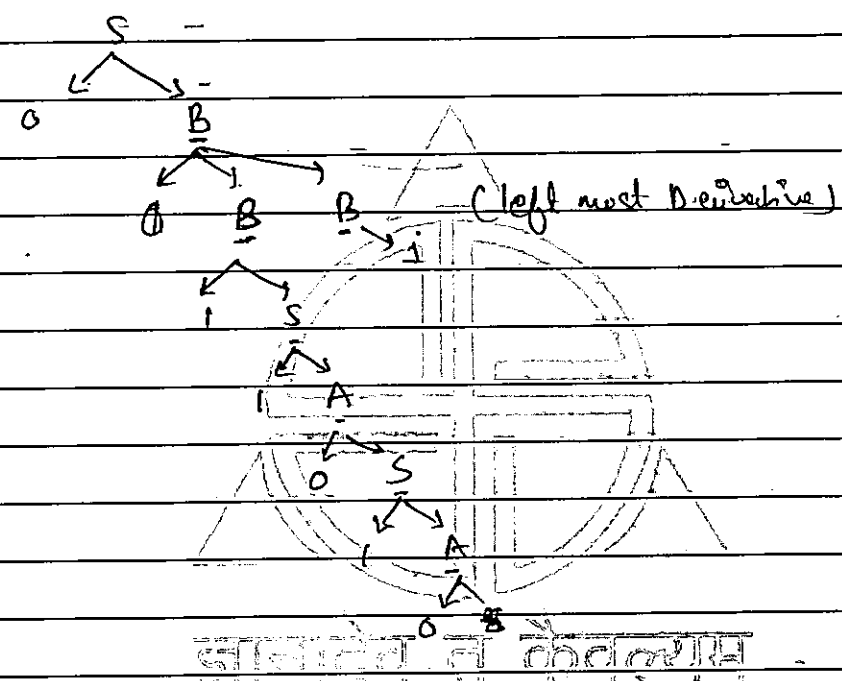


fig. 1

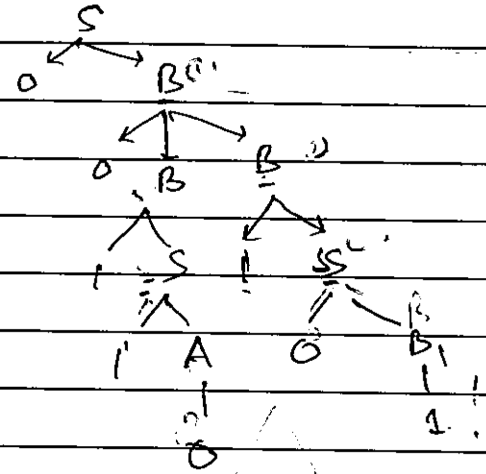
The left most tree is basically spitting out the left most production rule among the given rules, such that first left production and then right.

left most Derivative –

$S \to 0B \to 0BB \to 001SB \to 0011AB \to 00110SB$

$\boxed{00110101} \leftarrow 0011010B \leftarrow 001101AB$

Building Rightmost Derivation tree –



The Right most Derivation Tree is Basically spliting out the Right most ~~der~~ production Rule among the given Rules, such that ~~first~~ right production Rule and then left.

Derivation ID

~~For~~ left Most Derivative –

$S \to 0B \to 00BB \to 00B1 \to 001S1 \to 001.1A1$

$\downarrow$

$00110 \cdot 101 \leftarrow 00110 1A1 \leftarrow 0011\,0SA1$

Right most Derivative –

$S \to 0B \to 0\,0BB \to 00B1S \to 00B1\,0B \to 00B101$

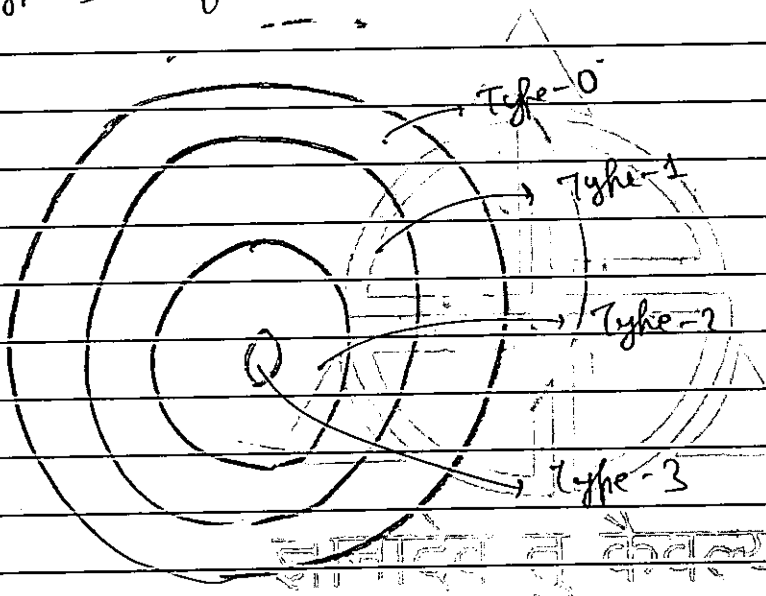
$\downarrow$

$\boxed{0011\,0101} \leftarrow 0011A101 \leftarrow 001S101$

b) Chomsky Classification -

Chomsky classified grammar based on four types, the types are as follows -

1) Type - 0 Grammar
2) Type - 1 Grammar
3) Type - 2 Grammar
4) Type - 3 Grammar



Type-0 Grammar -

It is also known as Recursively Enumerable grammar. It is recognized by Turing machine.

It is generated by Type-0 Grammar.

Type-1 Grammar -

It is generated by a Context Sensitive grammar.
It is recognized by Linear bounded automata.

It is a Subset of Type-0 Grammar.

Type-2 Grammar -

It is a Context free grammar generated by Context free Language.

It is recognized by Pushdown Automata.
It is a Subset of Type-1 Grammar.

Type-3 Grammar -

It is a Regular Grammar generated by regular language.

It is recognized by ~~RE.~~ Finite Automata.
It is a Subset of Type-2 Grammar.

Type-2 grammar is Superset of Type-3 Grammar.

Unit-4

(Q.4) b a) PDA for $L = \{a^n b^{2n}\}$, where $n \geq 1$

Now PDA is stands for Push down Automata which is used to Recognize Context-free Grammar.

taking ex-
for $n = 3$

$L = \{aaa\ bbbbbb\}$

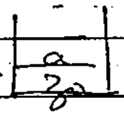
We have -
- $Q \rightarrow$ finite set of states $\{q_0, q_1, q_2, q_3\}$
- $\Sigma \rightarrow$ finite set of inputs $\{a, b, \epsilon\}$
- $\Gamma \rightarrow$ stack symbols
- $z_0 \rightarrow$ top of the stack
- $q_0 \rightarrow$ Initial state $\{q_0\}$
- $F \rightarrow$ final state $\{q_3\}$
- $\delta \rightarrow$ ~~translatio~~ transitions

There are 3 a's and 6 b's on pushing a onto the stack to the top of z_0 (which is top of the stack symbol) and all a's are pushed inside and.

transitions function →

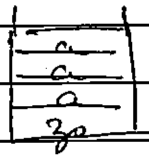
on pushing a (i.e. 3 a's) the function is —

$\delta(q_0, a, z_0) \rightarrow (q_0, a z_0)$



$\delta(q_0, a, a) \rightarrow (q_0, aa)$

$\delta(q_0, a, a) \rightarrow (q_0, aa)$



Now b's are not equal to no. of a's so ∴ on arriving at first b we will push a and after next b we will leave without any change and this process goes on.

So the for transitions are —

$\delta(q_0, b, a) \rightarrow (q_1, \epsilon)$

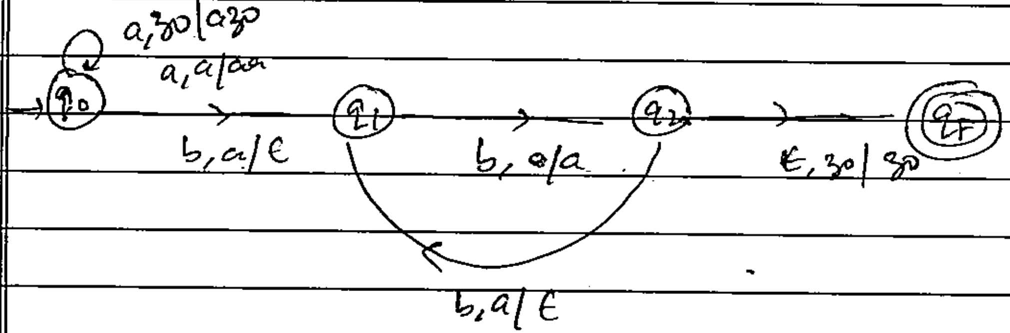
$\delta(q_1, b, a) \rightarrow (q_1, a)$

$\delta(q_2, b, a) \rightarrow (q_1, \epsilon)$

$\delta(q_2, \epsilon, z_0) \rightarrow (q_f, z_0)$ // accepted by final state

$\delta(q_2, \epsilon, z_0) \rightarrow (q_2, \epsilon)$ // accepted by empty state

PDA ~~is~~ for $a^n b^{2n}$ is —

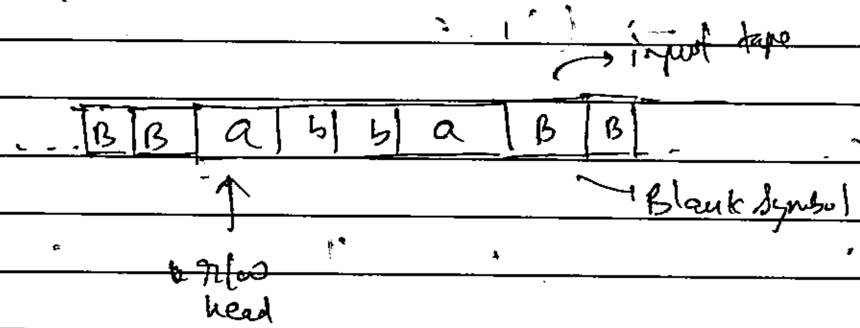


c) In turing machine we have seven' set tuples —

$Q \rightarrow$ finite set of states
$\Sigma \rightarrow$ finite set of input
$B \rightarrow$ Blank symbol

$q_0 \rightarrow$ Initial state
$F \rightarrow$ final state
$\delta \rightarrow$ transitions.

~~we have~~ In this machine we have an infinite length in input tape where we have a read/ write head.



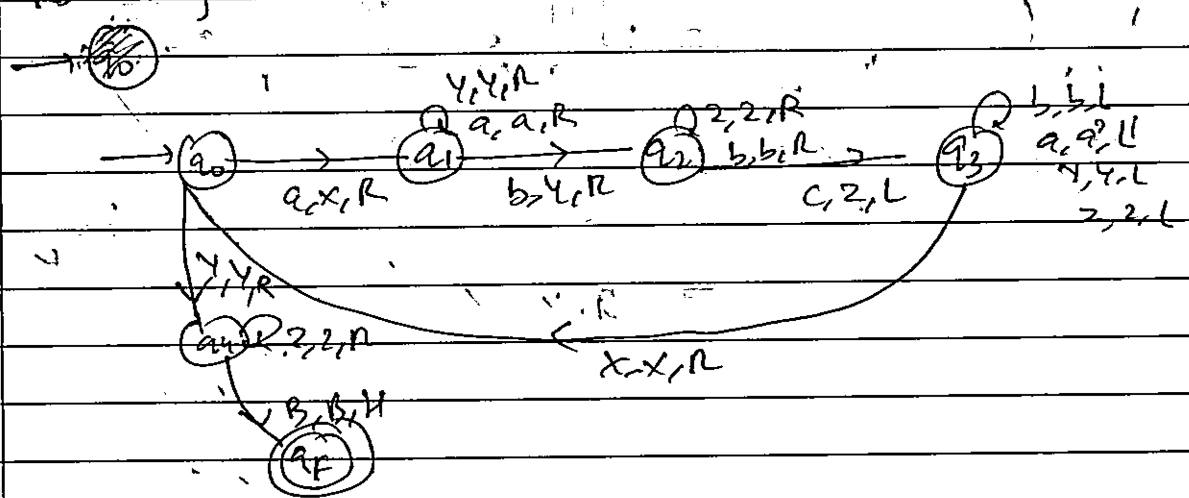
taking $n=3$

$L = \{aaabbbccc\}$ $n \geq 0$

For getting each a ~~with~~ we will modify it into X, same and on getting b we will modify it into Y and same for c as Z.

After getting the first Z we will read leftward to the ~~first~~ last X symbol, in this we can figure out that ~~same no. of X, Y,~~ a, b, c are occurred with same no. of X, Y, Z.

Turing Machine is —



The transitions are —

$\delta(q_0, a, X) \rightarrow (q_1, R)$ $\qquad$ aaabbbccc

$\delta(q_1, a, a) \rightarrow (q_1, R)$

$\delta(q_1, b, Y) \rightarrow (q_2, R)$

$\delta(q_2, b, b) \rightarrow (q_2, R)$

$\delta(q_2, c, z) \rightarrow (q_3, L)$

$\delta(q_3, b, b) \rightarrow (q_3, L)$

$\delta(q_3, Y, Y) \rightarrow (q_3, L)$

$\delta(q_3, a, a) \rightarrow (q_3, L)$

$\delta(q_3, X, X) \rightarrow (q_0, R)$

$\delta(q_1, Y, Y) \rightarrow (q_1, R)$

$\delta(q_2, \cancel{Z}, Z) \rightarrow (q_2, R)$

After ~~Conv~~ Converting all a, b, c into X, Y, Z respectively we will find the new index tape to be ~~X~~ Y instead of a.

So for this

the new transition is -

$\delta(q_0, Y, Y) \rightarrow (q_4, R)$

$\delta(q_4, Z, Z) \rightarrow (q_4, R)$

$\delta(q_4, B, B) \rightarrow (q_F, H)$

Where in the final state the machine gets halted.

a)

NPDA (Non-Deterministic)	DPDA
1) It is a Pushdown Automata.	1) It is a Deterministic Pushdown Automata.
2) In this Automata for one input it can ~~get~~ ~~non-~~derive to more than one states.	2) In this Automata for one unique input and state it will derive into a unique ~~and~~ single state.
3) example ~~Units~~ $ww^R \mid w \in (0,1)^*$	3) example $ww^R \mid w \in (0,1)^*$, $a^n b^n$

Unit-5.

a) Partial	Initial
1) It is a Recursive function which is defined for some of its arguments.	1) It is a Recursive function which is defined for all its arguments.
2)	2)

a) Partial - It is a recursive function which is defined for some of its arguments.

Ex - (1) Subtraction of two integers.
(2) ~~Mini~~ Minimization of Total Recursive function.

Initial - for set of Natural No - $N = \{0, 1, 2, \ldots$

If -
For, $z(x) = 0$, which is a zero function

$S(x) = x+1$, which is a Successor function

& $U_x^i(x_i, x) = x_i$, predecessor function

~~Unit~~.

b) $f(x,y) = x * y$

We know, ~~x+y~~ $f(x,y)$ is a primitive recursive function.

So $f(x,y) = x * y$

$f(x,0) = 0 = g(x)$

$f(x,y+1) = x * (y+1)$

$= x * y + x = f(x,y) + x$

$h(x,y,f(x,y)+x) = U_3^3(x,y,f(x,y)) + U_1^3(x,y,f(x,y))$

and we know "+" is a Recursive f^n ∴

$f(x,y) = x * y$ is a primitive Recursive function.

Now $f(x,y) = x^y$

$f(x,0) = 0 = g(x)$

$f(x,y+1) = x^{y+1} = x^y \cdot x$.

$= f(x,y) \cdot x$

$h(x,y,f(x,y) \cdot x) = U_3^3(x,y,f(x,y)) \cdot U_1^3(x,y,f(x,y))$

we proved "." is a ~~Re~~ Primitive Recursive function

∴ $f(x,y) = x^y$ is a Primitive recursive function

Unit-2

Q.2)

~~a)~~ b)

d) the equations are-

$q_1 \to \epsilon + q_1\cdot 0 + q_3\cdot 0$

$q_2 \to q_2\cdot 1 + q_1\cdot 1 + q_3\cdot 1$

$q_3 \to q_2\cdot 0$

putting the value of q_3 in q_2 equation we will get -

$q_2 \to q_2\cdot 1 + q_1\cdot 1 + q_2\cdot 0\cdot 1$

$q_2 \to q_1\cdot 1 + q_2\cdot(1+0\cdot 1)$

from arden's theorem

$\cancel{q_2 =} q_2 \doteq R = R + PQ$

$\boxed{R = PQ^*}$ unique equation,

$\therefore\ q_2 \to (q_1\cdot 1)\,(1+0\cdot 1)^*$

Now again putting the value

$$q_1 \to q_2 \cdot 0 \cdot 0 + q_1 \cdot 0$$

$$q_1 \to (q_2 \cdot 0 \cdot 0)(0)^*$$

putting the value of q_2 in q_3 equation

we get

$$q_3 \to (q_1 \cdot 1)(1+0 \cdot 1)^* 0$$

and putting the above equation in q_1.

we get.

$$q_1 \to (q_1 \cdot 1)(1+0 \cdot 1)^* 0 \cdot 0 + 0$$

$$q_1 \to q_1 \left[1 \cdot (1+0 \cdot 1)^* 0 \cdot 0 + 0 \right]$$

$$q_1 \to \left[1 \cdot (1+01)^* 0 \cdot 0 + 0 \right]^*$$ (from Arden's theorem)

$\therefore$ the Regular expression is –

$$\boxed{\left(1 \cdot (1+01)^* 0 \cdot 0 + 0\right)^*}$$

b) ~~F.A to R~~ Regular expression

Let the language a pumping length p
Such that $|S| \geq p$;

Where $p = 3$.

$\therefore L = \{aaabbb\}$

Now divide into three parts ~~Such that~~ xyz
Such that

① $|y| > 0$

② $|xy| \leq 0$

③ and $xy^iz \in A$

Now,

① $x = aa$ $y = ab$ $z = bb$

$S = aa\,(ab)^i\,bb$

for $i = 0$ aa ab bb ✓ a = b

for $i = 2$ aa (aabb) bb ✓ a = b

$|y| > 0$, $|xy| < 0$ (false)
(True)

② When
$x = aa$ $y = a$ $z = bbb$

$\therefore S = a\,a\,a^i\,bbb$

for $i=1$ aa a bbb ✓

$i=2$ aaaa bbb $a \neq b$ X

and $|y| \geq 0$ True.

$|xy| \leq 0$ True.

$\therefore$ from the above derivation we can say

$a^n b^n$ is not regular.

a) ~~Closure property is defined as of Regular grammar is defined as the transition of a state (current state) to the final state.~~

Closure property of a regular grammar is defined as consisting of all the languages of any combination and as well as ϵ (Epsilon).

ex - $L\{(a+b)^*\}$ (kleene-closure)

Now the grammar is -

$\{\epsilon, ab, aab, abb, aabb \ldots\}$

ज्ञानादेव तु कैवल्यम्