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Time Series Regression with ARIMA Noise and Missing Observations

Program TRAM

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Abstract

The present paper describes the program TRAM, which stands for "Time Series Regression with ARIMA Noise and Missing Observations". TRAM has been written in Fortran and is available from the authors for MS-DOS computers and mainframes. The program estimates the parameters of a regression model with possibly nonstationary noise and any sequence of missing observations, interpolates the missing values, and obtains forecasts for the series. The program incorporates several additional facilities, such as intervention analysis, and easter and/or trading day corrections. The methodology is described in the paper "Estimation, Prediction and Interpolation for Nonstationary Series with the Kalman Filter", by V. Gómez and A. Maravall, EUI Working Paper ECO No. 92/80.

The first part of this document presents a summary of the program. Part two contains the instructions for the user and a description of the parameters. Finally, part three illustrates the program for six well-known examples, which present different regression and time series models with different combinations of missing observations.

Preliminary Version

1. Introduction

TRAM ("Time series Regression with Arima noise and Missing Observations") is a program written in Fortran for mainframes and PCs under MSDos. The program performs estimation, forecasting, and interpolation of regression models with missing observations, and ARIMA errors. No restriction is imposed on the location of the missing observations in the series.

Given the vector of observations:

$$z = \left(z_{t_1}, \ldots, z_{t_M}\right)', \tag{1}$$

where $0 < t_1 < \ldots < t_M$, the program fits the regression model

$$z_t = y_t'\beta + \nu_t, \tag{2}$$

where $\beta = (\beta_1, \ldots, \beta_n)'$, is a vector of regression coefficients, $y'_t = \left[y_{1t}, \ldots, y_{nt}\right]$ denotes n regression variables, and ν_t follows the general ARIMA process

$$\phi(B)\delta(B)\nu_t = \theta(B)a_t + c, \tag{3}$$

where B is the backshift operator; $\phi(B)$, $\delta(B)$, and $\theta(B)$ are finite polynomials in B; a_t is assumed a n.i.i.d. $\left(o, \sigma_a^2\right)$ white-noise variable, and c is a constant.

The polynomial $\phi(B)$ contains the stationary autoregressive roots, $\delta(B)$ is the polynomial with the nonstationary autoregressive roots, and $\theta(B)$ denotes the (invertible) moving average polynomial. In TRAM, they assume the following multiplicative form:

$$\delta(B) = (1-B)^{IDR}\left(1-B^S\right)^{IDS}$$

$$\phi(B) = \left(1 + \phi_1 B + \ldots + \phi_{IPR} B^{IPR}\right)\left(1 + \Phi_1 B^S + \ldots + \Phi_{IPS} B^{S \times IPS}\right)$$

$$\theta(B) = \left(1 + \theta_1 B + \ldots + \theta_{IQR} B^{IQR}\right)\left(1 + \Theta_1 B^S + \ldots + \Theta_{IQS} B^{S \times IQS}\right)$$

As explained in the user instructions, initial estimates of the parameters can be input by the user, set to the default values, or computed by the program.

The regression variables can be input by the user (such as economic variables thought to be related with z_t), or generated by the program. The variables that can be generated are Trading Day, Easter Effect and intervention variables of the type:

a) dummy variables (additive outliers);

b) any possible sequence of ones and zeros;

c) $1/(1-\delta B)$ of any sequence of ones and zeros, where $0 < \delta \leq 1$;

d) $1/\left(1-\delta_S B^S\right)$ of any sequence of ones and zeros, where $0 < \delta_S \leq 1$;

e) $1/(1-B)\left(1-B^S\right)$ of any sequence of ones and zeros.

As indicated in the user instructions, missing observations can also be treated as additive outliers. In this case, the likelihood is corrected so that it coincides with that of the standard missing-observations case (see example 4 in section 3).

The program:

1) estimates by exact maximum likelihood (or unconditional least squares) the parameters in (2) and (3);

2) computes optimal forecasts for the series, together with their MSE; and

3) yields optimal interpolators of the missing observations and their associated MSE.

The methodology followed is described in the paper "Estimation, Prediction, and Interpolation for Nonstationary Series with the Kalman Filter" by V. Gomez and A. Maravall. This paper will be referred to as the "background paper"; references made in this Introduction can be found in the Reference section of the background paper.

Estimation of the regression parameters (including the missing observations among the initial values of the series) plus the ARIMA model parameters, can be made by concentrating the former out of the likelihood, or by joint estimation. Several algorithms are available for computing the likelihood or more precisely, the nonlinear sum of squares to be minimized. When the differenced series can be used, the algorithm of Morf, Sidhu and Kailath (1974), (as improved by Mélard, 1984) is employed.

For the nondifferenced series, it is possible to use the ordinary Kalman filter, as described in the background paper (default option), or its square root version (see Anderson and Moore, 1979). The latter is adequate when numerical difficulties arise; however it is markedly slower and does not permit (at present) to concentrate the regression parameters out of the likelihood. By default, the exact maximum likelihood method is employed, and the unconditional least squares method is available as an option. Nonlinear maximization of the likelihood function and computation of the parameter estimates standard errors is made using Marquardt's method and first numerical derivatives.

As detailed in the background paper, estimation of regression parameters is made by using first the Cholesky decomposition of the error covariance matrix to transform the regression equation (the Kalman filter provides an efficient algorithm to compute the variables in this transformed regression). Then, the resulting least squares problem is solved by orthogonal matrix factorization using the Householder transformation. This procedure yields a numerically stable method to compute GLS estimators of the regression parameters, which avoids matrix inversion.

For forecasting and interpolation, the ordinary Kalman filter or the square root filter options are available. Interpolation of missing values is made by the simplified Fixed Point Smoother, as described in the paper. When concentrating the regression parameters out of the likelihood, mean squared errors of the forecasts and interpolations are obtained following the approach of Kohn and Ansley (1985).

When some of the initial missing values are free parameters, the program detects them, and flags the forecasts or interpolations that depend on these free parameters. The user can then assign arbitrary values (typically, very large or very small) to the free parameters and rerun the program. Proceeding in this way, all parameters of the ARIMA model can be estimated because the function to minimize does not depend on the free parameters. Moreover, it will be evident which forecasts and interpolations are affected by these arbitrary values because they will strongly deviate from the rest of the estimates (see example 6 in section 3). However, if all unknown parameters are jointly estimated, the program may not flag all free parameters. It may happen, that there is convergence to a valid arbitrary set of solutions (i.e., that some linear combinations of the initial missing observations, including the free parameters, are estimable.)

2. Instruction for the user

INSTALLATION :

Insert the diskette in drive A or B, and
change the default drive (type "A:" or "B:").
When the prompt appears type:

INSTALL

The installation procedure creates a directory "TRAM";
be sure it doesn't already exist.
(If you have a partitioned diskette, you will be asked in which drive the program should be written.)

TO RUN THE PROGRAM :

Prepare the input file following the instructions in the next pages.
Once the input file has been prepared in the SERIES subdirectory, to execute the program simply type:

"TRAM filename"

where filename is the name of the input file, at the directory where the program has been installed (TRAM).
The results can be seen by editing or printing the file OUTPUT.

Typing **"GRAPH"**, several graphics can be readily obtained.
Moreover, from the subdirectory GRAPH (which is then created), the relevant arrays can be retrived for further use in other econometrics/statistics/graphics package.

The input starts with the series to be modelled, comprising no more than 250 observations, followed by one set of control parameters for the series model plus a list of instructions for the regression variables.

To specify the set of control parameters for the series model, as well as the instructions for the regression variables, the NAMELIST facility is used, so that only those parameters which are not at their default values (see below) need to be set.

The series is set up as:

Card 1	TITLE (no more than 72 characters)
Card 2	NZ NYER NPER NFREQ (free format)
Card 3 et seq	Z(I): I=1,NZ (free format),

where NZ is the number of observations, NYER the start year, NPER the start period, and NFREQ the observational frequency in the year. Z(.) is the array of observations. For each missing observation, the code -99999. must be entered.

This is followed by namelist DATEN. The namelist starts with &DATEN (in the second column) and terminates with /. The parameters in namelist DATEN are:

Parameter		Meaning	Default
LAMDA	=	1 No tranformation of data	1
	=	0 Take logs of data	
IMEAN	=	0 No mean correction	0
	=	1 Mean correction	
IDR	=	# of non-seasonal differences	1
IDS	=	# of seasonal differences	1
IPR	=	# of non-seasonal autoregressive terms	0
IPS	=	# of seasonal autoregressive terms	0
IQR	=	# of non-seasonal moving average terms	0
IQS	=	# of seasonal moving average terms	0
IREG	=	# of regression variables	0

Parameter		Meaning	Default
		(entered by the user or calculated by the program as intervention variables)	
ITRAD	=	0 No trading day adjustment	0
	=	1 Trading day adjustment	
IEAST	=	0 No Easter effect adjustment	0
	=	1 Easter effect adjustment	
IDUR	=	Duration of Easter affecting period	0
LAG	=	# of autocorrelations and partial autocorrelations printed	24
INCON	=	0 Exact maximum likelihood estimation	0
	=	1 Unconditional least squares	
NBACK	=	# of observations back from the end of the data that the multistep forecasts are to begin	0
NPRED	=	# of multistep forecast values to compute	0
INTERP	=	0 No interpolation of unobserved values	0
	=	1 Interpolation of unobserved values	
IESTIM	=	0 No estimation of unknown parameters	1
	=	1 Estimation of unknown parameters	
THR	=	IQR initial estimates of the regular moving average parameters (not input if there are not missing observations and INIC=0)	All -.1
THS	=	IQS initial estimates of the seasonal moving average parameters (not input if there are not missing observations and INIC=0)	All -.1
FIR	=	IPR initial estimates of the regular autoregressive parameters (not input if there are not missing observations and INIC=0)	All -.1
FIS	=	IPS initial estimates of the seasonal autoregressive parameters (not input if there are not missing observations and INIC=0)	All -.1
VA	=	Residual variance to be used for inter-	1.0

polation and prediction when no estimation is to be performed (not input if

Parameter		Meaning	Default
		IESTIM=1)	
IFILT	=	1 Square root filter	2 (No missing)
	=	2 Morf, Sidhu and Kailath algorithm, as improved by Mélard	3 (Missing)
	=	3 Kalman filter	
IGRBAR	=	1 Graph of autocorrelations printed	0
	=	0 " " " not printed	
IGRRES	=	1 Graph of model residuals printed	0
	=	0 " " " " not printed	
RG	=	IMEAN + IREG + ITRAD + IEAST initial estimates of the regression parameters, not including initial missing observations (not input if there are not missing observations or ICONCE=1)	All 0.1
IDENSC	=	1 Denominator of residual sum of squares is that of Ansley and Newbold = number of non-initial observations minus number of unknown parameters (AR and MA parameters plus regression parameters, including initial missing observations)	1
	=	0 Denominator of residual sum of squares is equal to the number of non-initial observations	
INVER	=	1 Parameters of the MA polynomial restricted to remain in the invertible region	0
	=	0 Parameters of the MA polynomial not restricted to remain in the invertible region	
INIC	=	1 Initial estimates of AR and MA parameters input	0
	=	0 Initial estimates of AR and MA parameters calculated At present, the program only calculates initial estimates of AR and MA parameters if there are no missing observations. With missing observations, the program always takes as initial estimates for the AR and MA parameters those input by the the user (or their default values if none are input)	

TOL	=	Convergence criterion in Marquardt's method	1.E-6

Parameter		Meaning	Default
ICONCE	=	1 σ^2 and regression parameters (included missing initial observations) concentrated out of the likelihood (not input if IFILT=1)	1
	=	0 only σ^2 concentrated out of the likelihood	

If IREG in namelist DATEN is greater than zero, then namelist DATEN should be followed by a certain number of namelists REG, to be described below. Each namelist REG starts with &REG (in the second column), terminates with / and contains the set of instructions for the corresponding regression variable/s.

Missing observations can also be treated as additive outliers. That is, each missing observation is assigned a tentative value (now the code -99999. should not be entered) and an additive outlier is specified for each missing observation. **In this case, one namelist REG corresponding to all missing observations should be written before the other namelists REG.** The determinantal term in the function to be minimized when this approach is used is adjusted so that it coincides with that of the function used in our approach.

The total number of namelists REG is as follows: There must be one namelist REG for all missing observations to be treated as additive outliers (in case there are any), specifying their time indices, and as many namelists REG following as there are regression variables, either input by the user or calculated by the program. It is not possible to treat some missing observations as additive outliers while specifying others with the code -99999., simultaneously. Only one procedure can be used.

The parameters in namelist REG are:

Parameter		Meaning	Default
IUSER	=	0 The program will generate one intervention variable if IAUS = -k (k = a positive integer): either k sequences of ones or the result of applying a filter to this intervention variable. Possible filters are: $1/(1-\delta B)$, $0<\delta\leq 1$, $1/(1-\delta B^S)$, S = NFREQ, $0<\delta\leq 1$, and $1/(1-B)(1-B^S)$, S = NFREQ. If IAUS = k (a positive integer), it means that missing observations are to be treated as additive outliers. In this case, the program will generate k intervention variables, one for each missing observation	0
	=	1 The user will enter a series for this regression variable. After the present namelist REG, the user will write the	

series X(I): I=1,ILONG (free format). After the series, next namelist REG should be written

Parameter		Meaning	Default
ILONG	=	Length of the series entered by the user if IUSER=1. The rest of the series, up to a total length of NZ + NPRED is filled up with zeros (not input if IUSER = 0)	0
IAUS	=	k (k = a positive integer) There are k missing observations to be treated as additive outliers. After the present namelist REG, the user must write the k time indices corresponding to these missing observations (free format). The program will generate k intervention variables of length NZ + NPRED, one for each additive outlier (=missing observation). The k time indices are to be followed by the next namelist REG. (no input if IUSER=1)	0
	=	-k (k = a positive integer) The program will generate one intervention varible of length NZ + NPRED consisting of k sequences of ones. After the present namelist REG, the user will write k pairs of numbers (free format); the j-th pair indicates the time index where the j-th sequence of ones is to beguin and its length, respectively (j=1,...,k). The k pairs of numbers are to be followed by the next namelist REG.(no input if IUSER=1)	
	=	0 The program will generate no regression variable (no input if IUSER=0)	
DELTA	=	δ ($0<\delta\leq1$); the filter $1/(1-\delta B)$ will be applied to the k sequences of ones generated by the program when IAUS = -k (no input if IUSER=1 or IAUS $\geq$ 0)	0
DELTAS	=	δ_S ($0<\delta_S\leq1$); the filter $1/(1-\delta_S B^S)$, S = NFREQ, will be applied to the k sequences of ones generated by the program when IAUS = -k (no input if IUSER=1 or IAUS $\geq$ 0)	0
ID1DS	=	1 The program will generate $1/(1-B)(1-B^S)$, S = NFREQ, of the k sequences of ones generated by the program when IAUS = -k (no input if IUSER=1 or IAUS $\geq$ 0)	0

The regression variables used to make the Trading Day or Easter Effect adjustment are generated by the program in the same way as that described in Hillmer, S.C., Bell, W. R., and Tiao,

G. E. (1983), "Modeling Considerations in the Seasonal Adjustment of Economic Time Series," in A. Zellner (ed.), Applied Time Series Analysis of Economic Data, Washington, D.C.: Bureau of the Census.

Memory Constraints

The user should be aware of the following memory constraints:

IFILT=2	2 + 2*IMRTE ≤ 42 IR + 2 + ICON ≤ 42
IFILT=3	1 + 2*IMRTE + IMISP ≤ 42 MAX {IR + 1, ICON} + ICON ≤ 42
IFILT=1	like IFILT=3
In all cases	MAX {N, ICON} + N + 1 ≤ 42,

where IMRTE = IMEAN + IREG + 7*ITRAD + IEAST, IMISP = number of initial missing values, ICON = ICONCE*(IMRTE + IMISP), IR = MAX { ID + IP, IQ + 1}, ID = IDR + NFREQ*IDS, IP = IPR + NFREQ*IPS, N = number of parameters to be estimated by Marquardt's method.

3. Examples

The input and output files of six examples are presented next. Some previous comments are in order:

a) Although only non-default parameter values need to be entered, for the more relevant parameters, the enclosed input files contain also the default values.

b) Since the state space representation we use directly provides [in x(t|t)] the (r–1)-periods-ahead forecast function (where r is the dimension of the state vector), the program computes (NPRED + r–1) forecasts. Standard errors, however, are only computed for the first NPRED forecasts.

c) When there is a missing observation among the initial values of the series, the missing observation is estimated by regression as explained in the background paper. In the output, the interpolated value appears as the coefficient of ZJ, under the heading "Estimates of Regression Parameters."

The six examples present different regression and time series models, with different combination of missing observations. They have been taken from the following references:

Box, G.E.P. and Jenkins, G.M. (1970), *Time Series Analysis*, San Francisco: Holden-Day.

Box, G.E.P. and Tiao, G.C. (1975), "Intervention Analysis with Applications to Economic and Environmental Problems", *Journal of the American Statistical Association* 70, 70–79.

Harvey, A.C. and Pierce, R.G. (1984), "Estimating Missing Observations in Economic Time Series", *Journal of the American Statistical Association* 79, 125–131.

Hillmer, S.C., Bell, W.R. and Tiao, G.C. (1983), "Modelling Considerations in the Seasonal Adjustment of Economic Time Series", in A. Zellner (ed.), *Applied Time Series Analysis of Economic Data*, Washington, D.C.: Bureau of the Census.

Kohn, R. and Ansley, C.F. (1986), "Estimation, Prediction and Interpolation for ARIMA Models with Missing Data", *Journal of the American Statistical Association* 81, 751–761.

Maddala, G.S. (1977), *Econometrics*, N.Y.: McGraw-Hill Book Co.

Example 1

The monthly series y_t of oxidant (O_3) level recordings in downtown Los Angeles, for the period January 1955 – December 1972, is considered. The series was analysed by Box and Tiao (1975) and the identified model is of the form

$$y_t = \frac{\omega_0}{1-B}\xi_{1t} + \frac{\omega_1}{1-B^{12}}\xi_{2t} + \frac{\omega_2}{1-B^{12}}\xi_{3t} + n_t,$$

$$\nabla_{12} n_t = (1-\theta_1 B)(1-\theta_{12}B^{12})a_t,$$

where ξ_{1t}, ξ_{2t} and ξ_{3t} are "intervention" variables such that

$$\xi_{1t} = \begin{cases} 0 & t \neq \text{Jan } 1960 \\ 1 & t = \text{Jan } 1960 \end{cases}$$

$$\xi_{2t} = \begin{cases} 1 & \text{months June} - \text{October, beginning in 1966} \\ 0 & \text{otherwise} \end{cases}$$

$$\xi_{3t} = \begin{cases} 1 & \text{months November} - \text{May, beginning in 1966} \\ 0 & \text{otherwise} \end{cases}$$

Eight observations were randomly removed and estimated as missing values, and one of them falls among the first 12 values. The missing observations can be identified by the number –99999 in the file. For this model, the three regression variables are constructed by the program. (For their particular meaning, see the paper by Box and Tiao.)

```
L.A. OXIDANT DATA (BOX-TIAO, JASA 75) WITH 3 INTERVENTIONS AND 8 M.O.
216   1955 1 12
2.7   2.0 -99999. 5.0   6.5   6.1   5.9   5.0   6.4   7.4   8.2   3.9
4.1   4.5   5.5   3.8   4.8   5.6   6.3   5.9 -99999. 5.3   5.7   5.7
3.0   3.4   4.9   4.5   4.0   5.7   6.3   7.1   8.0   5.2   5.0   4.7
3.7   3.1 -99999. 4.0   4.1   4.6 -99999. 4.2   5.1   4.6   4.4   4.0
2.9   2.4   4.7   5.1   4.0   7.5   7.7   6.3   5.3   5.7   4.8   2.7
1.7   2.0   3.4   4.0   4.3   5.0   5.5   5.0   5.4   3.8   2.4   2.0
2.2   2.5   2.6   3.3   2.9   4.3   4.2   4.2   3.9   3.9   2.5   2.2
2.4   1.9   2.1   4.5   3.3   3.4   4.1   5.7   4.8   5.0   2.8   2.9
1.7   3.2   2.7   3.0   3.4   3.8   5.0   4.8   4.9   3.5   2.5   2.4
1.6   2.3   2.5   3.1 -99999. 4.5   5.7   5.0   4.6   4.8   2.1   1.4
2.1   2.9   2.7   4.2   3.9   4.1   4.6   5.8   4.4   6.1   3.5   1.9
1.8   1.9   3.7   4.4   3.8   5.6   5.7   5.1   5.6 -99999. 2.5   1.5
1.8   2.5   2.6   1.8   3.7   3.7   4.9   5.1   3.7   5.4   3.0   1.8
2.1   2.6   2.8   3.2   3.5   3.5   4.9   4.2   4.7   3.7   3.2   1.8
2.0 -99999. 2.8   3.2   4.4   3.4   3.9   5.5   3.8   3.2   2.3   2.2
1.3   2.3   2.7   3.3   3.7   3.0   3.8   4.7   4.6   2.9   1.7   1.3
1.8   2.0   2.2   3.0   2.4   3.5   3.5   3.3   2.7   2.5   1.6   1.2
1.5   2.0   3.1   3.0   3.5 -99999. 4.0   3.8   3.1   2.1   1.6   1.3
 &DATEN IDR=0,IDS=1,IQR=1,IQS=1,LAG=24,INCON=0,
  NBACK=2,NPRED=14,LAMDA=1,IREG=3,IGRBAR=1,ICONCE=1,
  IESTIM=1,INTERP=1,IFILT=3,/
 &REG IAUS=-1, DELTA=1.D0,/
  61 1
 &REG IAUS=-8,DELTAS=1.D0, /
  138 5 150 5 162 5 174 5 186 5 198 5 210 5 222 5
 &REG IAUS=-8,DELTAS=1.D0, /
  143 7 155 7 167 7 179 7 191 7 203 7 215 7 227 7
```

TRAM

TIME SERIES REGRESSION MODELS WITH
ARIMA ERRORS AND MISSING VALUES.

BY VICTOR GOMEZ AND AGUSTIN MARAVALL.

PROGRAM DESIGNED AND WRITTEN BY VICTOR GOMEZ.

L.A. OXIDANT DATA (BOX-TIAO, JASA 75) WITH 3 INTERVENTIONS AND 8 M.O.
ORIGINAL SERIES

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1955	2.70	2.00	-99999.00	5.00	6.50	6.10	5.90	5.00	6.40	7.40	8.20	3.90
1956	4.10	4.50	5.50	3.80	4.80	5.60	6.30	5.90	-99999.00	5.30	5.70	5.70
1957	3.00	3.40	4.90	4.50	4.00	5.70	6.30	7.10	8.00	5.20	5.00	4.70
1958	3.70	3.10	-99999.00	4.00	4.10	4.60	-99999.00	4.20	5.10	4.60	4.40	4.00
1959	2.90	2.40	4.70	5.10	4.00	7.50	7.70	6.30	5.30	5.70	4.80	2.70
1960	1.70	2.00	3.40	4.00	4.30	5.00	5.50	5.00	5.40	3.80	2.40	2.00
1961	2.20	2.50	2.60	3.30	2.90	4.30	4.20	4.20	3.90	3.90	2.50	2.20
1962	2.40	1.90	2.10	4.50	3.30	3.40	4.10	5.70	4.80	5.00	2.80	2.90
1963	1.70	3.20	2.70	3.00	3.40	3.80	5.00	4.80	4.90	3.50	2.50	2.40
1964	1.60	2.30	2.50	3.10	-99999.00	4.50	5.70	5.00	4.60	4.80	2.10	1.40
1965	2.10	2.90	2.70	4.20	3.90	4.10	4.60	5.80	4.40	6.10	3.50	1.90
1966	1.80	1.90	3.70	4.40	3.80	5.60	5.70	5.10	5.60	-99999.00	2.50	1.50
1967	1.80	2.50	2.60	1.80	3.70	3.70	4.90	5.10	3.70	5.40	3.00	1.80
1968	2.10	2.60	2.80	3.20	3.50	3.50	4.90	4.20	4.70	3.70	3.20	1.80
1969	2.00	-99999.00	2.80	3.20	4.40	3.40	3.90	5.50	3.80	3.20	2.30	2.20
1970	1.30	2.30	2.70	3.30	3.70	3.00	3.80	4.70	4.60	2.90	1.70	1.30
1971	1.80	2.00	2.20	3.00	2.40	3.50	3.50	3.30	2.70	2.50	1.60	1.20
1972	1.50	2.00	3.10	3.00	3.50	-99999.00	4.00	3.80	3.10	2.10	1.60	1.30

```
INITIAL MISSING OBSERVATION NUMBER         3
MISSING OBSERVATION NUMBER         21
MISSING OBSERVATION NUMBER         39
MISSING OBSERVATION NUMBER         43
MISSING OBSERVATION NUMBER        113
MISSING OBSERVATION NUMBER        142
MISSING OBSERVATION NUMBER        170
MISSING OBSERVATION NUMBER        210
MODEL PARAMETERS:
IMEAN =          0
LAMDA =          1
IDR =          0
IDS =          1
IPR =          0
IPS =          0
IQR =          1
IQS =          1
IREG =          3
ITRAD =          0
IEAST =          0
IDUR =          0
LAG =         24
INCON =          0
NBACK =          2
NPRED =         14
INTERP =          1
IESTIM =          1
```

```
VA =        1.000000000000000

IFILT =          3

IGRBAR =          1

IGRRES =          0

IDENSC =          1

INVER =          0

INIC =          0

TOL =   1.000000000000000E-006

ICONCE =          4

THR =  -1.000000000000000E-001

THS =  -1.000000000000000E-001

NUMBER OF INITIAL OBSERVATIONS =          12
NUMBER OF MISSING INITIAL OBSERVATIONS =           1

NUMBER OF MISSING VALUES IN TIME SPAN
         13 -          216
=          7

ARIMA MODEL ESTIMATION BEGINS

INITIAL PARAMETER VALUES:
 -1.000000000000000E-001  -1.000000000000000E-001

 ITERATION, LAMBDA            1   0.000000000000000E+000
 FO FP      171.331186146082800      131.206576823649000
 FO-FP SUM S       40.124609322433770        29.164594366323460
       1.375798641957658
 ITERATION, LAMBDA            2   0.000000000000000E+000
 FO FP      131.206576823649000      120.236837906683100
 FO-FP SUM S       10.969738916965910         7.914529115534293
       1.386025467445054
```

```
ITERATION, LAMBDA          3   0.000000000000000E+000
FO FP       120.236837906683100       119.100785547575200
FO-FP SUM S        1.136052359107921    8.549258246775195E-001
      1.328831491944279
ITERATION, LAMBDA          4   0.000000000000000E+000
FO FP       119.100785547575200       119.011718865687900
FO-FP SUM S   8.906668188727451E-002   6.709646038024446E-002
      1.327442332762741
ITERATION, LAMBDA          5   0.000000000000000E+000
FO FP       119.011718865687900       119.002875693309000
FO-FP SUM S   8.843172378973918E-003   6.659776501750962E-003
      1.327848220829769
ITERATION, LAMBDA          6   0.000000000000000E+000
FO FP       119.002875693309000       119.001949221394300
FO-FP SUM S   9.264719146386824E-004   6.979055541350462E-004
      1.327503283430537
ITERATION, LAMBDA          7   0.000000000000000E+000

METHOD OF ESTIMATION: EXACT MAXIMUM LIKELIHOOD

PARAMETER      ESTIMATE          STD ERROR      T RATIO   LAG

MA1  1        .241425328         .071672167       3.37      1
MA2  1       -.767490202         .055975786     -13.71     12

REGULAR MA INVERSE ROOTS ARE
NO.     REAL P.     IMAG.P.     MODULUS
 1     .2414253    .0000000    .2414253
SEASONAL MA INVERSE ROOTS ARE
NO.     REAL P.     IMAG.P.     MODULUS
 1    -.7674902    .0000000    .7674902
```

```
CORRELATIONS OF THE ESTIMATES

 1.000   .116
  .116  1.000

AIC
    463.761

FINAL VALUE OF OBJECTIVE FUNCTION:
   119.0018507036

VARIANCE ESTIMATE:
   .5869994

ITERATIONS:          7

 NUMBER OF FUNCTION EVALUATIONS:          22

ESTIMATES OF REGRESSION PARAMETERS
CONCENTRATED OUT OF THE LIKELIHOOD
ZJ    3       4.356813054    (     .751488663)
REG   1      -1.366804804    (     .184767079)
REG   2       -.243046129    (     .057924772)
REG   3       -.094424049    (     .053171061)

 COVARIANCE MATRIX OF ESTIMATORS

 .565E+00 -.868E-02  .230E-03 -.107E-03
-.868E-02  .341E-01 -.967E-03 -.913E-03
 .230E-03 -.967E-03  .336E-02  .241E-03
-.107E-03 -.913E-03  .241E-03  .283E-02
```

CHECK OF WHITE NOISE RESIDUALS:

```
AUTOCORRELATIONS
----------------
     .0200   .0964   .0823   .1697  -.0268   .0802   .0465  -.0007   .0110   .0188  -.0134   .1419
SE   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720
     .1034   .0219   .0237   .1570  -.0718   .0841  -.0125   .1906  -.0019   .0437   .0666   .0481
SE   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720

 LJUNG-BOX OF ORDER Q IS      35.36 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUARED(11)
PIERCE OF ORDER QS IS         4.71 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS  CHI-SQUATED( 2)

               GRAPHIC INTERVAL IS      .0200

              -1.0000                                      .0000                                       1.0000  VALUES
               .+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.
          1                                           .     XX     .                                           .02003
          2                                           .     XXXXXX .                                           .09639
          3                                           .     XXXXX  .                                           .08233
          4                                           .     XXXXXXXXX                                          .16970
          5                                           .    XX      .                                          -.02680
          6                                           .     XXXXX  .                                           .08022
          7                                           .     XXX    .                                           .04647
          8                                           .     X      .                                          -.00066
          9                                           .     XX     .                                           .01095
         10                                           .     XX     .                                           .01882
         11                                           .    XX      .                                          -.01343
         12                                           .     XXXXXXXX                                           .14195
         13                                           .     XXXXXX .                                           .10338
         14                                           .     XX     .                                           .02186
         15                                           .     XX     .                                           .02366
         16                                           .     XXXXXXXXX                                          .15702
         17                                           . XXXXX      .                                          -.07181
         18                                           .     XXXXX  .                                           .08407
         19                                           .    XX      .                                          -.01248
         20                                           .     XXXXXXXXXXX                                        .19064
         21                                           .     X      .                                          -.00185
         22                                           .     XXX    .                                           .04374
         23                                           .     XXXX   .                                           .06661
         24                                           .     XXX    .                                           .04811
```

```
PARTIAL AUTOCORRELATIONS
---------------------
      .0200   .0960   .0794   .1607  -.0468   .0443   .0264  -.0347   .0071  -.0055  -.0198   .1517
SE    .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720
      .1015  -.0001  -.0090   .0986  -.1016   .0545  -.0466   .1689   .0345  -.0185   .0705  -.0222
SE    .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720   .0720

              GRAPHIC INTERVAL IS      .0200

             -1.0000                                            .0000                                       1.0000   VALUES
               .+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.
     1                                                .     XX     .                                              .02003
     2                                                .     XXXXXX .                                              .09603
     3                                                .     XXXXX  .                                              .07942
     4                                                .     XXXXXXXXX                                             .16068
     5                                                .   XXX      .                                             -.04685
     6                                                .     XXX    .                                              .04431
     7                                                .     XX     .                                              .02636
     8                                                .   XXX      .                                             -.03470
     9                                                .     X      .                                              .00711
    10                                                .     X      .                                             -.00553
    11                                                .    XX      .                                             -.01977
    12                                                .     XXXXXXXXX                                             .15172
    13                                                .     XXXXXX .                                              .10146
    14                                                .     X      .                                             -.00013
    15                                                .     X      .                                             -.00904
    16                                                .     XXXXXX .                                              .09864
    17                                                . XXXXXX     .                                             -.10158
    18                                                .     XXXX   .                                              .05454
    19                                                .   XXX      .                                             -.04662
    20                                                .     XXXXXXXXX                                             .16889
    21                                                .     XXX    .                                              .03446
    22                                                .    XX      .                                             -.01850
    23                                                .     XXXXX  .                                              .07048
    24                                                .    XX      .                                             -.02218
```

NUMBER OF WHITE NOISE RESIDUALS 193

WHITE NOISE RESIDUALS

-1.1342	-.1228	.3470	.6302	-1.6194	-1.6006	1.9023	-.7838
.2853	-.5285	.2367	-1.5258	.2402	.1170	1.4527	1.0108
-1.2125	-1.4369	.2553	.3521	-.3057	-.3921	-.7828	-.9422
-1.7290	-1.4943	-.8932	-1.4982	-.3584	-.4007	-.7413	-.2721
.8446	-.9228	2.2412	1.0670	.5322	-1.2134	.4763	-.9425
-1.5521	-.2541	-.0347	-.6483	.5597	.5454	-.1950	-.0418
.2172	.1856	-.8600	-1.8502	-.6394	.1809	.3800	-1.3464
-.0675	-.8616	-.5332	-1.2101	-.3485	-1.2195	-.2199	-1.3891
-.2801	.2872	-.3456	-1.3418	1.2210	-.5611	-1.3209	-.7777
1.1959	-.3719	.7958	-.9851	.4687	-.6434	1.2411	-.7734
-.6245	.0476	-.7213	.2430	-.1869	.0838	-.9777	-.6668
-.1624	-.4659	.0434	-.5694	-.3821	.1522	.7118	-.0697
-.2536	.6273	-1.2403	-.9461	.3412	.4642	-.3389	.7830
.2476	-.3375	-.4451	.9841	-.6432	1.8871	.1096	-.4802
-.0974	-.5571	.9600	.5057	.1048	1.5266	.5963	.0965
1.1336	-.3989	-.4971	.1122	.2835	-.3793	-1.7862	.6618
-.5988	.3519	.3675	-.8330	1.2838	.0389	-.0070	.4519
.3852	.0243	.1095	.0999	-.2937	.4980	-.4048	.7949
-.8051	.8087	-.0372	.4104	.2391	.1882	1.1032	-.3043
-.3307	1.4294	-.4420	-.5778	-.1315	.7088	-.4070	.4413
.1407	.4098	.2357	-.2183	-.0891	.5254	.8669	-.7602
-.4682	-.1113	.5019	-.0095	-.1537	.2336	-.9430	.8716
-.3510	-.6603	-.7097	-.3801	-.3520	-.0311	.2281	.1839
.8918	.0917	.5542	.6419	.0457	-.0098	-.5513	-.0514
.1794							

FORECASTS:

ORIGIN: 214 NUMBER: 14

OBS	FORECAST (TR. SERIES)	STD ERROR	ACTUAL	RESIDUAL	FORECAST (ORIGINAL SERIES)
215	1.8440	.7903	1.6000	-.2440	1.84
216	1.3392	.8113	1.3000	-.0392	1.34
217	1.3850	.8113			1.38
218	1.8705	.8152			1.87
219	2.4314	.8113			2.43
220	2.8188	.8113			2.82
221	3.1099	.8113			3.11
222	2.6225	.8470			2.62
223	3.3079	.8179			3.31
224	3.4632	.8179			3.46
225	2.9134	.8179			2.91
226	2.3364	.8181			2.34
227	1.8622	.8443			1.86
228	1.2448	.8456			1.24
229	1.2906				1.29
230	1.7761				1.78
231	4.3648				4.36
232	4.7521				4.75
233	5.0433				5.04
234	5.9336				5.93
235	6.6191				6.62
236	6.7743				6.77
237	6.2246				6.22
238	5.6475				5.65
239	3.9844				3.98
240	3.3670				3.37
241	3.4128				3.41

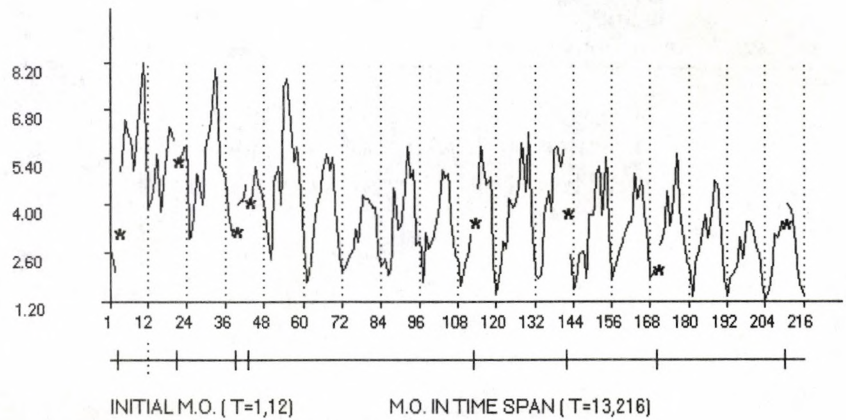
REGRESSION RESIDUALS

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1956	1.08	1.73	.49	-1.07	-1.09	-.13	.35	.63	-99999.00	-1.62	-1.60	1.91
1957	-.78	.29	-.11	.14	-1.50	.23	.12	1.45	1.01	-1.21	-1.44	.26
1958	.35	-.31	-99999.00	-.39	-.78	-.94	-99999.00	-1.73	-1.49	-.89	-1.50	-.36
1959	-.40	-.74	.00	.78	-.91	2.24	1.07	.53	-1.21	.48	-.94	-1.55
1960	.18	.29	-.12	.87	.90	.15	.30	.57	.53	-.51	-1.50	-.29
1961	.42	.65	-.97	.17	-.59	-.27	-.96	-.08	-.97	.04	-1.13	-.02
1962	.47	-.14	-1.06	1.40	-.36	-1.12	-.59	1.40	-.18	.99	-.79	.67
1963	-.51	1.39	-.56	-.49	.20	-.57	.38	-.03	.23	-.83	-.52	-.01
1964	-.36	.16	-.41	-.28	-99999.00	.29	.81	.05	-.14	.74	-1.13	-.83
1965	.42	.55	-.22	.86	.36	-.25	-.36	1.07	-.56	1.98	.20	-.39
1966	-.04	-.49	1.05	.57	.18	1.57	.64	.14	1.18	-99999.00	-.38	-.48
1967	.11	.29	-.36	-1.79	.67	-.58	.36	.38	-.82	1.31	-.01	-.04
1968	.40	.34	-.01	.06	.06	-.29	.48	-.41	.78	-.81	.71	-.11
1969	.32	-99999.00	.15	.10	1.02	-.32	-.36	1.40	-.47	-.60	-.26	.61
1970	-.53	.31	.04	.29	.12	-.24	-.14	.49	.82	-.80	-.62	-.24
1971	.36	-.15	-.28	.09	-1.08	.84	-.41	-.71	-.76	-.43	-.53	-.18
1972	.07	.03	.74	-.07	.40	-99999.00	.57	-.01	-.07	-.61	-.24	.02

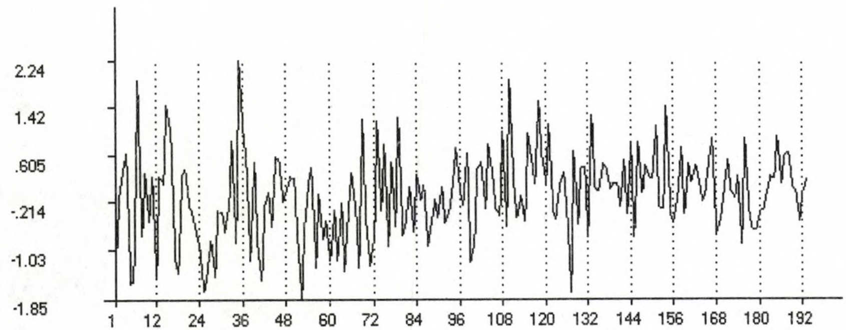
INTERPOLATED VALUES

OBS	INTERPOLATED VALUE	STD ERROR
21	6.1738	.7253
39	4.3180	.7158
43	5.7135	.7093
113	3.5070	.7012
142	4.6251	.7023
170	2.3317	.7077
210	3.0433	.7627

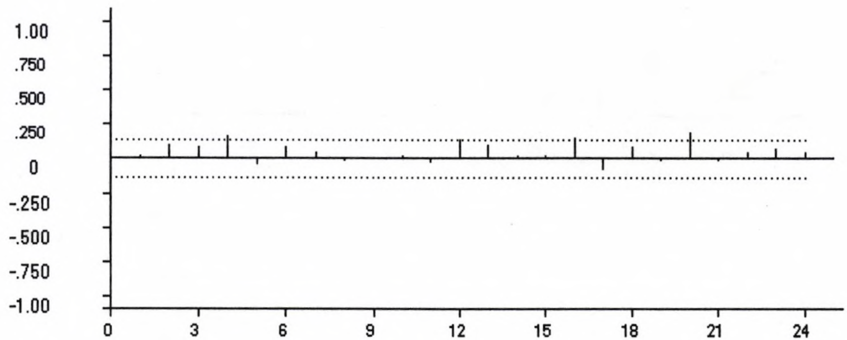
OZONOMI: ORIGINAL SERIES



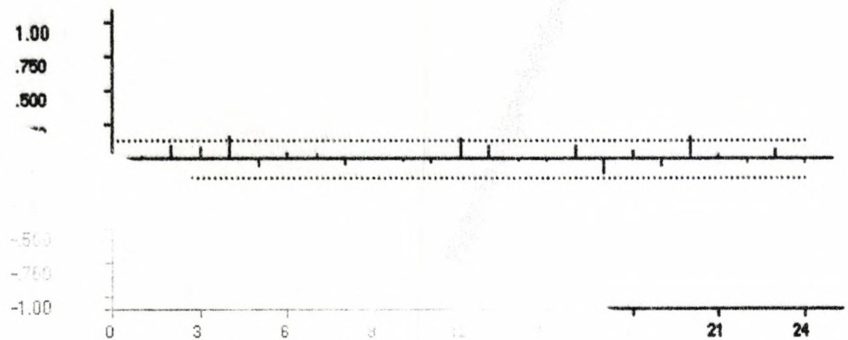
OZONOMI: RESIDUALS



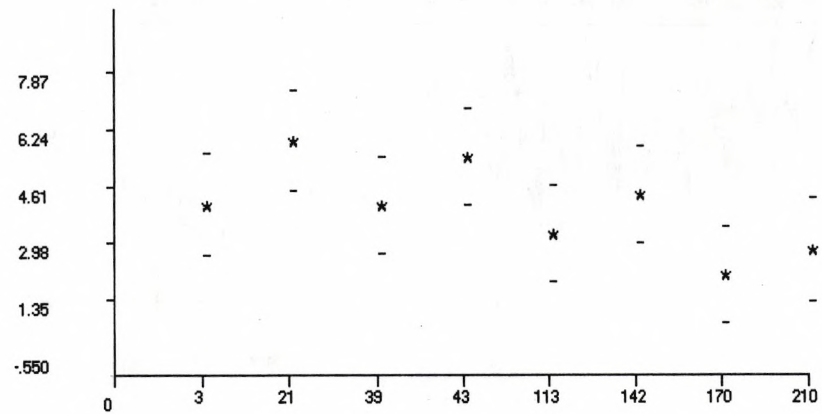
OZONOMI: ACF OF RESIDUALS

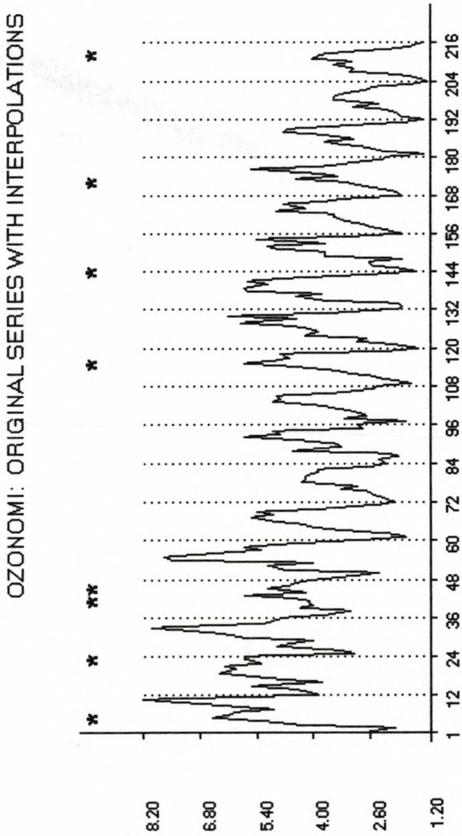


OZONOMI: PARTIAL ACF OF RESIDUALS

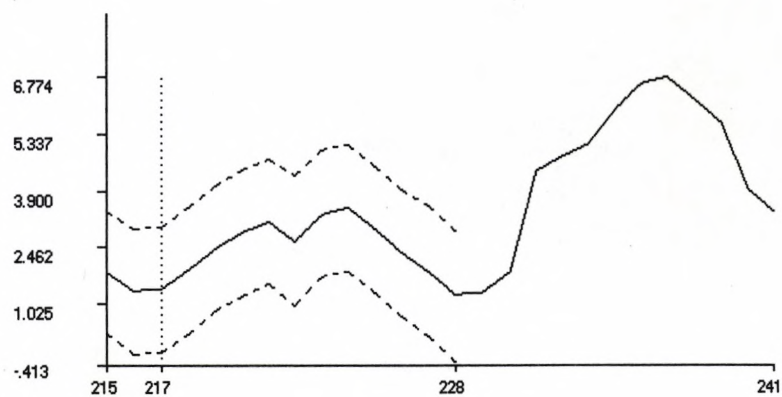


OZONOMI: INTERPOLATED VALUES




OZONOMI: ORIGINAL SERIES WITH INTERPOLATIONS
8.20
6.80
5.40
4.00
2.60
1.20
1
12
24
36
48
60
72
84
96
108
120
132
144
156
168
180
192
204
216

OZONOMI: FORECASTS



---- : 95 % C. I.

Example 2

The second example is the annual series y_t of Gross Investment in General Electric, for the period 1935–1954. The series is analysed in Maddala (1977).

The estimated model is of the form:

$$y_t = \omega_0 + \omega_1 x_{t-1} + \omega_2 z_{t-1} + n_t$$

$$\left(1 - \phi_1 B - \phi_2 B^2\right) n_t = a_t,$$

where x denotes a measure of the value of the firm, and z the stock of plant and equipment.

Two of the 20 observations were removed and treated as missing values. The regressors in this case are input by the user.

```
GEN. ELECTRIC: GROSS INVEST. (MADDALA, 1977 P.214), 2 REGRESSION VAR., 2 M.O.
20    1 1 1
 33.1 45. 77.2 44.6 48.1 74.4 113. -99999. 61.3 56.8 93.6
 159.9 147.2 166.3 98.3 93.5 135.2 157.3 -99999. 189.6
 &DATEN IDR=0,IDS=0,IPR=2,IPS=0,IQR=0,IQS=0,LAG=12,INCON=0,
  LAMDA=1,IREG=2,IGRBAR=1,IMEAN=1,IFILT=3,INTERP=1,ICONCE=1,
  IGRRES=0,/
 &REG ILONG=20, IUSER=1,/
 1170.6 2015.8 2803.3 2039.7 2256.2 2132.2 1834.1 1588.0 1749.4
 1687.2 2007.7 2208.3 1656.7 1604.4 1431.8 1610.5 1819.4 2079.7
 2371.6 2759.9
 &REG ILONG=20,IUSER=1, /
 97.8 104.4 118. 156.2 172.6 186.6 220.9 287.8 319.9 321.3 319.6
 346.0 456.4 543.4 618.3 647.4 671.3 726.1 800.3 888.9
```

```
                TRAM

TIME SERIES REGRESSION MODELS WITH
ARIMA ERRORS AND MISSING VALUES.

BY  VICTOR GOMEZ   AND   AGUSTIN MARAVALL.


PROGRAM DESIGNED AND WRITTEN BY  VICTOR GOMEZ.

GEN. ELECTRIC: GROSS INVEST. (MADDALA, 1977 P.214), 2 REGRESSION VAR., 2
ORIGINAL SERIES
    1          33.10
    2          45.00
    3          77.20
    4          44.60
    5          48.10
    6          74.40
    7         113.00
    8      -99999.00
    9          61.30
   10          56.80
   11          93.60
   12         159.90
   13         147.20
   14         166.30
   15          98.30
   16          93.50
   17         135.20
   18         157.30
   19      -99999.00
   20         189.60

MISSING OBSERVATION NUMBER          8

MISSING OBSERVATION NUMBER         19
```

```
MODEL PARAMETERS:
IMEAN =          1
LAMDA =          1
IDR =          0
IDS =          0
IPR =          2
IPS =          0
IQR =          0
IQS =          0
IREG =          2
ITRAD =          0
IEAST =          0
IDUR =          0
LAG =         12
INCON =          0
NBACK =          0
NPRED =          0
INTERP =          1
IESTIM =          1
VA =      1.000000000000000
IFILT =          3
```

```
IGRBAR =           1

IGRRES =           0

IDENSC =           1

INVER =           0

INIC =           0

TOL =   1.000000000000000E-006

ICONCE =           3

FIR =  -1.000000000000000E-001  -1.000000000000000E-001

NUMBER OF MISSING VALUES IN TIME SPAN
          1 -          20
=           2

ARIMA MODEL ESTIMATION BEGINS

INITIAL PARAMETER VALUES:
 -1.000000000000000E-001  -1.000000000000000E-001

 ITERATION, LAMBDA           1   0.000000000000000E+000
 FO FP    14379.024112017160000      9318.182158281445000
 FO-FP SUM S     5060.841953735715000     5701.689860005564000
  8.876038644674329E-001
 ITERATION, LAMBDA           2   0.000000000000000E+000
 FO FP     9318.182158281445000      9241.516109042313000
 FO-FP SUM S       76.666049239131100       83.359415209453180
  9.197047393686246E-001
 ITERATION, LAMBDA           3   0.000000000000000E+000
 FO FP     9241.516109042313000      9240.750931597517000
 FO-FP SUM S   7.651774447967910E-001   9.025917610451475E-001
  8.477558491235961E-001
 ITERATION, LAMBDA           4   0.000000000000000E+000
 FO FP     9240.750931597517000      9240.732903301729000
 FO-FP SUM S   1.802829578809906E-002   2.128342092131643E-002
  8.470581799208229E-001
 ITERATION, LAMBDA           5   0.000000000000000E+000
```

```
METHOD OF ESTIMATION: EXACT MAXIMUM LIKELIHOOD

PARAMETER      ESTIMATE           STD ERROR      T RATIO   LAG

AR1  1       -.633602709         .245064889      -2.59      1
AR1  2        .474179155         .229282721       2.07      2

REGULAR AR INVERSE ROOTS ARE
NO.      REAL P.     IMAG.P.     MODULUS
 1     .3168014   -.6114050    .6886067
 2     .3168014    .6114050    .6886067

CORRELATIONS OF THE ESTIMATES

 1.000  -.396
 -.396  1.000

AIC
    167.420

FINAL VALUE OF OBJECTIVE FUNCTION:
  9240.7324828389

VARIANCE ESTIMATE:
652.6323496

ITERATIONS:           5

 NUMBER OF FUNCTION EVALUATIONS:          16

ESTIMATES OF REGRESSION PARAMETERS
CONCENTRATED OUT OF THE LIKELIHOOD
MU          -15.256922081     (   34.489316834)
REG   1        .030411475     (     .017037059)
REG   2        .149197125     (     .029841631)

 COVARIANCE MATRIX OF ESTIMATORS

 .119E+04 -.539E+00 -.239E+00
-.539E+00  .290E-03 -.604E-04
-.239E+00 -.604E-04  .891E-03

 CHECK OF WHITE NOISE RESIDUALS:
```

```
AUTOCORRELATIONS
----------------
     -.1079   .0963  -.1838   .1120  -.0192   .0894  -.2811  -.1428  -.0639   .0953  -.0392  -.0134
SE    .2582   .2582   .2582   .2582   .2582   .2582   .2582   .2582   .2582   .2582   .2582   .2582

 LJUNG-BOX OF ORDER Q IS       5.65 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUARED(10)
PIERCE OF ORDER QS IS          5.65 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS  CHI-SQUATED(12)

               GRAPHIC INTERVAL IS      .0200

              -1.0000                                       .0000                                       1.0000   VALUES
               .+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.
          1                         .             XXXXXX                     .                               -.10791
          2                         .                  XXXXXX                .                                .09628
          3                         .         XXXXXXXXXX                     .                               -.18384
          4                         .                  XXXXXXX               .                                .11200
          5                         .                 XX                     .                               -.01925
          6                         .                  XXXXX                 .                                .08938
          7                         .    XXXXXXXXXXXXXXX                     .                               -.28111
          8                         .           XXXXXXXX                     .                               -.14277
          9                         .               XXXX                     .                               -.06386
         10                         .                  XXXXXX                .                                .09532
         11                         .                XXX                     .                               -.03921
         12                         .                 XX                     .                               -.01336
```

```
PARTIAL AUTOCORRELATIONS
---------------------
     -.1079   .0856  -.1683   .0752   .0266   .0476  -.2528  -.2204  -.0429   .0259  -.0453  -.0262
SE    .2582   .2582   .2582   .2582   .2582   .2582   .2582   .2582   .2582   .2582   .2582   .2582

          GRAPHIC INTERVAL IS      .0200

         -1.0000                                              .0000                                              1.0000  VALUES
           .+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.
      1                           .                      XXXXXX                          .                                -.10791
      2                           .                           XXXXX                      .                                 .08563
      3                           .                   XXXXXXXXX                          .                                -.16827
      4                           .                           XXXXX                      .                                 .07522
      5                           .                           XX                         .                                 .02655
      6                           .                           XXX                        .                                 .04760
      7                           .              XXXXXXXXXXXXXX                          .                                -.25284
      8                           .                XXXXXXXXXXXX                          .                                -.22044
      9                           .                         XXX                          .                                -.04289
     10                           .                          XX                          .                                 .02592
     11                           .                         XXX                          .                                -.04529
     12                           .                          XX                          .                                -.02622
```

NUMBER OF WHITE NOISE RESIDUALS 15

WHITE NOISE RESIDUALS

-29.1435	-14.5116	5.4513	26.9265	-18.4849	-4.4677	7.7990	45.2902
6.7531	51.8969	-35.4081	2.4689	8.6704	-10.6692	-2.8489	

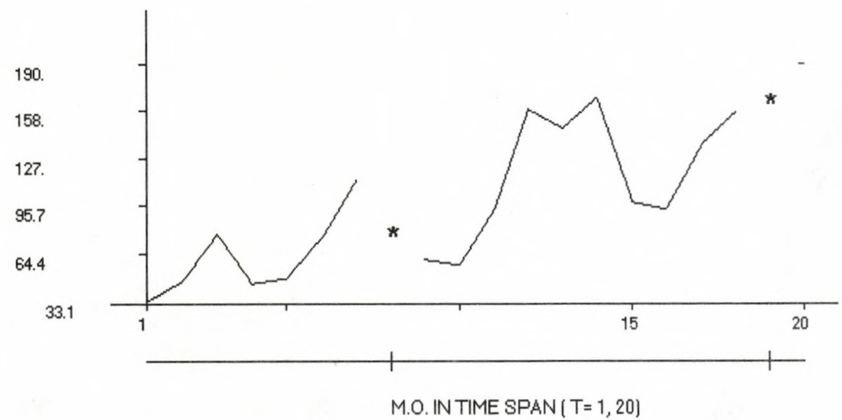
REGRESSION RESIDUALS

1	-1.46
2	-13.94
3	-.74
4	-26.77
5	-19.80
6	4.54
7	26.74
8	-99999.00
9	-18.93
10	-3.71
11	5.79
12	43.41
13	8.32
14	50.56
15	-34.13
16	1.79
17	7.75
18	-13.29
19	-99999.00
20	-11.10

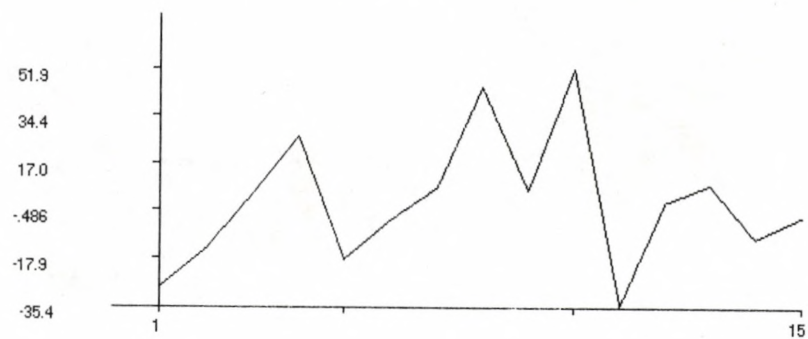
INTERPOLATED VALUES

OBS	INTERPOLATED VALUE	STD ERROR
8	93.4861	20.5678
19	173.3348	21.7403

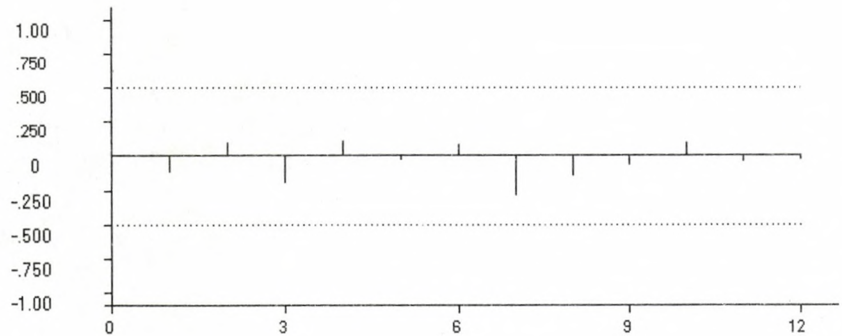
GEIMI: ORIGINAL SERIES



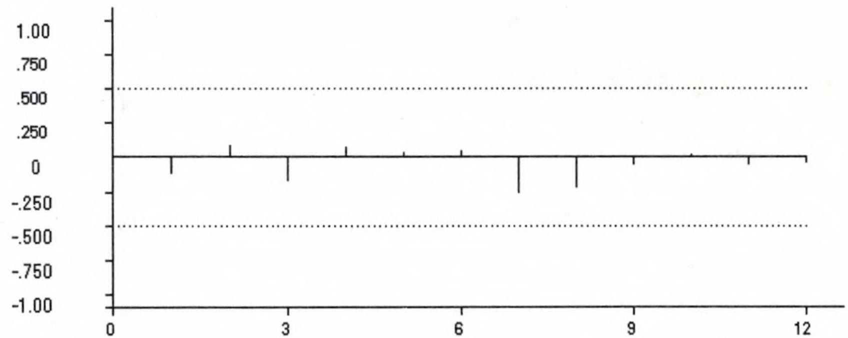
GEIMI: RESIDUALS



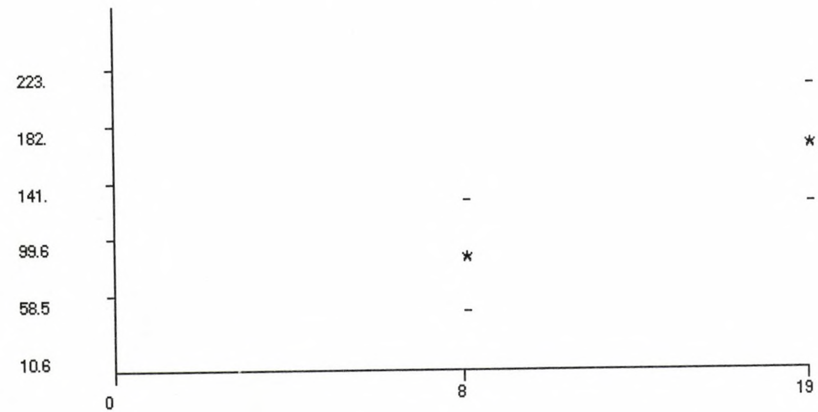
GEIMI: ACF OF RESIDUALS



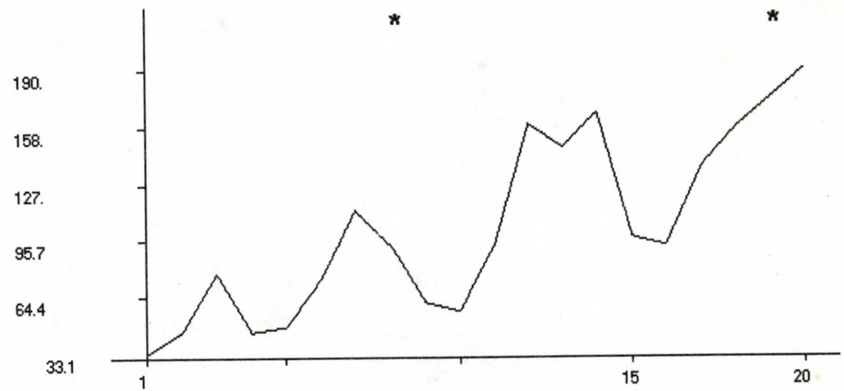
GEIMI: PARTIAL ACF OF RESIDUALS



GEIMI: INTERPOLATED VALUES



GEIMI: ORIGINAL SERIES WITH INTERPOLATIONS



Example 3

Example 3 is the same as in Harvey and Pierce (1984) and in Data Set 2 of Kohn and Ansley (1986). It consists of removing, from the 12 years of monthly data on a series of Airline passengers, all January through November data in the last 6 years. Since the estimation problem is identical when the missing values are placed at the beginning, the example illustrates an important possible application of the program: Interpolation of data for frequencies higher than the observed one.

The model has no regressors, and is given by

$$\nabla\nabla_{12} \log y_t = (1-\theta_1 B)(1-\theta_{12} B^{12}) a_t,$$

the so-called Airline Model of Box-Jenkins (1970).

```
EXAMPLE (HARVEY-PIERSE, JASA 84) MONTHLY INTERPOLATION, AIRLINE MODEL
144   1949 1 12
112 118 132 129 121 135 148 148 136 119 104 118
115 126 141 135 125 149 170 170 158 133 114 140
145 150 178 163 172 178 199 199 184 162 146 166
171 180 193 181 183 218 230 242 209 191 172 194
196 196 236 235 229 243 264 272 237 211 180 201
204 188 235 227 234 264 302 293 259 229 203 229
-99999. -99999. -99999. -99999. -99999. -99999. -99999. -99999.
-99999. -99999. -99999. 278
-99999. -99999. -99999. -99999. -99999. -99999. -99999. -99999.
-99999. -99999. -99999. 306
-99999. -99999. -99999. -99999. -99999. -99999. -99999. -99999.
-99999. -99999. -99999. 336
-99999. -99999. -99999. -99999. -99999. -99999. -99999. -99999.
-99999. -99999. -99999. 337
-99999. -99999. -99999. -99999. -99999. -99999. -99999. -99999.
-99999. -99999. -99999. 405
-99999. -99999. -99999. -99999. -99999. -99999. -99999. -99999.
-99999. -99999. -99999. 432
 &DATEN IDR=1,IDS=1,IQR=1,IQS=1,LAG=24,INCON=0,IESTIM=1,
   INTERP=1,IGRBAR=1,
  LAMDA=0,IFILT=3,NPRED=12,/
```

TRAM

TIME SERIES REGRESSION MODELS WITH
ARIMA ERRORS AND MISSING VALUES.

BY VICTOR GOMEZ AND AGUSTIN MARAVALL.

PROGRAM DESIGNED AND WRITTEN BY VICTOR GOMEZ.

EXAMPLE (HARVEY-PIERSE, JASA 84) MONTHLY INTERPOLATION, AIRLINE MODEL
ORIGINAL SERIES

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1949	112.00	118.00	132.00	129.00	121.00	135.00	148.00	148.00	136.00	119.00	104.00	118.00
1950	115.00	126.00	141.00	135.00	125.00	149.00	170.00	170.00	158.00	133.00	114.00	140.00
1951	145.00	150.00	178.00	163.00	172.00	178.00	199.00	199.00	184.00	162.00	146.00	166.00
1952	171.00	180.00	193.00	181.00	183.00	218.00	230.00	242.00	209.00	191.00	172.00	194.00
1953	196.00	196.00	236.00	235.00	229.00	243.00	264.00	272.00	237.00	211.00	180.00	201.00
1954	204.00	188.00	235.00	227.00	234.00	264.00	302.00	293.00	259.00	229.00	203.00	229.00
1955	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	278.00
1956	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	306.00
1957	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	336.00
1958	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	337.00
1959	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	405.00
1960	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	432.00

MISSING OBSERVATION NUMBER 73

MISSING OBSERVATION NUMBER 74

MISSING OBSERVATION NUMBER 75

MISSING OBSERVATION NUMBER 76

MISSING OBSERVATION NUMBER 77

MISSING OBSERVATION NUMBER 78

MISSING OBSERVATION NUMBER 79

MISSING OBSERVATION NUMBER 80

MISSING OBSERVATION NUMBER 81

MISSING OBSERVATION NUMBER 82

MISSING OBSERVATION NUMBER 83

MISSING OBSERVATION NUMBER 85
MISSING OBSERVATION NUMBER 86
MISSING OBSERVATION NUMBER 87
MISSING OBSERVATION NUMBER 88
MISSING OBSERVATION NUMBER 89
MISSING OBSERVATION NUMBER 90
MISSING OBSERVATION NUMBER 91
MISSING OBSERVATION NUMBER 92
MISSING OBSERVATION NUMBER 93
MISSING OBSERVATION NUMBER 94
MISSING OBSERVATION NUMBER 95
MISSING OBSERVATION NUMBER 97
MISSING OBSERVATION NUMBER 98
MISSING OBSERVATION NUMBER 99
MISSING OBSERVATION NUMBER 100
MISSING OBSERVATION NUMBER 101
MISSING OBSERVATION NUMBER 102
MISSING OBSERVATION NUMBER 103
MISSING OBSERVATION NUMBER 104
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MISSING OBSERVATION NUMBER 106
MISSING OBSERVATION NUMBER 107
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MISSING OBSERVATION NUMBER 110

MISSING OBSERVATION NUMBER 111
MISSING OBSERVATION NUMBER 112
MISSING OBSERVATION NUMBER 113
MISSING OBSERVATION NUMBER 114
MISSING OBSERVATION NUMBER 115
MISSING OBSERVATION NUMBER 116
MISSING OBSERVATION NUMBER 117
MISSING OBSERVATION NUMBER 118
MISSING OBSERVATION NUMBER 119
MISSING OBSERVATION NUMBER 121
MISSING OBSERVATION NUMBER 122
MISSING OBSERVATION NUMBER 123
MISSING OBSERVATION NUMBER 124
MISSING OBSERVATION NUMBER 125
MISSING OBSERVATION NUMBER 126
MISSING OBSERVATION NUMBER 127
MISSING OBSERVATION NUMBER 128
MISSING OBSERVATION NUMBER 129
MISSING OBSERVATION NUMBER 130
MISSING OBSERVATION NUMBER 131
MISSING OBSERVATION NUMBER 133
MISSING OBSERVATION NUMBER 134

```
MISSING OBSERVATION NUMBER       135
MISSING OBSERVATION NUMBER       136
MISSING OBSERVATION NUMBER       137
MISSING OBSERVATION NUMBER       138
MISSING OBSERVATION NUMBER       139
MISSING OBSERVATION NUMBER       140
MISSING OBSERVATION NUMBER       141
MISSING OBSERVATION NUMBER       142
MISSING OBSERVATION NUMBER       143
MODEL PARAMETERS:
IMEAN =          0
LAMDA =          0
IDR =          1
IDS =          1
IPR =          0
IPS =          0
IQR =          1
```

```
IQS =          1

IREG =          0

ITRAD =          0

IEAST =          0

IDUR =          0

LAG =         24

INCON =          0

NBACK =          0

NPRED =         12

INTERP =          1

IESTIM =          1

VA =        1.000000000000000

IFILT =          3

IGRBAR =          1

IGRRES =          0

IDENSC =          1

INVER =          0

INIC =          0

TOL =   1.000000000000000E-006

ICONCE =          0

THR =  -1.000000000000000E-001

THS =  -1.000000000000000E-001

NUMBER OF INITIAL OBSERVATIONS =          13
NUMBER OF MISSING INITIAL OBSERVATIONS =          0

NUMBER OF MISSING VALUES IN TIME SPAN
         14 -         144
=          66
```

TRANSFORMED SERIES (LOGARITHMS OF THE DATA)

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1949	4.72	4.77	4.88	4.86	4.80	4.91	5.00	5.00	4.91	4.78	4.64	4.77
1950	4.74	4.84	4.95	4.91	4.83	5.00	5.14	5.14	5.06	4.89	4.74	4.94
1951	4.98	5.01	5.18	5.09	5.15	5.18	5.29	5.29	5.21	5.09	4.98	5.11
1952	5.14	5.19	5.26	5.20	5.21	5.38	5.44	5.49	5.34	5.25	5.15	5.27
1953	5.28	5.28	5.46	5.46	5.43	5.49	5.58	5.61	5.47	5.35	5.19	5.30
1954	5.32	5.24	5.46	5.42	5.46	5.58	5.71	5.68	5.56	5.43	5.31	5.43
1955	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	5.63
1956	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	5.72
1957	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	5.82
1958	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	5.82
1959	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	6.00
1960	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	6.07

ARIMA MODEL ESTIMATION BEGINS

```
INITIAL PARAMETER VALUES:
 -1.000000000000000E-001  -1.000000000000000E-001

 ITERATION, LAMBDA          1   0.000000000000000E+000
 FO FP   2.075557758816538E-001    1.493805095746645E-001
 FO-FP SUM S   5.817526630698935E-002    5.483152048664167E-002
       1.060982183070453
 ITERATION, LAMBDA          2   0.000000000000000E+000
 FO FP   1.493805095746645E-001    1.463257044101909E-001
 FO-FP SUM S   3.054805164473540E-003    3.060104801093390E-003
  9.982681519214777E-001
 ITERATION, LAMBDA          3   0.000000000000000E+000
 FO FP   1.463257044101909E-001    1.462265819838008E-001
 FO-FP SUM S   9.912242639018021E-005    8.956109276718306E-005
       1.106757670407753
 ITERATION, LAMBDA          4   0.000000000000000E+000
 FO FP   1.462265819838008E-001    1.462227787122888E-001
 FO-FP SUM S   3.803271511976236E-006    3.618689306872155E-006
       1.051008027893842
 ITERATION, LAMBDA          5   0.000000000000000E+000
 FO FP   1.462227787122888E-001    1.462226219501260E-001
 FO-FP SUM S   1.567621627507343E-007    1.527323543117848E-007
       1.026384772611592
 ITERATION, LAMBDA          6   0.000000000000000E+000
```

METHOD OF ESTIMATION: EXACT MAXIMUM LIKELIHOOD

PARAMETER	ESTIMATE	STD ERROR	T RATIO	LAG
MA1 1	-.456909207	.096040703	-4.76	1
MA2 1	-.758389269	.227230388	-3.34	12

REGULAR MA INVERSE ROOTS ARE

NO.	REAL P.	IMAG.P.	MODULUS
1	-.4569092	.0000000	.4569092

SEASONAL MA INVERSE ROOTS ARE

NO.	REAL P.	IMAG.P.	MODULUS
1	-.7583893	.0000000	.7583893

CORRELATIONS OF THE ESTIMATES

1.000	-.009
-.009	1.000

AIC
-207.844

FINAL VALUE OF OBJECTIVE FUNCTION:
.1462226152

VARIANCE ESTIMATE:
.0017344

ITERATIONS: 6

NUMBER OF FUNCTION EVALUATIONS: 19

CHECK OF REGRESSION RESIDUALS:

AUTOCORRELATIONS

```
       .0461   .0568  -.2101  -.0257   .0463   .0611   .0050  -.0102   .0089   .0040  -.0983  -.0574
SE     .1240   .1240   .1240   .1240   .1240   .1240   .1240   .1240   .1240   .1240   .1240   .1240
      -.0443   .0948   .0276  -.0672   .1600  -.0153   .0357  -.0873  -.0346   .1101   .1564   .0187
SE     .1240   .1240   .1240   .1240   .1240   .1240   .1240   .1240   .1240   .1240   .1240   .1240

 LJUNG-BOX OF ORDER Q IS      13.54 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUARED(11)
PIERCE OF ORDER QS IS           .31 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS  CHI-SQUATED( 2)

               GRAPHIC INTERVAL IS      .0200

               -1.0000                                         .0000                                        1.0000   VALUES
                .+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.
         1                                     .           XXX        .                                              .04609
         2                                     .           XXXX       .                                              .05684
         3                                     .XXXXXXXXXXXX          .                                             -.21010
         4                                     .          XX          .                                             -.02571
         5                                     .           XXX        .                                              .04630
         6                                     .           XXXX       .                                              .06110
         7                                     .           X          .                                              .00503
         8                                     .          XX          .                                             -.01022
         9                                     .           X          .                                              .00892
        10                                     .           X          .                                              .00400
        11                                     .      XXXXXX          .                                             -.09825
        12                                     .        XXXX          .                                             -.05736
        13                                     .         XXX          .                                             -.04433
        14                                     .           XXXXXX     .                                              .09484
        15                                     .           XX         .                                              .02755
        16                                     .        XXXX          .                                             -.06721
        17                                     .           XXXXXXXXX  .                                              .16001
        18                                     .          XX          .                                             -.01530
        19                                     .           XXX        .                                              .03568
        20                                     .       XXXXX          .                                             -.08733
        21                                     .         XXX          .                                             -.03457
        22                                     .           XXXXXXX    .                                              .11008
        23                                     .           XXXXXXXXX  .                                              .15642
        24                                     .           XX         .                                              .01873
```

PARTIAL AUTOCORRELATIONS

	.0461	.0548	-.2162	-.0082	.0778	.0121	-.0164	.0114	.0302	-.0028	-.1139	-.0418
SE	.1240	.1240	.1240	.1240	.1240	.1240	.1240	.1240	.1240	.1240	.1240	.1240
	-.0203	.0644	-.0040	-.0922	.2331	-.0087	-.0477	-.0174	-.0106	.1207	.0993	-.0558
SE	.1240	.1240	.1240	.1240	.1240	.1240	.1240	.1240	.1240	.1240	.1240	.1240

GRAPHIC INTERVAL IS .0200

```
      -1.0000                                         .0000                                          1.0000    VALUES
      .+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.
  1                                         .           XXX         .                                           .04609
  2                                         .           XXXX        .                                           .05484
  3                                         .XXXXXXXXXXXX           .                                          -.21621
  4                                         .           X           .                                          -.00818
  5                                         .           XXXXX       .                                           .07784
  6                                         .           XX          .                                           .01210
  7                                         .          XX           .                                          -.01636
  8                                         .           XX          .                                           .01144
  9                                         .           XXX         .                                           .03020
 10                                         .           X           .                                          -.00282
 11                                         .     XXXXXXX           .                                          -.11393
 12                                         .         XXX           .                                          -.04176
 13                                         .          XX           .                                          -.02035
 14                                         .           XXXX        .                                           .06436
 15                                         .           X           .                                          -.00404
 16                                         .      XXXXXX           .                                          -.09225
 17                                         .           XXXXXXXXXXXXX                                           .23309
 18                                         .           X           .                                          -.00874
 19                                         .         XXX           .                                          -.04771
 20                                         .          XX           .                                          -.01736
 21                                         .          XX           .                                          -.01064
 22                                         .           XXXXXXX     .                                           .12075
 23                                         .           XXXXXX      .                                           .09927
 24                                         .        XXXX           .                                          -.05577
```

NUMBER OF WHITE NOISE RESIDUALS 65
WHITE NOISE RESIDUALS

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1950		.03	.01	-.01	-.02	.05	.05	.02	.02	-.02	-.03	.05
1951	.07	-.01	.05	-.03	.10	-.05	-.03	-.01	-.01	.02	.05	-.01
1952	.01	.00	-.06	-.04	.02	.08	-.02	.04	-.05	.03	.04	-.01
1953	-.01	-.06	.04	.07	.02	-.06	-.04	.00	-.04	-.01	-.04	-.05
1954	-.02	-.13	.03	.02	.06	.04	.06	-.02	-.02	-.01	.01	-.01
1955	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	.03
1956	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-.02
1957	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-.02
1958	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-.06
1959	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	.04
1960	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-99999.00	-.02

FORECASTS:

ORIGIN: 144 NUMBER: 12

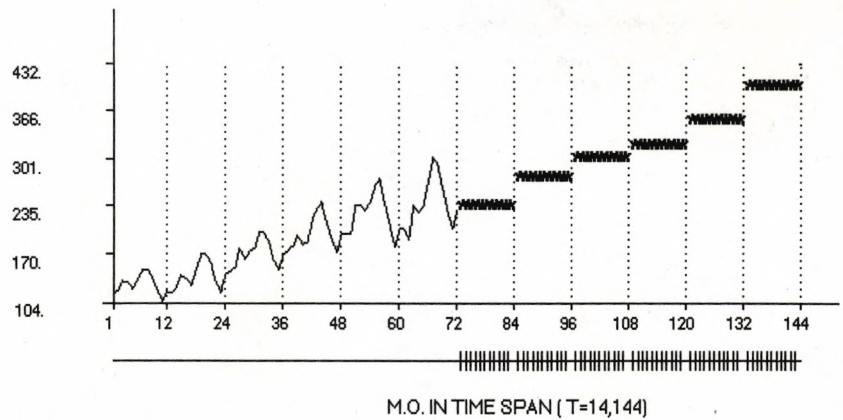
OBS	FORECAST (TR. SERIES)	STD ERROR	ACTUAL	RESIDUAL	FORECAST (ORIGINAL SERIES)
145	6.0838	.0531			438.71
146	6.0907	.0591			441.72
147	6.2468	.0644			516.35
148	6.2050	.0690			495.22
149	6.1991	.0731			492.32
150	6.3082	.0768			549.04
151	6.4091	.0802			607.33
152	6.4142	.0832			610.46
153	6.2990	.0860			544.05
154	6.1738	.0885			479.99
155	6.0432	.0907			421.23
156	6.1739	.0874			480.05
157	6.1861				485.97
158	6.1930				489.31
159	6.3491				571.98
160	6.3073				548.58
161	6.3014				545.36
162	6.4105				608.19
163	6.5114				672.76
164	6.5165				676.23
165	6.4013				602.66
166	6.2761				531.70
167	6.1455				466.61
168	6.2762				531.77
169	6.2885				538.32

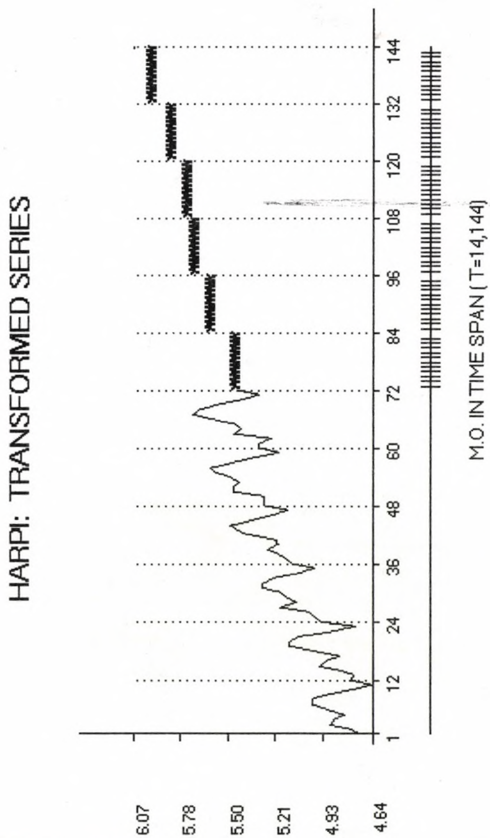
INTERPOLATED VALUES

OBS	INTERPOLATED VALUE (TRANSFORMED SERIES)	STD ERROR	INTERPOLATED VALUE (ORIGINAL SERIES)
73	5.4603	.0406	235.1685
74	5.4733	.0447	238.2494
75	5.6356	.0477	280.2248
76	5.6000	.0497	270.4224
77	5.6003	.0510	270.5002
78	5.7155	.0515	303.5301
79	5.8226	.0514	337.8327
80	5.8339	.0505	341.6749
81	5.7248	.0488	306.3855
82	5.6057	.0463	271.9844
83	5.4813	.0429	240.1640
85	5.6305	.0447	278.8135
86	5.6375	.0485	280.7540
87	5.7937	.0512	328.2167
88	5.7520	.0531	314.8157
89	5.7462	.0542	312.9976
90	5.8553	.0546	349.0881
91	5.9563	.0542	386.1843
92	5.9615	.0531	388.2091
93	5.8464	.0513	346.0035
94	5.7213	.0485	305.2925
95	5.5908	.0448	267.9413
97	5.7326	.0461	308.7738
98	5.7382	.0500	310.5187
99	5.8931	.0529	362.5413
100	5.8502	.0548	347.2868
101	5.8431	.0559	344.8324
102	5.9509	.0563	384.0936
103	6.0506	.0559	424.3574
104	6.0545	.0548	426.0279
105	5.9381	.0529	379.2171
106	5.8116	.0500	334.1632
107	5.6798	.0461	292.8986
109	5.8153	.0474	335.4061
110	5.8160	.0515	335.6110
111	5.9658	.0544	389.8736
112	5.9178	.0564	371.5973
113	5.9057	.0575	367.1218
114	6.0085	.0579	406.8714
115	6.1032	.0575	447.2700
116	6.1021	.0564	446.7802
117	5.9806	.0544	395.6960
118	5.8491	.0515	346.9367
119	5.7123	.0474	302.5707
121	5.8536	.0487	348.4839
122	5.8650	.0528	352.4783
123	6.0256	.0558	413.9084
124	5.9884	.0579	398.7836
125	5.9871	.0591	398.2532
126	6.1007	.0595	446.1599
127	6.2061	.0591	495.7782
128	6.2158	.0579	500.6058
129	6.1052	.0558	448.1753

130	5.9845	.0528	397.2106
131	5.8584	.0487	350.1722
133	6.0039	.0499	405.0071
134	6.0087	.0542	406.9611
135	6.1628	.0573	474.7506
136	6.1190	.0594	454.4009
137	6.1111	.0607	450.8186
138	6.2181	.0612	501.7343
139	6.3169	.0608	553.8745
140	6.3200	.0596	555.5978
141	6.2028	.0576	494.1438
142	6.0755	.0546	435.0778
143	5.9429	.0505	381.0382

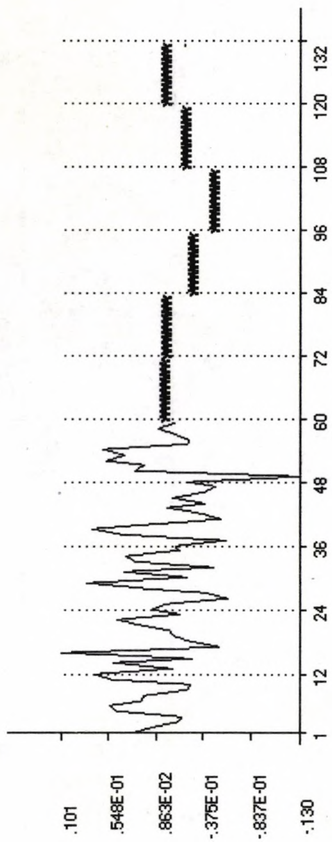
HARPI: ORIGINAL SERIES



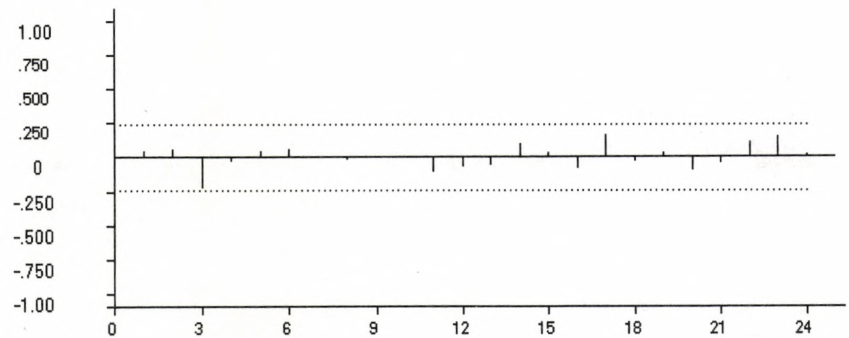


HARPI: TRANSFORMED SERIES
6.07
5.78
5.50
5.21
4.93
4.64
1
12
24
36
48
60
72
84
96
108
120
132
144
M.O. IN TIME SPAN (T=14,144)

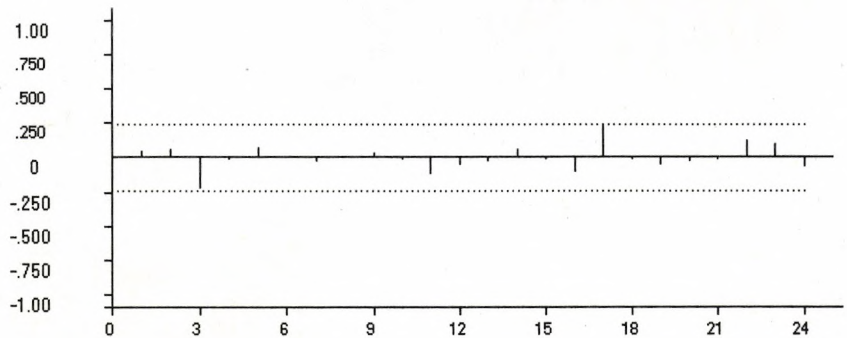
HARPI: RESIDUALS



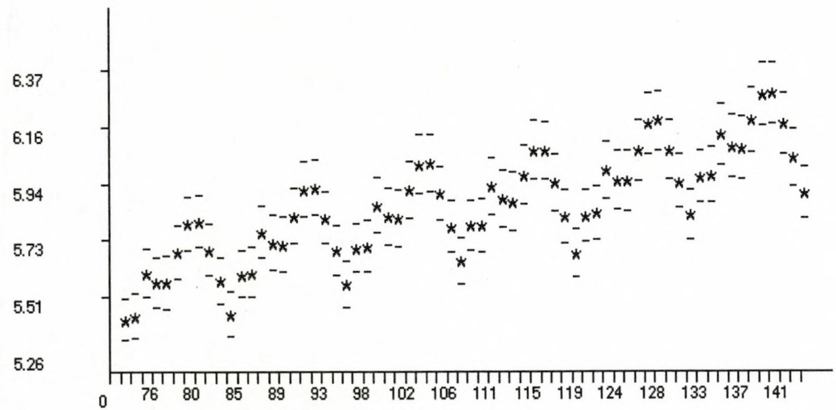
HARPI: ACF OF RESIDUALS



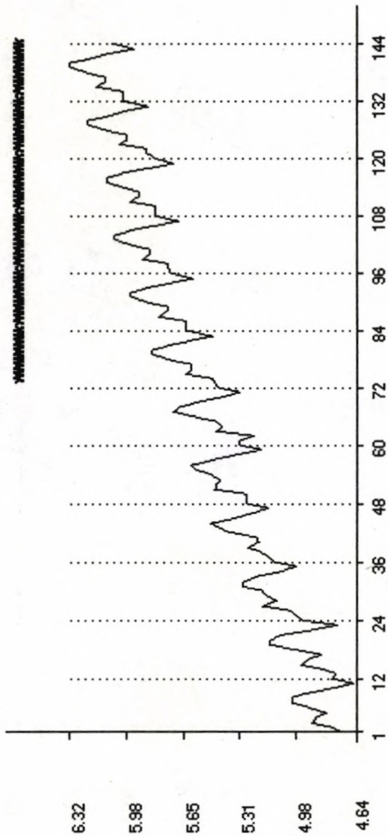
HARPI: PARTIAL ACF OF RESIDUALS



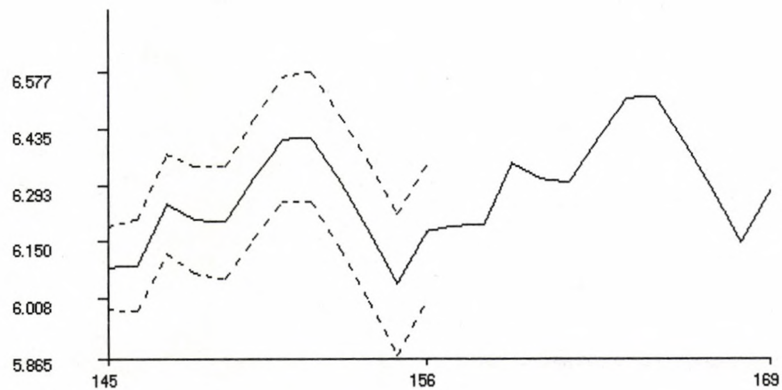
HARPI: INTERPOLATED VALUES



HARPI: TRANS. SERIES WITH INTERPOLATIONS

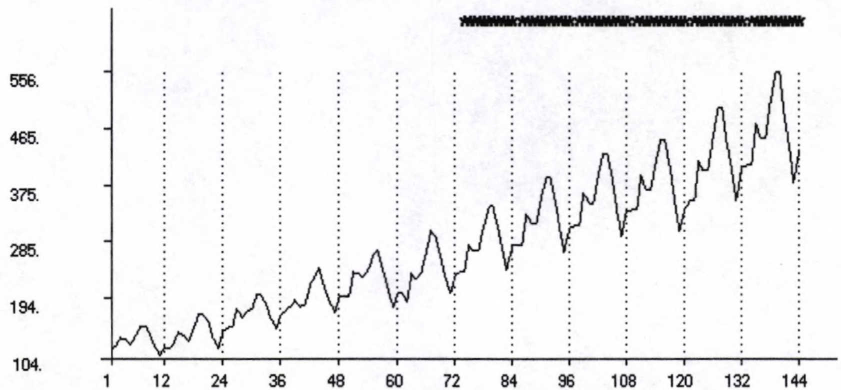


HARPI: FORECASTS



==== : 95 % C. I.

HARPI: ORIGINAL SERIES WITH INTERPOLATIONS



Example 4

This example consists of Data Set 3 in Kohn and Ansley (1986), a series with 144 monthly values. There are five missing observations and one of them falls among the initial values. The example illustrates two alternative ways of estimating the missing observations.

Example (4a) illustrates the standard approach: the initial missing observation is concentrated out of the likelihood, and estimated with regression. The other missing observations are obtained via the fixed point smoother.

Example (4b) treats all missing observations as additive outliers and estimates them with regression. Arbitrary (reasonable) numbers are plugged in the series holes, and then the following model is fit:

$$y_t = \sum_{i=1}^{5} \omega_i d_{it} + n_t,$$

$$\nabla\nabla_{12} n_t = (1-\theta_1 B)(1-\theta_{12} B^{12}) a_t$$

where $d_{1t},\ldots d_{5t}$ are the dummy variables associated with the additive outliers. The missing observation estimates are obtained as the fitted values, once the outlier effect $(\hat{\omega}_i)$ has been removed. When using additive outliers to estimate missing values, the likelihood function is modified by a determinantal term, so that it coincides with that of the standard missing observations case. Computation of the likelihood, however, is made easier, since the algorithm of Morf, Sidhu and Kailath can now be applied.

Comparing (4a) and (4b), it is seen that the forecasts and interpolators obtained with the two approaches are virtually indistinguishable.

4a

```
DATA SET 3 (KOHN-ANSLEY JASA 86), M.O. WITH F.P.S.
144   1949 1 12
112 118 132 129 121    135   -99999.     148 136 119 104 118
115 126 141 135 125    149      170      170 158 133 114 140
145 150 178 163 172    178      199      199 184 162 146 166
171 180 193 181 183    218      230      242 209 191 172 194
196 196 236 235 229    243      264      272 237 211 180 201
204 188 235 227 234    264      302      293 259 229 203 229
242 233 267 269 270    315      364      347 312 274 237 278
284 277 317 313 318    374      413      405 355 306 271 306
315 301 356 348 355 -99999. -99999. -99999. 404 347 305 336
340 318 362 348 363    435      491      505 404 359 310 337
360 342 406 396 420    472      548      559 463 407 362 405
417 391 419 461 472    535   -99999.     606 508 461 390 432
 &DATEN IDR=1,IDS=1,IQR=1,IQS=1,LAG=24,INCON=0,
  NPRED=12,LAMDA=0,INTERP=1,ICONCE=1,/
```

TRAM

TIME SERIES REGRESSION MODELS WITH
ARIMA ERRORS AND MISSING VALUES.

BY VICTOR GOMEZ AND AGUSTIN MARAVALL.

PROGRAM DESIGNED AND WRITTEN BY VICTOR GOMEZ.

DATA SET 3 (KOHN-ANSLEY JASA 86), M.O. WITH F.P.S.
ORIGINAL SERIES

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1949	112.00	118.00	132.00	129.00	121.00	135.00	-99999.00	148.00	136.00	119.00	104.00	118.00
1950	115.00	126.00	141.00	135.00	125.00	149.00	170.00	170.00	158.00	133.00	114.00	140.00
1951	145.00	150.00	178.00	163.00	172.00	178.00	199.00	199.00	184.00	162.00	146.00	166.00
1952	171.00	180.00	193.00	181.00	183.00	218.00	230.00	242.00	209.00	191.00	172.00	194.00
1953	196.00	196.00	236.00	235.00	229.00	243.00	264.00	272.00	237.00	211.00	180.00	201.00
1954	204.00	188.00	235.00	227.00	234.00	264.00	302.00	293.00	259.00	229.00	203.00	229.00
1955	242.00	233.00	267.00	269.00	270.00	315.00	364.00	347.00	312.00	274.00	237.00	278.00
1956	284.00	277.00	317.00	313.00	318.00	374.00	413.00	405.00	355.00	306.00	271.00	306.00
1957	315.00	301.00	356.00	348.00	355.00	-99999.00	-99999.00	-99999.00	404.00	347.00	305.00	336.00
1958	340.00	318.00	362.00	348.00	363.00	435.00	491.00	505.00	404.00	359.00	310.00	337.00
1959	360.00	342.00	406.00	396.00	420.00	472.00	548.00	559.00	463.00	407.00	362.00	405.00
1960	417.00	391.00	419.00	461.00	472.00	535.00	-99999.00	606.00	508.00	461.00	390.00	432.00

INITIAL MISSING OBSERVATION NUMBER 7

MISSING OBSERVATION NUMBER 102

MISSING OBSERVATION NUMBER 103

MISSING OBSERVATION NUMBER 104

MISSING OBSERVATION NUMBER 139

```
MODEL PARAMETERS:
IMEAN =          0
LAMDA =          0
IDR =          1
IDS =          1
IPR =          0
IPS =          0
IQR =          1
IQS =          1
IREG =          0
ITRAD =          0
IEAST =          0
IDUR =          0
LAG =         24
INCON =          0
NBACK =          0
NPRED =         12
INTERP =          1
IESTIM =          1
```

```
VA =        1.000000000000000

IFILT =         3

IGRBAR =          0

IGRRES =          0

IDENSC =          1

INVER =          0

INIC =          0

TOL =   1.000000000000000E-006

ICONCE =          1

THR =  -1.000000000000000E-001

THS =  -1.000000000000000E-001

NUMBER OF INITIAL OBSERVATIONS =          13
NUMBER OF MISSING INITIAL OBSERVATIONS =          1

NUMBER OF MISSING VALUES IN TIME SPAN
        14 -        144
=       4
TRANSFORMED SERIES (LOGARITHMS OF THE DATA)
```

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1949	4.72	4.77	4.88	4.86	4.80	4.91	-99999.00	5.00	4.91	4.78	4.64	4.77
1950	4.74	4.84	4.95	4.91	4.83	5.00	5.14	5.14	5.06	4.89	4.74	4.94
1951	4.98	5.01	5.18	5.09	5.15	5.18	5.29	5.29	5.21	5.09	4.98	5.11
1952	5.14	5.19	5.26	5.20	5.21	5.38	5.44	5.49	5.34	5.25	5.15	5.27
1953	5.28	5.28	5.46	5.46	5.43	5.49	5.58	5.61	5.47	5.35	5.19	5.30
1954	5.32	5.24	5.46	5.42	5.46	5.58	5.71	5.68	5.56	5.43	5.31	5.43
1955	5.49	5.45	5.59	5.59	5.60	5.75	5.90	5.85	5.74	5.61	5.47	5.63
1956	5.65	5.62	5.76	5.75	5.76	5.92	6.02	6.00	5.87	5.72	5.60	5.72
1957	5.75	5.71	5.87	5.85	5.87	-99999.00	-99999.00	-99999.00	6.00	5.85	5.72	5.82
1958	5.83	5.76	5.89	5.85	5.89	6.08	6.20	6.22	6.00	5.88	5.74	5.82
1959	5.89	5.83	6.01	5.98	6.04	6.16	6.31	6.33	6.14	6.01	5.89	6.00
1960	6.03	5.97	6.04	6.13	6.16	6.28	-99999.00	6.41	6.23	6.13	5.97	6.07

```
ARIMA MODEL ESTIMATION BEGINS

INITIAL PARAMETER VALUES:
 -1.000000000000000E-001  -1.000000000000000E-001

 ITERATION, LAMBDA          1   0.000000000000000E+000
 FO FP   2.427169606189864E-001   1.850019301320929E-001
 FO-FP SUM S   5.771503048689353E-002   5.037686886530479E-002
       1.145665298119444
 ITERATION, LAMBDA          2   0.000000000000000E+000
 FO FP   1.850019301320929E-001   1.836365518550908E-001
 FO-FP SUM S   1.365378277002061E-003   1.734210720702149E-003
  7.873197072898065E-001
 ITERATION, LAMBDA          3   0.000000000000000E+000
 FO FP   1.836365518550908E-001   1.835183656875106E-001
 FO-FP SUM S   1.181861675802198E-004   1.602129998009971E-004
  7.376815097839787E-001
 ITERATION, LAMBDA          4   0.000000000000000E+000
 FO FP   1.835183656875106E-001   1.835100546290567E-001
 FO-FP SUM S   8.311058453919218E-006   1.154511736458228E-005
  7.198764803739115E-001
 ITERATION, LAMBDA          5   0.000000000000000E+000
 FO FP   1.835100546290567E-001   1.835093393930473E-001
 FO-FP SUM S   7.152360094464516E-007   1.001114644845674E-006
  7.144396629585896E-001
 ITERATION, LAMBDA          6   0.000000000000000E+000

METHOD OF ESTIMATION: EXACT MAXIMUM LIKELIHOOD

PARAMETER      ESTIMATE          STD ERROR      T RATIO   LAG

MA1  1       -.404985037         .080995506      -5.00      1
MA2  1       -.566287014         .082722985      -6.85     12

REGULAR MA INVERSE ROOTS ARE
NO.      REAL P.     IMAG.P.     MODULUS
 1     -.4049850    .0000000    .4049850
SEASONAL MA INVERSE ROOTS ARE
NO.      REAL P.     IMAG.P.     MODULUS
 1     -.5662870    .0000000    .5662870
```

CORRELATIONS OF THE ESTIMATES

1.000	-.035
-.035	1.000

AIC
-466.129

FINAL VALUE OF OBJECTIVE FUNCTION:
.1835092797

VARIANCE ESTIMATE:
.0014040

ITERATIONS: 6

NUMBER OF FUNCTION EVALUATIONS: 19

ESTIMATES OF REGRESSION PARAMETERS
CONCENTRATED OUT OF THE LIKELIHOOD
ZJ 7 5.012837667 (.031406423)

COVARIANCE MATRIX OF ESTIMATORS

.986E-03

CHECK OF WHITE NOISE RESIDUALS:

AUTOCORRELATIONS

	.0093	.0199	-.1463	-.0853	.0616	-.0141	-.0028	.0027	.1629	-.0830	.0031	-.0483
SE	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891
	.0122	.0081	.0156	-.1293	.0298	.0018	-.1084	-.0569	-.0422	.0354	.1737	-.0499
SE	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891

LJUNG-BOX OF ORDER Q IS 19.81 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUARED(11)
PIERCE OF ORDER QS IS .72 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUATED(2)

PARTIAL AUTOCORRELATIONS

	.0093	.0198	-.1467	-.0844	.0703	-.0334	-.0313	.0169	.1744	-.1060	-.0031	.0130
SE	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891
	.0156	-.0365	.0317	-.1337	.0321	-.0180	-.1230	-.0891	.0041	-.0139	.1486	-.0752
SE	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891

NUMBER OF WHITE NOISE RESIDUALS 126

WHITE NOISE RESIDUALS

.0120	-.0130	-.0165	.0509	.0187	.0442	.0278	-.0224
-.0260	.0584	.0721	-.0122	.0517	-.0305	.1082	-.0650
-.0468	-.0016	-.0009	.0269	.0512	-.0208	.0096	-.0001
-.0697	-.0323	.0096	.0843	-.0342	.0458	-.0491	.0317
.0338	-.0166	-.0159	-.0596	.0525	.0787	.0078	-.0672
-.0361	-.0023	-.0298	-.0094	-.0460	-.0455	-.0189	-.1192
.0323	.0151	.0487	.0413	.0632	-.0275	-.0130	-.0106
.0102	-.0009	.0398	-.0032	-.0426	.0260	.0080	.0489
.0562	-.0248	.0059	-.0074	-.0193	.0284	.0004	.0021
-.0239	-.0054	.0085	.0374	-.0097	-.0026	-.0175	-.0313
.0017	-.0169	-.0055	-.0216	.0102	-.0035	.0087	.0135
-.0129	-.0044	-.0364	-.0311	-.0450	-.0457	-.0395	.0120
.0401	.0241	.0557	-.0743	-.0047	-.0193	-.0407	.0284
.0088	.0299	.0147	.0385	-.0303	.0184	.0216	-.0074
-.0001	.0197	.0181	-.0041	-.0163	-.0940	.0839	.0171
-.0099	-.0204	-.0042	.0317	-.0261	-.0148		

FORECASTS:

ORIGIN: 144 NUMBER: 12

OBS	FORECAST (TR. SERIES)	STD ERROR	ACTUAL	RESIDUAL	FORECAST (ORIGINAL SERIES)
145	6.1101	.0375			450.37
146	6.0540	.0436			425.81
147	6.1727	.0490			479.46
148	6.1991	.0538			492.33
149	6.2323	.0582			508.91
150	6.3671	.0624			582.36
151	6.4967	.0678			662.98
152	6.5028	.0699			666.99
153	6.3248	.0734			558.26
154	6.2088	.0767			497.09
155	6.0636	.0798			429.91
156	6.1682	.0829			477.35
157	6.2065				495.96
158	6.1504				468.91
159	6.2691				528.00
160	6.2956				542.16
161	6.3287				560.42
162	6.4635				641.31
163	6.5932				730.09
164	6.5992				734.51
165	6.4212				614.77
166	6.3052				547.41
167	6.1600				473.42
168	6.2647				525.67
169	6.3029				546.17

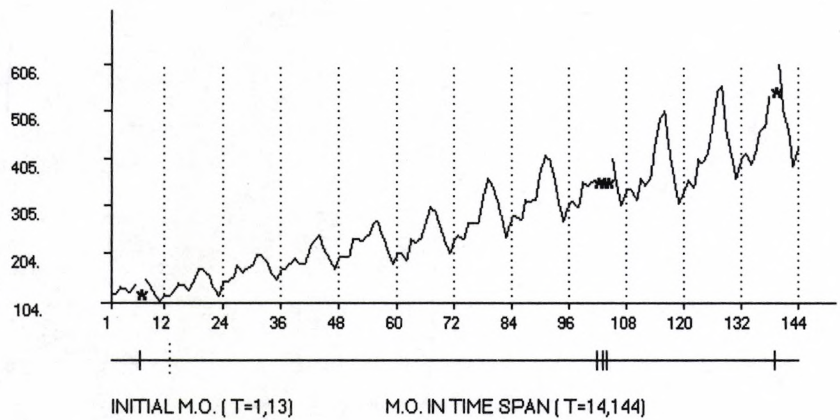
REGRESSION RESIDUALS

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1950		.03	.01	-.01	-.02	.05	.04	.03	.02	-.02	-.03	.06
1951	.07	-.01	.05	-.03	.11	-.07	-.04	-.01	.00	.03	.05	-.02
1952	.01	.00	-.07	-.03	.01	.08	-.03	.04	-.05	.03	.03	-.02
1953	-.02	-.06	.05	.08	.01	-.07	-.03	.00	-.03	-.01	-.05	-.05
1954	-.02	-.12	.03	.02	.05	.04	.07	-.03	-.01	-.01	.01	.00
1955	.04	.00	-.04	.03	.01	.05	.06	-.03	.01	-.01	-.02	.03
1956	.00	.00	-.02	-.01	.01	.04	-.01	.00	-.02	-.03	.00	-.02
1957	-.01	-.02	.01	.00	.01	-99999.00	-99999.00	-99999.00	.01	-.01	.00	-.04
1958	-.03	-.04	-.05	-.04	.01	.04	.02	.06	-.07	.00	-.02	-.04
1959	.03	.01	.03	.01	.04	-.03	.02	.02	-.01	.00	.02	.02
1960	.00	-.02	-.09	.08	.02	-.01	-99999.00	-.02	.00	.03	-.03	-.01

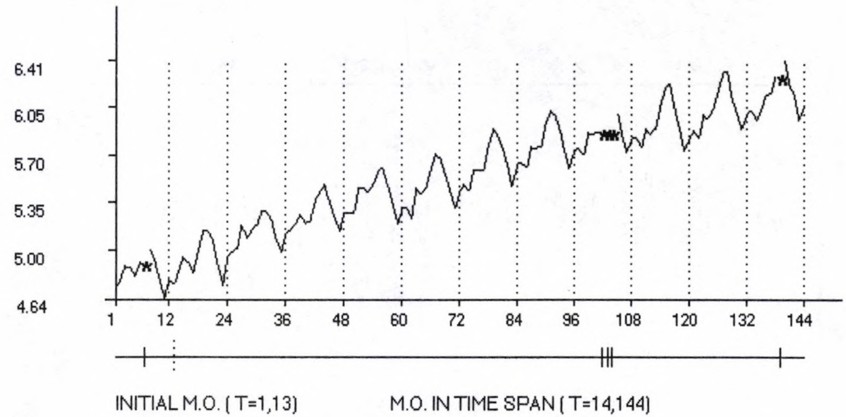
INTERPOLATED VALUES

OBS	INTERPOLATED VALUE (TRANSFORMED SERIES)	STD ERROR	INTERPOLATED VALUE (ORIGINAL SERIES)
102	6.0238	.0300	413.1473
103	6.1472	.0314	467.4085
104	6.1480	.0300	467.7699
139	6.4086	.0316	607.0696

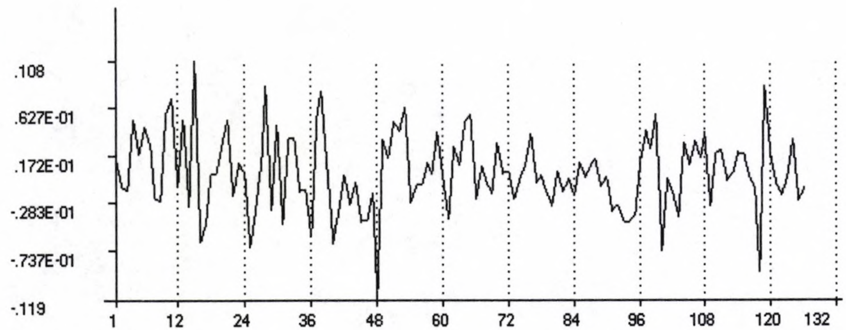
KADS3: ORIGINAL SERIES



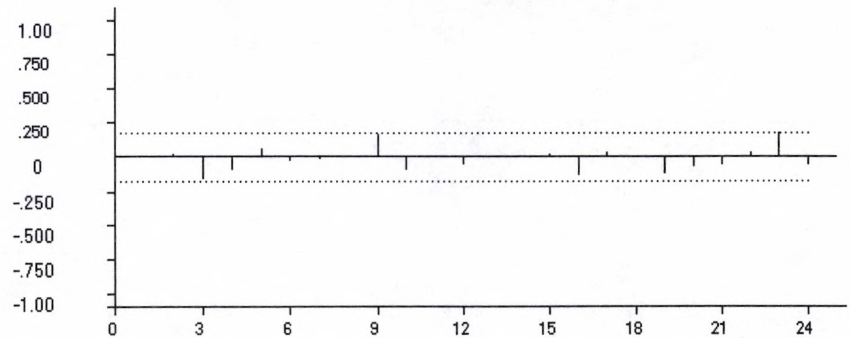
KADS3: TRANSFORMED SERIES



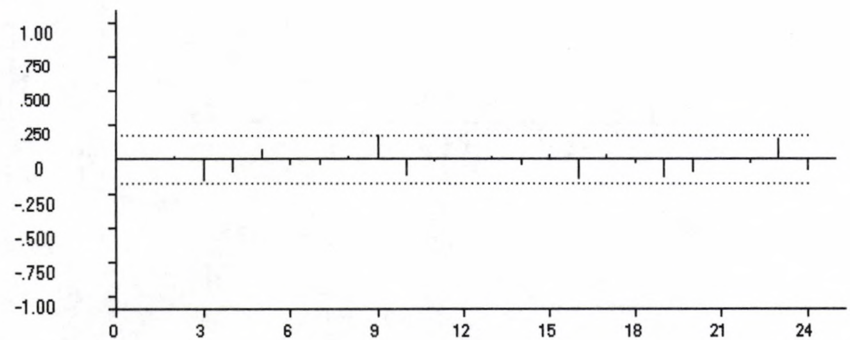
KADS3: RESIDUALS



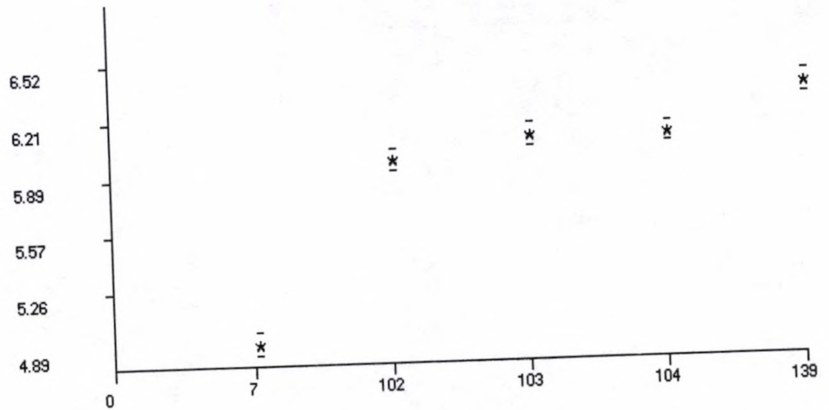
KADS3: ACF OF RESIDUALS



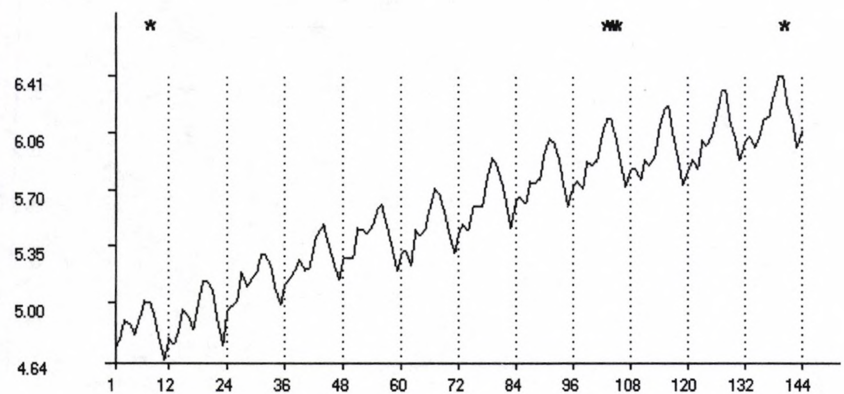
KADS3: PARTIAL ACF OF RESIDUALS



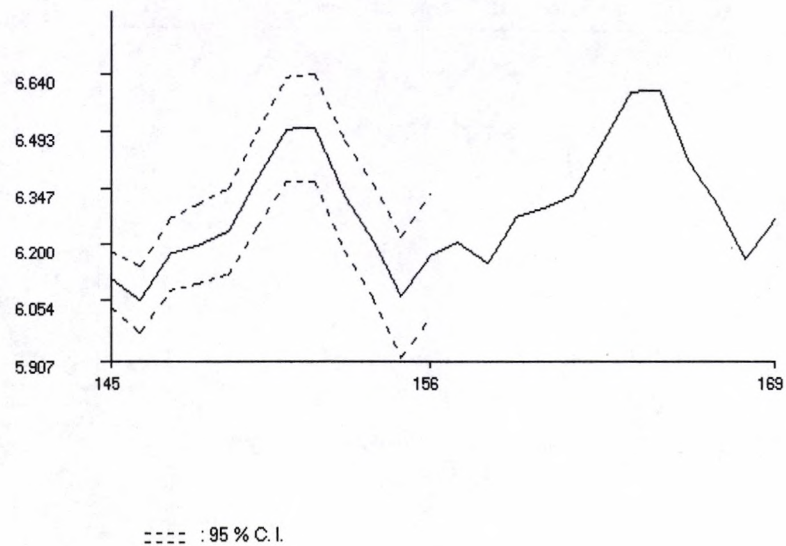
KADS3: INTERPOLATED VALUES



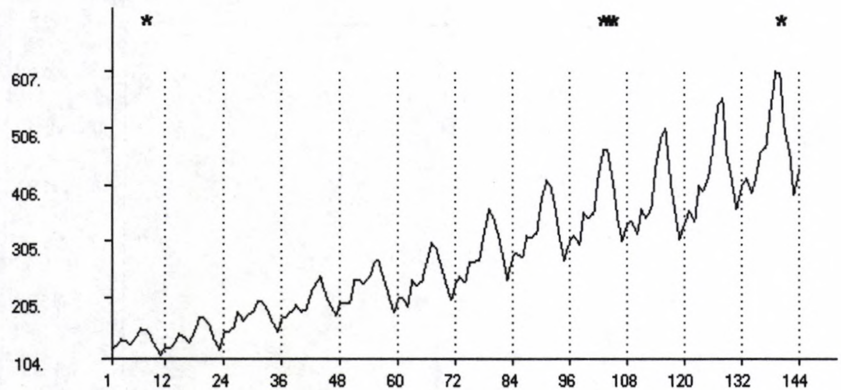
KADS3: TRANS. SERIES WITH INTERPOLATIONS



KADS3: FORECASTS



KADS3: ORIGINAL SERIES WITH INTERPOLATIONS



4b

```
DATA SET 3 (KOHN-ANSLEY, JASA 86) M.O. AS ADDITIVE OUTLIERS
144   1949 1 12
112 118 132 129 121 135 148 148 136 119 104 118
115 126 141 135 125 149 170 170 158 133 114 140
145 150 178 163 172 178 199 199 184 162 146 166
171 180 193 181 183 218 230 242 209 191 172 194
196 196 236 235 229 243 264 272 237 211 180 201
204 188 235 227 234 264 302 293 259 229 203 229
242 233 267 269 270 315 364 347 312 274 237 278
284 277 317 313 318 374 413 405 355 306 271 306
315 301 356 348 355 422 465 467 404 347 305 336
340 318 362 348 363 435 491 505 404 359 310 337
360 342 406 396 420 472 548 559 463 407 362 405
417 391 419 461 472 535 622 606 508 461 390 432
 &DATEN IDR=1,IDS=1,IQR=1,IQS=1,LAG=24,INCON=0,
  NPRED=12,LAMDA=0,IFILT=2,ICONCE=1,
  IGRBAR=1,IREG=5,/
 &REG IAUS=5,/
  7 102 103 104 139
```

TRAM

TIME SERIES REGRESSION MODELS WITH
ARIMA ERRORS AND MISSING VALUES.

BY VICTOR GOMEZ AND AGUSTIN MARAVALL.

PROGRAM DESIGNED AND WRITTEN BY VICTOR GOMEZ.

DATA SET 3 (KOHN-ANSLEY, JASA 86) M.O. AS ADDITIVE OUTLIERS
ORIGINAL SERIES

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1949	112.00	118.00	132.00	129.00	121.00	135.00	148.00	148.00	136.00	119.00	104.00	118.00
1950	115.00	126.00	141.00	135.00	125.00	149.00	170.00	170.00	158.00	133.00	114.00	140.00
1951	145.00	150.00	178.00	163.00	172.00	178.00	199.00	199.00	184.00	162.00	146.00	166.00
1952	171.00	180.00	193.00	181.00	183.00	218.00	230.00	242.00	209.00	191.00	172.00	194.00
1953	196.00	196.00	236.00	235.00	229.00	243.00	264.00	272.00	237.00	211.00	180.00	201.00
1954	204.00	188.00	235.00	227.00	234.00	264.00	302.00	293.00	259.00	229.00	203.00	229.00
1955	242.00	233.00	267.00	269.00	270.00	315.00	364.00	347.00	312.00	274.00	237.00	278.00
1956	284.00	277.00	317.00	313.00	318.00	374.00	413.00	405.00	355.00	306.00	271.00	306.00
1957	315.00	301.00	356.00	348.00	355.00	422.00	465.00	467.00	404.00	347.00	305.00	336.00
1958	340.00	318.00	362.00	348.00	363.00	435.00	491.00	505.00	404.00	359.00	310.00	337.00
1959	360.00	342.00	406.00	396.00	420.00	472.00	548.00	559.00	463.00	407.00	362.00	405.00
1960	417.00	391.00	419.00	461.00	472.00	535.00	622.00	606.00	508.00	461.00	390.00	432.00

MODEL PARAMETERS:

IMEAN = 0

LAMDA = 0

IDR = 1

IDS = 1

IPR = 0

IPS = 0

IQR = 1

IQS = 1

IREG = 5

ITRAD = 0

```
IEAST =          0

IDUR =           0

LAG =           24

INCON =          0

NBACK =          0

NPRED =         12

INTERP =         0

IESTIM =         1

VA =       1.000000000000000

IFILT =          2

IGRBAR =         1

IGRRES =         0

IDENSC =         1

INVER =          0

INIC =           0

TOL =   1.000000000000000E-006

ICONCE =         5

THR =  -1.000000000000000E-001

THS =  -1.000000000000000E-001

NUMBER OF INITIAL OBSERVATIONS =         13
NUMBER OF MISSING INITIAL OBSERVATIONS =          0

NUMBER OF MISSING VALUES IN TIME SPAN
         14 -          144
=           0
```

TRANSFORMED SERIES (LOGARITHMS OF THE DATA)

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1949	4.72	4.77	4.88	4.86	4.80	4.91	5.00	5.00	4.91	4.78	4.64	4.77
1950	4.74	4.84	4.95	4.91	4.83	5.00	5.14	5.14	5.06	4.89	4.74	4.94
1951	4.98	5.01	5.18	5.09	5.15	5.18	5.29	5.29	5.21	5.09	4.98	5.11
1952	5.14	5.19	5.26	5.20	5.21	5.38	5.44	5.49	5.34	5.25	5.15	5.27
1953	5.28	5.28	5.46	5.46	5.43	5.49	5.58	5.61	5.47	5.35	5.19	5.30
1954	5.32	5.24	5.46	5.42	5.46	5.58	5.71	5.68	5.56	5.43	5.31	5.43
1955	5.49	5.45	5.59	5.59	5.60	5.75	5.90	5.85	5.74	5.61	5.47	5.63
1956	5.65	5.62	5.76	5.75	5.76	5.92	6.02	6.00	5.87	5.72	5.60	5.72
1957	5.75	5.71	5.87	5.85	5.87	6.05	6.14	6.15	6.00	5.85	5.72	5.82
1958	5.83	5.76	5.89	5.85	5.89	6.08	6.20	6.22	6.00	5.88	5.74	5.82
1959	5.89	5.83	6.01	5.98	6.04	6.16	6.31	6.33	6.14	6.01	5.89	6.00
1960	6.03	5.97	6.04	6.13	6.16	6.28	6.43	6.41	6.23	6.13	5.97	6.07

INITIAL ESTIMATES OF REGRESSION PARAMETERS:
-1.995731788955890E-002 -3.875746990989691E-003 -2.217806738830563E-002
-2.735256894242799E-002 2.365566867883030E-002

DIFFERENCED TRANSFORMED SERIES

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1950		.04	.00	-.02	-.01	.07	.04	.00	.01	-.04	-.02	.08
1951	.06	-.06	.06	-.04	.13	-.14	-.02	.00	-.01	.04	.05	-.08
1952	-.01	.02	-.10	.02	-.04	.14	-.06	.05	-.07	.04	.00	-.01
1953	-.02	-.05	.12	.06	-.04	-.12	.03	-.02	.01	-.03	-.05	-.01
1954	.00	-.08	.04	-.03	.06	.06	.05	-.06	.01	-.01	.04	.01
1955	.04	.04	-.09	.04	-.03	.03	.01	-.02	.02	-.01	-.02	.04
1956	-.03	.01	.00	-.02	.01	.01	-.05	.03	-.03	-.02	.02	-.04
1957	.01	-.02	.03	-.01	.00	.01	.00	.02	-.01	.00	-.01	-.02
1958	-.02	-.02	-.04	-.02	.02	.01	.02	.02	-.08	.03	-.02	-.01
1959	.05	.02	.04	.01	.02	-.06	.03	-.01	.03	-.01	.03	.03
1960	-.04	-.01	-.10	.12	-.04	.01	.00	-.05	.01	.03	-.05	-.01

```
AUTOCORRELATIONS
----------------

    -.3423   .1151  -.2243   .0646   .0482   .0242  -.0556  -.0165   .1862  -.1024   .0807  -.3873
SE   .0874   .0971   .0981   .1019   .1023   .1024   .1025   .1027   .1027   .1053   .1060   .1065
     .1556  -.0570   .1568  -.1687   .0765   .0080   .0035  -.1034   .0255  -.0667   .1955  -.0115
SE   .1168   .1183   .1185   .1201   .1219   .1223   .1223   .1223   .1229   .1230   .1233   .1256

               GRAPHIC INTERVAL IS      .0200

     -1.0000                                          .0000                                            1.0000  VALUES
     .+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.
  1                                   XXXXXXXXXXXXXXXXXX        .                                             -.34230
  2                 .                        .         XXXXXXX   .                                             .11515
  3                                         XXXXXXXXXXXX         .                                            -.22429
  4                                          .         XXXX      .                                             .06464
  5                                          .         XXX       .                                             .04821
  6                                          .         XX        .                                             .02418
  7                                          .      XXXX         .                                            -.05561
  8                                          .        XX         .                                            -.01653
  9                                          .         XXXXXXXXXX.                                             .18619
 10                                          .    XXXXXX         .                                            -.10237
 11                                          .         XXXXX     .                                             .08065
 12                                 XXXXXXXXXXXXXXXXXXXX         .                                            -.38729
 13                                         .          XXXXXXXXX  .                                            .15563
 14                                        .        XXXX           .                                          -.05702
 15                                        .           XXXXXXXXX   .                                           .15682
 16                                        .   XXXXXXXXX           .                                          -.16866
 17                                        .           XXXXX       .                                           .07647
 18                                        .           X           .                                           .00798
 19                                        .           X           .                                           .00350
 20                                        .      XXXXXX           .                                          -.10336
 21                                        .           XX          .                                           .02550
 22                                        .        XXXX           .                                          -.06666
 23                                        .           XXXXXXXXXXX .                                           .19553
 24                                        .          XX           .                                          -.01153
```

PARTIAL AUTOCORRELATIONS

```
      -.3423  -.0023  -.2102  -.0898   .0621   .0254  -.0518  -.0261   .2193   .0025   .0364  -.3278
SE     .0874   .0874   .0874   .0874   .0874   .0874   .0874   .0874   .0874   .0874   .0874   .0874
      -.1230  -.0674  -.0035  -.1337   .0259   .0945  -.0121  -.1484   .1302  -.0826   .1381  -.0589
SE     .0874   .0874   .0874   .0874   .0874   .0874   .0874   .0874   .0874   .0874   .0874   .0874
```

```
              GRAPHIC INTERVAL IS      .0200

    -1.0000                                          .0000                                          1.0000  VALUES
     .+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.
  1                                   XXXXXXXXXXXXXXXXXX        .                                          -.34230
  2                                          .        X        .                                          -.00229
  3                                         XXXXXXXXXXXX        .                                          -.21020
  4                                          .    XXXXX        .                                          -.08984
  5                                          .        XXXX     .                                           .06208
  6                                          .        XX       .                                           .02543
  7                                          .     XXXX        .                                          -.05176
  8                                          .       XX        .                                          -.02611
  9                                          .        XXXXXXXXXXXX                                         .21930
 10                                          .        X        .                                           .00252
 11                                          .        XXX      .                                           .03635
 12                                    XXXXXXXXXXXXXXXXX        .                                          -.32783
 13                                          .  XXXXXXX        .                                          -.12305
 14                                          .     XXXX        .                                          -.06743
 15                                          .        X        .                                          -.00354
 16                                          . XXXXXXXX        .                                          -.13370
 17                                          .        XX       .                                           .02589
 18                                          .        XXXXXX   .                                           .09453
 19                                          .       XX        .                                          -.01213
 20                                          . XXXXXXXX        .                                          -.14842
 21                                          .        XXXXXXXX .                                           .13024
 22                                          .    XXXXX        .                                          -.08262
 23                                          .        XXXXXXXX .                                           .13813
 24                                          .     XXXX        .                                          -.05894
```

```
CONVERGENCE IN WILSON
ITERATIONS:          3
SUM OF SQUARES:   7.671208782927790E-005

ARIMA MODEL ESTIMATION BEGINS

INITIAL PARAMETER VALUES:
 -3.893778620965460E-001  -5.392975105419614E-001

 ITERATION, LAMBDA            1   0.000000000000000E+000
 FO FP   1.837516773697598E-001   1.835292471422154E-001
 FO-FP SUM S   2.224302275443957E-004   2.906449205179625E-004
  7.652988641535504E-001
 ITERATION, LAMBDA            2   0.000000000000000E+000
 FO FP   1.835292471422154E-001   1.835105453693007E-001
 FO-FP SUM S   1.870177291471431E-005   2.452280110987513E-005
  7.626279245556192E-001
 ITERATION, LAMBDA            3   0.000000000000000E+000
 FO FP   1.835105453693007E-001   1.835092822746050E-001
 FO-FP SUM S   1.263094695669320E-006   1.673132441776387E-006
  7.549281002096137E-001
 ITERATION, LAMBDA            4   0.000000000000000E+000

METHOD OF ESTIMATION: EXACT MAXIMUM LIKELIHOOD

PARAMETER      ESTIMATE          STD ERROR      T RATIO   LAG

MA1  1       -.404918225         .078432251      -5.16      1
MA2  1       -.565993169         .082100447      -6.89     12

REGULAR MA INVERSE ROOTS ARE
NO.     REAL P.     IMAG.P.     MODULUS
 1    -.4049182    .0000000    .4049182
SEASONAL MA INVERSE ROOTS ARE
NO.     REAL P.     IMAG.P.     MODULUS
 1    -.5659932    .0000000    .5659932

CORRELATIONS OF THE ESTIMATES

 1.000  -.046
 -.046  1.000

AIC
   -466.129

FINAL VALUE OF OBJECTIVE FUNCTION:
      .1835091891

VARIANCE ESTIMATE:
   .0014040

ITERATIONS:          4

 NUMBER OF FUNCTION EVALUATIONS:         13
```

ESTIMATES OF REGRESSION PARAMETERS
CONCENTRATED OUT OF THE LIKELIHOOD

REG 1	-.015623788	(.031406309)
REG 2	.021185049	(.030011908)
REG 3	-.005183469	(.031399702)
REG 4	-.001662390	(.030011971)
REG 5	.024290663	(.031648102)

INTERPOLATED VALUES

INT 7	5.012836062
INT 102	6.023820265
INT 103	6.147220874
INT 104	6.147991648
INT 139	6.408649430

COVARIANCE MATRIX OF ESTIMATORS

.986E-03	-.796E-09	.675E-05	-.447E-09	.320E-05
-.796E-09	.901E-03	.329E-03	.207E-03	-.120E-07
.675E-05	.329E-03	.986E-03	.329E-03	.110E-03
-.447E-09	.207E-03	.329E-03	.901E-03	-.241E-07
.320E-05	-.120E-07	.110E-03	-.241E-07	.100E-02

NUMBER OF WHITE NOISE RESIDUALS 126

WHITE NOISE RESIDUALS

.0648	.0168	.0167	-.0269	-.0278	.0576	.0717	-.0124
.0516	-.0305	.1082	-.0650	-.0249	-.0146	-.0062	.0248
.0503	-.0212	.0095	-.0002	-.0697	-.0323	.0096	.0844
-.0224	.0388	-.0519	.0305	.0333	-.0168	-.0160	-.0597
.0526	.0787	.0078	-.0672	-.0295	-.0062	-.0314	-.0100
-.0463	-.0456	-.0190	-.1192	.0323	.0151	.0487	.0413
.0669	-.0297	-.0139	-.0110	.0101	-.0009	.0398	-.0032
-.0427	.0259	.0079	.0489	.0583	-.0261	.0054	-.0076
-.0194	.0284	.0003	.0022	-.0239	-.0054	.0085	.0374
-.0085	-.0033	-.0177	-.0314	.0017	-.0169	-.0055	-.0216
.0102	-.0035	.0087	.0093	.0278	.0346	-.0233	-.0267
-.0102	-.0388	-.0321	-.0454	-.0459	-.0396	.0120	.0403
.0144	.0435	-.0669	-.0028	-.0186	-.0404	.0285	.0088
.0300	.0147	.0385	-.0286	.0156	.0193	-.0053	.0008
.0200	.0182	-.0040	-.0163	-.0940	.0839	.0171	-.0095
-.0544	.0067	.0073	.0364	-.0242	-.0140		

```
CHECK OF WHITE NOISE RESIDUALS:


AUTOCORRELATIONS
----------------
      .0201   .0301  -.1917  -.0448   .0567   .0346  -.0945  -.0133   .0886  -.0394  -.0304  -.0201
SE    .0891   .0891   .0891   .0891   .0891   .0891   .0891   .0891   .0891   .0891   .0891   .0891
      .0402  -.0070   .0471  -.1613   .0331   .0230  -.0707  -.1077  -.0409   .0147   .1548   .0026
SE    .0891   .0891   .0891   .0891   .0891   .0891   .0891   .0891   .0891   .0891   .0891   .0891

 LJUNG-BOX OF ORDER Q IS     19.74 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUARED(11)
PIERCE OF ORDER QS IS          .06 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS  CHI-SQUATED( 2)

              GRAPHIC INTERVAL IS      .0200

             -1.0000                                              .0000                                             1.0000   VALUES
              .+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.
          1                                               .        XX        .                                            .02012
          2                                               .        XXX       .                                            .03008
          3                                              XXXXXXXXXXX         .                                           -.19172
          4                                               .      XXX         .                                           -.04485
          5                                               .        XXXX      .                                            .05674
          6                                               .        XXX       .                                            .03460
          7                                               .   XXXXXX         .                                           -.09446
          8                                               .       XX         .                                           -.01333
          9                                               .        XXXXX     .                                            .08859
         10                                               .      XXX         .                                           -.03936
         11                                               .      XXX         .                                           -.03044
         12                                               .       XX         .                                           -.02012
         13                                               .        XXX       .                                            .04021
         14                                               .        X         .                                           -.00703
         15                                               .        XXX       .                                            .04705
         16                                               .XXXXXXXXX         .                                           -.16134
         17                                               .        XXX       .                                            .03311
         18                                               .        XX        .                                            .02296
         19                                               .    XXXXX         .                                           -.07067
         20                                               .   XXXXXX         .                                           -.10765
         21                                               .      XXX         .                                           -.04092
         22                                               .        XX        .                                            .01470
         23                                               .        XXXXXXXXX.                                             .15485
         24                                               .        X         .                                            .00255
```

PARTIAL AUTOCORRELATIONS

	.0201	.0297	-.1932	-.0386	.0734	.0021	.1219	.0136	.1189	-.0950	-.0589	.0504
SE	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891
	.0420	-.0724	.0487	-.1145	.0152	.0284	-.1210	-.1181	.0027	.0036	.0774	-.0273
SE	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891	.0891

```
            GRAPHIC INTERVAL IS      .0200

           -1.0000                                          .0000                                          1.0000   VALUES
             .+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.
       1                                             .          XX         .                                     .02012
       2                                             .          XX         .                                     .02969
       3                                             XXXXXXXXXXX           .                                    -.19315
       4                                             .        XXX          .                                    -.03861
       5                                             .          XXXXX      .                                     .07345
       6                                             .          X          .                                    -.00212
       7                                             .    XXXXXXX          .                                    -.12191
       8                                             .          XX         .                                     .01357
       9                                             .          XXXXXXX    .                                     .11885
      10                                             .     XXXXXX          .                                    -.09502
      11                                             .       XXXX          .                                    -.05891
      12                                             .          XXXX       .                                     .05035
      13                                             .          XXX        .                                     .04204
      14                                             .      XXXXX          .                                    -.07239
      15                                             .          XXX        .                                     .04873
      16                                             .    XXXXXXX          .                                    -.11447
      17                                             .          XX         .                                     .01523
      18                                             .          XX         .                                     .02844
      19                                             .    XXXXXXX          .                                    -.12097
      20                                             .    XXXXXXX          .                                    -.11813
      21                                             .          X          .                                     .00267
      22                                             .          X          .                                     .00357
      23                                             .          XXXXX      .                                     .07740
      24                                             .         XX          .                                    -.02729
```

FORECASTS:

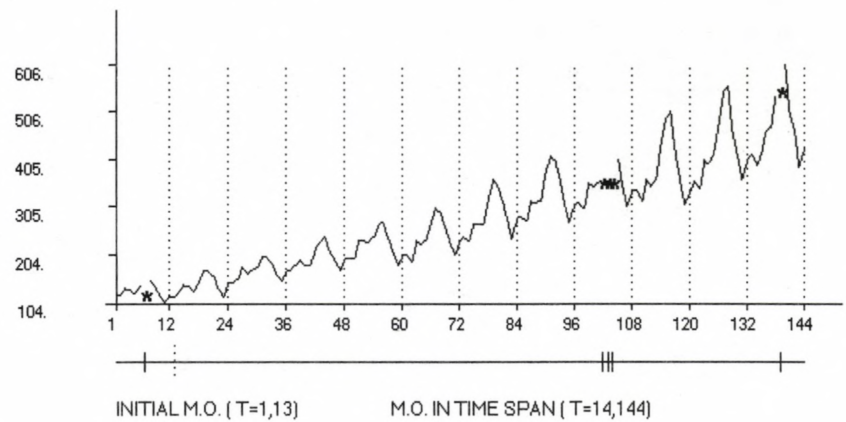
ORIGIN: 144 NUMBER: 12

OBS	FORECAST (TR. SERIES)	STD ERROR	ACTUAL	RESIDUAL	FORECAST (ORIGINAL SERIES)
145	6.1101	.0375			450.37
146	6.0540	.0436			425.80
147	6.1726	.0490			479.45
148	6.1991	.0538			492.33
149	6.2323	.0582			508.91
150	6.3671	.0624			582.36
151	6.4967	.0678			662.98
152	6.5028	.0699			666.99
153	6.3248	.0734			558.25
154	6.2088	.0767			497.10
155	6.0636	.0799			429.90
156	6.1682	.0829			477.34
157	6.2065				495.96
158	6.1504				468.90
159	6.2691				527.98
160	6.2956				542.16
161	6.3287				560.42
162	6.4635				641.31
163	6.5932				730.09
164	6.5992				734.51
165	6.4212				614.76
166	6.3052				547.41
167	6.1600				473.42
168	6.2647				525.66
169	6.3029				546.16

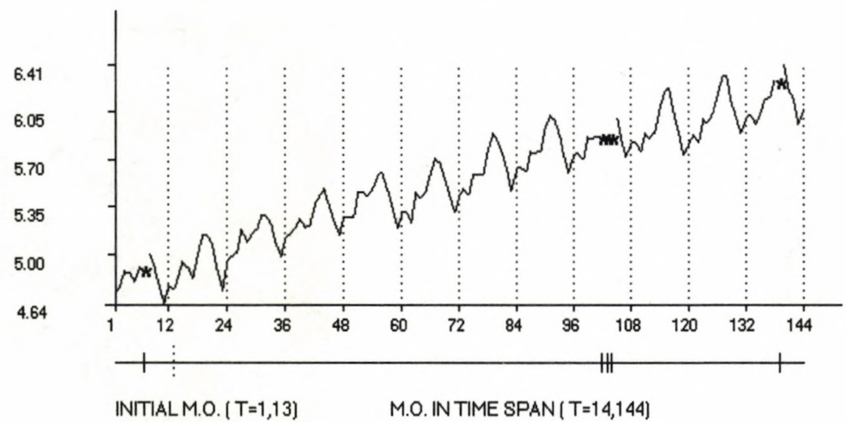
REGRESSION RESIDUALS

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1950		.03	.01	-.01	-.02	.05	.04	.03	.02	-.02	-.03	.06
1951	.07	-.01	.05	-.03	.11	-.07	-.04	-.01	.00	.03	.05	-.02
1952	.01	.00	-.07	-.03	.01	.08	-.03	.04	-.05	.03	.03	-.02
1953	-.02	-.06	.05	.08	.01	-.07	-.03	.00	-.03	-.01	-.05	-.05
1954	-.02	-.12	.03	.02	.05	.04	.07	-.03	-.01	-.01	.01	.00
1955	.04	.00	-.04	.03	.01	.05	.06	-.03	.01	-.01	-.02	.03
1956	.00	.00	-.02	-.01	.01	.04	-.01	.00	-.02	-.03	.00	-.02
1957	-.01	-.02	.01	.00	.01	.01	.02	.03	-.01	-.02	-.01	-.04
1958	-.03	-.05	-.05	-.04	.01	.04	.02	.05	-.07	.00	-.02	-.04
1959	.03	.01	.03	.01	.04	-.03	.02	.02	-.01	.00	.02	.02
1960	.00	-.02	-.09	.08	.02	-.01	-.01	-.02	.00	.03	-.03	-.01

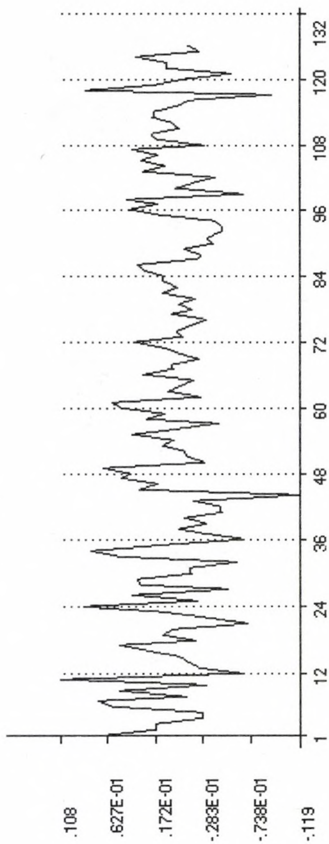
KADS3I: ORIGINAL SERIES



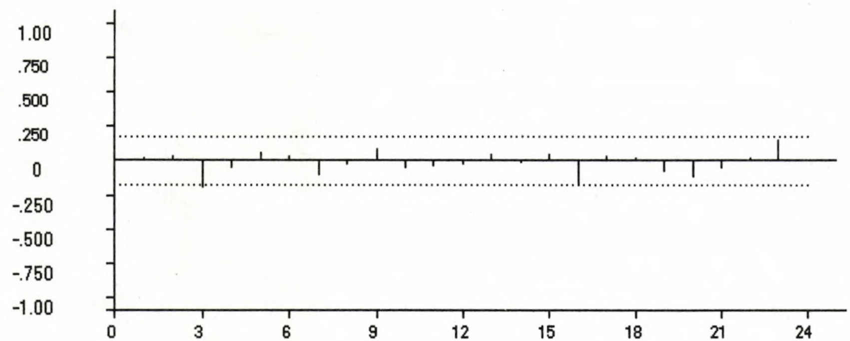
KADS3I: TRANSFORMED SERIES



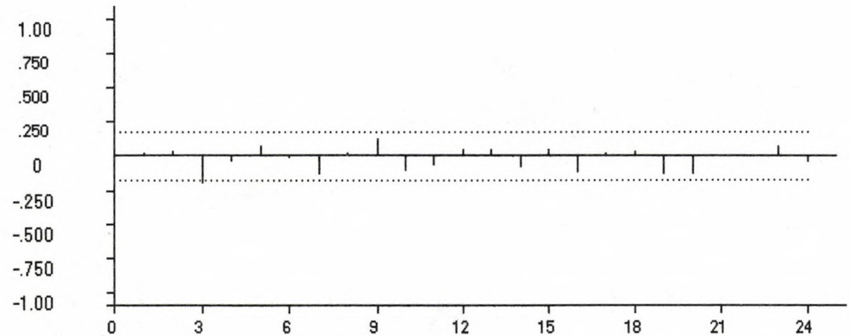
KADS3I: RESIDUALS



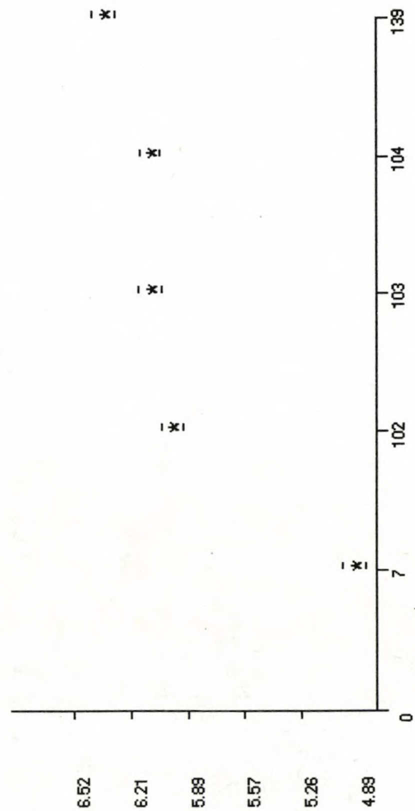
KADS3I: ACF OF RESIDUALS



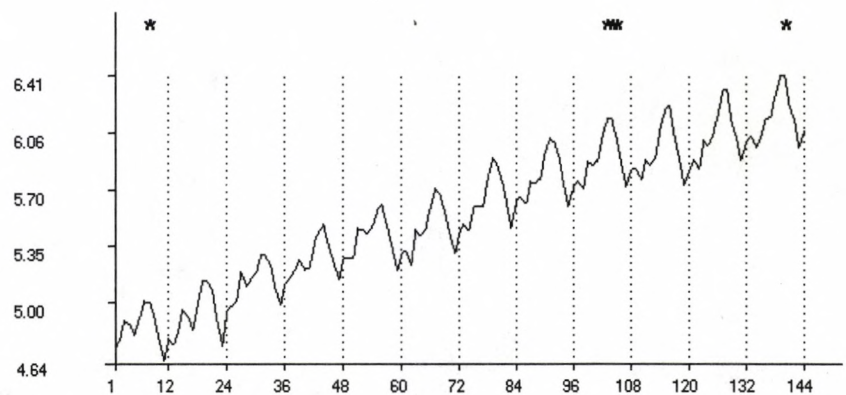
KADS3I: PARTIAL ACF OF RESIDUALS



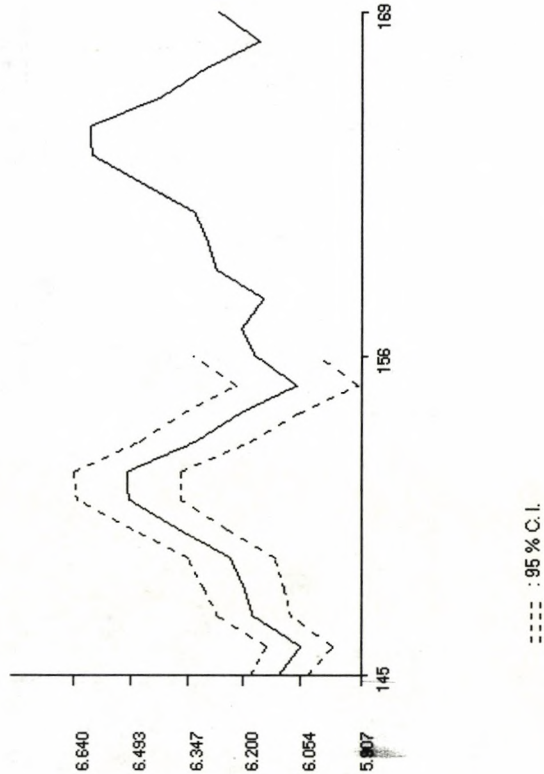
KADS3I: INTERPOLATED VALUES



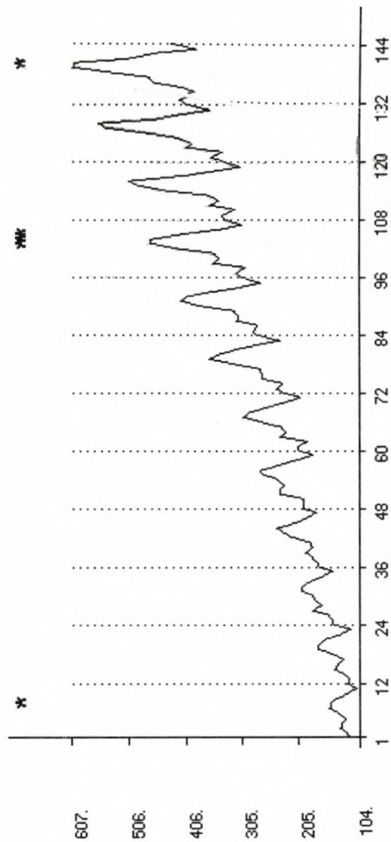
KADS3I: TRANS. SERIES WITH INTERPOLATIONS



KADS3I: FORECASTS



KADS3I: ORIGINAL SERIES WITH INTERPOLATIONS



Example 5

The example is taken from Hillmer, Bell and Tiao (1983), and uses the series of monthly retail sales of U.S. men's and boys' clothing stores from January 1967 through September 1979. The model is given by

$$y_t = \alpha H(\tau, t) + \sum_{i=1}^{7} \beta_i T_{it} + n_t,$$

$$\nabla\nabla_{12} n_t = \left(1 - \theta_1 B - \theta_2 B^2\right)\left(1 - \theta_{12} B^{12}\right) a_t,$$

where $H(\tau, t)$ denotes the Easter Effect and $\Sigma\beta_i T_{it}$ the overall Trading Day effect (see the paper by Hillmer, Bell, and Tiao), which are now the regressors. There are seven missing observations, and one of them falls among the initial values of the series.

```
EXAMPLE (HILLMER-BELL-TIAO, 83) M.O. WITH TRADING DAY AND EASTER EFFECT
153   1967 1 12
 237  -99999.  241    245  259  296  252  260  271  267  320  549
 266    216    252    297  302  310  270  288  280  316  372  594
 319    249    287    320  342  329  291  321  315  361  400  680
 338    268    304    313  348  350  321  317  333  364  396  719
 336    267    303    375  382  401  341  351  357  382  447  771
 364    310    379    408  439  451  390  413  424  469  534  884
 452    361    426    470  477  502  424  442  442  479  562  961
 437    368  -99999.  495  514  492  443  500  458  492  542  889
 459    403  -99999.  467  556  542  474  510  483  527  591 1044
 495    404  -99999.  540  518  552  505  502  496  558  629 1137
 511    440  -99999.  578  542  550  492  518  507  569  708 1141
 480    421  -99999.  536  542  563  508  554  552  609  763 1293
 561    462  -99999.  582  586  615  553  612  570
 &DATEN IDR=1,IDS=1,IQR=2,IQS=1,LAG=24,INCON=0,INVER=1,
  NBACK=14,NPRED=24,LAMDA=0,IFILT=3,IEAST=1,IDUR=9,ITRAD=1,
  INTERP=1,
  IGRBAR=0,/
```

TRAM

TIME SERIES REGRESSION MODELS WITH
ARIMA ERRORS AND MISSING VALUES.

BY VICTOR GOMEZ AND AGUSTIN MARAVALL.

PROGRAM DESIGNED AND WRITTEN BY VICTOR GOMEZ.

EXAMPLE (HILLMER-BELL-TIAO, 83) M.O. WITH TRADING DAY AND EASTER EFFECT
ORIGINAL SERIES

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1967	237.00	-99999.00	241.00	245.00	259.00	296.00	252.00	260.00	271.00	267.00	320.00	549.00
1968	266.00	216.00	252.00	297.00	302.00	310.00	270.00	288.00	280.00	316.00	372.00	594.00
1969	319.00	249.00	287.00	320.00	342.00	329.00	291.00	321.00	315.00	361.00	400.00	680.00
1970	338.00	268.00	304.00	313.00	348.00	350.00	321.00	317.00	333.00	364.00	396.00	719.00
1971	336.00	267.00	303.00	375.00	382.00	401.00	341.00	351.00	357.00	382.00	447.00	771.00
1972	364.00	310.00	379.00	408.00	439.00	451.00	390.00	413.00	424.00	469.00	534.00	884.00
1973	452.00	361.00	426.00	470.00	477.00	502.00	424.00	442.00	442.00	479.00	562.00	961.00
1974	437.00	368.00	-99999.00	495.00	514.00	492.00	443.00	500.00	458.00	492.00	542.00	889.00
1975	459.00	403.00	-99999.00	467.00	556.00	542.00	474.00	510.00	483.00	527.00	591.00	1044.00
1976	495.00	404.00	-99999.00	540.00	518.00	552.00	505.00	502.00	496.00	558.00	629.00	1137.00
1977	511.00	440.00	-99999.00	578.00	542.00	550.00	492.00	518.00	507.00	569.00	708.00	1141.00
1978	480.00	421.00	-99999.00	536.00	542.00	563.00	508.00	554.00	552.00	609.00	763.00	1293.00
1979	561.00	462.00	-99999.00	582.00	586.00	615.00	553.00	612.00	570.00			

INITIAL MISSING OBSERVATION NUMBER 2

MISSING OBSERVATION NUMBER 87

MISSING OBSERVATION NUMBER 99

MISSING OBSERVATION NUMBER 111

MISSING OBSERVATION NUMBER 123

MISSING OBSERVATION NUMBER 135

MISSING OBSERVATION NUMBER 147

DATES OF EASTER DURING THE REQUESTED TIME SPAN

YEAR	MONTH	DAY
1967	MARCH	26
1968	APRIL	14
1969	APRIL	6
1970	MARCH	29
1971	APRIL	11
1972	APRIL	2
1973	APRIL	22
1974	APRIL	14
1975	MARCH	30
1976	APRIL	18
1977	APRIL	10
1978	MARCH	26
1979	APRIL	15
1980	APRIL	6
1981	APRIL	19

MODEL PARAMETERS:

IMEAN = 0

LAMDA = 0

IDR = 1

IDS = 1

IPR = 0

IPS = 0

IQR = 2

IQS = 1

IREG = 8

ITRAD = 1

```
IEAST =          1
IDUR =           9
LAG =           24
INCON =          0
NBACK =         14
NPRED =         24
INTERP =         1
IESTIM =         1
VA =       1.000000000000000
IFILT =          3
IGRBAR =         0
IGRRES =         0
IDENSC =         1
INVER =          1
INIC =           0
TOL =    1.000000000000000E-006
ICONCE =         9
THR =  -1.000000000000000E-001  -1.000000000000000E-001
THS =  -1.000000000000000E-001
```

```
NUMBER OF INITIAL OBSERVATIONS =          13
NUMBER OF MISSING INITIAL OBSERVATIONS =           1

NUMBER OF MISSING VALUES IN TIME SPAN
         14 -          153
=          6
TRANSFORMED SERIES (LOGARITHMS OF THE DATA)
```

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1967	5.47	-99999.00	5.48	5.50	5.56	5.69	5.53	5.56	5.60	5.59	5.77	6.31
1968	5.58	5.38	5.53	5.69	5.71	5.74	5.60	5.66	5.63	5.76	5.92	6.39
1969	5.77	5.52	5.66	5.77	5.83	5.80	5.67	5.77	5.75	5.89	5.99	6.52
1970	5.82	5.59	5.72	5.75	5.85	5.86	5.77	5.76	5.81	5.90	5.98	6.58
1971	5.82	5.59	5.71	5.93	5.95	5.99	5.83	5.86	5.88	5.95	6.10	6.65
1972	5.90	5.74	5.94	6.01	6.08	6.11	5.97	6.02	6.05	6.15	6.28	6.78
1973	6.11	5.89	6.05	6.15	6.17	6.22	6.05	6.09	6.09	6.17	6.33	6.87
1974	6.08	5.91	-99999.00	6.20	6.24	6.20	6.09	6.21	6.13	6.20	6.30	6.79
1975	6.13	6.00	-99999.00	6.15	6.32	6.30	6.16	6.23	6.18	6.27	6.38	6.95
1976	6.20	6.00	-99999.00	6.29	6.25	6.31	6.22	6.22	6.21	6.32	6.44	7.04
1977	6.24	6.09	-99999.00	6.36	6.30	6.31	6.20	6.25	6.23	6.34	6.56	7.04
1978	6.17	6.04	-99999.00	6.28	6.30	6.33	6.23	6.32	6.31	6.41	6.64	7.16
1979	6.33	6.14	-99999.00	6.37	6.37	6.42	6.32	6.42	6.35			

```
ARIMA MODEL ESTIMATION BEGINS

INITIAL PARAMETER VALUES:
 -1.000000000000000E-001  -1.000000000000000E-001  -1.000000000000000E-001

 ITERATION, LAMBDA          1   0.000000000000000E+000
 FO FP    2.252110632453293E-001    1.718195890986127E-001
 FO-FP SUM S    5.339147414671658E-002    4.561428857578196E-002
        1.170498890013682
 ITERATION, LAMBDA          2   0.000000000000000E+000
 FO FP    1.718195890986127E-001    1.662986869825995E-001
 FO-FP SUM S    5.520902116013221E-003    4.789369825375178E-003
        1.152740823388125
 ITERATION, LAMBDA          3   0.000000000000000E+000
 FO FP    1.662986869825995E-001    1.661795954177431E-001
 FO-FP SUM S    1.190915648563440E-004    1.051201540436672E-004
        1.132908964411078
 ITERATION, LAMBDA          4   0.000000000000000E+000
 FO FP    1.661795954177431E-001    1.661740862181740E-001
 FO-FP SUM S    5.509199569125878E-006    4.027484551369964E-006
        1.367900856938583
```

```
ITERATION, LAMBDA          5   0.000000000000000E+000
FO FP   1.661740862181740E-001   1.661732458455445E-001
FO-FP SUM S   8.403726294570202E-007   6.007755752235643E-007
      1.398812908038572
ITERATION, LAMBDA          6   0.000000000000000E+000

METHOD OF ESTIMATION: EXACT MAXIMUM LIKELIHOOD

PARAMETER      ESTIMATE          STD ERROR      T RATIO   LAG

MA1  1       -.231177702          .086161280      -2.68      1
MA1  2       -.378357246          .084338790      -4.49      2
MA2  1       -.641684166          .086586230      -7.41     12

REGULAR MA INVERSE ROOTS ARE
NO.      REAL P.      IMAG.P.      MODULUS
 1     -.5102850     .0000000     .5102850
 2      .7414627     .0000000     .7414627
SEASONAL MA INVERSE ROOTS ARE
NO.      REAL P.      IMAG.P.      MODULUS
 1     -.6416842     .0000000     .6416842

CORRELATIONS OF THE ESTIMATES

 1.000  -.435  -.113
 -.435  1.000   .052
 -.113   .052  1.000

AIC
   -510.528

FINAL VALUE OF OBJECTIVE FUNCTION:
      .1661731109

VARIANCE ESTIMATE:
   .0012637

ITERATIONS:          6

 NUMBER OF FUNCTION EVALUATIONS:         25
```

ESTIMATES OF REGRESSION PARAMETERS
CONCENTRATED OUT OF THE LIKELIHOOD

ZJ	2	5.273050354	(.029836541)
TRAD	1	-.010901210	(.005125600)
TRAD	2	-.001032566	(.004726313)
TRAD	3	.006192338	(.005030442)
TRAD	4	-.003139052	(.004771628)
TRAD	5	.013193973	(.005096938)
TRAD	6	.010538226	(.004953727)
TRAD	7	.005569984	(.016909606)
EAST	1	.066316981	(.010965295)

COVARIANCE MATRIX OF ESTIMATORS

.890E-03	-.307E-05	.438E-05	-.138E-05	-.819E-05	.344E-05	.810E-05	-.124E-03	-.577E-04
-.307E-05	.263E-04	-.976E-05	-.971E-05	.585E-05	.621E-05	-.748E-05	.547E-05	-.163E-04
.438E-05	-.976E-05	.223E-04	-.891E-05	-.657E-05	.516E-05	.392E-05	-.793E-05	-.535E-06
-.138E-05	-.971E-05	-.891E-05	.253E-04	-.104E-04	-.902E-05	.693E-05	.831E-05	.144E-04
-.819E-05	.585E-05	-.657E-05	-.104E-04	.228E-04	-.938E-05	-.644E-05	-.562E-05	-.351E-05
.344E-05	.621E-05	.516E-05	-.902E-05	-.938E-05	.260E-04	-.120E-04	.123E-05	-.153E-04
.810E-05	-.748E-05	.392E-05	.693E-05	-.644E-05	-.120E-04	.245E-04	.362E-05	.137E-04
-.124E-03	.547E-05	-.793E-05	.831E-05	-.562E-05	.123E-05	.362E-05	.286E-03	.871E-05
-.577E-04	-.163E-04	-.535E-06	.144E-04	-.351E-05	-.153E-04	.137E-04	.871E-05	.120E-03

CHECK OF WHITE NOISE RESIDUALS:

AUTOCORRELATIONS

	-.0337	.0754	.0575	-.1057	-.0523	-.0521	-.0210	.0040	.2041	-.1137	.2562	-.0945
SE	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894
	-.0350	.0419	-.1610	.0190	-.1371	.0905	.0223	.0148	-.0504	.0706	-.2075	-.1241
SE	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894

LJUNG-BOX OF ORDER Q IS 40.03 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUARED(10)
PIERCE OF ORDER QS IS 3.67 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUATED(2)

PARTIAL AUTOCORRELATIONS

	-.0337	.0743	.0628	-.1085	-.0694	-.0444	-.0024	.0067	.2050	-.1191	.2273	-.1184
SE	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894
	-.0025	.0143	-.1006	.0219	-.1471	.0923	.0480	-.1064	.0088	-.0094	-.1979	-.0566
SE	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894	.0894

NUMBER OF WHITE NOISE RESIDUALS 125

WHITE NOISE RESIDUALS

.0066	.0034	.0310	.0084	-.0056	-.0247	.0140	-.0648
-.0302	.0356	-.0187	.0442	-.0190	.0263	-.0502	.0210
-.0697	-.0332	-.0059	-.0303	.0106	-.0402	.0121	-.0287
-.0025	.0229	-.0545	-.0026	.0067	.0185	.0478	.0096
-.0330	.0384	-.0431	-.0067	.0286	-.0290	.0152	.0325
.0112	.0728	.0125	-.0056	.0381	-.0094	.0121	.0791
-.0341	-.0106	.0600	-.0199	.0391	-.0471	-.0284	-.0169
.0063	-.0584	.0007	-.0323	-.0044	.0301	-.1031	.0366
-.0096	-.0052	-.0551	.0213	.0391	-.0319	-.0472	-.0777
-.0273	-.0057	.0754	-.0313	.0522	.0203	-.0207	.0204
-.0268	-.0214	.0058	.0318	-.0469	-.0041	-.0218	-.0507
.0127	.0062	-.0026	-.0305	.0374	.0212	.0241	-.0075
.0134	-.0165	-.0339	-.0591	-.0149	-.0056	-.0278	.0499
.0743	-.0660	-.0459	-.0269	.0175	-.0549	-.0063	.0260
.0121	.0235	.0528	.0565	.0379	-.0087	-.0283	-.0404
-.0206	.0010	.0423	.0028	-.0057			

FORECASTS:

ORIGIN: 139 NUMBER: 24

OBS	FORECAST (TR. SERIES)	STD ERROR	ACTUAL	RESIDUAL	FORECAST (ORIGINAL SERIES)
140	6.2991	.0363	6.3172	.0181	544.08
141	6.2819	.0455	6.3135	.0317	534.78
142	6.3364	.0473	6.4118	.0755	564.74
143	6.5197	.0494	6.6373	.1176	678.37
144	7.0472	.0512	7.1647	.1175	1149.68
145	6.2569	.0530	6.3297	.0728	521.62
146	6.0982	.0547	6.1356	.0374	445.06
147	6.2570	.0610	********	********	521.68
148	6.3529	.0585	6.3665	.0136	574.15
149	6.3662	.0600	6.3733	.0071	581.85
150	6.4023	.0616	6.4216	.0193	603.25
151	6.2458	.0630	6.3154	.0696	515.84
152	6.3439	.0685	6.4167	.0728	569.04
153	6.2902	.0726	6.3456	.0554	539.26
154	6.3938	.0748			598.12
155	6.5631	.0774			708.46
156	7.0595	.0793			1163.90
157	6.3011	.0817			545.17
158	6.1534	.0852			470.30
159	6.2871	.0889			537.60
160	6.3907	.0880			596.29
161	6.4212	.0900			614.72
162	6.3892	.0918			595.41
163	6.3110	.0939			550.60
164	6.3730				585.80
165	6.3190				555.01
166	6.4522				634.08
167	6.5851				724.24
168	7.1054				1218.53
169	6.3561				575.98
170	6.1710				478.67
171	6.2825				535.10
172	6.4545				635.55
173	6.4459				630.09
174	6.4395				626.07
175	6.3616				579.19
176	6.3853				593.05
177	6.3725				585.50

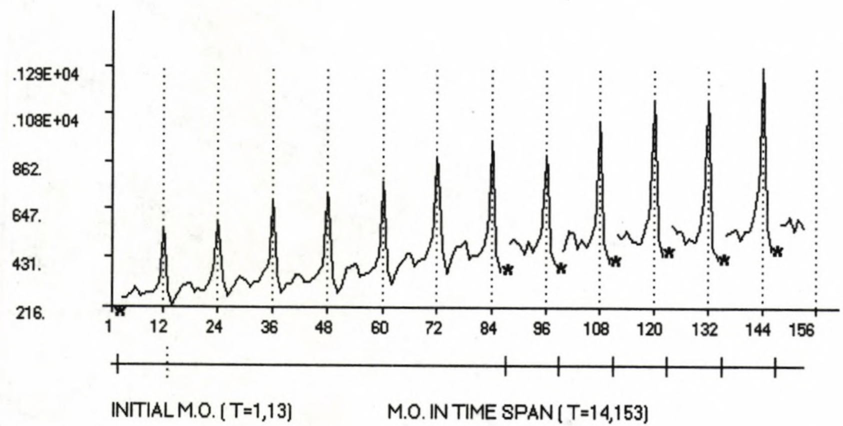
REGRESSION RESIDUALS

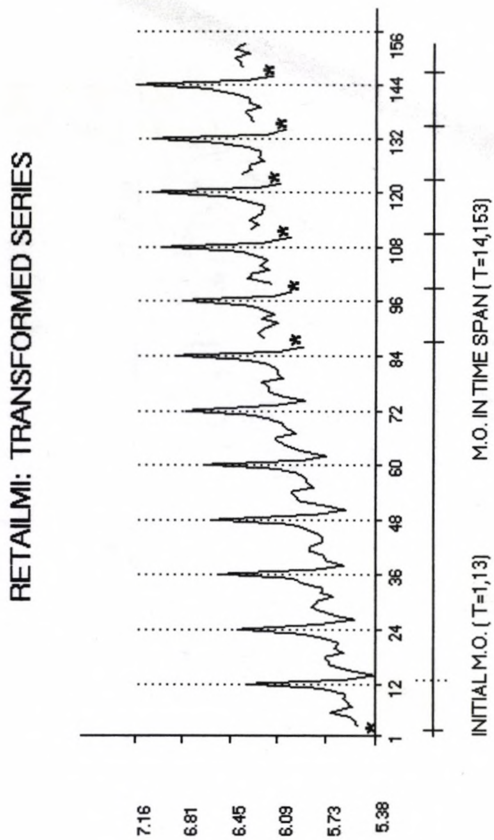
YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1968		.00	.01	.02	.01	-.05	-.01	.00	-.01	.05	.00	.00
1969	.04	.00	.00	-.03	.02	-.07	-.02	.03	-.02	.05	-.02	.02
1970	-.04	.01	-.06	-.04	.00	-.04	.02	-.04	.01	-.02	-.01	.03
1971	-.06	.00	-.01	.04	.04	.01	-.03	.03	-.03	-.01	.03	-.03
1972	.01	.03	.02	.06	.01	.00	.03	-.01	.01	.08	-.03	-.01
1973	.05	-.02	.04	-.04	-.03	-.02	.00	-.06	.00	-.04	.00	.03
1974	-.10	.03	-99999.00	-.01	.00	-.05	.01	.05	-.04	-.04	-.08	-.03
1975	.00	.07	-99999.00	-.04	.07	.02	-.01	.02	-.03	-.02	.01	.03
1976	-.04	.00	-99999.00	-.01	-.06	.01	.01	-.01	-.02	.03	.02	.03
1977	-.01	.01	-99999.00	-.01	-.04	-.05	-.02	-.01	-.02	.05	.07	-.06
1978	-.05	-.02	-99999.00	.01	-.06	.00	.02	.02	.02	.05	.06	.03
1979	-.01	-.03	-99999.00	-.04	-.02	.00	.04	.01	-.01			

INTERPOLATED VALUES

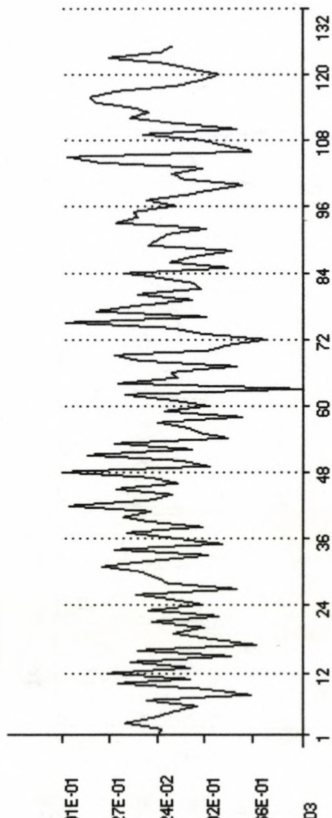
OBS	INTERPOLATED VALUE (TRANSFORMED SERIES)	STD ERROR	INTERPOLATED VALUE (ORIGINAL SERIES)
87	6.0663	.0287	431.0638
99	6.1420	.0316	465.0004
111	6.1432	.0323	465.5513
123	6.2066	.0334	496.0100
135	6.2915	.0355	539.9763
147	6.2867	.0359	537.3748

RETAILMI: ORIGINAL SERIES

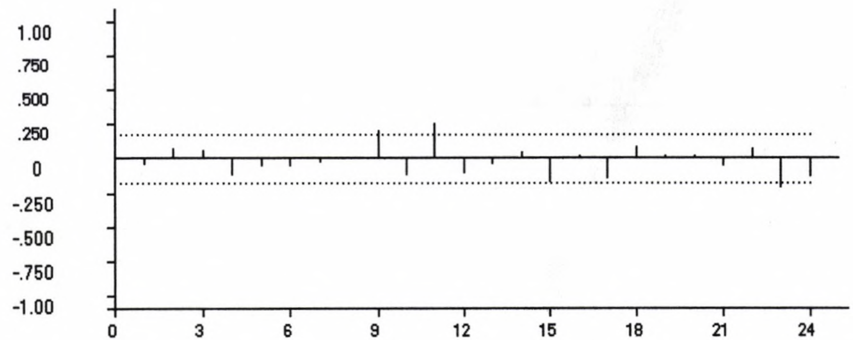



RETAILMI: TRANSFORMED SERIES
7.16
6.81
6.45
6.09
5.73
5.38
1
12
24
36
48
60
72
84
96
108
120
132
144
156
INITIAL M.O. (T=1,13)
M.O. IN TIME SPAN (T=14,153)

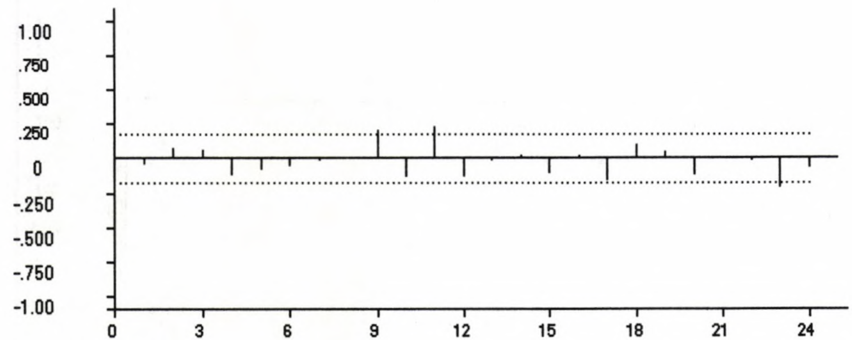
RETAILMI: RESIDUALS



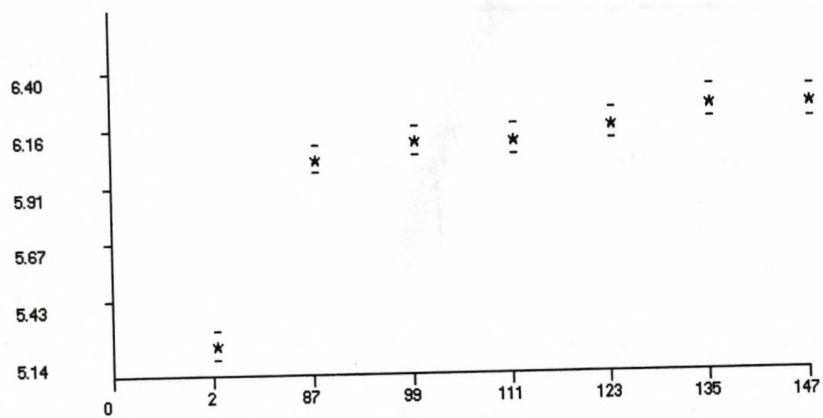
RETAILMI: ACF OF RESIDUALS



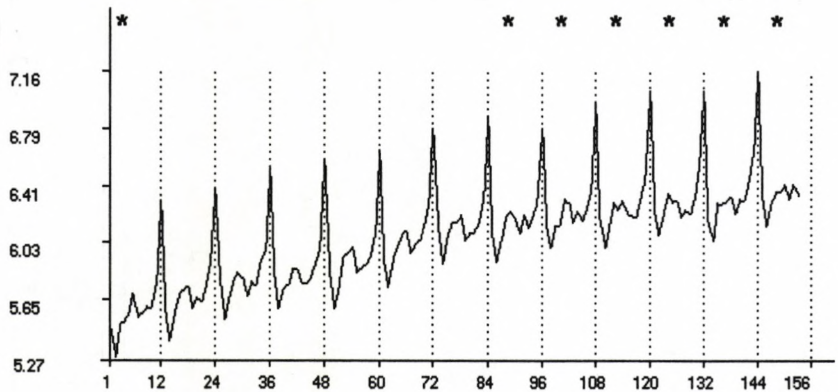
RETAILMI: PARTIAL ACF OF RESIDUALS



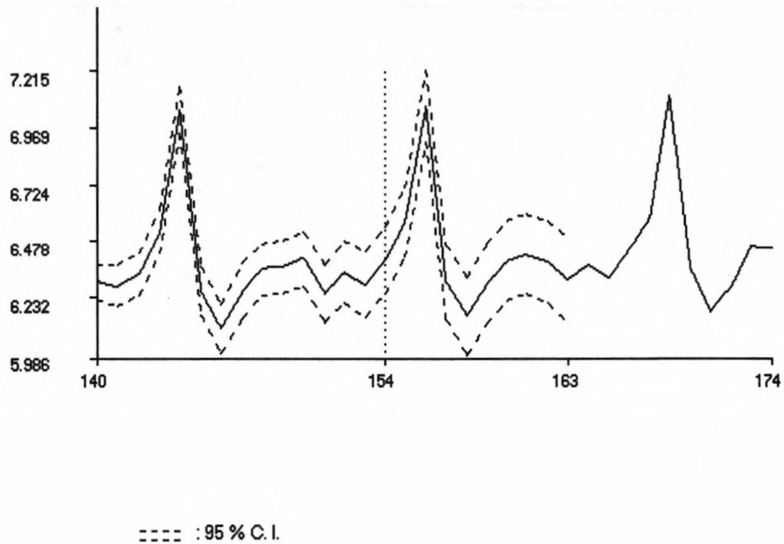
RETAILMI: INTERPOLATED VALUES



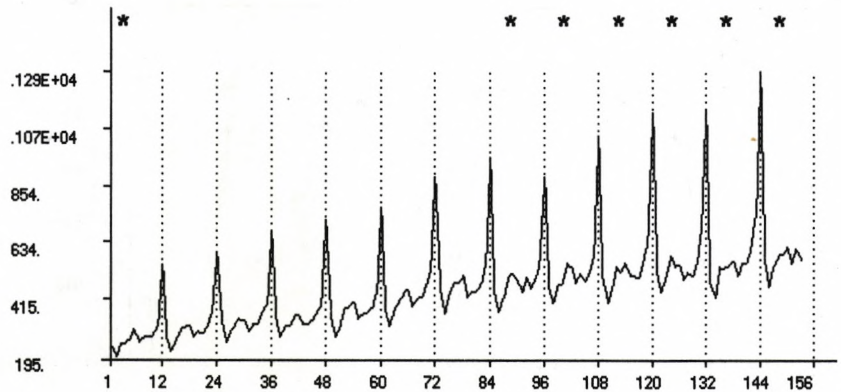
RETAILMI: TRANS. SERIES WITH INTERPOLATIONS



RETAILMI: FORECASTS



RETAILMI: ORIGINAL SERIES WITH INTERPOLATIONS



Example 6

The last example uses the same series and model of Example 4, and illustrates a case not considered by Kohn and Ansley (1986): when the number of nonestimable missing observations is larger than the number of free parameters. A total of 14 observations are removed from the airline passenger series; 2 are among the initial values and turn out to be nonestimable.

The first printout (with -99999. indicating the location of the missing observations) shows that z(13) is a free parameter, and that the vector (–1, 1) provides a linear combination of the two initial missing values [z(1) and z(13)] which is estimable. The missing observations estimators and the forecasts that cannot be estimated (independently of the free parameter) are listed.

By entering the value $z(1)=1$ [i.e., $\log z(1)=0$], the program is rerun. The ARIMA model is estimated and interpolators and forecasts are obtained. The interpolator of the free parameter, z(13), becomes the estimator of the annual rate of growth, and, in the output, the interpolators and forecasts that depend on the free parameter are easily identified.

```
DATA SET 5 IN PAPER, 2 NONESTIMABLE M.O., 1 FREE PARAMETER
144   1949 1 12
-99999.   118   132 129 121 135 148 148 136 119 104 118
-99999.   126   141 135 125 149 170 170 158 133 114 140
-99999. -99999. 178 163 172 178 199 199 184 162 146 166
-99999.   180   193 181 183 218 230 242 209 191 172 194
-99999.   196   236 235 229 243 264 272 237 211 180 201
-99999. -99999. 235 227 234 264 302 293 259 229 203 229
-99999.   233   267 269 270 315 364 347 312 274 237 278
-99999.   277   317 313 318 374 413 405 355 306 271 306
-99999.   301   356 348 355 422 465 467 404 347 305 336
-99999.   318   362 348 363 435 491 505 404 359 310 337
-99999.   342   406 396 420 472 548 559 463 407 362 405
-99999.   391   419 461 472 535 622 606 508 461 390 432
 &DATEN IDR=1,IDS=1,IQR=1,IQS=1,LAG=24,IDENSC=1,
  NPRED=12,LAMDA=0,IFILT=3,INTERP=1,ICONCE=1,/
```

TRAM

TIME SERIES REGRESSION MODELS WITH
ARIMA ERRORS AND MISSING VALUES.

BY VICTOR GOMEZ AND AGUSTIN MARAVALL.

PROGRAM DESIGNED AND WRITTEN BY VICTOR GOMEZ.

DATA SET 5 IN PAPER, 2 NONESTIMABLE M.O., 1 FREE PARAMETER
ORIGINAL SERIES

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1949	-99999.00	118.00	132.00	129.00	121.00	135.00	148.00	148.00	136.00	119.00	104.00	118.00
1950	-99999.00	126.00	141.00	135.00	125.00	149.00	170.00	170.00	158.00	133.00	114.00	140.00
1951	-99999.00	-99999.00	178.00	163.00	172.00	178.00	199.00	199.00	184.00	162.00	146.00	166.00
1952	-99999.00	180.00	193.00	181.00	183.00	218.00	230.00	242.00	209.00	191.00	172.00	194.00
1953	-99999.00	196.00	236.00	235.00	229.00	243.00	264.00	272.00	237.00	211.00	180.00	201.00
1954	-99999.00	-99999.00	235.00	227.00	234.00	264.00	302.00	293.00	259.00	229.00	203.00	229.00
1955	-99999.00	233.00	267.00	269.00	270.00	315.00	364.00	347.00	312.00	274.00	237.00	278.00
1956	-99999.00	277.00	317.00	313.00	318.00	374.00	413.00	405.00	355.00	306.00	271.00	306.00
1957	-99999.00	301.00	356.00	348.00	355.00	422.00	465.00	467.00	404.00	347.00	305.00	336.00
1958	-99999.00	318.00	362.00	348.00	363.00	435.00	491.00	505.00	404.00	359.00	310.00	337.00
1959	-99999.00	342.00	406.00	396.00	420.00	472.00	548.00	559.00	463.00	407.00	362.00	405.00
1960	-99999.00	391.00	419.00	461.00	472.00	535.00	622.00	606.00	508.00	461.00	390.00	432.00

INITIAL MISSING OBSERVATION NUMBER 1

INITIAL MISSING OBSERVATION NUMBER 13

MISSING OBSERVATION NUMBER 25

MISSING OBSERVATION NUMBER 26

MISSING OBSERVATION NUMBER 37

MISSING OBSERVATION NUMBER 49

MISSING OBSERVATION NUMBER 61

MISSING OBSERVATION NUMBER 62

MISSING OBSERVATION NUMBER 73

MISSING OBSERVATION NUMBER 85

```
MISSING OBSERVATION NUMBER        97
MISSING OBSERVATION NUMBER       109
MISSING OBSERVATION NUMBER       121
MISSING OBSERVATION NUMBER       133
MODEL PARAMETERS:
IMEAN =          0
LAMDA =          0
IDR =          1
IDS =          1
IPR =          0
IPS =          0
IQR =          1
IQS =          1
IREG =          0
ITRAD =          0
IEAST =          0
IDUR =          0
LAG =         24
INCON =          0
NBACK =          0
NPRED =         12
INTERP =          1
IESTIM =          1
```

```
VA =        1.000000000000000

IFILT =          3

IGRBAR =           0

IGRRES =           0

IDENSC =           1

INVER =          0

INIC =          0

TOL =   1.000000000000000E-006

ICONCE =           2

THR =  -1.000000000000000E-001

THS =  -1.000000000000000E-001

NUMBER OF INITIAL OBSERVATIONS =         13
NUMBER OF MISSING INITIAL OBSERVATIONS =          2

NUMBER OF MISSING VALUES IN TIME SPAN
        14 -        144
=          12
TRANSFORMED SERIES (LOGARITHMS OF THE DATA)
```

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1949	-99999.00	4.77	4.88	4.86	4.80	4.91	5.00	5.00	4.91	4.78	4.64	4.77
1950	-99999.00	4.84	4.95	4.91	4.83	5.00	5.14	5.14	5.06	4.89	4.74	4.94
1951	-99999.00	-99999.00	5.18	5.09	5.15	5.18	5.29	5.29	5.21	5.09	4.98	5.11
1952	-99999.00	5.19	5.26	5.20	5.21	5.38	5.44	5.49	5.34	5.25	5.15	5.27
1953	-99999.00	5.28	5.46	5.46	5.43	5.49	5.58	5.61	5.47	5.35	5.19	5.30
1954	-99999.00	-99999.00	5.46	5.42	5.46	5.58	5.71	5.68	5.56	5.43	5.31	5.43
1955	-99999.00	5.45	5.59	5.59	5.60	5.75	5.90	5.85	5.74	5.61	5.47	5.63
1956	-99999.00	5.62	5.76	5.75	5.76	5.92	6.02	6.00	5.87	5.72	5.60	5.72
1957	-99999.00	5.71	5.87	5.85	5.87	6.05	6.14	6.15	6.00	5.85	5.72	5.82
1958	-99999.00	5.76	5.89	5.85	5.89	6.08	6.20	6.22	6.00	5.88	5.74	5.82
1959	-99999.00	5.83	6.01	5.98	6.04	6.16	6.31	6.33	6.14	6.01	5.89	6.00
1960	-99999.00	5.97	6.04	6.13	6.16	6.28	6.43	6.41	6.23	6.13	5.97	6.07

```
ARIMA MODEL ESTIMATION BEGINS

INITIAL PARAMETER VALUES:
 -1.000000000000000E-001  -1.000000000000000E-001

REGRESSION VARIABLE NUMBER          2 IS A FREE PARAMETER
MATRIX IN ECHELON FORM IS
 -9.969524520828219E-001   9.969524520828212E-001
 OBSERVATION NUMBER          25
 CANNOT BE INTERPOLATED
 WITHOUT DEPENDENCE ON
 THE FREE PARAMETER(S)
 OBSERVATION NUMBER          37
 CANNOT BE INTERPOLATED
 WITHOUT DEPENDENCE ON
 THE FREE PARAMETER(S)
 OBSERVATION NUMBER          49
 CANNOT BE INTERPOLATED
 WITHOUT DEPENDENCE ON
 THE FREE PARAMETER(S)
 OBSERVATION NUMBER          61
 CANNOT BE INTERPOLATED
 WITHOUT DEPENDENCE ON
 THE FREE PARAMETER(S)
 OBSERVATION NUMBER          73
 CANNOT BE INTERPOLATED
 WITHOUT DEPENDENCE ON
 THE FREE PARAMETER(S)
 OBSERVATION NUMBER          85
 CANNOT BE INTERPOLATED
 WITHOUT DEPENDENCE ON
 THE FREE PARAMETER(S)
 OBSERVATION NUMBER          97
 CANNOT BE INTERPOLATED
 WITHOUT DEPENDENCE ON
 THE FREE PARAMETER(S)
 OBSERVATION NUMBER         109
 CANNOT BE INTERPOLATED
 WITHOUT DEPENDENCE ON
 THE FREE PARAMETER(S)
 OBSERVATION NUMBER         121
 CANNOT BE INTERPOLATED
 WITHOUT DEPENDENCE ON
 THE FREE PARAMETER(S)
 OBSERVATION NUMBER         133
 CANNOT BE INTERPOLATED
 WITHOUT DEPENDENCE ON
 THE FREE PARAMETER(S)
 OBSERVATION NUMBER         145
 CANNOT BE PREDICTED
 WITHOUT DEPENDENCE ON
 THE FREE PARAMETER(S)
```

```
DATA SET 5 IN PAPER, 2 NONESTIMABLE M.O., 1 FREE PARAMETER
144   1949 1 12
     1.    118   132 129 121 135 148 148 136 119 104 118
-99999.    126   141 135 125 149 170 170 158 133 114 140
-99999. -99999. 178 163 172 178 199 199 184 162 146 166
-99999.    180   193 181 183 218 230 242 209 191 172 194
-99999.    196   236 235 229 243 264 272 237 211 180 201
-99999. -99999. 235 227 234 264 302 293 259 229 203 229
-99999.    233   267 269 270 315 364 347 312 274 237 278
-99999.    277   317 313 318 374 413 405 355 306 271 306
-99999.    301   356 348 355 422 465 467 404 347 305 336
-99999.    318   362 348 363 435 491 505 404 359 310 337
-99999.    342   406 396 420 472 548 559 463 407 362 405
-99999.    391   419 461 472 535 622 606 508 461 390 432
 &DATEN IDR=1,IDS=1,IQR=1,IQS=1,LAG=24,IDENSC=1,
  NPRED=12,LAMDA=0,IFILT=3,INTERP=1,ICONCE=1,/
```

TRAM

TIME SERIES REGRESSION MODELS WITH
ARIMA ERRORS AND MISSING VALUES.

BY VICTOR GOMEZ AND AGUSTIN MARAVALL.

PROGRAM DESIGNED AND WRITTEN BY VICTOR GOMEZ.

DATA SET 5 IN PAPER, 2 NONESTIMABLE M.O., 1 FREE PARAMETER
ORIGINAL SERIES

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1949	1.00	118.00	132.00	129.00	121.00	135.00	148.00	148.00	136.00	119.00	104.00	118.00
1950	-99999.00	126.00	141.00	135.00	125.00	149.00	170.00	170.00	158.00	133.00	114.00	140.00
1951	-99999.00	-99999.00	178.00	163.00	172.00	178.00	199.00	199.00	184.00	162.00	146.00	166.00
1952	-99999.00	180.00	193.00	181.00	183.00	218.00	230.00	242.00	209.00	191.00	172.00	194.00
1953	-99999.00	196.00	236.00	235.00	229.00	243.00	264.00	272.00	237.00	211.00	180.00	201.00
1954	-99999.00	-99999.00	235.00	227.00	234.00	264.00	302.00	293.00	259.00	229.00	203.00	229.00
1955	-99999.00	233.00	267.00	269.00	270.00	315.00	364.00	347.00	312.00	274.00	237.00	278.00
1956	-99999.00	277.00	317.00	313.00	318.00	374.00	413.00	405.00	355.00	306.00	271.00	306.00
1957	-99999.00	301.00	356.00	348.00	355.00	422.00	465.00	467.00	404.00	347.00	305.00	336.00
1958	-99999.00	318.00	362.00	348.00	363.00	435.00	491.00	505.00	404.00	359.00	310.00	337.00
1959	-99999.00	342.00	406.00	396.00	420.00	472.00	548.00	559.00	463.00	407.00	362.00	405.00
1960	-99999.00	391.00	419.00	461.00	472.00	535.00	622.00	606.00	508.00	461.00	390.00	432.00

INITIAL MISSING OBSERVATION NUMBER 13

MISSING OBSERVATION NUMBER 25

MISSING OBSERVATION NUMBER 26

MISSING OBSERVATION NUMBER 37

MISSING OBSERVATION NUMBER 49

MISSING OBSERVATION NUMBER 61

MISSING OBSERVATION NUMBER 62

MISSING OBSERVATION NUMBER 73

MISSING OBSERVATION NUMBER 85

```
MISSING OBSERVATION NUMBER        97
MISSING OBSERVATION NUMBER       109
MISSING OBSERVATION NUMBER       121
MISSING OBSERVATION NUMBER       133
MODEL PARAMETERS:
IMEAN =          0
LAMDA =          0
IDR =          1
IDS =          1
IPR =          0
IPS =          0
IQR =          1
IQS =          1
IREG =          0
ITRAD =          0
IEAST =          0
IDUR =          0
LAG =         24
INCON =          0
NBACK =          0
NPRED =         12
INTERP =          1
IESTIM =          1
VA =      1.000000000000000
```

```
IFILT =         3

IGRBAR =          0

IGRRES =          0

IDENSC =          1

INVER =          0

INIC =          0

TOL =   1.000000000000000E-006

ICONCE =          1

THR =  -1.000000000000000E-001

THS =  -1.000000000000000E-001

NUMBER OF INITIAL OBSERVATIONS =         13
NUMBER OF MISSING INITIAL OBSERVATIONS =          1

NUMBER OF MISSING VALUES IN TIME SPAN
         14 -          144
=          12
TRANSFORMED SERIES (LOGARITHMS OF THE DATA)
```

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1949	.00	4.77	4.88	4.86	4.80	4.91	5.00	5.00	4.91	4.78	4.64	4.77
1950	-99999.00	4.84	4.95	4.91	4.83	5.00	5.14	5.14	5.06	4.89	4.74	4.94
1951	-99999.00	-99999.00	5.18	5.09	5.15	5.18	5.29	5.29	5.21	5.09	4.98	5.11
1952	-99999.00	5.19	5.26	5.20	5.21	5.38	5.44	5.49	5.34	5.25	5.15	5.27
1953	-99999.00	5.28	5.46	5.46	5.43	5.49	5.58	5.61	5.47	5.35	5.19	5.30
1954	-99999.00	-99999.00	5.46	5.42	5.46	5.58	5.71	5.68	5.56	5.43	5.31	5.43
1955	-99999.00	5.45	5.59	5.59	5.60	5.75	5.90	5.85	5.74	5.61	5.47	5.63
1956	-99999.00	5.62	5.76	5.75	5.76	5.92	6.02	6.00	5.87	5.72	5.60	5.72
1957	-99999.00	5.71	5.87	5.85	5.87	6.05	6.14	6.15	6.00	5.85	5.72	5.82
1958	-99999.00	5.76	5.89	5.85	5.89	6.08	6.20	6.22	6.00	5.88	5.74	5.82
1959	-99999.00	5.83	6.01	5.98	6.04	6.16	6.31	6.33	6.14	6.01	5.89	6.00
1960	-99999.00	5.97	6.04	6.13	6.16	6.28	6.43	6.41	6.23	6.13	5.97	6.07

```
ARIMA MODEL ESTIMATION BEGINS

INITIAL PARAMETER VALUES:
 -1.000000000000000E-001  -1.000000000000000E-001

 ITERATION, LAMBDA          1   0.000000000000000E+000
 FO FP   2.225502406909038E-001   1.682667726686799E-001
 FO-FP SUM S   5.428346802222392E-002   4.714957775229253E-002
       1.151303375555352
 ITERATION, LAMBDA          2   0.000000000000000E+000
 FO FP   1.682667726686799E-001   1.671762393314861E-001
 FO-FP SUM S   1.090533337193778E-003   1.037983713241614E-003
       1.050626636315951
 ITERATION, LAMBDA          3   0.000000000000000E+000
 FO FP   1.671762393314861E-001   1.670899704747714E-001
 FO-FP SUM S   8.626885671478290E-005   1.053787920093331E-004
  8.186548267429592E-001
 ITERATION, LAMBDA          4   0.000000000000000E+000
 FO FP   1.670899704747714E-001   1.670791999902531E-001
 FO-FP SUM S   1.077048451827989E-005   1.653256132333845E-005
  6.514710157509328E-001
 ITERATION, LAMBDA          5   0.000000000000000E+000
 FO FP   1.670791999902531E-001   1.670772284503561E-001
 FO-FP SUM S   1.971539896949981E-006   3.445689905922537E-006
  5.721756602534804E-001
 ITERATION, LAMBDA          6   0.000000000000000E+000
 FO FP   1.670772284503561E-001   1.670768283690235E-001
 FO-FP SUM S   4.000813325910624E-007   7.464279739429981E-007
  5.359945588288135E-001
 ITERATION, LAMBDA          7   0.000000000000000E+000

METHOD OF ESTIMATION: EXACT MAXIMUM LIKELIHOOD

PARAMETER      ESTIMATE         STD ERROR      T RATIO   LAG

MA1  1       -.400707079        .081990744      -4.89      1
MA2  1       -.563138634        .088727249      -6.35     12

REGULAR MA INVERSE ROOTS ARE
NO.      REAL P.     IMAG.P.     MODULUS
 1    -.4007071    .0000000    .4007071
SEASONAL MA INVERSE ROOTS ARE
NO.      REAL P.     IMAG.P.     MODULUS
 1    -.5631386    .0000000    .5631386
```

```
CORRELATIONS OF THE ESTIMATES

 1.000  -.147
 -.147  1.000

AIC
   -439.935

FINAL VALUE OF OBJECTIVE FUNCTION:
      .1670767383

VARIANCE ESTIMATE:
   .0013341

ITERATIONS:          7

 NUMBER OF FUNCTION EVALUATIONS:         22

ESTIMATES OF REGRESSION PARAMETERS
CONCENTRATED OUT OF THE LIKELIHOOD
ZJ   13        .068439292    (     .040437885)

 COVARIANCE MATRIX OF ESTIMATORS

 .164E-02

 CHECK OF WHITE NOISE RESIDUALS:


AUTOCORRELATIONS
----------------
      .0391  -.0639  -.0892  -.1766  -.0120   .0939  -.1344   .1484  -.0670  -.0159  -.0045   .0384
SE    .0921   .0921   .0921   .0921   .0921   .0921   .0921   .0921   .0921   .0921   .0921   .0921
      .0732   .0315  -.1333   .0230  -.1953  -.0699  -.1078   .0450   .1897   .1098   .0322   .0530
SE    .0921   .0921   .0921   .0921   .0921   .0921   .0921   .0921   .0921   .0921   .0921   .0921

 LJUNG-BOX OF ORDER Q IS      31.60 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS CHI-SQUARED(11)
PIERCE OF ORDER QS IS          .62 AND IF RESIDUALS ARE RANDOM IT SHOULD BE DISTRIBUTED AS  CHI-SQUATED( 2)
```

PARTIAL AUTOCORRELATIONS

	.0391	-.0655	-.0845	-.1763	-.0124	.0682	-.1783	.1444	-.0974	.0153	-.0484	.0768
SE	.0921	.0921	.0921	.0921	.0921	.0921	.0921	.0921	.0921	.0921	.0921	.0921
	.0724	-.0316	-.0664	.0227	-.1785	-.0910	-.1765	.0268	.1025	.0178	.1026	.0395
SE	.0921	.0921	.0921	.0921	.0921	.0921	.0921	.0921	.0921	.0921	.0921	.0921

NUMBER OF WHITE NOISE RESIDUALS 118

WHITE NOISE RESIDUALS

-.0001	-.0179	-.0184	.0503	.0549	.0220	.0187	-.0262
-.0274	.0580	.0620	-.0322	.1076	-.0660	-.0296	-.0119
-.0050	.0253	.0505	-.0214	-.0062	-.0586	-.0259	.0122
.0855	-.0247	.0404	-.0515	.0309	.0333	-.0170	-.0668
.0588	.0800	.0076	-.0674	-.0306	-.0053	-.0308	-.0098
-.0463	-.0453	-.0383	-.0095	.0387	.0371	.0644	-.0303
-.0139	-.0110	.0102	-.0008	-.0225	-.0034	.0415	.0138
.0512	.0585	-.0257	.0058	-.0075	-.0193	.0285	-.0188
-.0070	.0014	.0112	.0382	-.0087	-.0029	-.0176	-.0312
.0019	-.0169	-.0327	.0205	.0003	.0102	.0338	-.0022
.0233	-.0130	-.0224	-.0084	-.0380	-.0610	-.0405	-.0368
.0132	.0301	.0273	.0484	-.0714	-.0043	-.0191	-.0404
.0189	.0363	.0168	.0391	-.0346	.0229	.0220	-.0076
-.0002	.0197	.0182	-.0181	-.0895	.0865	.0175	-.0127
.0170	-.0319	-.0100	.0294	-.0272	-.0151		

FORECASTS:

ORIGIN: 144 NUMBER: 12

OBS	FORECAST (TR. SERIES)	STD ERROR	ACTUAL	RESIDUAL	FORECAST (ORIGINAL SERIES)
145	1.3438	.0632			3.83
146	6.0552	.0426			426.34
147	6.1724	.0479			479.33
148	6.1992	.0526			492.37
149	6.2324	.0570			508.98
150	6.3688	.0611			583.36
151	6.5070	.0649			669.84
152	6.5028	.0685			666.99
153	6.3248	.0719			558.26
154	6.2089	.0751			497.15
155	6.0636	.0783			429.91
156	6.1682	.0813			477.33
157	1.4401				4.22
158	6.1516				469.48
159	6.2688				527.83
160	6.2956				542.19
161	6.3288				560.48
162	6.4652				642.39
163	6.6034				737.62
164	6.5992				734.48
165	6.4212				614.74
166	6.3053				547.46
167	6.1600				473.41
168	6.2646				525.63
169	1.5365				4.65

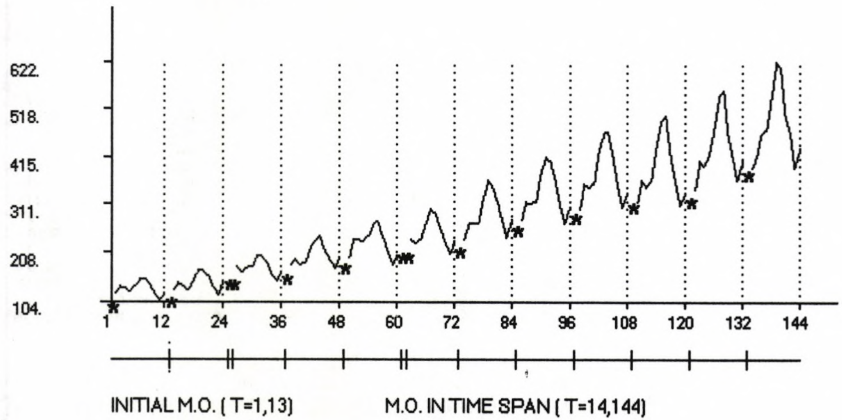
REGRESSION RESIDUALS

YEAR	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1950		.00	.00	-.02	-.02	.05	.05	.02	.02	-.03	-.03	.06
1951	-99999.00	-99999.00	.06	-.03	.11	-.07	-.03	-.01	-.01	.03	.05	-.02
1952	-99999.00	-.01	-.06	-.03	.01	.09	-.02	.04	-.05	.03	.03	-.02
1953	-99999.00	-.07	.06	.08	.01	-.07	-.03	-.01	-.03	-.01	-.05	-.05
1954	-99999.00	-99999.00	-.04	-.01	.04	.04	.06	-.03	-.01	-.01	.01	.00
1955	-99999.00	-.02	.00	.04	.01	.05	.06	-.03	.01	-.01	-.02	.03
1956	-99999.00	-.02	-.01	.00	.01	.04	-.01	.00	-.02	-.03	.00	-.02
1957	-99999.00	-.03	.02	.00	.01	.03	.00	.02	-.01	-.02	-.01	-.04
1958	-99999.00	-.06	-.04	-.04	.01	.03	.03	.05	-.07	.00	-.02	-.04
1959	-99999.00	.02	.04	.02	.04	-.03	.02	.02	-.01	.00	.02	.02
1960	-99999.00	-.02	-.09	.09	.02	-.01	.02	-.03	-.01	.03	-.03	-.02

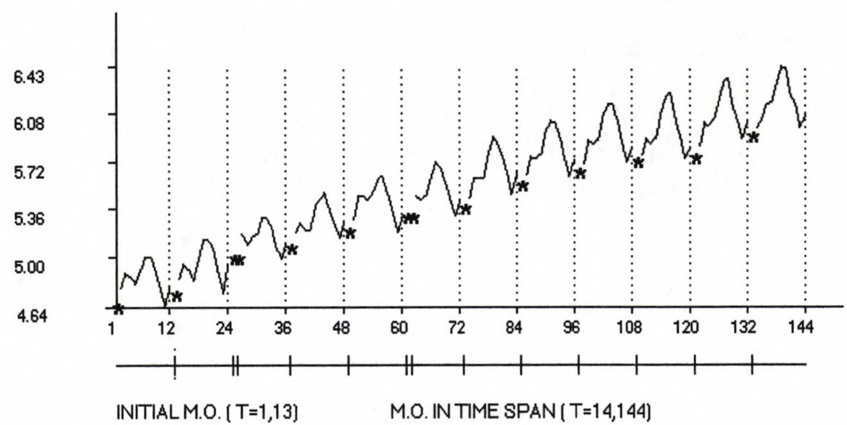
INTERPOLATED VALUES

OBS	INTERPOLATED VALUE (TRANSFORMED SERIES)	STD ERROR	INTERPOLATED VALUE (ORIGINAL SERIES)
25	.2410	.0439	1.2726
26	5.0200	.0289	151.4071
37	.4065	.0446	1.5016
49	.5536	.0466	1.7395
61	.5926	.0493	1.8087
62	5.3268	.0284	205.7785
73	.7187	.0503	2.0517
85	.8947	.0520	2.4466
97	.9969	.0537	2.7099
109	1.0688	.0553	2.9119
121	1.1199	.0569	3.0646
133	1.2586	.0585	3.5206

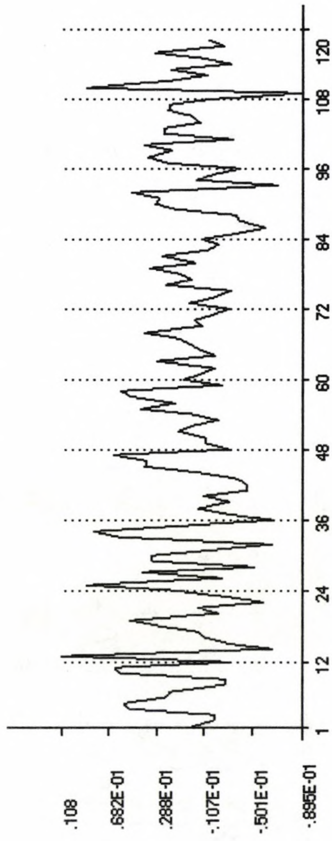
DS51: ORIGINAL SERIES



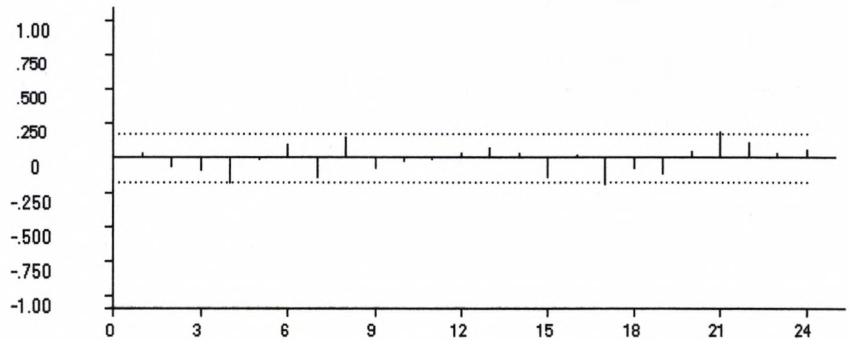
DS51: TRANSFORMED SERIES



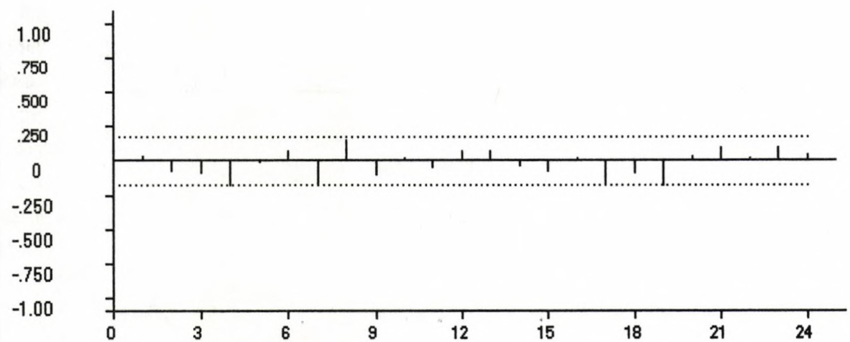
DS51: RESIDUALS



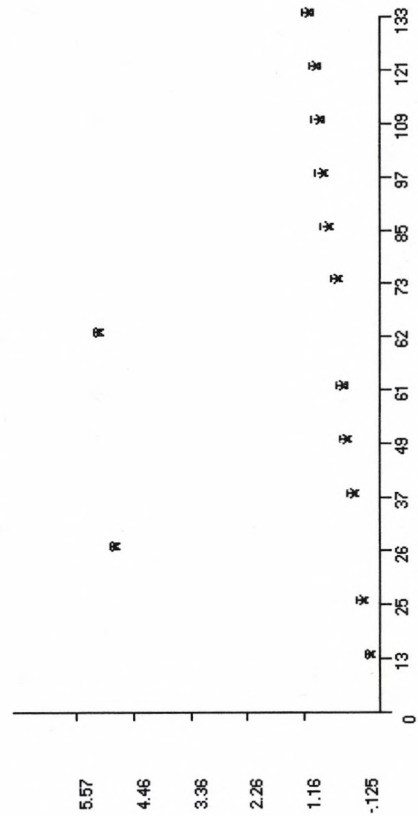
DS51: ACF OF RESIDUALS



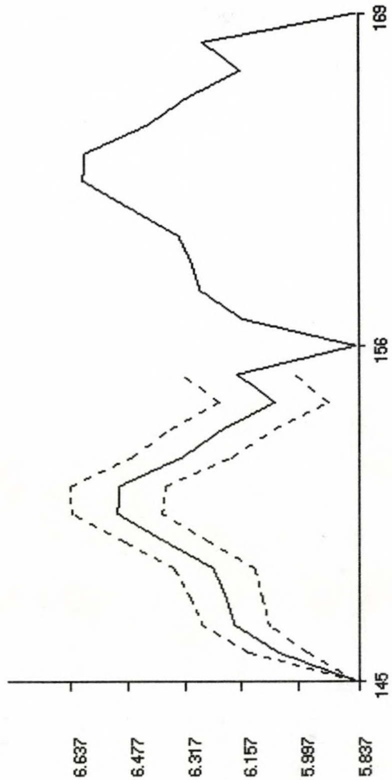
DS51: PARTIAL ACF OF RESIDUALS



DS51: INTERPOLATED VALUES

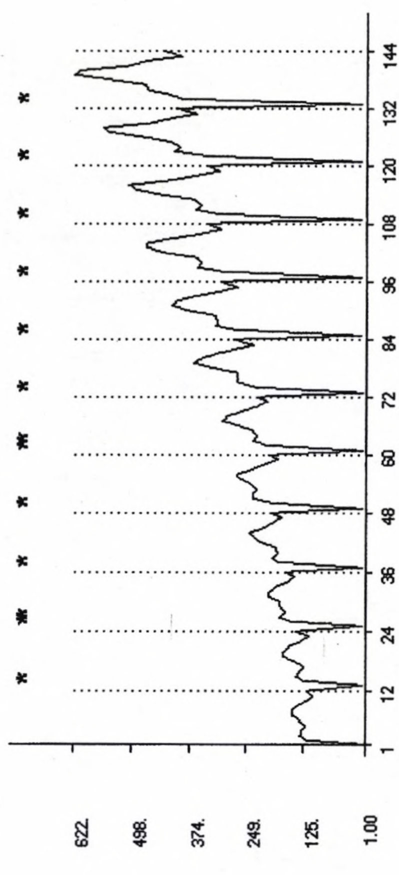


DS51: FORECASTS

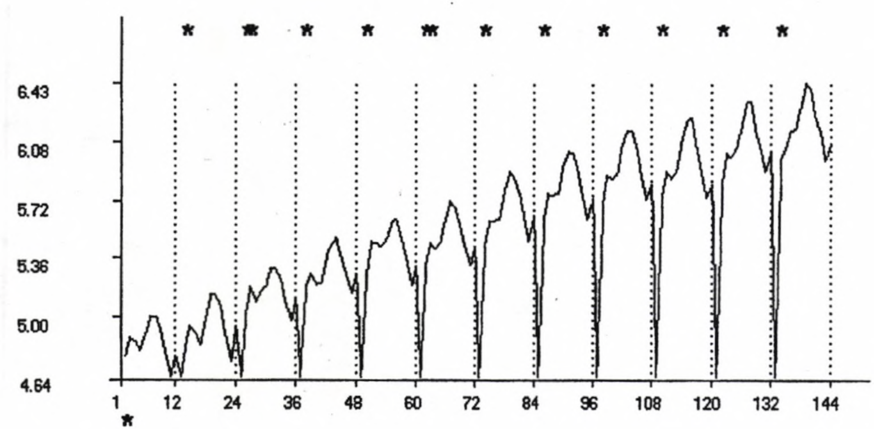


---- : 95 % C. I.

DS51: ORIGINAL SERIES WITH INTERPOLATIONS



DS51: TRANS. SERIES WITH INTERPOLATIONS



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